Research on the Axionic Domain Wall Problem by Lattice Simulation

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Abstract

Particle Theory

Axion

Cosmology

Chaotic Inflation

These are consistent in Linde’s model.

Contents

Strong CP Problem

QCD Lagrangian has a CP violating term

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^{a\mu\nu}F^{a}_{\mu\nu} + \boxed{\frac{g_s^2\theta}{32\pi^2}\tilde{F}^{a\mu\nu}F^{a}_{\mu\nu}}$$

CP violation

Neutron electric dipole moment

$$d_n \simeq 4.5 \times 10^{-15}\theta e\,{\rm cm}$$

Experimental constraint C. A. Baker et al. 2006

$$|d_n| < 2.9 \times 10^{-26} e\,{\rm cm}$$

$$\Rightarrow \quad |\theta| < 0.7 \times 10^{-11}$$

J. E. Kim and G. Carosi 2010

Unnatural small dimensionless parameter θ : Strong CP Problem

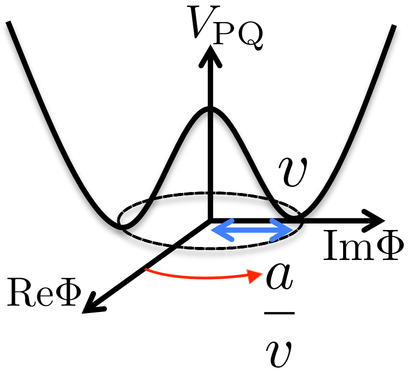
Peccei-Quinn Mechanism R. D. Peccei and H. R. Quinn 1977

Complex scalar field (PQ field)

$$\Phi = |\Phi| \, e^{i\frac{a}{v}}$$

axion

$$V_{\mathrm{PQ}} = -m^2 |\Phi|^2 + \frac{\lambda}{2} |\Phi|^4$$



After $U(1)_{\mathrm{PQ}}$ symmetry is broken down,

Parameter θ ➡ Physical d.o.f θ_{eff}

The effective potential from the non-perturbative effect of QCD

$$V_\theta \propto 1 - \cos\theta_{\mathrm{eff}}$$

θ_{eff} evolves to 0.

PQ mechanism can solve strong CP problem.

Domain Wall Problem

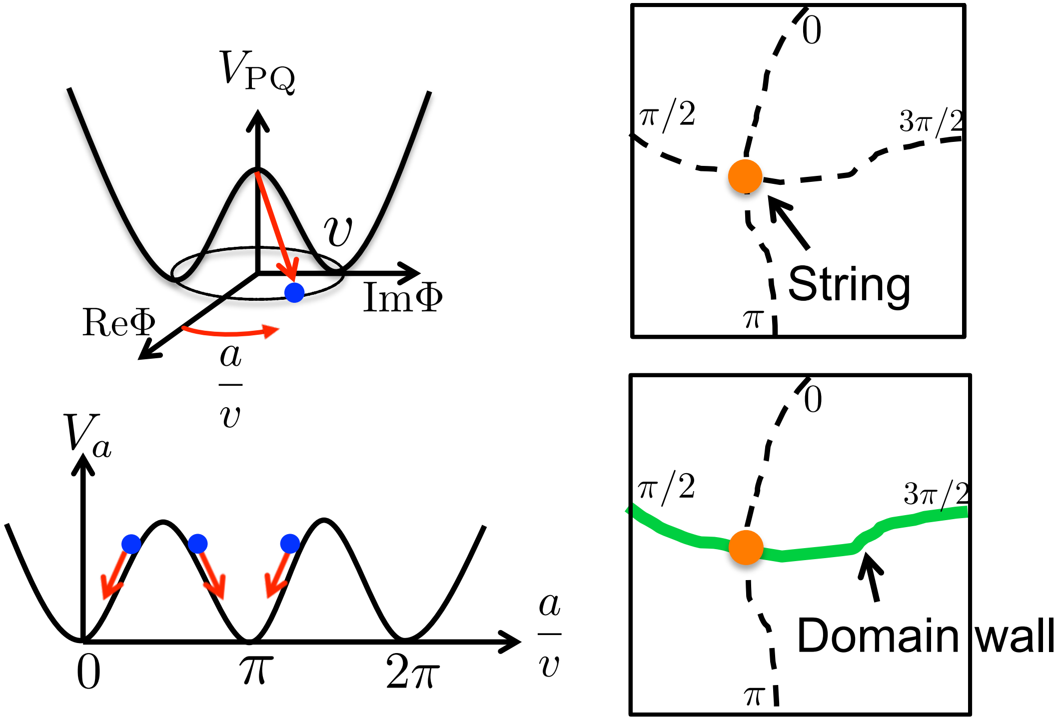
Cosmic temperature : T

$$\boxed{100\,\mathrm{MeV} \lesssim T < v}$$

Spontaneous breaking down of $U(1)_{\mathrm{PQ}}$ symmetry

$$\boxed{T \lesssim 100\,\mathrm{MeV}}$$

Spontaneous breaking down of Z_N symmetry



Soon after the formation of domain walls the universe is dominated by domain wall.

