Developing “1D+” simulation of core-collapse supernovae

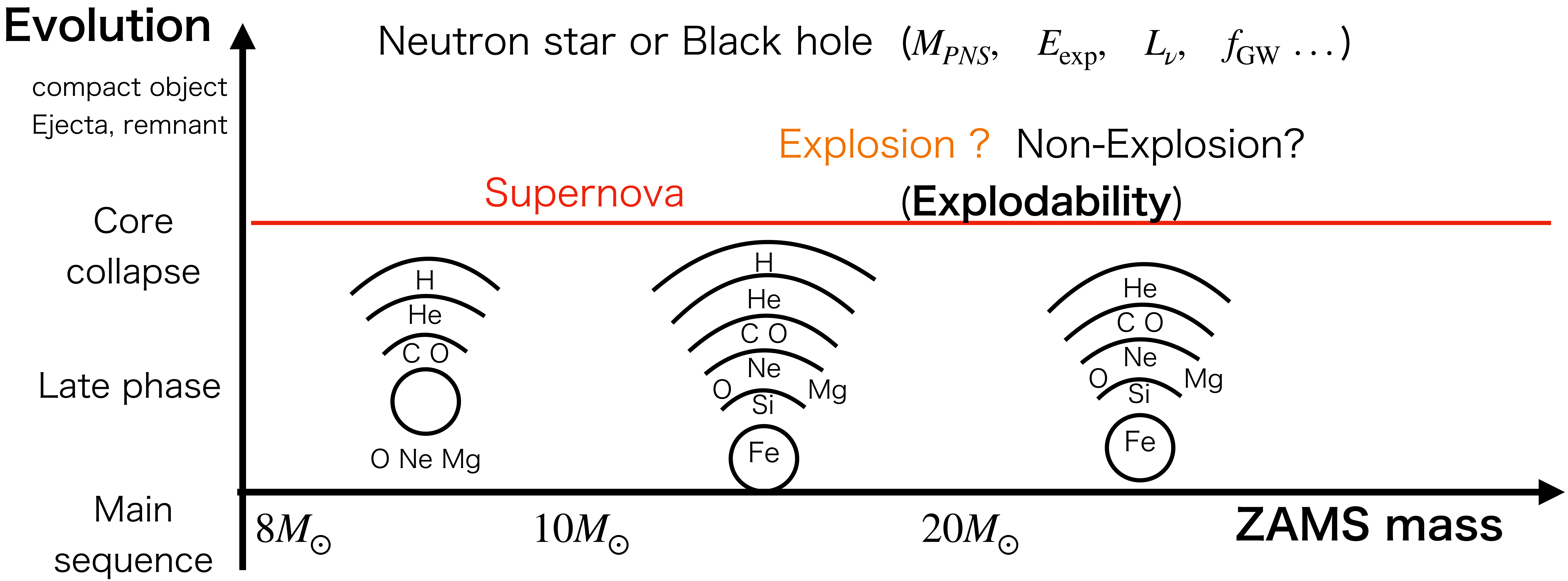
“1D+”シミュレーションの開発
— 超新星爆発の系統的な理解を目指して—

DIVISION OF SCIENCE
National Astronomical Observatory of Japan

NAOJ/SOKENDAI D2 : Shunsuke Sasaki
coauthor: Tomoya Takiwaki

2023/11/02 マルチメッセンジャー天文学の進展@東京大学柏キャンパス

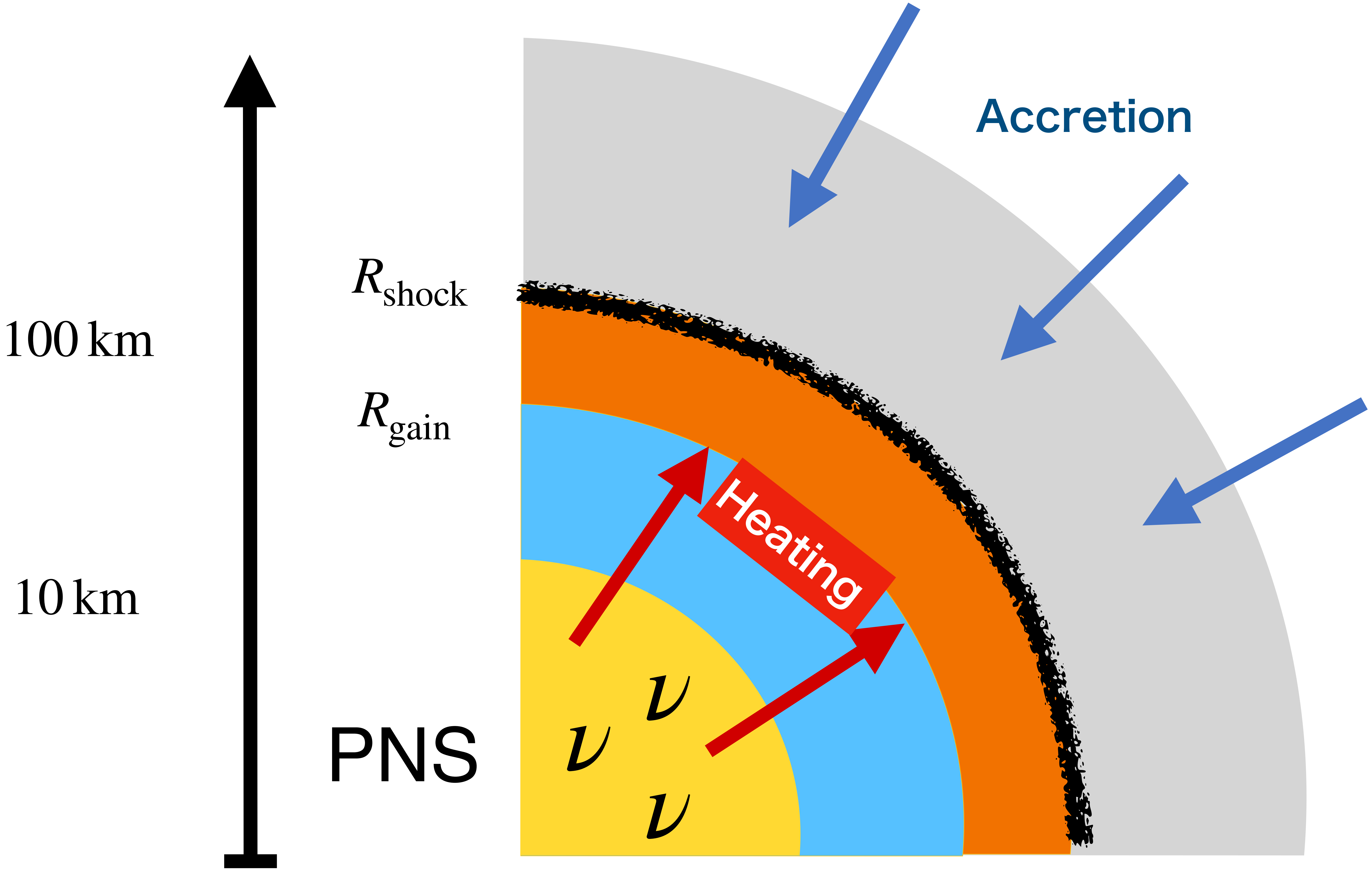
Stellar evolution and Supernovae



Explosion Mechanism

Neutrino driven mechanism

Core of massive star

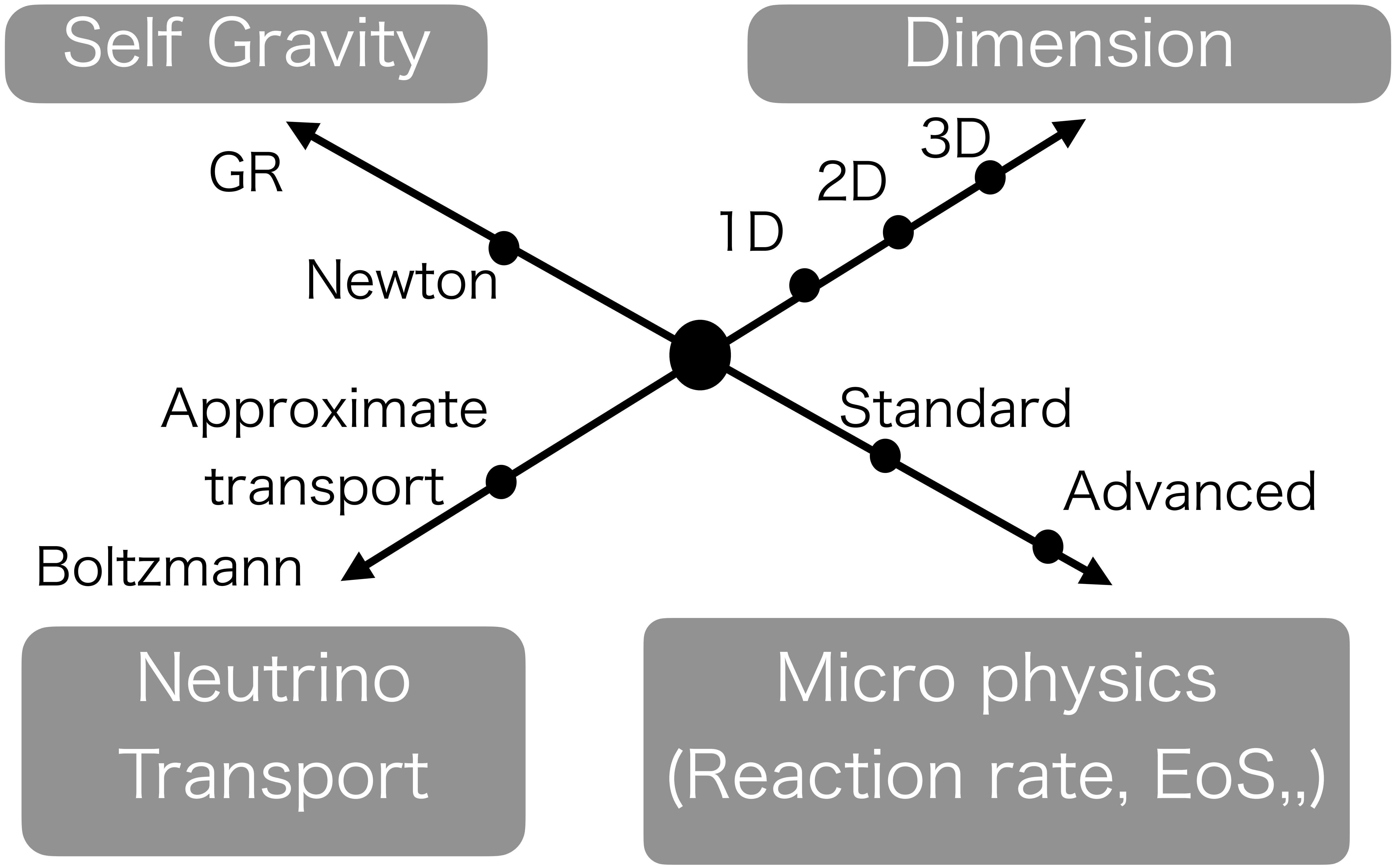


ν : Neutrino

Explosion : Shock propagate

Non-explosion : Shock is stalled

About core collapse supernova simulation



Initial condition

- progenitor model (mass)
- Rotation
- Magnetic field
- Perturbation

Numerical setups

- resolution

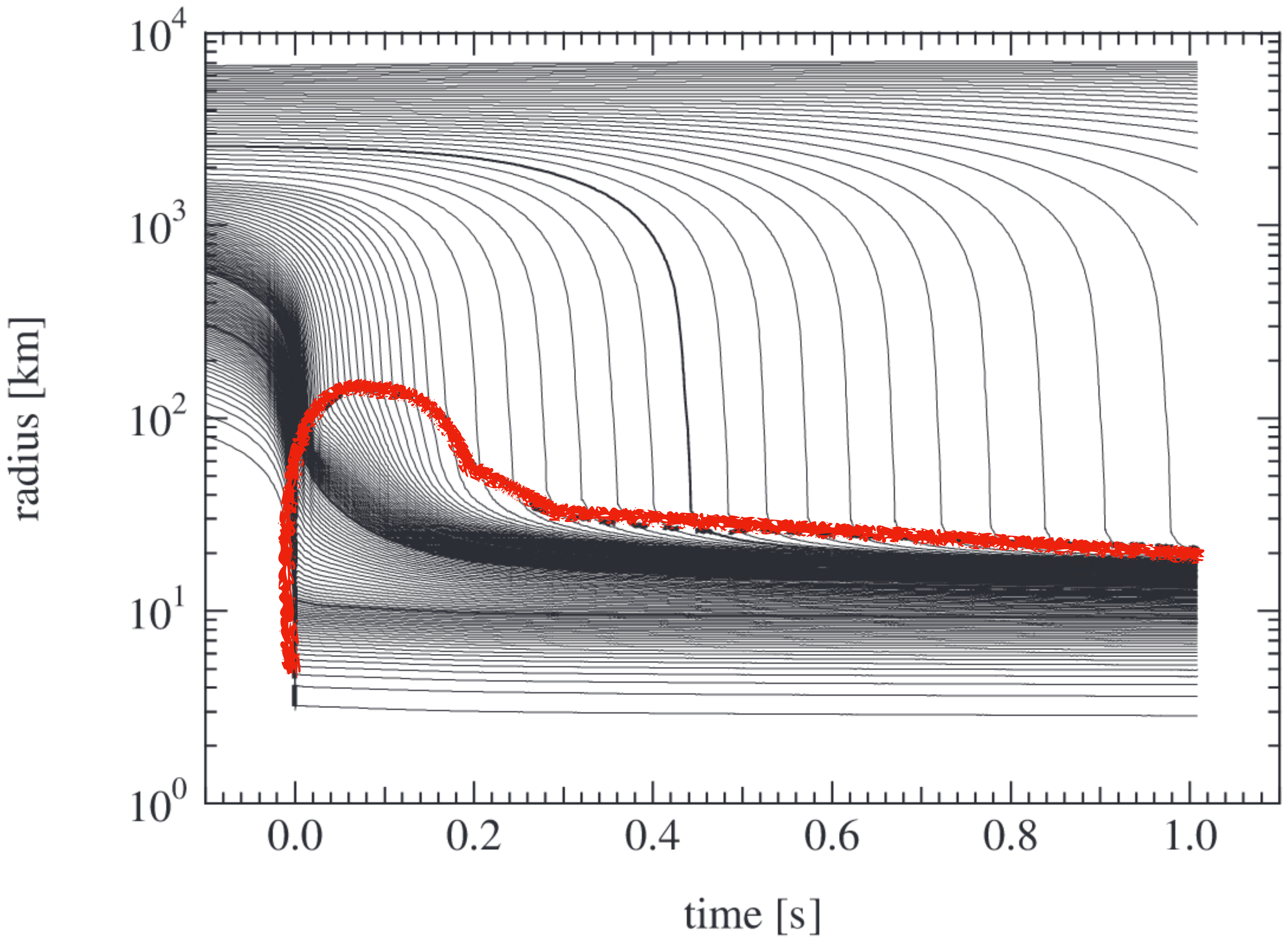
……

Output

$M_{PNS}(t_{\rm cb}),\quad E_{\rm exp}(t_{\rm cb}),\quad L_\nu(t_{\rm cb}),\quad f_{\rm GW}(t_{\rm cb})\ \ldots$

