Neutrino Mixing, Oscillations, Leptonic CP-Violation and Leptogenesis

S. T. Petcov

SISSA/INFN, Trieste, Italy,
IPMU, University of Tokyo, Tokyo, Japan and
INRNE, Bulgarian Academy of Sciences, Sofia, Bulgaria

ICRR, Tokyo University
April 15, 2009

Plan of the talk

1. Introduction.

2. Neutrino Mixing: Current Status.

3. Determining the Type of Neutrino Mass Spectrum.

4. High Precision Measurement of $\Delta m^2_\odot$ and $\sin^2\theta_\odot$.

5. Dirac and Majorana CP-Violation and Leptogenesis.

6. Conclusions.

Compelling Evidences for $\nu-$Oscillations

$-\nu_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z-, $L/E-$ **dependences of $\mu-$like events**

Dominant $\nu_\mu \longrightarrow \nu_\tau$ K2K, MINOS; CNGS (OPERA)

$-\nu_\odot$: **Homestake, Kamiokande, SAGE, GALLEX/GNO**

Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \longrightarrow \nu_{\mu,\tau}$ **BOREXINO; KamLAND..., LowNu**

$-$ LSND

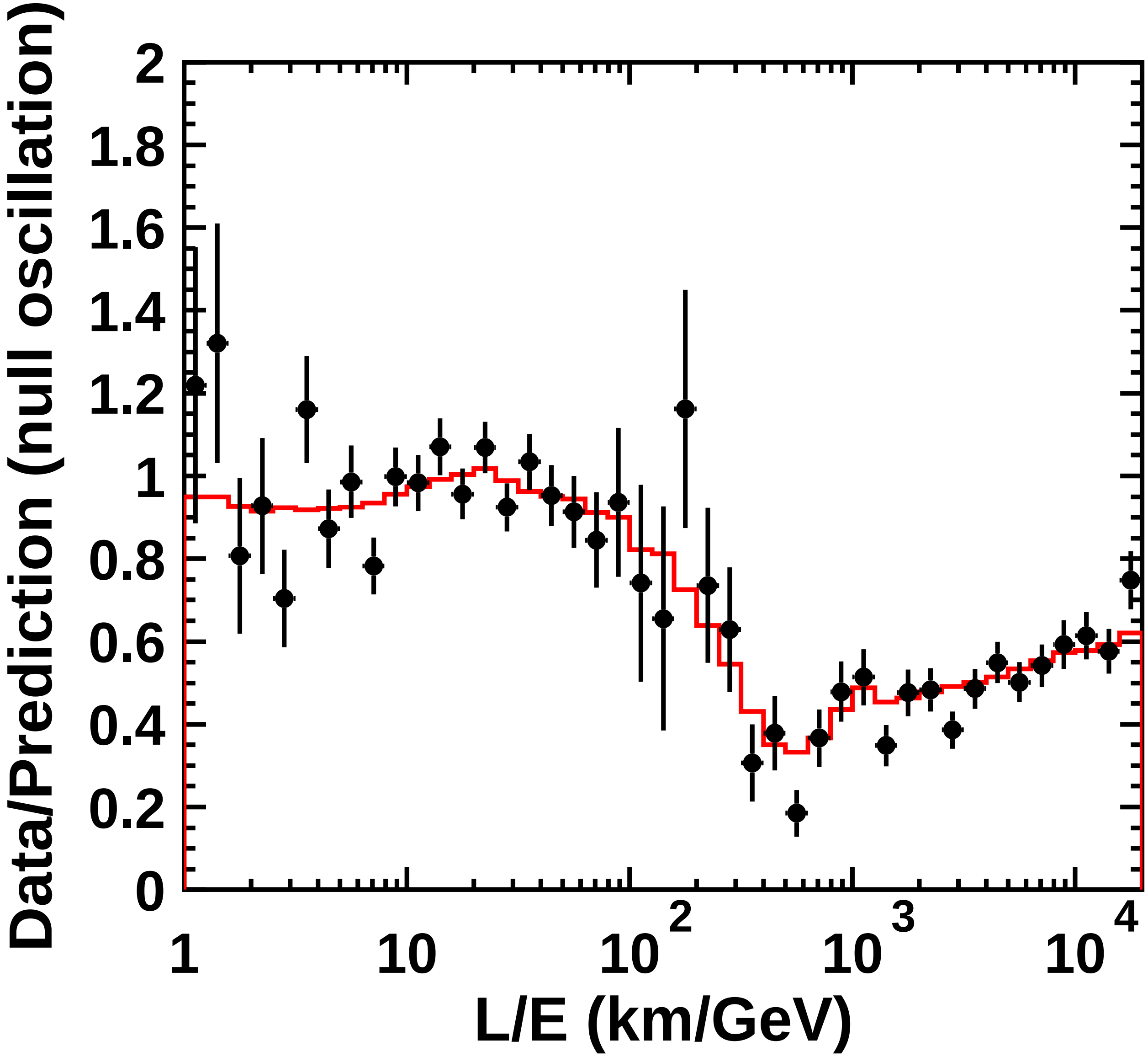
Dominant $\bar{\nu}_\mu \longrightarrow \bar{\nu}_e$; **MiniBOONE 11/04/07: negative result**

$$\nu_{l\mathrm{L}} = \sum_{j=1} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;

Z. Maki, M. Nakagawa, S. Sakata, 1962;

SK: L/E Dependence, μ−Like Events



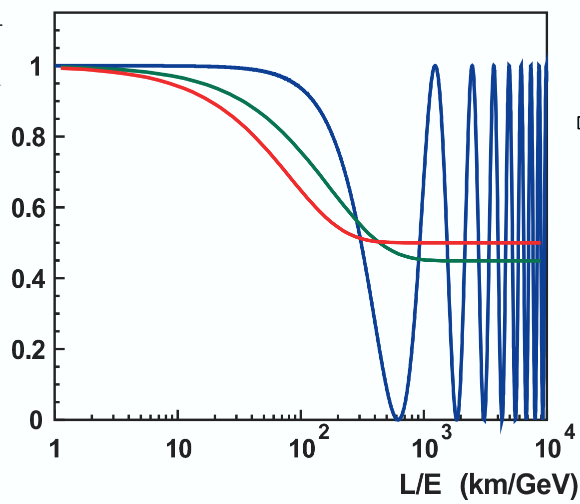
L/E dependence of $P_{\rm osc}$: V. Gribov, B. Pontecorvo, 1969

L/E analysis

Neutrino oscillation : $P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(1.27 \frac{\Delta m^2 L}{E})$

