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Temporal Instability of Supernova Neutrinos

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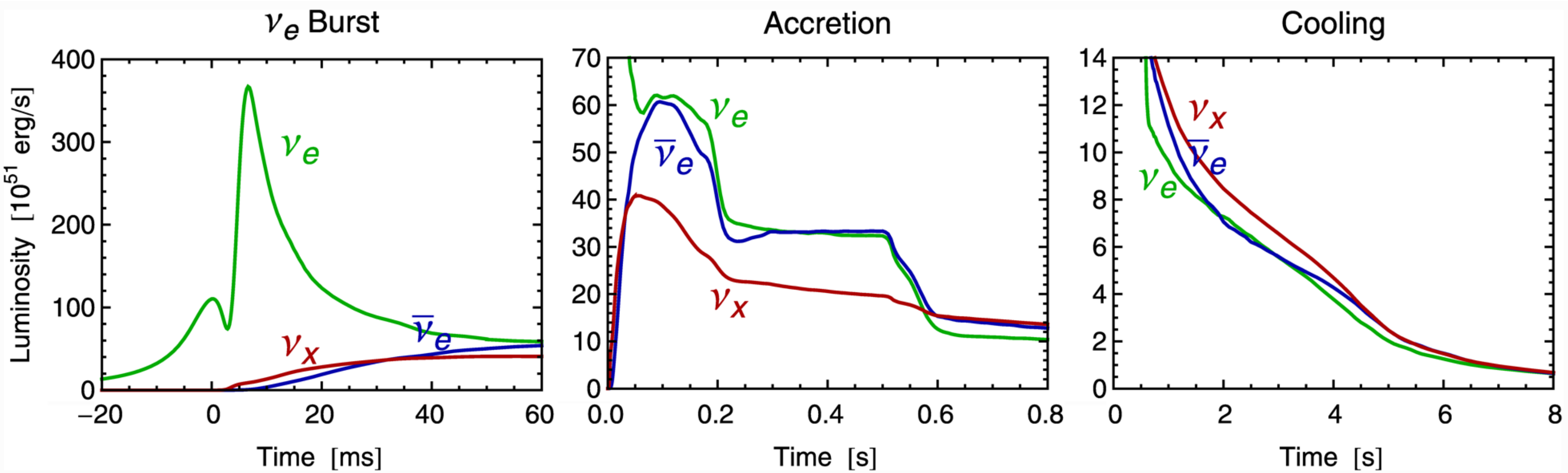
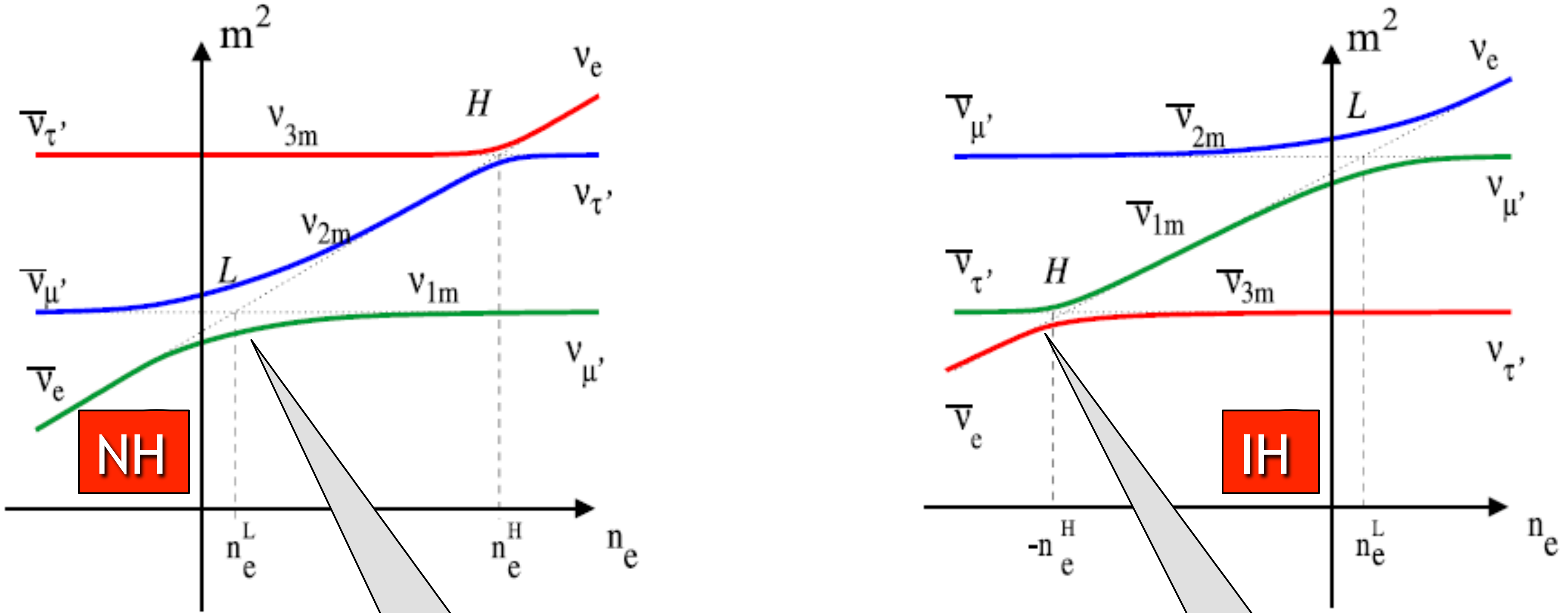
Sanduleak –69 202
Supernova 1987A
23 February 1987
29 YEARS AGO
IN A GALAXY NEAR US

Aim of the Game



Burst	Accretion	Cooling
SN standard candle?	Astrophysics	Nuclear physics
SN theory	Collective effects?	Nucleosynthesis
Timing	Shock revival?	Exotics/Axions
Mass hierarchy	Mass hierarchy?	…
…	…	

MSW Effects



Dighe and Smirnov

L resonance : $\nu_1 \Longleftrightarrow \nu_2$

$(\Delta m_{sol}^2, \theta_{12})$ at 10^1 - 10^2 g/cc

Always in neutrinos.

Always adiabatic.

H resonance : $\nu_1/\nu_2 \Longleftrightarrow \nu_3$

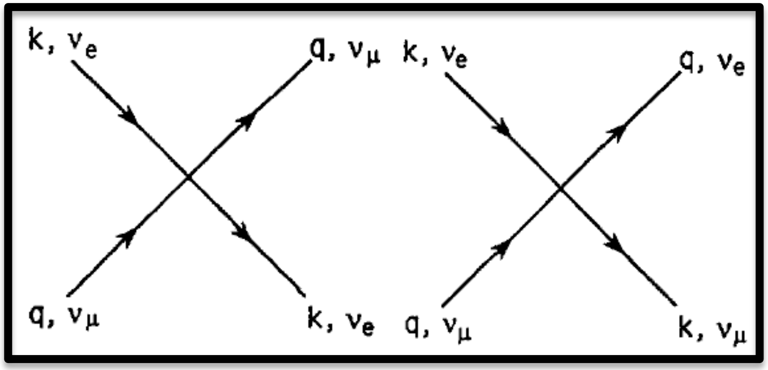
$(\Delta m_{atm}^2, \theta_{13})$ at 10^3 - 10^4 g/cc

In neutrinos for NH and in antineutrinos for IH.

