

# Oscillating Composite Asymmetric Dark Matter

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(Dated: July 29, 2019)

## Abstract

The asymmetric dark matter (ADM) scenario can solve the coincidence problem between the baryon and the dark matter (DM) abundance when the DM mass is of  $\mathcal{O}(1)$  GeV. In the ADM scenarios, composite dark matter is particularly motivated, as it can naturally provide the DM mass in the  $\mathcal{O}(1)$  GeV range and a large annihilation cross section simultaneously. In this paper, we discuss the indirect detection constraints on the composite ADM model. The portal operators connecting the  $B - L$  asymmetries in the dark and the Standard Model(SM) sectors are assumed to be generated in association with the seesaw mechanism. In this model, composite dark matter inevitably obtains a tiny Majorana mass which induces a pair-annihilation of ADM at late times. We show that the model can be efficiently tested by the searches for gamma-ray from the dwarf spheroidal galaxies.

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## I. INTRODUCTION

Asymmetric dark matter (ADM) scenario sheds light on the coincidence problem between the observed baryon and dark matter (DM) abundances in the universe [1–11] (see also [12–14] for reviews). If the DM abundance is provided by a mechanism which is unrelated to the baryogenesis, it is quite puzzling why those abundances are close with each other despite the fact that the baryon abundance is dominated by the contribution from the matter-antimatter asymmetry. In the ADM scenario the coincidence problem can be explained when the DM mass is of  $\mathcal{O}(1)$  GeV, where the matter-antimatter asymmetry is thermally distributed between the dark and the Standard Model (visible) sectors.

Among various ADM scenarios, composite baryonic DM in QCD-like dynamics is particularly motivated since it can naturally provide a large annihilation cross section and the DM mass in the GeV range simultaneously [7, 8, 15–23]. Recently, a minimal composite ADM model and its ultraviolet (UV) completion [24–26] have been proposed where the asymmetry generated by the thermal leptogenesis [27] (see also [28–30] for review) is thermally distributed between the two sectors through a portal operator associated with the seesaw mechanism [31–35]. The dark sector of the model consists of QCD-like dynamics and QED-like interaction, which are called as dark QCD and dark QED, respectively. The lightest baryons of dark QCD play the role of ADM. The dark QED photon (dark photon) obtains a mass of  $\mathcal{O}(10-100)$  MeV, which plays a crucial role to transfer the excessive entropy of the dark sector into the visible sector before neutrino decoupling [24, 36].

In this paper, we discuss the indirect detection of the composite ADM model in [24–26]. The portal operator in this model is generated in association with the seesaw mechanism. In this model, the dark-neutron, one of the lightest dark baryons, inevitably obtains a tiny Majorana mass. Such a tiny Majorana mass induces the oscillation between DM particle and the antiparticle, which induces a pair-annihilation of ADM at late times [37–42]. A pair of DM particle and the antiparticle annihilates into multiple dark pions, and the (neutral) dark pion subsequently decays into a pair of the dark photons. The dark photon eventually decays into an electron-positron pair. Thus, the late time annihilation of ADM results in

multiple soft electrons/positrons. In addition, soft photons are also emitted as final state radiation. As we will see, the model can be efficiently tested by the searches for gamma-ray from the dwarf spheroidal galaxies (dSphs) by the Fermi-LAT.

The organization of the paper is as follows. In section II, we review the composite ADM model in [24–26] and show how the tiny Majorana mass of the dark neutron appears associated with the seesaw mechanism. In section III, we derive the expected gamma-ray flux from the dSphs and discuss the constraints on the model by comparing the flux with the Fermi-LAT results. The final section is devoted to the conclusions and discussion.

## II. DM ANTI-DM OSCILLATION IN THE COMPOSITE ADM MODEL

### A. A Model of Composite ADM

In this subsection, we briefly review the composite ADM model in [24–26]. The model is based on  $N_g$ -generation dark quarks with  $SU(3)_D \times U(1)_D$  gauge symmetry.  $SU(3)_D$  provides the dark QCD dynamics and  $U(1)_D$  the dark QED interaction. The dark quarks are the fundamental representations of  $SU(3)_D$ . They are charged under the dark QED and the  $B - L$  in analogy to the up-type and the down-type quarks in the visible sector (see Tab. I). They have tiny masses,

$$\mathcal{L}_{\text{mass}} = m'_U \bar{U}' U' + m'_D \bar{D}' D' + \text{h.c.} , \quad (1)$$

with  $m'_U$  and  $m'_D$  being the mass parameters. Hereafter, we put primes on the parameters and the fields in the dark sector when there are counterparts in the visible sector.

The dark QCD exhibits confinement below the dynamical scale of  $SU(3)_D$ ,  $\Lambda'_{\text{QCD}}$ , which leads to the emergence of the dark baryons and the dark mesons. Throughout this paper, we assume that only one generation of the dark quarks have masses smaller than  $\Lambda'_{\text{QCD}}$ .<sup>1</sup>

<sup>1</sup> For  $N_g > 1$ , we assume the heavier dark quarks decay into the lighter ones by emitting the dark Higgs boson which has the dark QED charge of 1. It should be noted that the dark quark masses are not generated by the vacuum expectation value of the dark Higgs boson [24–26], and hence, the dark Higgs couplings generically violate the flavor symmetry in the dark sector.

Table I. The charge assignment of dark quarks. We assume  $N_g$  generations of the dark quarks, although only one generation has a mass smaller than  $\Lambda'_{\text{QCD}}$ . The  $U(1)_{B-L}$  symmetry is the global symmetry which is shared with the visible sector.

	$SU(3)_D$	$U(1)_D$	$U(1)_{B-L}$
$U'$	$\mathbf{3}$	$2/3$	$1/3$
$\bar{U}'$	$\bar{\mathbf{3}}$	$-2/3$	$-1/3$
$D'$	$\mathbf{3}$	$-1/3$	$1/3$
$\bar{D}'$	$\bar{\mathbf{3}}$	$1/3$	$-1/3$

The lightest dark baryons, i.e. the dark nucleons,

$$p' \propto U'U'D' , \quad \bar{p}' \propto \bar{U}'\bar{U}'\bar{D}' , \quad n' \propto U'D'D' , \quad \bar{n}' \propto \bar{U}'\bar{D}'\bar{D}' , \quad (2)$$

are stable in the decoupling limit from the visible sector due to their  $B - L$  charges. Once the  $B - L$  asymmetry is shared between the visible and the dark sector, the dark nucleon abundance is dominated by the asymmetric component due to their large annihilation cross section. Therefore, the dark nucleon with a mass in the GeV range is a good candidate of ADM.

When the  $B - L$  asymmetry is thermally distributed between the visible and the dark sectors, the ratio of the  $B - L$  asymmetry stored in each sector is given by  $A_{\text{DM}}/A_{\text{SM}} = 44N_g/237$  for the  $B - L$  charges given in Tab. I [43].<sup>2</sup> Thus, the observed ratio of the DM and the baryon abundance can be reproduced when the dark nucleon mass is

$$m'_N \simeq \frac{\Omega_{\text{DM}}}{\Omega_{\text{B}}} \frac{A_{\text{B}}}{A_{\text{SM}}} \frac{A_{\text{SM}}}{A_{\text{DM}}} \times m_N \simeq \frac{8.5 \text{ GeV}}{N_g} . \quad (3)$$

Here, we have used the ratio of the baryon asymmetry to the  $B - L$  asymmetry in the visible sector,  $A_{\text{B}}/A_{\text{SM}} = 30/97$  [44]. The dark nucleon mass in this range can be naturally realized when  $\Lambda'_{\text{QCD}}$  is in the GeV range.

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<sup>2</sup> In the presence of additional  $B - L$  charged fields in the dark sector, such as dark leptons, the ratio can be modified. Besides, the neutrality condition of  $U(1)_D$  and the contributions from the dark Higgs sector also change the ratio by some tens percent for a given  $N_g$ .

