

# SK でのステライルおよび非標準的 相互作用に関する制限

## Limits on Sterile Neutrinos and Non-Standard Neutrino Interactions at SK

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- SK searches:
  - **Sterile neutrino oscillation** (A. Himmel, Duke U.)  
<http://arxiv.org/abs/1410.2008>
  - **MeV sterile decay** (E. Richard, RCCN)
  - **Non Standard Neutrino Interactions** (G. Mitsuka, RCCN)  
<http://arxiv.org/abs/1109.1889>
  - **Lorentz Invariance Violation** (T. Akiri and A. Himmel, Duke U.)  
<http://arxiv.org/abs/1410.4267>

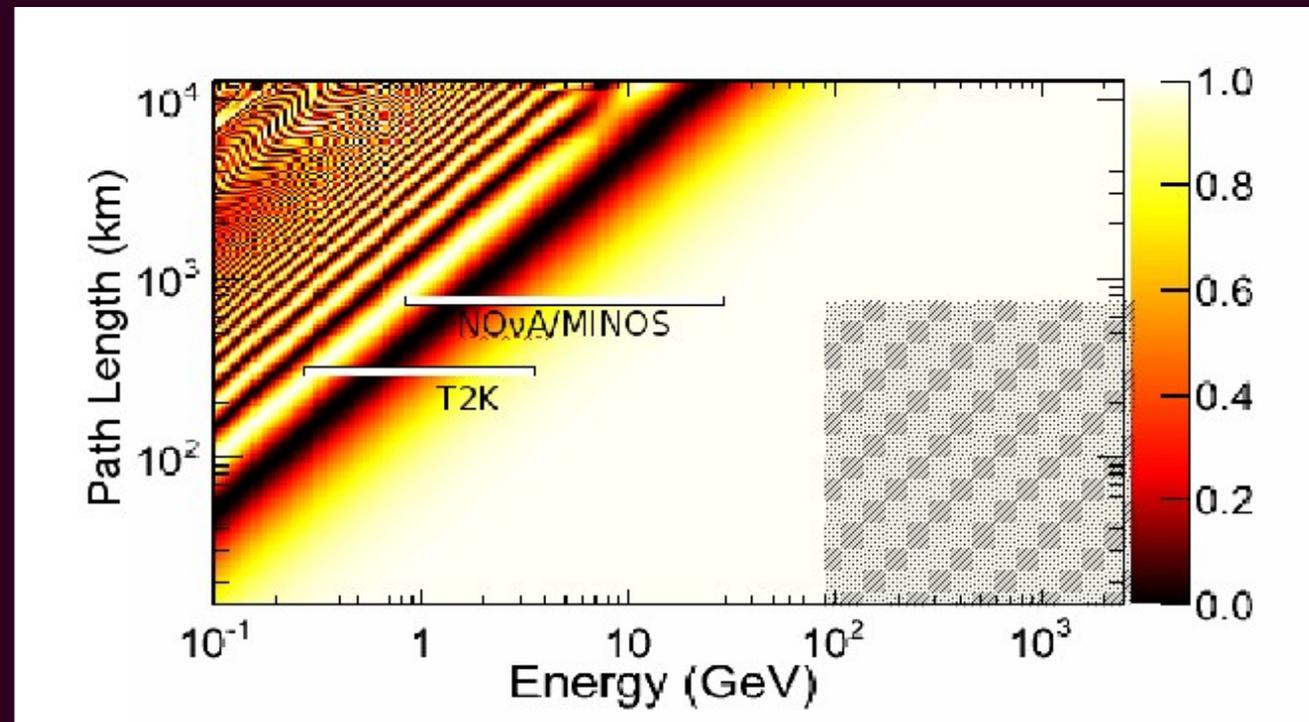
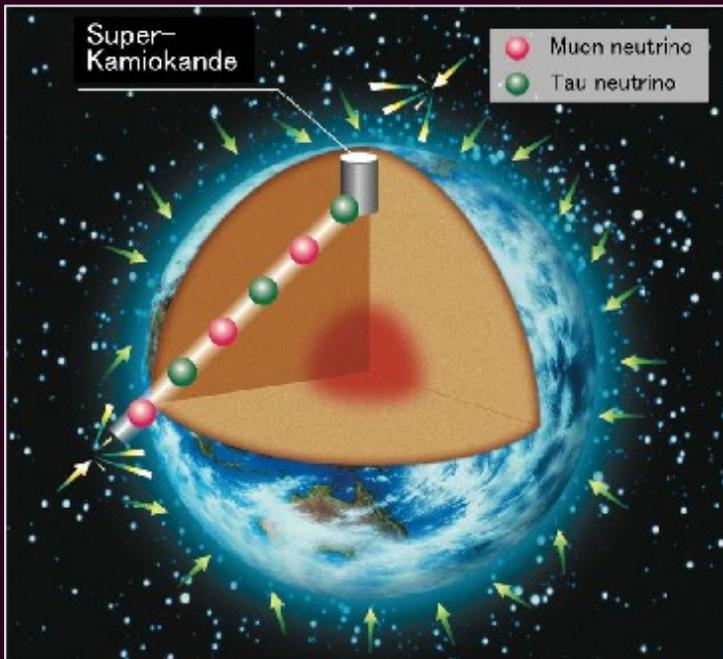
Thanks especially to A. Himmel for providing many of the slides and figures.

# Introduction to SK

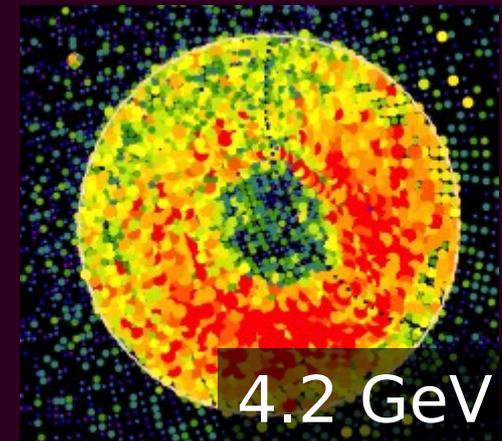
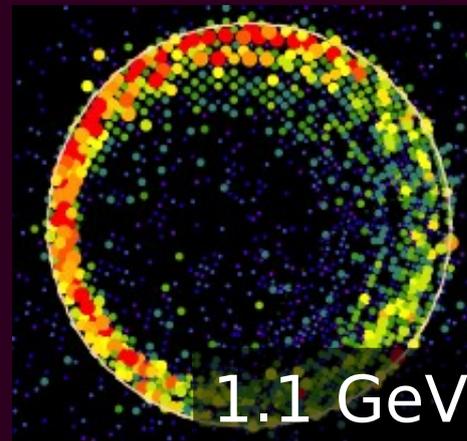
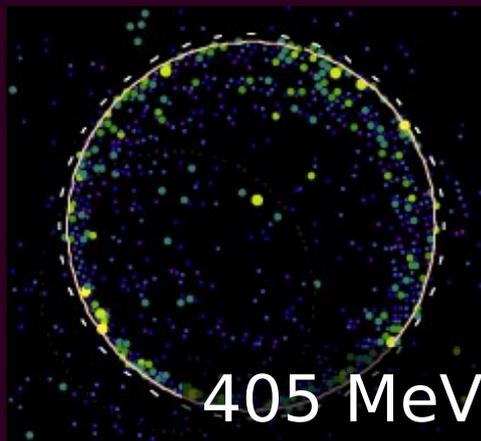
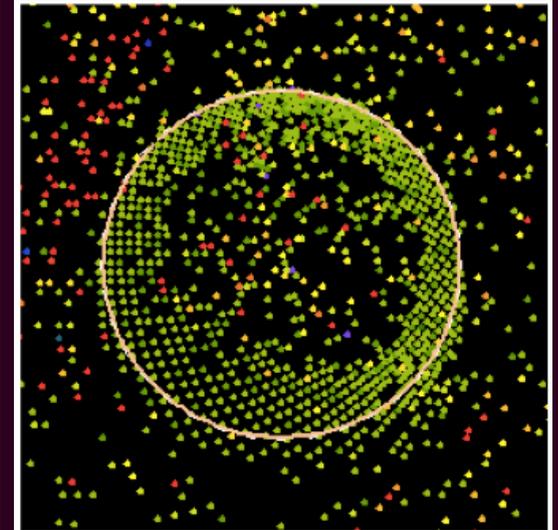
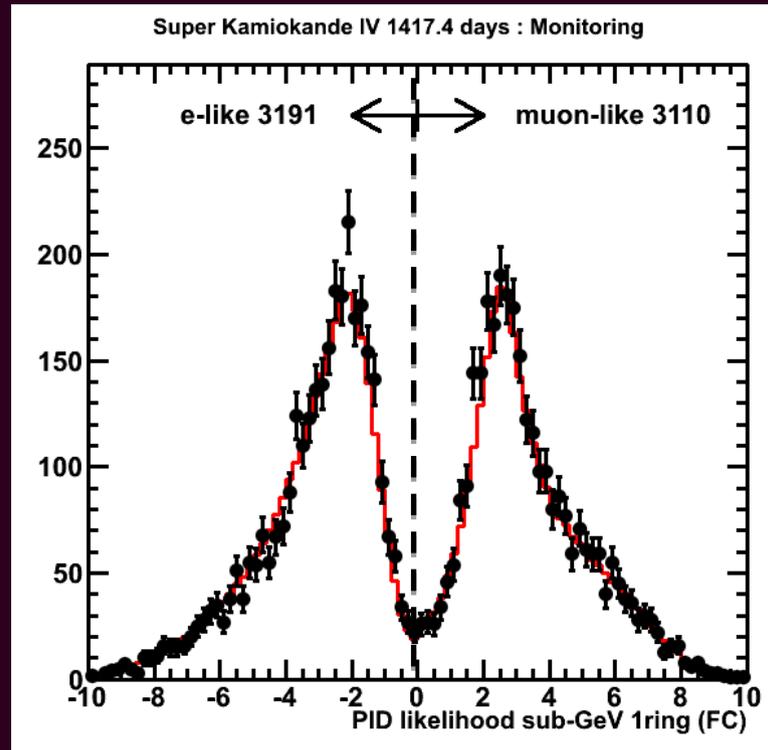
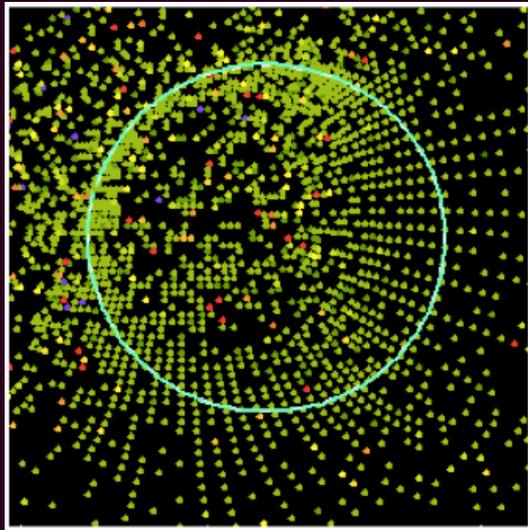
# Super-Kamiokande

- Atmospheric dataset provides access to wide range of  $L$  and  $E$ .

$$P(\nu_{\mu} \rightarrow \nu_{\mu})$$

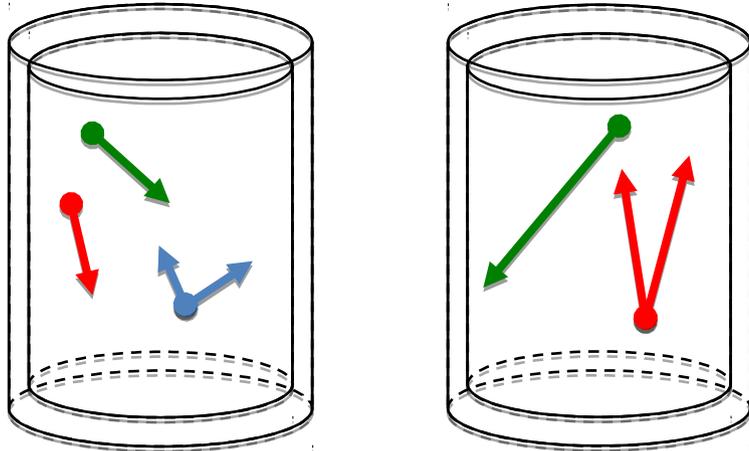


# Particle Identification

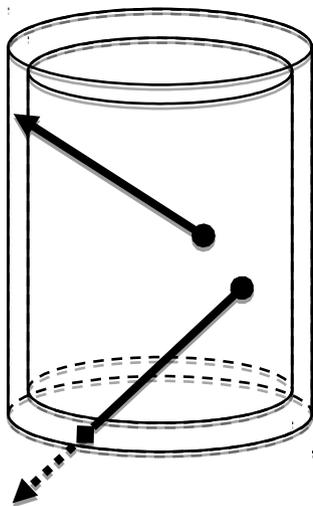


# Data Samples

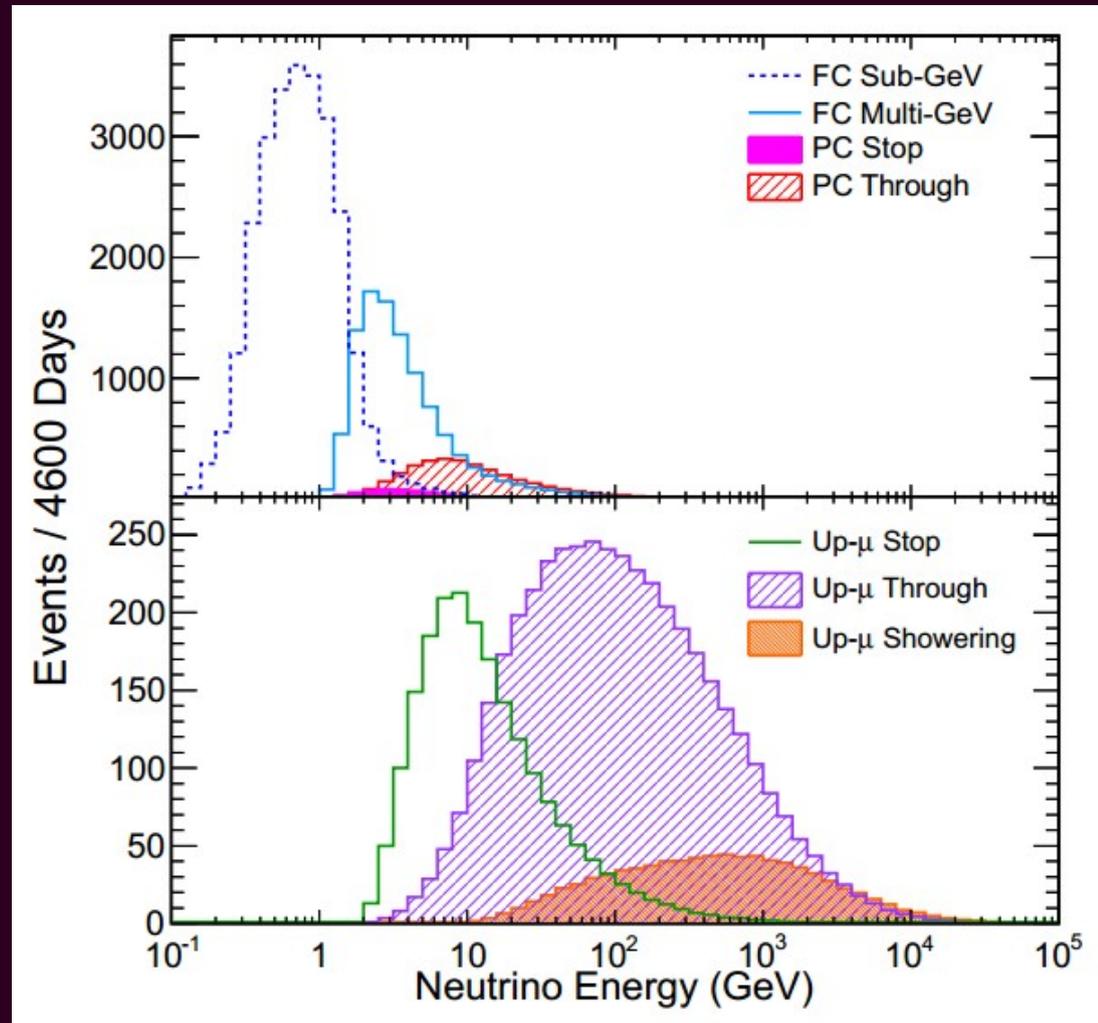
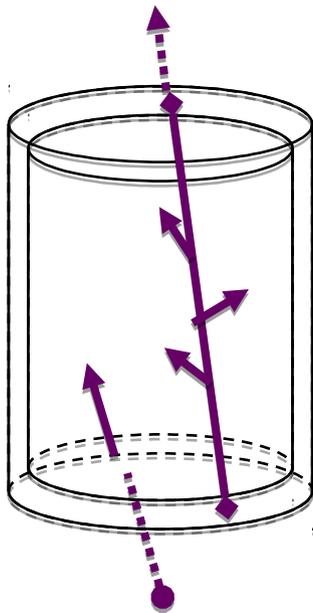
## Fully Contained



## Partially Contained



## Up-going $\mu$



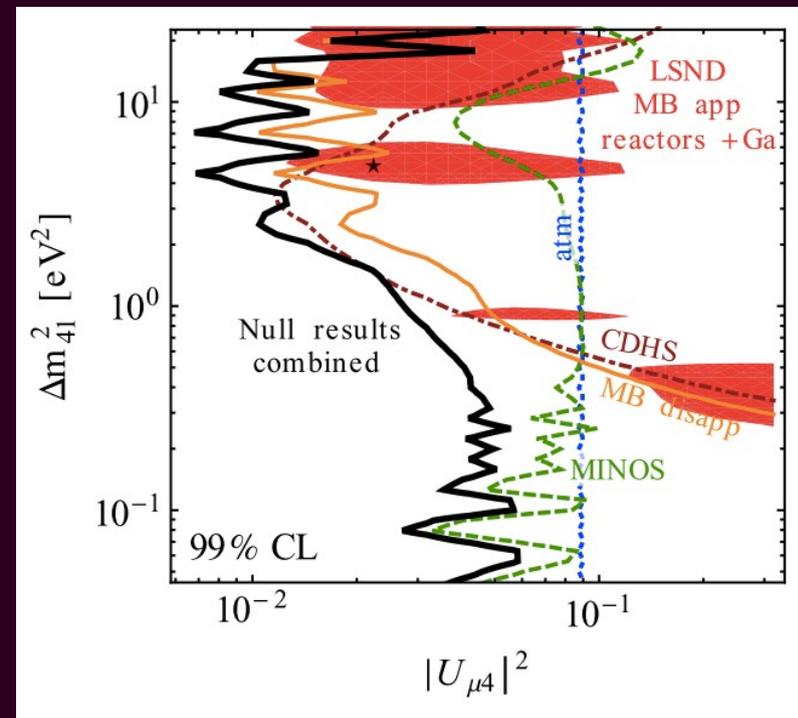
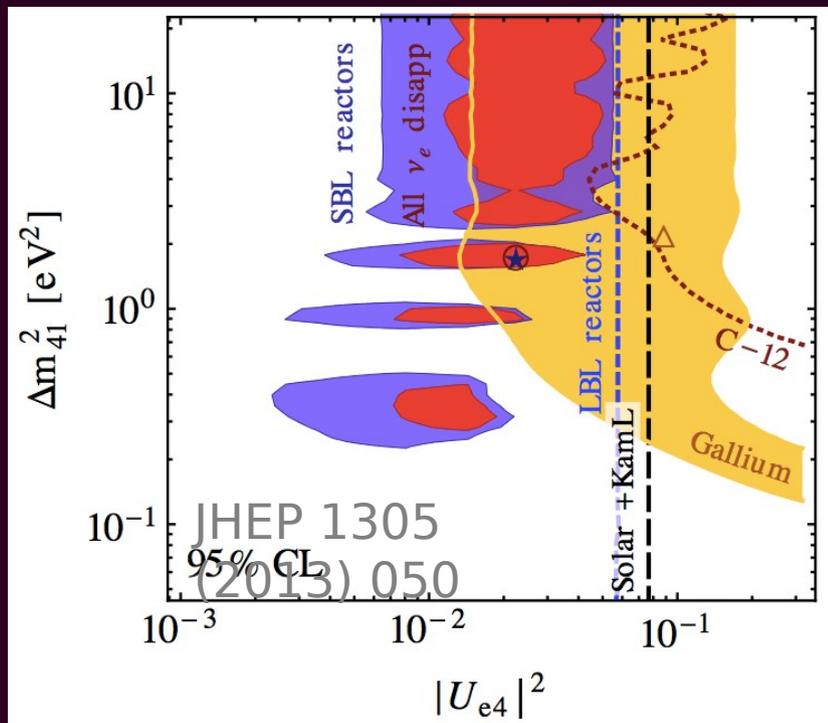
Sterile neutrino oscillation search

# Introduction

- 4<sup>th</sup> (5<sup>th</sup>, 6<sup>th</sup>...) neutrino mass state in the eV-scale may participate in neutrino oscillation.

$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

- Assuming 1 sterile, there is significant tension in current measurements (LSND, MiniBooNE, radioactive source, reactor).



# SK Approach

- Generally follow the approach in [1] with some simplifying assumptions for atmospheric sterile neutrinos.
  - Mass difference is large enough that oscillations are “fast”, i.e. the  $\sin(L / E)$  term can be approximated by a constant.
  - No  $\nu_e$ - $\nu_s$  oscillations, i.e.  $|U_{e4}| \sim 0$ .
  - Complex phases are negligible.
- The validity of these assumptions is discussed in the backup.
- Firstly consider only 1 sterile, but in a way that can be easily extended at the end to  $N$  steriles ( $3+N$  neutrinos).