Adiabaticity ≈ $\sin^2\theta_{13}$

Neutrino-Neutrino Interactions

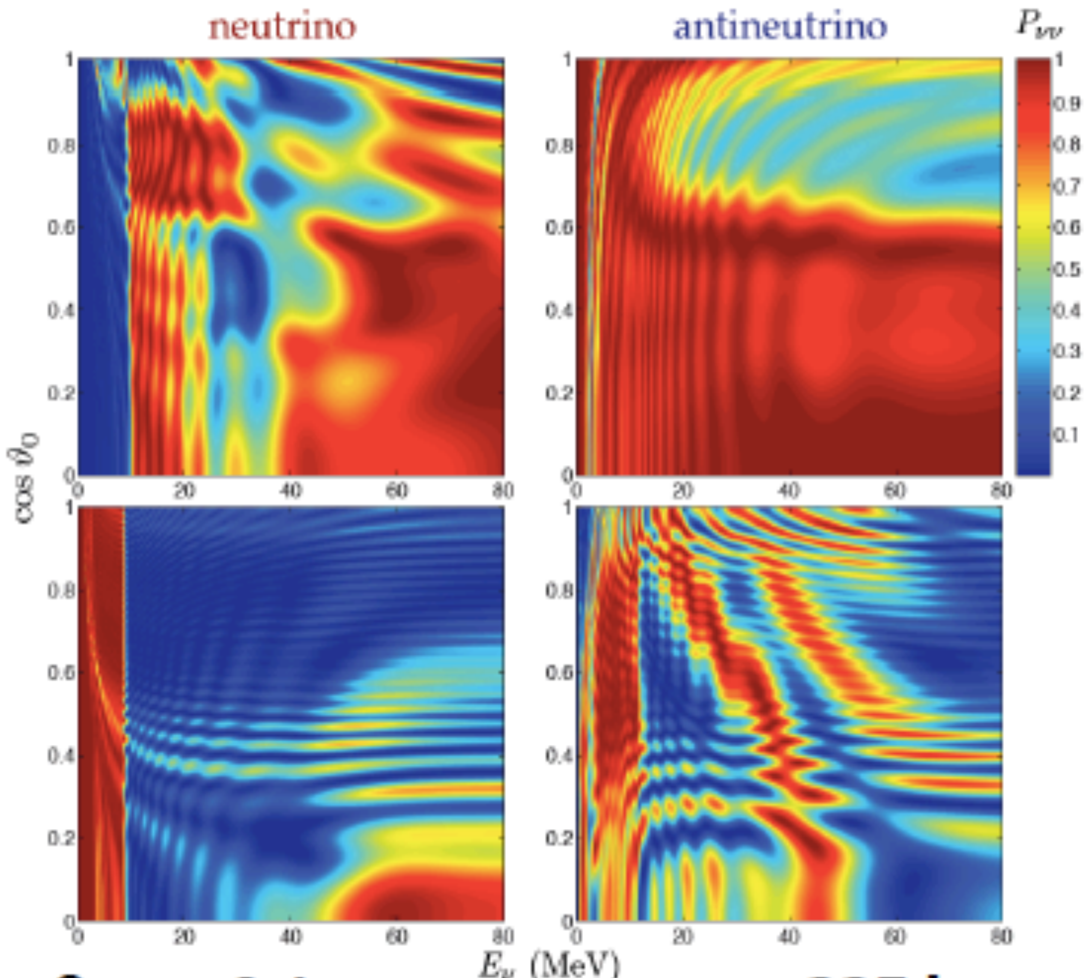
$$i\frac{d}{dt}\begin{vmatrix} |\nu_e(\boldsymbol{k})\nu_e(\boldsymbol{q})\rangle \\ |\nu_e(\boldsymbol{k})\nu_\mu(\boldsymbol{q})\rangle \\ |\nu_\mu(\boldsymbol{k})\nu_e(\boldsymbol{q})\rangle \\ |\nu_\mu(\boldsymbol{k})\nu_\mu(\boldsymbol{q})\rangle \end{vmatrix} = V_2 \begin{vmatrix} |\nu_e(\boldsymbol{k})\nu_e(\boldsymbol{q})\rangle \\ |\nu_e(\boldsymbol{k})\nu_\mu(\boldsymbol{q})\rangle \\ |\nu_\mu(\boldsymbol{k})\nu_e(\boldsymbol{q})\rangle \\ |\nu_\mu(\boldsymbol{k})\nu_\mu(\boldsymbol{q})\rangle \end{vmatrix}$$



Pantaleone (1992)

$$V_{2\nu} = \sqrt{2}G_F\xi\frac{1}{V}\begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

Collective Effects

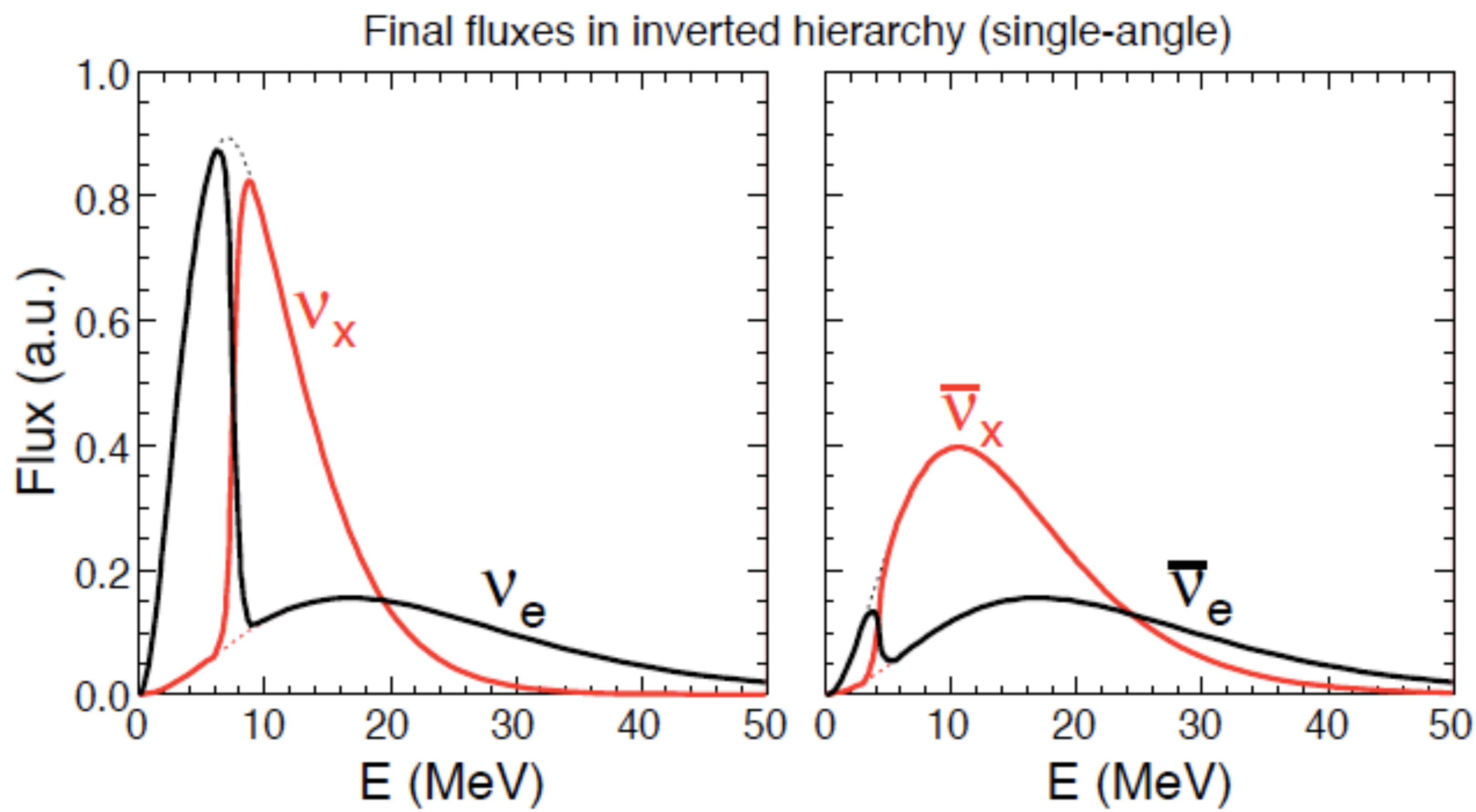


θ_{13} ~ 0.1 **r ~ 225 km**

Duan, Fuller, Carlson, Qian (2005, 2006,…)