The lightest dark mesons,

$$\pi'^0 \propto U'\bar{U}' - D'\bar{D}' , \quad \pi'^+ \propto U'\bar{D}' , \quad \pi'^- \propto D'\bar{U}' , \quad (4)$$

annihilate or decay into the dark photons. As a result, they do not contribute to the effective number of neutrino degrees of freedom nor to the dark matter abundance significantly even if they are stable. In the following analysis, we assume that the dark charged pions are stable for simplicity.<sup>3</sup> The decay of the dark neutral pion into a pair of the dark photons, on the other hand, is inevitable due to the chiral anomaly. As we will see, the decay of the neutral pion plays a central role for the indirect detection of ADM.

The dark photon obtains its mass by the dark Higgs mechanism, and it decays into the visible fermions through the kinetic mixing with the visible QED photon,

$$\mathcal{L}_{\gamma'} = \frac{\epsilon}{2} F_{\mu\nu} F'^{\mu\nu} + \frac{m_{\gamma'}^2}{2} A'_\mu A'^\mu . \quad (5)$$

Here,  $F_{\mu\nu}$  and  $F'_{\mu\nu}$  denote the field strengths of the visible and the dark QED with  $A'_\mu$  being the dark photon gauge field. In the following, we assume the kinetic mixing parameters of  $\epsilon = 10^{-10}$ – $10^{-8}$  and the dark photon mass in  $\mathcal{O}(10\text{--}100)$  MeV range which satisfies all the constraints [24] (see also [45–47]).<sup>4</sup> In this parameter range, the dark photon decays when the cosmic temperature is above  $\mathcal{O}(1)$  MeV.

Finally, let us comment on the ratio between the abundances of the dark protons and the dark neutrons. In the present model, there is no dark leptons nor dark weak gauge bosons. Besides, it is expected that the mass difference between the dark neutron and the dark proton is smaller than the mass of the dark pion when the dark quark masses are smaller than the dynamical scale of  $SU(3)_D$ . Thus, the dark neutron is stable in the limit of the vanishing  $B - L$  portal interactions (see below). The ratio between the dark proton

<sup>3</sup> If  $U(1)_D$  is broken by the vacuum expectation value of a dark Higgs with the dark QED charge of 2, a  $\mathbb{Z}_2$  symmetry remains unbroken which makes the dark charged pion stable. If  $U(1)_D$  is broken by the dark Higgs with the charge 1, the neutral and the charged pions can mix with each other, and hence, the charged pions decay.

<sup>4</sup> See [24] for discussion on the origin of the tiny kinetic mixing parameters.

abundance and the dark neutron abundance is given by [25],

$$n'_n/n'_p \sim e^{-(m'_n - m'_p)/T_F} . \quad (6)$$

Here,  $n'_{n,p}$  and  $m'_{n,p}$  are the number densities and the masses of the dark neutron and the dark proton, respectively.  $T_F$  denotes the freeze-out temperature of the dark pion annihilation,  $T_F \simeq m'_\pi/\mathcal{O}(10)$ . Thus, for  $m'_n - m'_p \ll m'_\pi$ , the dark neutron abundance is comparable to that of the dark proton. In the following, we take  $n'_n = n'_p$ .

### B. The $B - L$ portal operator

The  $B - L$  asymmetry generated by thermal leptogenesis is thermally distributed between the visible and the dark sectors. For this purpose, there need to be portal interactions which connect the  $B - L$  symmetry in the two sectors. In the model in [24] (see also [43, 48]), the following operators are assumed as the portal operators,

$$\mathcal{L}_{\text{portal}} \sim \frac{1}{M_*^3} (\bar{U}' \bar{D}' \bar{D}') (LH) + \frac{1}{M_*^3} (U'^\dagger D'^\dagger \bar{D}') (LH) + \text{h.c.} , \quad (7)$$

where  $L$  and  $H$  are the lepton and the Higgs doublets in the visible sector, and  $M_*$  is a dimensional parameter.<sup>5</sup> Here, we omit the  $\mathcal{O}(1)$  coefficients. The effects of the above operators decouple at the cosmic temperature below  $T_* \sim M_*(M_*/M_{\text{PL}})^{1/5}$ . Here,  $M_{\text{PL}} = 2.4 \times 10^{18}$  GeV denotes the reduced Planck mass. For successful ADM with thermal leptogenesis, the decoupling temperature,  $T_*$ , is required to be lower than the temperature,  $T_{B-L}$ , at which leptogenesis completes. In the following, we consider the so-called strong washout regime of thermal leptogenesis, where the leptogenesis completes at the temperature about  $T_{B-L} \simeq M_R/z_{B-L}$  with  $z_{B-L} \simeq 10$  [29].

In [24, 25], the UV model has been proposed in which the portal operators in Eq. (7) are generated by integrating out the right-handed neutrinos,  $\bar{N}$ , and the dark colored Higgs

<sup>5</sup> The portal operators require the gauge invariant operators which are charged under the  $B - L$  symmetry. This is the reason why we need both the up-type and the down-type quarks in the dark sector.

boson,  $H'_C$ . The gauge charges of  $H'_C$  are identical to those of  $D'$ , while  $H'_C$  has the  $B - L$  charge  $-2/3$ . The right-handed neutrinos couple to both sectors via,

$$\mathcal{L} = \frac{M_R}{2} \bar{N} N + y_N L H \bar{N} + \frac{1}{2} M_C^2 |H'_C|^2 - Y_N H'_C \bar{D}' \bar{N} - Y_C H'_C U' D' - Y_{\bar{C}} H'_C \bar{U}' \bar{D}' + \text{h.c.} \quad (8)$$

Here,  $M_C$  denotes the dark colored Higgs mass,  $M_R$  the mass of the right-handed neutrinos, and  $y_N$  and  $Y$ 's are the Yukawa coupling constants. The flavor and the gauge indices are suppressed. It should be noted that the mass terms of the right-handed neutrino break  $B - L$  symmetry explicitly. The first two terms are relevant for the seesaw mechanism.

By integrating out  $\bar{N}$  and  $H'_C$  from Eq. (8), the portal operators in Eq. (7) are obtained where  $M_*$  corresponds to

$$\frac{1}{M_*^3} = \frac{y_N Y_N Y_{\bar{C}}}{2 M_C^2 M_R}, \quad \frac{1}{M_*^3} = \frac{y_N Y_N Y_C^*}{2 M_C^2 M_R}, \quad (9)$$

for each term of Eq. (7), respectively. From the condition of  $T_* < T_{B-L}$ , the mass of the dark colored Higgs should satisfy,<sup>6</sup>

$$\frac{M_R}{z_{B-L}} \lesssim M_C \lesssim \frac{10}{z_{B-L}^{5/4}} \left( \frac{\hat{m}_\nu}{0.1 \text{eV}} \right)^{1/4} \sqrt{Y_N Y_C} M_R. \quad (10)$$

The first inequality comes from a consistency condition of the decoupling limit of the dark colored Higgs at the temperature  $T_{B-L}$ . In the right hand side, we have reparameterized the neutrino Yukawa coupling by using a tiny neutrino mass parameter,  $\hat{m}_\nu$ ,

$$|y_N^2| \sim 10^{-5} \left( \frac{\hat{m}_\nu}{0.1 \text{eV}} \right) \left( \frac{M_R}{10^9 \text{GeV}} \right). \quad (11)$$

Incidentally, the dark nucleon can decay into the dark pion and the anti-neutrino in the visible sector through the  $B - L$  portal operator in Eq. (7) [43]. The lifetime is roughly given

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<sup>6</sup> Hereafter, we take  $M_R > 0$  and neglect the complex phases of  $Y_N$ ,  $Y_C$  and  $Y_{\bar{C}}$ . We also assume  $Y_C = Y_{\bar{C}}$  for simplicity.

by,

$$\tau'_N \simeq 10^{33} \text{ sec} \left( \frac{2 \text{ GeV}}{\Lambda'_{\text{QCD}}} \right)^4 \left( \frac{0.1 \text{ eV}}{\hat{m}_\nu} \right) \left( \frac{M_R}{10^9 \text{ GeV}} \right) \left( \frac{\tilde{M}_C}{3 \times 10^9 \text{ GeV}} \right)^4 \left( \frac{10 \text{ GeV}}{m_{\text{DM}}} \right), \quad (12)$$

where  $\tilde{M}_C = M_C/\sqrt{Y_N Y_C}$ . Thus, the lifetime of the dark nucleons is much longer than the age of the universe for  $M_R \sim M_C \sim 10^9 \text{ GeV}$ .