1D simulation

Sumiyoshi et al. 2005

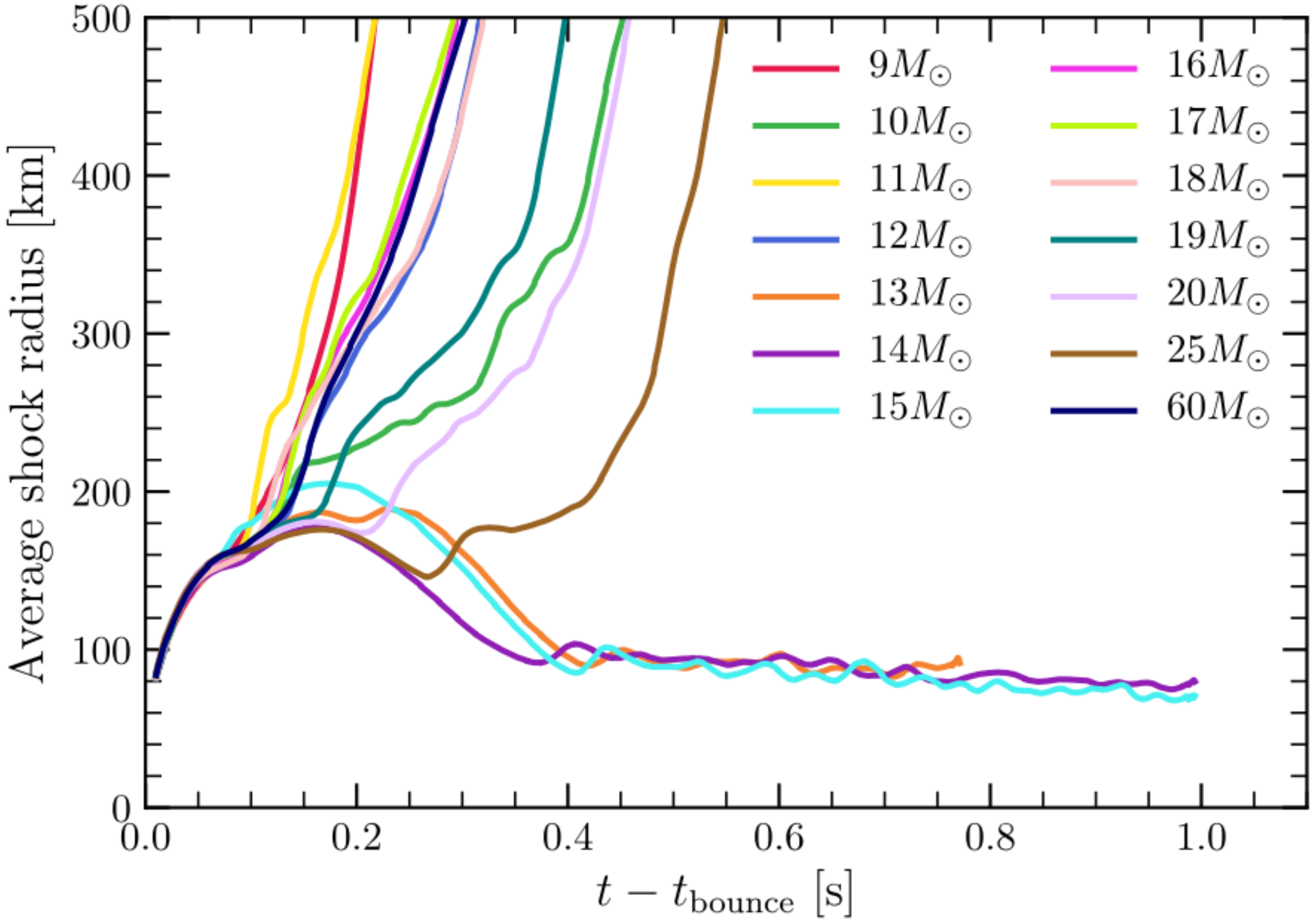


Red line: shock radius

1D simulation CANNOT get successful explosion results

3D simulation

Entropy profile



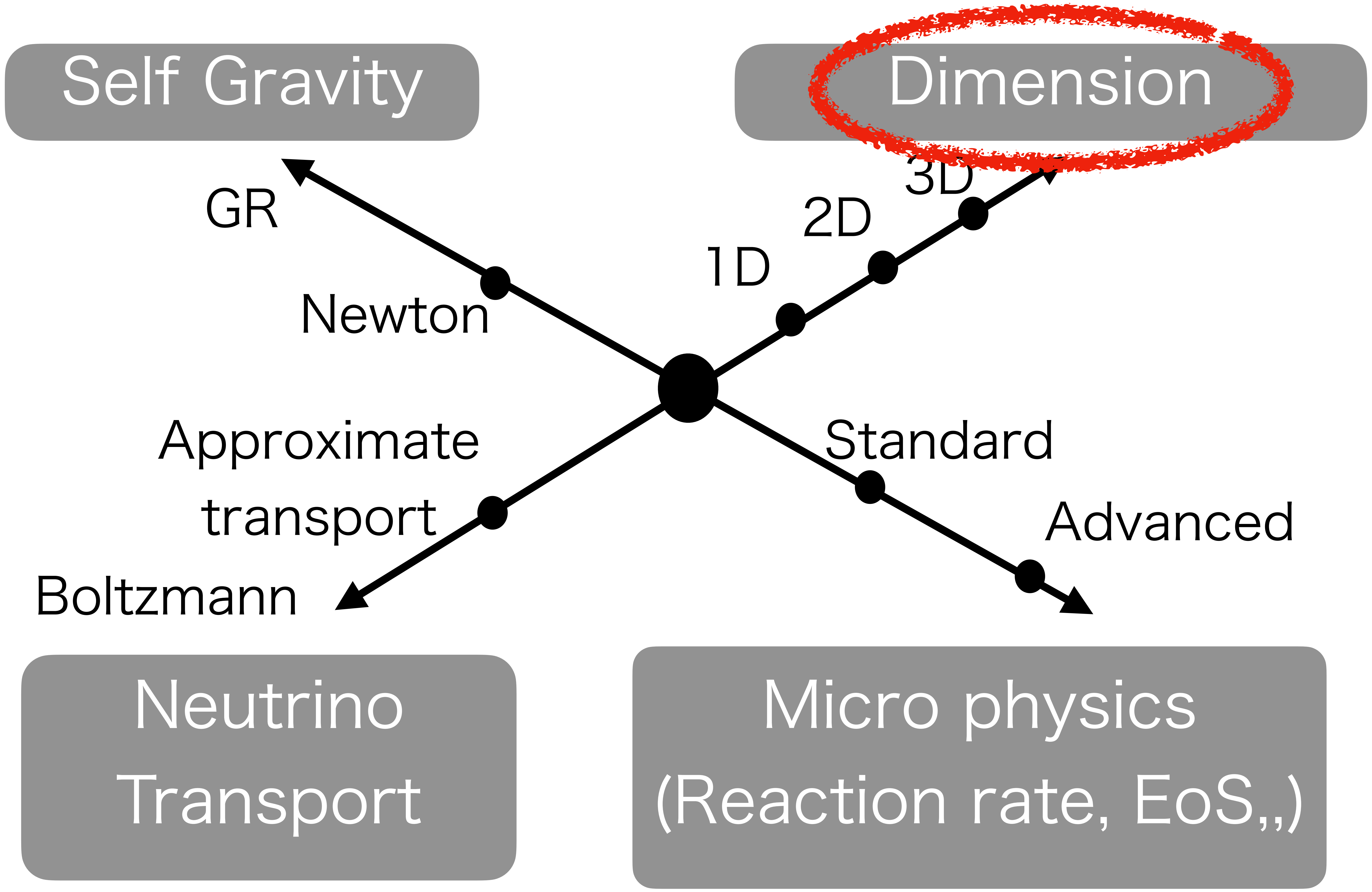
Burrows et al. 2020

3D simulation can get successful explosion results

One of the reasons : multi dimensional turbulent effects

My work

My works focus on turbulence



Initial condition

- progenitor model
- Rotation
- Magnetic field
- Perturbation

Numerical setups

- resolution

……

Output

$$M_{PNS}(t_{\rm cb}), \quad E_{\rm exp}(t_{\rm cb}), \quad L_\nu(t_{\rm cb}), \quad f_{\rm GW}(t_{\rm cb}) \ldots$$

Motivation

Goal: progenitor dependence collaboration with other field

Huge computational cost:
a few month/1model

Low computational cost:
a week/1model

1D+

3D

Including 3D turbulent effects

1st step

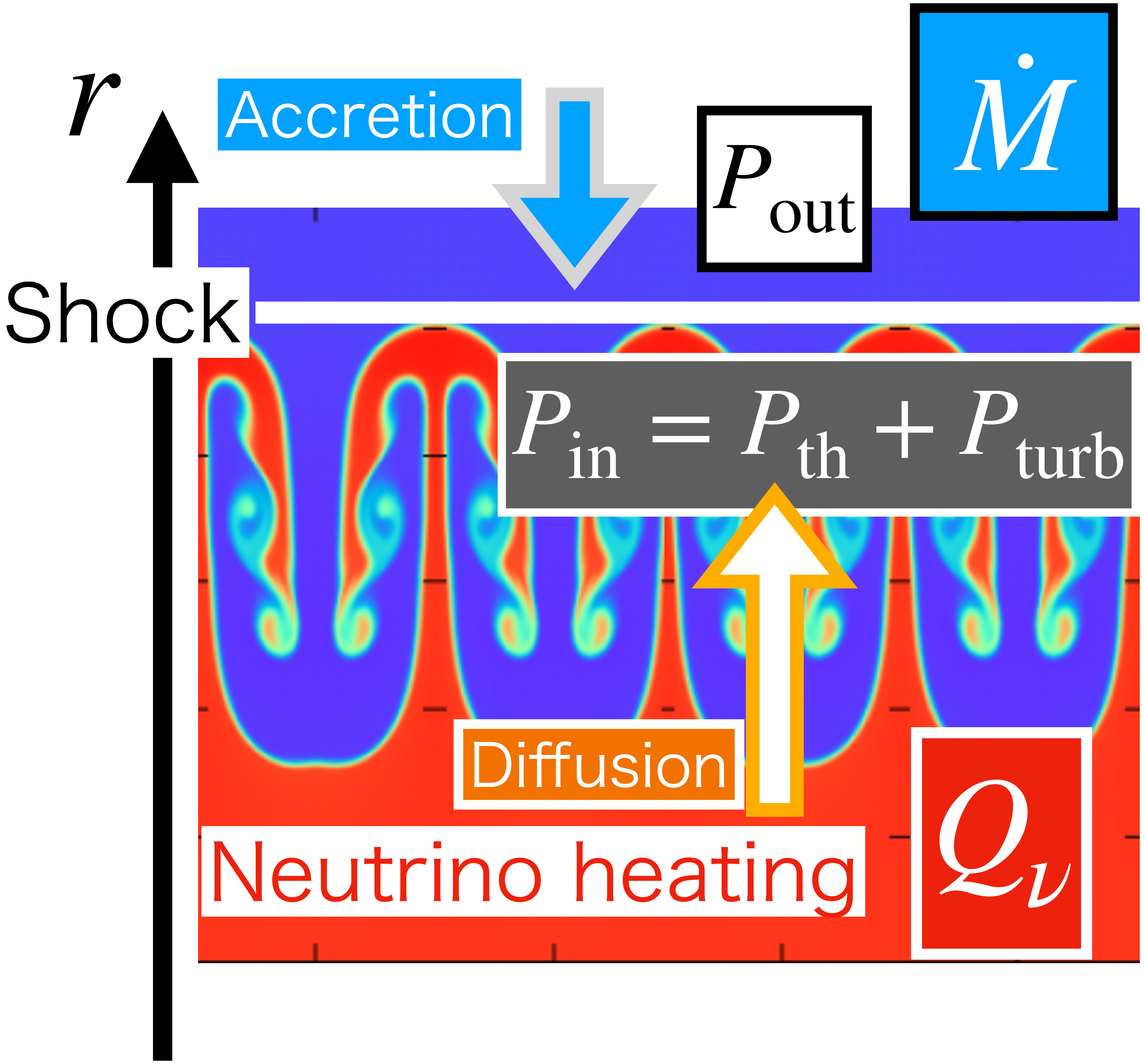
It needs to reproduce the 3D results with 1D+

Turbulent effects

Reynolds decomposition

$$\hat{A} = \langle A \rangle = \frac{1}{4\pi}\int A(r,\theta,\varphi)\, d\Omega$$

$$A' = A - \hat{A} \qquad \langle A' \rangle = 0$$



Turbulence plays an important role in the explosion mechanism.

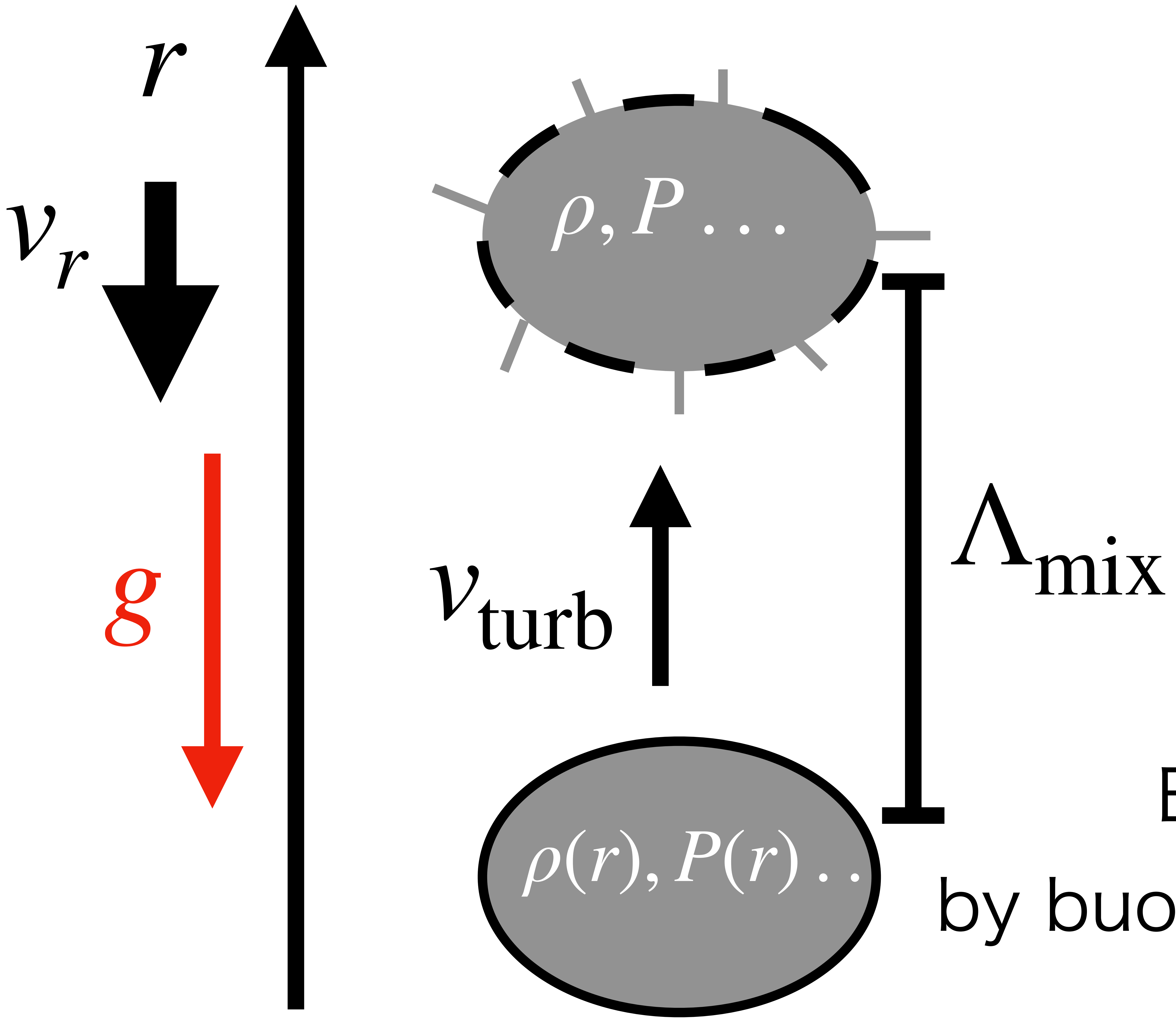
(1) Turbulent Pressure $P_{\mathrm{turb}} = \langle \rho v' v' \rangle$

(2) Diffusion $F_e = \langle e' v' \rangle$, F_K, F_{Y_e}

(3) Dissipation $\dot{e}_{\mathrm{dis}} = \rho v'^3 / L$

We should consider the turbulent effects in CCSN mechanism

Model： Mixing Length Theory



Scale of turbulence : $\Lambda_{\rm mix}$

Turbulent velocity : $v_{\rm turb}$

Element is powered

by buoyancy force : $\omega_{\rm BV}$ [1/sec]

Governing equation (e.g. Müller 2019, Couch et al. 2020)

Turbulent pressure

Diffusion

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}r^2[\rho v_r] = 0$$ Mass conservation

$$\frac{\partial \rho v_r}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2(\rho v_r^2 + P + P_{\rm turb})] = \frac{2\hat{P} + cP_{\rm turb}}{r} - \rho g + S_\nu$$ Euler equation

Source of turbulence

$$\frac{\partial(\rho e)}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 v_r(\rho e + P + P_{\rm turb}) - r^2\rho D_\epsilon\left(\frac{\partial \epsilon}{\partial r} + P\frac{\partial}{\partial r}\left(\frac{1}{\rho}\right)\right) - r^2\rho D_K \nabla v_{\rm turb}^2] = -\rho v_r g + \rho v_{\rm turb}\omega_{\rm BV}^2\Lambda_{\rm mix} + Q_\nu$$ Energy conservation

$$\partial_t e_{\rm turb} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[e_{\rm turb}v_r - r^2\rho D_K\nabla v_{\rm turb}^2\right] = \rho v_{\rm turb}\omega_{\rm BV}^2\Lambda_{\rm mix} - \rho\frac{v_{\rm turb}^3}{\Lambda_{\rm mix}}$$ Turbulent energy conservation

Dissipation

Mixing length parameter

$$\Lambda_{\rm mix} = \alpha_\Lambda H_P = \alpha_\Lambda \frac{P}{\rho g}$$ mixing length

Diffusion parameters

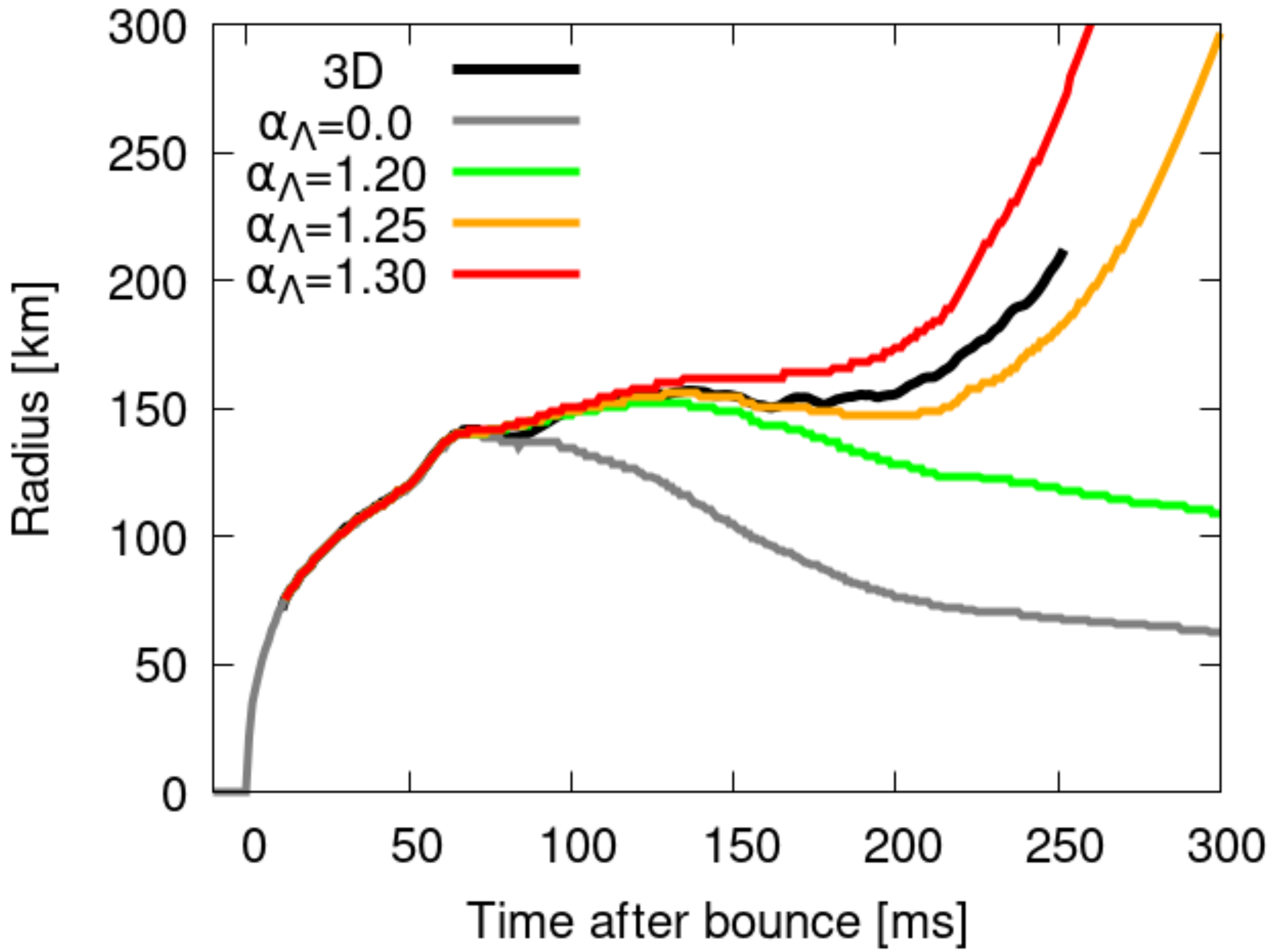
$$D_u = \alpha_u v_{\rm turb}\Lambda_{\rm mix} \quad (u = \epsilon, Y_e, K)$$

Results

Evolution of Shock : parameter survey

Mixing length parameters

$$\Lambda_{\rm mix} = \alpha_\Lambda H_P = \alpha_\Lambda \frac{P}{\rho g}$$ mixing length



We need to set turbulent parameters to mimic 3D simulation.