Spectral Splits

Nontrivial Evolution only for Inverted Hierarchy

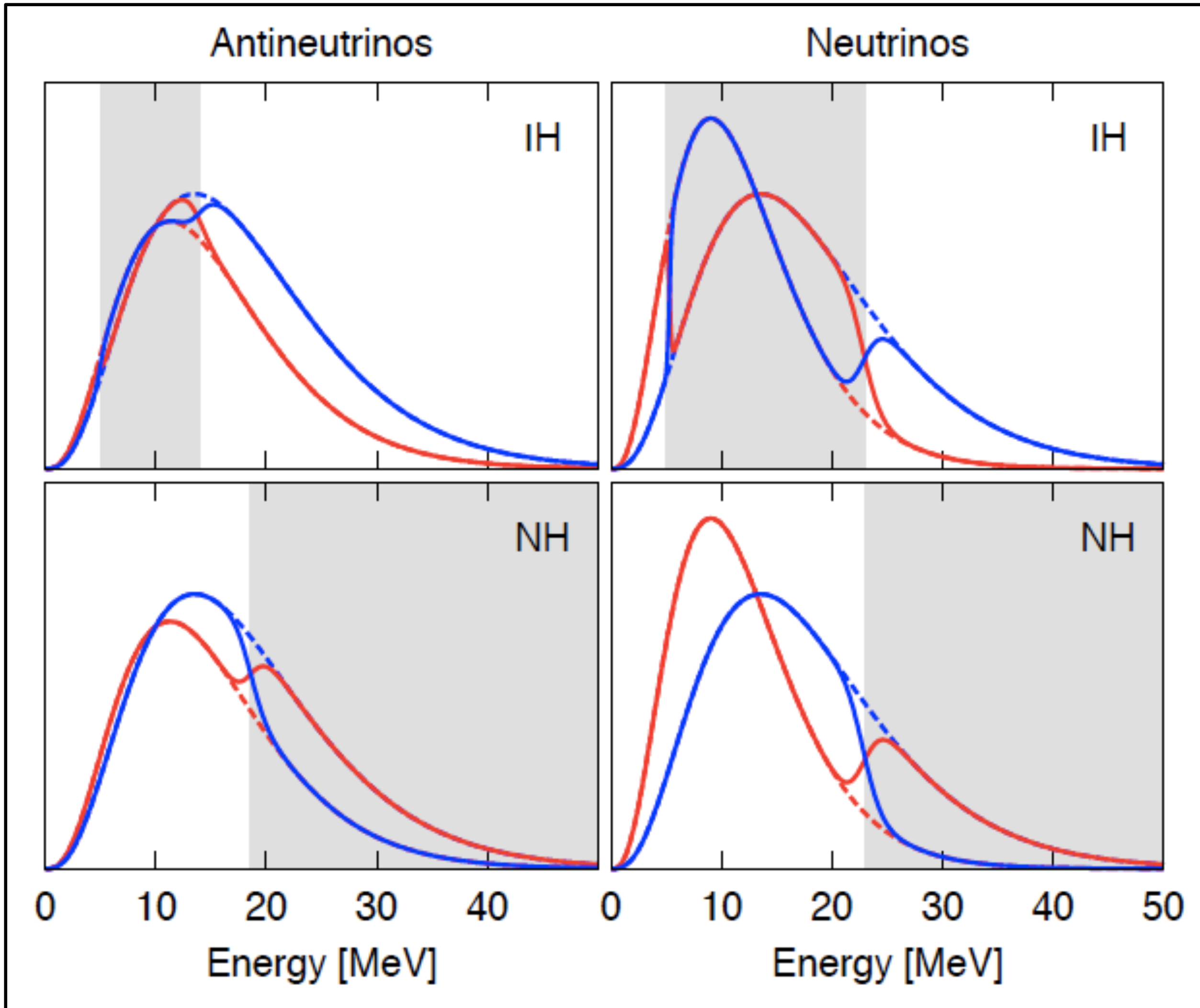


Fogli, Lisi, Marrone and Mirizzi

Multiple Splits

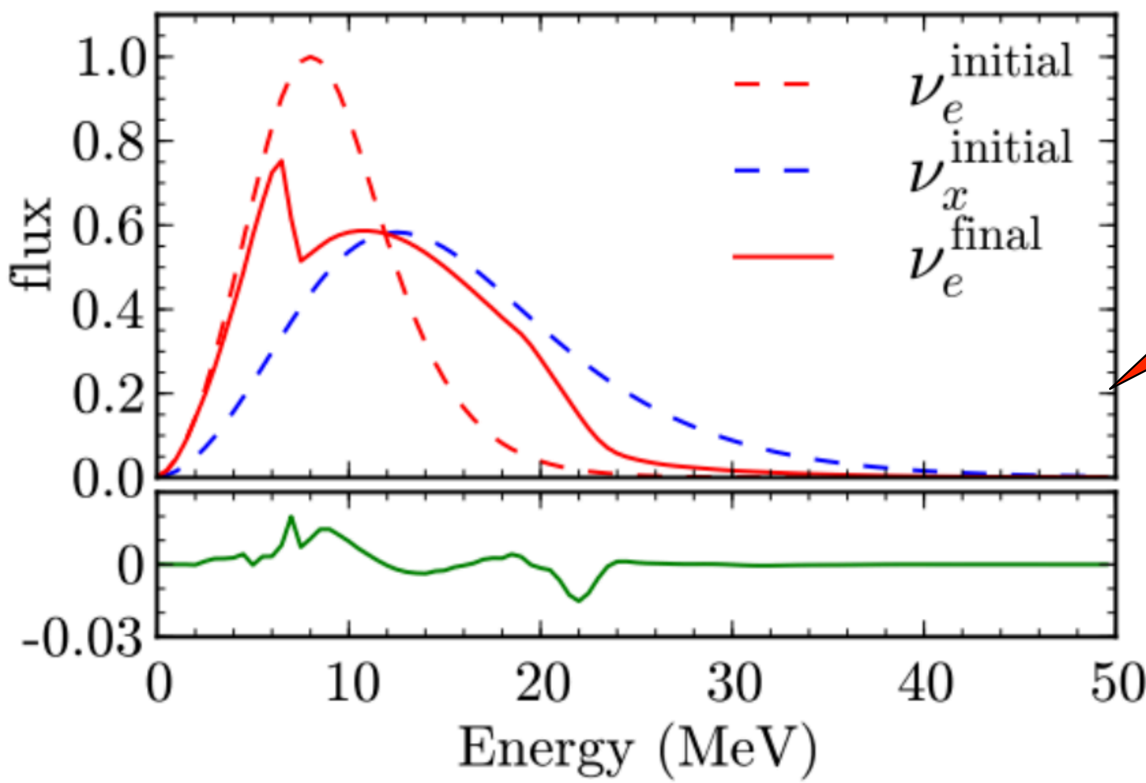
Spectral Splits in Inverted Hierarchy

Spectral Splits in Normal Hierarchy



Dasgupta, Dighe, Raffelt and Smirnov (PRL 2009)

