# A Dynamical Solution to the Axion Domain Wall Problem

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(Dated: September 5, 2019)

# Abstract

The domain wall problem and the isocurvature problem restrict possible combinations of axion models and inflation models. In this paper, we considered a new mechanism which solves those problems by dynamics of multiple scalar fields during/after inflation. The mechanism makes axion models with a non-trivial domain wall number compatible with inflation models with a large Hubble parameter,  $H_I \gg 10^{7-8}$  GeV. The mechanism also avoids the isocurvature problem. This mechanism increases the freedom of choice of combinations of axion models and inflation models.

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### I. INTRODUCTION

The Peccei-Quinn (PQ) mechanism is the most plausible solution to the Strong CP problem [1, 2]. In this mechanism, the effective  $\theta$ -angle of QCD is canceled by the vacuum expectation value (VEV) of the pseudo-Nambu-Goldstone boson, axion a, which is associated with the spontaneous breaking of the global U(1) symmetry (PQ symmetry) [3, 4]. The mechanism is particularly attractive as the invisible axion [5–8] is a good candidate for cold dark matter [9–11].

The domain wall problem and the isocurvature problem, however, restrict possible combinations of axion models and inflation models. For example, when spontaneous symmetry breaking of the PQ symmetry takes place after the end of inflation, it triggers the formation of the cosmic (global) strings [12]. Around the cosmic string, the axion goes round its domain in  $a/f_a = [0, 2\pi N_{\rm DM})$ . Here,  $f_a$  is the axion decay constant, and the integer  $N_{\rm DM} \ge 1$ is the so-called domain wall number (see, e.g. [13, 14]). As the universe cools down below the QCD scale, the axion obtains a periodic scalar potential due to non-perturbative QCD effects, which leads to the formation of the axion domain wall around the cosmic string. The formed string-wall network is stable unless  $N_{\rm DM} = 1$ , which dominates over the energy density of the universe soon after its formation.<sup>1</sup>

Until today, there are a few solutions to the domain wall problem. For example, the trivial domain wall number,  $N_{\rm DW} = 1$ , is possible in the KSVZ model [5, 6]. In this case, only one domain wall attaches to each of the cosmic string, and hence, the string-wall network collapses immediately after the QCD phase transition [16]. As a notable feature of this scenario, the abundance of the axion dark matter is dominated by the contribution from the decay of the string-wall network, and the observed dark matter density is explained for  $f_a \sim 10^{10} \,\text{GeV}$  [17]. The required decay constant is much smaller than  $f_a \sim 10^{12} \,\text{GeV}$  which is appropriate for the so-called misalignment mechanism for the axion dark matter. These scenarios can be distinguished by the axion search experiments (see, e.g. [18]).

Another possibility to evade the domain wall problem is to assume the PQ symmetry breaking before inflation. In this case, the axion takes a single field value in our universe with a tiny quantum fluctuation, and hence, no domain wall is formed below the QCD

<sup>&</sup>lt;sup>1</sup> If the PQ symmetry is explicitly broken, the string-wall network is not exactly stable even for  $N_{\rm DW} > 1$ . However, the explicit breaking which is required to make the string-wall network collapses fast enough encounters the strong CP problem again [15].

scale. The quantum fluctuation of the axion, on the other hand, induces the isocurvature perturbations of cold dark matter. To avoid too large isocurvature perturbations, the Hubble parameter during inflation,  $H_I$ , must be smaller than about  $10^{7-8}$  GeV [19] (see also [13, 20]). This constraint significantly restricts the variety of inflation models.

In this paper, we discuss a new mechanism which solves those problems by dynamics of multiple scalar fields during/after inflation. The mechanism makes the axion models with a non-trivial domain wall number compatible with the inflation models with a large Hubble parameter,  $H_I \gg 10^{7-8}$  GeV. The mechanism also avoids the isocurvature problem. This mechanism increases the freedom of choice of combination of axion models and inflation models. In particular, this mechanism makes the axion model with  $f_a \sim 10^{12}$  GeV compatible with the models of inflation with  $H_I \sim 10^{13}$  GeV. Such a large axion decay constant and the large Hubble parameter during inflation can be tested by the axion search experiments (see, e.g. [18]) and the searches for the primordial *B*-mode polarization in the cosmic microwave background (CMB) (see, e.g. [21]), respectively.

In the new mechanism, we may consider any type of the axion model. We call the PQ charged field which spontaneously breaks the PQ symmetry, the PQ field. Then, we introduce an additional PQ charged scalar field which obtains a vanishing VEV. We call this additional field the spectator PQ field. We assume that the spectator PQ field obtains a large field value during/after inflation. The large field value of the spectator PQ field provides a non-trivial scalar potential of the axion when the PQ field obtains its VEV after inflation. The non-trivial axion potential prohibits the formation of the cosmic string, and hence, prohibits the string-wall network below the QCD scale. The isocurvature perturbation which stems from the quantum fluctuation of the spectator PQ field is suppressed by its large field value during inflation [22]. In this way, the new mechanism solves the domain wall problem without causing the isocurvature problem. This mechanism may be regarded as a multi-field version of the mechanism discussed in [23].<sup>2</sup>

The organization of the paper is as follows. In section II, we summarize the setup of our model. In section III, we discuss how the spectator PQ field evolves. In section IV, we discuss how the axion behaves in the presence of the spectator PQ field. We also discuss

 $<sup>^{2}</sup>$  See also [24–27] for other realization of dynamics which solves the domain wall and the isocurvature problems. The solutions in the context of the axion predicted in the string theory [28–30] have also been discussed, where the axion dark matter abundance is suppressed dynamically [31, 32].

viable parameter region of the new mechanism. In section V, we discuss supersymmetric extension of the model. The final section is devoted to our conclusions.

# II. PECCEI-QUINN MECHANISM WITH A SPECTATOR PQ FIELD

### A. General Recipe of the Dynamical Solution

Before discussing the details of the mechanism, we summarize the general recipe for the dynamical solution of the domain wall and the isocurvature problems.

- 1. Bring an axion model where the PQ symmetry is spontaneously broken by the VEV of the PQ field, *P*.
- 2. Add a spectator PQ field, S, which obtains a vanishing VEV but has a large field value in the early universe until P obtains the VEV (section III).
- 3. Introduce a mixing term between P and S so that P feels a strong PQ symmetry breaking effects when it obtains the VEV (section IV).
- 4. Make the effects of the mixing term inefficient before S starts coherent oscillation around its origin (section IV).

With the large field value of S, P feels a strong PQ symmetry breaking, and no cosmic strings are formed when P obtains the VEV. The fourth condition is important not to randomize the axion field value even after S starts the coherent oscillation (subsection IV C). Without cosmic strings and with the uniform axion field value, the domain walls are not formed after the QCD phase transition. The quantum fluctuation of the phase component of S is imprinted in the axion through the mixing term. The isocurvature problem can be avoided by requiring that the field value of S is of  $\mathcal{O}(M_{\rm Pl})$  during inflation (subsection IV D). In any successful implementation of this mechanism, the domain wall and the isocurvature problems are solved dynamically.