# SK Approach

- Some care must be taken with “sterile matter effects”, e.g. the modified oscillation probabilities in the Earth:
  - $\nu_e$  has CC and NC interactions
  - $\nu_\mu$  and  $\nu_\tau$  have only NC interactions
  - $\nu_s$  have no interactions
- Difficult computationally to calculate sterile and standard ( $\nu_e$ ) matter effects at the same time, so two fits are performed:

$$H = UM^{(3+N)}U^\dagger + V_e + V_s.$$

$$M^{(3+N)} = \frac{1}{2E} \text{diag} \left( 0, \Delta m_{21}^2, \dots, \Delta m_{(3+N)1}^2 \right),$$

$$V_e = \pm(G_F/\sqrt{2}) \text{diag} (2N_e, 0, \dots)$$

$$V_s = \pm(G_F/\sqrt{2}) \text{diag} (0, 0, 0, N_n, N_n, \dots)$$

No- $\nu_e$ Fit	Sterile Vacuum Fit
<ul style="list-style-type: none"> <li>– Sterile matter effects</li> <li>– Fit for <math> U_{\tau 4} ^2 +  U_{\mu 4} ^2</math></li> <li>– Over-constrains <math> U_{\mu 4} ^2</math></li> </ul>	<ul style="list-style-type: none"> <li>– <math>\nu_e</math> matter effects</li> <li>– Fit for <math> U_{\mu 4} ^2</math> only</li> <li>– Most accurate <math> U_{\mu 4} ^2</math> limit, but no <math> U_{\tau 4} ^2</math> limit</li> </ul>

- The  $\nu_\mu$  survival probability in the no- $\nu_e$  approximation (3+1):

$$P_{\mu\mu} = (1 - |U_{\mu 4}|^2)^2 \overset{\nearrow}{P_{\mu\mu}^{(2)}} + |U_{\mu 4}|^4$$

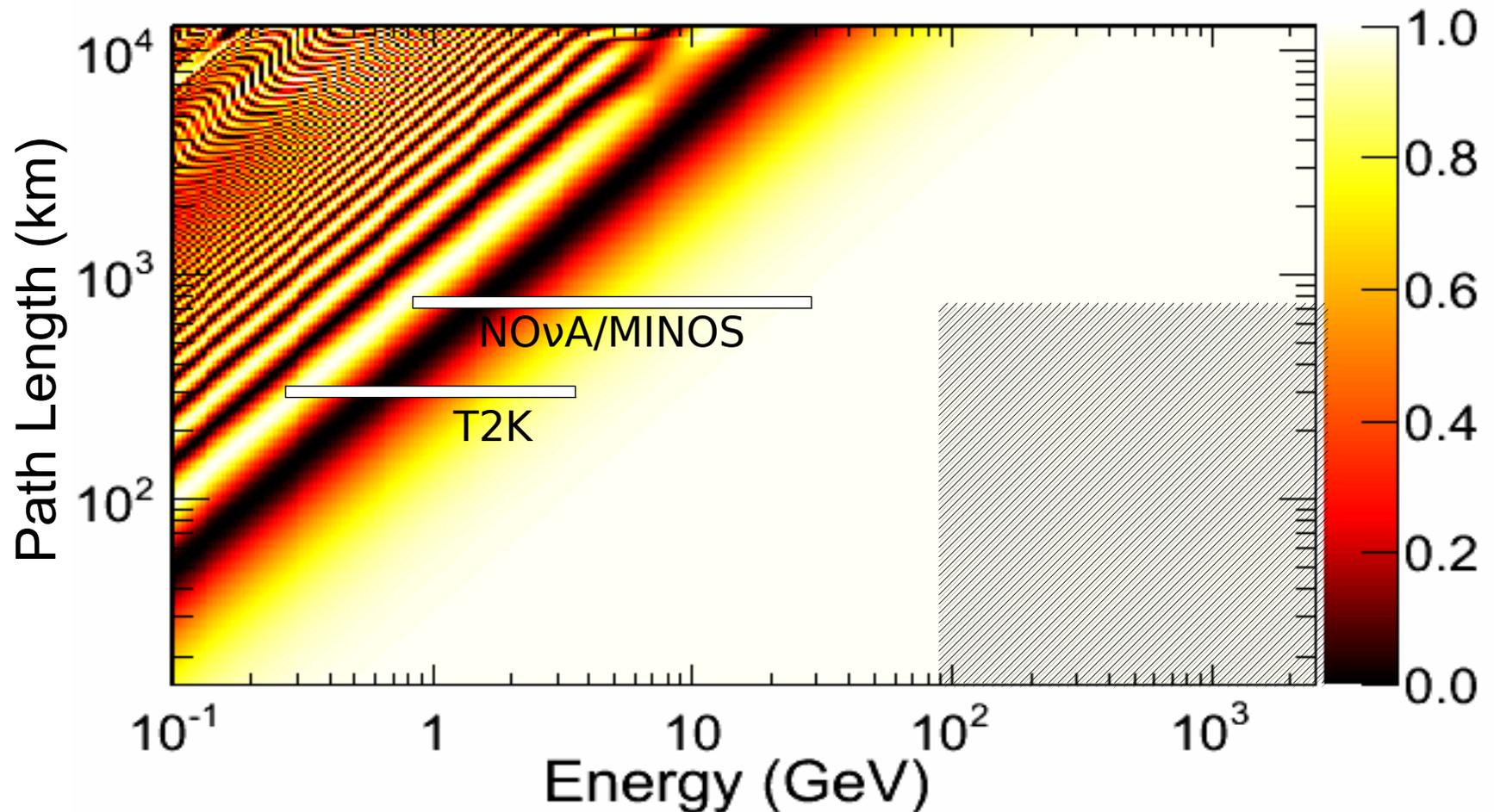
$$\begin{aligned} H^{(2)} &= H_{sm}^{(2)} + H_s^{(2)} : \\ &= \frac{\Delta m_{31}^2}{4E} \begin{pmatrix} -\cos 2\theta_{23} & \sin 2\theta_{23} \\ \sin 2\theta_{23} & \cos 2\theta_{23} \end{pmatrix} \pm \frac{G_F N_n}{\sqrt{2}} \begin{pmatrix} |\tilde{U}_{s2}|^2 & \tilde{U}_{s2}^* \tilde{U}_{s3} \\ \tilde{U}_{s2} \tilde{U}_{s3}^* & |\tilde{U}_{s3}|^2 \end{pmatrix} \end{aligned}$$

- $P^{(2)}$  represents the “standard” 2-flavour probability, plus the sterile matter effects
- $U_{si}$  can be written in terms of  $|U_{\tau 4}|^2$  and  $|U_{\mu 4}|^2$  in a 3+1 framework.

No- $\nu_e$  Fit

# $P(\nu_\mu \text{ to } \nu_\mu)$ Oscillogram

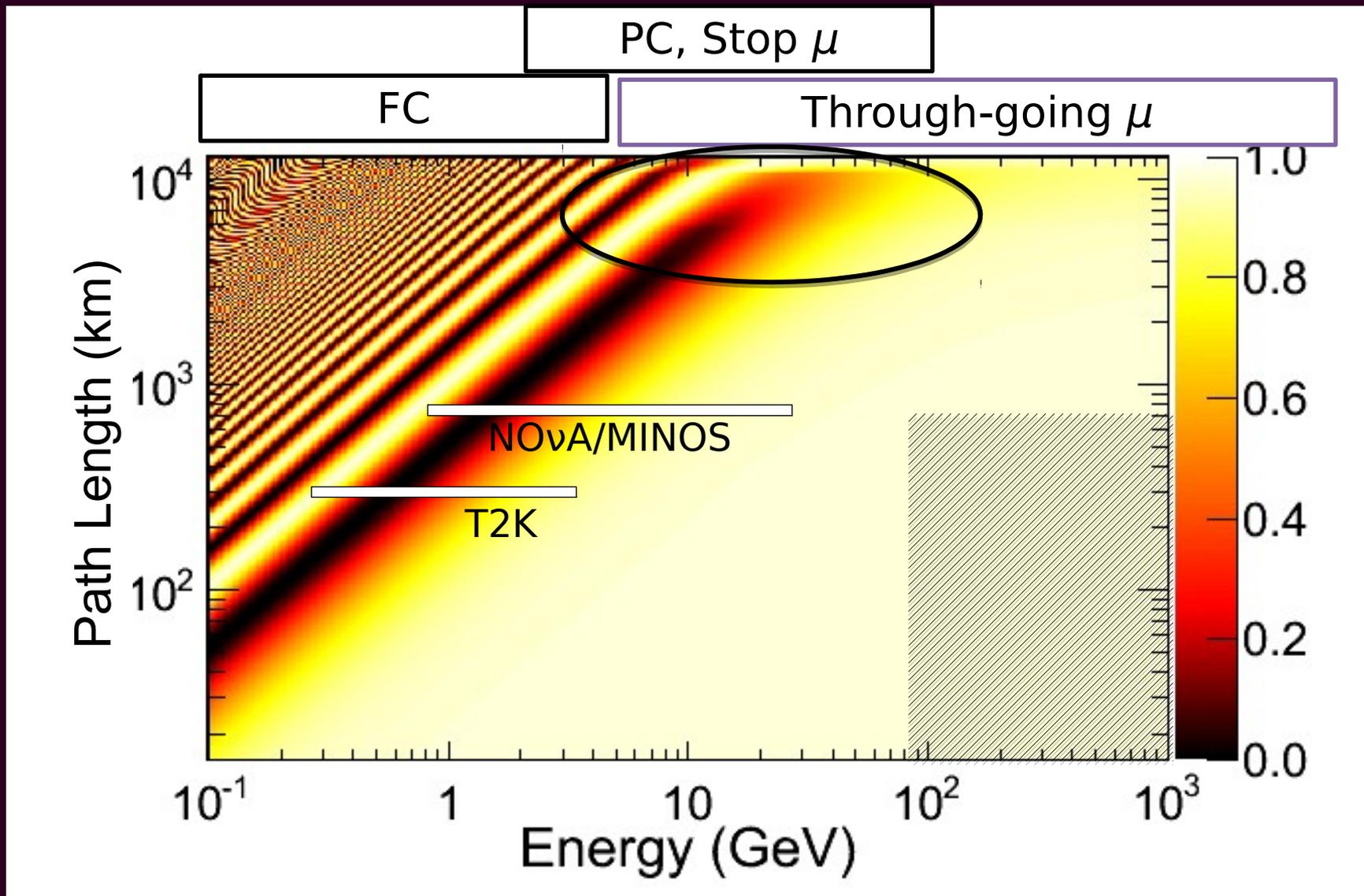
Standard oscillation - **no sterile effects**



No- $\nu_e$  Fit

# $P(\nu_\mu \text{ to } \nu_\mu)$ Oscillogram

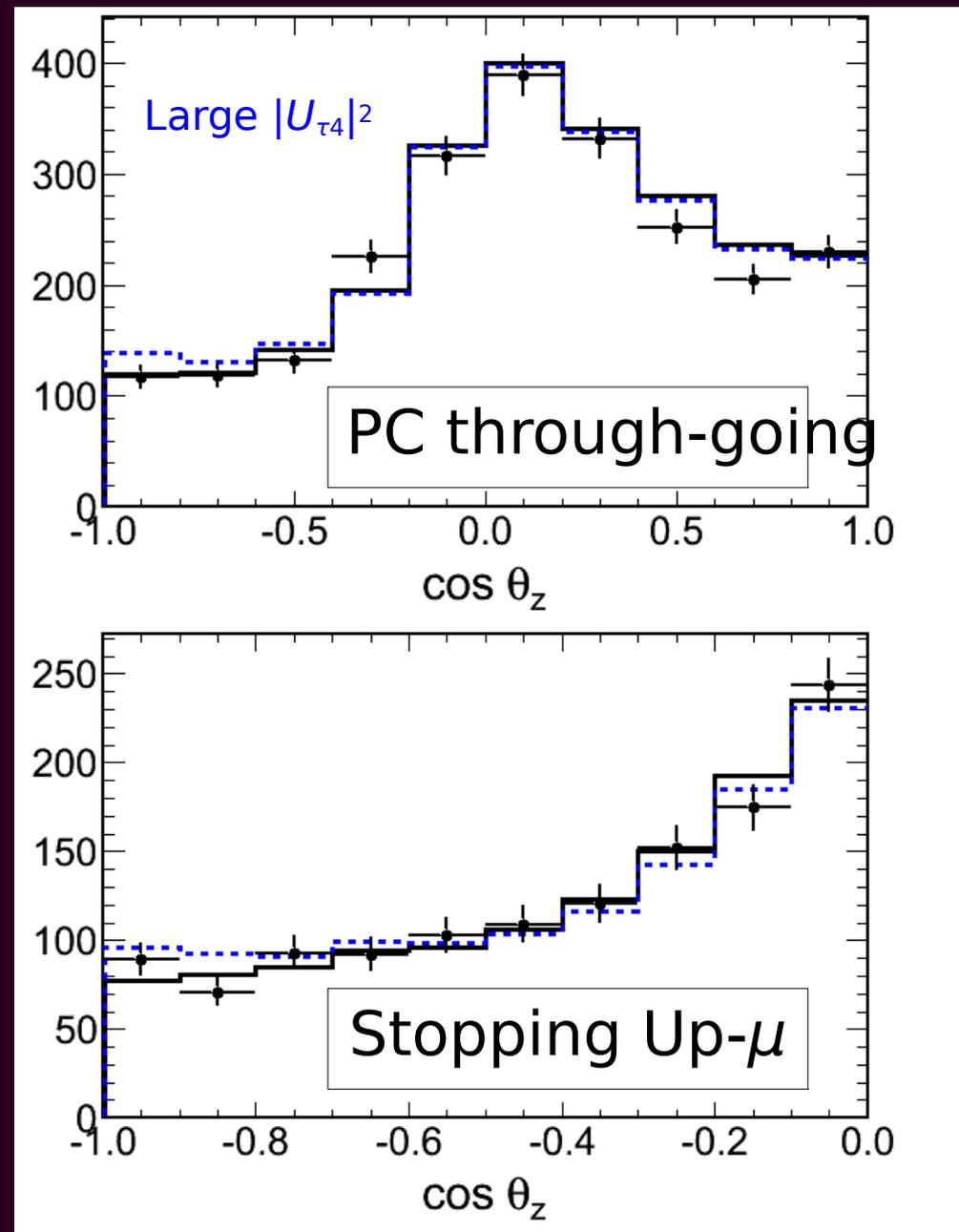
With sterile effects



# No- $\nu_e$ Fit

- Example distributions in some of the more sensitive samples.
- The fit procedure minimizes over all systematic errors to find the best fit for each hypothesis.
- Exclusion regions are found using the distance in  $\Delta\chi^2$  to the best fit point, and difference in dimensionality of the parameter space (Wilks' theorem).

# Fit Procedure



# No- $\nu_e$ Fit

# Results

Limit:

90 and 99% C.L. shown  
 $|U_{\tau 4}|^2 < 0.23$  at 99% C.L.

Best fit at:

$$|U_{\tau 4}|^2 = 0.021$$

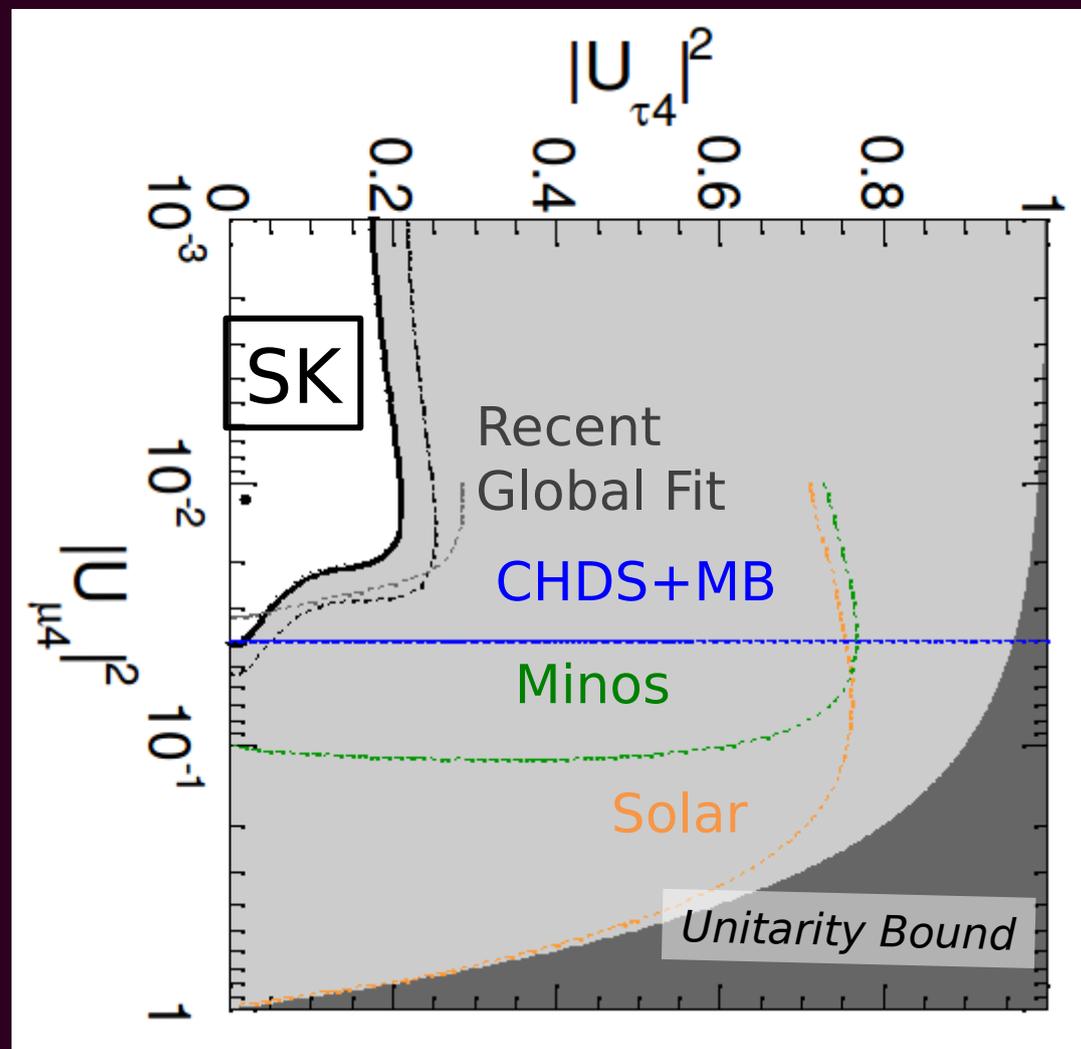
$$|U_{\mu 4}|^2 = 0.012$$

Goodness-of-fit:

$$\chi^2/\text{dof} = 531.1/480 \text{ (0.05)}$$

Favors  $\nu_\mu$  to  $\nu_\tau$  oscillations  
over  $\nu_\mu$  to  $\nu_s$  oscillations.

The  $|U_{\mu 4}|^2$  constraint is likely  
overestimated here.



- Matter term becomes

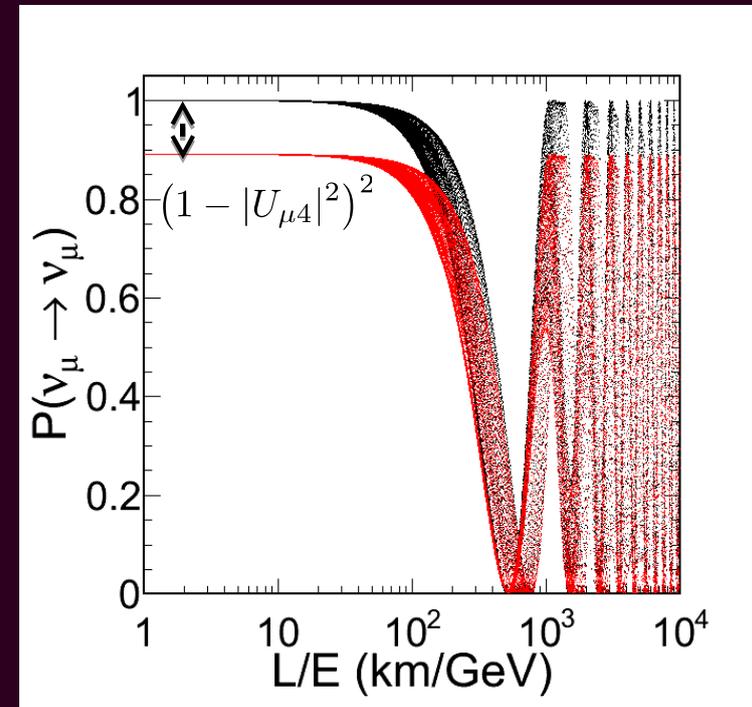
$$H^{(2)} = H_{\text{SM}} \pm \underbrace{\frac{G_F N_n}{\sqrt{2}} \sum_{\alpha} \begin{pmatrix} |\tilde{U}_{\alpha 2}|^2 & \tilde{U}_{\alpha 2}^* \tilde{U}_{\alpha 3} \\ \tilde{U}_{\alpha 2} \tilde{U}_{\alpha 3}^* & |\tilde{U}_{\alpha 2}|^2 \end{pmatrix}}_{H_s(A_s, \theta_s)}$$

- Extend to multiple sterile neutrinos with a sum over sterile species  $\alpha$
- Because it is a 2-level system, any number of sterile parameters reduce to 3:  $A_s, \theta_s, |U_{\mu 4}|^2$
- We also make the results of this fit available in these parameters.