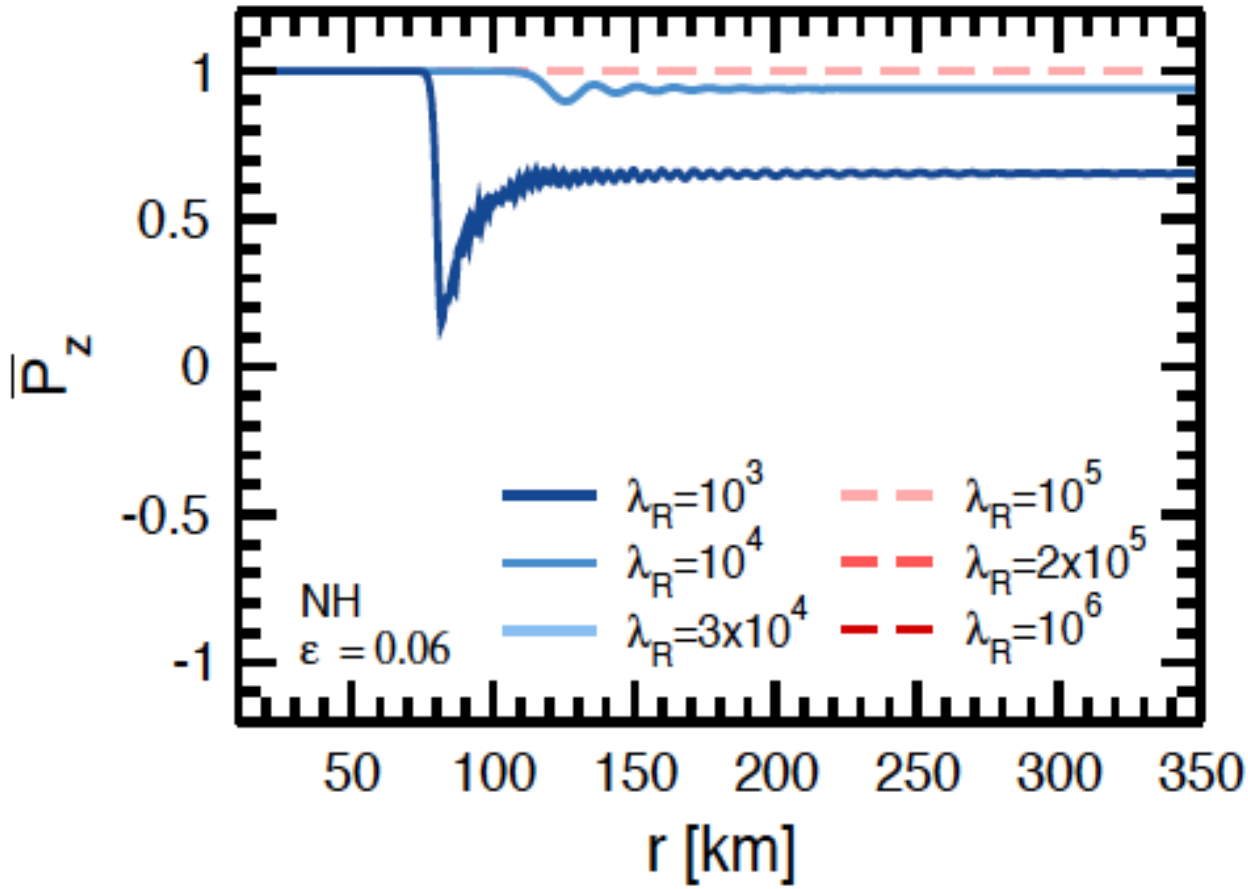
Multi-Angle Effects



Smeared Spectral Splits

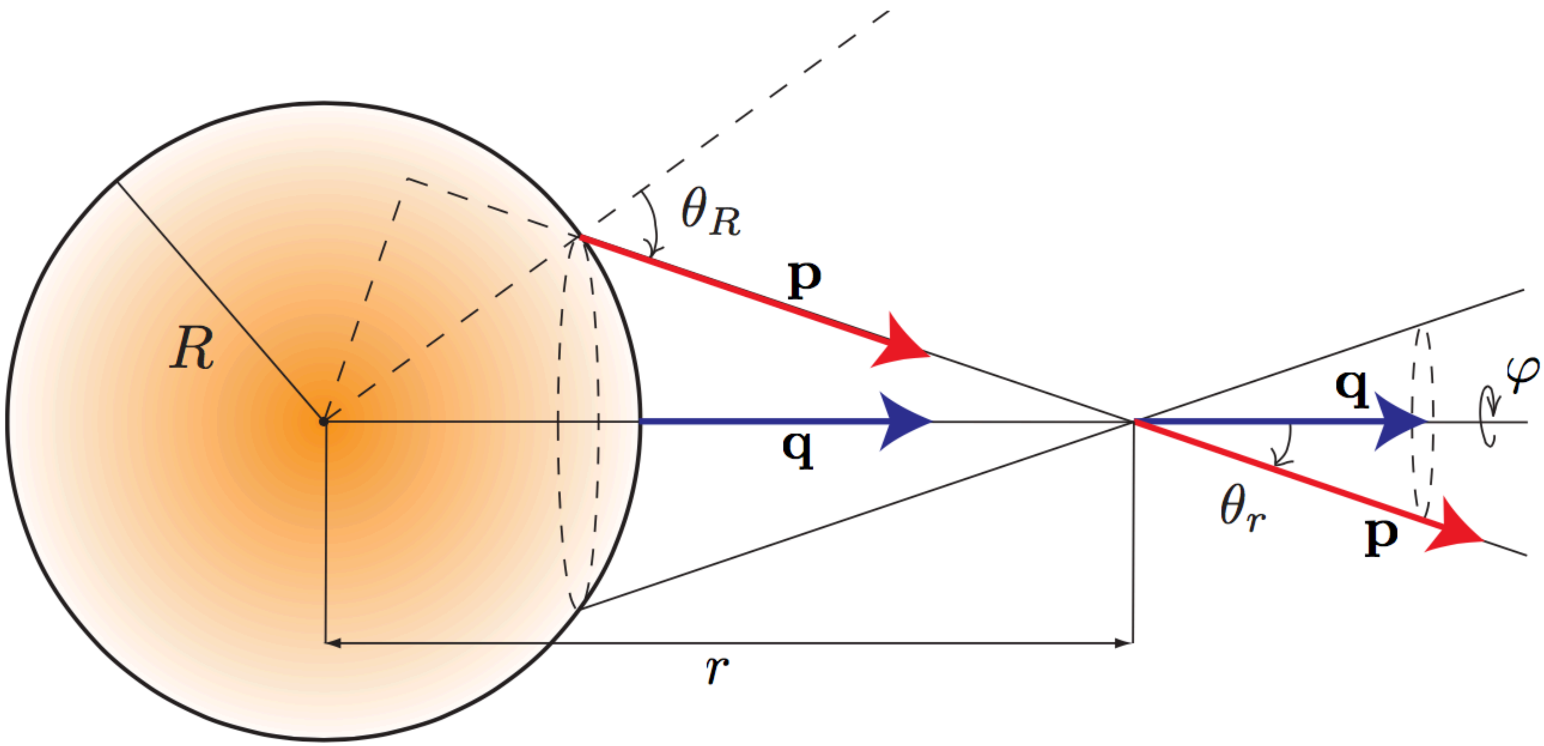
Many works, e.g., Esteban-Pretel et al, Friedland, Duan and Shalgar

Multi-Angle Matter Effect



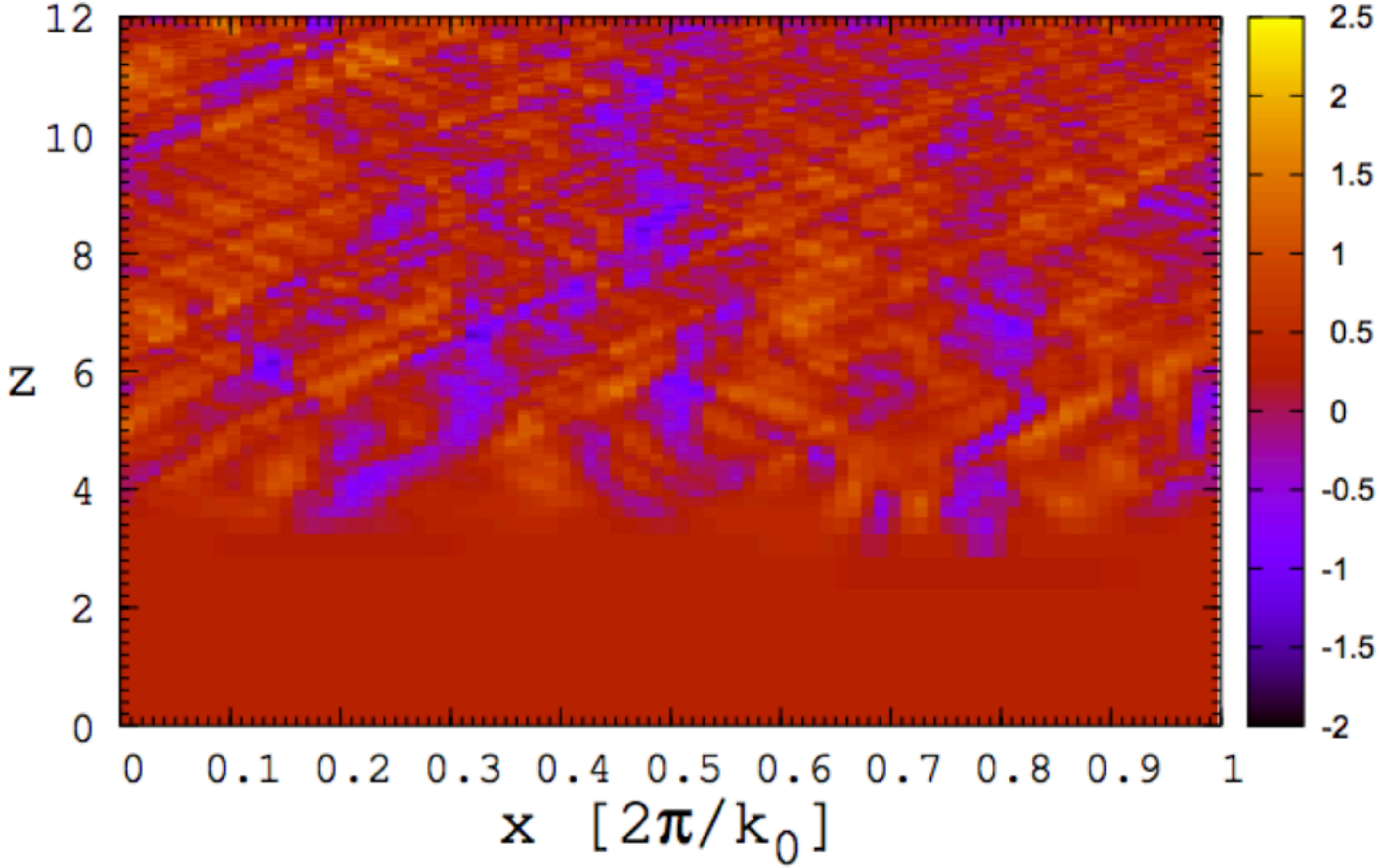
Esteban-Pretel, Mirizzi, Pastor, Tomas, Raffelt, Serpico, Sigl