### B. KSVZ Axion Model

As a concrete example of the axion model, we consider the KSVZ axion model [5, 6] in which the PQ field, P, obtains the VEV via the scalar potential,

$$V(P) = \lambda_p \left( |P|^2 - \frac{v_{\rm PQ}^2}{2} \right)^2 \,. \tag{1}$$

Here,  $\lambda_p$  is a coupling constant of  $\mathcal{O}(1)$  and  $v_{PQ}$  is a parameter with mass dimension. The VEV of the PQ field is given by,  $\langle P \rangle = v_{PQ}/\sqrt{2}$ . The axion field, *a*, corresponds to the phase component of *P*,

$$P = \frac{1}{\sqrt{2}} v_{\mathrm{PQ}} e^{ia/v_{\mathrm{PQ}}} , \qquad (2)$$

where we omit the radial component of P for brevity.

The PQ field couples to the  $N_f$  vector-like quarks in the fundamental representation of the SU(3) gauge group of QCD,  $(Q_L, \bar{Q}_R)$  via

$$\mathcal{L} = y_{\text{KSVZ}} P Q_L \bar{Q}_R + h.c. , \qquad (3)$$

with  $y_{\text{KSVZ}}$  being the coupling constant. Below the mass scale of the KSVZ quarks,  $y_{\text{KSVZ}}v_{\text{PQ}}$ , the QCD anomaly induces the axion couplings to QCD,

$$L = \frac{g_s^2}{32\pi^2} \frac{N_f}{v_{PQ}} a \, G\tilde{G} \,\,, \tag{4}$$

Here,  $g_s$  denotes the QCD gauge coupling constant and G and  $\tilde{G}$  are the QCD field strength and its hodge dual, respectively. The Lorentz and color indices are understood. We define the origin of the axion field space at which the effective  $\theta$ -angle of QCD is vanishing.

Below the QCD scale, the above interaction term in Eq. (4) leads to the scalar potential of the axion,

$$V(a) \sim m_a^2 f_a^2 \left[ 1 - \cos \frac{a}{f_a} \right] .$$
(5)

Here,  $f_a = v_{PQ}/N_f$  is the effective decay constant of the axion, and  $m_a$  denotes the mass of the axion which is estimated to be,

$$m_a \simeq 6\,\mu \mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right) \;,$$
 (6)

(see e.g. [33, 34]). It should be noted that the domain of the axion field is given by  $a/f_a = [0, 2\pi N_f)$ , and hence,  $N_f$  corresponds to the domain wall number,  $N_{\text{DW}} = N_f$ .

As we will see below, the axion field value settles to a non-zero value  $a_i$  of  $\mathcal{O}(f_a)$  after a complex dynamics of the new mechanism. Below the QCD scale, the axion starts coherent oscillation from the non-zero field value around its origin which behaves as cold dark matter as in the conventional misalignment mechanism [9–11]. The axion dark matter density is given by [35],

$$\Omega_a h^2 \simeq 0.2 \times \left(\frac{a_i}{f_a}\right)^2 \left(\frac{f_a}{10^{12} \,\mathrm{GeV}}\right)^{1.19} \,. \tag{7}$$

Based on this estimate, we focus on the case with  $f_a = \mathcal{O}(10^{12})$  GeV in the following discussion.

## C. Spectator PQ Field

Now let us introduce another PQ charged scalar field, the spectator PQ field, S. We assume that S has a PQ charge which is -1/m of that of the PQ field ( $m \in \mathbb{N}$ ). With this assumption, S couples to P via,

$$V(P,S) = \lambda_p \left( |P|^2 - \frac{v_{\rm PQ}^2}{2} \right)^2 + m_S^2 |S|^2 + \frac{1}{(n!)^2} \frac{\lambda_s^2}{M_{\rm Pl}^{2n-4}} |S|^{2n} + \frac{\lambda}{m! M_{\rm Pl}^{m-3}} S^m P + \text{h.c.}$$
(8)

Here,  $m_S$  is the mass parameter of S,  $\lambda_s$  and  $\lambda$  are dimensionless coupling constants, and  $M_{\rm Pl} \simeq 2.4 \times 10^{18} \,\text{GeV}$  the reduced Planck scale. Due to the positive mass squared, S does not obtain a non-vanishing VEV. As we will see in the following two sections, n is required to be larger than 5 for a successful mechanism. The absence of the lower dimensional scalar potential terms of S than  $|S|^{2n}$  will be justified in the supersymmetric extension discussed in section V.

In this mechanism, a large field value of S during/after inflation plays a crucial role to solve the domain wall problem and the isocurvature problem. For that purpose, we introduce interactions between the (spectator) PQ fields with the inflaton field  $\phi$ ,

$$V(P, S, \phi) = V(\phi) + V(P, S) + \frac{c_p}{3} \frac{V(\phi)}{M_{\rm Pl}^2} |P|^2 - \frac{c_s}{3} \frac{V(\phi)}{M_{\rm Pl}^2} |S|^2 , \qquad (9)$$

where  $c_p$  and  $c_s$  are positive valued coupling constant.  $V(\phi)$  denotes the inflaton potential with which the Hubble parameter during inflation is given by,

$$H_I^2 = \frac{V(\phi)}{3M_{\rm Pl}^2} \,. \tag{10}$$

Through the interactions with the inflaton, P and S obtain the positive and the negative Hubble-induced mass terms during inflation,

$$\tilde{m}_P^2 = c_p H_I^2 (> 0) , \quad \tilde{m}_S^2 = -c_s H_I^2 (< 0) ,$$
(11)

respectively. Generally, scalar fields obtain Hubble-induced mass terms. We also discuss how the interactions with the inflaton in Eq. (9) can be obtained in the supersymmetric extension.

In the following scenario, we assume that S is never in the thermal equilibrium. Such a situation can be easily realized when the inflaton field mainly decays into the Standard Model particles. Late time interactions of S with thermal bath particles are negligible as it only couples to other fields through the Planck suppressed operators.