- The  $\nu_\mu$  survival probability in 3+1:

$$P_{\mu\mu} = (1 - |U_{\mu 4}|^2)^2 P_{\mu\mu}^{(3)} + |U_{\mu 4}|^4$$

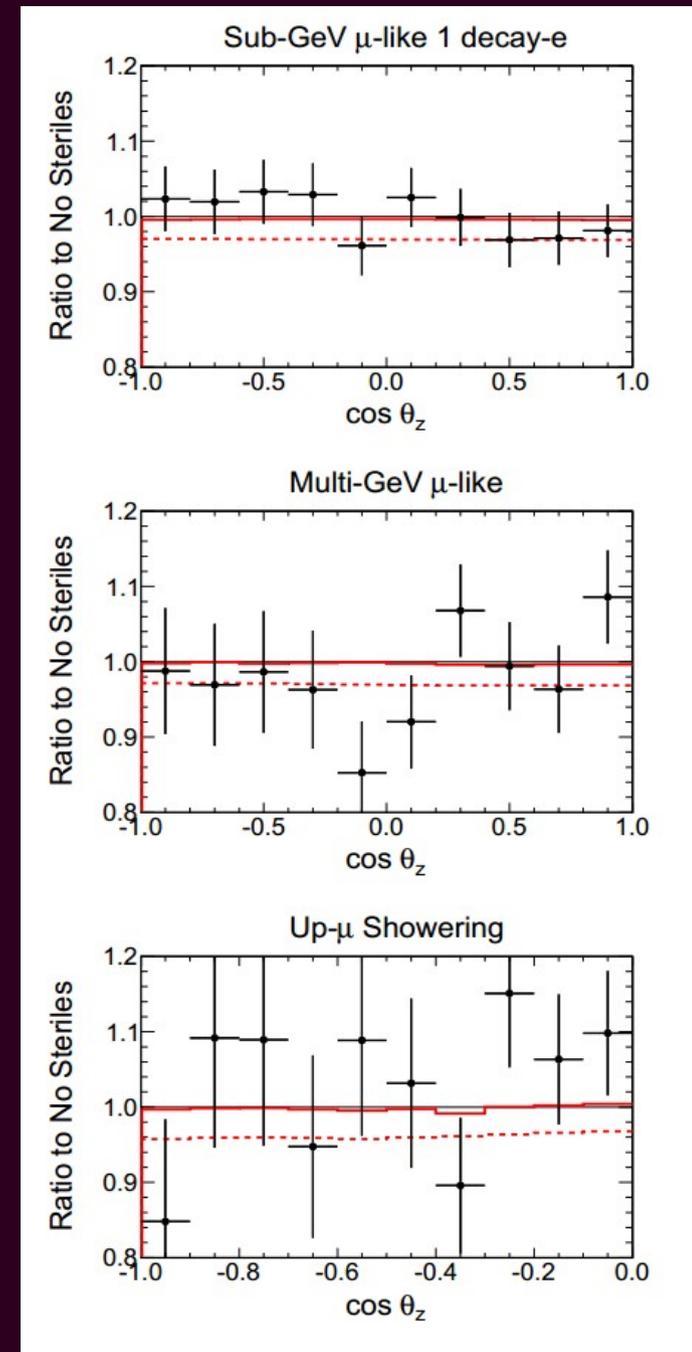
- Here,  $P^{(3)}$  is just the standard 3-flavor oscillation probability, including  $\nu_e$  matter effects.
- Can't derive a limit for  $|U_{\tau 4}|^2$
- No oscillogram - just a drop in normalization of all  $\mu$  samples.
- Probability is not unique at each L/E due to matter effects being dependent on L and E individually.



## Sterile Vacuum Fit

- Fit is then essentially systematics-limited.
  - The normalization of the  $e$  samples are used to constrain the normalization of the  $\mu$  samples more accurately.
- Best fit  $|U_{\mu 4}|^2 = 0.016$ 
  - Figure shows the best fit including minimization over systematics (red line), and the same sterile parameters without shifting the systematics (red dotted).

## Fit Procedure



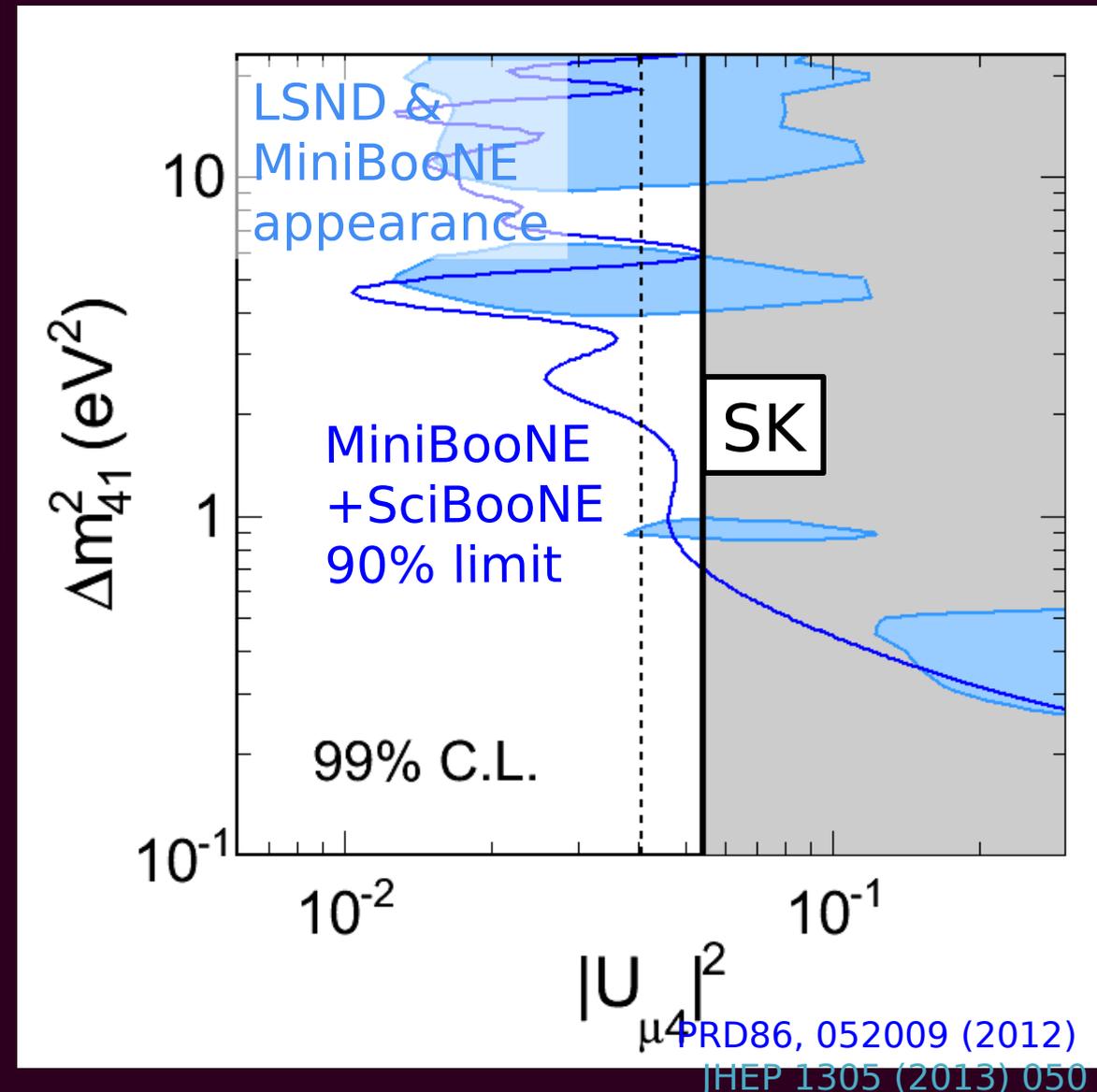
Best fit:  $|U_{\mu 4}|^2 = 0.016$

$|U_{\mu 4}|^2 < 0.041$  at 90% C.L.

$|U_{\mu 4}|^2 < 0.054$  at 99% C.L.

Sensitivity: 0.024 at 90%

- No strong sterile-driven  $\nu_\mu$  disappearance.
- $\Delta\chi^2$  of 1.1 between the best fit and no sterile neutrinos.
- Analysis is systematics limited.



- The  $\nu_\mu$  survival probability in 3+N:

$$P_{\mu\mu} = (1 - d_\mu^2)^2 P_{\mu\mu}^{(3)} + \sum_{\alpha} |U_{\alpha 4}|^4$$

$$d_\mu = \sum_{\alpha} |U_{\alpha 4}|^2$$

- Very similar to the 3+1 formula with  $|U_{\mu 4}|^2 \rightarrow d_\mu$

MeV sterile decay search

# “Heavy” Sterile Neutrinos

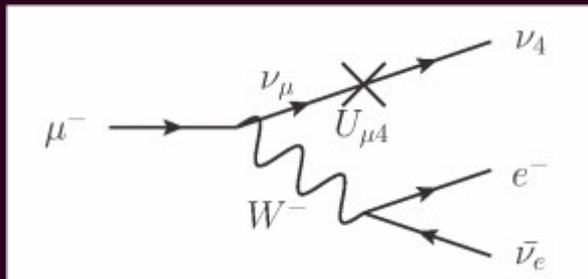
- A state  $m_4 \gtrsim \text{keV}$  is separated from the oscillation effects.
- The phenomenology varies depending on the mass, and in some cases we may have observable decay products.
  - For example, take  $m_4 \sim \text{MeV}$ 
    - Motivated by e.g.  $\nu\text{MSM}$  - standard Seesaw mechanism, but Majorana masses  $M_I$  are chosen below electroweak scale.

# MeV Sterile Decay

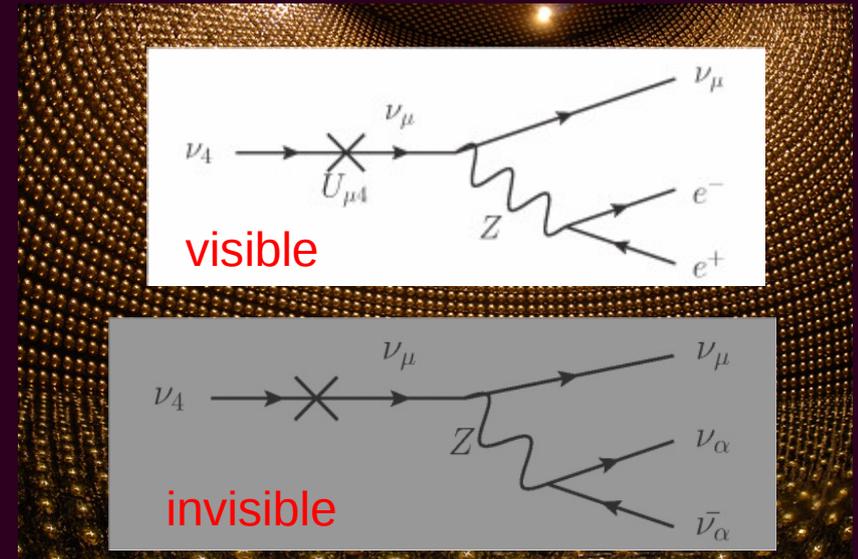
- Considering  $10 \lesssim m_4 \lesssim 100$  MeV, a heavy neutrino  $\nu_4$  that mixes with  $\nu_\mu$  may be produced in atmosphere by  $\mu$ ,  $K$ , or  $\pi$  decay.
  - We consider only the mixing parameter  $|U_{\mu 4}|^2$  as electron-mixing is already excluded at 2~3 orders of magnitude lower in this region, and atmospheric decay has negligible  $\tau$  component.
  - Visible decay products, for example Super-Kamiokande can see the decay below, with two electron-like Čerenkov rings.

SK

Cosmic ray



Atmosphere

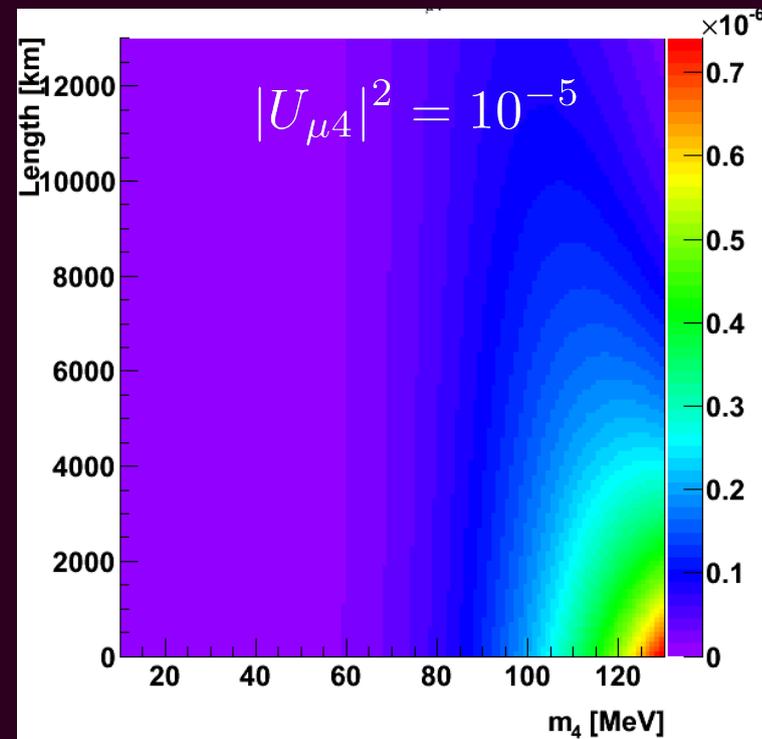


# Sterile Simulation

- Using Honda-flux atmospheric MC, events are reweighted by “sterile creation probability”, e.g. for muon decay:

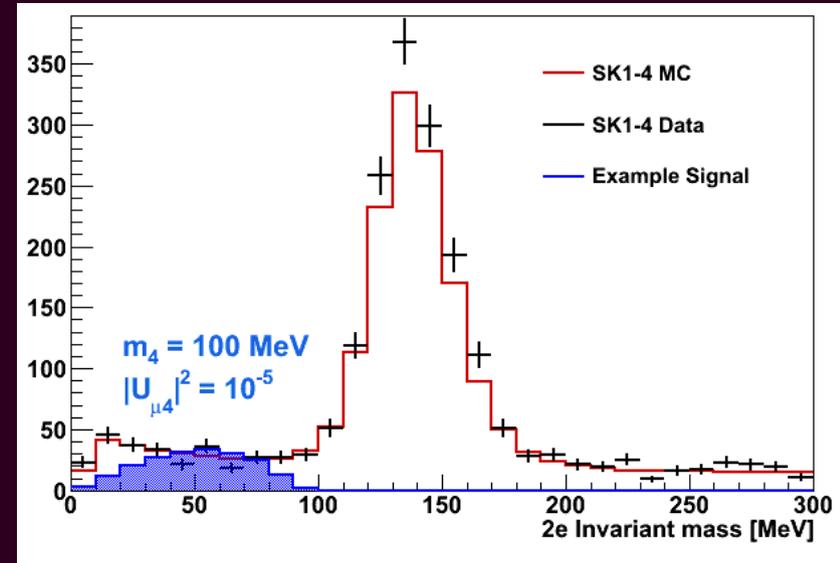
$$\frac{\Gamma(\mu^- \rightarrow e^- \nu_e \nu_4)}{\Gamma(\mu^- \rightarrow e^- \nu_e \nu_\mu)} = |U_{\mu 4}|^2 (1 - 8r + 8r^3 - r^4 - 24r^2 \ln(r)) \quad r = \left(\frac{m_4}{m_\mu}\right)^2$$

- Neutrinos from pion and kaon decays also reweighted.
- Then track event probability to decay inside of SK.
  - Path-length dependency  
= zenith angle dependency

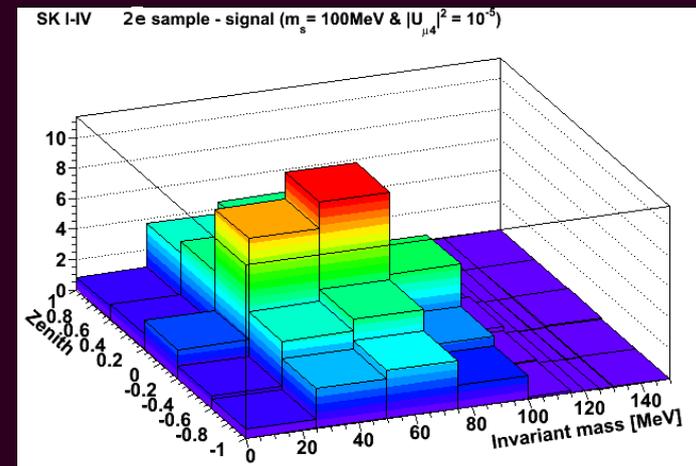
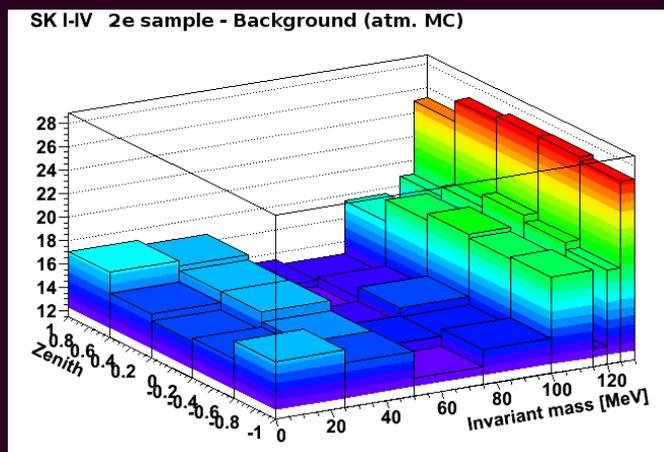


# Fit Method

- The visible decay is a 3-body decay ( $e^+$ ,  $e^-$ ,  $\nu_\mu$ ) so we should see a signal distribution (not peak) in the invariant mass distribution of the e-like two-ring event sample.
  - Figure shows MC truth.



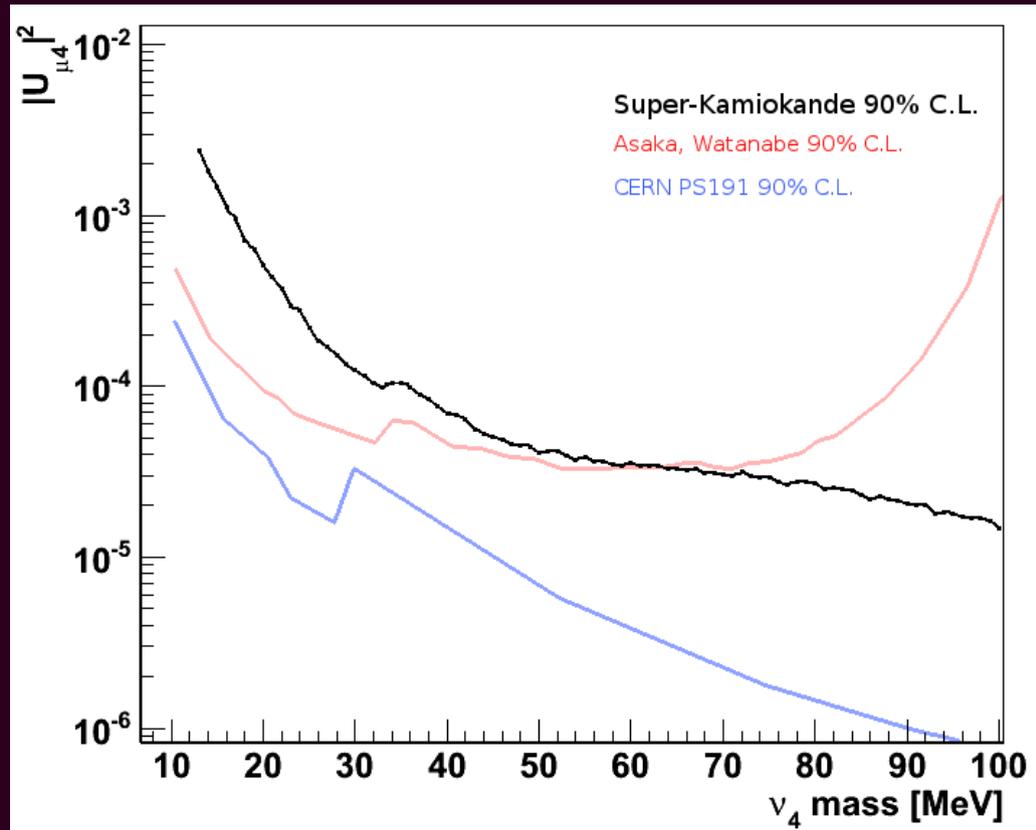
- So we see signal in  $e^+ e^-$  invariant mass and zenith angle.



- Fit procedure similar to oscillation case (minimizing over systematics).

# Results

- Compared to **SK study by T. Asaka**, our internal SK study uses the Honda-flux prediction and full knowledge of the detector and data-set. Contribution to steriles from Kaon-decay is also estimated.
- The final extracted limit is still below **CERN PS191**.
- There are some easy extensions to the phenomenology for higher masses  $\sim 400$  MeV.
- Depending on the mass region, need to consider phenomenology more, but SK (and especially HK) may have some good search power regions.



# Non Standard Neutrino Interactions

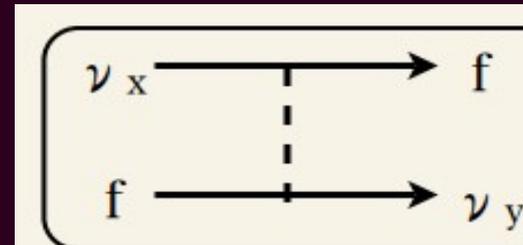
# NSI - Introduction

- A very general model for non-standard neutrino interactions (NSI) with matter can be introduced with the Hamiltonian

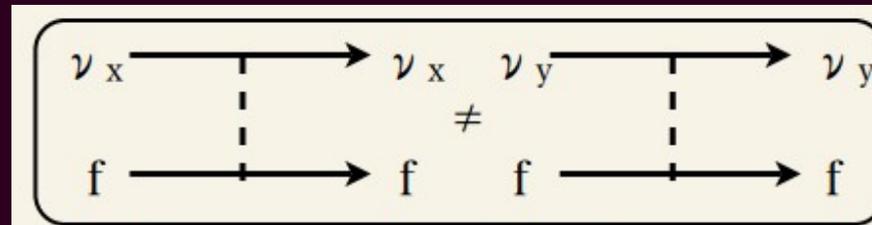
$$H_{\alpha\beta} = \frac{1}{2E} U_{\alpha j} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} (U^\dagger)_{k\beta} + V_{\text{MSW}} + \sqrt{2} G_F N_f(\vec{r}) \begin{pmatrix} \varepsilon_{ee} & \varepsilon_{e\mu}^* & \varepsilon_{e\tau}^* \\ \varepsilon_{e\mu} & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau}^* \\ \varepsilon_{e\tau} & \varepsilon_{\mu\tau} & \varepsilon_{\tau\tau} \end{pmatrix}$$

which represents respectively: the standard neutrino oscillation, the standard matter effect, and the NSI; for a flavour change  $\nu_\alpha \rightarrow \nu_\beta$ .

- The NSI matrix includes
  - Flavour Changing Neutral Current (FCNC, the off-diagonal  $\varepsilon_{xy}$ ).



- Lepton Non-Universality (NU, the on-diagonal  $\varepsilon_{xx}$ ).



- Motivations from R-parity violating SUSY, neutral heavy leptons...