### C. The Majorana mass of the dark neutron

The portal operators in Eq. (7) are generated in association with the seesaw mechanism. As a notable feature of the UV completion model in Eq. (8), it also leads to the Majorana mass term of the dark neutron. This can be observed by integrating out  $H'_C$  and  $\bar{N}$  one by one. In the case of  $M_C > M_R$ , we first integrate out  $H'_C$  from Eq. (8), which reads

$$\begin{aligned} \mathcal{L} = & \bar{N}^\dagger i\sigma^\mu \partial_\mu \bar{N} + \frac{M_R}{2} \bar{N} \bar{N} + y_N L H \bar{N} \\ & - \frac{|Y_N|^2}{2M_C^2} \bar{D}' \bar{N} (\bar{D}' \bar{N})^\dagger - \frac{Y_N}{2M_C^2} \bar{D}' \bar{N} \left[ (Y_C U' D')^\dagger + Y_{\bar{C}} \bar{U}' \bar{D}' \right] + \text{h.c.} \\ & + (\text{quartic in dark quark fields}). \end{aligned} \quad (13)$$

Here, we show the kinetic term of  $\bar{N}$  explicitly which were implicit in Eq. (8). This formula is of the form

$$\mathcal{L} = (A \bar{N} \bar{N} + B \bar{N} + \text{h.c.}) - C \bar{N} \bar{N}^\dagger, \quad (14)$$

where<sup>7</sup>

$$A = \frac{M_R}{2}, \quad B = y_N L H - \frac{Y_N}{2M_C^2} \bar{D}' \left[ Y_{\bar{C}} \bar{U}' \bar{D}' + (Y_C U' D')^\dagger \right], \quad C = -\frac{i}{2} \sigma^\mu \overleftrightarrow{\partial}_\mu + \frac{|Y_N|^2}{2M_C^2} \bar{D}' \bar{D}'^\dagger. \quad (15)$$

<sup>7</sup> Here,  $\chi^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \eta = \chi^\dagger \sigma^\mu \partial_\mu \eta - \partial_\mu \chi^\dagger \sigma^\mu \eta$  for the Weyl fermions,  $\chi$  and  $\eta$ .

To make  $\bar{N}$  integrated out, it is convenient to complete the square of Eq. (15) with respect to  $\bar{N}$ . For this purpose, we shift  $\bar{N}$  by  $\bar{N} \rightarrow \bar{N} + \bar{\psi}$ , with which we can eliminate the linear term in Eq. (14). The condition  $\bar{\psi}$  must satisfy is  $2A\bar{\psi} + B - C\bar{\psi}^\dagger = 0$ , which reads

$$\bar{\psi} = -\frac{2A^*B + CB^\dagger}{4|A|^2 - C^2} \simeq -\frac{1}{2|M_R|^2} \left(1 + \frac{C^2}{|M_R|^2}\right) (M_R^*B + CB^\dagger) \quad (16)$$

After the shift, we integrate out  $\bar{N}$  to obtain

$$\begin{aligned} \mathcal{L} &= (A\bar{\psi}\bar{\psi} + B\bar{\psi} + \text{h.c.}) - C\bar{\psi}\bar{\psi}^\dagger \\ &= \frac{1}{2}B\bar{\psi} + \text{h.c.} \\ &= -\left(1 + \frac{C^2}{|M_R|^2}\right) \left(\frac{1}{2M_R}BB + \frac{1}{2|M_R|^2}CB^\dagger B\right) + \text{h.c.} \end{aligned} \quad (17)$$

From Eq. (15), we find that  $BB$  term includes the Majorana mass term of the dark neutron

$$\frac{1}{2M_R}BB \supset \frac{Y_N^2 Y_C^2}{8M_R M_C^4} (\bar{U}' \bar{D}' \bar{D}')^2 \sim \frac{Y_N^2 Y_C^2 \Lambda_{\text{QCD}}^6}{8M_R M_C^4} \bar{n}' \bar{n}'. \quad (18)$$

In this way, Eq. (8) leads to the Majorana mass,

$$m_M = \frac{Y_N^2 Y_C^2 \Lambda_{\text{QCD}}^6}{4M_R M_C^4} = \frac{\Lambda_{\text{QCD}}^6}{4M_R \tilde{M}_C^4}, \quad (19)$$

in addition to the  $B - L$  portal operators in Eq. (7).

Once the dark neutron obtains the Majorana mass, the dark neutron and the anti-dark neutron oscillate with a time scale of  $t_{\text{osc}} = m_M^{-1}$  [37–42]. The probability to find an anti-dark neutron at a time  $t$  is given by,

$$P(n' \leftrightarrow \bar{n}') = \sin^2(m_M t). \quad (20)$$

Here, we assume that the initial state at  $t = 0$  is a pure dark neutron state. As we will see in the next section, the oscillation induces a pair-annihilation of ADM which ends up with

multiple soft electrons/positrons/photons.

#### D. Washout Interactions and On-Shell Portal

Before closing this section, let us discuss the  $B - L$  washout interactions which are also induced from Eq. (8). In fact, the term  $CB^\dagger B$  in Eq. (17) includes

$$\frac{1}{2|M_R|^2}CB^\dagger B \supset \frac{y_N Y_N^*}{2M_C^2|M_R|^2} \left[ Y_{\bar{C}}^*(\bar{U}^\dagger \bar{D}'^\dagger \bar{D}'^\dagger)(i\sigma^\mu \partial_\mu)(LH) + Y_C(U' D' \bar{D}'^\dagger)(i\sigma^\mu \partial_\mu)(LH) \right]. \quad (21)$$

In these interaction terms, and those in Eq. (7),  $L$  couples to the dark sector operators which have the opposite  $B - L$  charges with each other. Thus, if these operators are also in equilibrium at  $T_{B-L}$ , the  $B - L$  asymmetry generated by leptogenesis is washed out. To avoid such problems, it is required that

$$\frac{10}{z_{B-L}^{7/4}} \left( \frac{\hat{m}_\nu}{0.1\text{eV}} \right)^{1/4} M_R \lesssim \tilde{M}_C. \quad (22)$$

By comparing Eqs. (10) and (22), we find that the allowed parameter region for the ADM scenario is highly restricted due to the washout interaction when the portal operators are generated from the UV model in Eq. (8).