### III. DYNAMICS OF THE SPECTATOR PQ FIELD

### A. Inflation Era

During inflation, S obtains a negative Hubble-induced mass term in Eq. (11), which is much larger than  $m_S^2$  in size. Thus, the potential of S can be approximated by

$$V(S) = \frac{\lambda_s^2}{(n!)^2 M_{\rm Pl}^{2n-4}} |S|^{2n} - c_s H_I^2 |S|^2 .$$
(12)

Due to the Hubble-induced mass term, S obtains a large expectation value,

$$\langle S_I \rangle \simeq \left( \sqrt{\frac{c_s}{n}} \frac{n!}{\lambda_s} \right)^{\frac{1}{n-1}} \left( \frac{H_I}{M_{\rm Pl}} \right)^{\frac{1}{n-1}} M_{\rm Pl} \ .$$
 (13)

Hereafter, the expectation value,  $\langle S \rangle$ , denotes its absolute value if not otherwise specified. In the following analysis, we assume

$$\lambda_s \gtrsim n! \sqrt{\frac{c_s}{n}} \left(\frac{H_I}{M_{\rm Pl}}\right)$$
, (14)

so that  $\langle S_I \rangle$  not to exceed  $\mathcal{O}(M_{\rm Pl})$ . For example,  $\langle S_I \rangle \sim M_{\rm Pl}$ , for n = 6,  $H_I = 10^{12} \,{\rm GeV}$ ,  $c_s = 1$  and  $\lambda_s = 10^{-4}$ .

# B. Inflaton Oscillation Era

After inflation, the inflaton starts coherent oscillation around its minimum. As the inflaton oscillation time scale becomes much shorter than the Hubble time, the dynamics of S can be analyzed by taking the time average of the inflaton oscillation. Thus, the inflaton potential in Eq. (9) can be approximated by

$$\overline{V(\phi)} = \frac{3}{2} H^2 M_{\rm Pl}^2 , \qquad (15)$$

where the bar denotes the time average and H is the Hubble parameter at that time.<sup>3</sup>

Now, let us focus on the dynamics of the radial component of S,

$$S = \frac{\chi}{\sqrt{2}} , \quad (\chi \in \mathbb{R}) .$$
 (16)

Due to the PQ symmetry, the potential of S does not induce the torque in the complex plane of S. Thus, the motion of S is confined on a straight line passing through S = 0 (see e.g. Fig. 2). We will discuss how the phase component of S behaves in the next section. In

<sup>&</sup>lt;sup>3</sup> At the beginning of the inflaton oscillation, in the case of chaotic inflation, the time scale of the inflaton oscillation is comparable to the Hubble time. However, this does not change the dynamics of S significantly.

the above approximation, the equation of motion (EOM) of the zero-mode of  $\chi$  is given by,

$$\ddot{\chi} + 3H\dot{\chi} + \frac{n\lambda_s^2}{2^{n-1}(n!)^2 M_{\rm Pl}^{2n-4}} \chi^{2n-1} - \frac{c_s}{2} H^2 \chi = 0 , \qquad (17)$$

where the dot denotes the time derivative. We neglect  $m_s^2$  by assuming that it is still much smaller than  $H^2$  in this period.

Following [23], we introduce the *e*-folding number,  $N \equiv \ln R$ , as a time variable, where R is the scale factor of the universe. We define R such that R = 1 when the inflaton starts to oscillate. With the *e*-folding number, the EOM is rewritten by,

$$\frac{\mathrm{d}^2}{\mathrm{d}N^2}\chi + \frac{3}{2}\frac{\mathrm{d}}{\mathrm{d}N}\chi + \frac{n\lambda_s^2}{2^{n-1}(n!)^2M_{\mathrm{Pl}}^{2n-4}H^2}\chi^{2n-1} - \frac{c_s}{2}\chi = 0.$$
(18)

Next, we set  $\chi$  in the form of

$$\chi = \sigma \sqrt{2} M_{\rm Pl} \left( \frac{c_s(n!)^2 H_i^2}{2n\lambda_s^2 M_{\rm Pl}^2} \right)^{\frac{1}{2(n-1)}} \exp\left[ -\frac{3N}{2(n-1)} \right] \,. \tag{19}$$

Here,  $H_i$  denotes the Hubble parameter at the on-set of the inflaton oscillation. This leads to the EOM of  $\sigma$ 

$$\frac{\mathrm{d}^2}{\mathrm{d}N^2}\sigma + \frac{3(n-3)}{2(n-1)}\frac{\mathrm{d}}{\mathrm{d}N}\sigma + \frac{c_s}{2}\sigma^{2n-1} - \left(\frac{9(n-2)}{4(n-1)^2} + \frac{c_s}{2}\right)\sigma = 0.$$
(20)

The EOM of  $\sigma$  represents a motion of a particle in a potential,

$$V(\sigma) = -\frac{1}{2} \left( \frac{9(n-2)}{4(n-1)^2} + \frac{c_s}{2} \right) \sigma^2 + \frac{c_s}{4n} \sigma^{2n} , \qquad (21)$$

which has a minimum at,

$$\sigma_0 = \left(1 + \frac{9(n-2)}{2(n-1)^2 c_s}\right)^{\frac{1}{2(n-1)}}.$$
(22)

The second term of the EOM is a velocity dependent force. The initial position of  $\sigma$  is given

by Eq. (13),

$$\sigma_i \sim 2^{\frac{1}{2(n-1)}} , \qquad (23)$$

which is close to  $\sigma_0$  for  $c_s = \mathcal{O}(1)$ .

The velocity dependent force in Eq. (20) plays the role of friction for  $n \ge 4$  (see also [23]). Thus, for  $n \ge 4$ ,  $\sigma$  which starts oscillation from  $\sigma_0$  immediately settles down to  $\sigma_0$  and stays there during the inflaton oscillation era. In this case, the field value of S is roughly given by

$$\langle S \rangle \simeq \left( \sqrt{\frac{c_s}{n}} \frac{n!}{\lambda_s} \right)^{\frac{1}{n-1}} \left( \frac{H}{M_{\rm Pl}} \right)^{\frac{1}{n-1}} M_{\rm Pl} , \qquad (24)$$

which has the same dependency on the Hubble parameter in Eq. (13). This behavior of the scalar field is called as the scaling solution [36].

For n = 3,  $\sigma$  keeps oscillating around  $\sigma_0$  but does not go over  $\sigma = 0$  [23]. Thus, in this case, S again keeps a large field value during the inflaton oscillation era.<sup>4</sup> As pointed out in [37], however, the scalar field shows a peculiar behaviour for n = 3, the pseudo-scaling solution, in which the field value gradually decreases in a zigzag manner.

For n < 3, the velocity dependent force accelerates the motion of  $\sigma$ , and eventually, the oscillation of S goes over S = 0, which results in  $\langle S \rangle = 0$ . For a successful solution to the domain wall problem, we require that S has a large field value when P obtains the VEV in the radiation dominated (RD) era. Thus, we at least require  $n \ge 3$  so that S keeps a large field value during the inflaton oscillating period. As we will immediately see, however, we eventually require  $n \ge 5$  for S to have a large field value in the RD era (see also [36]).

### C. Radiation Dominated Era

In the RD era, the time dependence of the Hubble parameter changes from that in the inflaton oscillation era. Besides, the PQ fields no longer obtain the Hubble-induced mass terms through the interactions with the inflaton.