# SK Search

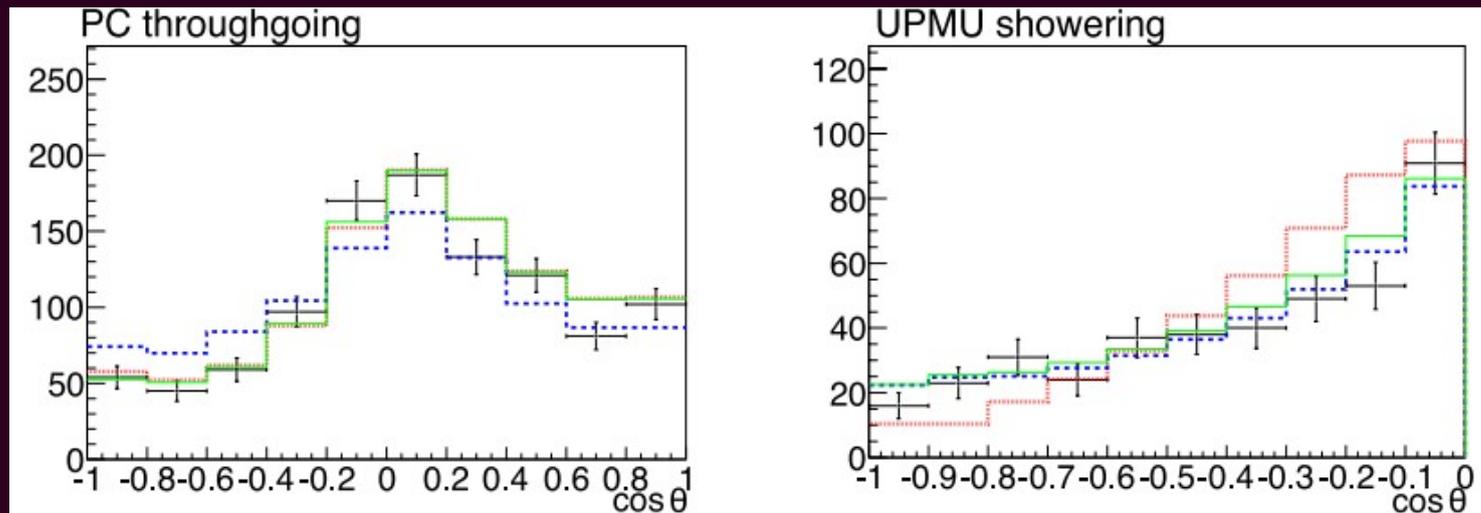
- Two methods are adopted that can simultaneously test NSI and neutrino oscillation in the atmospheric data.
- Two-flavour approach
  - NSI coexists with the dominant  $\nu_\mu \leftrightarrow \nu_\tau$  atmospheric oscillations.
  - $\nu_e$  is completely decoupled and ignored (no MSW effect).
    - Constrains  $\varepsilon_{\mu\mu}$ ,  $\varepsilon_{\mu\tau}$ , and  $\varepsilon_{\tau\tau}$ .
- Three-flavour “hybrid” approach
  - $\nu_\mu \leftrightarrow \nu_\tau$  atmospheric oscillations and  $\nu_e \leftrightarrow \nu_\tau$  NSI.
    - Constrains  $\varepsilon_{ee}$ ,  $\varepsilon_{e\tau}$ , and  $\varepsilon_{\tau\tau}$ .

# Two-Flavour approach

- Following [2], assuming NSI is dominated by d-quark interactions, we define  $\varepsilon = \varepsilon_{\mu\tau}$  (FCNC part) and  $\varepsilon' = \varepsilon_{\tau\tau} - \varepsilon_{\mu\mu}$  (NU part).
- Survival probability is somewhat complicated (backup), but we have an effective mixing angle  $\Theta$  and oscillation wavelength correction factor  $R$ , which are dependent on  $\varepsilon$ ,  $\varepsilon'$  and the neutrino energy  $E$ .

$$P_{\nu_{\mu} \rightarrow \nu_{\mu}} = 1 - P_{\nu_{\mu} \rightarrow \nu_{\tau}} = 1 - \sin^2 2\Theta \sin^2 \left( \frac{\Delta m_{23}^2 L}{4E} R \right)$$

- Predicted effect thus in different samples for  $\varepsilon$  (FCNC) and  $\varepsilon'$  (NU):

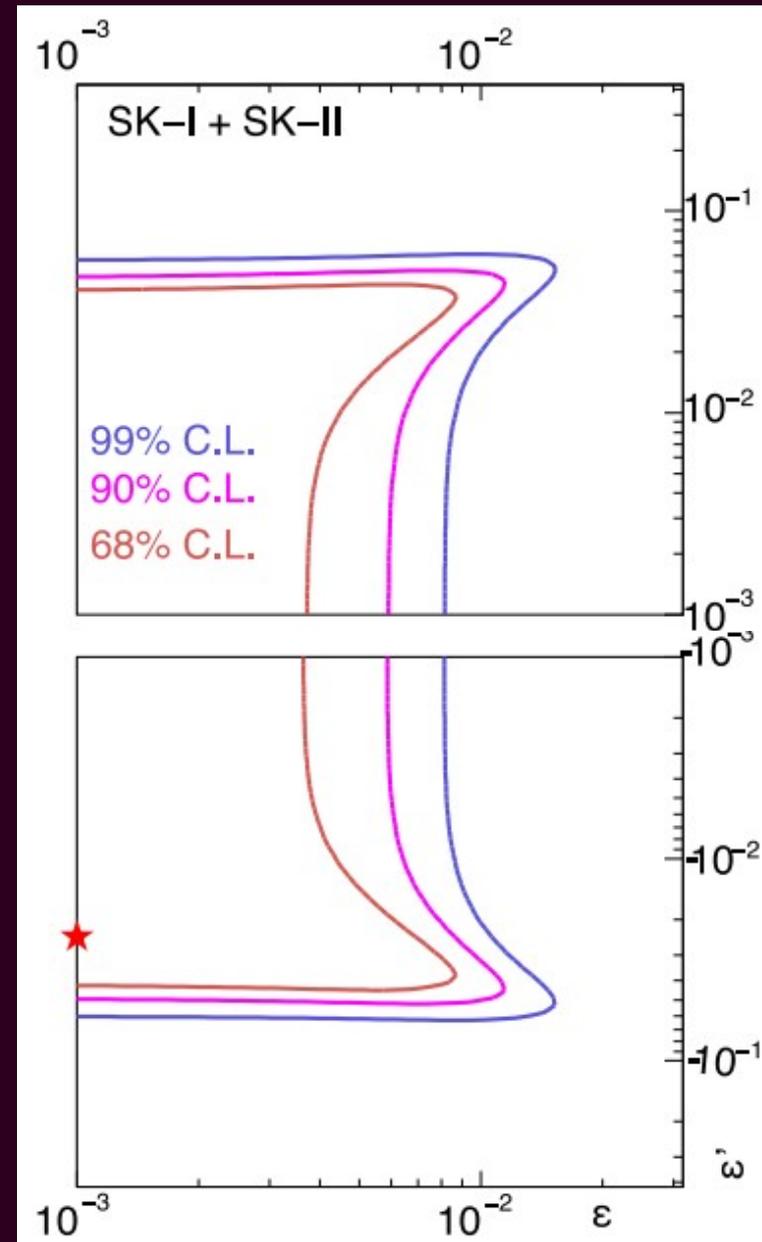


# Two-flavour Results

- Best fit information:

$$\begin{aligned}\sin^2 2\theta_{23} &= 1.00, & \Delta m_{23}^2 &= 2.2 \times 10^{-3} \text{eV}^2, \\ \varepsilon &= 1.0 \times 10^{-3}, & \varepsilon' &= -2.7 \times 10^{-2} \\ \chi_{min}^2 &= 838.9 / 746.0 \text{ d.o.f.}\end{aligned}$$

- $\varepsilon$  (FCNC) limits are tighter compared to other experiments, while  $\varepsilon'$  (NU) limits are not quite as strong.



$\varepsilon' = +1$

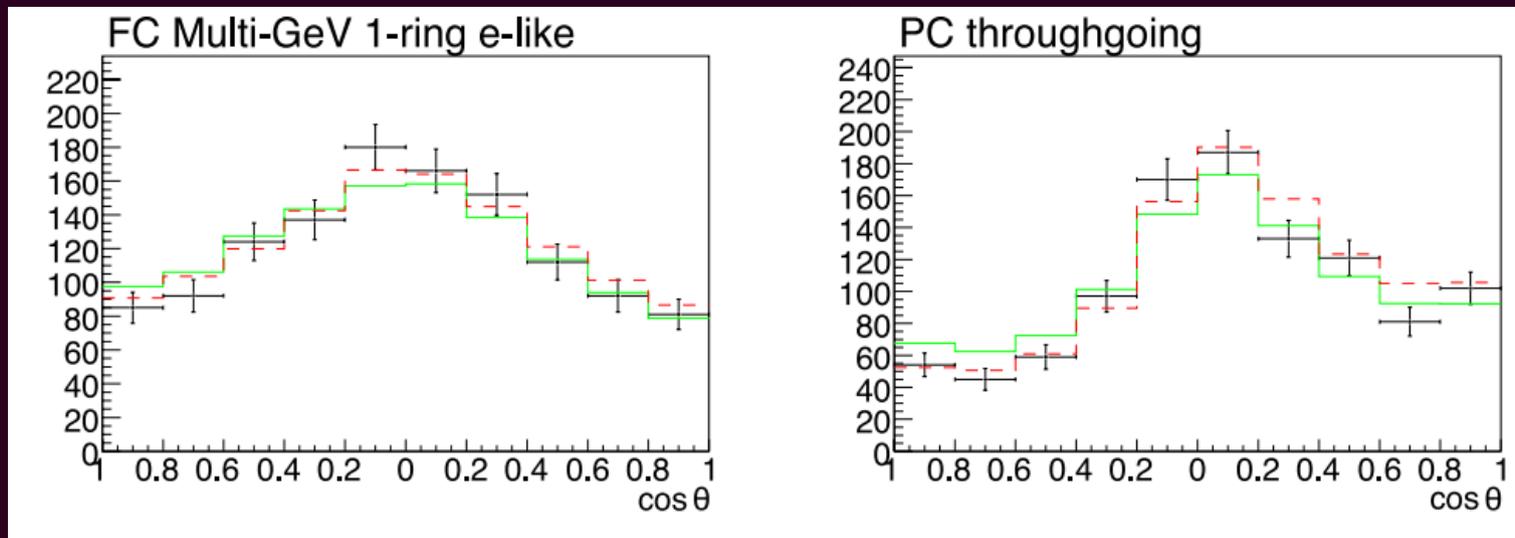
$\varepsilon' = -1$

# Three-Flavour approach

- In the three-flavour case, the effective Hamiltonian becomes

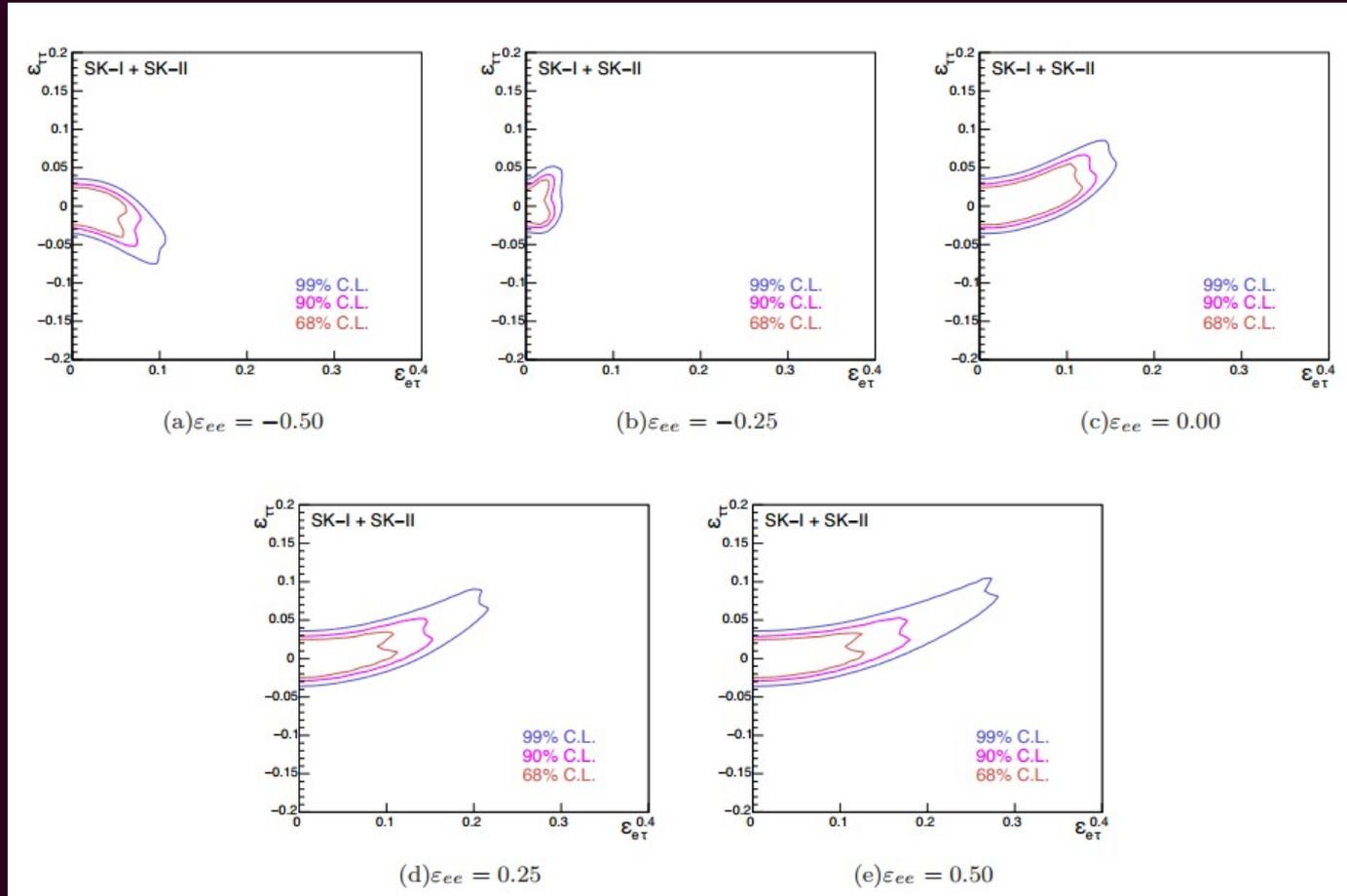
$$H_{\text{hybrid}} = \underbrace{\frac{1}{2E} U_{\alpha j} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix}_{jk} (U)^\dagger_{k\beta}}_{\text{vacuum term}} + \underbrace{V_{MSW} + \sqrt{2}G_F N_f \begin{pmatrix} \epsilon_{ee} & 0 & \epsilon_{e\tau} \\ 0 & 0 & 0 \\ \epsilon_{e\tau}^* & 0 & \epsilon_{\tau\tau} \end{pmatrix}}_{\text{matter term}}$$

- One drawback of this model is that as  $\epsilon_{e\tau} \rightarrow 0$ , eigenstates revert back to the vacuum ones, and there is no ability to constrain  $\epsilon_{ee}$ .
- Example distributions:  $\epsilon_{ee}=0, \epsilon_{e\tau}=0, \epsilon_{\tau\tau}=0$  and  $\epsilon_{ee}=0, \epsilon_{e\tau}=0.2, \epsilon_{\tau\tau}=0.2$



# Three-Flavour Results

- Results given at fixed  $\varepsilon_{ee}$  for  $\varepsilon_{e\tau}$  and  $\varepsilon_{\tau\tau}$ ,  $\theta_{23}$  and  $\Delta m^2_{23}$  integrated out.



- Modern values of  $\theta_{13}$  and the SK III-IV dataset should result in some improvements of these constraints.

# Lorentz Invariance Violation search

# Introduction

- Violations of Lorentz invariance are predicted at the Planck scale by a variety of models, such as space-time foam interactions.
- The Standard Model Extension (SME) adds to the Standard Model all possible Lorentz-Violating (LV) terms.
  - Terms may be directional (indicating a preferred spatial direction) or isotropic.
  - Neutrino oscillations are a sensitive probe of these coefficients.
    - In this analysis, we focus on isotropic coefficients (effects relating to L and E).

Coefficient	Unit	$d$	$CPT$	Oscillation Effect
<b>Isotropic</b>				
$a_{\alpha\beta}^T$	GeV	3	odd	$\propto L$
$c_{\alpha\beta}^{TT}$	-	4	even	$\propto LE$
<b>Directional</b>				
$a_{\alpha\beta}^X, a_{\alpha\beta}^Y, a_{\alpha\beta}^Z$	GeV	3	odd	sidereal variation
$c_{\alpha\beta}^{XX}, c_{\alpha\beta}^{YZ}, \dots$	-	4	even	sidereal variation

TABLE I. Lorentz-violating coefficients and their properties. The last row includes all possible combinations of  $X, Y, Z$ , and  $T$  except  $TT$ .  $d$  refers to the dimension of the operator.  $\alpha$  and  $\beta$  range over the neutrino flavors,  $e, \mu$ , and  $\tau$ . The  $X, Y$ , and  $Z$  indicate coefficients which introduce effects in a particular direction in a Lorentz-violating preferred reference frame. The  $T$  and  $TT$  terms are not associated with any direction and thus introduce isotropic distortions in the oscillation pattern.

# Oscillations

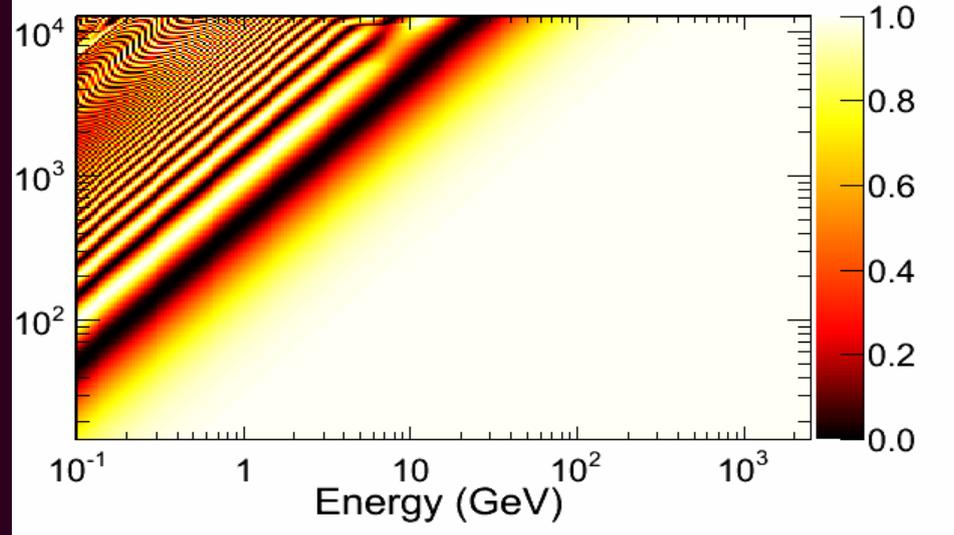
- The neutrino Hamiltonian (3-flavour oscillation, matter and LV terms):

$$H = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_{31}^2}{2E} \end{pmatrix} U^\dagger \pm \sqrt{2} G_F \begin{pmatrix} N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \pm \begin{pmatrix} 0 & a_{e\mu}^T & a_{e\tau}^T \\ (a_{e\mu}^T)^* & 0 & a_{\mu\tau}^T \\ (a_{e\tau}^T)^* & (a_{\mu\tau}^T)^* & 0 \end{pmatrix} - E \begin{pmatrix} 0 & c_{e\mu}^{TT} & c_{e\tau}^{TT} \\ (c_{e\mu}^{TT})^* & 0 & c_{\mu\tau}^{TT} \\ (c_{e\tau}^{TT})^* & (c_{\mu\tau}^{TT})^* & 0 \end{pmatrix}$$

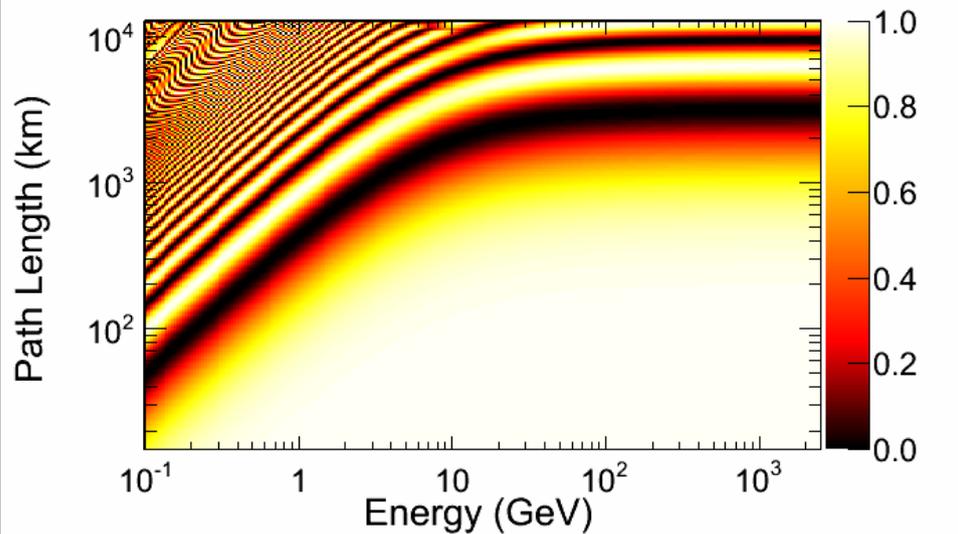
- Unlike NSI, the new terms are not matter effects.
- Diagonal terms cannot be observed.
- Perturbation method on the Hamiltonian was found to be inappropriate over the large range of L and E in the SK dataset.
  - First analysis to use the exact diagonalization of  $H$ .

