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# IGEC toolbox for coincidence search

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# Abstract

The standard IGEC approach to detection of gravitational waves with many detectors is a simple time coincidence search. We discuss the problems of false alarm and false dismissal assessment, in the case of both stationary and non-stationary noise. The significance of any cumulative excess of found coincidences over the background is determined by maximum likelihood methods.

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(Some figures in this article are in colour only in the electronic version)

# 1. Introduction

Gravitational wave (GW) resonant detectors operating so far are narrow band detectors and their typical bandwidths are no more than a few Hz with a central frequency of about 900 Hz [1]. Therefore, they are insensitive to any structure present in the waveform of the impinging GW, they just sample the amplitude of the Fourier component of the signal at a few discrete frequencies. The impulse response template is routinely used as a first approximation to describe all burst classes whose duration is of the order of 1 ms.

The search for bursts is performed by threshold crossing (the signal overcomes a certain amplitude threshold) [2–4] or maximum hold (the signal is locally at maximum amplitude) [5] methods. This search usually allows very low ( $\sim$ 3–5) signal-to-noise ratios (SNRs). Estimating false alarms by relying on the nominal probability of local fluctuation of a Gaussian stochastic process is often not viable, because outliers (for instance, due to environmental transient noise) are unavoidable and they mimic the effect of GW signals. The puzzle can be solved when two or more detectors work at comparable sensitivity. Given that the delay

between successive triggers is much longer than the time uncertainty on the arrival time of the burst, the chances of an accidental coincidence are proportionally low; the more detectors there are the better.

A fundamental step before performing a coincidence search is to make uniform the sensitivity of the detectors by restricting their periods of operation such that they overlap and they all work with minimum detectable amplitudes below a *common search threshold*. We assume that this step has already been done (further details can be found in [6]). Also, we will not discuss here the synthesis of the information on the GW burst provided by different detectors once a coincidence is found.

In this paper we report on coincidence search analysis as carried on within the International Gravitational Event Collaboration (IGEC) [1–6]. First we give an operative definition of coincidence and explain how the choice of a time window is linked to estimates of timing uncertainty through the requirement of a minimum detection efficiency. We will describe how the accidental coincidences are estimated in section 3. The maximum likelihood criterion will eventually guide us to the assessment of detection confidence (section 4). The procedure we propose avoids unphysical (i.e. negative) values for the GW rate and estimates the correct non-Bayesian confidence intervals for any number of found and expected coincidences, overcoming the criticisms of previous analyses of upper limits for GW rate (recently raised, for instance, by [7]).

# 2. Triggers, coincidences and time windows

The basic ingredients of the IGEC GW search recipe are the self-consistent '*event files*', which are temporally ordered records of every *event* (or *trigger*) selected by each group of the Collaboration as candidate GW signals [1]. It is understood that, before the event search, the data are processed by a Wiener filter matched to the impulse response of the system, what we call a  $\delta$ -*like* GW. The parameters that fully describe such a signal are just the estimated time of arrival (ETA) and amplitude together with their error statistics, and—when available—a test statistic to assess compliance of the signal with the template. Bundled in the same exchanged files, there is asynchronous information about amplitude precision and accuracy and the values of the amplitude selection threshold.

The sequence of ETA in an event list can be described by a (possibly non-homogeneous) Poisson point series, which requires the triggers to be *rare* and *independent*. All search algorithms associate with every found trigger a dead time of the order of the reciprocal of the Wiener filter bandwidth, yet, as it is much shorter than the average delay between events, it does not significantly affect ETA statistics.