In this mechanism, we assume that  $m_s$  is much smaller than H at the beginning of the

<sup>&</sup>lt;sup>4</sup> The parametric resonance due to the oscillation of S is not effective [23].

RD era. Then, the EOM of S can be written as

$$\ddot{\chi} + 3H\dot{\chi} + \frac{n\lambda_s^2}{2^{n-1}(n!)^2 M_{\rm Pl}^{2n-4}} \chi^{2n-1} = 0 .$$
<sup>(25)</sup>

As in the inflaton oscillation era, we introduce the *e*-folding number  $N \equiv \ln R$  which is vanishing at the beginning of the RD era. By using the *e*-folding number, the EOM is rewritten by,

$$\frac{\mathrm{d}^2}{\mathrm{d}N^2}\sigma + \frac{n-5}{n-1}\frac{\mathrm{d}}{\mathrm{d}N}\sigma + \sigma^{2n-1} - \frac{2(n-3)}{(n-1)^2}\sigma = 0.$$
 (26)

Here, we set

$$\chi = \sigma \sqrt{2} M_{\rm Pl} \left( \frac{(n!)^2 H_{r,i}^2}{n \lambda_s^2 M_{\rm Pl}^2} \right)^{\frac{1}{2(n-1)}} \exp\left[ -\frac{2N}{n-1} \right] \,, \tag{27}$$

with  $H_{r,i}$  being the Hubble parameter at the beginning of the RD era.

As in the case of the inflaton oscillation era, the EOM corresponds to a participle motion in a potential

$$V(\sigma) = -\frac{(n-3)}{(n-1)^2}\sigma^2 + \frac{1}{2n}\sigma^{2n} , \qquad (28)$$

which has the minimum at,

$$\sigma_{r,0} = \left[\frac{2(n-3)}{(n-1)^2}\right]^{\frac{1}{2(n-1)}}.$$
(29)

The initial position of  $\sigma$  is roughly given by,

$$\sigma_{r,i} \sim c_s^{\frac{1}{2(n-1)}}$$
 (30)

Similarly to the case of inflaton oscillation era, the friction term has a wrong sign for  $n \leq 4$ . For n > 5, on the other hand, S again behaves as the scaling solution [36],

$$\langle S \rangle \simeq \left[ \frac{2(n-3)(n!)^2}{n(n-1)^2 \lambda_s^2} \right]^{\frac{1}{2(n-1)}} \left( \frac{H}{M_{\rm Pl}} \right)^{\frac{1}{n-1}} M_{\rm Pl} \simeq \left( \frac{n!}{\lambda_s} \right)^{\frac{1}{n-1}} \left( \frac{H}{M_{\rm Pl}} \right)^{\frac{1}{n-1}} M_{\rm Pl} \,. \tag{31}$$

For n = 5, S behaves as the pseudo-solution [37]. In summary, S keeps a large field value during the RD era for  $n \ge 5$ . In the following analysis, we take n = 6 as the minimal model, since the zigzag behavior in the pseudo scaling solution makes the analysis complicated.

While  $H \gtrsim m_S$ , S obeys the scaling solution.<sup>5</sup>

When the Hubble parameter decreases further and becomes smaller than  $m_s$ , S finally starts to oscillate around its origin. For  $n = 6, \lambda_s = 10^{-4}$  and  $m_S \leq 10^{-15} \,\text{GeV}$ , the contribution of the coherent oscillation of S to the DM abundance is negligibly small.<sup>6</sup> For a larger  $m_s$ , we assume that S decays into massless fermions  $\psi_s$  through

$$\mathcal{L}_{S-\text{decay}} = -y_s S \bar{\psi}_s \psi_s + \text{h.c.}$$
(32)

With this assumption, the energy density of S does not cause any cosmological problem. The number density of the massless fermions is also negligibly small and does not contribute to the dark radiation (see the Appendix A).

As we will see below, the behavior of S discussed in this section successfully solves the domain wall problem and the isocurvature problem.

# IV. DYNAMICS OF AXION AND CONSTRAINTS

Now, let us consider the dynamics of the PQ field and the axion. A notable feature of this mechanism is that the large field value of S provides a non-vanishing effective linear term of P through the P-S mixing term (see Eq. (9)).

### A. Before PQ Breaking

During inflation, the minimum of the scalar potential of P is shifted due to the effective linear term, and P also obtains a non-zero field value which is determined by ballancing

 $<sup>^{5}</sup>$  When S obeys the scaling solution, S does not lose its potential energy through particle emission.

<sup>&</sup>lt;sup>6</sup> For  $m_S = \mathcal{O}(10^{-15})$  GeV, the coherent oscillation of S can be the dominant component of DM. Interestingly, this case provides an ultra-light DM model whose initial condition of the coherent oscillation is dynamically determined.

between the quartic term  $\lambda_p |P|^4$  and the effective linear term  $\lambda \langle S \rangle^m P$ , which is given by

$$\langle P_I \rangle \sim M_{\rm Pl} \left(\frac{\lambda}{m!\lambda_p}\right)^{1/3} \left(\frac{\langle S_I \rangle}{M_{\rm Pl}}\right)^{m/3} \\ \simeq M_{\rm Pl} \left[\frac{\lambda}{m!\lambda_p} \left(\frac{n!}{\lambda_s}\right)^{\frac{m}{n-1}}\right]^{1/3} \left(\frac{H_I}{M_{\rm Pl}}\right)^{\frac{m}{3(n-1)}}.$$
(33)

As we will see in the following, the minimal model which successfully solves the domain wall problem is the one with n = 6, m = 11. In this case,  $\langle P_I \rangle$  is rather large during inflation,

$$\langle P_I \rangle \sim 10^{14} \,\mathrm{GeV}$$
, (34)

where the parameters are set to  $\lambda_p = 1$ ,  $\lambda_s = 10^{-4}$ ,  $\lambda = 10^{-6}$  and  $H_I = 10^{12}$  GeV as a benchmark point (see section IV E). Accordingly, the KSVZ quarks obtain heavy masses through the Yukawa coupling in Eq. 3,

$$M_{KSVZ} \simeq y_{KSVZ} \langle P_I \rangle$$
, (35)

during inflation.