This constraint can be easily relaxed by introducing additional  $B - L$  portals. For example, we may introduce a pair of gauge singlet fermions,  $(X, \bar{X})$  with new scalar fields,  $H_p$ , and  $H'_{Cp}$ , whose gauge and  $B - L$  charges are the same with those of the Higgs doublet of the SM and the dark colored Higgs, respectively. In this case, there can be additional operators,

$$\mathcal{L} = M_X X \bar{X} + \frac{1}{2} M_H^2 |H_p|^2 + \frac{1}{2} M_{Cp}^2 |H'_{Cp}|^2 + y_X L H_p \bar{X} - Y_X H'_{Cp} \bar{D}' \bar{X}. \quad (23)$$

Here,  $M_X$ ,  $M_H$  and  $M_{Cp}$  are the mass parameters of  $(X, \bar{X})$ ,  $H_p$  and  $H'_{Cp}$ , respectively, and

$y_X$  and  $Y_X$  are Yukawa coupling constants.<sup>8</sup> As the mass of  $\bar{X}$  is the Dirac type, the interaction terms in Eq. (23) do not violate the  $B - L$  symmetry. Thus, these interactions do not washout the asymmetry generated by leptogenesis but thermally distribute the asymmetry between the visible and the dark sector for  $M_{X,H,Cp} < T_{B-L}$ .

In the following analysis, we divide the parameter region into two.

- Off-shell  $B - L$  portal scenario:

$$\frac{10}{z_{B-L}^{7/4}} \left( \frac{\hat{m}_\nu}{0.1\text{eV}} \right)^{1/4} M_R \lesssim \tilde{M}_C \lesssim \frac{10}{z_{B-L}^{5/4}} \left( \frac{\hat{m}_\nu}{0.1\text{eV}} \right)^{1/4} M_R . \quad (24)$$

- On-Shell  $B - L$  portal scenario:

$$\frac{10}{z_{B-L}^{5/4}} \left( \frac{\hat{m}_\nu}{0.1\text{eV}} \right)^{1/4} M_R \lesssim \tilde{M}_C . \quad (25)$$

In the on-shell portal scenario, we assume that there are lighter particles than  $T_{B-L}$  which mediate the  $B - L$  asymmetry between two sectors as in Eq. (23).<sup>9</sup> It should be emphasized that the  $B - L$  asymmetries in the two sectors are thermally distributed in both the scenarios.<sup>10</sup>

### III. GAMMA-RAY FLUX FROM THE DWARF SPHEROIDAL GALAXIES

As we have seen in the previous section, the dark neutron obtains a Majorana mass when the portal operator is generated in association with the seesaw mechanism. Due to the Majorana mass of the dark neutron, the dark neutron can oscillate into the anti-dark

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<sup>8</sup>  $\bar{N}$  and  $\bar{X}$  can be distinguished by an approximate discrete symmetry under which  $(X, \bar{X})$ ,  $H_p$  and  $H'_{Cp}$  are charged. With the discrete symmetry, we can avoid unnecessarily mixing between  $\bar{N}$  and  $\bar{X}$ .

<sup>9</sup> In the on-shell scenario, we may take  $Y_N = 0$ , and hence, the Majorana dark neutron mass is not inevitable.

<sup>10</sup> In the absence of the on-shell portal, the region with  $M_C < T_{B-L}$  results in a dark sector asymmetry which depends on the branching ratio of  $\bar{N}$  for small  $Y_N$ 's [49]. If  $Y_N$ 's are large for  $M_C < T_{B-L}$ , on the other hand, the  $B - L$  asymmetry is washed out very strongly, and results in too small asymmetry.

neutron. The typical time scale of the oscillation,  $t_{\text{osc}} = m_M^{-1}$ , is estimated as

$$t_{\text{osc}} \simeq 3.3 \times 10^{21} \text{ sec} \left( \frac{\Lambda'_{\text{QCD}}}{2 \text{ GeV}} \right)^{-6} \left( \frac{\tilde{M}_C}{3 \times 10^9 \text{ GeV}} \right)^4 \left( \frac{M_R}{10^9 \text{ GeV}} \right). \quad (26)$$

We now see that some fraction of  $n'$  can convert into  $\bar{n}'$  at late time, and then  $n'/p'$  and  $\bar{n}'$  annihilate into the dark pions. The neutral dark pions decay into the dark photons, and the dark photons finally decay into  $e^+e^-$  pairs.  $\gamma$  can be also emitted by the final state radiation (FSR) process as depicted in figure 1.

Among the soft  $e^+e^-$  and  $\gamma$  produced by the late time annihilation, the  $\gamma$ -ray signal is the most promising channel to search for dark matter annihilation (e.g., [50, 51] for review). In particular, the Dwarf Spheroidal galaxies (dSphs) in our galaxy are the ideal targets to search for the  $\gamma$ -ray signal, since they have high dynamical mass-to-light ratios, ( $M/L \sim 10-1000$ ), while they lack contaminating astrophysical  $\gamma$ -ray sources [52, 53]. In the following, we focus on the  $\gamma$ -ray spectrum and discuss the testability of the ADM scenario.

First, we calculate the  $\gamma$ -ray spectrum from the  $n'\bar{n}'$  annihilation processes:

$$n'\bar{n}' \rightarrow m\pi^{l0} + l\pi'^+ + l\pi'^-, \quad (m, l = 0, 1, 2, \dots). \quad (27)$$

The cascade spectrum can be calculated by using the technique developed by [54–56].

We start to calculate the  $\gamma$ -ray spectrum at the rest frame of  $\gamma'$ . For  $m_{\gamma'} \gg m_e$ , the spectrum is given by the Altarelli-Parisi approximation formula [54],<sup>11</sup>

$$\frac{d\tilde{N}_\gamma}{dx_0} = \frac{\alpha_{\text{EM}}}{\pi} \frac{1 + (1 - x_0)^2}{x_0} \left[ -1 + \ln \left( \frac{4(1 - x_0)}{\epsilon_0^2} \right) \right], \quad (28)$$

where  $\epsilon_0 = 2m_e/m_{\gamma'}$  and  $x_0 = 2E_0/m_{\gamma'}$  with  $E_0$  being the energy of  $\gamma$  at the rest frame of  $\gamma'$ .  $\alpha_{\text{EM}}$  denotes the fine structure constant of SM QED.

Next step is to translate the spectrum in the rest frame of  $\gamma'$  to that in the rest frame of

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<sup>11</sup> In the appendix A, we compare the direct calculation of the FSR with the Altarelli-Parisi approximation formula, and confirm the validity of the approximation in the parameter region we are interested in.

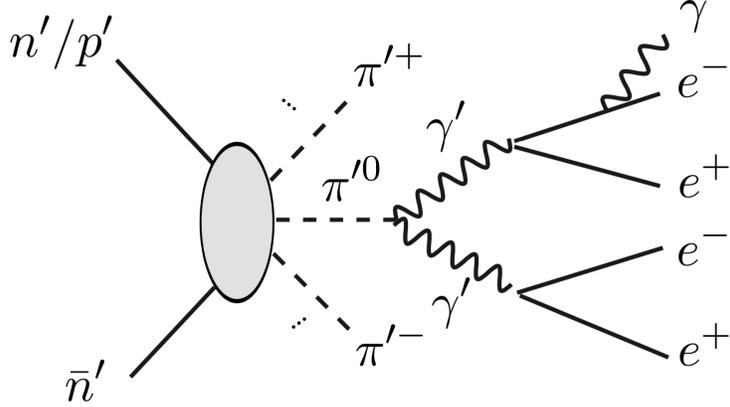


Figure 1. ADM annihilation which happens at late time:  $\bar{n}'$  can be generated from the ADM oscillation. Once the  $\bar{n}'$  is generated, dark nucleons ( $n'/p'$ ) and  $\bar{n}'$  annihilate into dark pions ( $\pi'^{\pm}$  and  $\pi'^0$ ).  $\pi'^0$  subsequently decays into a pair of dark photons ( $\gamma'$ ).  $\gamma'$  eventually decays into  $e^+e^-$ , and emits  $\gamma$  through the FSR process.