Domain Wall Problem

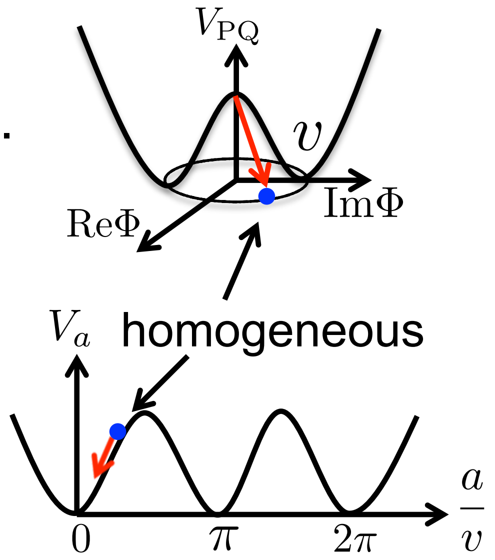
Linde's Model A. Linde 1991

Assumption

$U(1)_{\mathrm{PQ}}$ symmetry is broken down before inflation.

➡ Exponential expansion makes the phase of PQ field homogeneous.

➡ No domain wall



During inflation quantum fluctuation of axion field δa

➡ Isocurvature perturbation $\propto \dfrac{H_{\mathrm{inf}}}{v} > 1$ in the chaotic inflation model

excluded by CMB observation

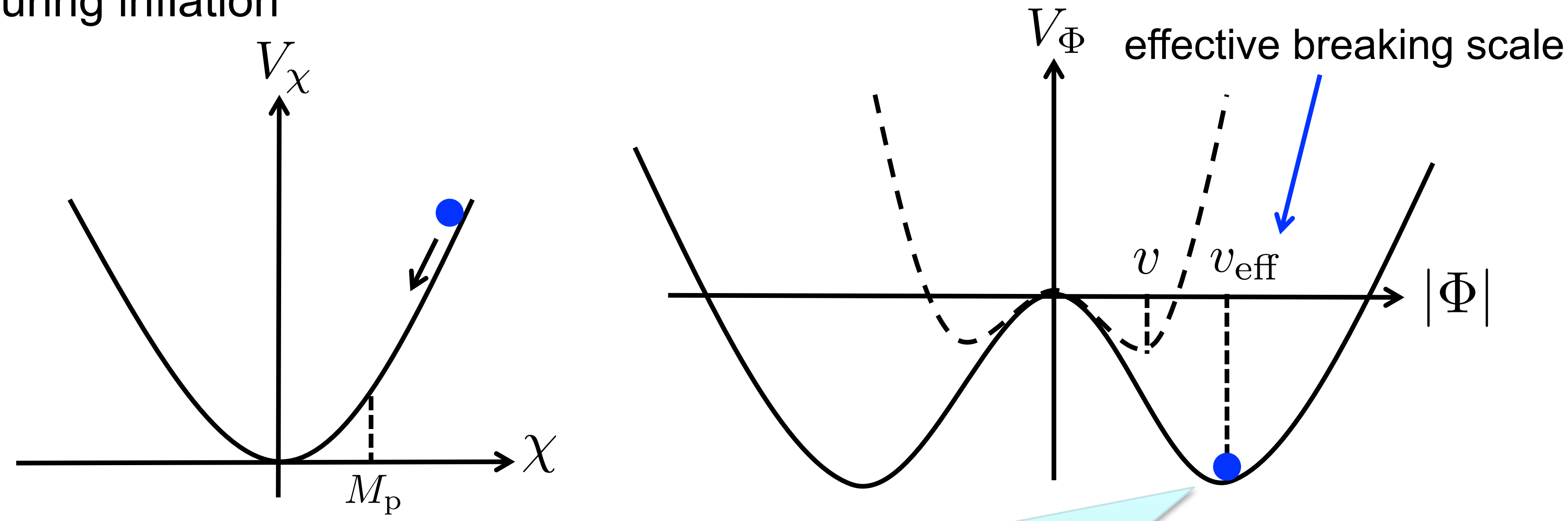
G. Hinshaw et al. 2012 (WMAP 9 year)

Linde's Model A. Linde 1991

The potential of infaton χ and PQ field Φ

$$V = \underbrace{\frac{M^2}{2}\chi^2}_{\text{chaotic inflation}} \underbrace{- m^2|\Phi|^2 + \frac{\lambda}{2}|\Phi|^4}_{\text{PQ potential}} \underbrace{- \lambda'\chi^2|\Phi|^2}_{\text{coupling between inflaton and PQ}}$$