In the inflaton oscillation era, on the other hand, S decreases as the scaling solution, and hence, the minimum of the potential of P also decreases. By the time of the completion of reheating process, the minimum position of P becomes much smaller than the temperature of the universe, T.<sup>7</sup> Therefore, the KSVZ quarks and P are thermalized by the completion of the reheating process. Once the KSVZ quarks and P are thermalized, P obtains a thermal potential.<sup>8</sup>

Due to the thermal mass of P of  $\mathcal{O}(T^2)$ , P is settled to

$$\langle P \rangle \sim \frac{\lambda}{m! T^2 M_{\rm Pl}^{m-3}} \langle S \rangle^m$$

$$\simeq \frac{\lambda}{m!} \left(\frac{n!}{\lambda_s}\right)^{\frac{m}{n-1}} \left(\frac{T}{M_{\rm Pl}}\right)^{\frac{2m}{n-1}-2} M_{\rm Pl} .$$
(36)

<sup>&</sup>lt;sup>7</sup> Here, P does not necessarily follow the minimum of the potential, although such a behavior does not affect the following argument.

<sup>&</sup>lt;sup>8</sup> S and P mix through  $S^m P$  term, however the mixing is not large enough for S to be thermalized.

<sup>&</sup>lt;sup>9</sup> We assume  $\lambda_p$  and  $y_{\text{KSVZ}}$  are of  $\mathcal{O}(1)$ .



Figure 1. A schematic picture of the axion evolution when P obtains the VEV. The tilted Mexican hat corresponds the scalar potential of the PQ field on a complex plane of P. In each Hubble patch, the axion rolls down from the hilltop of the potential in a random direction. The axion settles to a unique field value due to the bias term made by the large field value of S.

This expectation value is much smaller than  $v_{PQ} = \mathcal{O}(10^{12})$  GeV. Thus, we can consider that P is stabilized at the origin by the thermal mass term at the beginning of the RD era.

# B. After PQ Breaking

When P obtains the VEV,  $\langle P \rangle = v_{PQ}/\sqrt{2}$ , the axion rolls down from the hilltop of the potential. The direction of the axion is random in each Hubble patch (see Fig. 1), which results in the formation of the cosmic strings. However, the axion settles down to its minimum of the cosine potential induced by the P-S mixing term, which forms the domain walls around the strings. This situation is analogous to the string-wall network formation below the QCD scale in the  $N_{\rm DW} = 1$  scenario. In  $N_{\rm DW} = 1$  scenario, the string-wall network collapses by itself when the energy of a domain wall exceeds that of a string [16].

The condition for the collapse of the string-wall network is given by

$$\frac{\sigma_{\rm w} d_H^2}{T_{\rm s} d_H} > 1 , \qquad (37)$$

where  $\sigma_{\rm w}$  is the surface tension of the domain wall,  $T_{\rm s}$  the tension of the string, and  $d_H \sim 1/H$ the Hubble length. The energy density inside the domain wall is of  $\mathcal{O}(m_a^2 v_{\rm PQ}^2)$ . The typical thickness of the domain wall is given by  $m_a(T)^{-1}$  [38, 39], where

$$m_a(T)^2 \simeq \frac{\sqrt{2\lambda}}{m!} \frac{M_{\rm Pl}^3}{v_{\rm PQ}} \left[ \frac{2(n-3)(n!)^2}{n(n-1)^2 \lambda_s^2} \right]^{\frac{m}{2(n-1)}} \left( \frac{H}{M_{\rm Pl}} \right)^{\frac{m}{n-1}}$$
(38)

is induced by the P-S mixing. The typical radius of the cosmic string is given by  $v_{\rm PQ}^{-1}$ for  $\lambda_p = \mathcal{O}(1)$  [40], with the energy density inside the cosmic string of  $v_{PQ}^4$ . From these quantities, we obtain,

$$\sigma_{\rm w} \simeq 8m_a(T)v_{\rm PQ}^2 ,$$
  
$$T_{\rm s} \simeq 2\pi v_{\rm PQ}^2 \ln(v_{\rm PQ}/H(T)) .$$
(39)

By plugging Eq. (39) into Eq. (37), the condition Eq. (37) is reduced to

$$m_a(T) > \frac{\pi}{4} H(T) \ln\left(\frac{v_{\rm PQ}}{H(T)}\right) \,. \tag{40}$$

The string-wall network immediately shrinks and collapses when this condition is satisfied for some temperature after P obtains the VEV.

It should be noted that the phase component of S also participates in the scalar potential induced by the P-S mixing. Unlike the axion, however, it does not move during the above process. Thus, we can neglect the motion of the phase component of S in the above argument. To see this behavior, let us decompose S field by

$$S = \frac{\chi}{\sqrt{2}} \exp\left(i\frac{b}{\chi}\right) \,, \tag{41}$$

where b is the phase component of S. The field value of the radial component  $\chi$  slowly decreases according to the scaling solution (see Eqs.(24) and (31)). The scalar potential of a and b induced by the P-S mixing term is given by,<sup>10</sup>

$$V(a,b) = 2\frac{\lambda}{m!M_{\rm Pl}^{m-3}} \left(\frac{\chi}{\sqrt{2}}\right)^m \frac{v_{\rm PQ}}{\sqrt{2}} \left[1 - \cos\left(\frac{a}{v_{\rm PQ}} + m\frac{b}{\chi}\right)\right]$$
(42)

This scalar potential implies that the axion oscillates much faster than b since  $\chi \gg v_{\rm PQ}$  at  $T \sim v_{\rm PQ}$ . Thus, a oscillates around  $-mbv_{\rm PQ}/\chi \pmod{2\pi}$ , while b does not feel the force from the potential since it is averaged out by the oscillation of  $a^{11}$ 

In Fig. 2, we show the behaviors of a and b during the RD era at a benchmark point,  $n = 6, m = 11, \lambda_p = c_s = c_p = 1, \lambda_s = 10^{-4}, \lambda = 10^{-6} \text{ and } v_{PQ} = 10^{12} \text{ GeV}$  (see subsection <sup>10</sup> Here, we redefine the origin of b so that the minimum of the potential is at a = b = 0. <sup>11</sup> More precisely, b settles to  $b_i - ma_i v_{\rm PQ}/\chi + \mathcal{O}(b_i m^2 v_{\rm PQ}^2/\chi^2)$ .



Figure 2. The behaviors of a and b during the RD era at a benchmark point, n = 6, m = 11,  $\lambda_p = c_s = c_p = 1$ ,  $\lambda_s = 10^{-4}$ ,  $\lambda = 10^{-6}$  and  $v_{PQ} = 10^{12}$  GeV. The initial conditions of the phase components are  $a/v_{PQ} = \pi/2$  (left) and  $a/v_{PQ} = -3\pi/2$  while  $b/\chi = \pi/5$  in the both panels. In both cases,  $a/v_{PQ}$  settles around  $-m\pi/5 \pmod{2\pi} = -\pi/5$ , while  $b/\chi$  is a constant in time.