# $P(\nu_\mu \rightarrow \nu_\mu)$ Oscillations

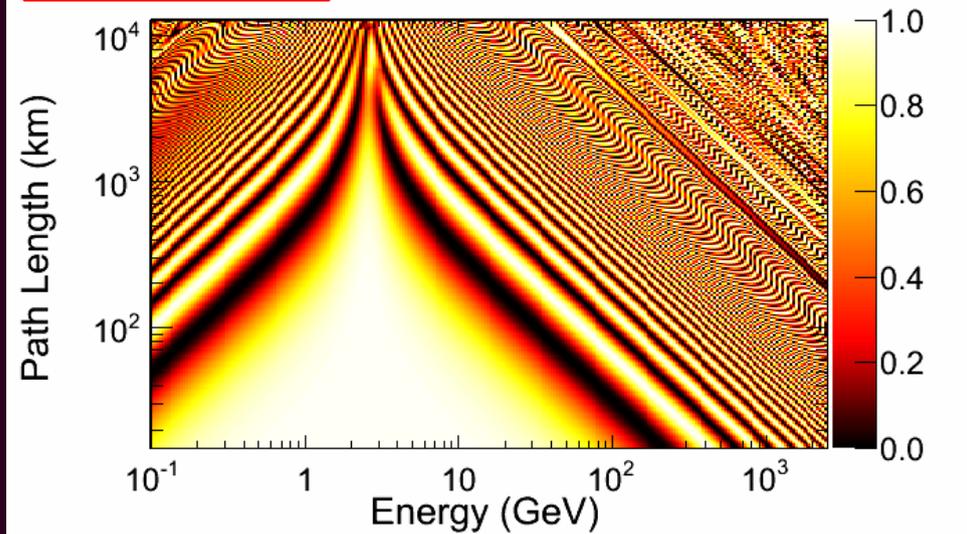
## Standard 3-flavour



$$\text{Re } a_{\mu\tau}^T = 10^{-22}$$



$$\text{Re } c_{\mu\tau}^{TT} = 10^{-22}$$



# Sensitive Samples

$$\text{Re } a_{\mu\tau}^T$$

$$\text{Re } c_{\mu\tau}^{TT}$$

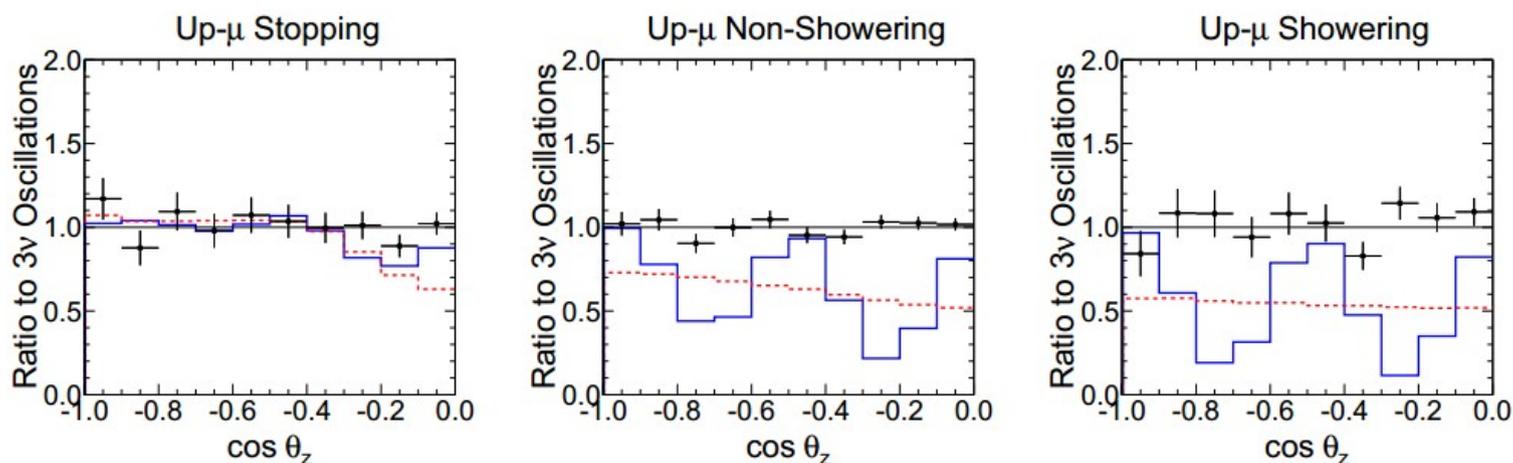
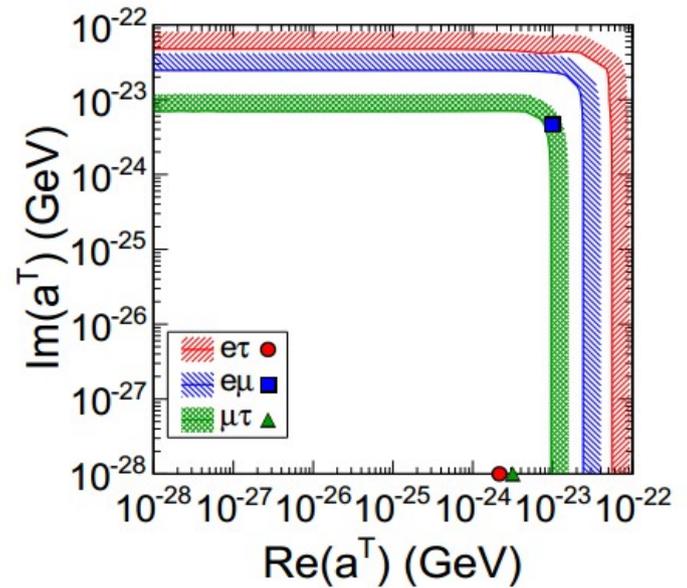


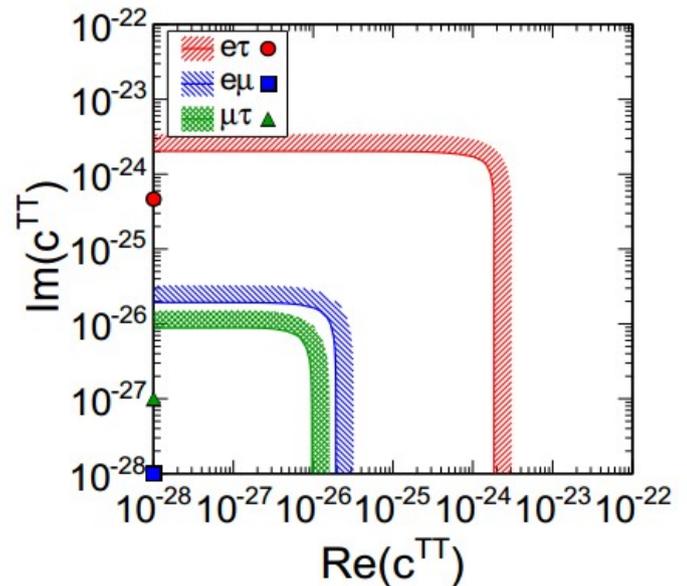
FIG. 2. (color online) Ratios of the summed SK-I through SK-IV  $\cos \theta_z$  distributions relative to standard three-flavor oscillations for the UP- $\mu$  sub-samples, which are the most sensitive to the effects of LV. The stopping sub-sample (left) has a peak energy around 10 GeV, the non-showering sub-sample (center) peaks around 100 GeV, and the showering sub-sample (right) peaks around 1 TeV. The black points represent the data with statistical errors. The lines corresponds to the MC prediction including Lorentz-violating effects, with  $a_{\mu\tau}^T = 10^{-22}$  GeV in solid blue and  $c_{\mu\tau}^{TT} = 10^{-22}$  in dashed red.

# Results

- No evidence of LV.
  - Limits set on the isotropic LV parameters in the  $e\mu$ ,  $\mu\tau$ , and  $e\tau$  sectors.
    - First limits in the  $\mu\tau$  sector.
    - $a^T$  limits improved by  $\sim 3$  orders of magnitude
    - $c^{TT}$  limits improved by  $\sim 7$  orders of magnitude.



(a)

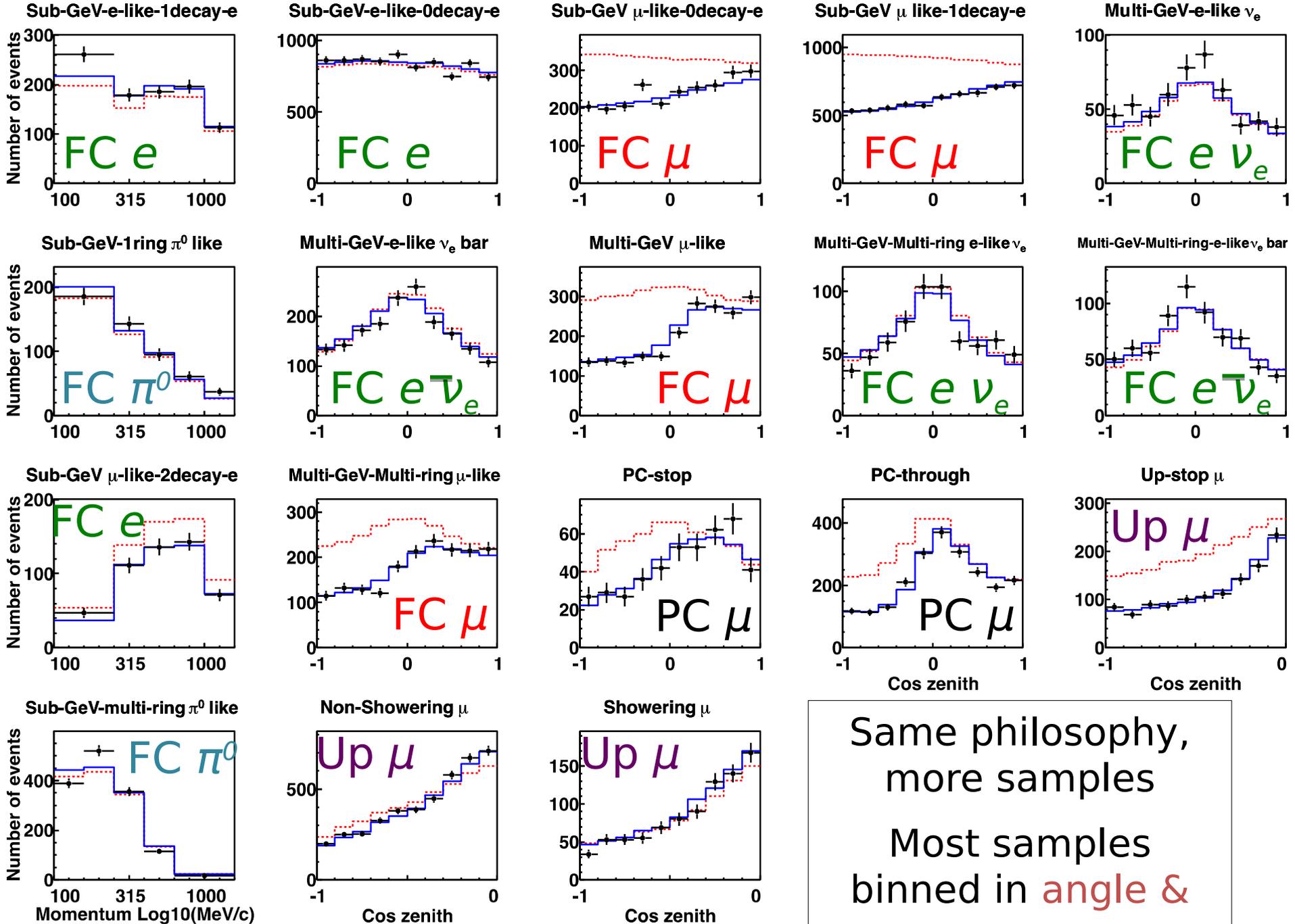


# Summary

# Results

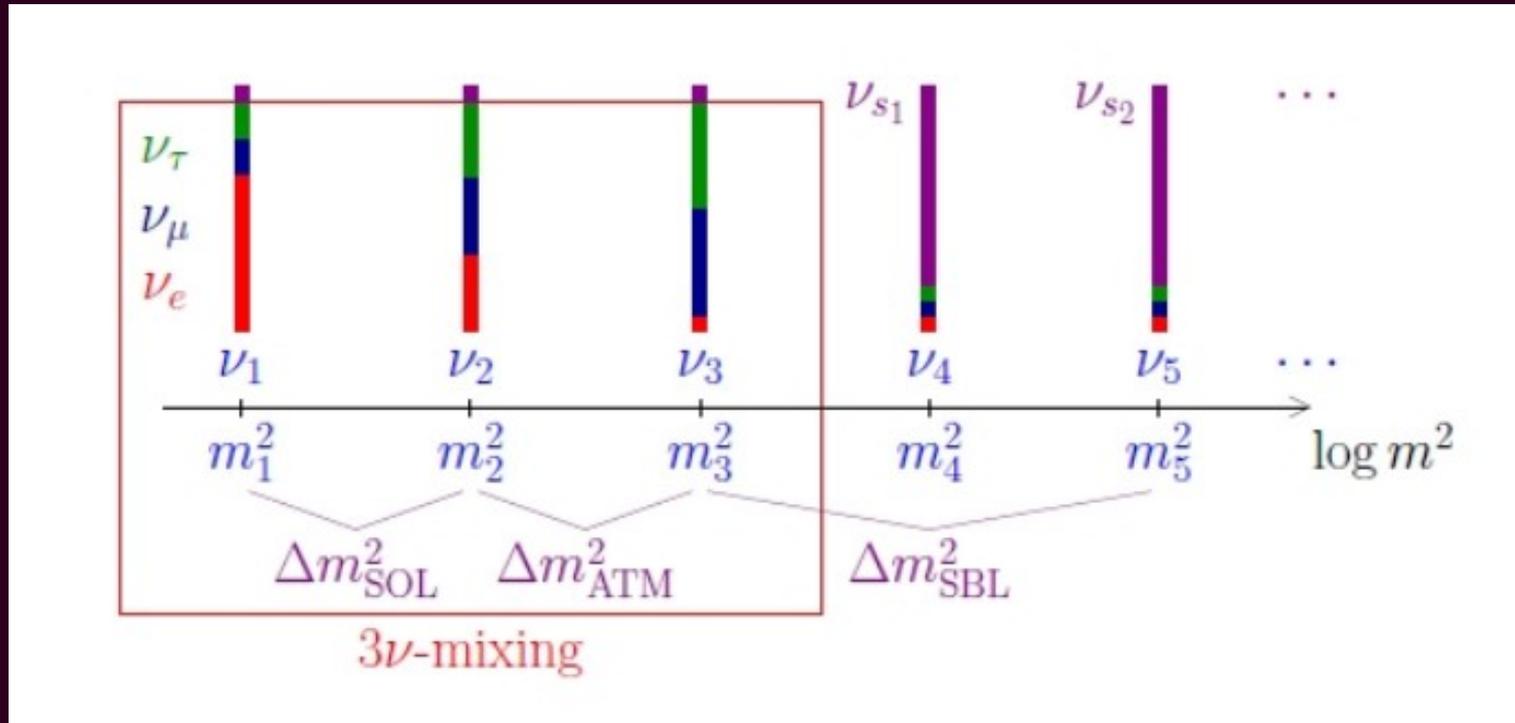
- LIV violation
  - Limits set on the isotropic LV parameters in  $e\mu$ ,  $\mu\tau$ , and  $e\tau$  sectors.
    - First limits in the  $\mu\tau$  sector.
    - $a^T$  and  $c^{TT}$  limits in other sectors improved by orders of magnitude.
- NSI
  - Limits on real parts of  $\varepsilon_{e\tau}$ ,  $\varepsilon_{\mu\mu}$ ,  $\varepsilon_{\mu\tau}$ , and  $\varepsilon_{\tau\tau}$ .
- Sterile Neutrinos
  - Limits on  $|U_{\mu 4}|^2$  and  $|U_{\tau 4}|^2$ 
    - at MeV scale (15~100 MeV range).
    - at eV scale, limits in the 3+1 case
      - extensions to 3+N shown.

Backup



Same philosophy,  
 more samples  
 Most samples  
 binned in **angle & energy**

# Sterile Neutrino - Basic Theory

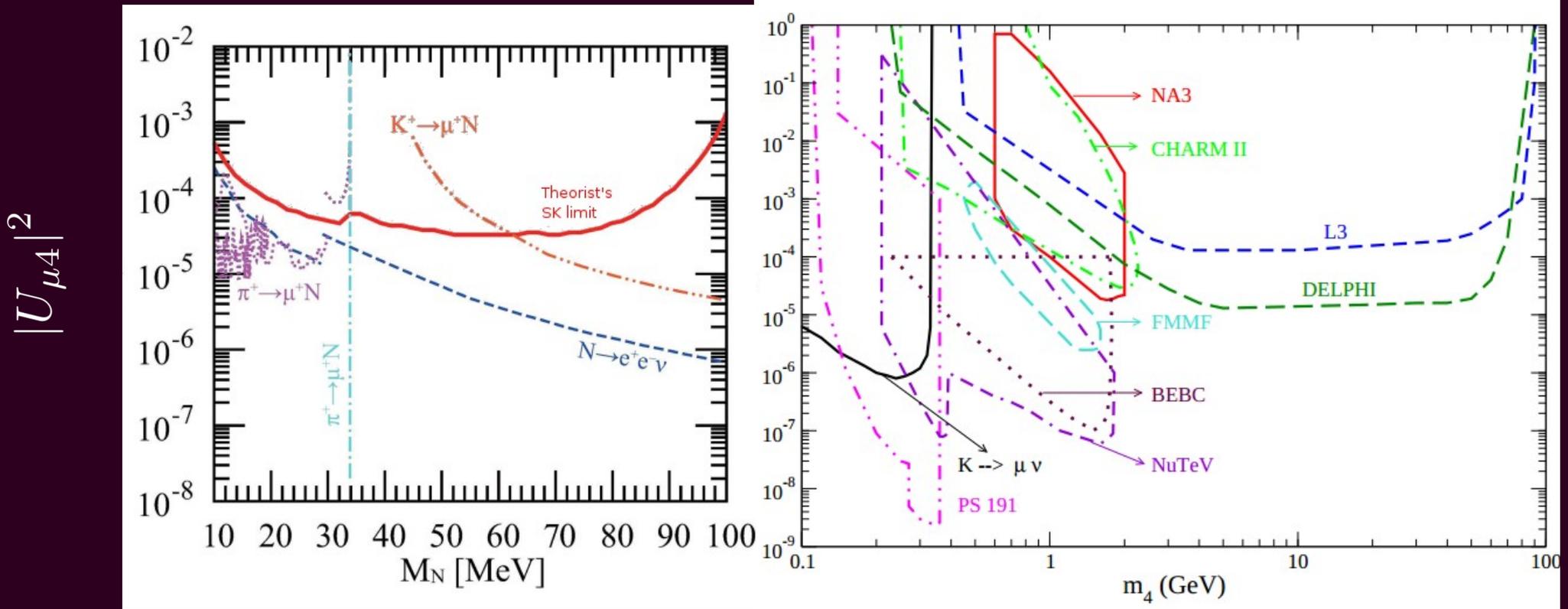


PMNS

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

# Current Limits

- Previous limits on the parameter  $|U_{\mu 4}|^2$  (~2 years old)



- Red line is a limit using public SK data, by T. Asaka and A. Watanabe<sup>[3]</sup> (not a Super-K collaboration paper).

# Flux Simulation

- **In summary**, we will simulate the expected number of heavy-neutrino decays detected in SK, using modifications to the atmospheric neutrino flux simulation by M. Honda<sup>[4]</sup> (and detector response by Monte-Carlo).
- *Heavy neutrino creation in the atmosphere:*
  - The creation is similar to the muon (anti-)neutrino, subject to the extra mass requirements.
  - Each creation of a muon neutrino in the simulation can be reweighted, e.g. for muon decay:

$$\frac{\Gamma(\mu^- \rightarrow e^- \nu_e \nu_4)}{\Gamma(\mu^- \rightarrow e^- \nu_e \nu_\mu)} = |U_{\mu 4}|^2 (1 - 8r + 8r^3 - r^4 - 24r^2 \ln(r)) \quad [3]$$

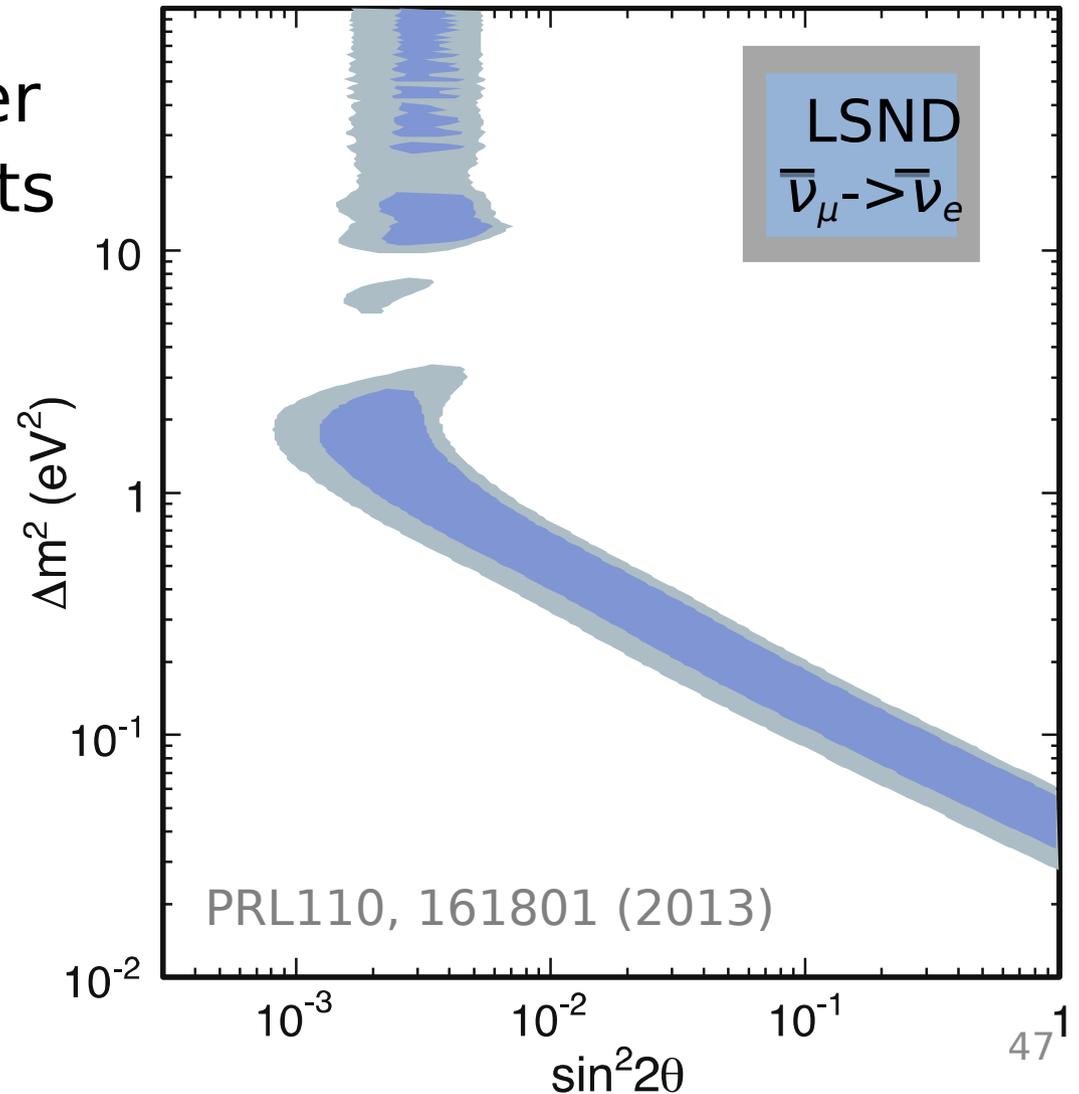
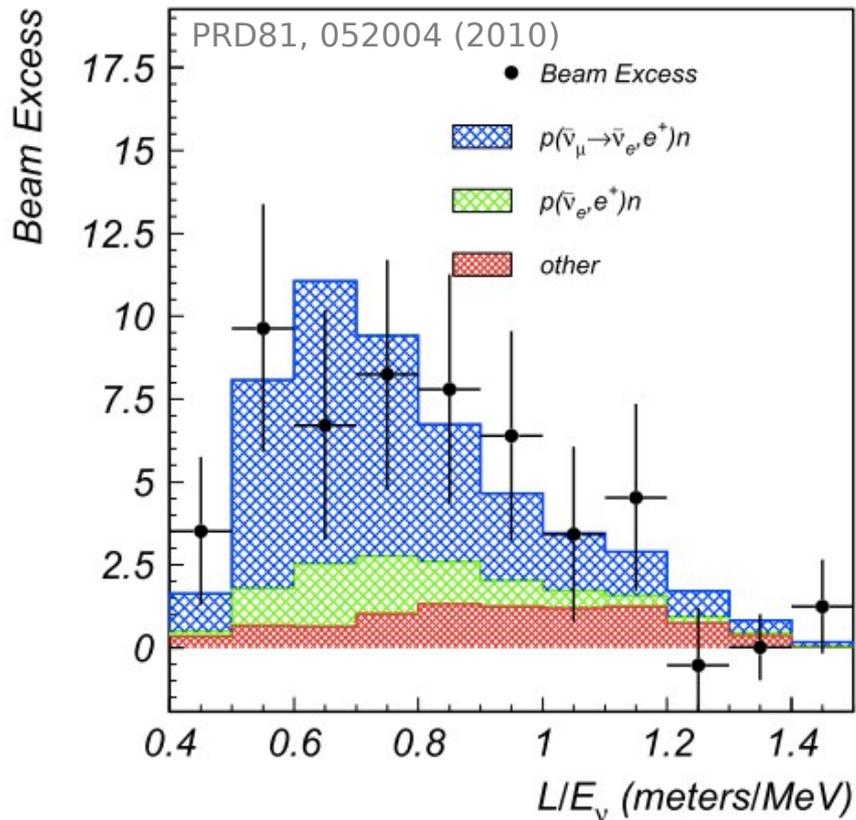
where

$$r = \left( \frac{m_4}{m_\mu} \right)^2$$

- Similar reweighting for pion & kaon decays.