During inflation



effective breaking scale

Large $v_{\rm eff}$ ➡ Small isocurvature perturbation

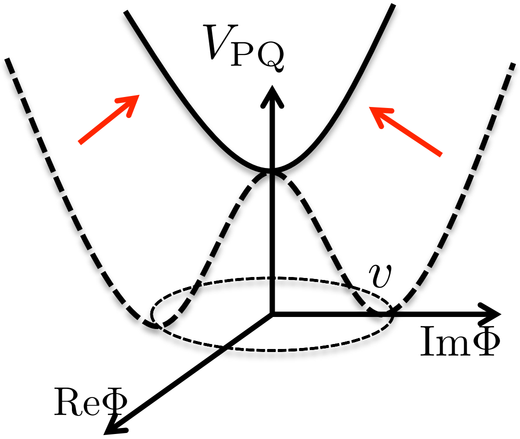
Domain Wall Problem Again

But it is possible that the domain wall problem comes again if the nonlinear dynamics of these fields after inflation is considered.

PQ fluctuations $\delta\Phi$ grow up by the nonlinear dynamics (parametric resonance).

$$|\Phi|^4 \rightarrow \left\langle |\delta\Phi|^2 \right\rangle |\Phi|^2$$

PQ fluctuations lift up the effective potential.



- ➡ Nonthermal restoration of PQ symmetry
- ➡ String formation by spontaneous breaking down of PQ symmetry
- ➡ Domain wall formation

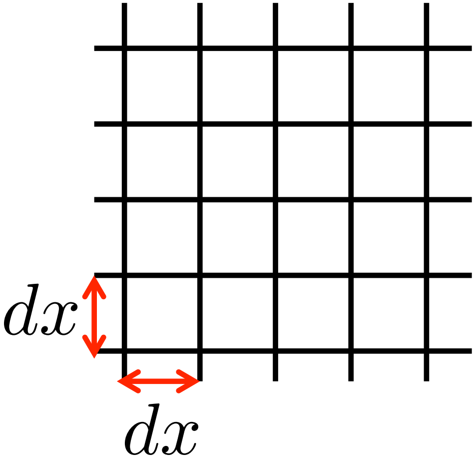
Numerical simulation is necessary.

Lattice Simulation

Solving E.O.M at each lattice points

$$\ddot{\Phi} + 3H\dot{\Phi} - \frac{\nabla^2}{R^2}\Phi + \left(\lambda\left|\Phi\right|^2 - m^2 - \lambda'\chi^2\right)\Phi = 0$$

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2}{R^2}\chi + \left(M^2 - 2\lambda'\left|\Phi\right|^2\right)\chi = 0$$



Dimensionless parameters

$$\underline{\alpha \equiv \frac{m}{M}} \qquad \underline{\beta \equiv \frac{\sqrt{\lambda'}M_p}{M}} \qquad \underline{\gamma \equiv \frac{\lambda'}{\lambda}}$$