IV E). We start analysis just after P obtains the VEV at  $T \sim 10^{12}$  GeV.  $\chi$  is assumed to follow the scaling solution in Eq. (31). As the initial conditions of the phase components, we take  $a/v_{\rm PQ} = \pi/2$  (left) and  $a/v_{\rm PQ} = -2\pi/3$  (right), while  $b/\chi = \pi/5$ , respectively. The initial velocities are taken to be zero. The figure shows that  $a/v_{\rm PQ}$  settles around  $-mv_{\rm PQ}b/\chi$ (mod  $2\pi$ ). The figure also shows that  $b/\chi$  is an almost constant in time as expected. The oscillation period of  $a/v_{\rm PQ}$  becomes longer at the later time, as  $\chi$  decreases according to the scaling solution. In this way, the random field value of a in each Hubble patch settles in the vicinity of the uniform field value,  $-mbv_{\rm PQ}/\chi$  (mod  $2\pi$ ).

### C. The Axion Dynamics After the Onset of S Oscillation

As we have mentioned earlier, S starts to oscillate around its origin when Hubble parameter becomes smaller than  $m_s$ . In this subsection, we consider the dynamics of the axion after the onset of the S oscillation.

Because the axion potential is induced by the mixing term, the sign of the axion potential also flips when S oscillates (see Fig. 3). If the axion mass exceeds the Hubble parameter at that time, the axion falls from the top of the flipped potential. If this happens, the axion field value in each Hubble patch is again randomized and can no longer be uniform in our universe. Such a behaviour brings back the domain wall problem.

To avoid this situation, we require that the Hubble friction on the axion is effective when



Figure 3. A schematic picture of the behavior of the scalar potential of the PQ field. As the spectator PQ field oscillates, the sign of the axion potential flips.

S starts to oscillate. This condition can be expressed in the form

$$m_a(T) < 3H = m_s , \qquad (43)$$

where we estimate the onset of the S oscillation by  $3H = m_s$ .<sup>12</sup>

As we have seen in Eq. (40), the axion mass should exceed the Hubble parameter when P obtains the VEV. On the other hand, the axion mass needs to be smaller than the Hubble friction when  $3H = m_s$ . These two requirements lead to the condition that the axion mass in Eq. (38),  $m_a \propto H^{\frac{m}{2(n-1)}}$ , must decrease faster than Hubble parameter. Thus, the above condition is satisfied for

$$m \ge 2n - 1 . \tag{44}$$

To this point, we have ignored the back reaction to S from P. As we have seen,  $\langle S_I \rangle \gg \langle P_I \rangle$  during inflation. Thus, the back reaction from P through the mixing term is negligible during inflation. As the field value of P decreases much faster than that of S, the back reaction is also negligible in the inflaton oscillation era and the RD era until P obtains the VEV.

After P obtains the VEV,  $\langle P \rangle \simeq v_{\rm PQ}$ , the back reaction could modify the behavior of S

<sup>&</sup>lt;sup>12</sup> Below the temperature of  $m_a(T) \simeq 3H$ , the field value of *a* is frozen to a different field value in each Hubble patch. The difference of the frozen field value, however, does not cause the domain wall problem as long as all the difference is smaller than  $2\pi f_a$ .

in Eq. (31) if <sup>13</sup>

$$\frac{\lambda_s^2}{(n!)^2 M_{\rm Pl}^{2n-4}} |S|^{2n} \sim \frac{\lambda}{(2n-1)! M_{\rm Pl}^{2n-4}} |S|^{2n-1} v_{\rm PQ}$$
$$\therefore S \sim \frac{(n!)^2 \lambda}{(2n-1)! \lambda_s^2} v_{\rm PQ}. \tag{45}$$

However, such a situation can be avoided if the mass of S is larger so that S starts oscillation before Eq. (45) is satisfied. Thus, so long as

$$\lambda v_{\rm PQ} M_{\rm Pl} \left( \frac{(n!)^2 \lambda}{m! \lambda_s^2} \frac{v_{\rm PQ}}{M_{\rm Pl}} \right)^{2n-3} < m_s^2 , \qquad (46)$$

we can safely neglect the back reaction of P to the dynamics of S.

### D. Isocurvature Perturbations of the Axion

Because P obtains the VEV during the RD era, it seems that the axion does not suffer from the isocurvature problem. However, the phase component b of S has a flat potential when S takes a large field value, and it obtains quantum fluctuation during inflation. In the presence of the mixing term, the fluctuation of the phase component of S is imprinted in the axion, which leads to the isocurvature perturbations of the axion dark matter.

During inflation, the fluctuation of b is given by [41–45],

$$\delta b_I \simeq \frac{H_I}{2\pi} , \qquad (47)$$

where  $\langle \chi_I \rangle = \sqrt{2} \langle S_I \rangle$ .<sup>14</sup> After inflation, S follows the scaling solution which is along the straight line passing through S = 0 in the complex plane of S. Thus, the fluctuation of  $\delta b$  decreases as

$$\delta b \simeq \frac{\chi}{\langle \chi_I \rangle} \times \delta b_I \simeq \frac{H_I}{2\pi} \frac{\chi}{\langle \chi_I \rangle} ,$$
 (48)

Once P obtains the VEV, the P-S mixing term leads to the potential of a and b in

<sup>&</sup>lt;sup>13</sup> Here, we use the lower limit on m in Eq. (44) since the back reaction is weaker for a larger m.

<sup>&</sup>lt;sup>14</sup> Since P also takes a large field value, a also fluctuates during inflation. However, the axion eventually settles around  $-mv_{PQ}/\chi b$  regardless of its initial value as we have seen in subsection IV B. Thus, the fluctuation of a does not affect the following arguments.

Eq. (42). Then, as we discussed in subsection IV B,  $a/v_{PQ}$  settles around  $-m \times b/\chi$  (mod  $2\pi$ ) while  $b/\chi$  does not move. As a result, the fluctuation of b is imprinted in a as<sup>15</sup>

$$\frac{\delta a}{f_a} \simeq m \frac{v_{\rm PQ}}{\chi} \frac{\delta b}{f_a} \simeq \frac{m N_{\rm DW} H_I}{2\pi \langle \chi_I \rangle} \ . \tag{49}$$

Below the QCD scale, the axion starts coherent oscillation and the axion fluctuation results in the uncorrelated isocurvature perturbations. The power spectrum of the isocurvature perturbations is given by

$$\mathcal{P}_I \simeq 8 \left(\frac{\delta a}{f_a}\right)^2 \left(\frac{f_a}{10^{12} \,\mathrm{GeV}}\right)^{1.19} \left(\frac{\Omega_a h^2}{0.1}\right) , \qquad (50)$$

where we assume that the observed dark matter density is dominated by the axion. From the CMB observations, the uncorrelated isocurvature perturbations of cold dark matter are constrained by [19],

$$\beta_{\rm iso} = \frac{\mathcal{P}_I}{\mathcal{P}_{\zeta} + \mathcal{P}_I} \le 0.038 , \qquad (51)$$

where  $\mathcal{P}_{\zeta} \simeq 2 \times 10^{-9}$  denotes the power spectrum of the curvature perturbations. By combining Eqs. (13), (49) and (50), we find the upper limit on  $H_I$  is given by,

$$H_{I} \lesssim M_{\rm Pl} \times \left(8 \times 10^{-10} \frac{1}{m^2 N_{\rm DW}^2} \left(\frac{10^{12} \,{\rm GeV}}{f_a}\right)^{1.19} \left(\frac{0.1}{\Omega_a h^2}\right)\right)^{\frac{n-1}{2(n-2)}} \left(\sqrt{\frac{c_s}{n}} \frac{n!}{\lambda_s}\right)^{\frac{1}{n-2}}$$
(52)