Symmetry Breaking

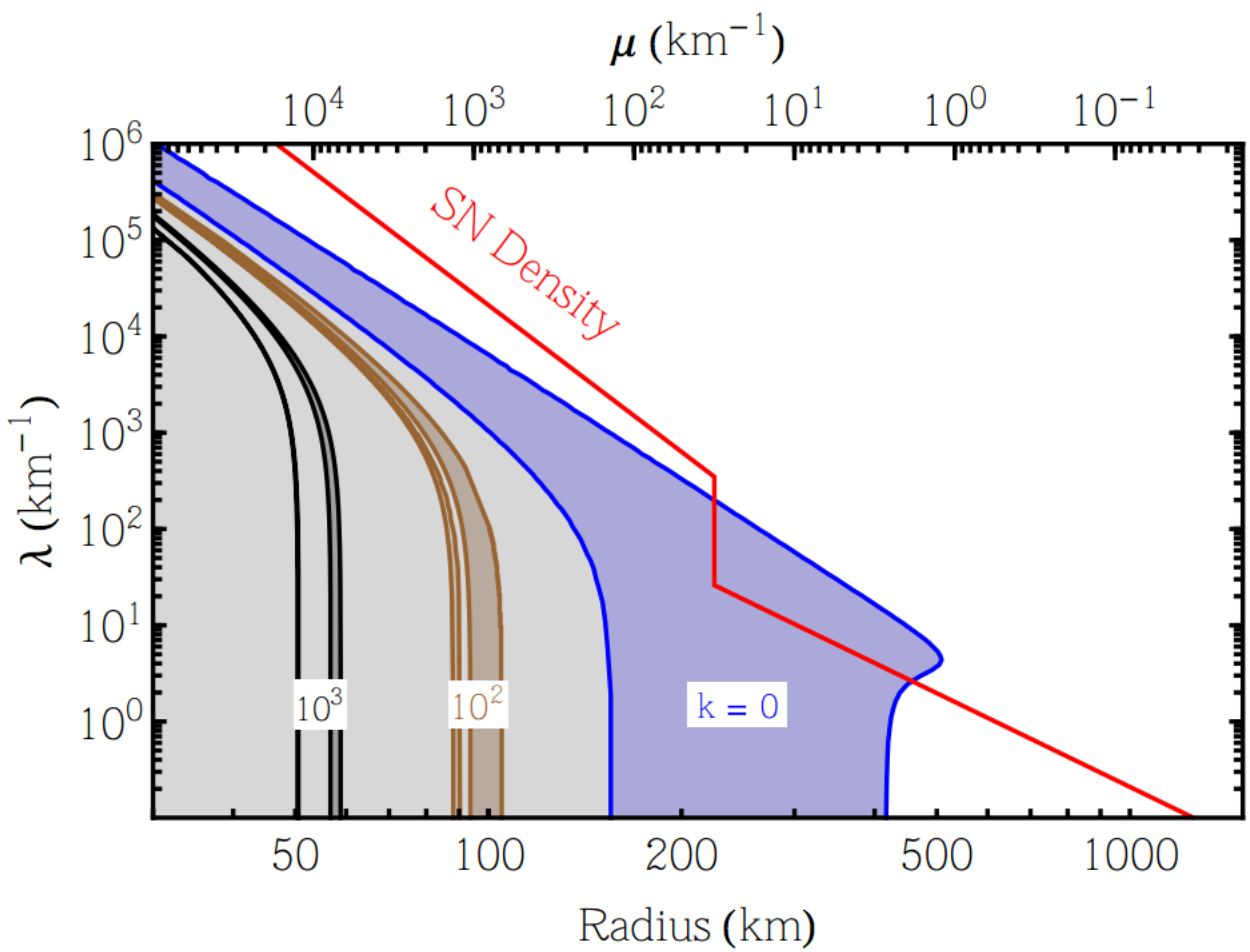


Symmetry Breaking

Mirizzi, Mangano, Saviano



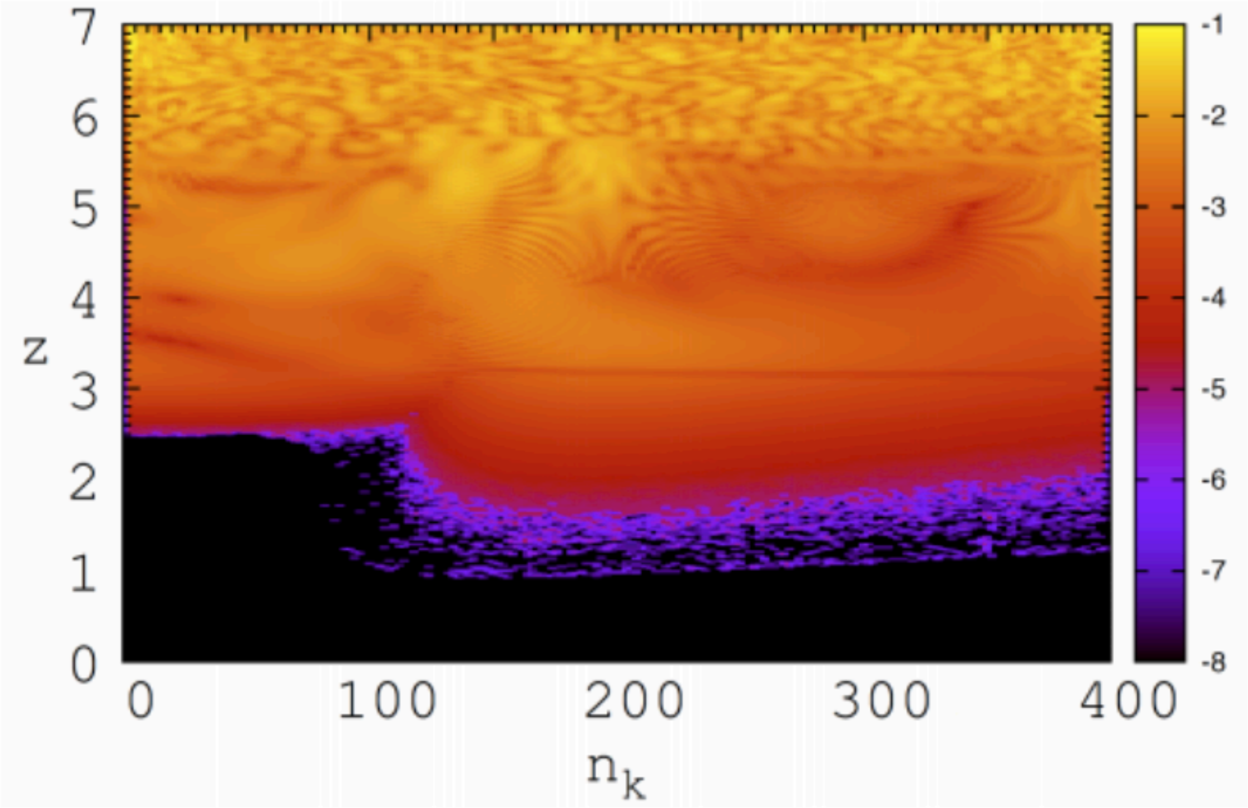
Instability Footprint

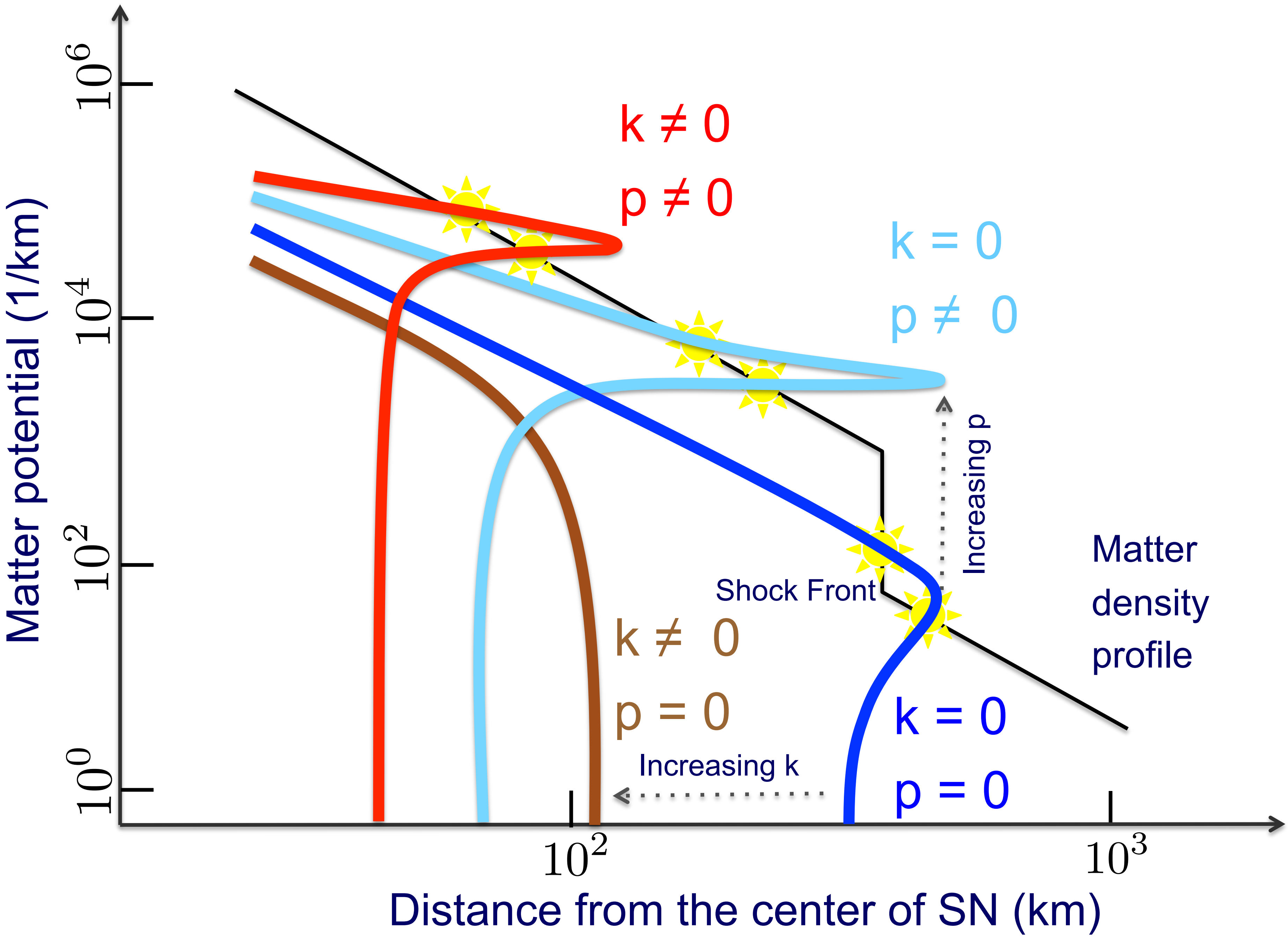


Chakraborty, Hansen, Izagguire, Raffelt

Temporal Instability

Dasgupta and Mirizzi





Matter potential (1/km)
10^6
10^4
10^2
10^0
$k \neq 0$
$p \neq 0$
$k = 0$
$p \neq 0$
Increasing p
Matter density profile
Shock Front
$k \neq 0$
$p = 0$
$k = 0$
$p = 0$
Increasing k
10^2
10^3
Distance from the center of SN (km)

Neutrino Flavor Evolution

$$i(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}})\varrho_{E,\mathbf{v}} = [\mathsf{H}_{E,\mathbf{v}}, \varrho_{E,\mathbf{v}}]$$

$$\mathsf{H}_{E,\mathbf{v}} = \mathsf{H}_{\rm vac} + \mathsf{H}_{\rm mat} + \mathsf{H}_{\nu\nu}$$

$$\mathsf{H}_{\rm vac} = \frac{1}{2E}\mathsf{U}_\theta \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \mathsf{U}_\theta^\dagger$$

$$\mathsf{H}_{\rm mat} = \sqrt{2}G_F \begin{pmatrix} n_e & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathsf{H}_{\nu\nu} = \sqrt{2}G_F \int \frac{1}{(2\pi)^3} E'^2 dE'^2 d\mathbf{v}' (1 - \mathbf{v}\cdot\mathbf{v}')\varrho_{E',\mathbf{v}'}$$

What is the "spectrum" ?