mass of PQ

coupling between inflaton and PQ

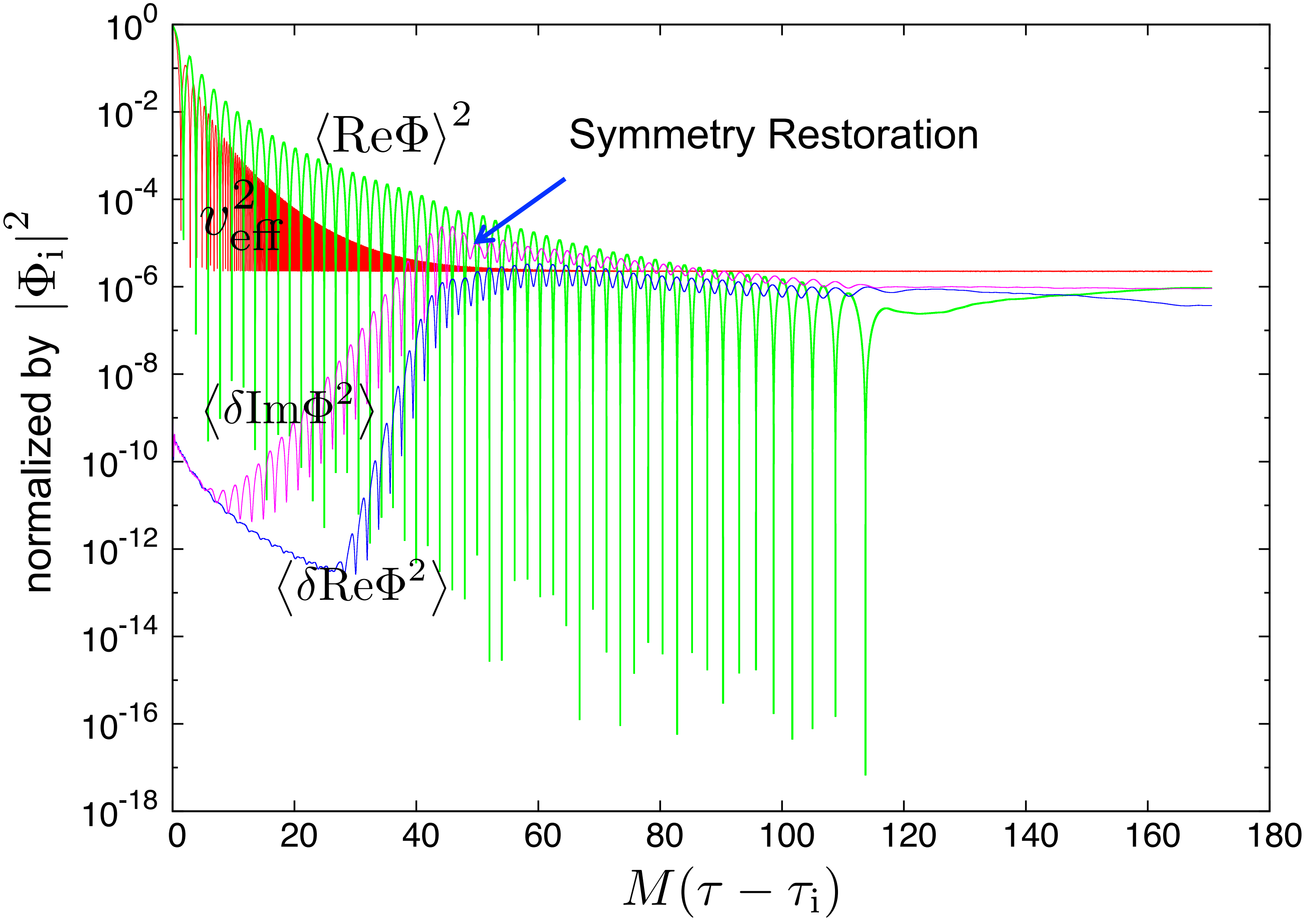
initial value of PQ

Our criterion

Formation of stable strings ➡ Domain wall problem