Progenitor: $12M_\odot$ (Woosley & Heger 2007)

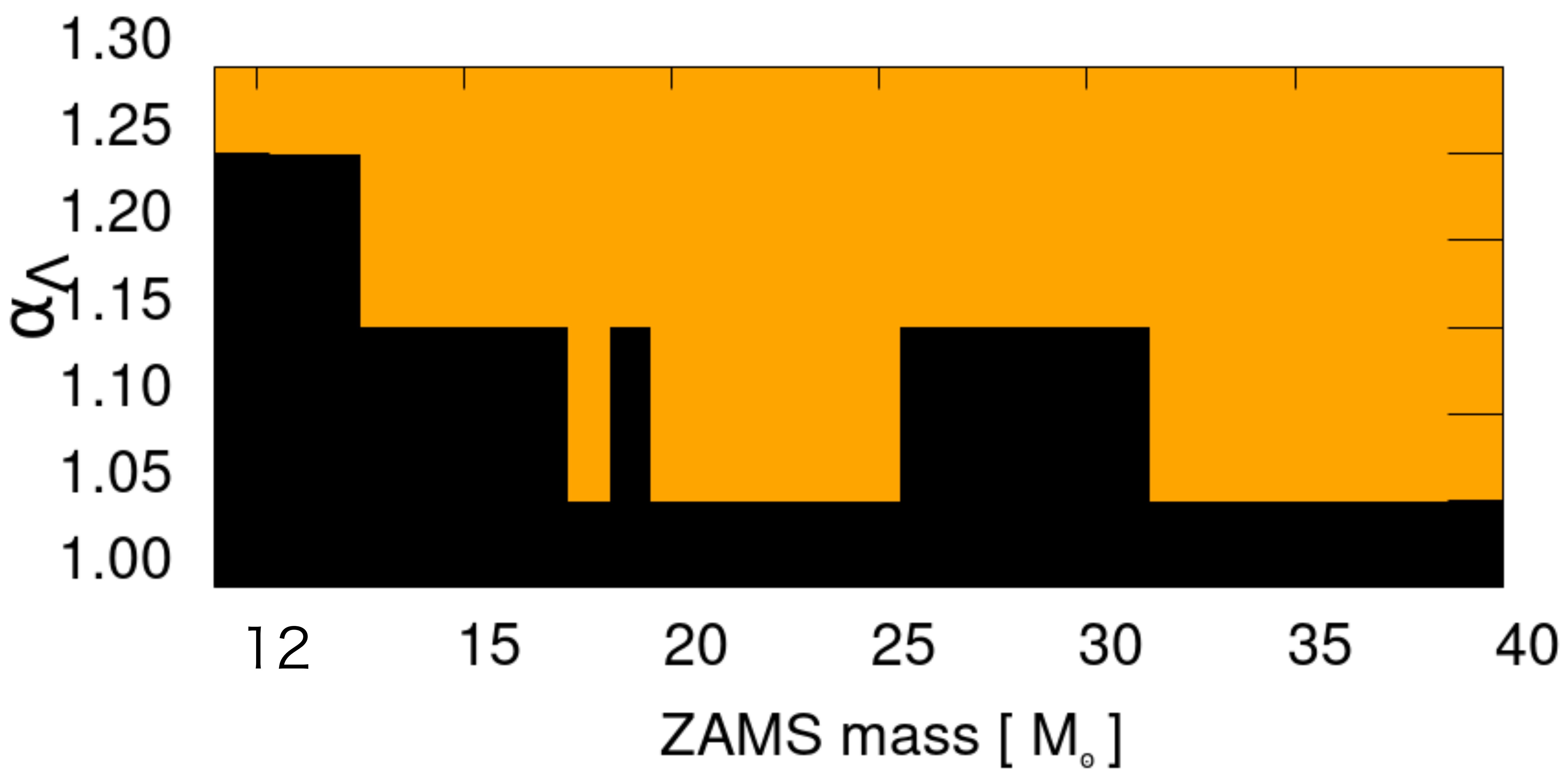
At a later phase, our model with $\alpha_\Lambda = 1.25$ can mimic the evolution of shock . (submitted)

For multi-messenger

Explodability

α_Λ Mixing length parameters

$$\Lambda_{\rm mix} = \alpha_\Lambda H_P = \alpha_\Lambda \frac{P}{\rho g}$$ mixing length



Explosion , non-explosion

Progenitor : $12,13,14,15,,,,40\, M_\odot$

(Woosly&Hegar 2007)

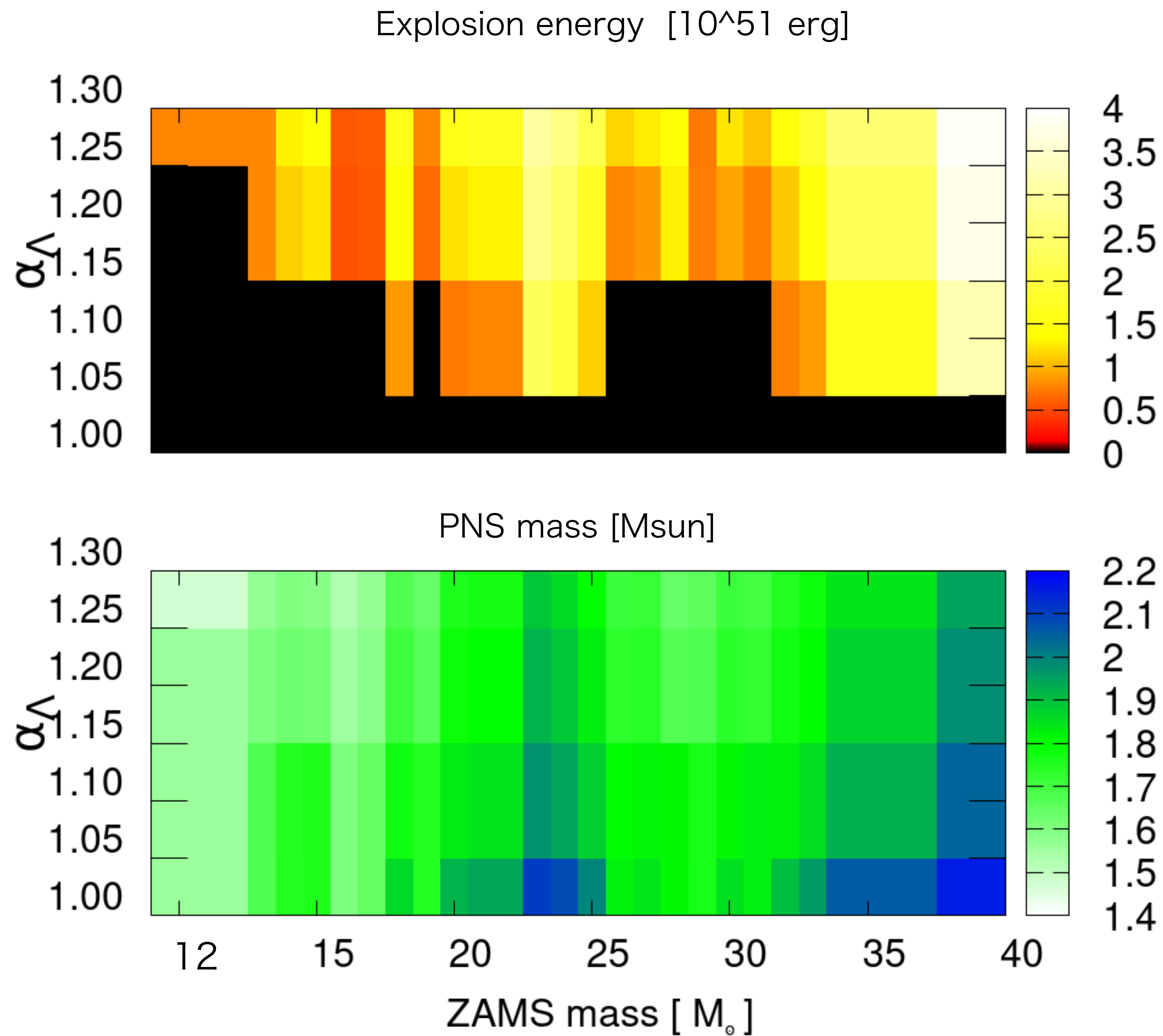
$\alpha_\Lambda = 1.0\,, 1.1\,, 1.2\,, 1.3$

total 4x23= 92 models

Best fit parameter of $12M_\odot$ is

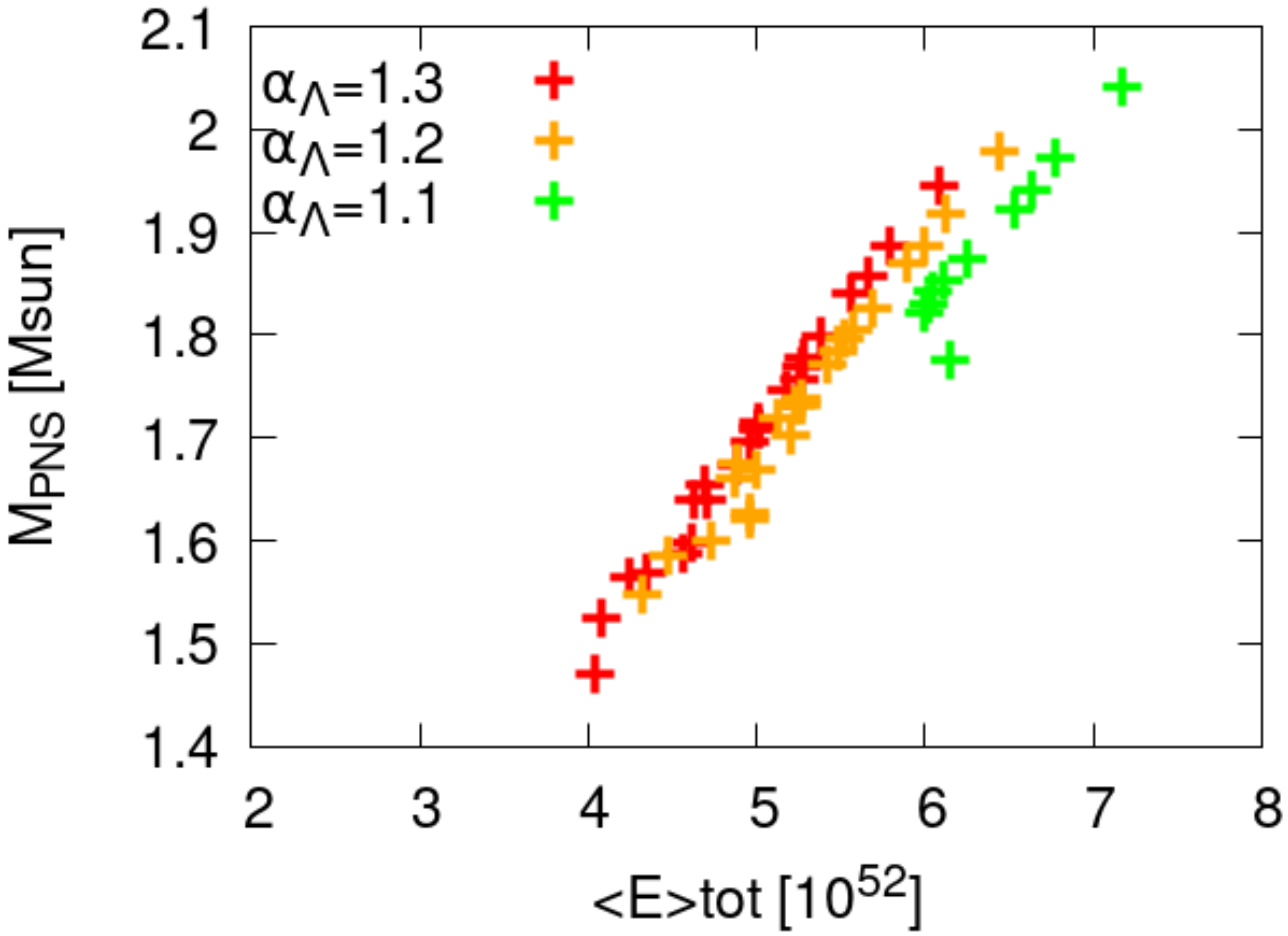
$\alpha_\Lambda = 1.25$ -> Most models are

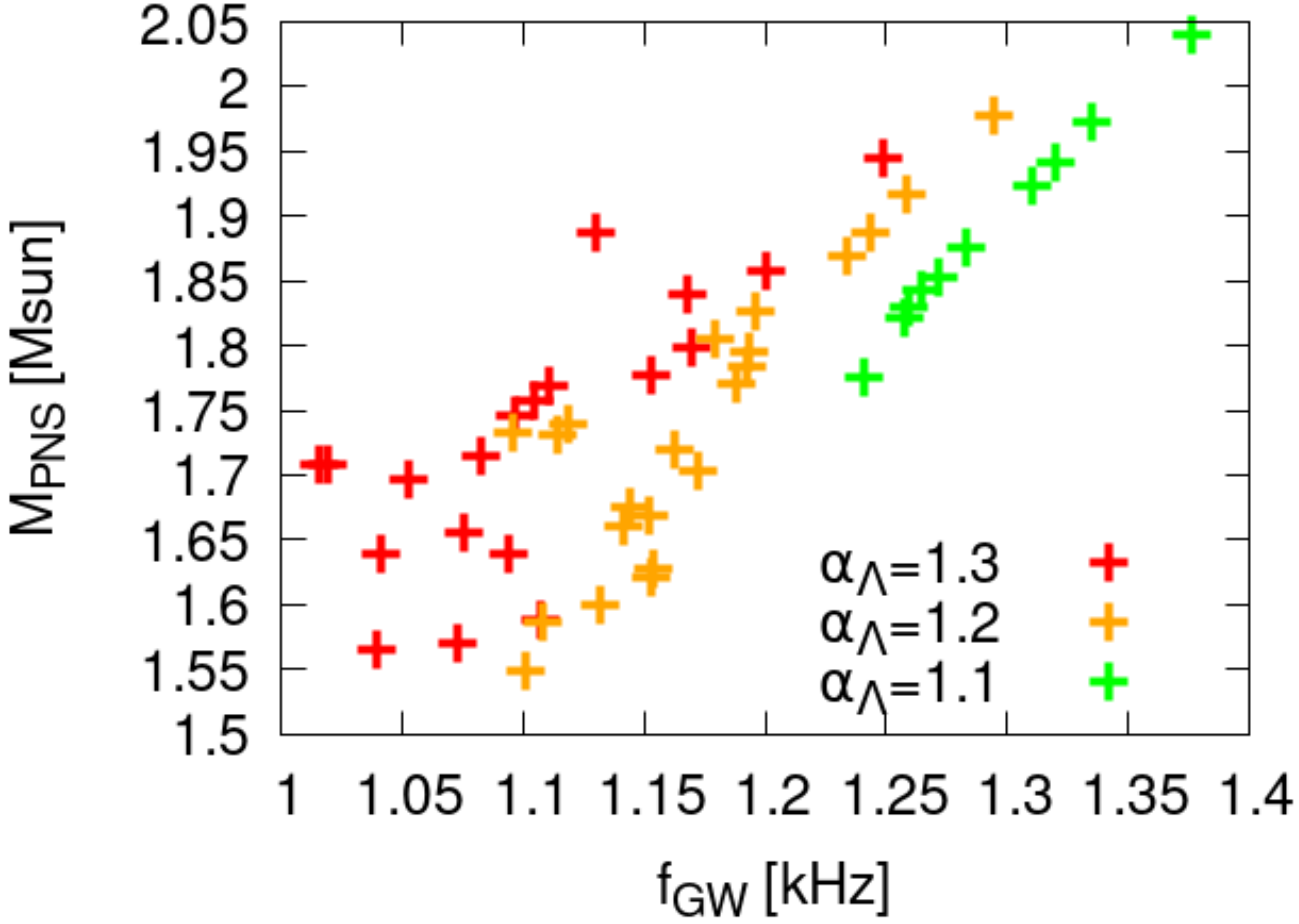
Explosion model…



Explosion energy [10^51 erg]
1.30
1.25
1.20
1.15
1.10
1.05
1.00
α_Λ
4
3.5
3
2.5
2
1.5
1
0.5
0
PNS mass [Msun]
1.30
1.25
1.20
1.15
1.10
1.05
1.00
α_Λ
2.2
2.1
2
1.9
1.8
1.7
1.6
1.5
1.4
12
15
20
25
30
35
40
ZAMS mass [$M_\odot$]

Correlation





Summary
Our 1D+

The next step in the theoretical study of supernova explosions is the development of 1D+ simulations that can reproduce 3D simulations. 1D+ is expected to be an important tool to facilitate collaborative research with other fields.

Results ：

Our 1D+ can mimic shock evolution of 3D

Next researches:

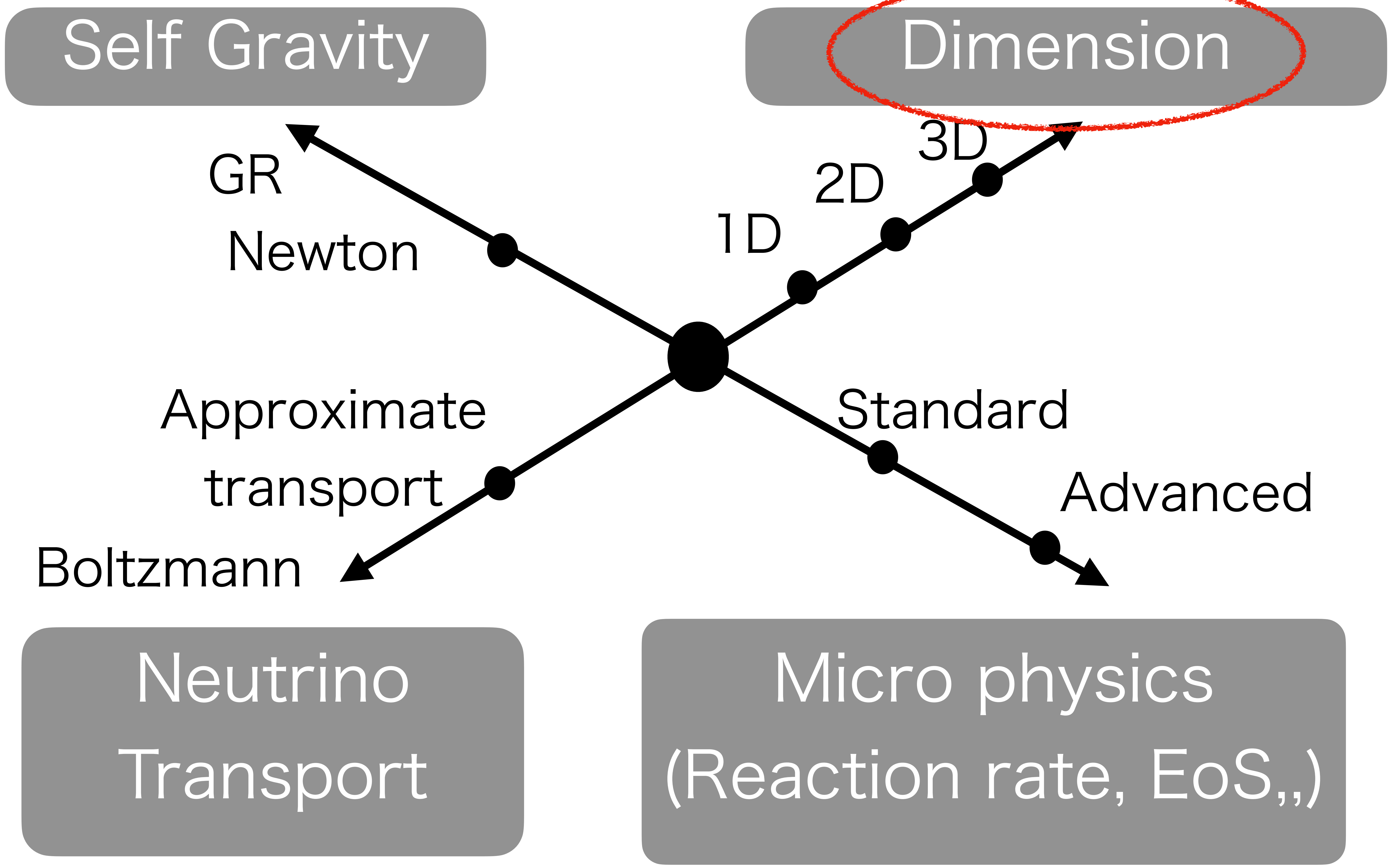
We are researching progenitor dependence and observational value with 1D+

