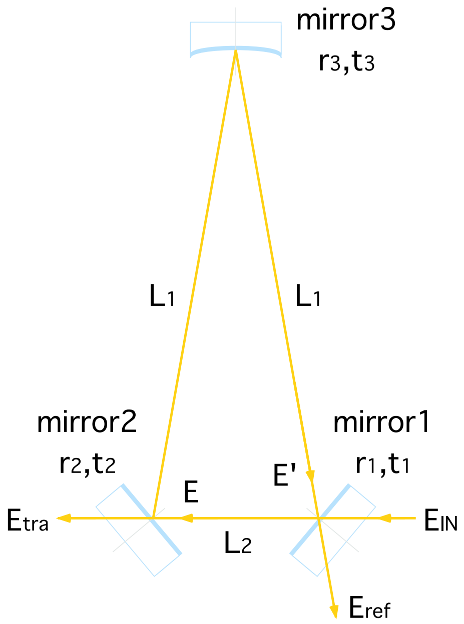
CLIO MC Transmissivity, Reflectivity



L_1: length between mirror 2 and 3 (,3 and 1)
L_2: length between mirror 1 and 2
T_1,T_2: transit time
$T_1 = L_1/c$
$T_2 = L_2/c$

Input field: $E_{\text{IN}}(t) = \mathrm{e}^{i\Omega t}$
Intracavity fields: $E = A\mathrm{e}^{i\Omega t}$, $E' = A'\mathrm{e}^{i\Omega t}$
(Amplitudes of intracavity field A and A')

$$\begin{aligned}
A &= t_1\mathrm{e}^{-i\Omega T_1} + t_1(-r_1r_2r_3)\mathrm{e}^{-i\Omega(2T_1+T_2)} + t_1(-r_1r_2r_3)^2\mathrm{e}^{-i\Omega 2(2T_1+T_2)} + \dots \\
&= t_1\mathrm{e}^{-i\Omega T_2}\sum_{n=0}^{\infty}\left(-r_1r_2r_3\mathrm{e}^{-i\Omega(2T_1+T_2)}\right)^n \\
&= \frac{t_1t_2\mathrm{e}^{-i\Omega T_2}}{1+r_1r_2r_3\mathrm{e}^{-i\Omega(2T_1+T_2)}}
\end{aligned}$$

$$\begin{aligned}
A' &= t_1(r_2r_3)\mathrm{e}^{-i\Omega(2T_1+T_2)} + t_1(r_2r_3)(-r_1r_2r_3)t_1\mathrm{e}^{-i\Omega 2(2T_1+T_2)} + t_1(r_2r_3)(-r_1r_2r_3)^2t_1\mathrm{e}^{-i\Omega 3(2T_1+T_2)} + \dots \\
&= t_1(r_2r_3)\mathrm{e}^{-i\Omega(2T_1+T_2)}\left(1 + (-r_1r_2r_3)\mathrm{e}^{-i\Omega(2T_1+T_2)} + (-r_1r_2r_3)^2\mathrm{e}^{-i\Omega 2(2T_1+T_2)} + \dots\right) \\
&= t_1(r_2r_3)\mathrm{e}^{-i\Omega(2T_1+T_2)}\sum_{n=0}^{\infty}\left(-r_1r_2r_3\mathrm{e}^{-i\Omega(2T_1+T_2)}\right)^n \\
&= \frac{t_1(r_2r_3)\mathrm{e}^{-i\Omega(2T_1+T_2)}}{1+r_1r_2r_3\mathrm{e}^{-i\Omega(2T_1+T_2)}}
\end{aligned}$$

Resonance Condition: $\mathrm{e}^{-i\Omega(2T_1+T_2)} = -1 \Rightarrow \Omega(2T_1+T_2) = (2n+1)\pi$, $(n = 0, 1, 2, \dots)$
Then

$$E(t) = \frac{t_1\mathrm{e}^{-i\Omega T_2}}{1-r_1r_2r_3}\mathrm{e}^{i\Omega t} \quad , \qquad E'(t) = -\frac{t_1^2r_2r_3}{1-r_1r_2r_3}\mathrm{e}^{i\Omega t}$$

$$E_{\text{tra}}(t) = \frac{t_1t_2\mathrm{e}^{-i\Omega T_2}}{1-r_1r_2r_3}\mathrm{e}^{i\Omega t} \quad , \qquad E_{\text{ref}}(t) = \left(r_1 - \frac{t_1^2r_2r_3}{1-r_1r_2r_3}\right)\mathrm{e}^{i\Omega t}$$