Neutrino decay : $P_{\mu\mu} = (\cos^2\theta + \sin^2\theta \times \exp(-\frac{m}{2\tau}\frac{L}{E}))^2$

Neutrino decoherence : $P_{\mu\mu} = 1 - \frac{1}{2}\sin^2 2\theta \times (1 - \exp(-\gamma_0 \frac{L}{E}))$



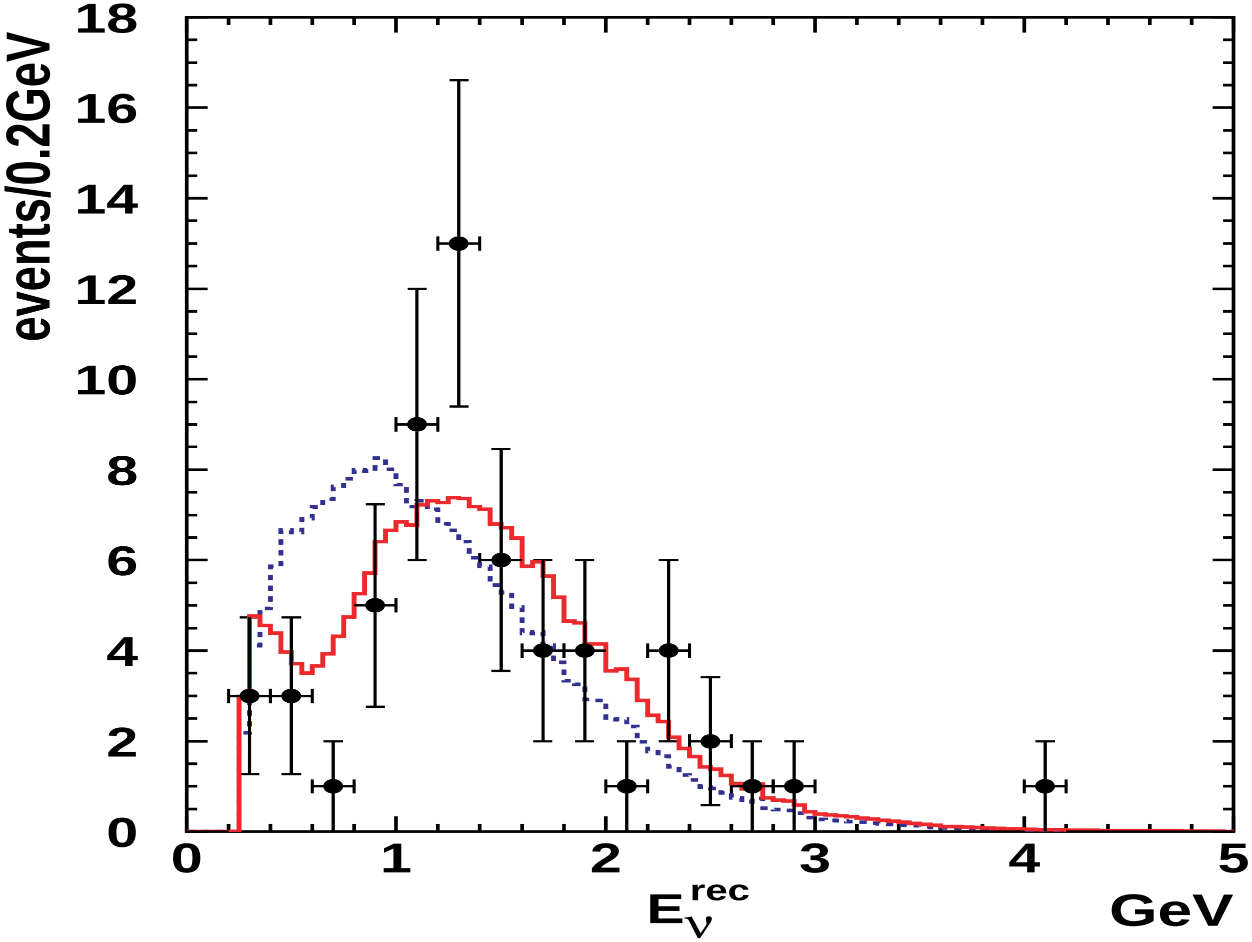
Use events with high resolution in L/E

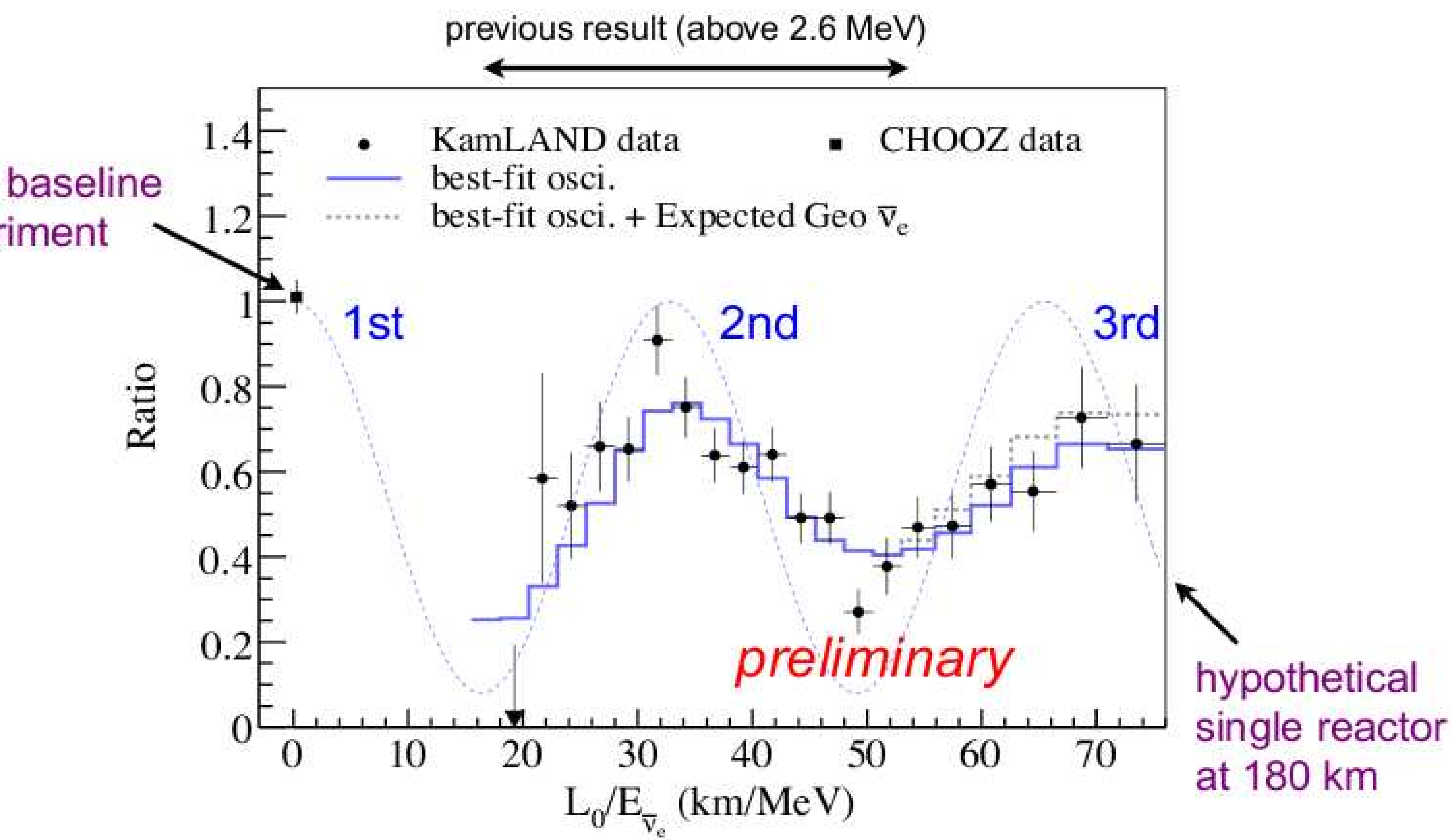
The first dip can be observed

→ Direct evidence for oscillations

→ Strong constraint to oscillation parameters, especially Δm^2 value

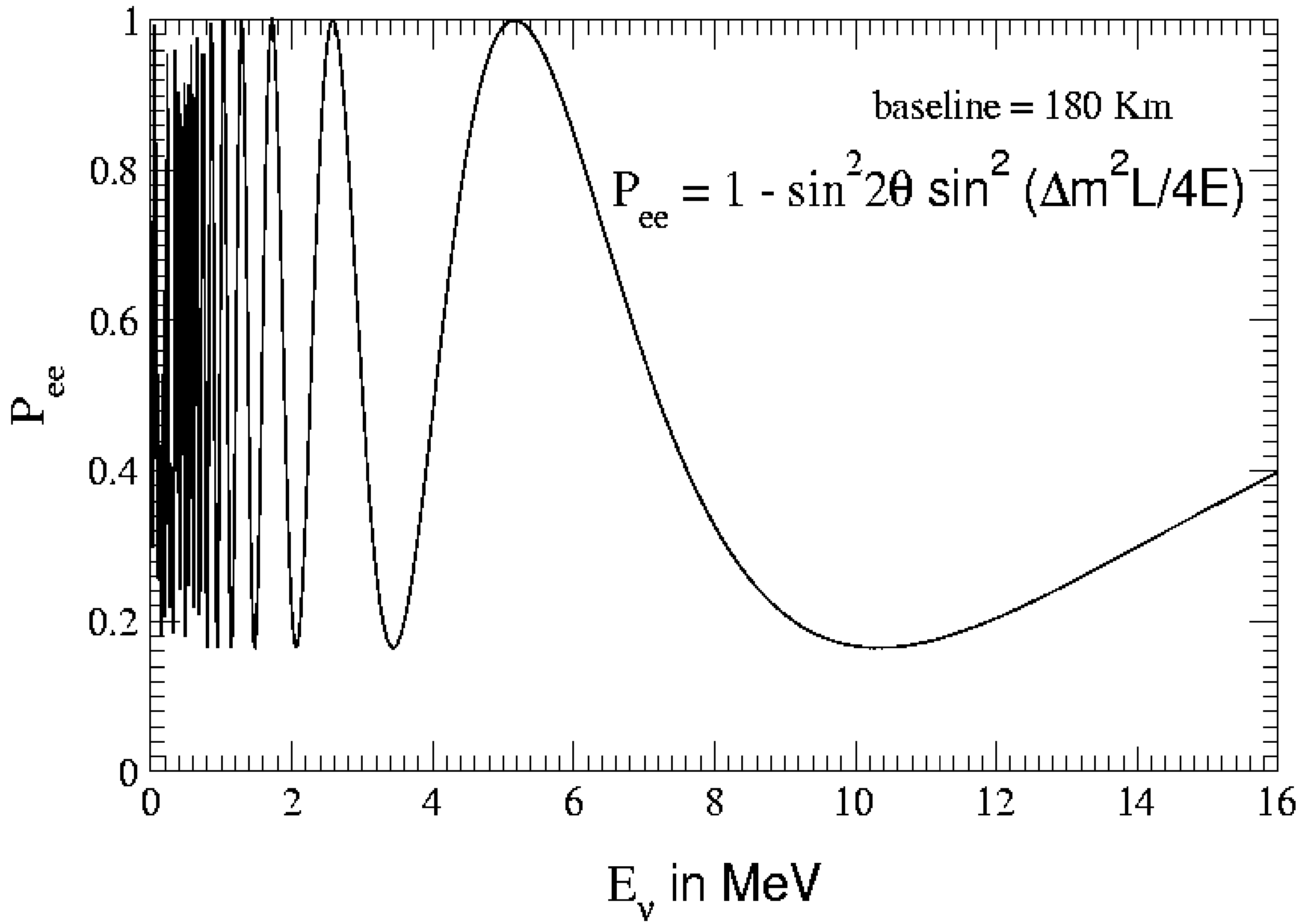
K2K: ν_μ Spectrum



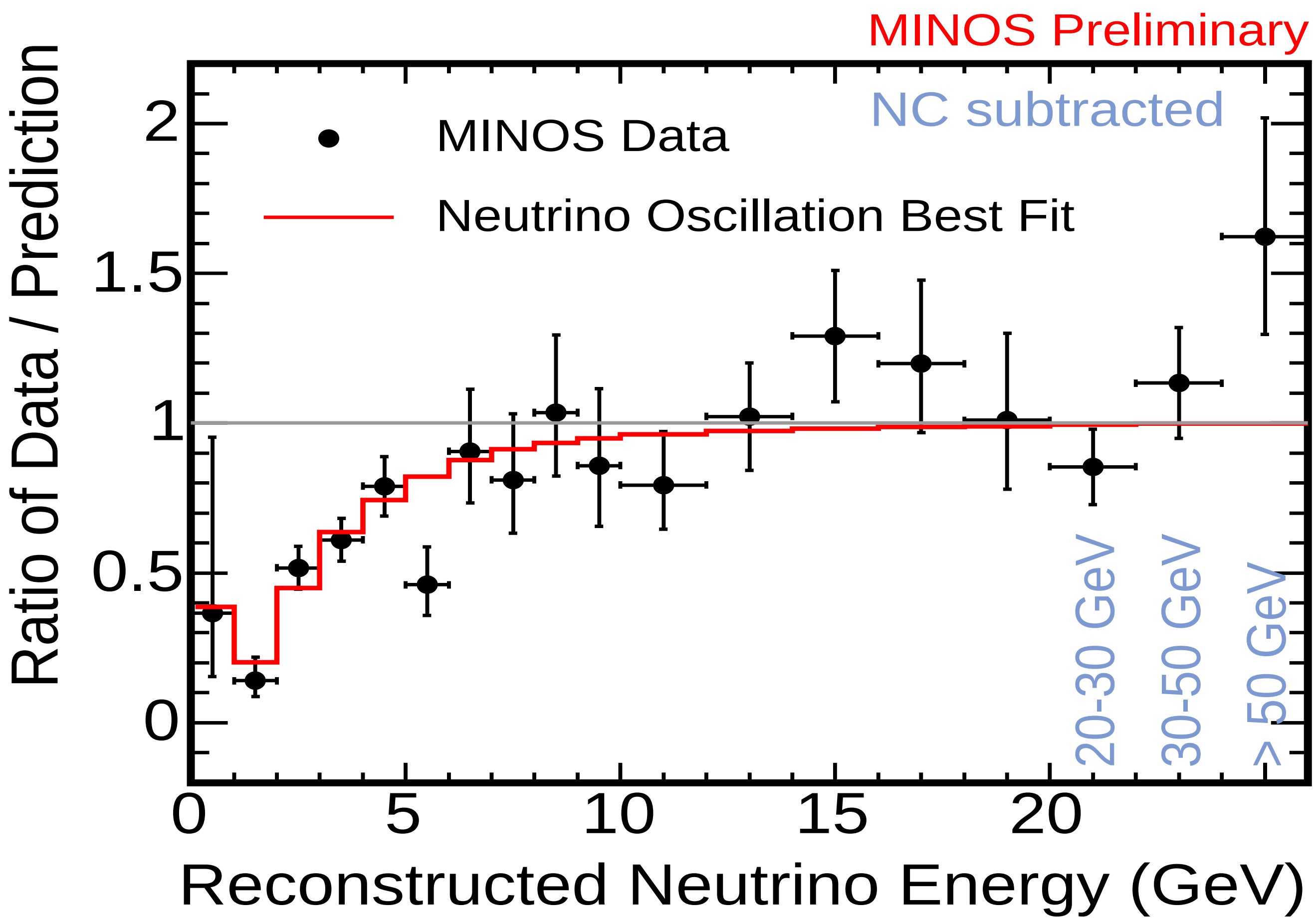


KamLAND: $L/E-$Dependence

$\bar{\nu}_e \longrightarrow \bar{\nu}_e$



MINOS: ν_μ Spectrum