$\pi'^0$ . For the case where  $m_{\pi'} \gg m_{\gamma'}$ , the spectrum is calculated as

$$\frac{d\tilde{N}_{\gamma}}{dx_1} = 2 \int_{x_1}^1 \frac{dx_0}{x_0} \frac{d\tilde{N}_{\gamma}}{dx_0} f\left(\frac{2x_1}{x_0} - 1\right) + \mathcal{O}\left(\frac{m_{\gamma'}^2}{m_{\pi'}^2}\right), \quad (29)$$

where  $x_1 = 2E_1/m_{\pi'}$  with  $E_1$  being the energy of  $\gamma$  at the rest frame of  $\pi'$ . The function  $f$  represents the effect of the anisotropy of the  $\gamma'$  decay. According to [55, 57], we take

$$f(\cos\theta) = \frac{3}{8}(1 + \cos^2\theta), \quad (30)$$

with  $\theta$  being the angle between the  $\gamma$  emission line and the boost axis of  $\gamma'$ . Note that the angle  $\theta$  is kinematically constrained as

$$\cos\theta = \frac{2x_1}{x_0} - 1 + \mathcal{O}\left(\frac{m_{\gamma'}^2}{m_{\pi'}^2}\right). \quad (31)$$

This is the reason why we put  $f(2x_1/x_0 - 1)$  in Eq. (29).

We next translate the spectrum Eq. (29) to that in the center of mass (CM) frame for the ADM annihilation. In order to do that, we need to know how much  $\pi'^0$  is boosted. If the

total number of the dark pions is two ( $m + 2l = 2$ ), we can exactly know the energy/boost of the dark pions since they should be emitted back to back in the CM frame. In this case, the  $\gamma$  spectrum is calculated as

$$\frac{d\tilde{N}_\gamma^{(m,l)}}{dx_2} = 2 \int_{x_2}^1 \frac{dx_1}{x_1} \frac{d\tilde{N}_\gamma}{dx_1} + \mathcal{O}\left(\frac{m_{\pi'}^2}{m_{\text{DM}}^2}\right), \quad \text{for } m + 2l = 2, \quad (32)$$

where  $x_2 = E_2/m_{\text{DM}}$  with  $E_2$  being the energy of  $\gamma$  at the CM frame.

On the other hand, in the case of  $m + 2l \geq 3$ , it becomes highly non-trivial to know how much the  $\pi'^0$  can be boosted even when we assume that the matrix element of the annihilation is constant as a function of the final state momenta. This is because, in this case, the energy spectrum of the dark pion is given as

$$\frac{dN_{\pi'}}{d\xi} = \frac{1}{R_n} \frac{dR_n}{d\xi}, \quad (33)$$

where  $\xi = E_{\pi'}/m_{\text{DM}}$  and  $R_n$  is the  $n = m + 2l$  body phase space integration [58].  $E_{\pi'}$  denotes the energy of the dark pion in the CM frame. In general, it is difficult to perform the phase space integration for  $n \geq 3$ . However, as discussed in [56, 58], under the assumption that  $m_{\pi'^0} = m_{\pi'+} \equiv m_{\pi'} \ll m_{\text{DM}}$ , we can perform the phase space integrations analytically as

$$\frac{dN_{\pi'}}{d\xi} = (n-1)(n-2)(1-\xi)^{n-3}\xi + \mathcal{O}\left(\frac{m_{\pi'}^2}{m_{\text{DM}}^2}\right), \quad (34)$$

for  $n = m + 2l \geq 3$ . Using the results, we finally obtain

$$\frac{d\tilde{N}_\gamma^{(m,l)}}{dx_2} = 2(n-1)(n-2) \int_{x_2}^1 (1-\xi)^{n-3} \int_{x_2/\xi}^1 \frac{dx_1}{x_1} \frac{d\tilde{N}_\gamma}{dx_1} + \mathcal{O}\left(\frac{m_{\pi'}^2}{m_{\text{DM}}^2}\right), \quad \text{for } n = m + 2l \geq 3, \quad (35)$$

where we assume  $m_{\pi'^0} = m_{\pi'+} \equiv m_{\pi'}$ .

Finally, we sum over the possible intermediate states and take into account the number of the final states. It turns out that the total  $\gamma$  spectrum from the  $n'\bar{n}'$  annihilation is

expressed as

$$\frac{dN_\gamma^{(n'\bar{n}')}}{dx_2} = \sum_{m,l} 2m \left( \text{Br}^{(n'\bar{n}')} (m, l) \frac{d\tilde{N}_\gamma^{(m,l)}}{dx_2} \right), \quad (36)$$

where  $\text{Br}^{(n'\bar{n}')} (m, l)$  denotes the branching ratio for the  $n'\bar{n}' \rightarrow m\pi^0 + l\pi^{'+} + l\pi'^{-}$  annihilation process. The factor  $2m$  corresponds to the number of  $e^+e^-$  pairs in the annihilation process.

In the same way, we can estimate the  $\gamma$  spectrum from the  $p'\bar{n}'$  annihilation processes:

$$p'\bar{n}' \rightarrow m\pi^0 + l\pi^{'+} + (l-1)\pi'^{-}, \quad (m = 0, 1, 2, \dots, l = 1, 2, \dots). \quad (37)$$

The  $\gamma$  spectrum is calculated as

$$\frac{dN_\gamma^{(p'\bar{n}')}}{dx_2} = \sum_{m,l} 2m \left( \text{Br}^{(p'\bar{n}')} (m, l) \frac{d\tilde{N}_\gamma^{(m,l)}}{dx_2} \right), \quad (38)$$

with replacing  $n = m + 2l$  by  $n = m + 2l - 1$  in the calculation of  $d\tilde{N}_\gamma^{(m,l)}/dx_2$ .

In the following analysis, we simply assume that the branching ratio of the dark nucleon annihilations can be estimated as that of nucleon-antinucleon annihilations. According to [59], we approximate the branching ratios by the fireball model,<sup>12</sup>

$$\text{Br}^{(n'\bar{n}')} (m, l) = \frac{2\alpha^{2l}}{(1+\alpha)^n + (1-\alpha)^n} {}_n C_{2l} P_n, \quad \text{with } n = m + 2l, \quad (39)$$

$$\text{Br}^{(p'\bar{n}')} (m, l) = \frac{2\alpha^{2l-1}}{(1+\alpha)^n + (1-\alpha)^n} {}_n C_{2l-1} P_n, \quad \text{with } n = m + 2l - 1, \quad (40)$$

where

$$P_n = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(n - \langle n \rangle)^2}{2\sigma^2}\right), \quad (41)$$

<sup>12</sup> In this approximation, the Parity violating mode,  $(m, l) = (2, 0)$ , is allowed, although it is not significant numerically.

with  $a = 1/4$ ,  $\langle n \rangle = 5.05$ ,  $\sigma^2 = a\langle n \rangle$  and

$$\alpha = \begin{cases} \sqrt{2} & \text{for } n = 2, \\ 1.5 & \text{for } n \neq 2. \end{cases} \quad (42)$$

We are now ready to estimate the  $\gamma$ -ray spectrum emitted from the ADM annihilations. Figure 2 shows the value of the  $\gamma$ -ray spectrum. Here, we take  $m_{DM} = 10 \text{ GeV}$ ,  $m_{\pi'} = 1 \text{ GeV}$  and  $m_{\gamma'} = 40 \text{ MeV}$ . The black solid and the dashed lines correspond to the spectra predicted from the  $n'\bar{n}'$  and  $p'\bar{n}'$  annihilations, respectively. In the analysis, we ignore the contributions from the annihilations with large  $(m, l)$  since the branching ratios of them are much suppressed. We stop taking the sum over  $(m, l)$  if the size of contribution is less than 1% of the total amount.

The figure shows that the ADM annihilations predict the continuous  $\gamma$ -ray spectrum peaked at the energy of  $\mathcal{O}(m_{DM}/10)$ . This is expected as the typical number of the dark pions for an annihilation is five, and the neutral dark pion decays into two pairs of  $e^+e^-$ .