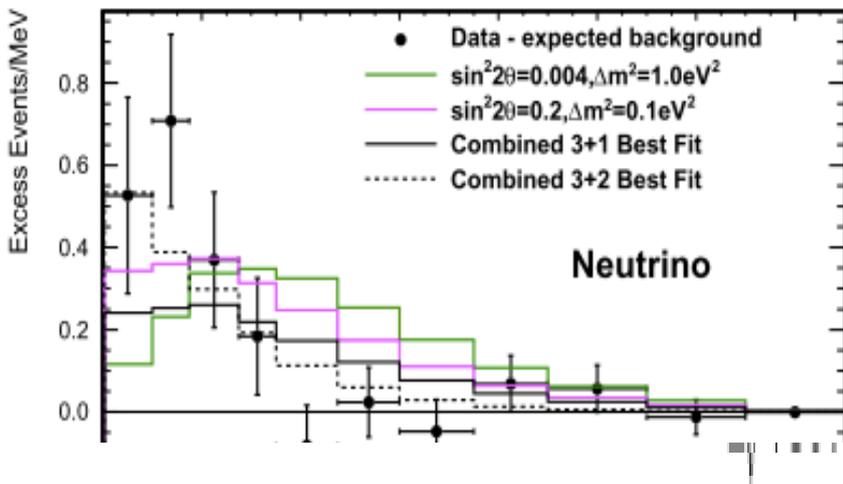
# $\nu_e$ Appearance at 1 m/MeV?

- LSND
  - Anti- $\nu_e$  appearance in a stopped- $\pi$  beam
  - $L \sim 30$  m,  $E \sim 30$  MeV  $\rightarrow \Delta m^2 \sim \mathbf{1 \text{ eV}^2}$
- Not consistent with other oscillation measurements

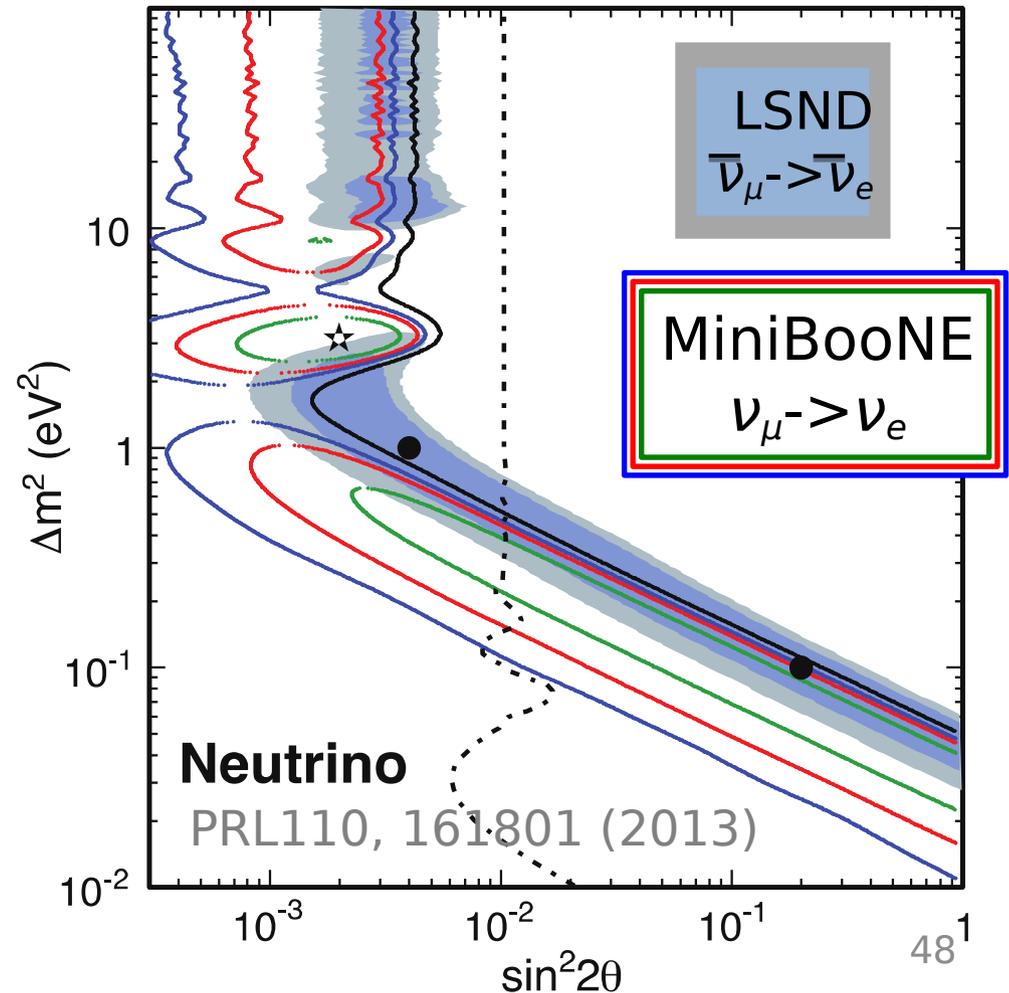


# $\nu_e$ Appearance at 1 m/MeV?

- MiniBooNE
  - $\nu_e$  and anti- $\nu_e$  appearance, different beam
  - $L \sim 500$  m,  $E \sim 500$  MeV
- Does not confirm LSND, but does not exclude sterile oscillations



Alex Himmel



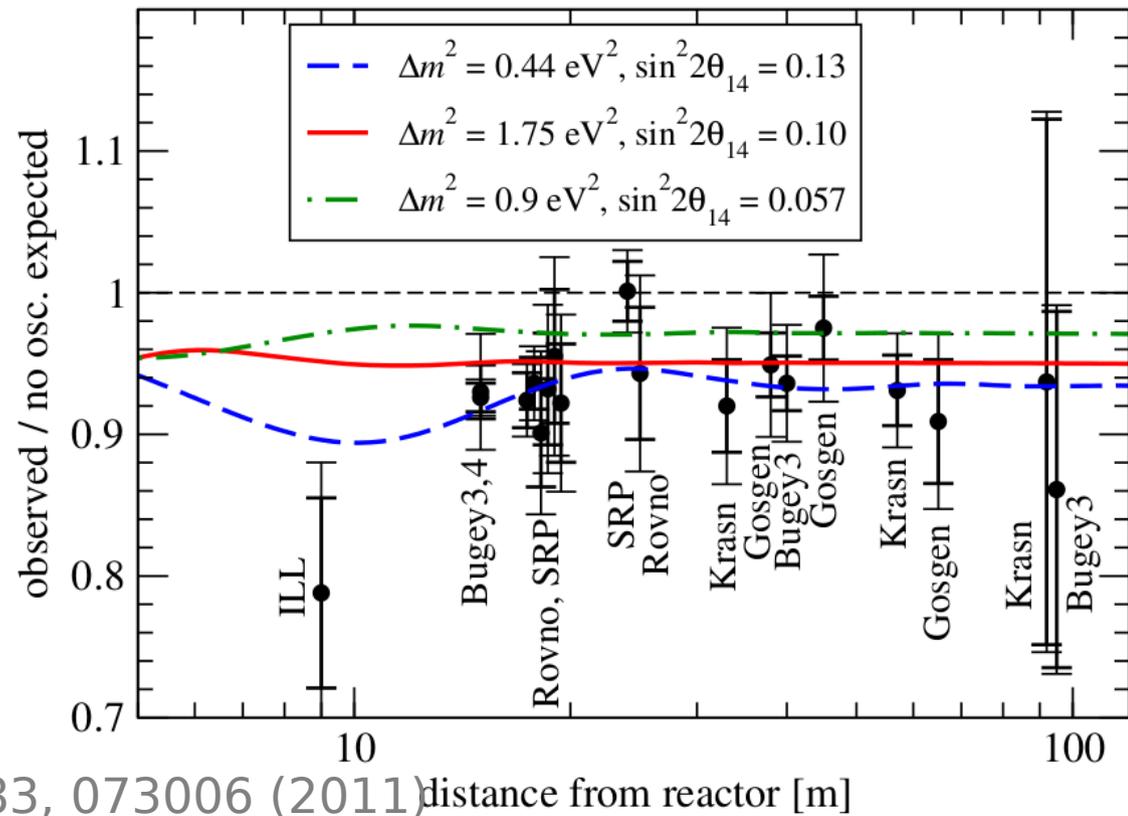
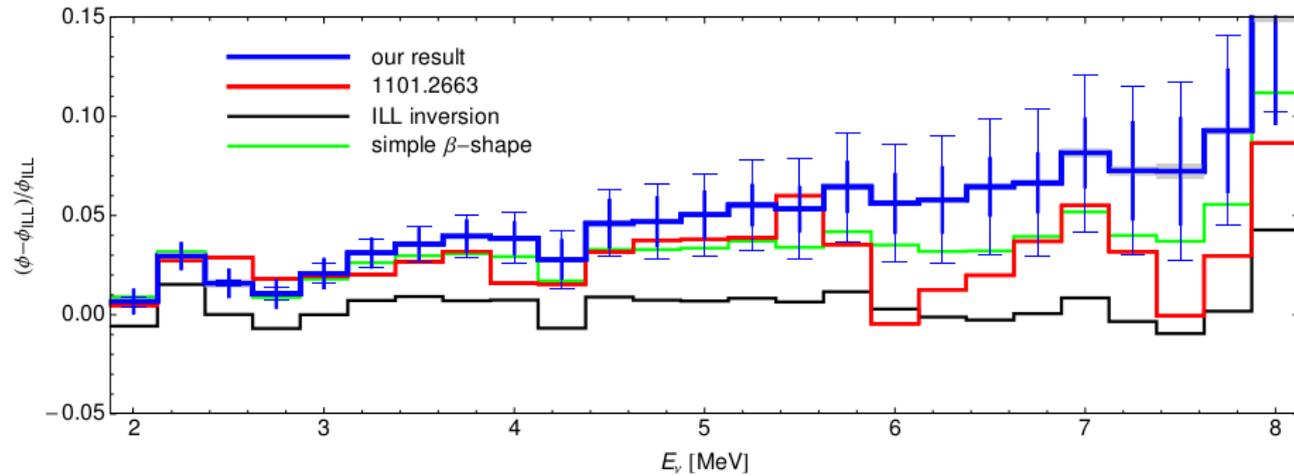
# $\nu_e$ Disappearance at 1 m/MeV?

- Reactor anti- $\nu_e$  flux recalculated in 2010
- With the new flux, most short-baseline reactor experiments have deficits

–  $R_{avg} = 0.927 \pm 0.023$

–  $L \sim 10\text{-}100$  m

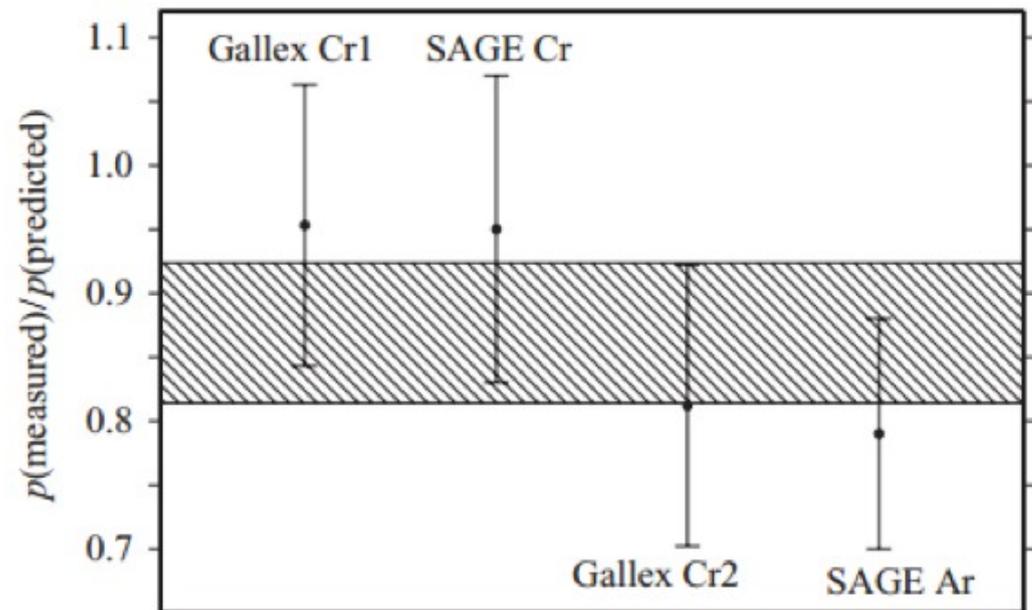
–  $E \sim 5$  MeV



# $\nu_e$ Disappearance at 1 m/MeV?

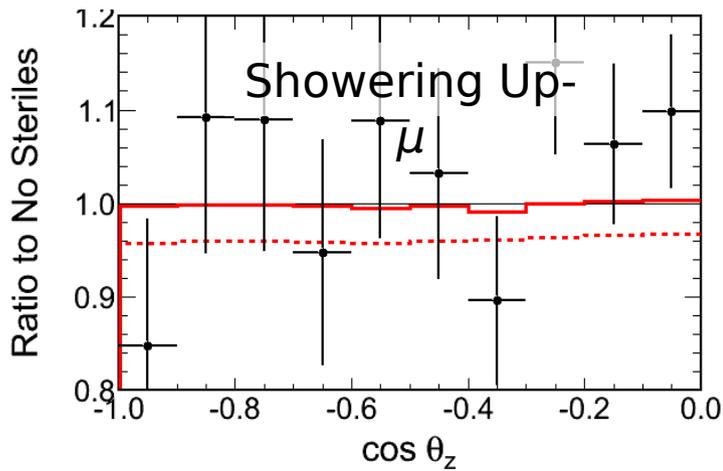
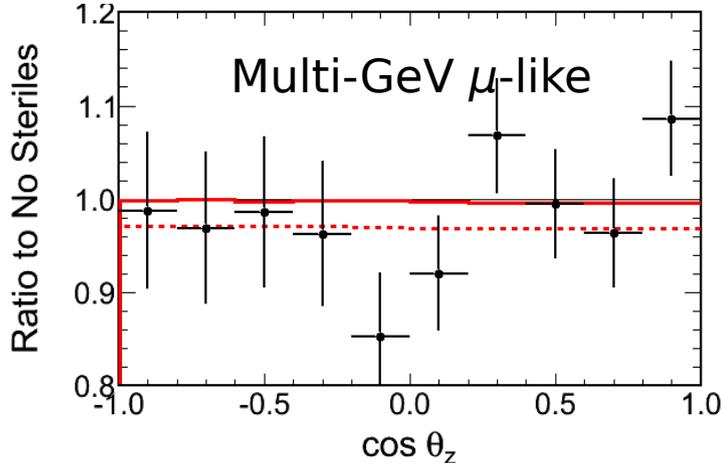
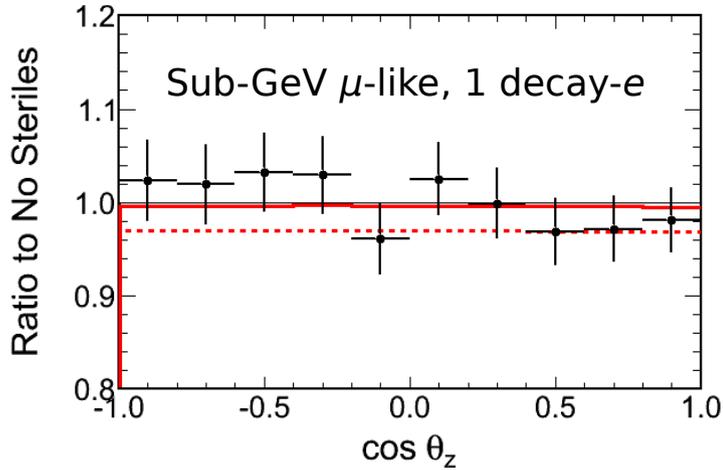
- Another anomaly in Gallium-based solar experiments
- Gallex and SAGE used radioactive calibration sources  $^{51}\text{Cr}$  and  $^{37}\text{Ar}$
- The rates from these sources were, again, lower than expected.

$$-R_{avg} = 0.87 \pm 0.05$$



# Sterile Vacuum Results

- Best fit:  $|U_{\mu 4}|^2 = 0.016$ 
  - Shown as solid line at right
  - Dashed line shows fit without minimizing systematics
- All of the  $\chi^2$  improvement at best fit is in systematics.
  - Fit is systematically limited



Systematic	No Steriles	Best Fit
$\nu_\mu/\nu_e$ flux, $E < 1$ GeV	$-0.52\sigma$	$-0.07\sigma$
$\nu_\mu/\nu_e$ flux, $E 1-10$ GeV	$-0.50\sigma$	$-0.11\sigma$
<b>CCQE <math>\nu_\mu/\nu_e</math></b>	$0.38\sigma$	<b><math>-0.01\sigma</math></b>

# Approximations

## 1. No sterile-electron neutrino mixing

Following the method in Appendix C2 of [60], we can approximate the primary effect of a non-zero  $|U_{e4}|^2$  by considering only its effect on the  $\nu_e$  survival probability  $P_{ee}$ , taken as analogous to  $P_{\mu\mu}$ :

$$P_{ee} = (1 - |U_{e4}|^2)^2 P_{ee}^{(3)} + |U_{e4}|^4, \quad (\text{B1})$$

where  $P_{ee}^{(3)}$  is the standard three-flavor  $\nu_e$  survival probability. When this extra free parameter is introduced, the limit on  $|U_{\mu 4}|^2$  turns out to be correlated with the limit on  $|U_{e4}|^2$ , as shown in the sensitivity contours in

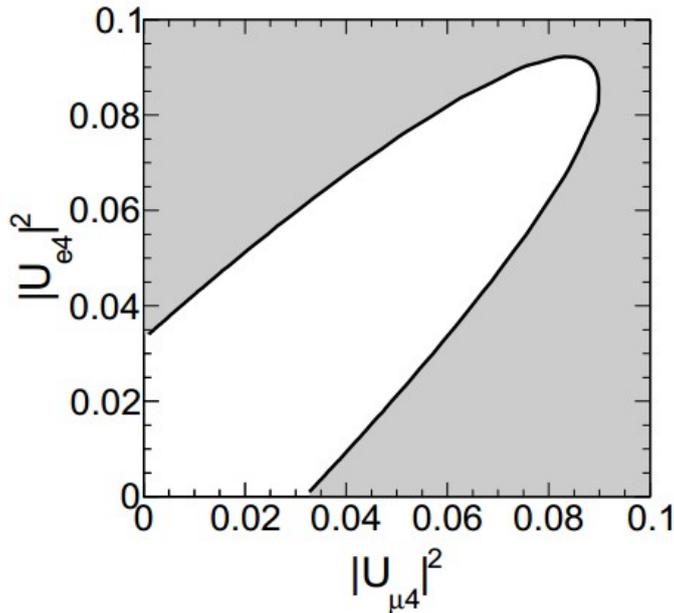


FIG. 9. The 90% sensitivity contour for the sterile vacuum fit with the effect  $P_{ee}$  from Eq. (B1) included. Allowing the freedom in the electron sample normalization reduces the sensitivity to  $|U_{\mu 4}|^2$  as can be seen from the bowing outward on the right side of the contour. Note that on this plot  $|U_{\mu 4}|^2$  is shown in linear scale so the correlation with  $|U_{e4}|^2$  is clear.

Fig. 9. With  $|U_{e4}|^2$  unconstrained, the 90% MC sensitivity to  $|U_{\mu 4}|^2$  becomes 0.067, significantly weaker than the 0.024 sensitivity with the assumption of  $|U_{e4}|^2 = 0$ . However, when the Bugey constraint is included by adding the penalty term

$$\chi_{\text{penalty}}^2 = (|U_{e4}|^2/0.012)^2, \quad (\text{B2})$$

again following the technique of [60], the sensitivity becomes 0.029, very close to the original sensitivity.