Result

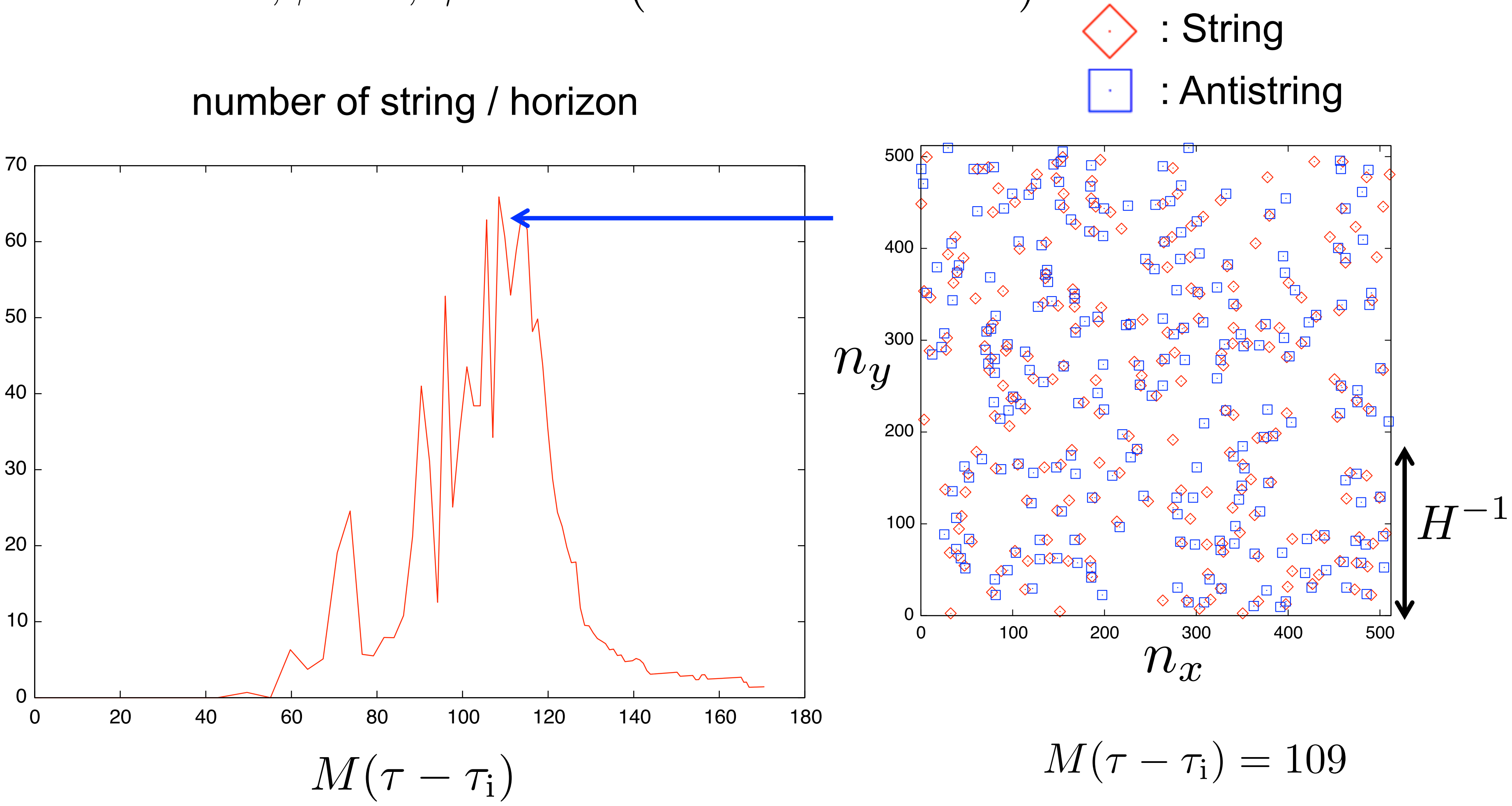
$\alpha = 3 \times 10^{-3},\ \beta = 2,\ \gamma = 10^{-4}\ \left(v = 3.7 \times 10^{13}\,\mathrm{GeV}\right)$