It should be reminded that the  $\gamma$ -ray emission from the ADM annihilations can happen at the present universe since the ADM oscillation effectively happens at the late time scale. The ADM signals can therefore be tested by  $\gamma$ -ray telescope experiments from nearby sources, while evading the constraints from the observations of the cosmic microwave observations (see e.g. [56]).

The  $\gamma$ -ray flux from the dSphs for an energy bin from  $E_{\min}$  to  $E_{\max}$  is calculated as

$$\Phi = \int_{E_{\min}}^{E_{\max}} dE E \int_{\Delta\Omega} \frac{d\Omega}{4\pi} \int_{\text{l.o.s.}} dl \left( n_{n'} n_{\bar{n}'} \langle \sigma v \rangle_{n'\bar{n}'} \frac{dN_{\gamma}^{(n'\bar{n}')}}{dE} + n_{p'} n_{\bar{n}'} \langle \sigma v \rangle_{p'\bar{n}'} \frac{dN_{\gamma}^{(p'\bar{n}')}}{dE} \right), \quad (43)$$

where we perform the integrations over a solid angle,  $\Delta\Omega$ , and the line-of-sight (l.o.s.). Here  $n_i$  and  $\langle \sigma v \rangle_{ij}$  denote the number density of a particle  $i$  at the dSphs and the kinematically averaged cross section for  $ij$  annihilation, respectively.  $N_{\gamma}^{(n'\bar{n}')}$  and  $N_{\gamma}^{(p'\bar{n}')}$  are the photon spectra from  $n'\bar{n}'$  and  $p'\bar{n}'$  annihilations which can be calculated from Eqs. (36) and (38), respectively.

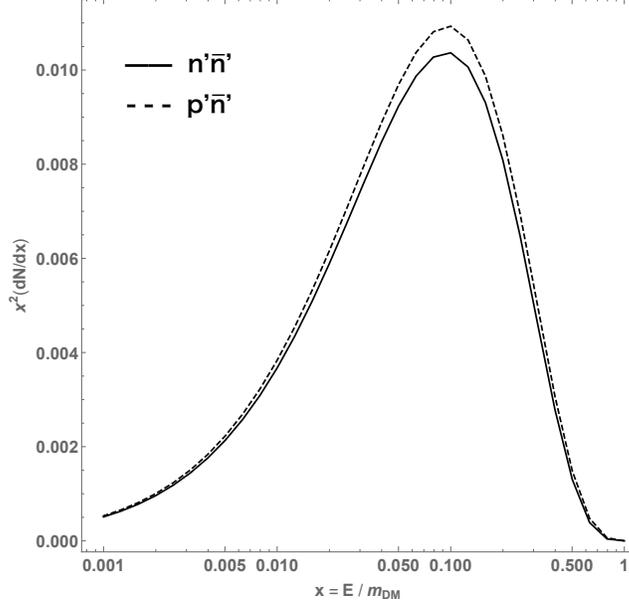


Figure 2. Photon spectrum predicted from the  $n'\bar{n}'$  (Solid line) and  $p'\bar{n}'$  (Dashed line) annihilations. We take  $m_{\pi'} = 1$  GeV and  $m_{\gamma'} = 40$  MeV.

It should be noted that the total amount of the  $\gamma$ -ray flux can be large enough to be tested by the gamma-ray searches on the dSphs although the flux is suppressed by the factor,

$$\begin{aligned} \frac{n_{\bar{n}'}}{n_{n'}} &\simeq \left( \frac{t_0}{t_{\text{osc}}} \right)^2 \\ &\simeq 1.6 \times 10^{-8} \left( \frac{\Lambda'_{\text{QCD}}}{2 \text{ GeV}} \right)^{12} \left( \frac{\tilde{M}_C}{3 \times 10^9 \text{ GeV}} \right)^{-8} \left( \frac{M_R}{10^9 \text{ GeV}} \right)^{-2}. \end{aligned} \quad (44)$$

where  $t_0 \simeq 4.3 \times 10^{17}$  sec is the age of the universe. This is because the thermally-averaged cross section can be large due to the strong interaction. In the following analysis, we take the annihilation cross sections to be

$$\langle \sigma v \rangle_{n'\bar{n}'} = \langle \sigma v \rangle_{p'\bar{n}'} = \frac{4\pi}{m_{\text{DM}}^2}, \quad (45)$$

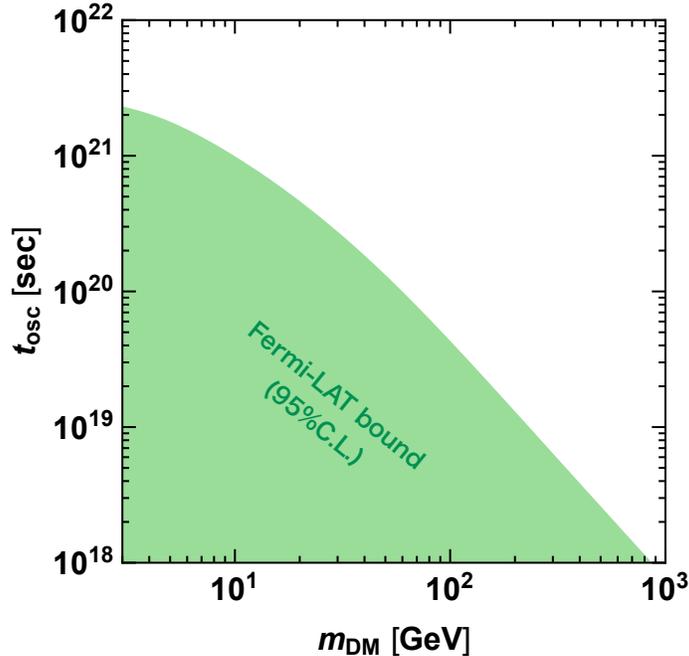


Figure 3. Fermi-LAT constraint on the oscillation time scale: We here assume  $m_{n'} = m_{p'} = m_{\text{DM}}$  and take  $n_{n'} = n_{p'}$ ,  $m_{\pi'} = 1$  GeV and  $m_{\gamma'} = 40$  MeV. Our approximation is valid for  $m_{\pi'}^2/m_{\text{DM}}^2 \ll 1$  and  $m_{\pi'}^2/m_{\text{DM}}^2 \ll 1$ .

to give rough estimation. Such a large annihilation cross section multiplied by the relative velocity is supported by the cross section measurements of the non-relativistic nucleon and anti-nucleon annihilation [60, 61] (see also [62, 63]).<sup>13</sup>

In the figure 3, we show the constraint on the oscillation time scale. In our analysis, we use the bin-by-bin likelihoods for the  $\gamma$ -ray flux based on 6 years of Pass 8 data by the Fermi-LAT collaboration [64]. To obtain the predicted  $\gamma$ -ray spectrum, we use the  $J$ -factors estimated in [65] which takes into account the effects of the non-sphericity of the dSphs.<sup>14</sup> For a conservative estimate, we only consider the 8-classical dSphs given in [64]. The green region corresponds to the 95% C.L. excluded region obtained by the above procedure (see also [56, 67]). Here, we assume  $m_{n'} = m_{p'} = m_{\text{DM}}$  and fix  $m_{\pi'} = 1$  GeV and  $m_{\gamma'} = 40$  MeV.

<sup>13</sup> The cross section multiplied by the relative velocity in Eq. (45) is much smaller than the unitarity limit.

<sup>14</sup> As for the  $J$ -factor of the Ursa Minor classical dSphs, we use the value given in [66] as it is not analyzed in [65].