Compelling Evidences for $\nu-$Oscillations: 3-ν mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1} U_{lj}\,\nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

		n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$		1	3	6
CP-violating phases:					
• $\nu_j -$ Dirac:	$\frac{1}{2}(n-1)(n-2)$		0	1	3
• $\nu_j -$ Majorana:	$\frac{1}{2}n(n-1)$		1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C\ (\bar{\chi}_k(x))^{\mathsf{T}} = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1$$

– Invariant under proper Lorentz transformations.
– Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1):\ \chi_k(x) \to e^{i\alpha}\chi_k(x) - \text{ impossible}$$

– $\chi_k(x)$ cannot absorb phases.
– $Q_{U(1)} = 0:\ Q_{\text{el}} = 0,\ L_l = 0,\ L = 0, ...$
– $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
– $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$–Dirac, $\chi(x)$–Majorana

$$< 0|T(\Psi_\alpha(x)\bar{\Psi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\Psi_\alpha(x)\Psi_\beta(y))|0 >= 0\ ,\quad < 0|T(\bar{\Psi}_\alpha(x)\bar{\Psi}_\beta(y))|0 >= 0\ .$$

$$< 0|T(\chi_\alpha(x)\bar{\chi}_\beta(y))|0 >= S^F_{\alpha\beta}(x-y)\ ,$$

$$< 0|T(\chi_\alpha(x)\chi_\beta(y))|0 >= -\xi^* S^F_{\alpha\kappa}(x-y)C_{\kappa\beta}\ ,$$

$$< 0|T(\bar{\chi}_\alpha(x)\bar{\chi}_\beta(y))|0 >= \xi\ C^{-1}_{\alpha\kappa}S^F_{\kappa\beta}(x-y)$$

$$U_{CP}\ \chi(x)\ U^{-1}_{CP} = \eta_{CP}\ \gamma_0\ \chi(x'),\quad \eta_{CP} = \pm i\ .$$