When $R_1 = R_2 = 0.9986$, $R_3 = 0.9999$, $t_{\text{CAV}} = 0.965563$, $r_{\text{CAV}} = 0.0344601$.

	hand	e2e		$\times T_{\text{AR}}$	
T_{CAV}	0.932312	0.93231195236661	T'_{CAV}	0.9323×0.925^2	$= 0.798$
R_{CAV}	0.00118757	0.00118756794501	R'_{CAV}	0.0012×0.925	$= 1.11 \times 10^{-3}$

Considering AR coating, MC Transmissivity is about 80% at Loss less.

Phase Modulation

$$E_{\text{out}} = E_{\text{in}} \mathrm{e}^{i\Gamma_\varphi \sin \Omega t} = E_{\text{in}} \sum_{l=-\infty}^{\infty} J_l(\Gamma_\varphi) \mathrm{e}^{i(\Omega t) l} \simeq E_{\text{in}} \sum_{l=-n}^{n} J_l(\Gamma_\varphi) \mathrm{e}^{i(\Omega t) l}$$

When $n = 1$,

$$\begin{aligned} E_{\text{out}} &= E_{\text{in}} \times \sum_{l=-1}^{1} J_l(\Gamma_\varphi) \mathrm{e}^{i(\Omega t) l} = E_{\text{in}} \times \left(J_{-1}(\Gamma_\varphi) \mathrm{e}^{-i(\Omega t)} + J_0(\Gamma_\varphi) + J_1(\Gamma_\varphi) \mathrm{e}^{i(\Omega t)} \right) \\ &= E_{\text{in}} \times \left(J_0(\Gamma_\varphi) + J_1(\Gamma_\varphi)(\mathrm{e}^{i(\Omega t)} - \mathrm{e}^{-i(\Omega t)}) \right) = E_{\text{in}} \times \left(J_0(\Gamma_\varphi) + 2iJ_1(\Gamma_\varphi) \sin(\Omega t) \right) \end{aligned}$$

$$P_{\text{out}} = P_{\text{in}} \times \left(J_0(\Gamma_\varphi)^2 + 4J_1(\Gamma_\varphi) \sin^2(\Omega t) \right) \Rightarrow P_{\text{in}} \times \left(J_0(\Gamma_\varphi)^2 + 2J_1(\Gamma_\varphi) \right)$$

Amplitude Modulation

$$E_{\text{out}} = E_{\text{in}} \mathrm{e}^{\Gamma_{\text{amp}} \sin \Omega t} = E_{\text{in}} \sum_{l=-\infty}^{\infty} (-i)^l I_l(\Gamma_{\text{amp}}) \mathrm{e}^{i(\Omega t) l} \simeq E_{\text{in}} \sum_{l=-n}^{n} (-i)^l I_l(\Gamma_{\text{amp}}) \mathrm{e}^{i(\Omega t) l}$$

When $n = 1$,

$$\begin{aligned} E_{\text{out}} &= E_{\text{in}} \times \sum_{l=-1}^{1} (-i)^l I_l(\Gamma_{\text{amp}}) \mathrm{e}^{i(\Omega t) l} = E_{\text{in}} \times \left(iI_{-1}(\Gamma_{\text{amp}}) \mathrm{e}^{-i(\Omega t)} + I_0(\Gamma_{\text{amp}}) - iI_1(\Gamma_{\text{amp}}) \mathrm{e}^{i(\Omega t)} \right) \\ &= E_{\text{in}} \times \left(I_0(\Gamma_{\text{amp}}) - iI_1(\Gamma_{\text{amp}})(\mathrm{e}^{i(\Omega t)} + \mathrm{e}^{-i(\Omega t)}) \right) = E_{\text{in}} \times \left(I_0(\Gamma_{\text{amp}}) - 2iI_1(\Gamma_{\text{amp}}) \cos(\Omega t) \right) \end{aligned}$$

$$P_{\text{out}} = P_{\text{in}} \times \left(I_0(\Gamma_{\text{amp}})^2 + 4I_1(\Gamma_{\text{amp}}) \cos^2(\Omega t) \right) \Rightarrow P_{\text{in}} \times \left(I_0(\Gamma_{\text{amp}})^2 + 2I_1(\Gamma_{\text{amp}}) \right)$$

Detected Power Table by e2e (W) (Initial Leser Power 1W)