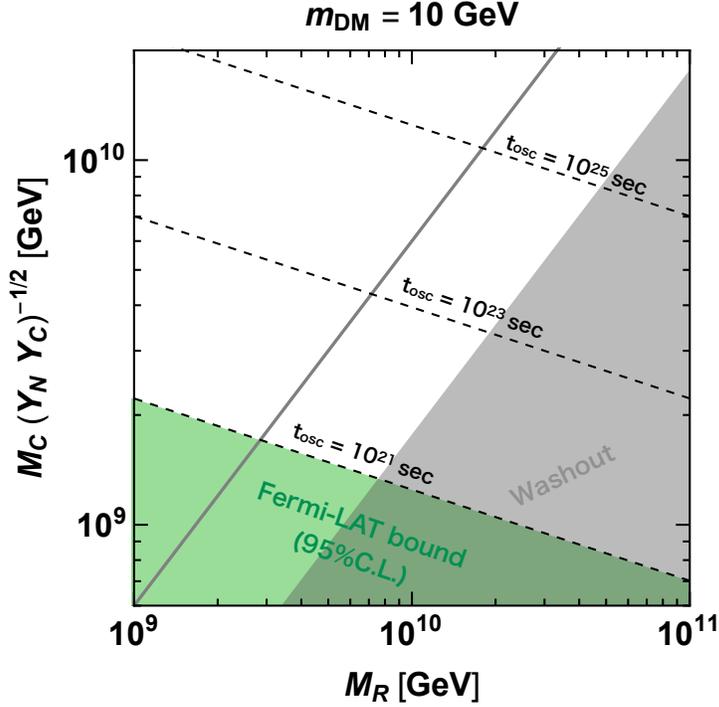


Figure 4. Fermi-LAT constraint on our ADM scenario: We here take  $m_{\pi'} = 1$  GeV,  $m_{\gamma'} = 40$  MeV,  $m_{DM} = 10$  GeV,  $\Lambda'_{QCD} = 2$  GeV. The green region corresponds the 95% C.L. excluded region. In the lower gray region is excluded in which the  $B - L$  asymmetry is washed out (see Eq. (22)). Above the solid line, we require an on-shell  $B - L$  portal sector (see Eq. (25)).

We see that, for  $m_{DM} \simeq 10$  GeV, the oscillation time scale should be longer than  $\sim 10^{21}$  sec to avoid the Fermi-LAT constraint. This lower limit corresponds to the effective annihilation cross section,<sup>15</sup>

$$\left(\frac{t_0}{t_{osc}}\right)^2 \langle \sigma v \rangle \sim 10 \text{ pb} \left(\frac{10 \text{ GeV}}{m_{DM}}\right)^2 \left(\frac{10^{21} \text{ sec}}{t_{osc}}\right)^2. \quad (46)$$

It should be also noted that the Fermi-LAT lose its sensitivity for  $m_{DM} \lesssim 5$  GeV, as it is sensitive to the  $\gamma$ -ray with energy higher than 500 MeV.

Figure 4 shows the impact of the Fermi-LAT limit on our ADM scenario. In the analysis, we fix  $m_{\pi'} = 1$  GeV,  $m_{\gamma'} = 40$  MeV,  $m_{DM} = 10$  GeV, and  $\Lambda'_{QCD} = 2$  GeV. The kinetically-

<sup>15</sup> The effective cross section into the  $\gamma$ -ray is further suppressed by Eq. (28).

averaged cross sections are taken as Eq. (45). The green region corresponds to the 95% C.L. excluded region as discussed above. In the lower gray region, the  $B - L$  washout interactions become effective (see Eq. (22)). Above the solid line, we require an on-shell  $B - L$  portal sector (see Eq. (25)). We now see that our ADM scenario can be efficiently tested by the  $\gamma$ -ray searches from the dSphs by the Fermi-LAT.

Several comments are in order. In our discussion, we consider only the  $\gamma$ -ray emitted by the FSR. This should be justified as the  $\gamma$ -rays made by the Synchrotron radiation and the inverse Compton scattering from the sub-GeV  $e^+e^-$  are very soft and below the Fermi-LAT sensitivity [68]. It should be also noted that the gamma-ray signal from the galactic center do not lead to more stringent constraints, despite the signal strength is higher than that from the dSphs. This is because the  $\gamma$ -ray background is much higher for the galactic center, and hence, it is difficult to distinguish the continuous signal spectrum from the background spectrum.

Let us also comment on the cosmic electron/positron rays from the late time ADM annihilation. As we have discussed, the primary final states of the ADM annihilation are the soft electrons/positrons in the sub-GeV region. The predicted flux of them are, however, lower than the constraints in [69] for the effective annihilation rate satisfying the Fermi-LAT constraint discussed above.<sup>16</sup> Therefore, we find that the soft electrons/positrons searches are not sensitive to test the present model.

#### IV. CONCLUSIONS

The composite ADM model is particularly motivated as it provides the DM mass of  $\mathcal{O}(1)$  GeV and a large annihilation cross section simultaneously. In this paper, we discussed the indirect detection of the composite ADM where the portal operators of the  $B - L$  asymmetry is generated in association with the seesaw mechanism. In this model, the dark-neutrino obtains a tiny Majorana mass, and hence, ADM can pair-annihilate at later times.

<sup>16</sup> Here, we estimate the electron/positron fluxes from the ADM annihilation by using the approximated approach to solve the electron/positron propagation given in [68]. Then, we confirmed that the predicted interstellar electron/positron fluxes are lower than those estimated from the observed fluxes after removing the solar modulation [69].

As we have discussed, the late time annihilation of ADM results in multiple soft electrons/positrons and soft photons emitted as the FSR. As a result, some parameter region of the composite ADM has been excluded by the  $\gamma$ -ray searches from the dSphs by the Fermi-LAT. The obtained constraint is tighter than that from the anti-neutrino flux made by the decay of ADM via the  $B - L$  portal operator [43] (see Eq. (12)). Future experiments which are sensitive to sub-GeV  $\gamma$ -rays such as SMILE [70], GRAINE [71], and GRAMS [72] projects will be important to test the oscillating ADM model further.

## ACKNOWLEDGEMENTS

MI thanks K. Kohri for useful comments on the possible constraints on the model from the cosmic electron/positron rays. This work is supported by JSPS KAKENHI Grant Numbers, 19K14701 (R. N.), JP17H02878, No. 15H05889 and No. 16H03991 (M. I.) and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

## Appendix A: Final State Radiation In the Dark Photon Decay

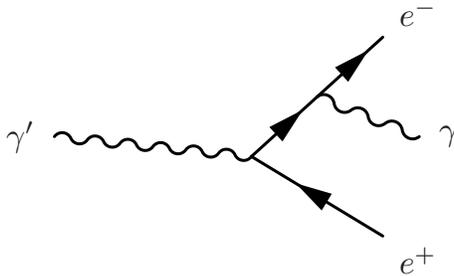


Figure 5. One of the Feynman diagrams of the final state radiation.

This appendix is devoted to the photon energy spectrum of the final state radiation in the dark photon decay,  $\gamma' \rightarrow e^+e^-\gamma$ . One of the diagram is shown in the figure 5.

The invariant amplitude for this process is

$$\mathcal{M} = -4\pi\epsilon\alpha_{\text{EM}}\bar{u}(p_1) \left[ \not{\epsilon}^*(p_3) \frac{\not{p}_1 + \not{p}_3 + m_e}{(p_1 + p_3)^2 - m_e^2} \not{\epsilon}(p_0) + \not{\epsilon}(p_0) \frac{-\not{p}_2 - \not{p}_3 + m_e}{(p_2 + p_3)^2 - m_e^2} \not{\epsilon}^*(p_3) \right] v(p_2), \quad (\text{A1})$$

where  $\epsilon$  represents the strength of kinetic mixing,  $\alpha_{\text{EM}}$  the fine structure constant of QED,  $\epsilon$  the polarization vector,  $m_e$  the electron mass,  $u$  and  $v$  spinors and  $p$  momentum vector. Here the subscripts (0, 1, 2, 3) denote the  $(\gamma', e^-, e^+, \gamma)$ .