PMNS Matrix: Standard Parametrization

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

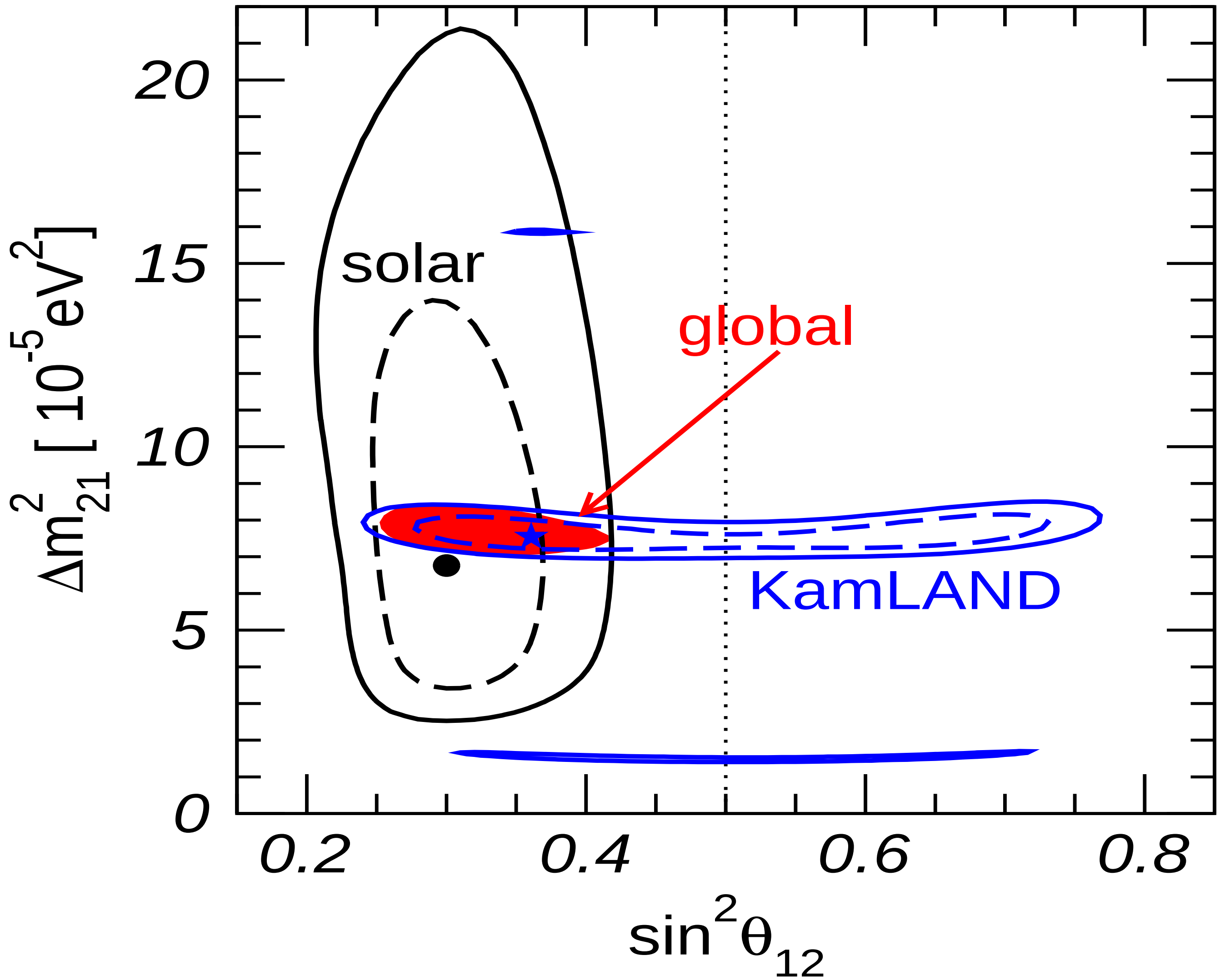
$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CP-violation phase, $\delta = [0, 2\pi]$,
- α_{21}, α_{31} - the two Majorana CP-violation phases.
- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.6 \times 10^{-5}$ eV$^2 > 0$, $\sin^2\theta_{12} \cong 0.305$, $\cos 2\theta_{12} \gtrsim 0.26$ (3σ),
- $|\Delta m^2_{\text{atm}}| \equiv |\Delta m^2_{31}| \cong 2.4$ $(2.5) \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} \cong 1$,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} < 0.040$ $(0.056$ $(0.063))$ 2σ (3σ).

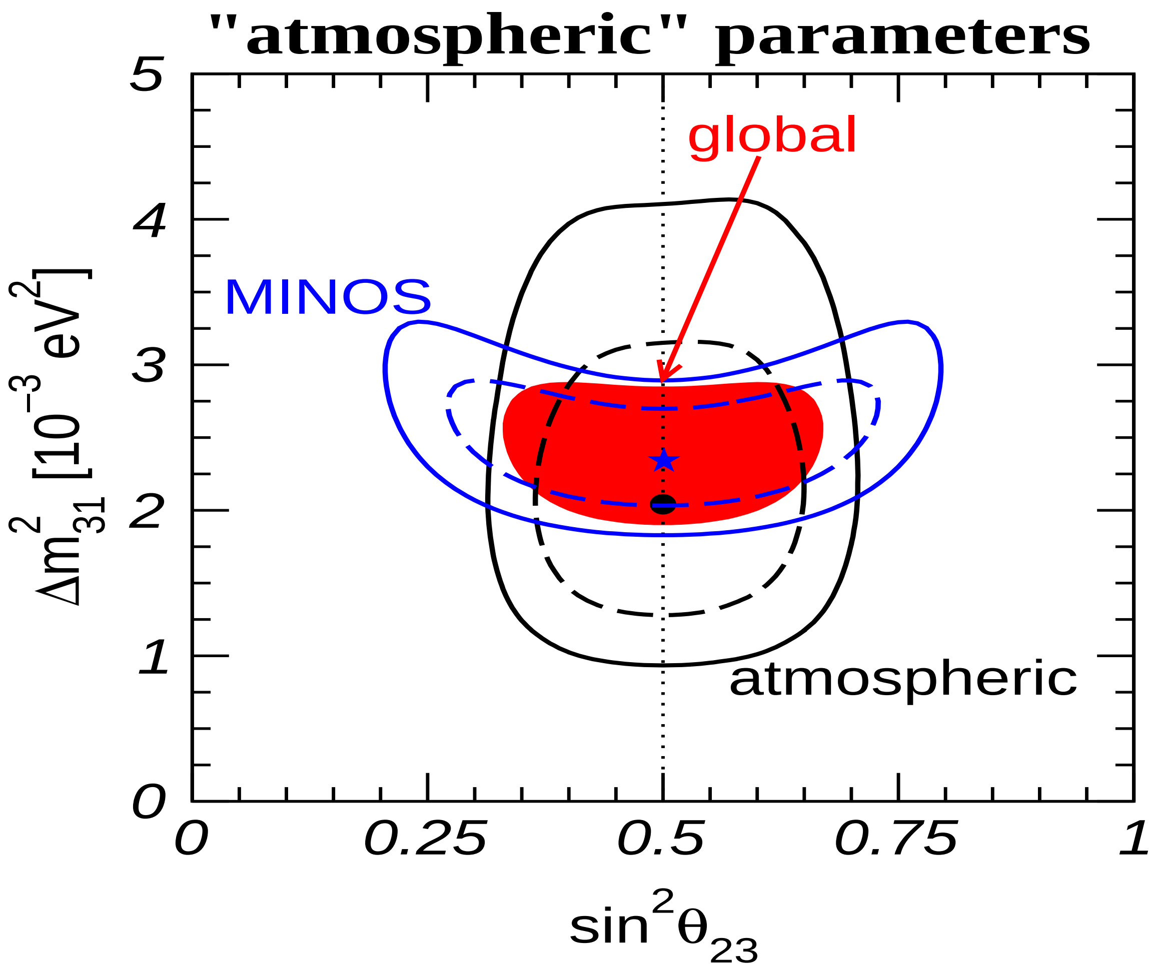
A.Bandyopadhyay, S.Choubey, S.Goswami, S.T.P., D.P.Roy, arXiv:0804.4857;