$$\varrho_{\omega,\mathbf{v}} = \frac{\mathrm{Tr}\,\varrho_{\omega,\mathbf{v}}}{2}\mathrm{I} + \frac{\Phi_\nu}{2}\, g_{\omega,\mathbf{v}} \begin{pmatrix} s_{\omega,\mathbf{v}} & S_{\omega,\mathbf{v}} \\ S^*_{\omega,\mathbf{v}} & -s_{\omega,\mathbf{v}} \end{pmatrix}$$

$$\int_{-\infty}^{0} d\Gamma\, \Phi_\nu g_{\omega,\mathbf{v}} = -(\Phi_{\bar{\nu}_e} - \Phi_{\bar{\nu}_x})$$

Fourier Modes

Precession-like motion for the modes

$$(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}})\mathbf{P}_{\omega,\mathbf{v}} = \left[-\omega\mathbf{B} + \lambda\mathbf{L} + \mu\int d\Gamma'(1-\mathbf{v}\cdot\mathbf{v}')\mathbf{P}_{\omega',\mathbf{v}'}\right]\times\mathbf{P}_{\omega,\mathbf{v}}$$

Fourier decomposition

$$\mathbf{P}_{\omega,\mathbf{v}}(t,x,z) = \sum_{p,k} e^{-i(pt+kx)}\mathbf{P}^{p,k}_{\omega,\mathbf{v}}(z)$$

Equation for Fourier modes

$$v_z\partial_z\mathbf{P}^{p,k}_{\omega,\mathbf{v}} = i(p+v_x k)\mathbf{P}^{p,k}_{\omega,\mathbf{v}} - (\omega\mathbf{B}-\lambda\mathbf{L})\times\mathbf{P}^{p,k}_{\omega,\mathbf{v}} + \sum_{p',k',\omega',\mathbf{v}'}\mu_v\,\mathbf{P}^{p-p',k-k'}_{\omega',\mathbf{v}'}\times\mathbf{P}^{p',k'}_{\omega,\mathbf{v}}$$

Linearization

$$i(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{x}})S_{\omega,\mathbf{v}} = (-\omega + \lambda + \epsilon\mu_v)S_{\omega,\mathbf{v}} - \mu_v \int d\Gamma' g_{\omega',\mathbf{v}'} S_{\omega',\mathbf{v}'}$$