We are trying to develop more realistic 1D+ by 3D analysis

Appendix

Research Plan2

1D+ is useful to calculate a lot of models: progenitor dependence



Initial condition

- progenitor model
- Rotation
- Magnetic field
- Perturbation

Numerical setups

- resolution

……

1D simulation

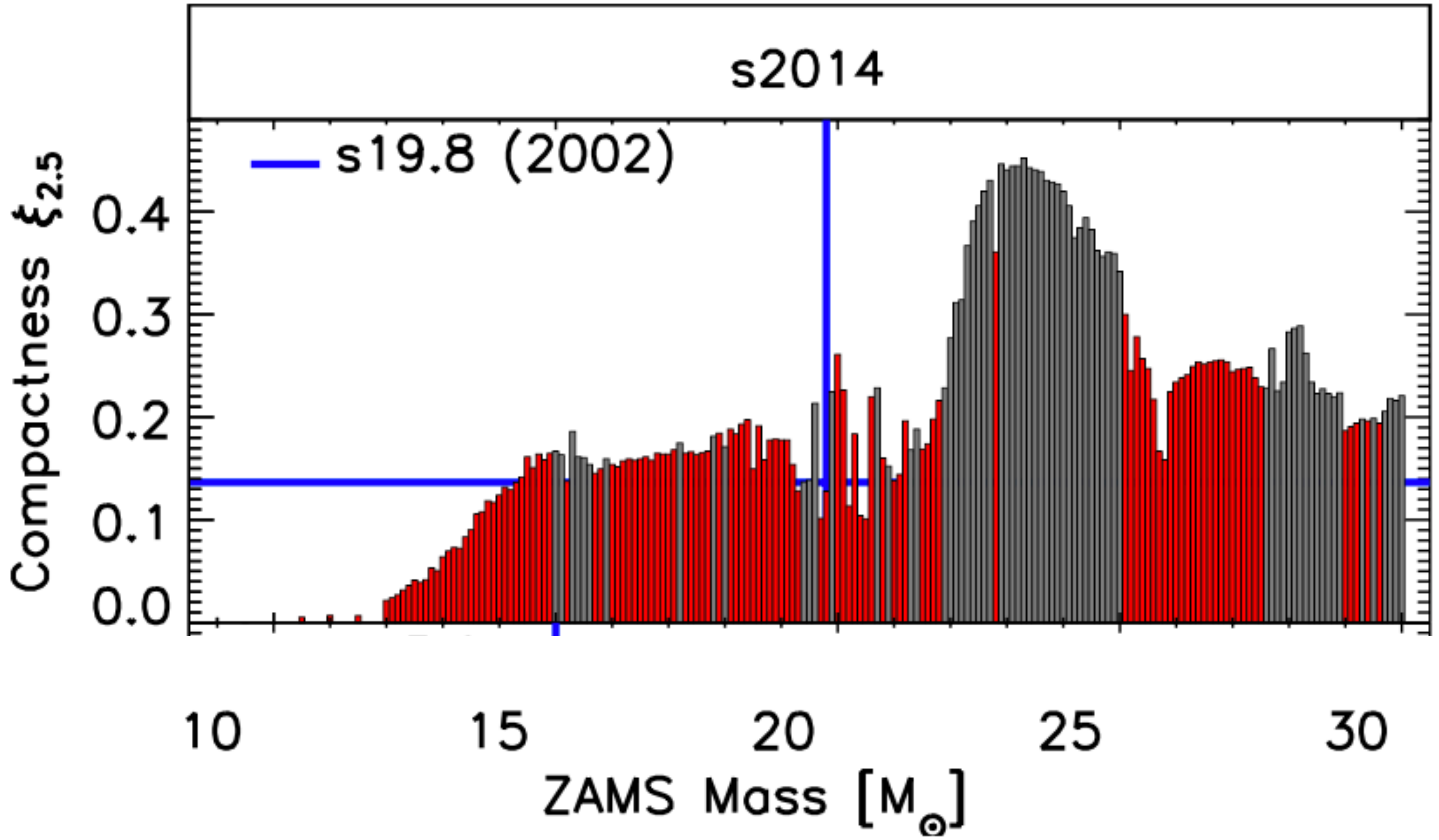
Systematic study

Compactness: $\xi_M = \dfrac{M/M_\odot}{R(M)/1000\mathrm{km}}$

$$M(r) = \int_0^r 4\pi\rho\, r'^2\, dr'$$

$\xi_M > 0.2$: Black hole (black)

$\xi_M < 0.2$: Supernova (Red)



O'Connor & Ott 2011
Ertl et al. 2016

1D : phenomenological simulation

Prediction : property of progenitor like ξ_M governs supernova explosion

-> However, this prediction is NOT consistent with 3D results

PNS mass

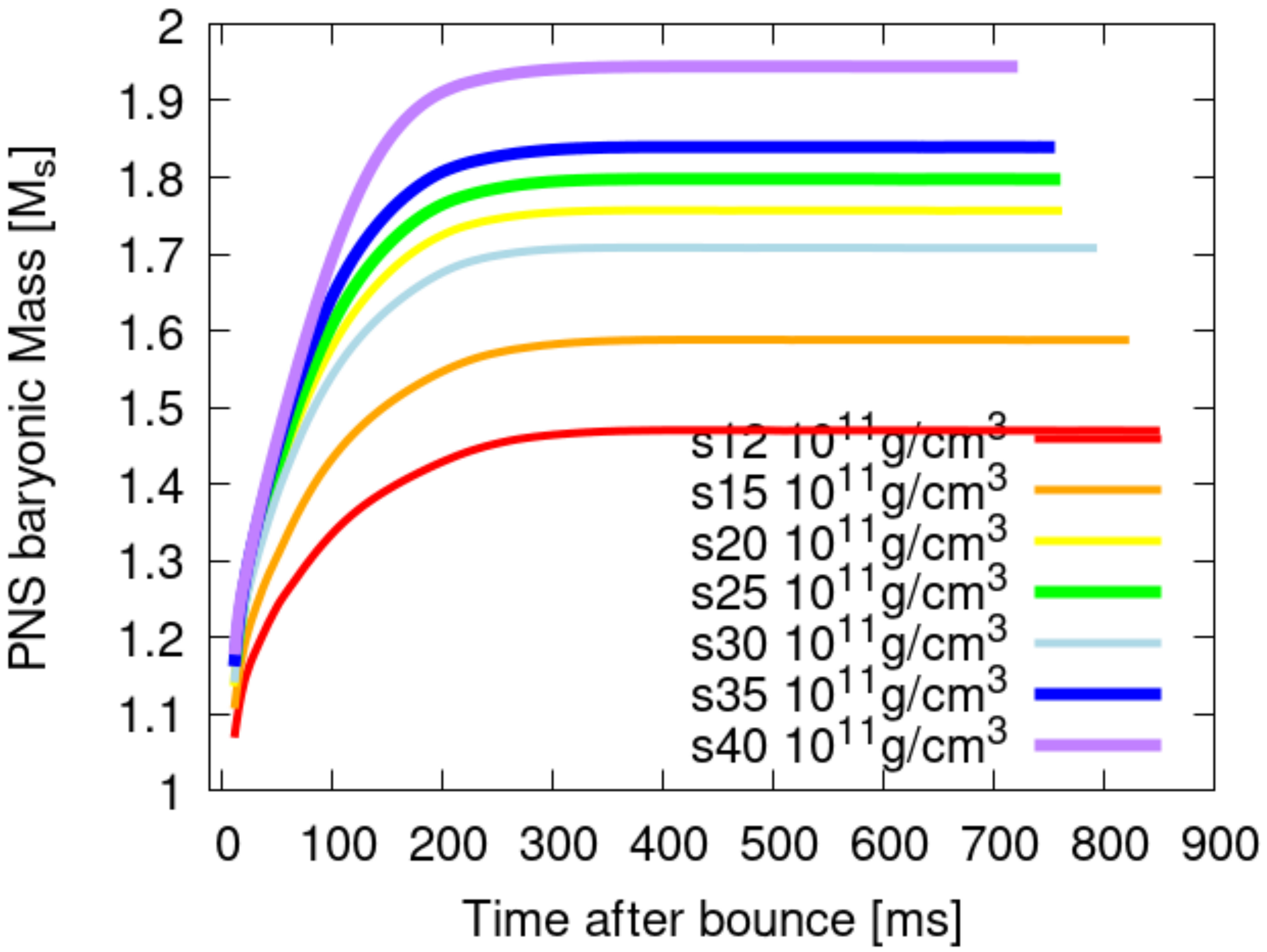


Figure: models with $\alpha_\Lambda = 1.3$

Explosion: s12 s15 s20 s25 s30 s35 s40

Non-Explosion: nothing

Density at PNS radius : 10^{11} [g/cm^3]

Estimating GW

Eq. 3 Sotani et al. 2021

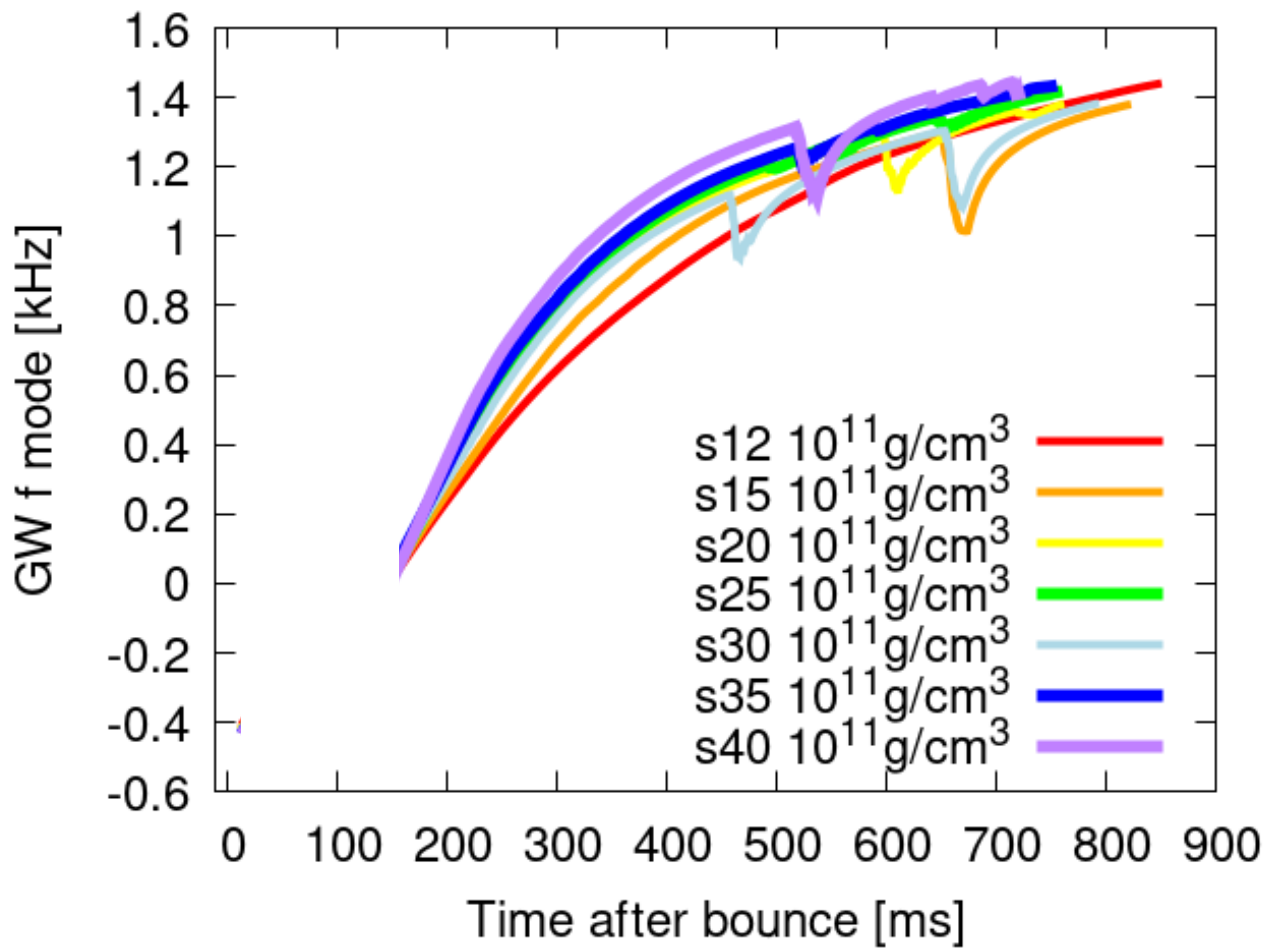


Figure: models with $\alpha_\Lambda = 1.3$

Explosion: s12 s15 s20 s25 s30 s35 s40

Non-Explosion: nothing

$$f[\mathrm{kHz}] = -1.410 - 0.443\ln(x) + 9.337x - 6.714x^2$$

$$x \equiv \left(\frac{M_{\mathrm{PNS}}}{1.4M_\odot}\right)^{1/2}\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-3/2}$$

Neutrino luminosity

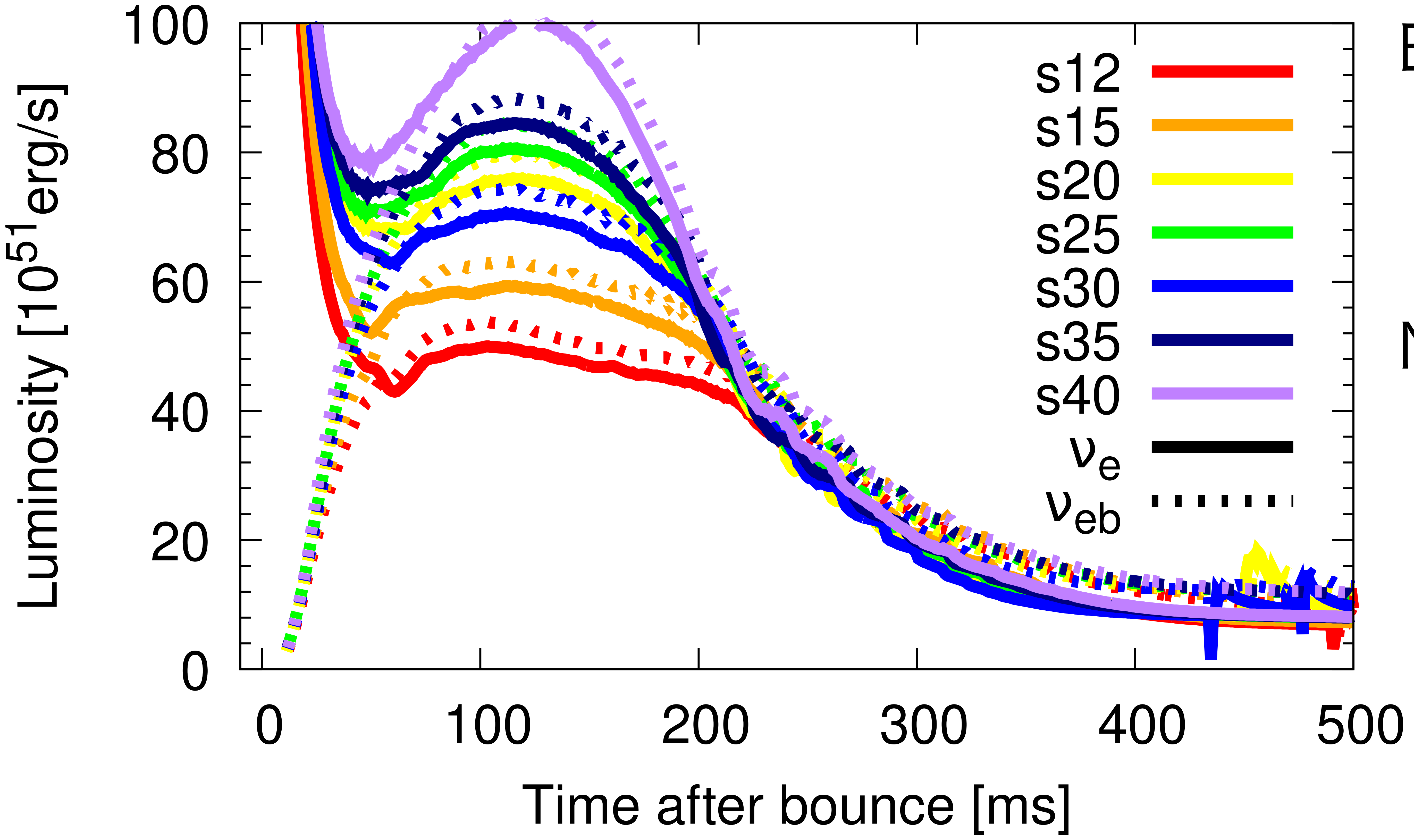


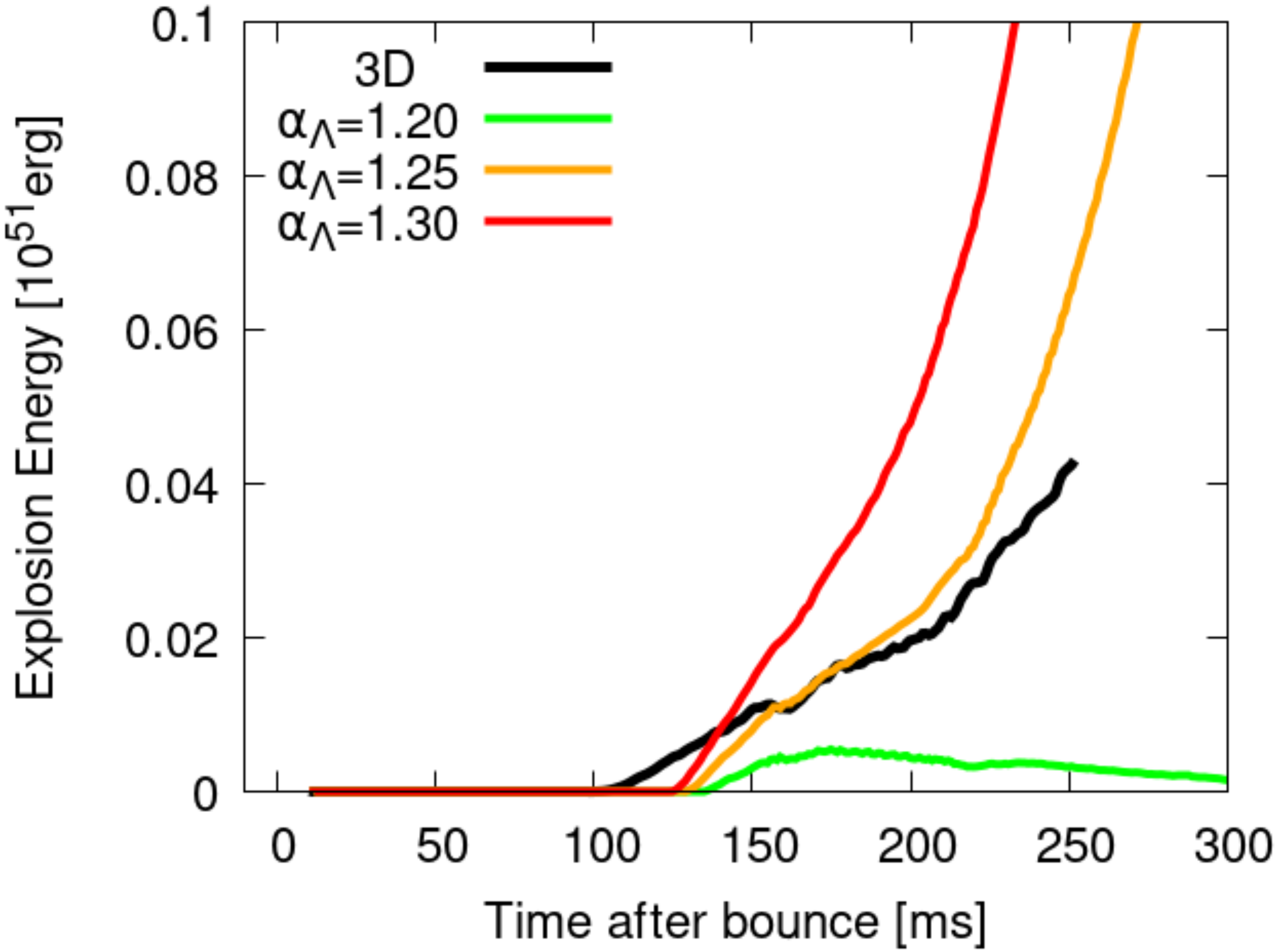
Figure: models with $\alpha_\Lambda = 1.3$