T. Schwetz et al., arXiv:0808.2016

"solar" parameters



T. Schwetz, arXiv:0710.5027[hep-ph]



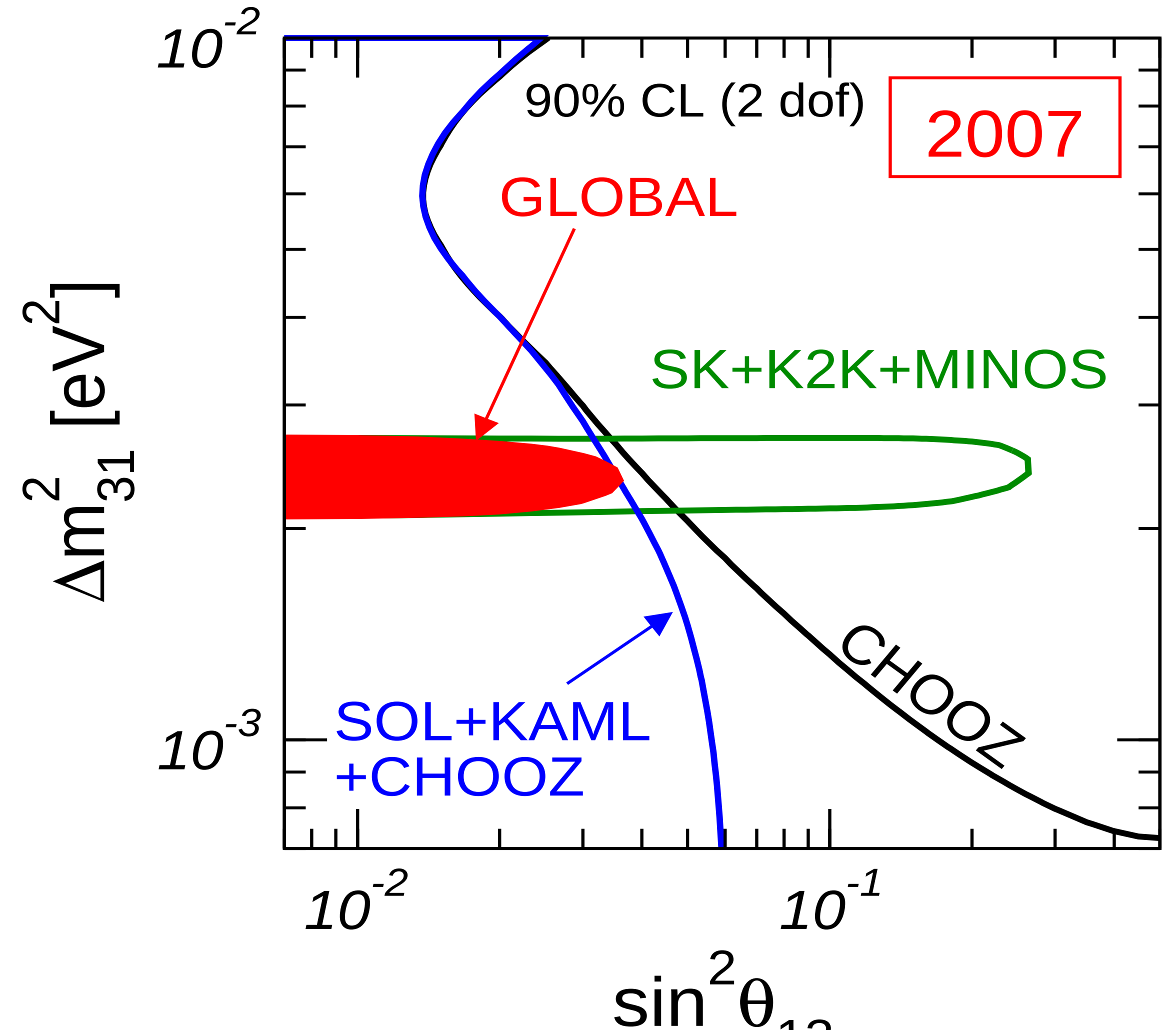
T. Schwetz, arXiv:0710.5027[hep-ph]

- sign of Δm^2_{atm} not determined;

  3-ν mixing: $\Delta m^2_{31} > 0$, $m_1 < m_2 < m_3$ (normal ordering (NO));

  $\Delta m^2_{31} < 0$, $m_3 < m_1 < m_2$ (inverted ordering (IO)).

- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23}, $(\frac{\pi}{4} - \theta_{23})$ ambiguity.



- $\sin^2\theta_{13} < 0.033$ (0.050) at 95% (99.73%) C.L.

Neutrino Oscillation Parameters

parameter	bf	1σ acc.	2σ range	3σ range
Δm^2_{21} [10^{-5} eV^2]	7.65	3%	7.25 – 8.11	7.05 – 8.34
$\|\Delta m^2_{31}\|$ [10^{-3} eV^2]	2.4	5%	2.18 – 2.64	2.07 – 2.75
$\sin^2\theta_{12}$	0.304	7%	0.27 – 0.35	0.25 – 0.37
$\sin^2\theta_{23}$	0.50	14%	0.39 – 0.63	0.36 – 0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	–	≤ 0.040	≤ 0.056

Best fit values (bf), relative accuracies at 1σ, and 2σ and 3σ allowed ranges of three-flavor neutrino oscillation parameters from a combined analysis of global data.

T. Schwetz et al., arXiv:0808.2016[hep-ph]

$3-\nu$ Mixing Analysis: $\Delta m^2_\odot \ll |\Delta m^2_{\text{atm}}|$

$P^{3\nu}_\odot \cong \sin^4\theta_{13} + \cos^4\theta_{13}\; P^{2\nu}_\odot$,

$P^{2\nu}_\odot = \bar{P}^{2\nu}_\odot + P^{2\nu}_{\odot\,\text{osc}}$,

$\bar{P}^{2\nu}_\odot = \frac{1}{2} + (\frac{1}{2} - P')\cos 2\theta^m_{12}(t_0)\cos 2\theta_{12} \qquad (\theta_{12} \equiv \theta_\odot)$,

$P' = 0$: S. Mikheyev, A. Smirnov, 1985;

$P' \neq 0$ (general or LZ): S. Parke, W. Haxton, 1986;

$P^{2\nu}_{\odot\,\text{osc}}$: S.T.P., 1988

$N_e \rightarrow N_e\cos^2\theta_{13}$,

$$P' = \frac{e^{-2\pi r_0\frac{\Delta m^2}{2E}\sin^2\theta_{12}} - e^{-2\pi r_0\frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0\frac{\Delta m^2}{2E}}}, \qquad r_0 \sim 0.1 R_\odot$$