	No Sideband	$f = 15.75$MHz	$f = 12$MHz	$f = 15.75 + 12$MHz
input	1	0.999997	0.999997	
trans	0.932312	0.932263	0.927659	0.927610
trans (15.75MHz)	0	0.002302	0	0.002290
trans (12.00MHz)	0	0	0.264×10^{-8}	0.263×10^{-8}
ref	0.001188	0.001237	0.006169	0.006219
ref (15.75MHz)	0	0.000028	0	0.000028
ref (12.00MHz)	0	0	0.002494	0.002481

input: input to MC after sideband-gen
trans: trans power through OUT mirror
ref : ref power from IN mirror

	ini (power)	tr(%)	tra(power)	Ref(%)	ref(power)
$\Gamma_\varphi = 0$,	$\Gamma_{\rm amp} = 0$				
car15.75	1	93.23119523666	0.9323119523666	0.11875679450	0.001187567945
12	1	93.23119523666	0.9323119523666	0.11875679450	0.001187567945
side15.75	0	0	0	0	0
12	0	0	0	0	0
$\Gamma_\varphi = 0.1$					
car15.75	0.9950093663242	93.23119523667	0.9276591249408	0.11875679450	0.001181641228
12	0.9950093663242	93.23119523667	0.9276591249408	0.11875679451	0.001181641228
side15.75	0.0024937565066	92.29888881011	0.0023017095452	1.11756330684	0.000027869308
12	0.0024937565066	0.00010605606	$2.64478e^{-9}$	99.99988637904	0.002493753673
$\Gamma_\varphi = 0.2$					
car15.75	0.9801494456580	93.23119523667	0.9138050432925	0.11875679450	0.001163994062
12	0.9801494456580	93.23119523667	0.9138050432925	0.11875679450	0.001163994063
side15.75	0.0099004156959	92.29888880993	0.0091379736749	1.11756330682	0.000110643413
12	0.0099004156959	0.00010605605	$1.049999e^{-8}$	99.99988637922	0.009900404447
$\Gamma_\varphi = 0.4$					
car15.75	0.9223647537708	93.23119523666	0.8599316843822	0.11875679450	0.001095370815
12	0.9223647537708	93.23119523666	0.8599316843822	0.11875679450	0.001095370815
side15.75	0.0384264192649	92.29888880988	0.0354671579909	1.11756330685	0.000429439562
12	0.0384264192649	0.00010605602	$4.075353e^{-8}$	99.99988637916	0.038426375605
$\Gamma_{\rm amp} = 0.1$					
car15.75	1.0050093807291	93.23119523665	0.9369822578942	0.11875679450	0.001193516925
12	1.0050093807291	93.23119523665	0.9369822578942	0.11875679450	0.001193516925
side15.75	0.0025062565120	92.29888880992	0.0023132469113	1.11756330669	0.000028009003
12	0.0025062565120	0.00010605618	$2.65804e^{-9}$	99.99988637915	0.002506253664
$\Gamma_{\rm amp} = 0.2$					
car15.75	1.0201505453097	93.23119523666	0.9510985466055	0.11875679450	0.001211498087
12	1.0201505453097	91.40351155866	0.9324534215980	2.07680653826	0.021186553225
side15.75	0.0101004176064	92.29888880990	0.0093225732158	1.11756330688	0.000112878561
12	0.0101004176064	0.00010605601	$1.07121e^{-8}$	99.9998863792	0.010100406130
$\Gamma_{\rm amp} = 0.4$					
car15.75	1.0824358281566	93.23119523666	1.0091678602603	0.11875679450	0.001285466092
12	1.0824358281566	86.13218090387	0.9323255856761	7.72413367820	0.083608790348
side15.75	0.0416269165700	92.29888880989	0.0384211814399	1.11756330685	0.000465207145
12	0.0416269165700	0.00010605602	$4.414785e^{-8}$	99.99988637917	0.041626869273
$\Gamma_\varphi = 0.1$,	$\Gamma_{\rm amp} = 0.1$				
car15.75	1.0000187470431	93.23119523665	0.9323294304588	0.11875679450	0.001187590208
12	1.0000187470431	93.23119523665	0.9323294304588	0.11875679450	0.001187590208
side15.75	0.0049999973863	92.29888880991	0.0046149420281	1.11756330680	0.000055878136
12	0.0049999973863	0.00010605606	$5.3028e^{-9}$	99.9998863791	0.004999991705