Explosion:

s12 s15 s20 s25 s30 s35 s40

Non-Explosion: nothing

Diagnostic Explosion energy



Discussion

Parameter of turbulence

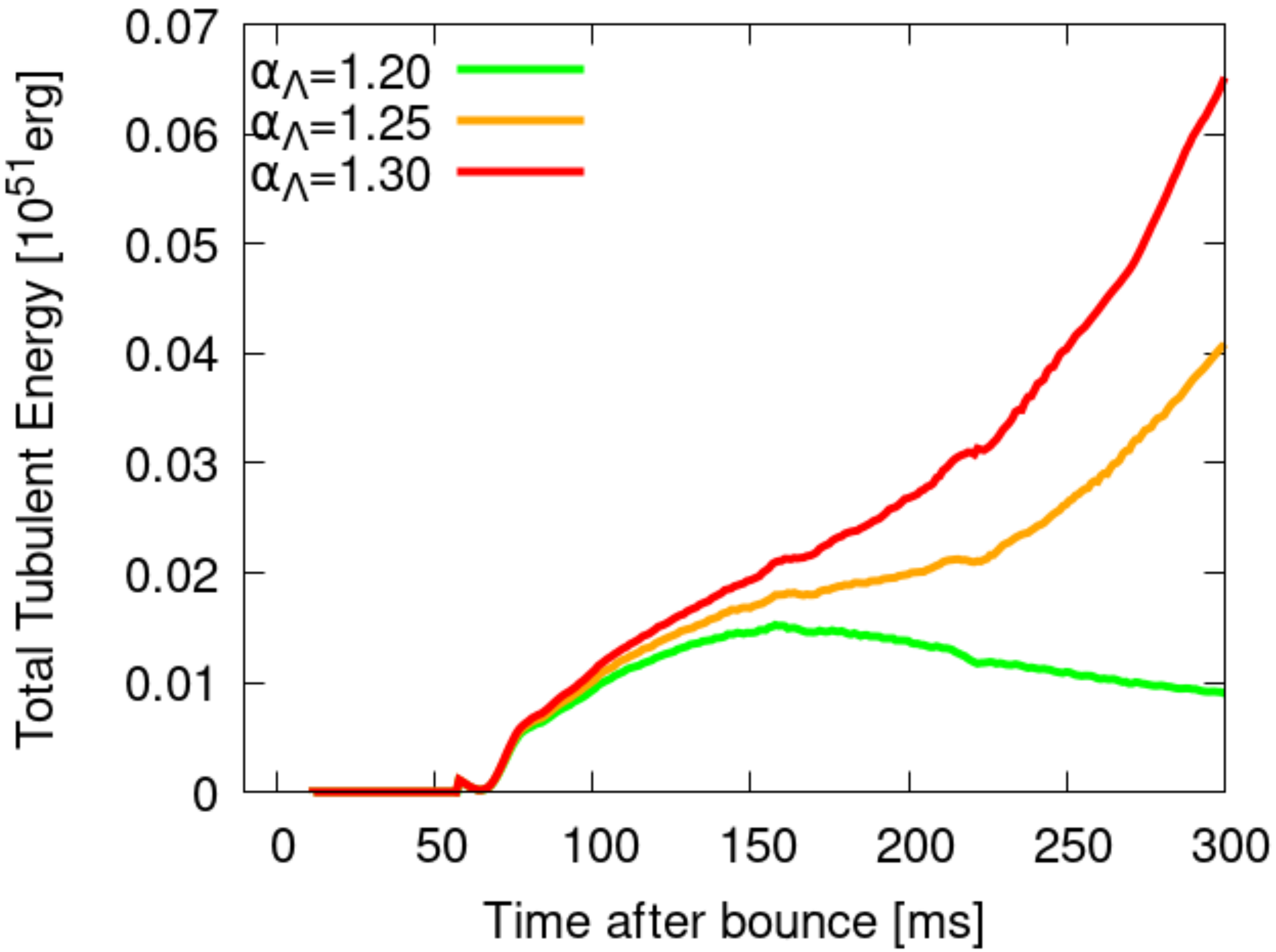


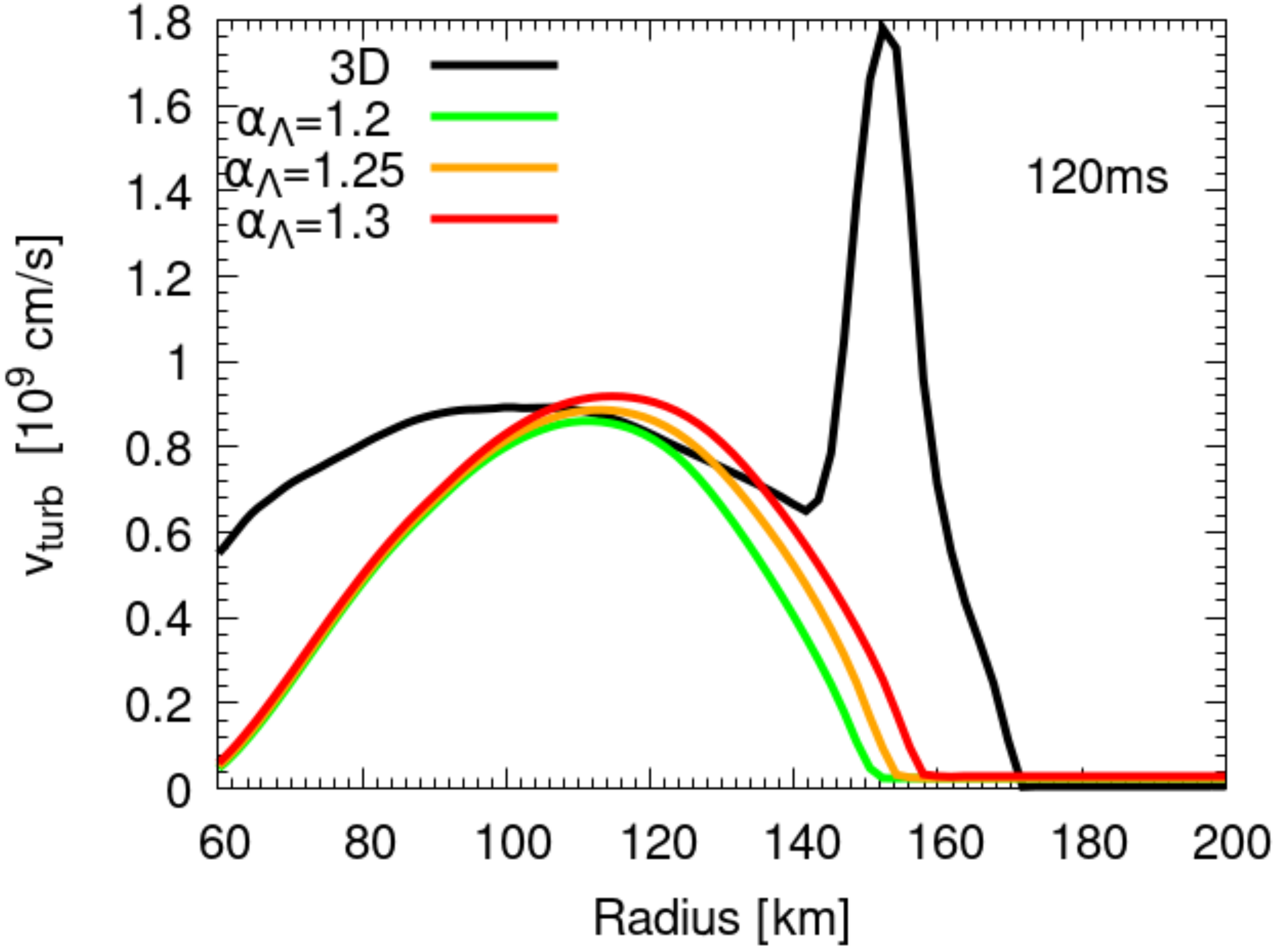
Figure: models with $\alpha_\Lambda = 1.0 - 1.3$

Progenitor : 12 Msun

Explosion: $\alpha_\Lambda \geq 1.25$

$\alpha_\Lambda \uparrow$ means that larger turbulent energy is generated. Turbulent effects are stronger.

Turbulent velocity



Reynolds stress tensor

$$R_{ij} = v_i' v_j' = (v_i - < v_i >)(v_j - < v_j >)$$

Summary
Our 1D+

The next step in the theoretical study of supernova explosions is the development of 1D+ simulations that can reproduce 3D simulations. 1D+ is expected to be an important tool to facilitate collaborative research with other fields.

Results ：

Our 1D+ can mimic shock evolution of 3D

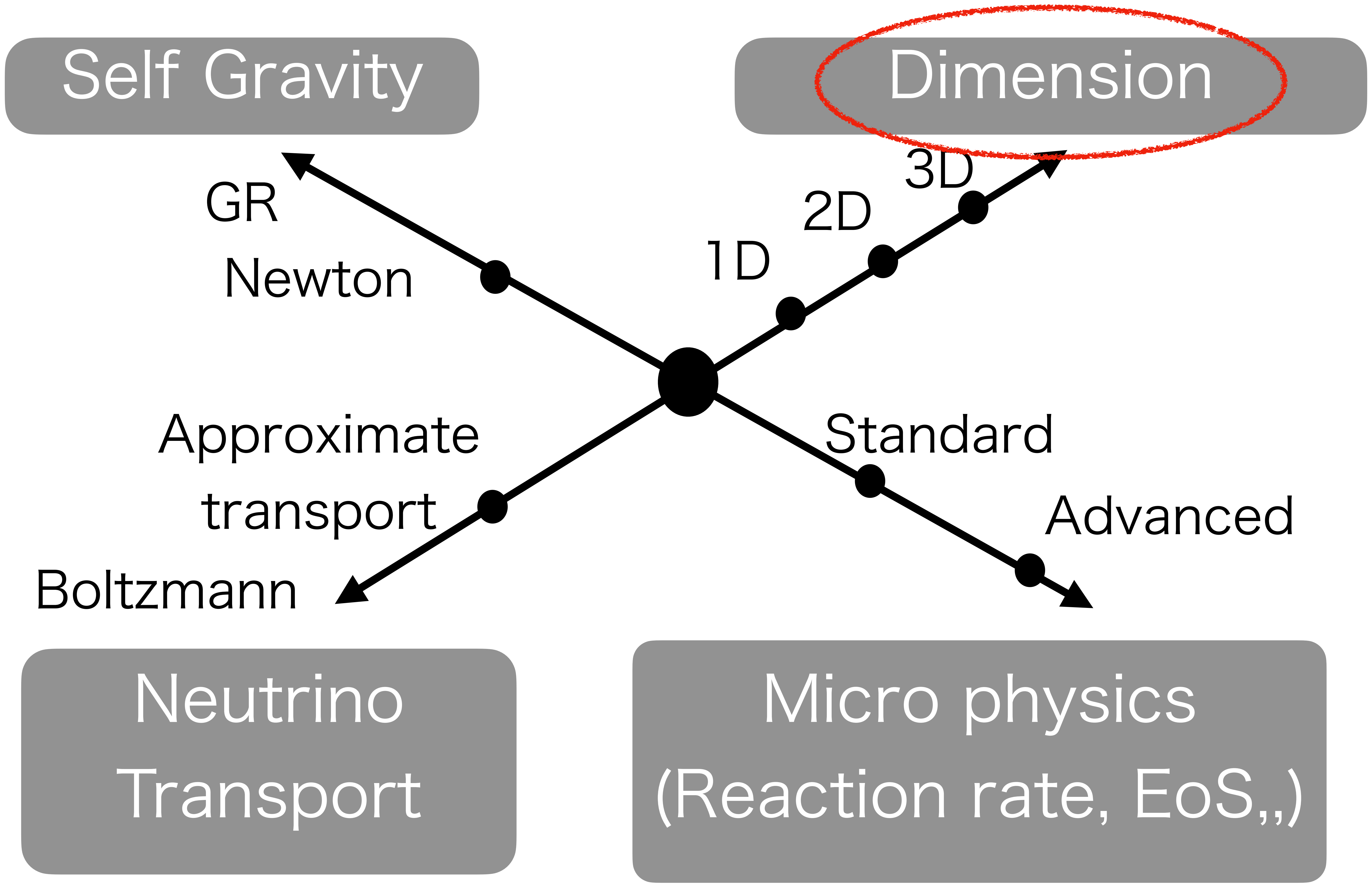
Next researches:

We are researching progenitor dependence with 1D+

We are trying to develop more realistic 1D+ by 3D analysis

Research Plan3

Turbulence is related to them



Initial condition

- progenitor model
- Rotation
- Magnetic field
- Perturbation

Numerical setups

- resolution

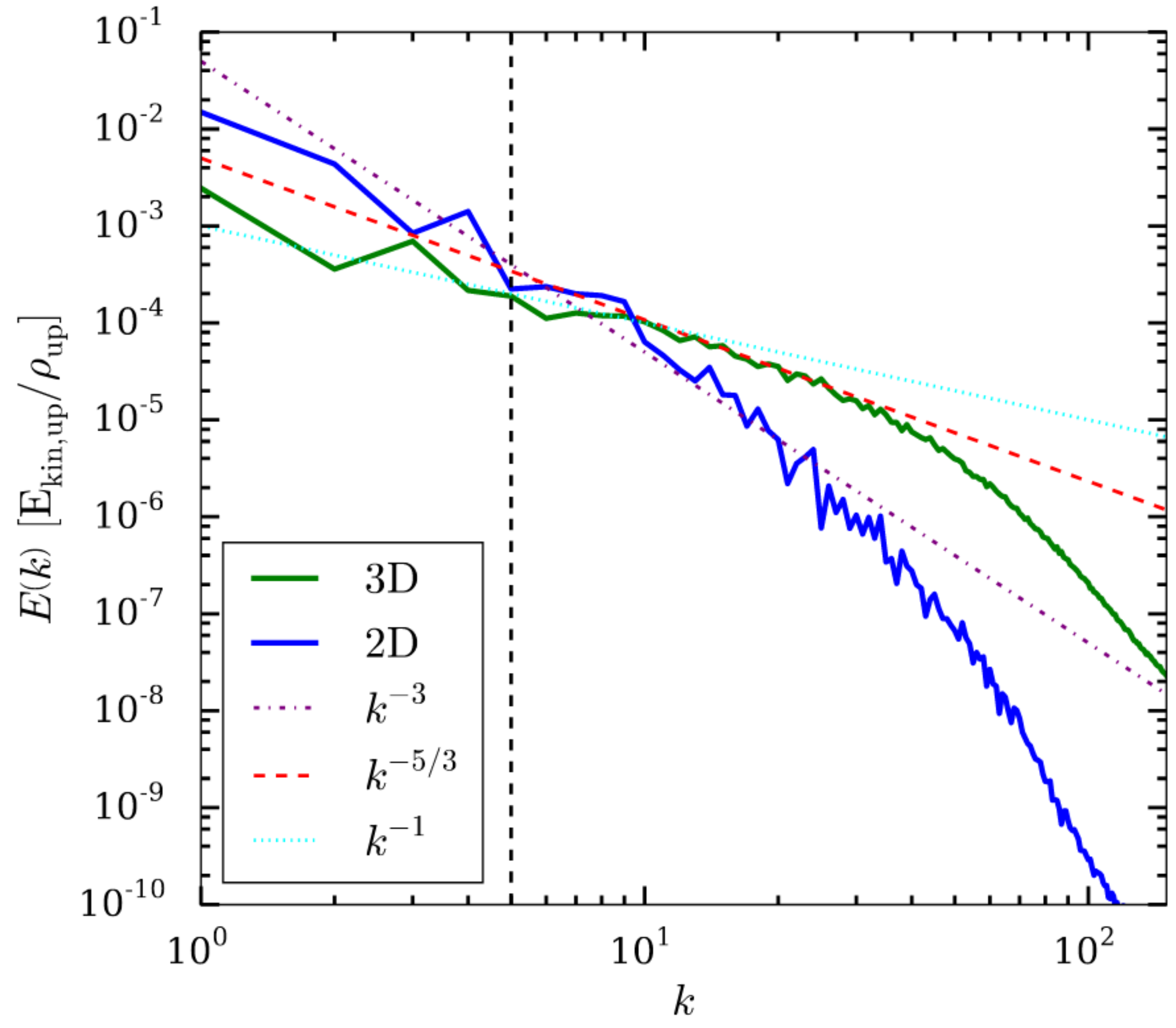
……

Turbulence in CCSNe

Phenomenological turbulent model of 1D+ is simple model. This is based on Mixing Length Theory(MLT). MLT select only one eddy length of turbulence: $\Lambda_{\rm mix}$

However, turbulence of 3D distributed over a wide range of scale hight. : $\Lambda_{\rm min} \leq \Lambda \leq \Lambda_{\rm max}$

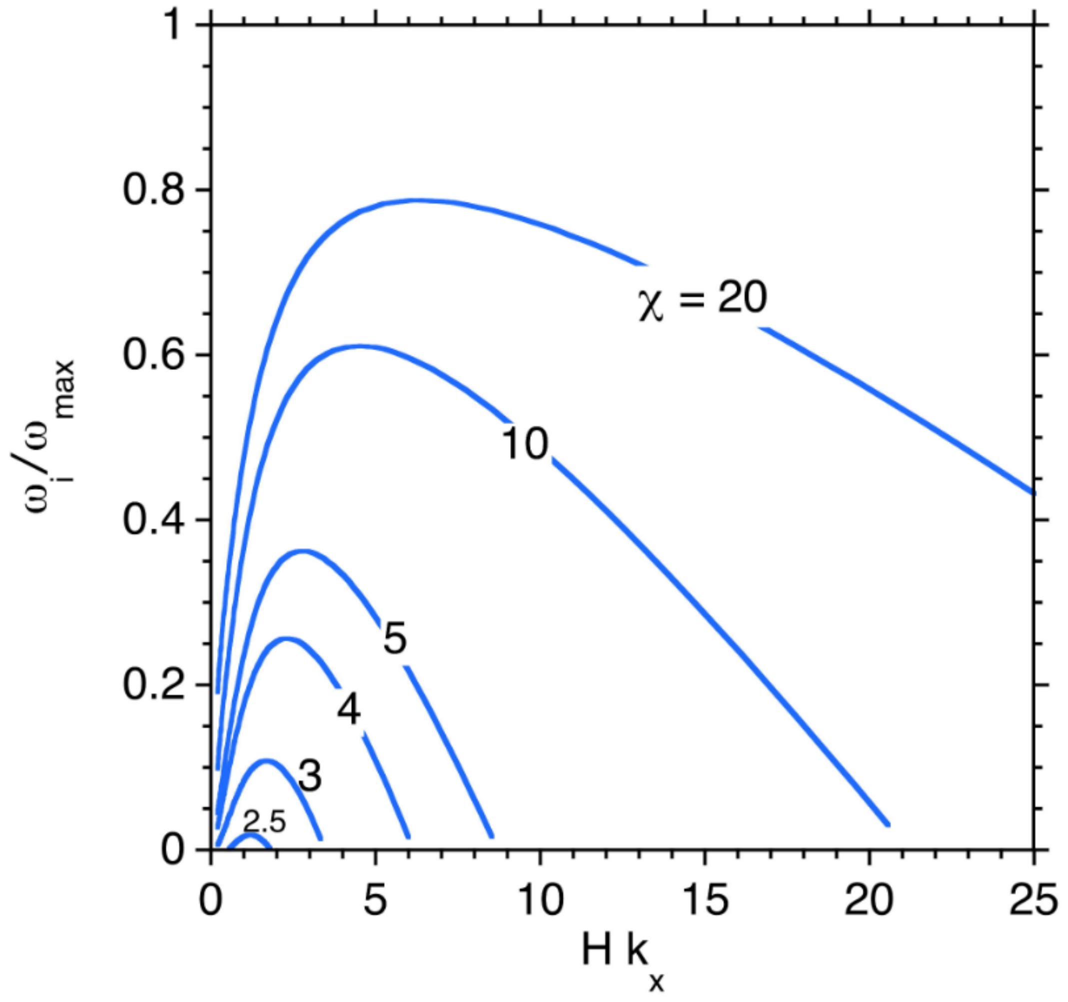
Source of turbulent energy should be also corrected.