S.T.P., 1988

LMA: $P' \ll 1, \quad < P^{2\nu}_{\odot\,\text{osc}} > \cong 0$

J. Rich, S.T.P., 1988

$$P^{3\nu}_{\text{KL}} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2(\frac{\Delta m^2_\odot}{4E}L)\right] \quad \left(P^{3\nu}_{\text{SNO}} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\sin^2\theta_{12}\right)$$

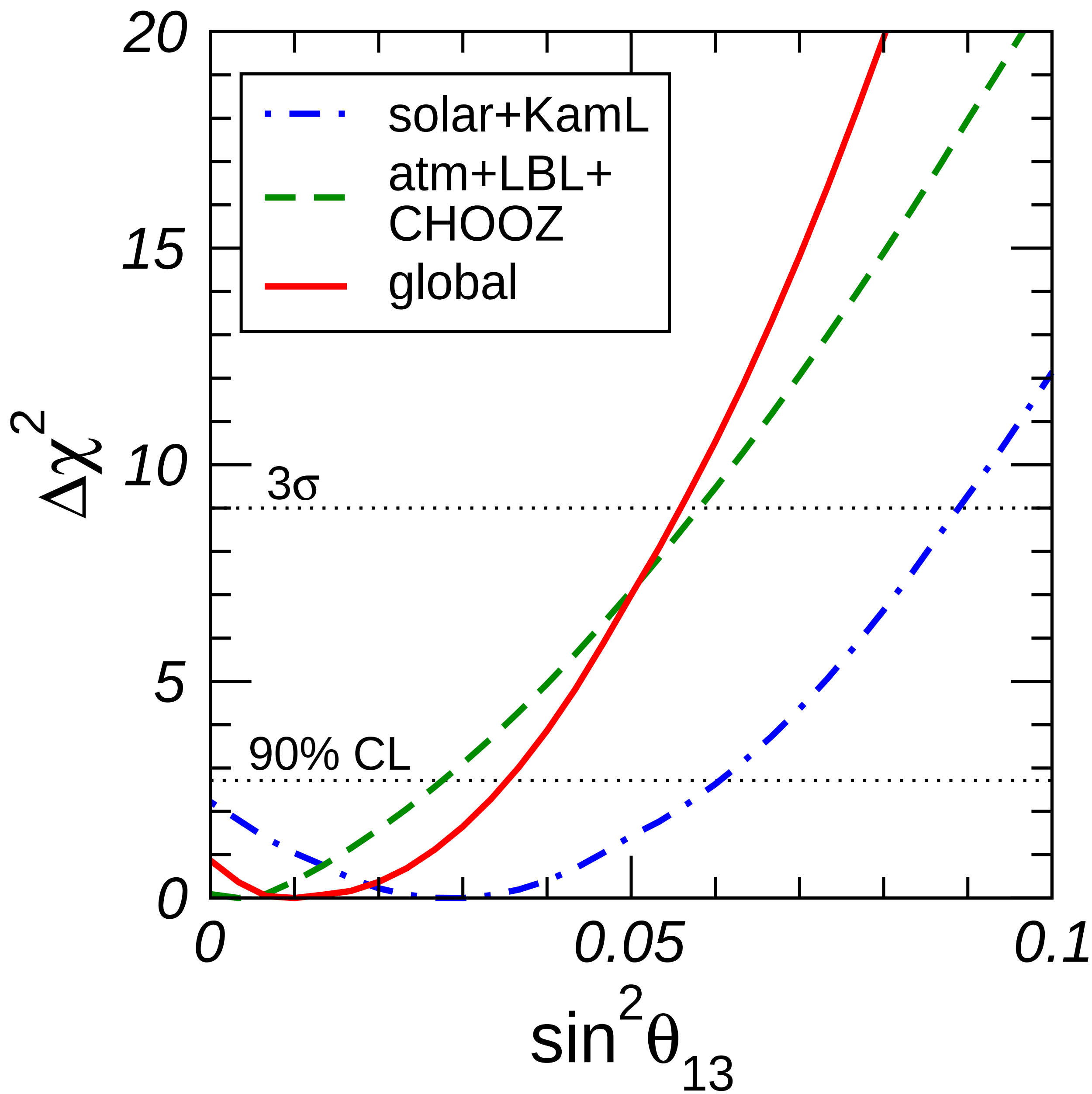
$$P^{3\nu}_{\text{CHOOZ}} \cong 1 - \sin^2 2\theta_{13}\sin^2(\frac{\Delta m^2_{\text{atm}}}{4E}L)$$

$\sin^2\theta_{13} = 0.016 \pm 0.010$, $\sin\theta_{13} = (0.077 - 0.161)$, 1σ

E. Lisi *et al.*, arXiv:0806.2649

Atmospheric ν data: $\cos\delta = -1$ favored over $\cos\delta = +1$

J. Escamilla *et al.*, arXiv:0805.2924



T. Schwetz *et al.*, arXiv:0808.2016[hep-ph]

MSW Transitions of Solar Neutrinos in the Sun and the Hydrogen Atom

$$i\frac{d}{dt}\begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} = \begin{pmatrix} -\epsilon(t) & \epsilon'(t) \\ \epsilon'(t) & \epsilon(t) \end{pmatrix} \begin{pmatrix} A_\alpha(t,t_0) \\ A_\beta(t,t_0) \end{pmatrix} \quad (1)$$

where $\alpha = \nu_e$, $\beta = \nu_{\mu(\tau)}$,

$\epsilon(t) = \frac{1}{2}\,[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e(t)]$,

$\epsilon'(t) = \frac{\Delta m^2}{4E}\sin 2\theta$, with $\Delta m^2 = m_2^2 - m_1^2$.

- Standard Solar Models

$N_e(t) \; = \; N_e(t_0)\exp\left\{-\frac{t-t_0}{r_0}\right\}$, $r_0 \sim 0.1R_\odot$, $R_\odot = 6.96 \times 10^5$km

Introducing the dimensionless variable

$$Z = ir_0\sqrt{2}G_F N_e(t_0)e^{-\frac{t-t_0}{r_0}}, \quad Z_0 = Z(t = t_0),$$

and making the substitution

$$A_e(t,t_0) = (Z/Z_0)^{c-a}\; e^{-(Z-Z_0)+i\int_{t_0}^{t}\epsilon(t')dt'}\; A'_e(t,t_0),$$

$A'_e(t,t_0)$ satisfies the confluent hypergeometric equation (CHE):

$$\left\{\; Z\frac{d^2}{dZ^2} + (c - Z)\;\frac{d}{dZ} - a\right\} A'_e(t,t_0) = 0,$$

where

$$a = 1 + ir_0\;\frac{\Delta m^2}{2E}\sin^2\theta, \qquad c = 1 + ir_0\;\frac{\Delta m^2}{2E}.$$