### E. Viable Parameter Region

Let us summarize the constraints on the model parameters. In Fig. 4, we show the constraints on the  $(m_s, \lambda)$  plane, for  $N_{\rm DM} = 1$  (the minimal KSVZ axion model) and for  $N_{\rm DM} = 6$  (DFSZ axion model [7, 8]).<sup>16</sup> We take n = 6, m = 11,  $\lambda_p = 1$ ,  $c_s = 1$  and  $c_p = 1$  as a benchmark point. The blue shaded region is excluded where the condition in Eq. (40) is not satisfied after P obtains the VEV, and hence, the randomness of the axion direction is not resolved. The orange shaded region is excluded where the condition in Eq. (43) is not

<sup>&</sup>lt;sup>15</sup> Only the fluctuation modes longer than the Hubble length at the QCD temperature are relevant for the isocurvature perturbations of the axion dark matter, which are superhorizon mode when a settles around  $-mv_{PQ}/\chi b$ .

<sup>&</sup>lt;sup>16</sup> Here, we assume that the two Higgs doublets in the DFSZ model couple to the PQ field via  $P^2H_1H_2$ .



Figure 4. Various constraints on the model parameters. The blue, orange and green shaded areas are excluded by the conditions in Eqs. (40), (43), (46). The red one is excluded by a conservative condition  $m_a(T \sim v_{\rm PQ}) \gtrsim 3H(T \sim v_{\rm PQ})$ . As a benchmark, we set n = 6, m = 11, and  $\lambda_p = c_p = c_s = 1$  and  $\lambda_s = 10^{-4}$ . Left  $N_{\rm DW} = 1$ , Right  $N_{\rm DW} = 6$ .

satisfied at  $3H \simeq m_s$ , and hence, the axion is randomized as S oscillates. The green shaded region is excluded where the condition in Eq. (46) is not satisfied, where the back reaction from P to S becomes sizable. The red shaded region shows a more conservative constraint,  $m_a(T \sim v_{\rm PQ}) \gtrsim 3H(T \sim v_{\rm PQ})$ , which is weaker than that in Eq. (40). This weaker constraint is good enough if no cosmic strings are formed when P obtains the VEV at  $T \sim v_{\rm PQ}$ . The figure shows that all the conditions are satisfied for a wide range of  $m_s$ . For the minimal KSVZ model, i.e.  $N_{\rm DW} = 1$ ,  $m_s \lesssim 1 \,\text{GeV}$ , and for the DFSZ model,  $m_s \lesssim 10^2 \,\text{GeV}$ .

For the benchmark scenario, n = 6, m = 11, the constraint on the isocurvature perturbations leads to

$$H_I \lesssim 10^{12} \,\mathrm{GeV} \times \frac{1}{N_{\mathrm{DW}}^{5/4}} \left(\frac{\sqrt{c_s}}{\lambda_s}\right)^{1/4} \,.$$
 (53)

Thus, we find that the present mechanism allows  $H_I \sim 10^{13} \,\text{GeV}$  for  $\lambda_s \sim 10^{-6}$ .<sup>17</sup>

As a result, we find that the present mechanism makes the axion model with  $N_{\rm DM} \neq 1$  compatible with the inflation model in which a large Hubble parameter is rather large, i.e.,  $H_I \gg 10^{7-8}$  GeV. Therefore, this mechanism increases the freedom of choice of the combinations of the axion models and inflation models.

<sup>&</sup>lt;sup>17</sup> For  $\lambda_s \sim 10^{-6}$ , the expectation value of S during inflation slightly exceeds  $M_{\rm Pl}$ , which requires a small  $c_s$  to avoid too much potential energy of S during inflation.

Table I. Charge assignments of the supersymmetric model. In addition to the PQ symmetry, we show the *R*-symmetry.

	X	Y	Z	S	P	$\bar{P}$	$Q_L \bar{Q}_R$	$\psi_s \bar{\psi}_s$
$U(1)_{\rm PQ}$	0	-6	-9	1	7	-7	-7	-1
R	2	2	2	0	0	0	2	2

# V. SUPERSYMMETRIC REALIZATION

As we have seen in the previous section, the model requires the higher dimensional interaction terms with specific exponents. Such scalar potentials are not easily justified in nonsupersymmetric theories. In this section, we briefly discuss a supersymmetric realization of the mechanism to make the scenario more viable. A detailed analysis of the supersymmetric extension will be given elsewhere.

For example, a model with n = 6 and m = 11 can be easily realized by assuming a superpotential,

$$W = X(P\bar{P} - v_{\rm PQ}^2) + \frac{1}{M_{\rm Pl}^4}YS^6 + Z\left(S^2P + \frac{1}{M_{\rm Pl}^6}S^9\right) + \frac{1}{M_{\rm Pl}^6}X\bar{P}S^7 .$$
(54)

Here, P and S are the chiral superfields corresponding to the (spectator) PQ fields, while X, Y and Z are chiral superfields whose F-components lead to the scalar potential in Eq. (8).<sup>18</sup> We omit the coupling constants for brevity. The PQ charge assignment is given in Table I. This model justifies why the potential term with a lower dimension than  $|S|^{2n}$  are absent. The mass term of the scalar component of S is generated by the supersymmetry breaking effects. It should be noted that the unwanted superpotential terms such as  $YP^iS^j$  with i > 1 (i + j = 6) are suppressed by PQ and R-symmetry.

The last term of Eq. (54) induces

$$V = \frac{1}{M_{\rm Pl}^6} (P\bar{P} - v_{\rm PQ}^2) \bar{P}^* S^7 + \text{h.c.}$$
(55)

When  $T \gtrsim v_{PQ}$ , P and  $\bar{P}$  are settled to the origin due to thermal mass terms, thus this term does not affect the dynamics of S and P. In addition, after  $\langle P\bar{P} \rangle$  settles to  $v_{PQ}^2$ , this term

<sup>&</sup>lt;sup>18</sup> In this realization, the *P-S* mixing term in the scalar potential is given by  $|S|^4 S^7 P^*$  + h.c. Accordingly, the cosine potential of the phase components in Eq. (42) is modified to  $\cos(a/v_{PQ} + 7b/\chi)$ .

vanishes. Therefore, this term does not have an influence on the dynamics all through the epoch of interest.