A coincidence is defined as a *M*tuple of triggers, one for each detector, with ETA such that there is a common overlap between their *time windows*. More precisely, if  $t^{(m)}$  is the set of ETAs from the detector labelled *m*, the following relation defines the set of all coincidences,

$$\left\{ c_{n} = (t_{1}, t_{2}, \dots, t_{M}) | t_{m} \in \mathbf{t}^{(m)} \& \forall h, k: |t_{h} - t_{k} + \Delta t_{hk}| < (\Delta t_{h} + \Delta t_{k}) \right\}_{n=1,2...}$$
(1)

where  $\Delta t_m$  is half the length of the time window associated with the event occurring at time  $t_m$  in the *m*th detector and  $\Delta t_{hk}$  corrects for time delay in signal propagation at the speed of light.  $\Delta t_m$  is a function of the probability density of the timing error once a minimum confidence level (or a maximum false dismissal probability) has been fixed. This function is non-analytical and depends on the implementation of trigger selection and on the SNR of the signal, and has to be determined by Monte Carlo methods. Figure 1 shows a typical scaling of the time error standard deviation  $\sigma$  with respect to SNR. The exact relation between  $\sigma_m$  and  $\Delta t_m$  depends on the robustness of the model for the statistics of the noise. The Gaussian



**Figure 1.** Standard deviation of the time error versus SNR for triggers exchanged within IGEC by the AURIGA Collaboration (courtesy of the AURIGA Group). The continuous line is the asymptotic scaling of the standard deviation as 1/SNR expected at high SNR, the dashed horizontal line represents the lower bound due to present systematic calibration errors. Note that 88% of the events have SNR between 5 and 6. Spreading at low SNR is due to variations of the Wiener filter bandwidth.

model would give  $\Delta t_m = 1.96 \sigma_m$  (for a confidence level c = 95%) but in general this model does not apply. Using the very general Bienaymè–Tchebicheff inequality,  $c \leq 95\%$  when  $\Delta t_m = \sigma_m/\sqrt{1-c} \approx 4.5 \sigma_m$ . This relation can be refined by including moments of higher order [8].

If the source direction is not available, its determination and consistency tests on the ETA sequence are deferred to a later analysis, and the relation  $|t_h - t_k + \Delta t_{hk}| < (\Delta t_h + \Delta t_k)$  in (1) is substituted by  $|t_h - t_k| < (\Delta t_h + \Delta t_k + \Delta t_{hk}^{MAX})$ , where  $\Delta t_{hk}^{MAX}$  is the light travel time between the labelled pair of detectors.

#### 3. False alarm estimates

A simple analytical formula for the expected background rate of accidental coincidences with the definition (1) is viable only by fixing for each detector the value of the standard deviation of the timing noise.

If the background triggers can be modelled as a homogeneous Poisson point process, and  $\lambda^{(k)}$  and  $\Delta t_k$  being, respectively, the background rate and time window of the *k*th (over *M*) detector, the expected false alarm rate  $\lambda_b$  is given by<sup>5</sup>

$$\lambda_b = C_M(\Delta t_1, \dots, \Delta t_M) \times \prod_{k=1}^M \lambda^{(k)}$$
<sup>(2)</sup>

<sup>5</sup> To figure out how this formula can be derived, the reader should refer to figure 2, where the cases M = 2 and M = 3 are depicted. The measure of the fiducial volume is the product of its transverse section and length. The first factor is given by the measure of the (M - 1)-dimensional surface obtained by projecting the *M*-dimensional error box in the plane transverse to the bisector. This means multiplying  $1/\sqrt{M}$  by the measure  $\prod_{h \neq k} 2\Delta t_h$  of each hyperface and summing over all directions  $k = 1, \ldots, M$ .

The second factor is equal to the common observation time of the *M* detectors divided by the same quantity  $1/\sqrt{M}$ , which then cancels out in the product. In order to get the number of accidental coincidences, the measure of the fiducial volume has to be multiplied by the density of the lattice nodes representing each possible coincidence, which is given by  $\prod_{k=1}^{M} \lambda^{(k)}$ . Dividing by the common observation time at last gives (2).