Use Fourier decomposition

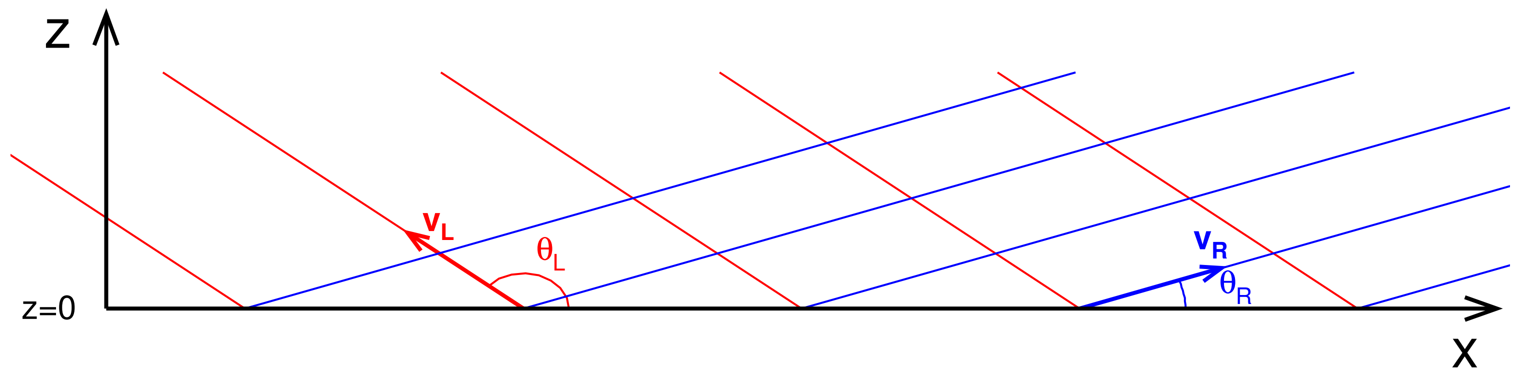
$$S_{\omega,\mathbf{v}}(t,x,z) = \sum_{p,k} e^{-i(pt+kx)} S^{p,k}_{\omega,\mathbf{v}}(z)$$

Get an eigenvalue equation

$$\left(\frac{-\omega - kv_x - p + \lambda + \epsilon\mu_v}{v_z}\right) Q^{p,k}_{\omega,\mathbf{v}} - \frac{\mu_v}{v_z}\int d\omega'\, g_{\omega',\mathbf{v}'} Q^{p,k}_{\omega,\mathbf{v}'} = \Omega^{p,k}_{\omega,\mathbf{v}} Q^{p,k}_{\omega,\mathbf{v}}$$

In principle this is a solvable eigenvalue problem

A simple example



Neutrinos emitted from a surface, along two directions L and R

The initial condition along x are noisy and not steady

How does the flavor content evolve along z?

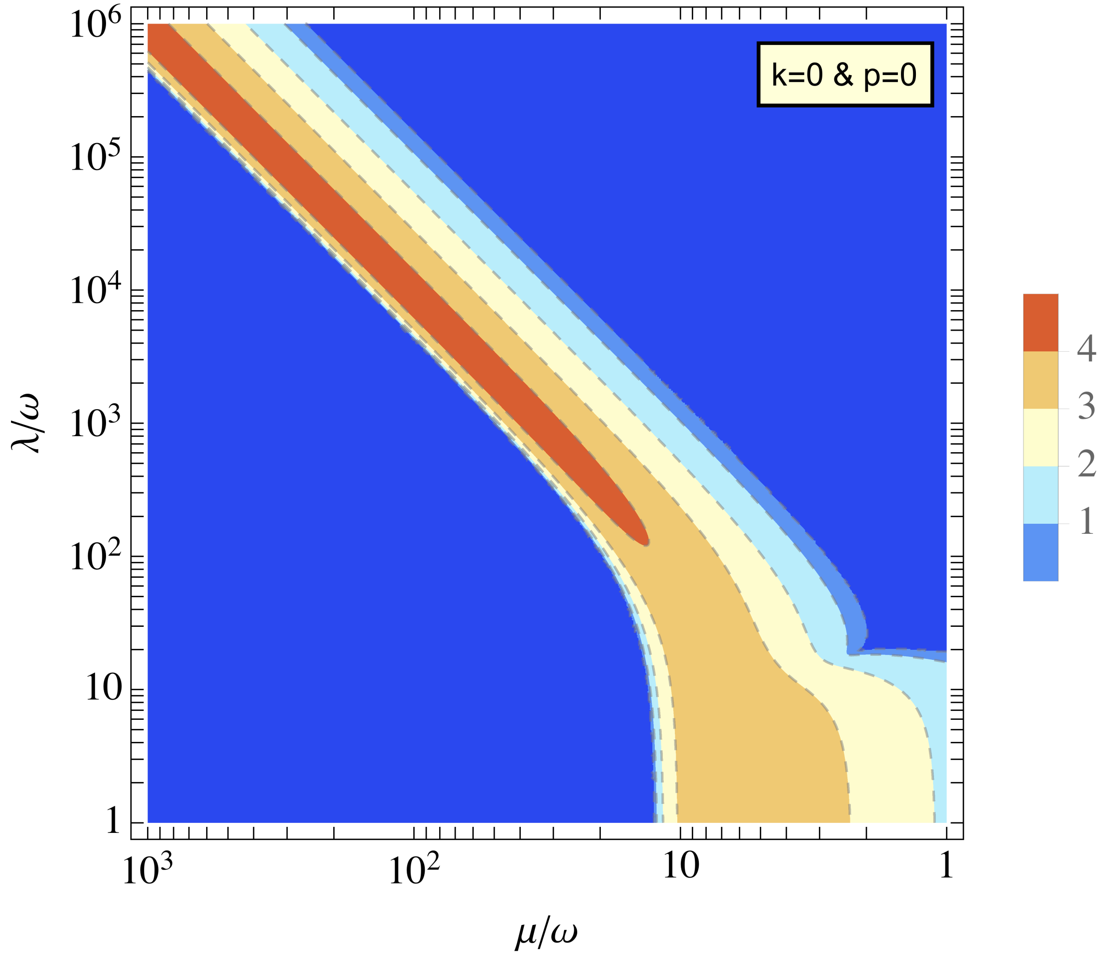
How does it break the x and t translation symmetry?

Explicit Computation

$$\mathrm{Det}\begin{pmatrix} \omega_{+L}+\Omega & 0 & (1+\epsilon)\mu_L & -\mu_L \\ 0 & \omega_{-L}+\Omega & (1+\epsilon)\mu_L & -\mu_L \\ (1+\epsilon)\mu_R & -\mu_R & \omega_{+R}+\Omega & 0 \\ (1+\epsilon)\mu_R & -\mu_R & 0 & \omega_{-R}+\Omega \end{pmatrix} = 0$$