Result

$$\alpha = 3 \times 10^{-3},\ \beta = 2,\ \gamma = 10^{-4}\ \left(v = 3.7 \times 10^{13}\,\mathrm{GeV}\right)$$

number of string / horizon



$M(\tau - \tau_\mathrm{i}) = 109$

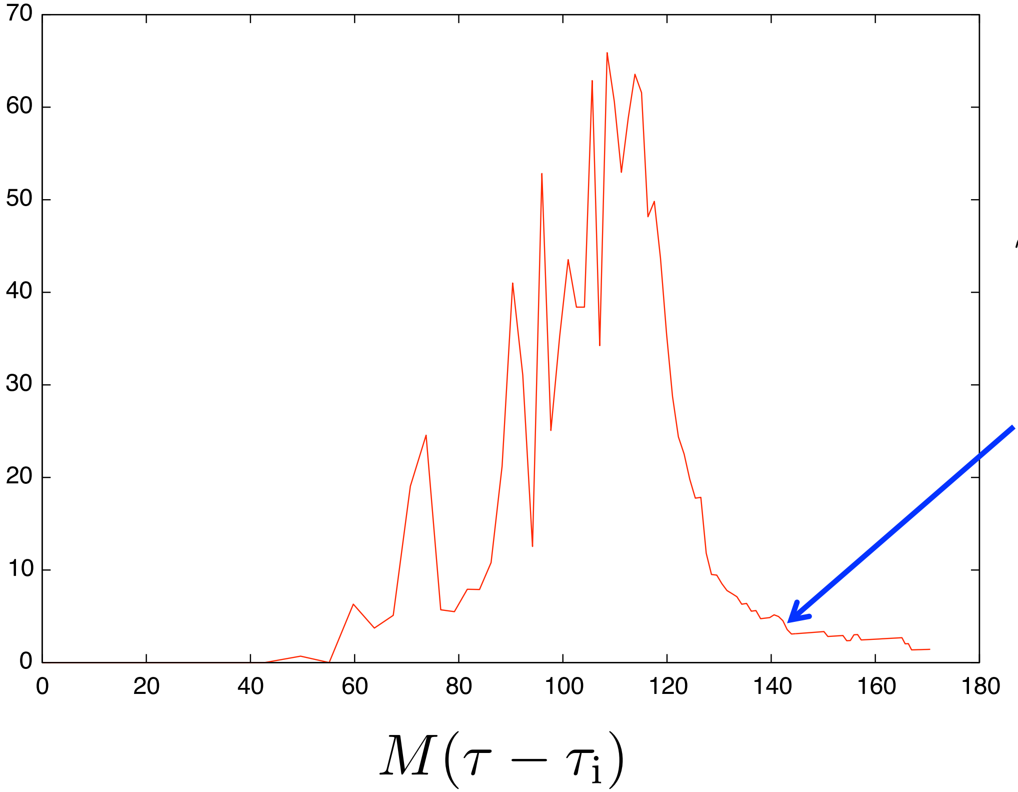
Result

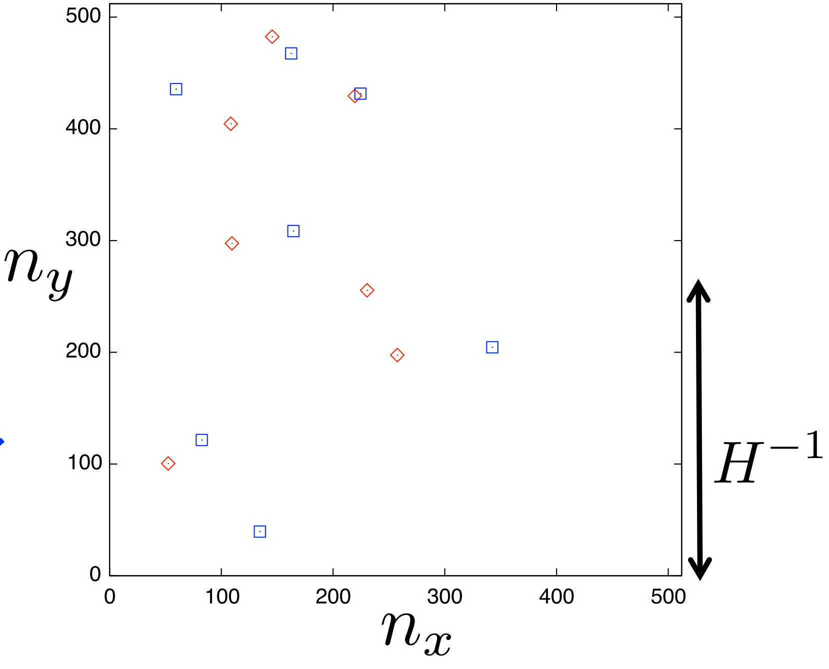
$$\alpha = 3 \times 10^{-3},\ \beta = 2,\ \gamma = 10^{-4}\ \left(v = 3.7 \times 10^{13}\,\mathrm{GeV}\right)$$