**Figure 2.** (Left) The set of all possible pairings between two triggers in a two-detector configuration can be represented with a lattice in a two-dimensional plane, obtained by the tensor product of the original single detector event lists. In the simplified case when every trigger of one detector has the same time window, coincidence search can be seen as a particular selection of events in a stripe (*fiducial volume*) along the bisector, which corresponds to synchronous detector timescales. A time-shifted search means counting trigger pairs that fall inside a new translated stripe. (Right) In a similar way, in a three-detector configuration, the geometrical representation of the fiducial volume for coincidence search is a tube obtained when developing the three-dimensional error box along the bisector.

where

$$C_M(\Delta t_1, \dots, \Delta t_M) \equiv \sum_{k=1}^M \prod_{h \neq k} 2\Delta t_h.$$
(3)

Equation (2) also holds as an *instantaneous* background rate predictor when  $\lambda_b$  is a function of time. The average rate  $\bar{\lambda}_b$  in the total observation time  $T_{obs}$  is given by

$$\lambda_b(t) = C_M(t) \times \prod_{k=1}^M \lambda^{(k)}(t) \quad \Leftrightarrow \quad \bar{\lambda}_b = \frac{1}{T_{\text{obs}}} \int dt \, C_M(t) \prod_{k=1}^M \lambda^{(k)}(t). \tag{4}$$

In practical use of this result, the observation time is divided into smaller time intervals where the background process can be considered stationary, but long enough to estimate the statistics of the detector noise parameters. We remark that if the statistics of false alarms in each time interval are Poisson, then the total number of false alarms is again a Poisson random variable (RV), because it results from linear operations (like integrals). An issue still undecided with (4) is the implementation of a robust unbiased estimator of  $\lambda^{(k)}(t)$  capable of dealing with low event rates.

A different well-known numerical method to estimate the false alarms probability is a time-shifted search strategy. It consists in generating independent detector configurations by adding small delays dt to the ETA of all but one detector trigger lists. These configurations are supposed to be independent when the delay is applied in steps longer than the maximum total time coincidence window: ergodicity of the system is then required. In fact, referring to figure 2, the counts of coincidences inside each translated stripe are, in principle, different RV, but we use them as independent instantiations of the counts inside the synchronous stripe. This is correct only if detector performances at shifted times are *on average* the same as at zero delay. More precisely, the longest time shift should be shorter than the timescale of the

structures of the cross-correlation between  $\lambda^{(h)}(t)$  and  $\lambda^{(k)}(t + dt)$  as a function of dt. The variations of the measured observation time  $T_{obs}$  itself are less critical, as they are easily taken into account by proper normalization of the results.

The time shift analysis is computationally heavier than simply using (4), but allows estimatation of the false alarm rate even when the value of the coincidence window varies from trigger to trigger, dealing with variations of SNR and the Wiener filter bandwidth (see figure 1). Moreover, it naturally takes care of the non-homogeneous character of the trigger rate. Finally, we remark that, by repeated independent time shifts, the Poisson statistics of the coincidence counts can be directly tested, instead of being blindly assumed. For these reasons, the number of accidental coincidences is presently estimated within IGEC by time shift analysis, though the analytical approach is perhaps worth a further thought.

Note that either method (analytical or numerical) is biased by the assumption that the event list contains no real signal, yet since the latter is supposed to be rarer, this effect is negligible.

#### 4. Computation of average GW burst rate

Finally, when the number of predicted and found coincidences has been determined, a decision has to be made about the presence among the latter of GW signals. In the usual non-Bayesian approach, a straightforward result in terms of confidence intervals is obtained by maximum likelihood methods.