## 2. No three-flavor matter effects in the no- $\nu_e$ fit

The main effect of setting  $\theta_{13}$  to zero in the no- $\nu_e$  fit, eliminating Multi-GeV  $\nu_e$  appearance, was already discussed in Eq. (5.3). However, this assumption has a second effect: it eliminates the distortion in the  $\nu_\mu$  survival probability from matter effects in the Earth. These distortion can be seen in the few-GeV region for the most upward going events ( $\cos\theta_z \cong -1$ ) in Fig. 2(a).

Neglecting this matter effect turns out to have little effect on the  $|U_{\tau 4}|^2$  limit. A sensitivity fit using the no- $\nu_e$  model to a MC prediction made using the full three-flavor oscillation probability which includes these distortions finds a best fit at  $|U_{\tau 4}|^2 = 0$  and  $|U_{\mu 4}|^2$  equal to its minimum value (it is binned in log scale and so does not go to zero). The three flavor distortions in the  $\nu_\mu$  survival probability turn out to be relatively small (at most a few percent in the PC through-going and stopping UP- $\mu$  samples) and to not affect the through-going UP- $\mu$  samples which are distorted significantly by the sterile matter effects.

# Approximations

### 3. Sterile-induced fast oscillations

This assumption posits that the oscillations driven by  $\Delta m^2$  are so fast that individual oscillation periods cannot be resolved in the experiment and that functions of  $\Delta m^2$  can be replaced with their average values:

$$\sin\left(\frac{\Delta m^2 L}{4E}\right) \rightarrow \langle \sin \rangle = 0 \quad (\text{B3})$$

$$\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \rightarrow \langle \sin^2 \rangle = \frac{1}{2}. \quad (\text{B4})$$

However, since the phase in these terms depends on  $L$  and  $E$  as well as  $\Delta m^2$ , the ranges over which they are valid could vary for the different samples used in the analysis. For a sufficiently small  $\Delta m^2$ , this fast oscillation assumption will break down, and the higher the energy and shorter the path length, the larger of a value of  $\Delta m^2$  that is invalid. We can estimate this lower limit by calculating the value of  $\sin^2(\Delta m^2 L/4E)$  for many MC events in the various SK samples (FC Sub- and Multi-GeV, PC, and UP- $\mu$ ) at a range of possible values of  $\Delta m^2$ . The average is then calculated from the event-by-event values at each  $\Delta m^2$  and the point where the actual average

deviates significantly from one half can be found. These averages vs.  $\Delta m^2$  for the four samples can be seen in Fig. 10.

Setting a threshold of 5% error on the value of  $\sin^2$ , we find that the fast oscillation sample is valid until approximately  $10^{-1}$  in all four samples. The highest limit is 0.13 in the PC sample where there are both high energies and the very short track lengths from down-going events.

Meeting this assumption only sets the bottom of the valid  $\Delta m^2$  range. The upper limit on the mass for which the limits are valid is set by the requirement that the mass-splitting is sufficiently small that the neutrinos remain coherent. A sufficiently heavy neutrino, approximately 1 keV or so, will separate from the other light neutrinos and thus not be able to participate in oscillations.

# “Heavy” Sterile Neutrinos

- A state  $m_4 \gtrsim \text{keV}$  is separated from the oscillation effects.
- The phenomenology varies depending on the mass, and in some cases we may have observable decay products.
  - For example, take  $m_4 \sim \text{MeV}$ 
    - Motivated by e.g.  $\nu\text{MSM}$  - standard Seesaw mechanism, but Majorana masses  $M_I$  are chosen below electroweak scale.

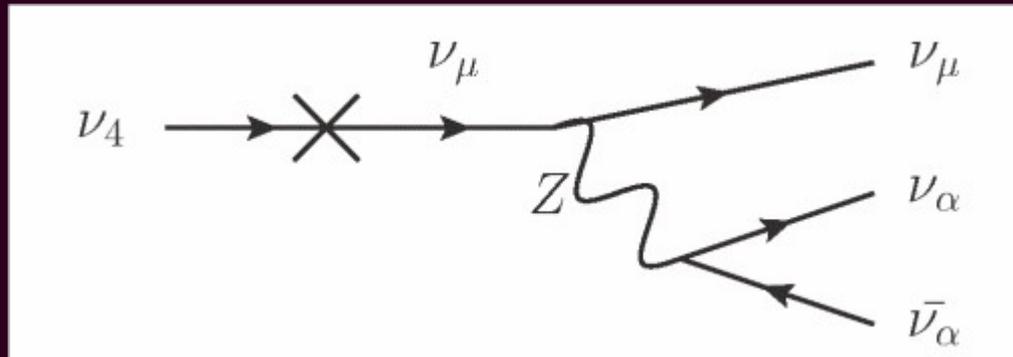
$$\mathcal{L} = \mathcal{L}_{SM} + i\bar{\nu}_{sI}\partial_\mu\gamma^\mu\nu_{sI} - \left( F_{\alpha I} \bar{L}_\alpha\nu_{sI}\tilde{\phi} - \frac{M_I}{2} \bar{\nu}_{sI}^c\nu_{sI} + h.c. \right)$$

with left-handed leptons  $L_\alpha$ , Yukawa couplings  $F_{\alpha I}$ , and Higgs  $\phi$ .

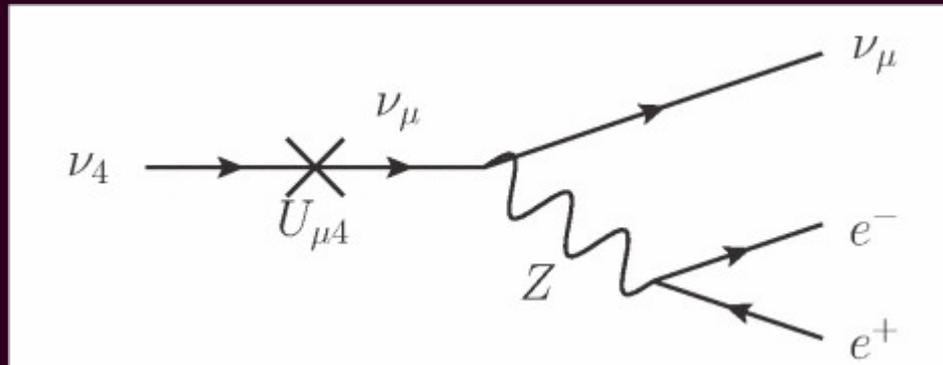
# Flux Simulation

- *Decay of the heavy neutrino:*
  - Approximately 12% of the decays are in the visible mode.

$$\Gamma(\nu_4 \rightarrow 3\nu) = \frac{G_F^2 m_4^5 |U_{\mu 4}|^2}{192\pi^3}$$

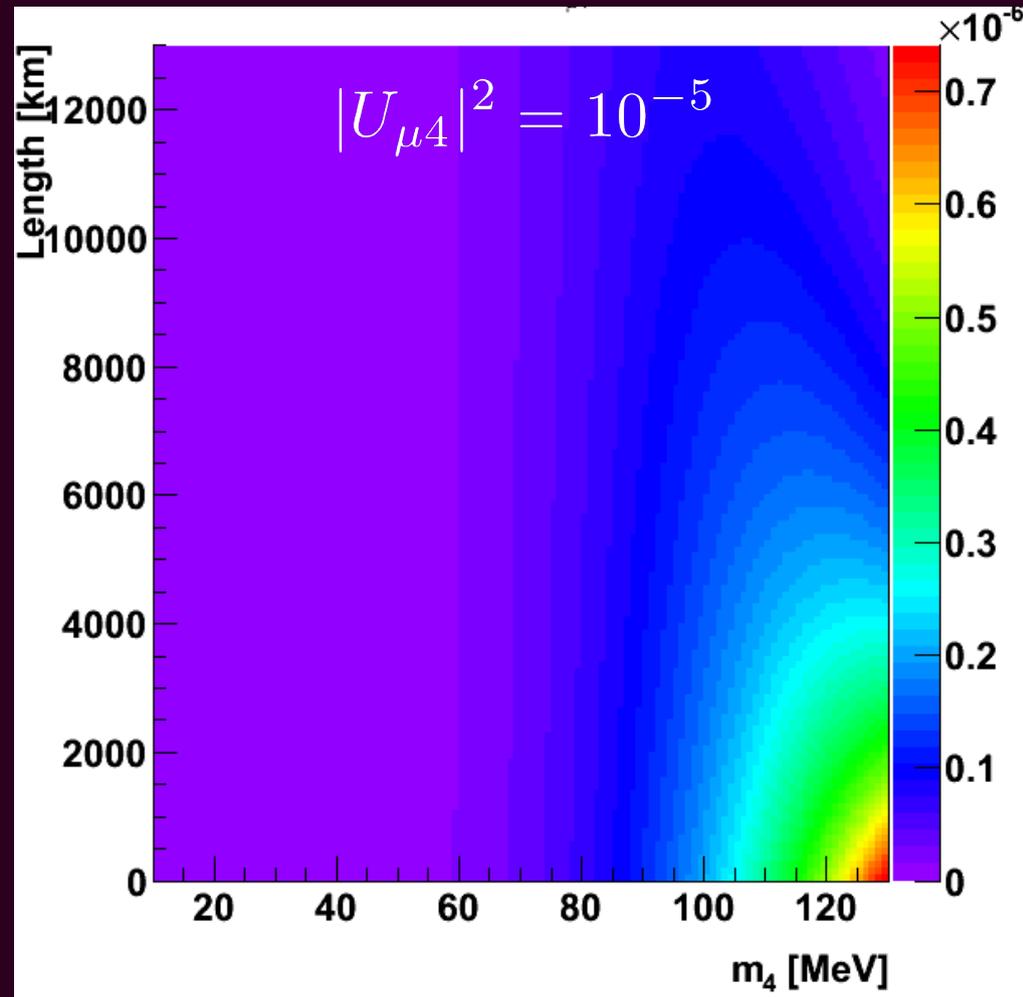


$$\Gamma(\nu_4 \rightarrow e^+ e^- \nu) = \Gamma(\nu_4 \rightarrow 3\nu) \left( \frac{1}{4} - \sin^2 \theta_W + 2 \sin^4 \theta_W \right)$$



# Flux Simulation

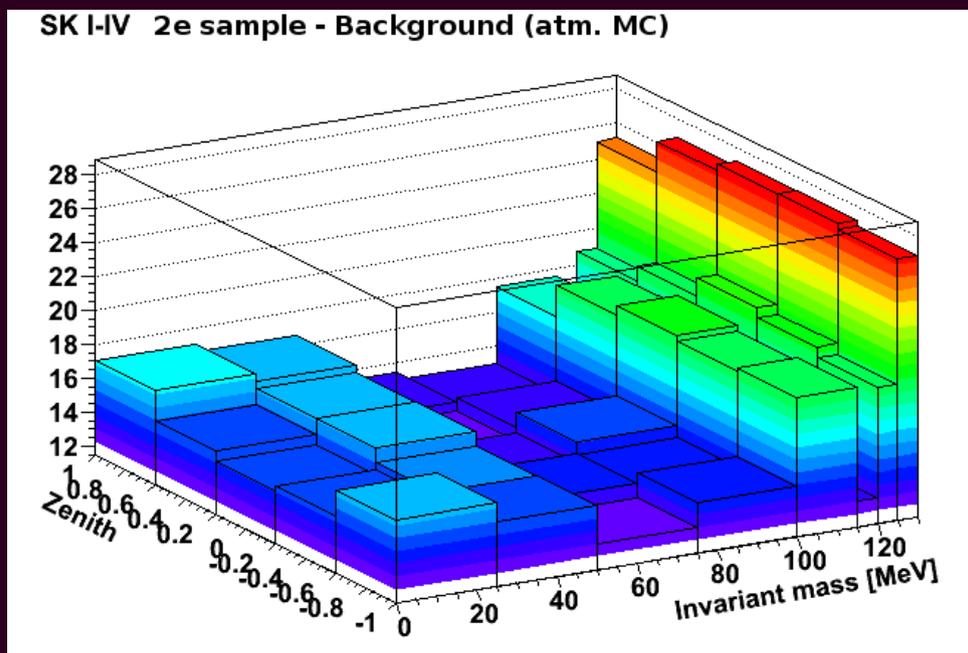
- Thus, we can estimate the probability of a heavy-neutrino to decay to the visible mode inside of SK, depending on the mass, travel distance to SK, and  $|U_{\mu 4}|^2$ .
  - For this plot, we set
    - $|U_{\mu 4}|^2 = 10^{-5}$
  - It can be seen that there will be a dependence on travel length that gets stronger with increasing mass.
    - In SK, this means a zenith angle dependence of the signal.
  - A similar dependence is seen for  $|U_{\mu 4}|^2$  for a fixed mass.



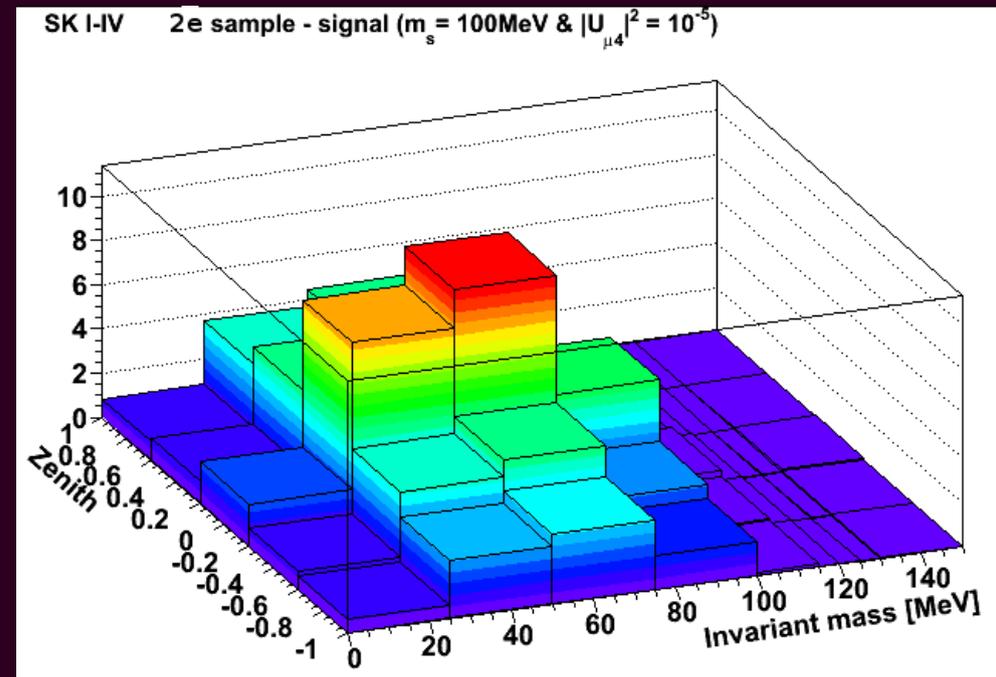
# 2D distributions

- Events are binned in 2d.
  - Not actually the final binning (can't find those plots...)

## Background (atmospheric MC)



## Signal (sterile decay)



# Fitting Procedure

- A fit is performed for  $|U_{\mu 4}|^2$  separately at each mass point.
  - The binning of the zenith & invariant mass distributions was optimized using the Monte-Carlo.
  - For this study, 44 (21) systematic errors from the SK MC are applicable to the atmospheric background MC (*sterile MC*).
- The fit is a  $\chi^2$ -minimization, with systematic errors included in the fit using the method of penalty terms (“error pulls”).

$$\chi^2 = 2 \sum_{i=1}^{\text{nbins}} \left( z_i - N_i^{\text{obs}} + N_i^{\text{obs}} \ln \frac{N_i^{\text{obs}}}{z_i} \right) + \sum_{j=1}^{N_{\text{syserr}}} \left( \frac{\epsilon_j}{\sigma_j} \right)^2$$
$$z_i = \alpha \cdot N_i^{\text{back}} \left( 1 + \sum_{j=1}^{N_{\text{syserr}}} f_i^j \frac{\epsilon_j}{\sigma_j} \right) + \beta \cdot N_i^{\text{sig}} \left( 1 + \sum_{j=1}^{N_{\text{syserr}}} f_i^j \frac{\epsilon_j}{\sigma_j} \right)$$

$N_i^{\text{obs}}$ ,  $N_i^{\text{sig}}$ ,  $N_i^{\text{back}}$  are the data, signal MC, background MC

$\epsilon_j$  are the pull terms for each error

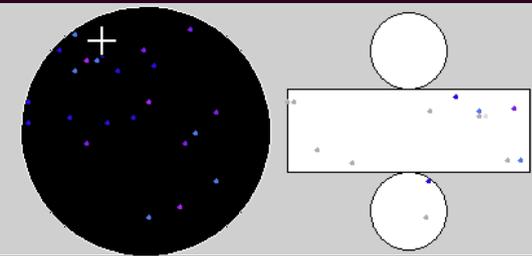
$f_i^j$  are the fractional changes by a  $\sigma_j = 1$  pull

$\alpha$  is background MC normalization, and  $\beta$  is related to  $|U_{\mu 4}|^2$

# Detection Efficiency

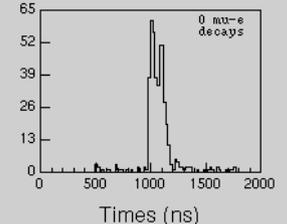
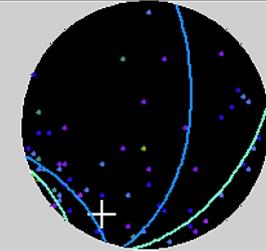
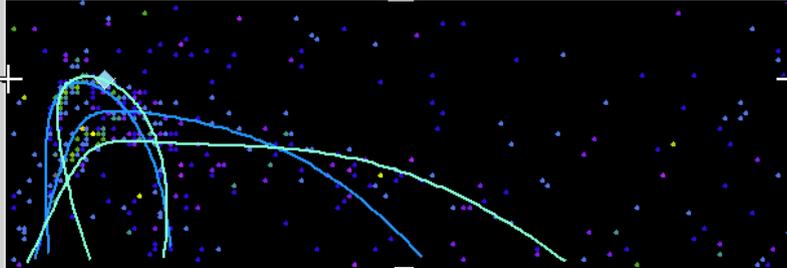
- Some event displays at 30 MeV  
 → detection efficiency is ~25%,  
 but mis-reconstruction rate  
 is also a little high.

**Super-Kamiokande IV**  
 Run 999999 Sub 0 Event 10  
 14-05-17:12:43:15  
 Inner: 447 hits, 668 pe  
 Outer: 5 hits, 5 pe  
 Trigger: 0x00  
 D\_wall: 335.9 cm  
 Evis: 60.5 MeV  
 2 e-like rings: mass = 31.2 MeV/c<sup>2</sup>

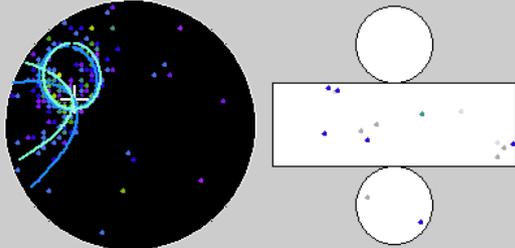


Charge(pe)

- >26.7
- 23.3-26.7
- 20.2-23.3
- 17.3-20.2
- 14.7-17.3
- 12.2-14.7
- 10.0-12.2
- 8.0-10.0
- 6.2- 8.0
- 4.7- 6.2
- 3.3- 4.7
- 2.2- 3.3
- 1.3- 2.2
- 0.7- 1.3
- 0.2- 0.7
- < 0.2

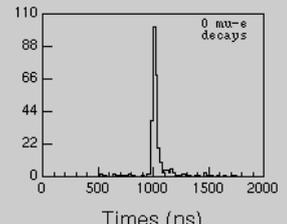
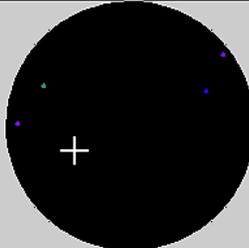
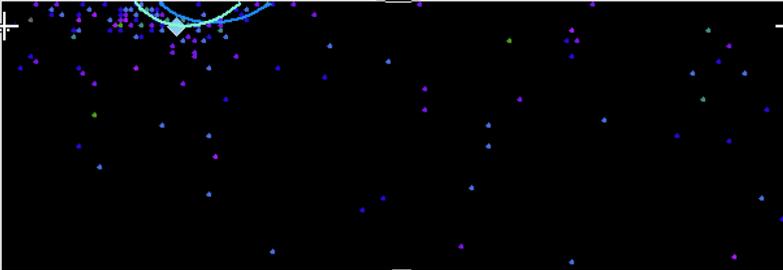


**Super-Kamiokande IV**  
 Run 999999 Sub 0 Event 8  
 14-05-17:12:43:12  
 Inner: 296 hits, 486 pe  
 Outer: 7 hits, 8 pe  
 Trigger: 0x00  
 D\_wall: 339.9 cm  
 Evis: 47.5 MeV  
 2 e-like rings: mass = 20.5 MeV/c<sup>2</sup>

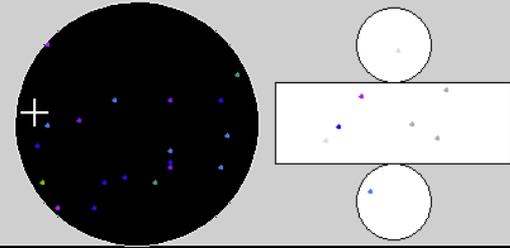


Charge(pe)

- >26.7
- 23.3-26.7
- 20.2-23.3
- 17.3-20.2
- 14.7-17.3
- 12.2-14.7
- 10.0-12.2
- 8.0-10.0
- 6.2- 8.0
- 4.7- 6.2
- 3.3- 4.7
- 2.2- 3.3
- 1.3- 2.2
- 0.7- 1.3
- 0.2- 0.7
- < 0.2

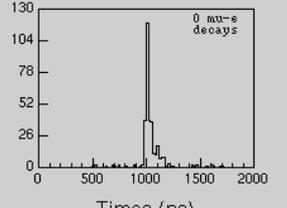
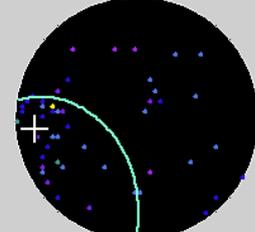
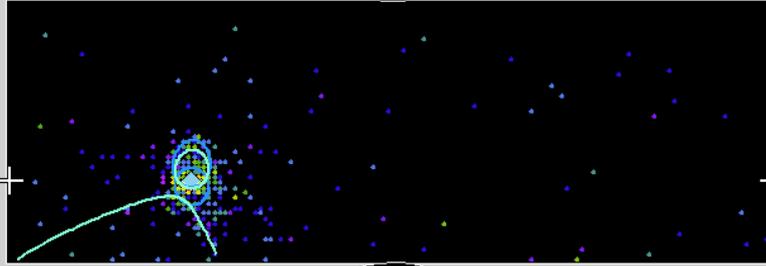