Summing over the spins of the final state  $e^-, e^+$  and averaging over the helicity of initial state  $\gamma'$ , we obtain

$$\begin{aligned} \frac{1}{3} \sum_{\text{spin}} |\mathcal{M}|^2 &= \frac{8(4\pi\epsilon\alpha_{\text{EM}})^2}{3} \frac{1}{(m_{13}^2 - m_e^2)^2 (m_{23}^2 - m_e^2)^2} [m_{13}^2 m_{23}^2 \{2m_{12}^4 + 2m_{12}^2 (m_{13}^2 + m_{23}^2) + m_{13}^4 + m_{23}^4\} \\ &\quad - m_e^2 (m_{13}^2 + m_{23}^2) \{2m_{12}^4 + 4m_{12}^2 (m_{13}^2 + m_{23}^2) + 3(m_{13}^2 + m_{23}^2)^2\} \\ &\quad + m_e^4 \{2m_{12}^4 + 10m_{12}^2 (m_{13}^2 + m_{23}^2) + 11(m_{13}^2 + m_{23}^2)^2\} \\ &\quad - 4m_e^6 \{2m_{12}^2 + 3(m_{13}^2 + m_{23}^2)\} + 2m_e^8], \end{aligned} \quad (\text{A2})$$

by using the Mandelstam invariants,  $m_{ij}^2 = (p_i - p_j)^2$ , with the subscripts defined above. There is a relation between the invariants,  $m_{\gamma'}^2 + 2m_e^2 = m_{12}^2 + m_{13}^2 + m_{23}^2$ , with  $m_{\gamma'}$  being the dark photon mass. This expression is symmetric under the exchange between  $m_{13}^2$  and  $m_{23}^2$  as expected.

Now, let us calculate the decay rate with the final state radiation. In the following calculation, we use the center of mass frame in which three out-going particles lie in a same plane. Thus, we can transform the three-body phase space integral into integration over energy of two particles and three angles. By taking account the energy-momentum conservation, the three-body phase space has  $9 - 4 = 5$  d.o.f. After fixing the energy of  $e^-$ , three d.o.f. remain. Two of them are angles  $(\alpha, \beta)$  that specify the direction of  $\vec{p}_3$ . The last one is an angle  $\delta$  which determines the plane of decay around  $\vec{p}_3$ . Thus,  $\Gamma_{\gamma' \rightarrow e^+ e^- \gamma}$  can be

written as

$$\Gamma_{\gamma' \rightarrow e^+ e^- \gamma} = \int \frac{1}{16m_{\gamma'}} \frac{1}{3} \sum_{\text{spin}} |\mathcal{M}|^2 \frac{dE_3 dE_1 d\alpha d(\cos \beta) d\delta}{(2\pi)^5}, \quad (\text{A3})$$

$$= \frac{m_{\gamma'}}{32(2\pi)^3} \int \frac{1}{3} \sum_{\text{spin}} |\mathcal{M}|^2 dx dy, \quad (\text{A4})$$

$$= \frac{m_{\gamma'}}{32(2\pi)^3} \int dx \sum_{n=0}^2 \epsilon_0^{2n} [f_n(x, y_{\max}(x, \epsilon_0)) - f_n(x, y_{\min}(x, \epsilon_0))] . \quad (\text{A5})$$

Here we define  $x = 2E_3/m_{\gamma'}$ ,  $y = 2E_1/m_{\gamma'}$  and  $\epsilon_0 = 2m_e/m_{\gamma'}$ . Each  $f_n(x, y)$  is defined as the integration of the invariant scattering amplitude over  $E_1$ , i.e.,  $y$ . The analytical formula for each  $f_n(x, y)$  is as follows:

$$f_0(x, y) = \frac{8}{3} (4\pi\epsilon\alpha_{\text{EM}})^2 \left[ 2(1-y) - \frac{1+(1-x)^2}{x} \ln \left( \frac{1-y}{1-x-y} \right) \right], \quad (\text{A6})$$

$$f_1(x, y) = \frac{4}{3} (4\pi\epsilon\alpha_{\text{EM}})^2 \left[ \frac{x+2y-2}{(1-y)(1-x-y)} + 2 \ln \left( \frac{1-y}{1-x-y} \right) \right], \quad (\text{A7})$$

$$f_2(x, y) = \frac{2}{3} (4\pi\epsilon\alpha_{\text{EM}})^2 \left[ \frac{x+2y-2}{(1-y)(1-x-y)} + \frac{2}{x} \ln \left( \frac{1-y}{1-x-y} \right) \right]. \quad (\text{A8})$$

Here  $y_{\min}$  and  $y_{\max}$  are the lower and the upper bounds of the integration region of  $y$  corresponding to the Dalitz region. The explicit forms of  $y_{\min}$  and  $y_{\max}$  are

$$y_{\min} = \max \left[ 1 - \frac{x}{2} - \frac{x}{2} \sqrt{1 - \frac{\epsilon_0^2}{1-x}}, \epsilon_0 \right], \quad (\text{A9})$$

$$y_{\max} = \min \left[ 1 - \frac{x}{2} + \frac{x}{2} \sqrt{1 - \frac{\epsilon_0^2}{1-x}}, 1 \right]. \quad (\text{A10})$$

From above, we obtain the energy spectrum of the final state radiation photon. The energy spectrum is expressed as [55]

$$\frac{1}{N_\gamma} \frac{dN_\gamma}{dx} = \frac{1}{\Gamma_{\gamma' \rightarrow e^+ e^-}} \frac{d\Gamma_{\gamma' \rightarrow e^+ e^- \gamma}}{dx}. \quad (\text{A11})$$

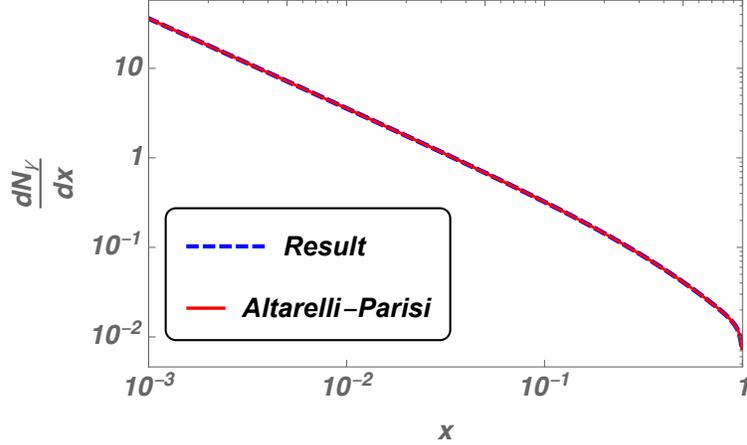


Figure 6. The plot of the analytic formula and the approximation. Here we take  $m_{\gamma'} = 40$  MeV. Two expressions are in good agreement.

Here,  $\Gamma_{\gamma' \rightarrow e^+e^-} = \frac{1}{3}\epsilon^2\alpha_{\text{EM}}m_{\gamma'}$  is the decay rate of the process  $\gamma' \rightarrow e^+e^-$ . We compare the result with twice the Altarelli-Parisi approximation formula [54]

$$\frac{1}{\Gamma_{\gamma' \rightarrow e^+e^-}} \frac{d\Gamma_{\gamma' \rightarrow e^+e^- \gamma}}{dx} = \frac{\alpha_{\text{EM}}}{\pi} \frac{1 + (1-x)^2}{x} \ln \left( \frac{4(1-x)}{\epsilon_0^2} \right), \quad (\text{A12})$$

in the figure 6. We take  $m_{\gamma'} = 40$  MeV. We see that two formulae are in good agreement in a wide range of the photon momentum.

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