◇ : String

□ : Antistring

number of string / horizon



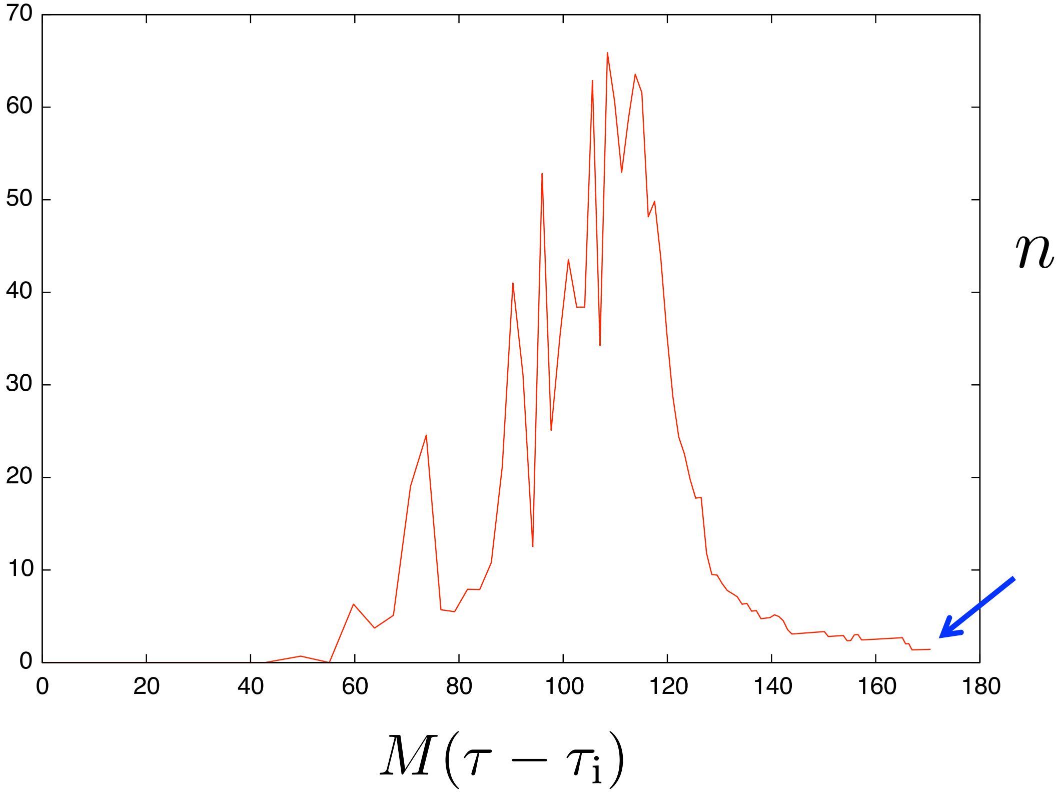


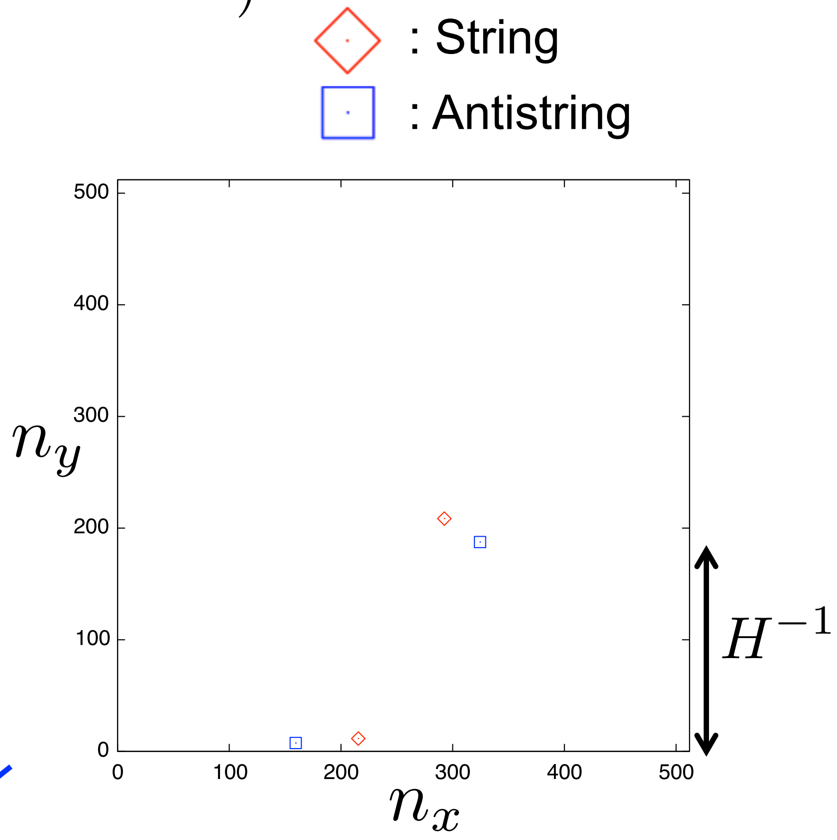
$$M(\tau - \tau_\mathrm{i}) = 143$$

Result

$$\alpha = 3 \times 10^{-3},\ \beta = 2,\ \gamma = 10^{-4}\ \left(v = 3.7 \times 10^{13}\,\mathrm{GeV}\right)$$