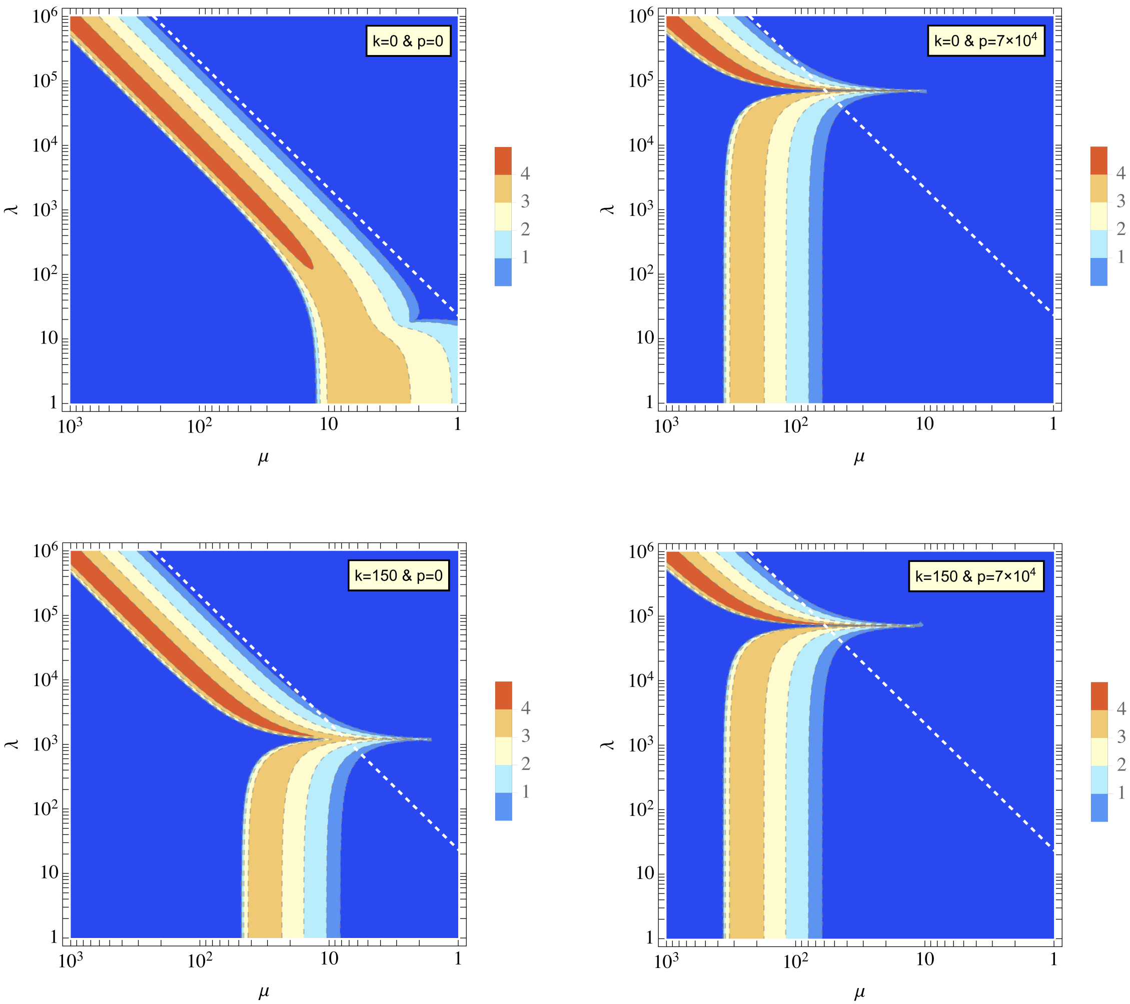
$$\omega_{\pm(L,R)} = (\pm\omega_0 + k v_{x,(L,R)} + p - \lambda - \epsilon\mu_v)/v_{z,(L,R)}$$
$$\mu_{(L,R)} = \mu_v / v_{z,(L,R)}\,.$$

Without Noise



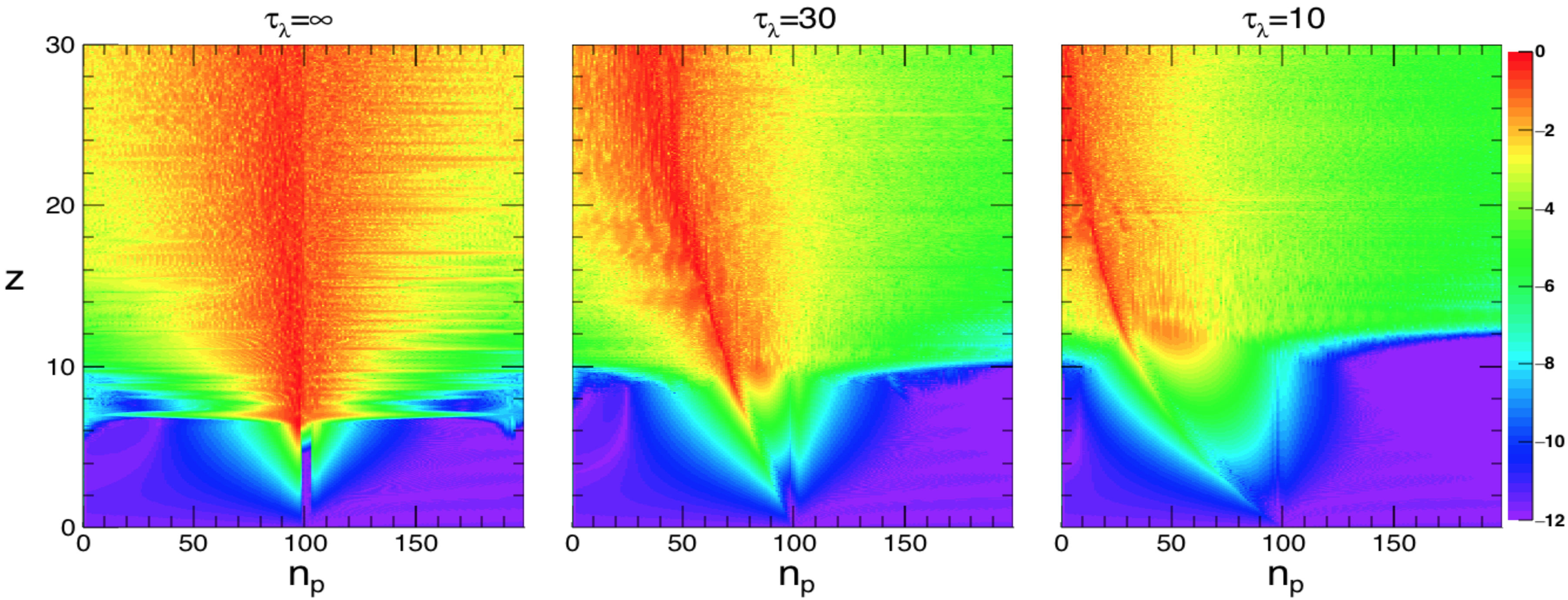
Capozzi, Dasgupta, Mirizzi (2016, JCAP)

With Noise

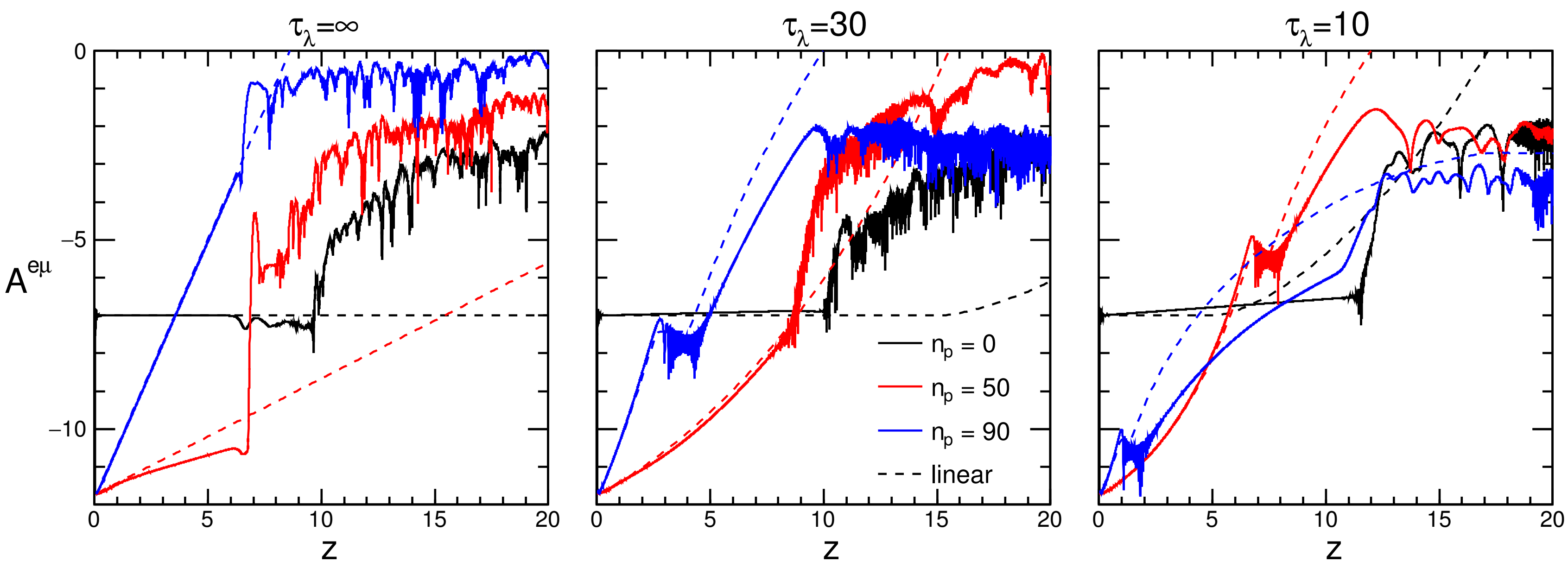


Capozzi, Dasgupta, Mirizzi (2016, JCAP)

Fully Nonlinear Evolution



Linear vs. Nonlinear



More work in progress

Moving Footprints