Given the mean number of expected background coincidences  $\tilde{\mathbf{N}}_b \equiv E \{\mathbf{N}_b\}$  over a time  $T_{\text{obs}}$ , let  $\Lambda$  be the unknown mean rate of triggers produced by random (and rare) sources, which is natural to the model with a Poisson point process. Even if it happens to be non-homogeneous, the number of coincidences  $N_c$  counted in a typical experiment is a statistical sample of a Poisson RV  $\mathbf{N}_c$  with mean  $\tilde{\mathbf{N}}_c = \tilde{\mathbf{N}}_b + T_{\text{obs}}\Lambda$ . Let  $N_{\Lambda} \equiv T_{\text{obs}}\Lambda$ . The probability density function of  $\mathbf{N}_c$  computed at the observed value is

$$P_{N_c}(\tilde{\mathbf{N}}_b + N_\Lambda) \equiv \frac{1}{N_c!} (\tilde{\mathbf{N}}_b + N_\Lambda)^{N_c} \mathrm{e}^{-(\tilde{\mathbf{N}}_b + N_\Lambda)}$$
(5)

and the corresponding likelihood function is

$$\ell(N_{\Lambda}; N_c, \bar{\mathbf{N}}_b) \equiv P_{N_c}(N_{\Lambda} + \bar{\mathbf{N}}_b) \tag{6}$$

with the obvious bound  $N_{\Lambda} \ge 0$ . The most likely value of  $N_{\Lambda}$  supported by the observations is obtained when  $\ell$  is maximum, and for any *confidence level* (CL)  $c \in [0, 1]$ , its associated *confidence interval*  $[N_{inf}, N_{sup}]$  is defined by  $\ell(N_{inf}) = \ell(N_{sup})$  and  $c = \left[\int_{0}^{\infty} \ell(N) dN\right]^{-1} \int_{N_{inf}}^{N_{sup}} \ell(N) dN$ . We will say that the number of found coincidences  $N_c$  'agrees at confidence level c with a rate  $\Lambda$  between  $N_{inf}/T_{obs}$  and  $N_{sup}/T_{obs}$ '. In the case  $N_{inf} = 0$  the null hypothesis is not ruled out by the experiment, and we shall say ' $N_{sup}/T_{obs}$  is the upper limit at confidence level c'. The non-Bayesian interpretation of this statement is that the actual outcome is c/(1-c) times more likely to be observed over many repetitions of the experiment if the value of  $N_{\Lambda}$  is inside the interval  $[N_{inf}, N_{sup}]$ , rather than outside.

Figure 3 shows an example of confidence intervals set as a function of the number of found coincidences for the case  $\bar{\mathbf{N}}_b = 7.0$ . Assuming that 1000 time shifts were performed (a typical figure for M = 2 in the current IGEC analysis), the standard deviation of the estimate of  $\bar{\mathbf{N}}_b$  is 0.083, which propagates to the confidence intervals as depicted in figure 3. Up to  $N_c = 12$ , no positive detection claim is allowed at 95% CL, but only upper limits.

This procedure can be retraced with a different density function in the case of known deviations of the data from the Poisson statistics.



**Figure 3.** Example of CL contour plot for a background  $\tilde{N}_b = 7.0$ . For each possible integer value of  $N_c$  in the abscissa, the corresponding confidence interval can be determined by finding along the vertical real axis the intercepts with a pair of contours enclosing the desired CL. In this example, the 50%, 95%, 99% and 99.9% CL contours are plotted, along with the most likely value (*darkest color*). The dashed regions show the amount of fluctuation of the contours when the estimate of  $\tilde{N}_b$  falls within 6.83 and 7.17, corresponding to four times the standard deviation after 1000 independent shifts. Note that when  $N_c < \tilde{N}_b$ , the likelihood function becomes single-tailed and the most likely value is zero.

# 5. Conclusions

Within IGEC, the presently used coincidence search method varies the time window aperture in order to guarantee a specific maximum false dismissal. The corresponding false alarm statistics are then empirically investigated by assuming the ergodic approximation and taking time-shifted configurations of the observatory as independent instantiations of the observations at zero time delay. Finally, by feeding the standard maximum likelihood with the estimated false alarm rates, the significance of the estimated GW average burst rate can be assessed.

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