number of string / horizon





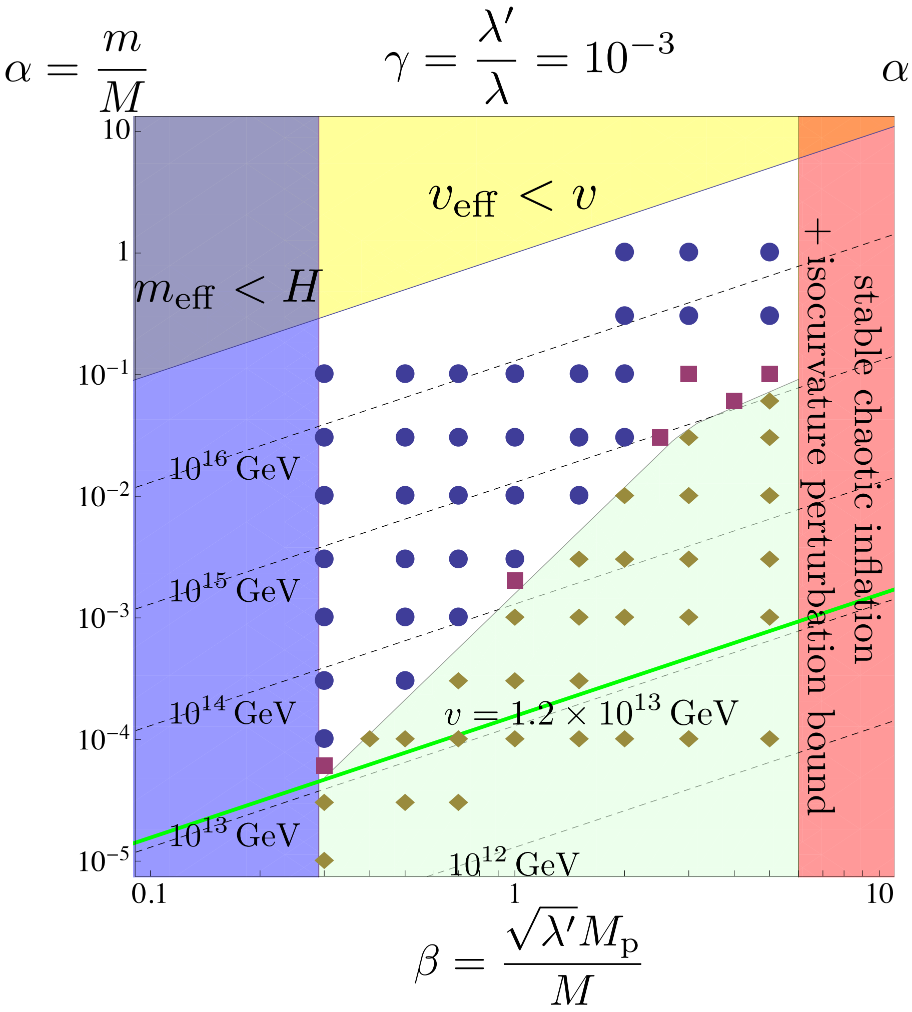
$M(\tau-\tau_{\rm i}) = 171$

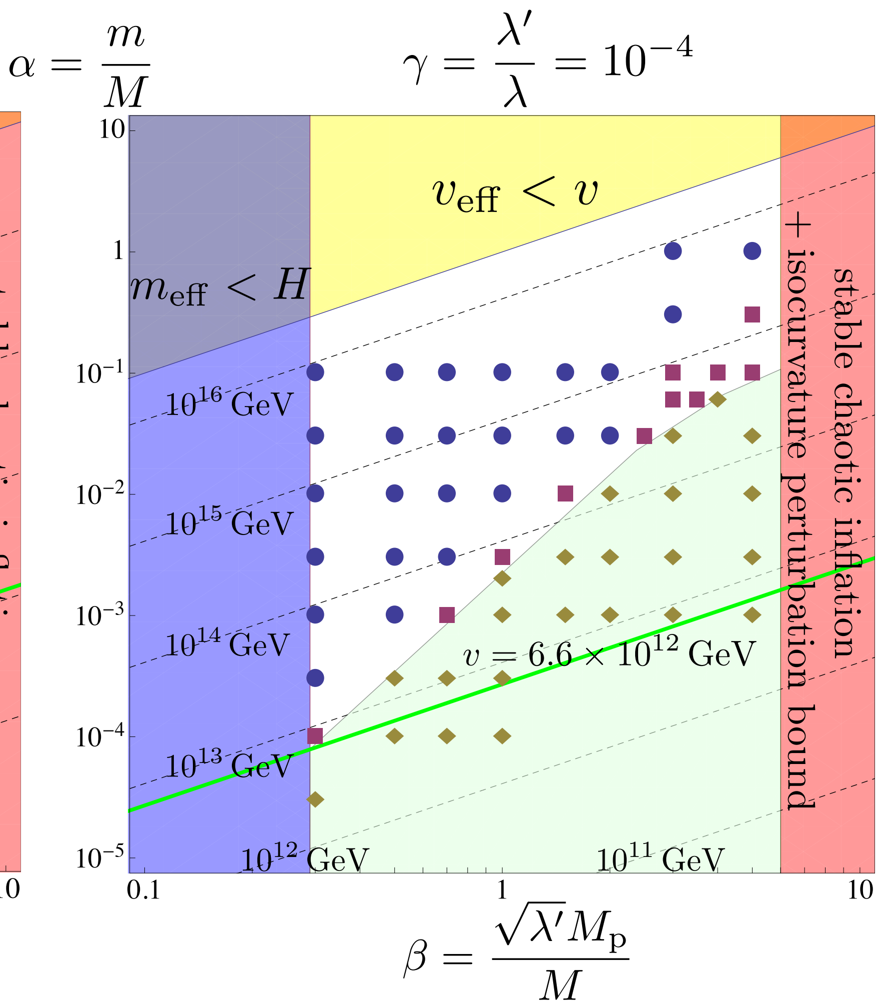
Result

● No String

■ Unstable String

◆ Stable String





Discussion

Constraints on the PQ breaking scale

$$v \gtrsim \begin{cases} 1.2 \times 10^{13}\,\mathrm{GeV} & \gamma = 10^{-3} \\ 6.6 \times 10^{12}\,\mathrm{GeV} & \gamma = 10^{-4} \end{cases}$$

(cf.) Axion can become a good candidate of dark matter.

Present energy density of axion $\lesssim$ Present energy density of dark matter
(Theoretical) (Observational)

⬇

$$v \lesssim 10^{12-13}\,\mathrm{GeV}$$

Linde’s model do not need any fine tuning.

Conclusion

By the lattice simulation of the dynamics
of Linde’s model after inflation,
it is proved that axion is consistent
with the chaotic inflation model without any parameter tuning.