Kazeroni et al. 2018

$k = 1/\Lambda$

$$\chi = \frac{\omega_{\rm BV} {\rm d}z}{v_z}$$



Turbulence in CCSNe

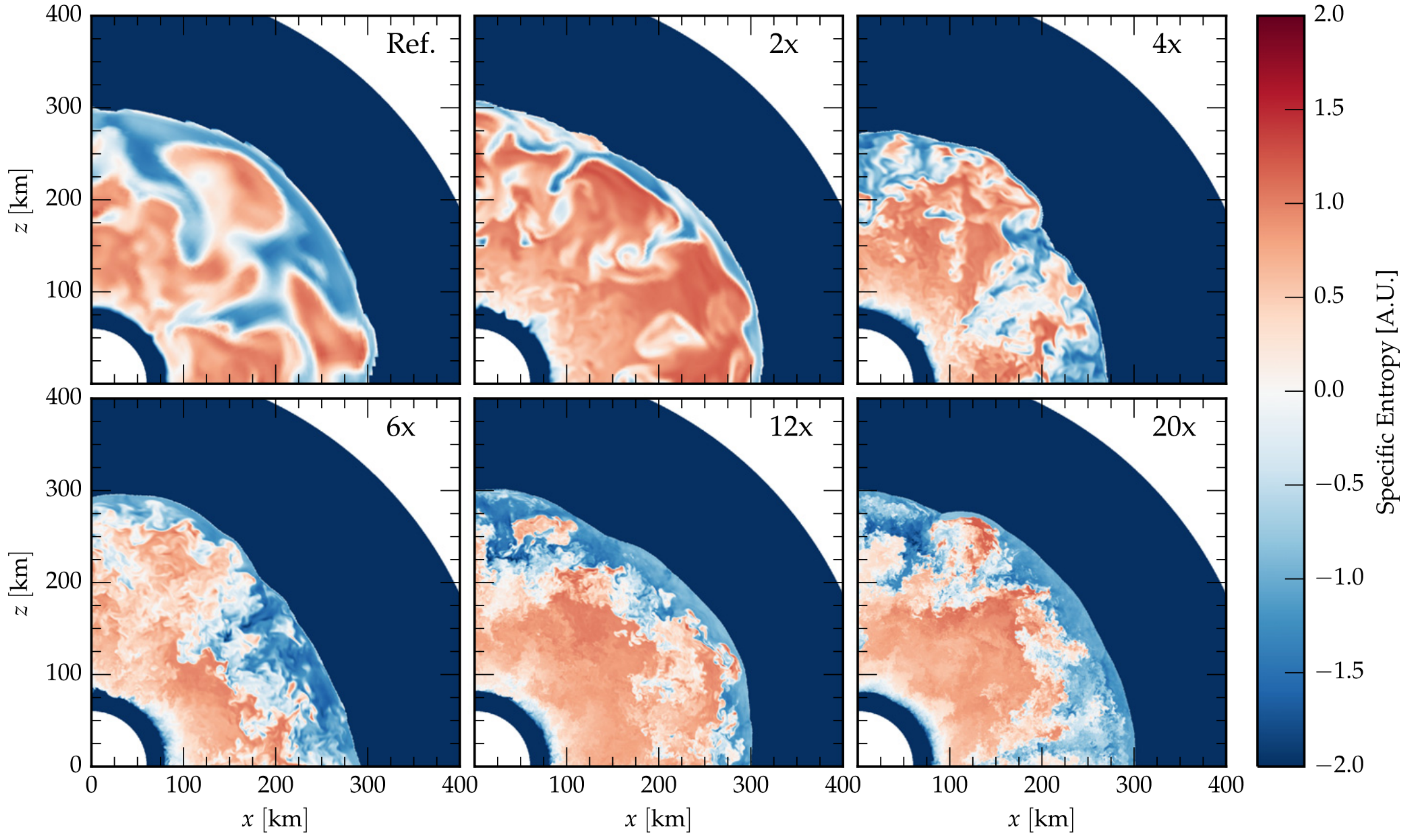
Turbulence of 3D distributed over a wide range of scale hight. :

$\Lambda_{\min} \leq \Lambda \leq \Lambda_{\max}$

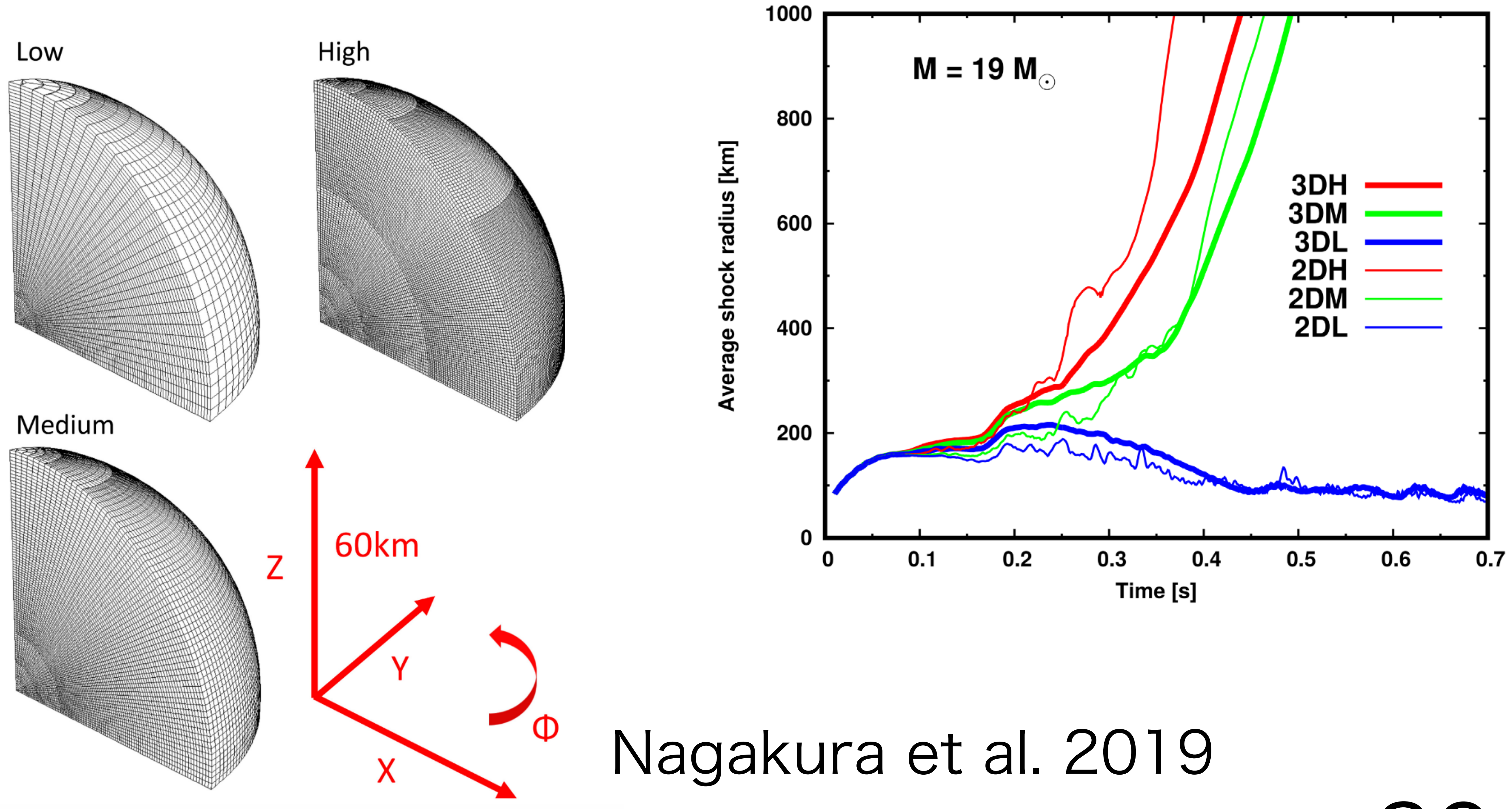
Scale hight depends on resolution.

Growth rate of turbulence also depends on resolution.

-> Explosion phenomenon depends on resolution



Radice et al. 2018



Nagakura et al. 2019

Research Plan3

In order to develop more realistic 1D+

1st step: Modeling phenomenological turbulent effects

-> Analyzing turbulence of 3D (I'm working now)

2nd step: Developing and testing more realistic 1D+

3rd step: Predicting progenitor dependence with 1D+

Analyzing turbulent effects with 3D

Setup (Radice et al.2016)

Code : 3DnSNe
(Takiwaki+2016 ,Matsumoto+2022)

Grid number : N_r x N_th x N_phi

Area： $40 < r < 400\,[\mathrm{km}]$,$\frac{\pi}{4} < \theta < \frac{3\pi}{4}$, $0 < \varphi < \frac{\pi}{2}$

θ, φ :periodic boundary condition,

r: steady flow

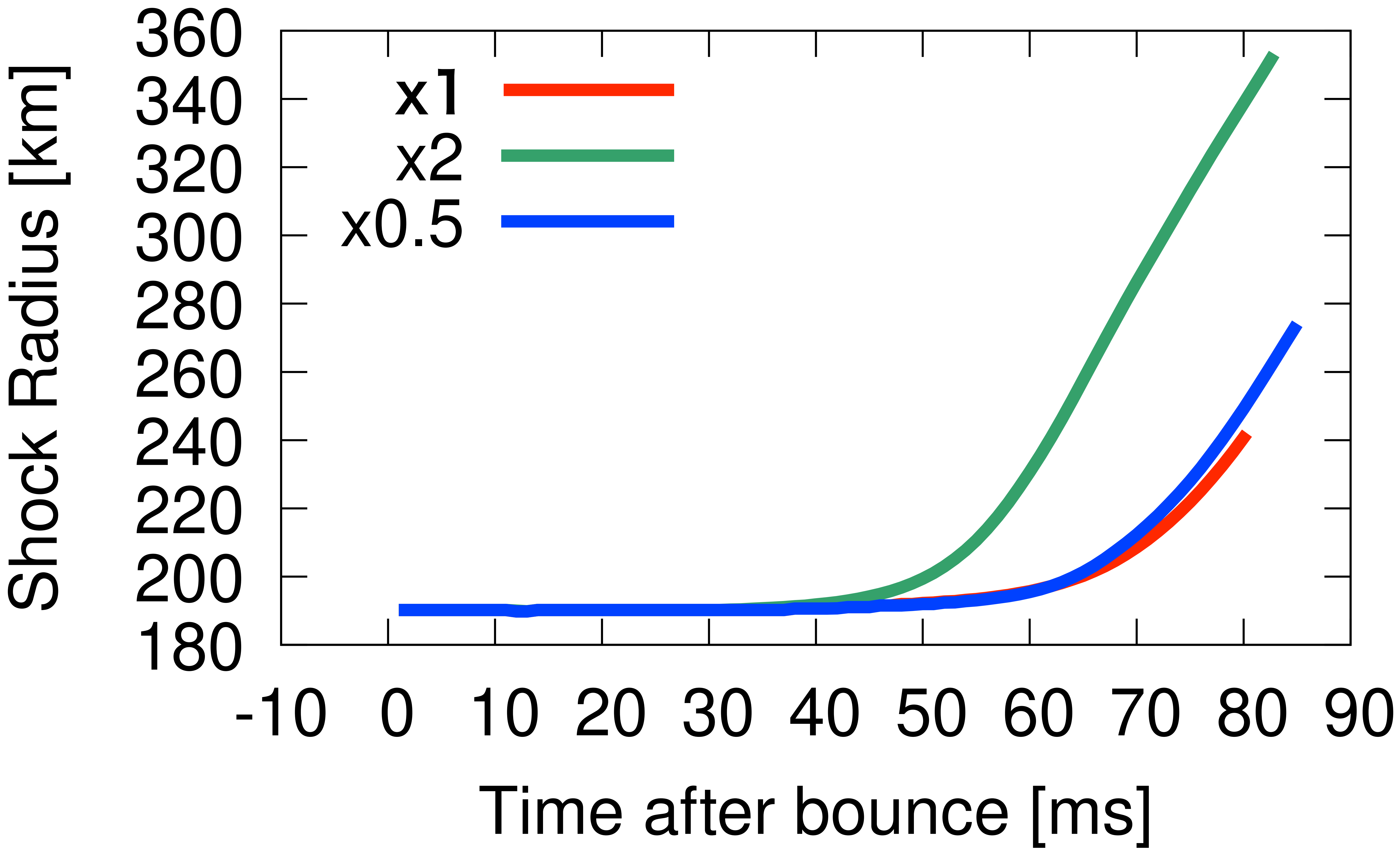
Perturbation $\frac{\delta\rho}{<\rho>} \sim 1\,\%$

Other setup is same as setup of Radice (2016)

Model name	N_r [grid]	N_th x N_phi [grid]
X0.5	1000	50 x 50
X1	1000	100 x 100
X2	1000	200 x 200