We briefly comment on the effects of higher dimensional terms in the Kähler potential. For example, a higher dimensional operator,

$$K = \frac{1}{M_{\rm Pl}^2} |X|^2 |S|^2 , \qquad (56)$$

induces a scalar potential,

$$V = \frac{|P\bar{P} - v_{\rm PQ}^2|^2}{1 + |S|^2/M_{\rm Pl}^2} .$$
(57)

This term leads to an additional effective mass of S for a given  $\langle P\bar{P} \rangle$ . During inflation, this term is smaller than the Hubble induced mass term (see Eqs. (33) and (34)). After inflation,  $\langle P\bar{P} \rangle$  immediately vanishes, and the mass term in Eq. (57) leads to a mass of  $\mathcal{O}(v_{PQ}^2/M_{Pl})$ , which is smaller than the Hubble constant until  $T \sim v_{PQ}$ . After  $\langle P\bar{P} \rangle$  settles to  $v_{PQ}^2$ , the induced mass vanishes.<sup>19</sup> Therefore, we find that the induced mass does not affect the dynamics of the scalar fields.

In addition to the higher dimensional term in Eq. (56), there are terms

$$K = \frac{1}{M_{\rm Pl}^2} (|Y|^2 |S|^2 + |Z|^2 |S|^2 + |S|^4) + \cdots$$
 (58)

These terms just induce additional terms with higher dimension than those in Eq. (8), and hence, they do not affect the scaling behavior of  $S^{20}$  In the case of  $|S|^4$ , we redefine Sso that it has a canonical kinetic term. With the redefinition, the effects appear as the additional terms with higher dimension than those in Eq. (8). The same is true for the higher dimensional Kähler term of  $|S|^{2k}$  (k > 1).

Supersymmetric extension is also advantageous to explain the interactions between the

<sup>&</sup>lt;sup>19</sup> In the presence of the supersymmetry breaking effects, the VEV of  $P\bar{P}$  is slightly shifted from  $v_{PQ}^2$ , and hence, the effective mass in Eq. (57) does not vanish completely. However, we can set this mass term small enough not to affect the dynamics.

<sup>&</sup>lt;sup>20</sup> We assume that X, Y, Z obtain the positive Hubble mass terms, hence they settle at the origin.

PQ fields and the inflaton in Eq. (9). In fact, a Kähler potential,

$$K = \frac{c_p}{3M_{\rm Pl}^2} |I|^2 |P|^2 - \frac{c_s}{3M_{\rm Pl}^2} |I|^2 |S|^2 , \qquad (59)$$

explains the interactions between the PQ fields and the inflaton. Here, I denotes the chiral superfield of the inflaton  $\phi$  whose F-component provides the inflaton potential, i.e.,  $V(\phi) = |F_I|^2$ .

In addition to Eq. (59), we can write higher order Kähler terms such as

$$K = |I|^2 \left( \frac{|S|^4}{M_{\rm Pl}^4} + \frac{|S|^6}{M_{\rm Pl}^6} + \cdots \right).$$
(60)

In the case of  $\langle S_I \rangle \sim M_{\rm Pl}$ , these terms slightly modify the expectation value of S during inflation. However, after inflation,  $\langle S \rangle$  starts to decrease, then the higher order contributions become negligible, therefore they do not affect the dynamics.

## VI. CONCLUSIONS

The domain wall problem and the isocurvature problem restrict possible combinations of axion models and inflation models. In this paper, we considered a new mechanism which solves those problems by introducing the spectator PQ field which obtains a large field value before the PQ field obtains the VEV. The mechanism makes the axion model with a non-trivial domain wall number compatible with the inflation model with a large Hubble parameter,  $H_I \gg 10^{7-8}$  GeV. The mechanism is also free from the isocurvature problem. It should be emphasized that this mechanism can be added to any conventional axion models. Thus, this mechanism increases the freedom of choice of combinations of axion models and inflation models.

We also find that the present mechanism can be consistent with a large Hubble parameter during inflation, of  $H_I \sim 10^{13}$  GeV. Thus, the scenario can be tested by combining future axion search experiments and the searches for the primordial *B*-mode polarization in the CMB.

The model also predicts the existence of the spectator PQ field. As we have discussed in

III C, the coherent oscillation of the spectator PQ field can play a role of the dark matter when it is very light.<sup>21</sup> As the initial amplitude of the coherent oscillation is dynamically determined, this model realizes a very light scalar dark matter without fine-tuning of the initial condition in an alternative way to the axion-like ultra-light dark matter in [46, 47]. Such a very light dark matter can be tested via astronomical ephemeris [48].

### ACKNOWLEDGMENTS

The authors thank M. Kawasaki for an important comment on the isocurvature problem. This work is supported in part by JSPS Grant-in-Aid for Scientific Research No. 16H02176 (T.T.Y), No. 17H02878 (M.I., and T.T.Y.), No. 15H05889, No. 16H03991, No. 18H05542 (M.I.) and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan (M.I., and T.T.Y.). T.T.Y. is supported in part by the China Grant for Talent Scientific Start-Up Project. T.T.Y. thanks to Hamamatsu Photonics. The work of M.S. is supported in part by a Research Fellowship for Young Scientists from the Japan Society for the Promotion of Science (JSPS).

### Appendix A: Energy Density of the Spectator PQ Field

In this appendix, we discuss the energy density of the spectator PQ field, S. When S follows the scaling solution in Eqs. (13), (24) and (31), the potential energy density of S is of  $\mathcal{O}(H^2|S|^2)$ . Thus, it is sub-dominant compared with the dominant energy density of  $\mathcal{O}(H^2 M_{\rm Pl}^2)$  as long as  $\langle S \rangle \ll M_{\rm Pl}$ .

Once S starts coherent oscillation around its origin, S behaves as a massive matter with an energy density

$$\rho_S = m_s^2 |S|^2 . \tag{A1}$$

The radiation density at that time is  $m_s^2 M_{\rm Pl}^2/3$  where we have used  $H \simeq m_s/3$ . Thus, again, the energy density of S is sub-dominant since  $S \ll M_{\rm Pl}$  at the onset of the coherent oscillation.

<sup>&</sup>lt;sup>21</sup> In this case, we do not need to introduce the light fermions in Eq. (32).

As we considered in II, S immediately decays into the massless fermions which behave as radiation. Thus, the energy density of S does not causes any cosmological problems. As the energy density of S is sub-dominant, the energy density of the massless fermions are also sub-dominant. Furthermore, the relative entropy of the massless fermions is diluted when all the entropy in the thermal bath goes into the particles in the standard cosmology (i.e. the photons and the neutrinos). Thus, the contributions of the massless fermions to the dark radiation is also negligible.

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