**Super-Kamiokande IV**  
 Run 999999 Sub 0 Event 12  
 14-05-17:12:43:16  
 Inner: 327 hits, 810 pe  
 Outer: 3 hits, 2 pe  
 Trigger: 0x00  
 D\_wall: 261.0 cm  
 Evis: 72.3 MeV  
 2 e-like rings: mass = 48.0 MeV/c<sup>2</sup>



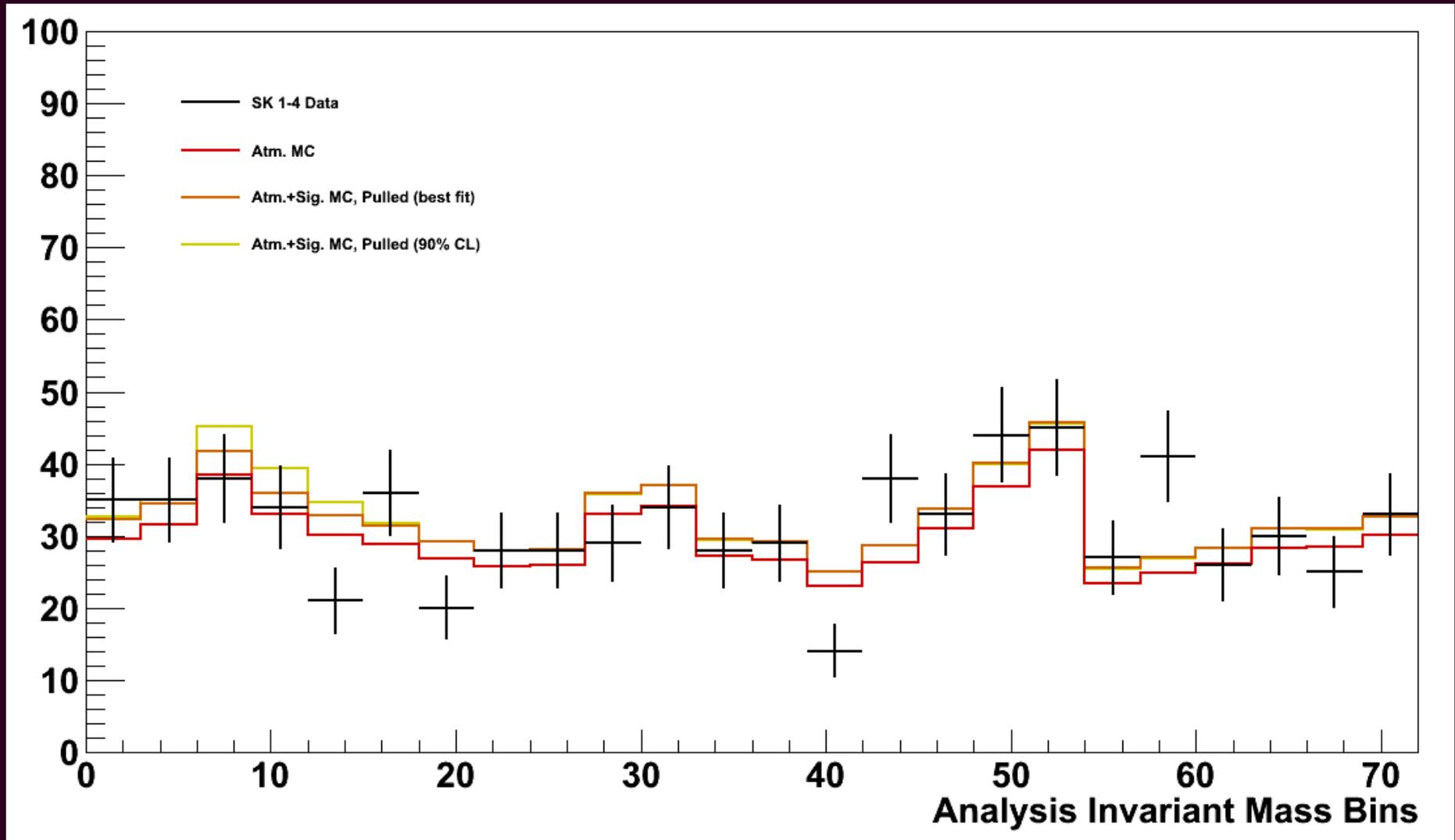
Charge(pe)

- >26.7
- 23.3-26.7
- 20.2-23.3
- 17.3-20.2
- 14.7-17.3
- 12.2-14.7
- 10.0-12.2
- 8.0-10.0
- 6.2- 8.0
- 4.7- 6.2
- 3.3- 4.7
- 2.2- 3.3
- 1.3- 2.2
- 0.7- 1.3
- 0.2- 0.7
- < 0.2



## Final Fit to Data

- An example fit at sterile mass = 50 MeV, shown by *final analysis bins*
  - Data / MC comparison at best-fit point, and 90%-confidence exclusion point



## Justification for Simplified Matrix Element

- From previous limits, electron-mixing is ruled out at 2-3 orders of magnitude below muon- mixing (thanks to double-beta decay experiments), so seems negligible.
- Atmospheric decays (from Pion, Kaon, Muon) involve the charged current, so Tau mixing cannot be involved (in this energy range).
- Sterile decay can involve a Tau, with the same channels as Muon, so the final results can be interpreted instead as a limit on  
 $\text{sqrt } |U_{\mu 4}|^2 (|U_{\mu 4}|^2 + |U_{\tau 4}|^2)$

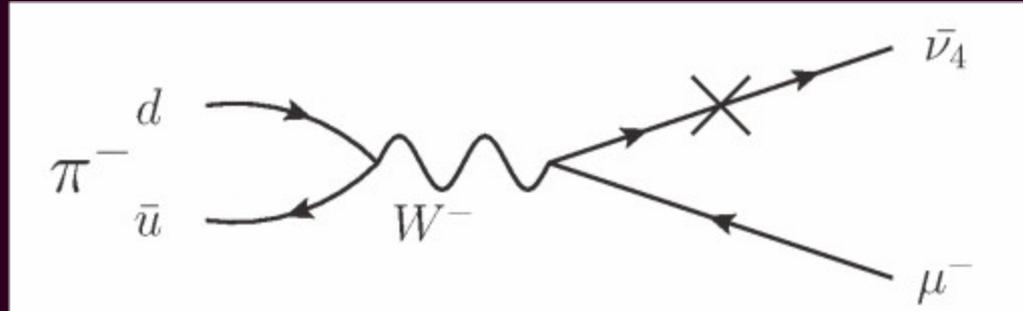
## More Phenomenology

- Pion decay rates

$$\frac{\Gamma(\pi^- \rightarrow \mu^- \nu_4)}{\Gamma(\pi^- \rightarrow \mu^- \nu_\mu)} = |U_{\mu 4}|^2 \sqrt{1 - 2(r_4 + r_\mu) + (r_4 - r_\mu)^2} \frac{r_4 + r_\mu - (r_4 - r_\mu)^2}{r_\mu(1 - r_\mu)^2}$$

$$r_4 = \left(\frac{m_4}{m_\pi}\right)^2$$

$$r_\mu = \left(\frac{m_\mu}{m_\pi}\right)^2$$

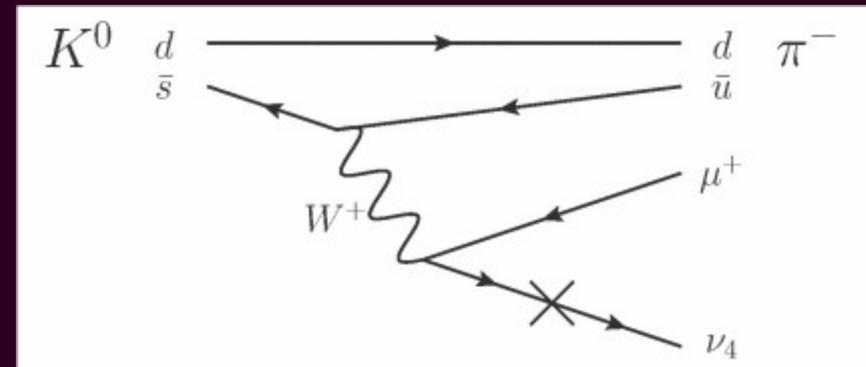


- Kaon decay:

$K^\pm \rightarrow$  mostly 2-body decays (as the pion decay)

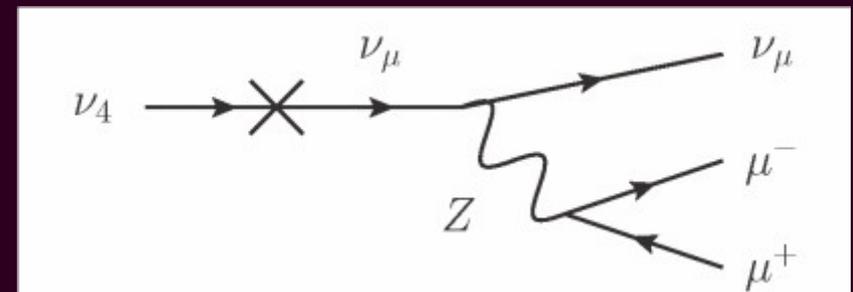
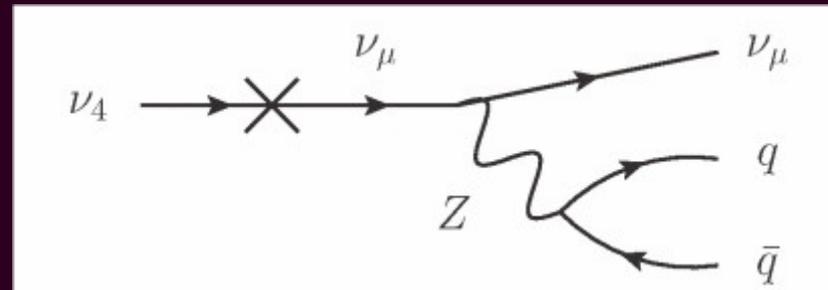
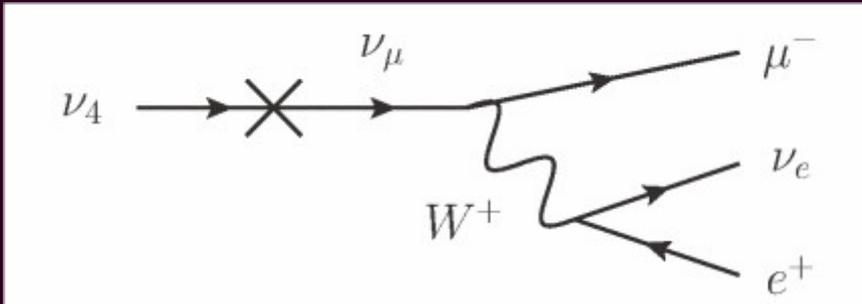
$K^0 \rightarrow$  3-body decay

Just using an approximation  
for this one...



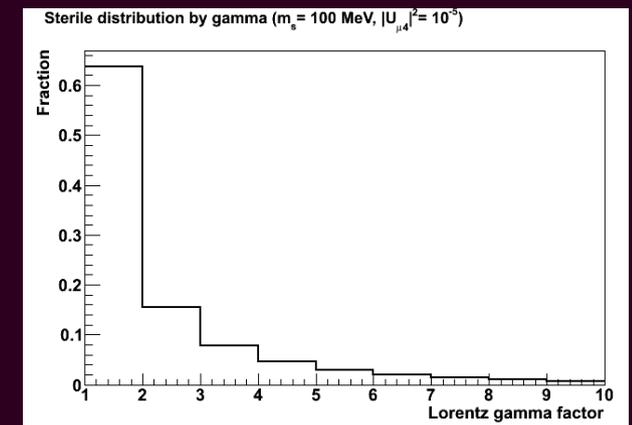
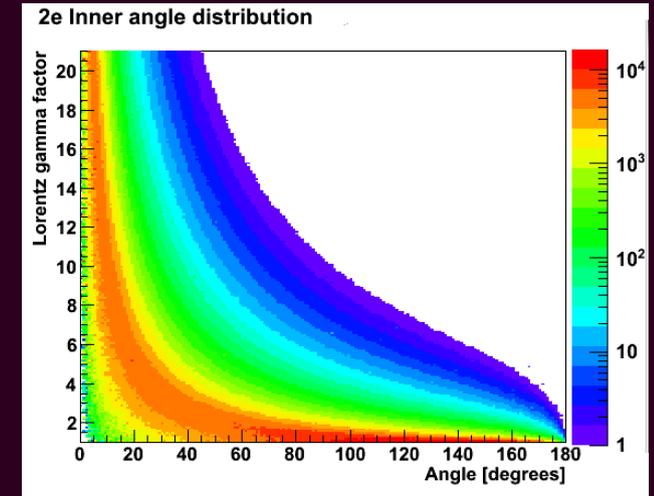
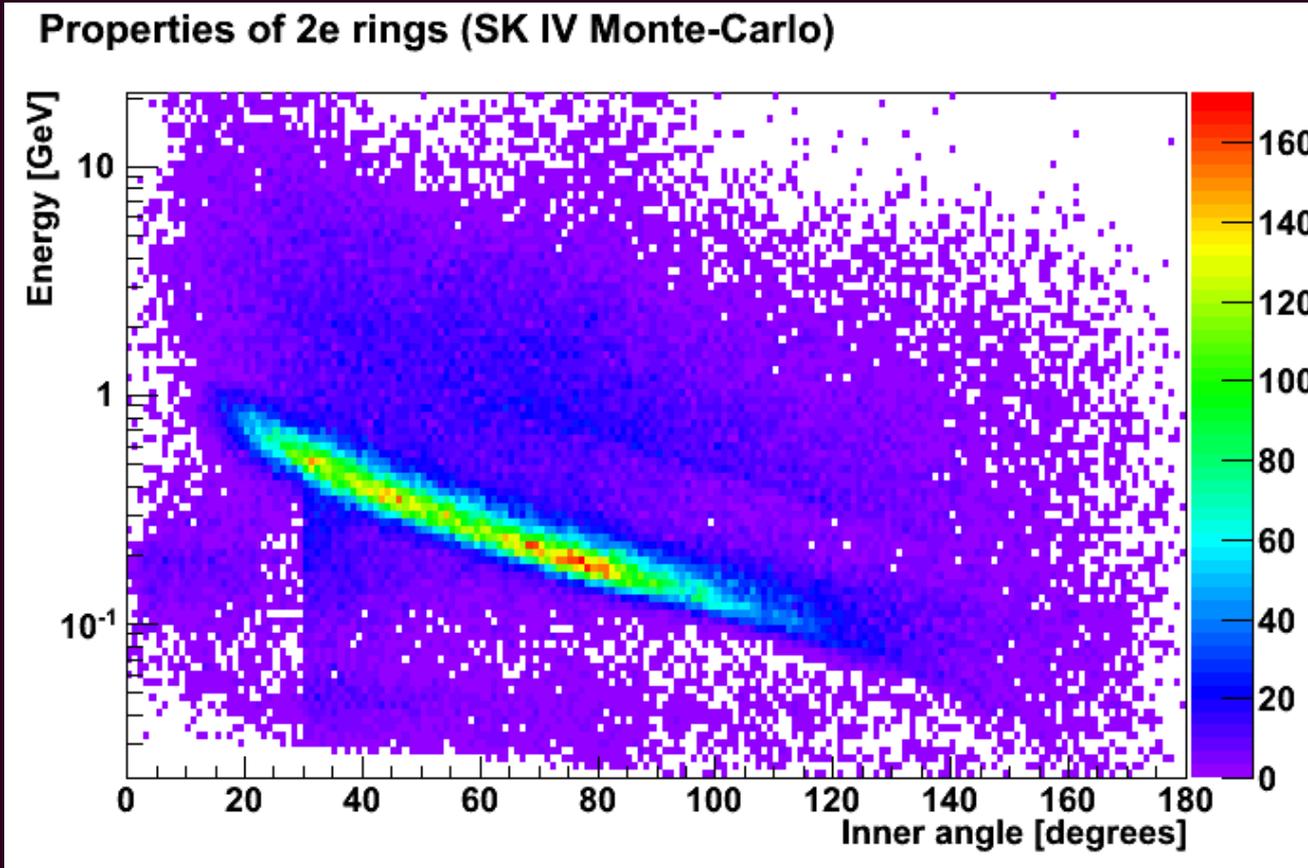
## More Phenomenology

- Extension to higher sterile masses fairly simple using just W / Z branching ratios
  - would have to consider more decay samples in the analysis



## Other Possible Variables

- Use inner angle as a separation variable
  - Need to use e.g. Poisson likelihood due to low events per bin.



# References

References for the sterile - decay study

- [1] T. Asaka and M. Shaposhnikov, Phys. Lett. B620 17-26 (2005)
- [2] C. F. Wong, Durham University PhD Thesis 4931
- [3] T. Asaka and A. Watanabe, arXiv:1202.0725 [hep-ph]
- [4] M. Honda, Phys. Rev. D83 123001 (2011)
- [5] G. Bernardi et al., Phys. Lett. B166 479 (1986)

# Two-flavour approach

- Survival probability is somewhat complicated

$$P_{\nu_\mu \rightarrow \nu_\mu} = 1 - P_{\nu_\mu \rightarrow \nu_\tau} = 1 - \sin^2 2\Theta \sin^2 \left( \frac{\Delta m_{23}^2 L}{4E} R \right). \quad (2)$$

The effective mixing angle,  $\Theta$ , and the correction factor to the oscillation wavelength,  $R$ , are given by

$$\begin{aligned} \sin^2 2\Theta &= \frac{1}{R^2} (\sin^2 2\theta + R_0^2 \sin^2 2\xi + 2R_0 \sin 2\theta \sin 2\xi), \\ R &= \sqrt{1 + R_0^2 + 2R_0 (\cos 2\theta \cos 2\xi + \sin 2\theta \sin 2\xi)}, \\ R_0 &= \sqrt{2} G_F N_f \frac{4E}{\Delta m^2} \sqrt{|\epsilon|^2 + \frac{\epsilon'^2}{4}}, \\ \xi &= \frac{1}{2} \tan^{-1} \left( \frac{2\epsilon}{\epsilon'} \right), \end{aligned} \quad (3)$$

# Calculation of LV Oscillation Probabilities

## Hamiltonian

$$H = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_{31}^2}{2E} \end{pmatrix} U^\dagger \pm \sqrt{2} G_F \begin{pmatrix} N_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \pm \begin{pmatrix} 0 & a_{e\mu}^T & a_{e\tau}^T \\ (a_{e\mu}^T)^* & 0 & a_{\mu\tau}^T \\ (a_{e\tau}^T)^* & (a_{\mu\tau}^T)^* & 0 \end{pmatrix} - E \begin{pmatrix} 0 & c_{e\mu}^{TT} & c_{e\tau}^{TT} \\ (c_{e\mu}^{TT})^* & 0 & c_{\mu\tau}^{TT} \\ (c_{e\tau}^{TT})^* & (c_{\mu\tau}^{TT})^* & 0 \end{pmatrix}$$

## Eigenvalue

$$S: E_i = -2\sqrt{Q} \cos\left(\frac{\theta_i}{3}\right) - \frac{a}{3}$$

$$Q = \frac{a^2 - 3b}{9}$$

$$\theta_0 = \cos^{-1}\left(RQ^{-\frac{3}{2}}\right)$$

$$\theta_1 = \theta_0 + 2\pi$$

$$\theta_2 = \theta_0 - 2\pi,$$

$$a = -\text{Tr}(H)$$

$$b = \frac{\text{Tr}(H)^2 - \text{Tr}(H^2)}{2}$$

$$c = -\det(H)$$

$$R = \frac{2a^3 - 9ab + 27c}{54}.$$

Special Thanks:  
J.S. Diaz of IU

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LV Parameter	95% Upper Limit	Best Fit	No LV $\Delta\chi^2$	Previous Limit	
$e\mu$	$\text{Re}(a^T)$	$1.8 \times 10^{-23}$ GeV	$1.0 \times 10^{-23}$ GeV	1.4	$4.2 \times 10^{-20}$ GeV [51]
	$\text{Im}(a^T)$	$1.8 \times 10^{-23}$ GeV	$4.6 \times 10^{-24}$ GeV		
	$\text{Re}(c^{TT})$	$1.1 \times 10^{-26}$	$1.0 \times 10^{-28}$	0.0	$9.6 \times 10^{-20}$ [51]
	$\text{Im}(c^{TT})$	$1.1 \times 10^{-26}$	$1.0 \times 10^{-28}$		
$e\tau$	$\text{Re}(a^T)$	$4.1 \times 10^{-23}$ GeV	$2.2 \times 10^{-24}$ GeV	0.0	$7.8 \times 10^{-20}$ GeV [52]
	$\text{Im}(a^T)$	$2.8 \times 10^{-23}$ GeV	$1.0 \times 10^{-28}$ GeV		
	$\text{Re}(c^{TT})$	$1.2 \times 10^{-24}$	$1.0 \times 10^{-28}$	0.3	$1.3 \times 10^{-17}$ [52]
	$\text{Im}(c^{TT})$	$1.4 \times 10^{-24}$	$4.6 \times 10^{-25}$		
$\mu\tau$	$\text{Re}(a^T)$	$6.5 \times 10^{-24}$ GeV	$3.2 \times 10^{-24}$ GeV	0.9	–
	$\text{Im}(a^T)$	$5.1 \times 10^{-24}$ GeV	$1.0 \times 10^{-28}$ GeV		
	$\text{Re}(c^{TT})$	$5.8 \times 10^{-27}$	$1.0 \times 10^{-28}$	0.1	–
	$\text{Im}(c^{TT})$	$5.6 \times 10^{-27}$	$1.0 \times 10^{-27}$		

TABLE II. Summary of the results of the six fits for Lorentz-violating parameters (the real and imaginary parts of each parameter are fit simultaneously). The 95% upper limits and best fits are shown, as well as the  $\Delta\chi^2$  for no Lorentz violation. The most significant exclusion of No LV is  $a_{e\mu}^T$ , which still includes No LV within the 68% C.L. Since the parameters are scanned on a logarithmic scale,  $10^{-28}$  is the minimum value considered and is equivalent to no LV.