Analyzing turbulent effects with 3D

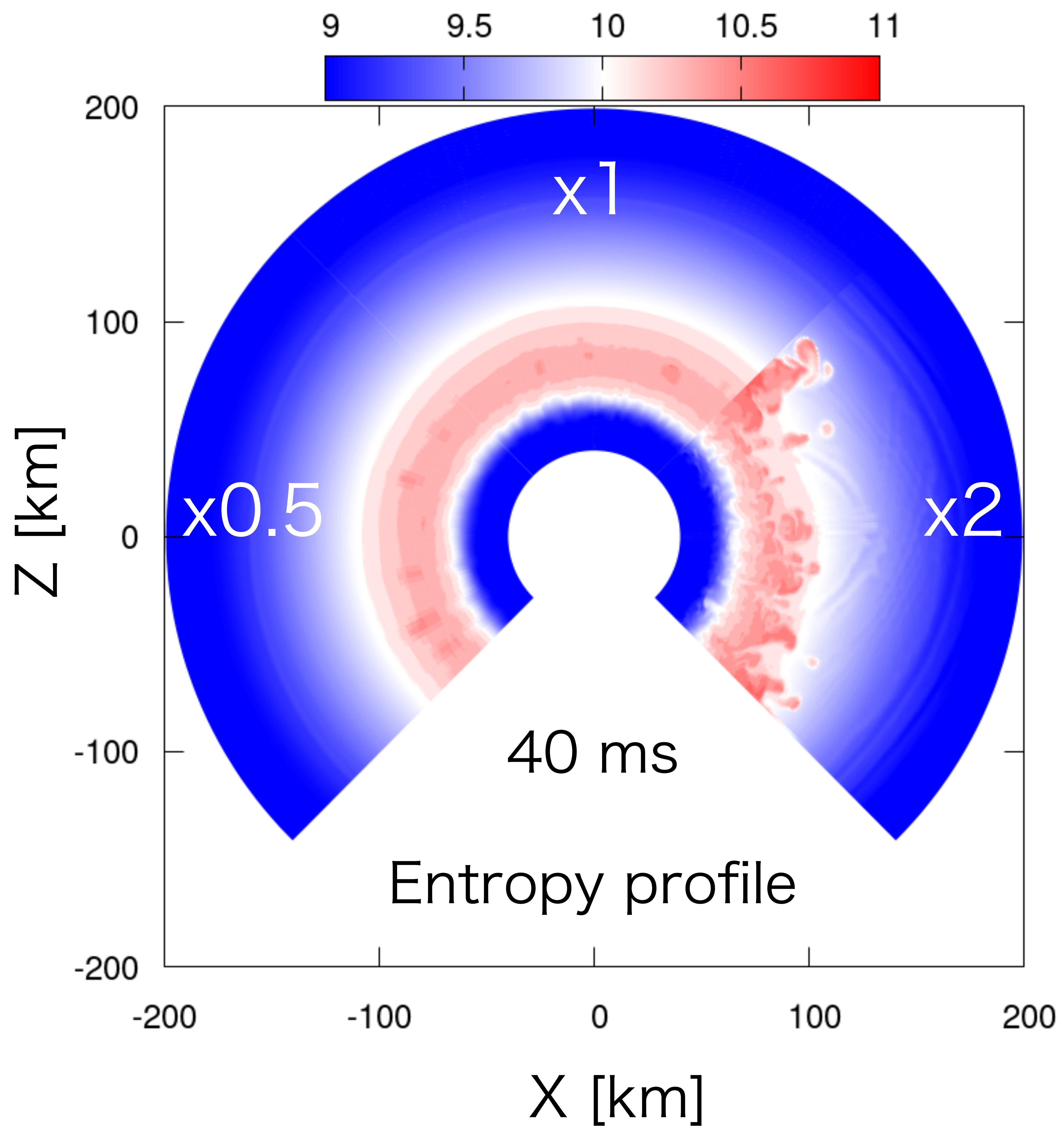
Shock evolution



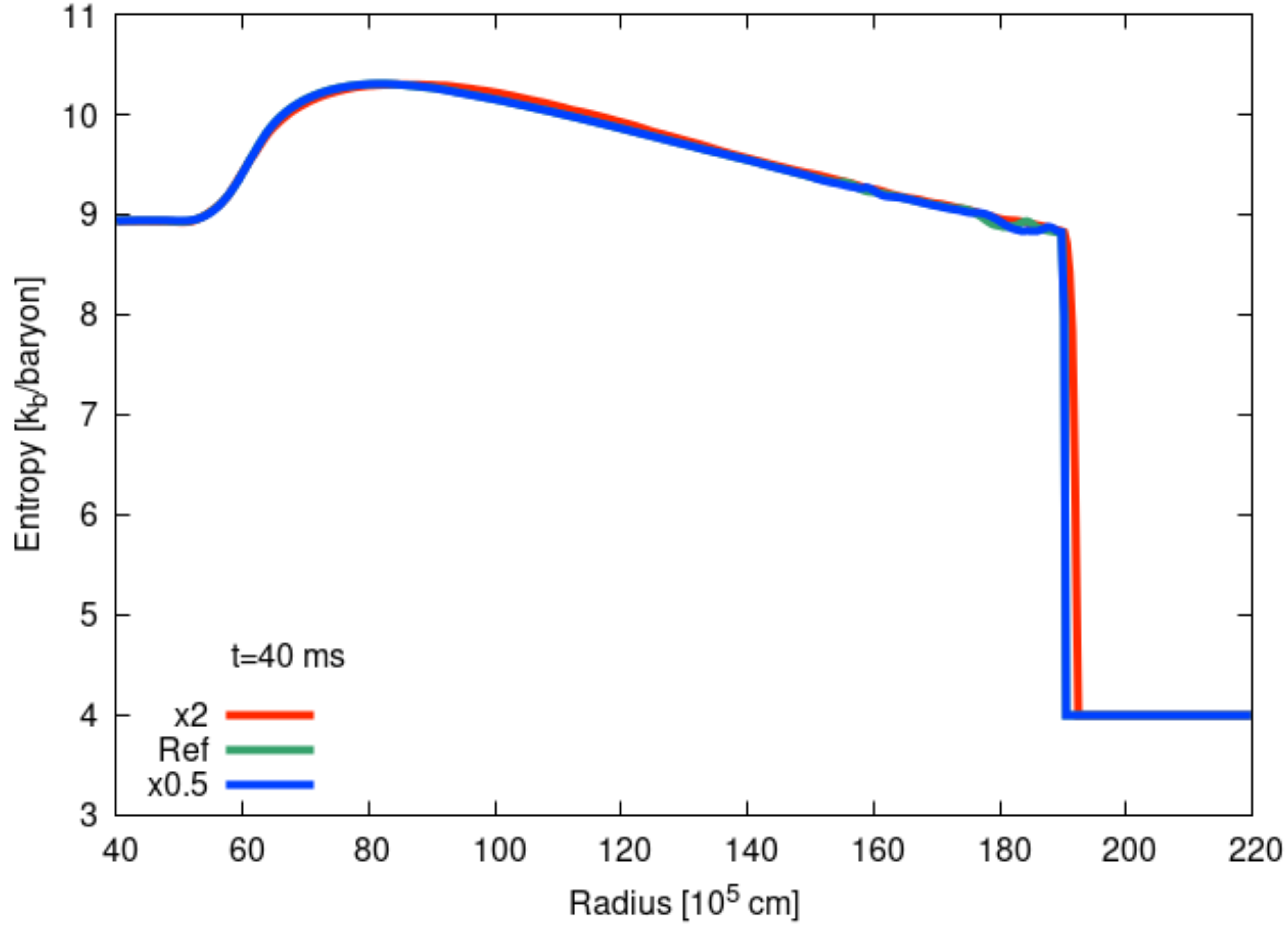
	N_r [grid]	N_th x N_phi [grid]
X0.5	1000	50 x 50
X1	1000	100 x 100
X2	1000	200 x 200

Analyzing turbulent effects with 3D

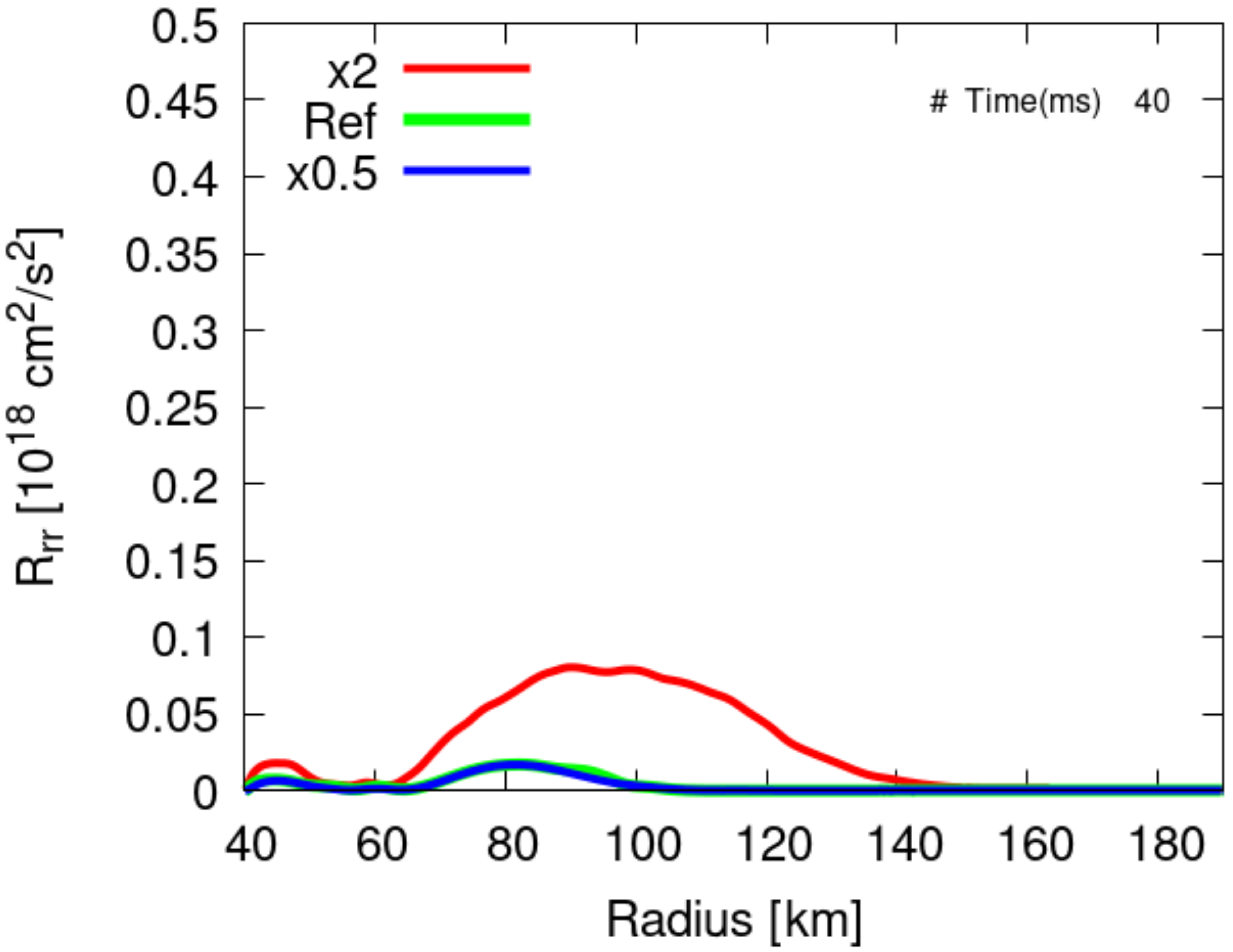
Entropy



Angular average : $\frac{\int s \, d\Omega}{4\pi}$



Analyzing turbulent effects with 3D

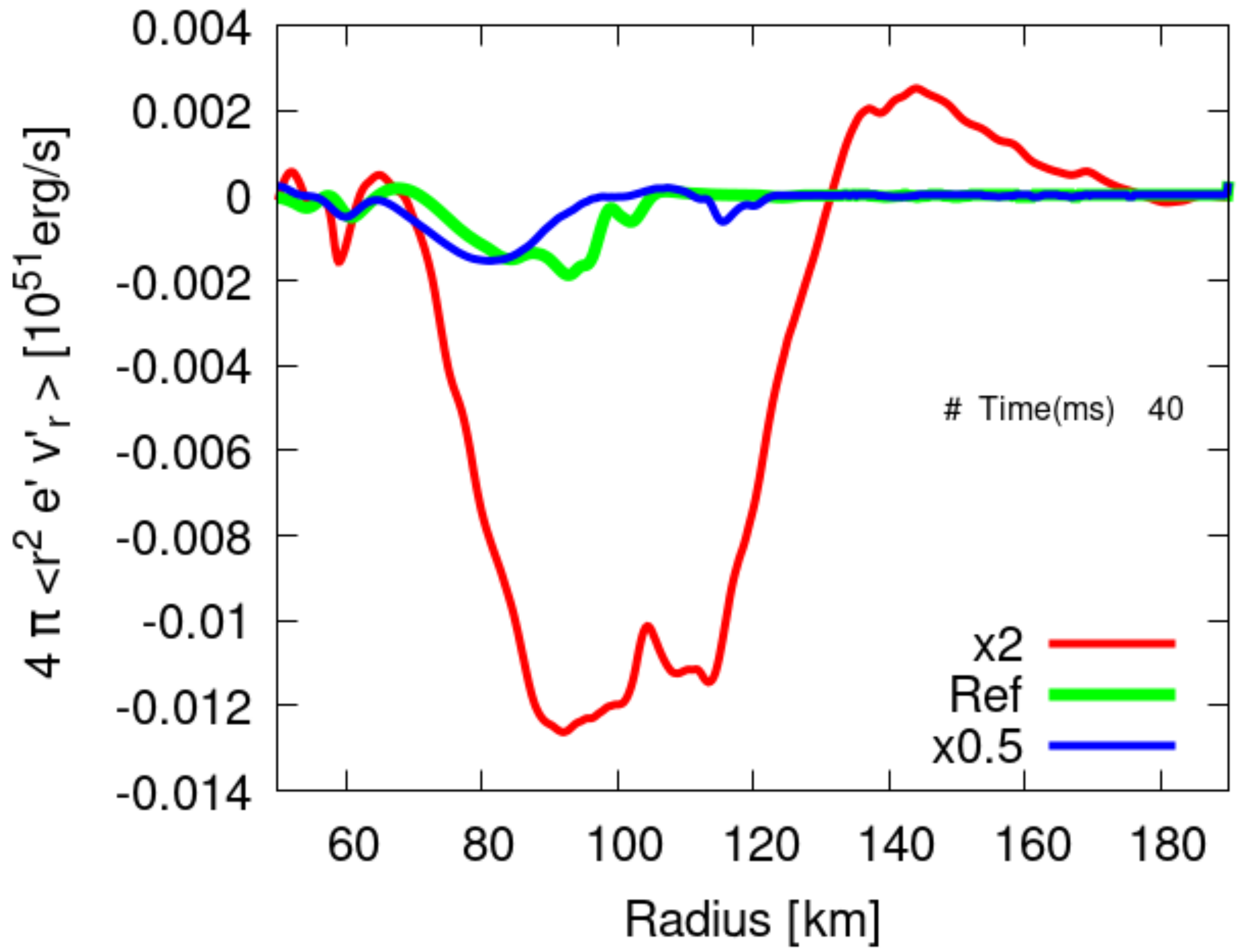


We try to analyze turbulent effects based on Reynolds decomposition.

Ex: Reynolds stress tensor

$$R_{ij} = v_i' v_j' = (v_i - < v_i >)(v_j - < v_j >)$$

Analyzing turbulent effects with 3D



Internal energy transport by turbulence:

$$e' = e - < e >$$

$$v_r' = v_r - < v_r >$$

X2 has positive internal energy transport.

Summary
Our 1D+

The next step in the theoretical study of supernova explosions is the development of 1D+ simulations that can reproduce 3D simulations. 1D+ is expected to be an important tool to facilitate collaborative research with other fields.

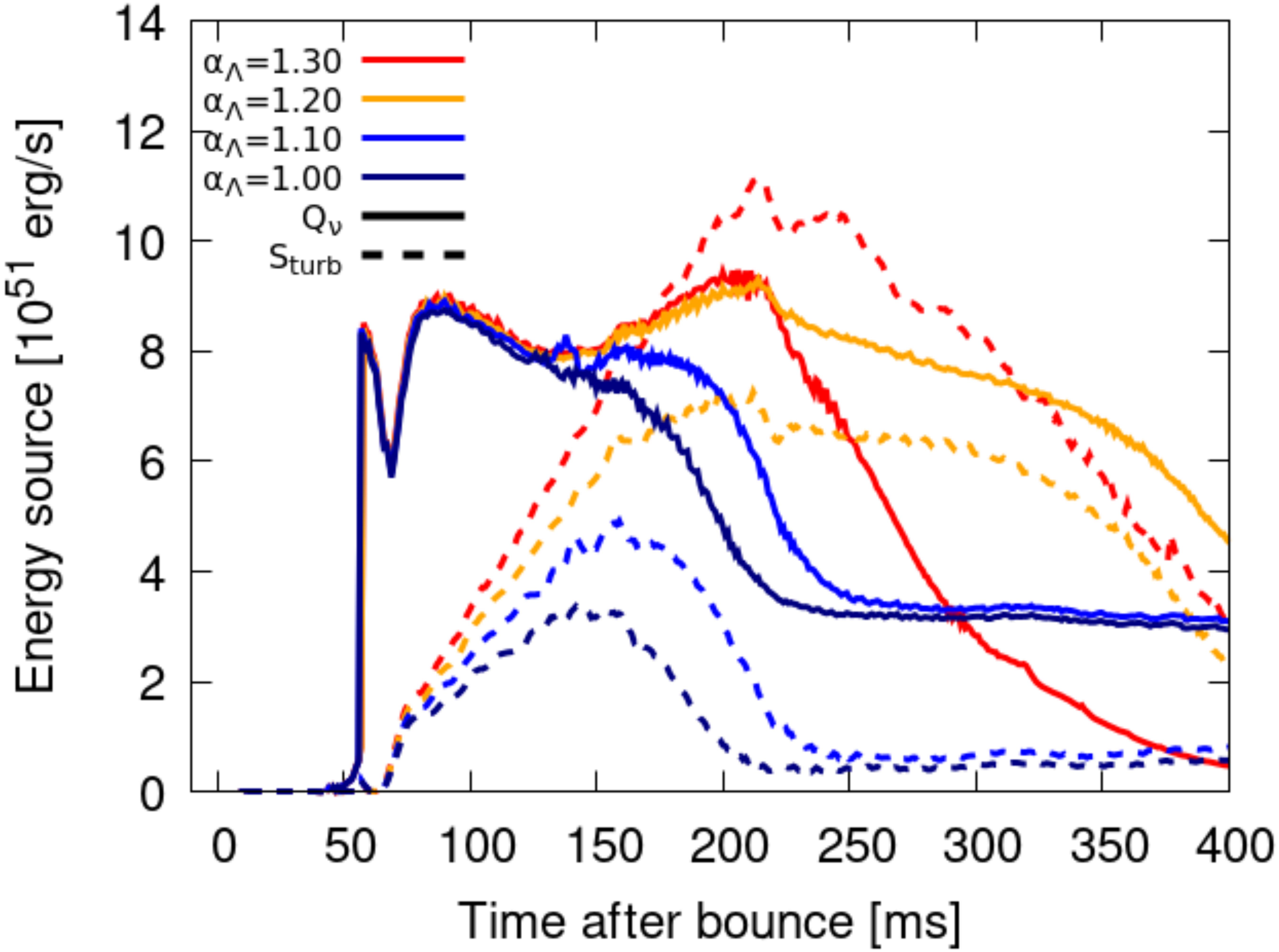
Results ：

Our 1D+ can mimic shock evolution of 3D

Next researches:

We are researching progenitor dependence with 1D+

We are trying to develop more realistic 1D+ by 3D analysis

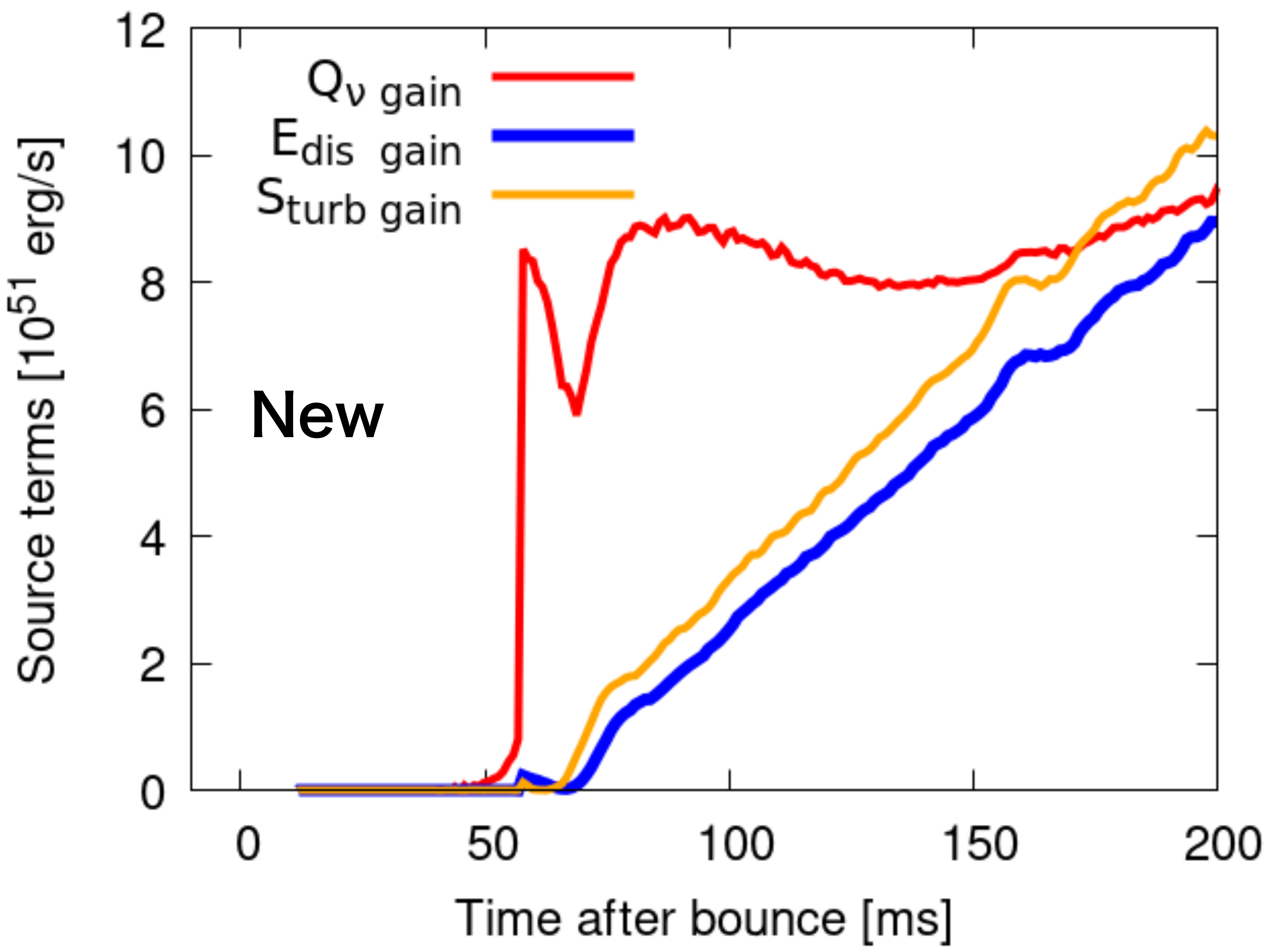


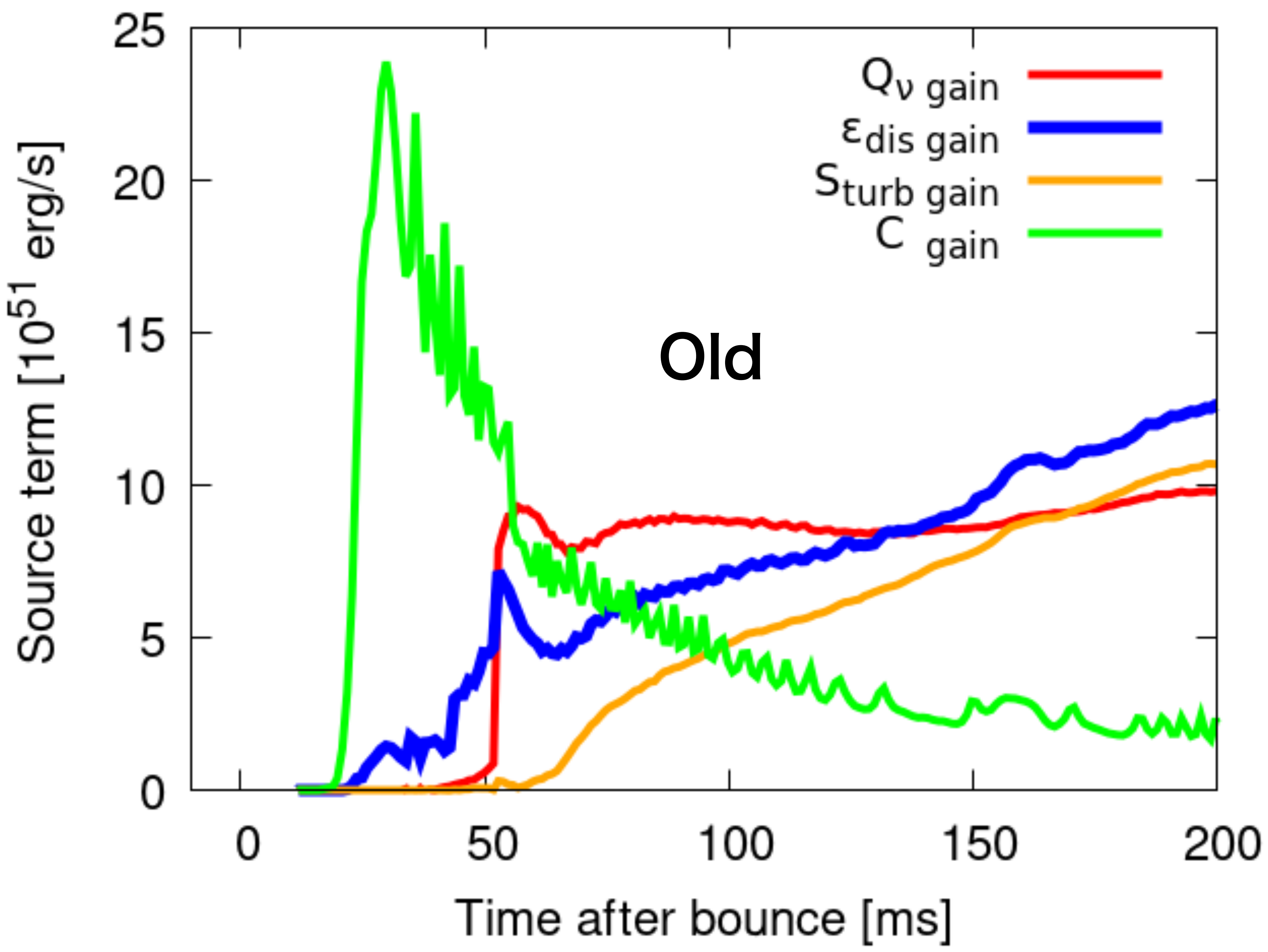
$$\frac{dE_{turb}}{dt} = S_{\mathrm{turb}} - E_{\mathrm{dis}}$$

Trigger of turbulent energy generation is neutrino heating Q_ν.

Discussion of turbulent effects

New model and old model



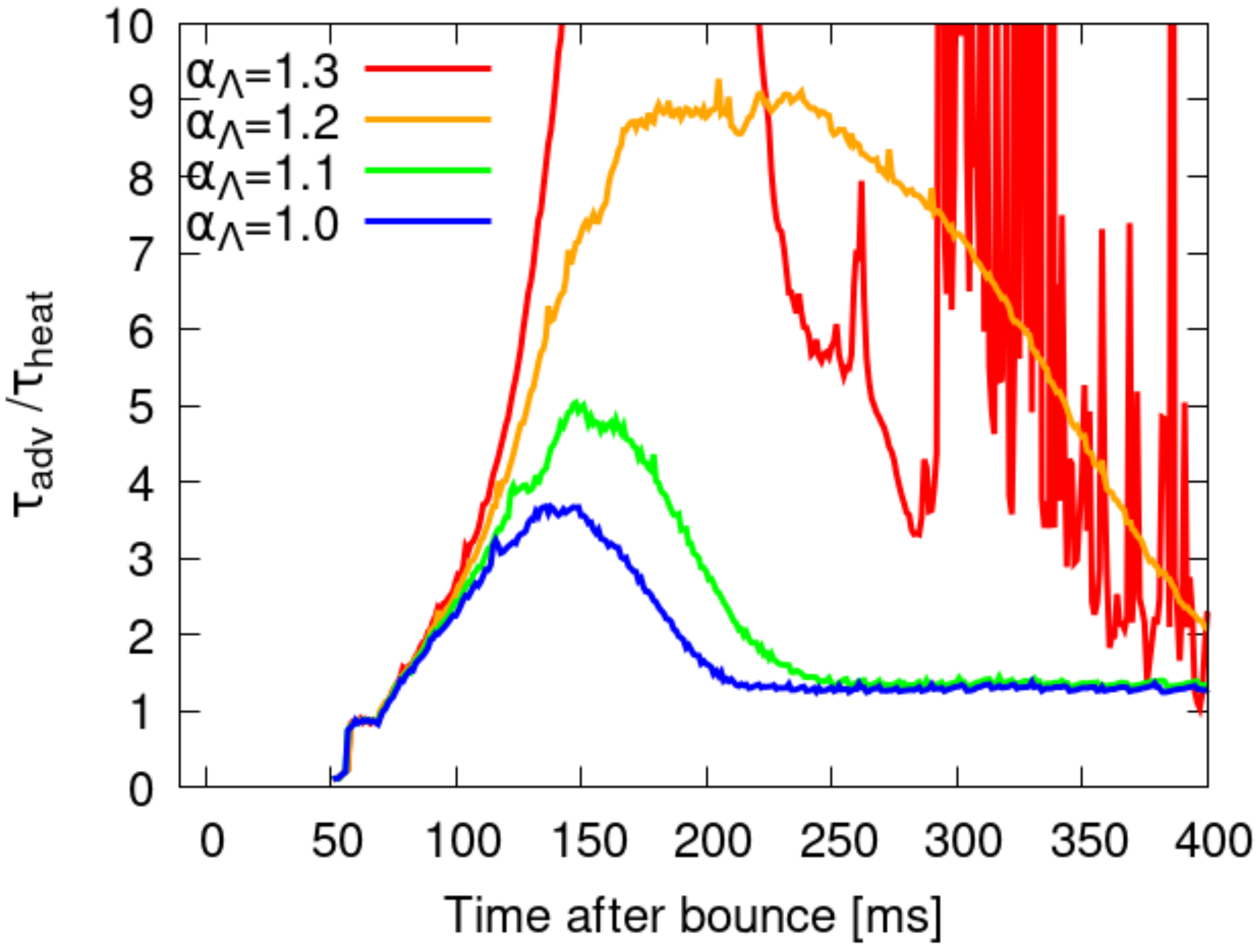


Two panel shows time evolution of total turbulent energy source in turbulent regime.

Compression C_{gain} is the reason that shock propagation of old model is faster than 3D results

Discussion

Effects of Mixing length parameter α_Λ



α_Λ determines the scale of turbulence. Large α_Λ means that turbulent affects the mechanism strongly.

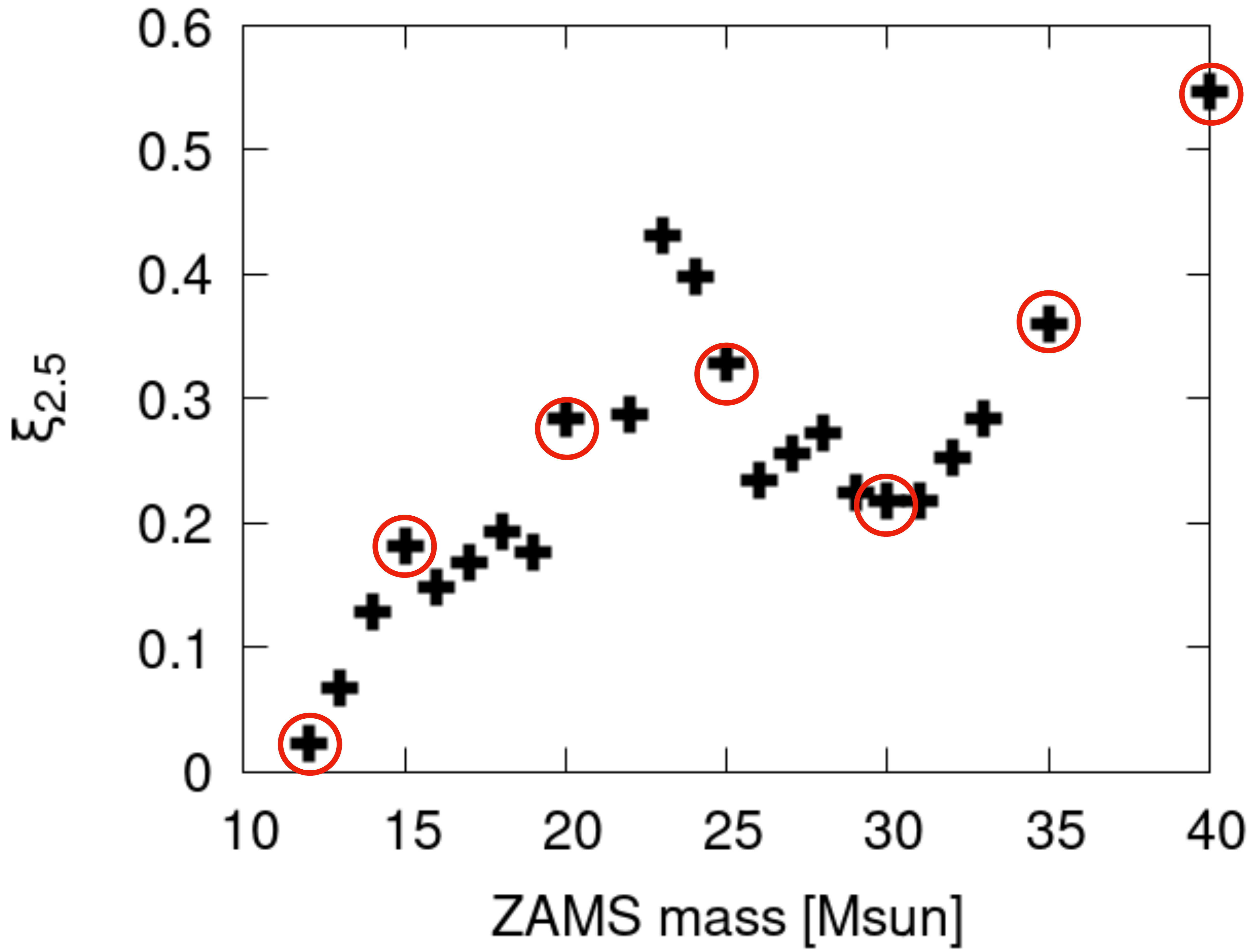
Turbulence affects advection time and heating time.

compactness

Compactness : $\xi_M = \dfrac{M/M_\odot}{R/1000\mathrm{km}}$

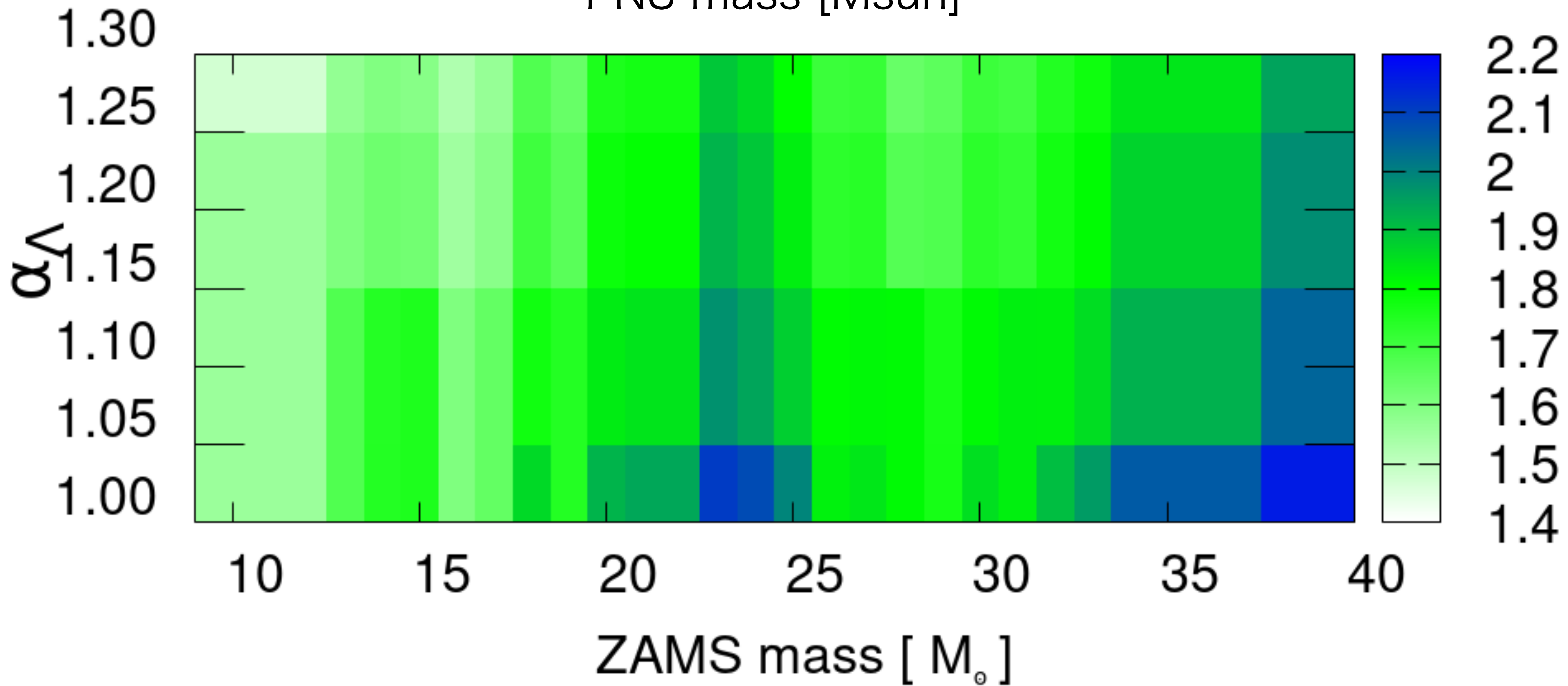
Progenitor :

$12, 13, 14, 15 .... 33, 35, 40 \, M_\odot$



Neutrino emitted energy [10^51 erg]

PNS mass [Msun]



Appendix : calculation of $f_{\rm GW}$ with 1D+

Equation 3 in Sotani et al. 2021

1D+ can evaluate PNS mass $M_{\rm PNS}$ and PNS radius R_{PNS} from density distribution.

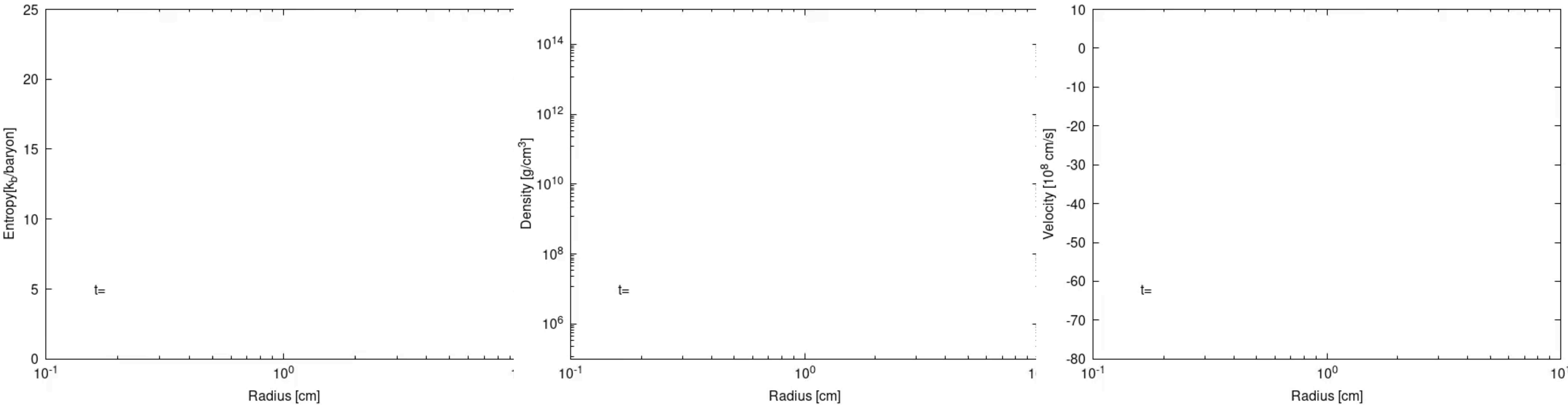
Finding the radius that is $\rho = 10^{11}$ [g/cm^3] $\rightarrow$ R_{PNS} $M_{\rm PNS} = \int_0^{R_{\rm PNS}} 4\pi r^2 \rho dr$

Derive the GW frequency $f_{\rm GW}$ from the following equation (Sotani et al. 2021)

$$f[\mathrm{kHz}] = -1.410 - 0.443\ln(x) + 9.337x - 6.714x^2 \ , \ x \equiv \left(\frac{M_{\rm PNS}}{1.4M_\odot}\right)^{1/2} \left(\frac{R_{\rm PNS}}{10 \ \mathrm{km}}\right)^{-3/2}$$

Movie

1D+ simulation



Entropy density velocity