The confluent hypergeometric equation describing the ν_e oscillations in the Sun, coincides in form with the Schroedinger (energy eigenvalue) equation obeyed by the radial part, $\psi_{kl}(r)$, of the non-relativistic wave function of the hydrogen atom,

$\Psi(\overrightarrow{r}) = \frac{1}{r}\psi_{kl}(r) Y_{lm}(\theta', \phi')$,

r, θ' and ϕ' are the spherical coordinates of the electron in the proton's rest frame, l and m are the orbital momentum quantum numbers ($m = -l, ..., l$), k is the quantum number labeling (together with l) the electron energy (the principal quantum number is equal to $(k+l)$), E_{kl} ($E_{kl} < 0$), and $Y_{lm}(\theta', \phi')$ are the spherical harmonics. The function

$\psi'_{kl}(Z) = Z^{-c/2}\ e^{Z/2}\ \psi_{kl}(r)$

satisfies the confluent hypergeometric equation in which the variable Z and the parameters a and c are in this case related to the physical quantities characterizing the hydrogen atom:

$$Z = 2\,\frac{r}{a_0}\sqrt{-E_{kl}/E_I},\ a \equiv a_{kl} = l+1-\sqrt{-E_I/E_{kl}},\ c \equiv c_l = 2(l+1),$$

$a_0 = \hbar/(m_e e^2)$ is the Bohr radius and $E_I = m_e e^4/(2\hbar^2) \cong 13.6\ eV$ is the ionization energy of the hydrogen atom.

Quite remarkably, the behavior of such different physical systems as solar neutrinos undergoing MSW transitions in the Sun and the non-relativistic hydrogen atom are governed by one and the same differential equation.

Any solution - linear combination of two linearly independent solutions:

$\Phi(a,c;Z)$, $Z^{1-c}\ \Phi(a-c+1, 2-c; Z)$; $\Phi(a',c';Z=0)=1$, $a',c' \neq 0,-1,-2,....$

$$A(\nu_e \rightarrow \nu_{\mu(\tau)}) = \frac{1}{2}\ \sin 2\theta\ \left\{\Phi(a-c, 2-c; Z_0) - e^{i(t-t_0)\frac{\Delta m^2}{2E}}\ \Phi(a-1, c; Z_0)\ \right\}.$$

Sun: $N_e(x) \cong N_e(x_0) e^{-\frac{x}{r_0}}$, $r_0 \cong 0.1 R_{\odot}$, $R_{\odot} \cong 7 \times 10^5$ km

The region of $\nu_{\odot}$ production:

$20\ N_A\ cm^{-3} \lesssim N_e(x_0) \lesssim 100\ N_A\ cm^{-3}$: $|Z_0| > 500$ (!)

The solar ν_e survival probability:

$$\bar{P}(\nu_e \rightarrow \nu_e) = \tfrac{1}{2} + (\tfrac{1}{2} - P') \cos 2\theta_m^0 \cos 2\theta,$$

$$P' = \frac{e^{-2\pi r_0 \frac{\Delta m^2}{2E} \sin^2\theta} - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}{1 - e^{-2\pi r_0 \frac{\Delta m^2}{2E}}}$$

S.T.P., 1988

- sgn(Δm^2_{atm}) = sgn(Δm^2_{31}) not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering}$$

Convention: $m_1 < m_2 < m_3$ - NMO, $m_3 < m_1 < m_2$ - IMO

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \; m^2_{1,2,3} >> \Delta m^2_{\text{atm}}, \; \text{QD}; \; m_j \gtrsim 0.10 \text{ eV}.$$

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\text{CP}} \propto J_{\text{CP}} \propto \sin\theta_{13} \sin\delta$
- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

    S.M. Bilenky, J. Hosek, S.T.P.,1980;
    P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

  - $|<m>|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \to e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\alpha_{21,31}$!

$$A(\beta\beta)_{0\nu} \sim <m>$$ M(A,Z), M(A,Z) - NME,

$$|<m>| \cong \left|\sqrt{\Delta m^2_\odot}\sin^2\theta_{12}e^{i\alpha} + \sqrt{\Delta m^2_{31}}\sin^2\theta_{13}e^{i\beta}\right|, \quad m_1 \ll m_2 \ll m_3 \ \text{(NH)},$$

$$|<m>| \cong \sqrt{m_3^2 + \Delta m^2_{13}}\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|, \quad m_3 < (\ll) m_1 < m_2 \ \text{(IH)},$$

$$|<m>| \cong m\left|\cos^2\theta_{12} + e^{i\alpha}\ \sin^2\theta_{12}\right|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV} \ \text{(QD)},$$

$\theta_{12} \equiv \theta_\odot$, θ_{13}-CHOOZ; $\alpha \equiv \alpha_{21}$, $\beta + 2\delta \equiv \alpha_{31}$.

CP-invariance: $\alpha = 0, \pm\pi$, $\beta_M = 0, \pm\pi$;

$$|<m>| \lesssim 5 \times 10^{-3} \text{ eV, NH;}$$

$$\sqrt{\Delta m^2_{13}}\cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |<m>| \lesssim \sqrt{\Delta m^2_{13}} \cong 0.055 \text{ eV}, \quad \text{IH;}$$

$$m\cos 2\theta_{12} \lesssim |<m>| \lesssim m, \ m \gtrsim 0.10 \text{ eV}, \quad \text{QD}\ .$$

Absolute Neutrino Mass Measurements

The Troitzk and Mainz ^{3}H β-decay experiments

$$m_{\nu_e} < 2.3 \text{ eV} \qquad (95\% \text{ C.L.})$$

There are prospects to reach sensitivity

$$\text{KATRIN}: \qquad m_{\nu_e} \sim 0.2 \text{ eV}$$

Cosmological and astrophysical data: the WMAP result combined with data from large scale structure surveys (2dFGRS, SDSS)

$$\sum_j m_j \equiv \Sigma < (0.4 - 1.4) \text{ eV}$$

The WMAP and future PLANCK experiments can be sensitive to

$$\sum_j m_j \cong 0.4 \text{ eV}$$

Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and PLANCK experiments may allow to determine

$$\sum_j m_j : \qquad \delta \cong 0.04 \text{ eV}.$$

Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .

- Determination of sgn(Δm^2_{atm}), type of $\nu-$ mass spectrum

$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> \Delta m^2_{\text{atm}},\ \text{QD};\ m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j-masses, or min(m_j).

- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21}, α_{31} (Majorana)?

- Measurement of, or improving by at least a factor of (5 - 10) the existing upper limit on, $\sin^2\theta_{13}$.

- High precision determination of $\Delta m^2_{\odot}$, $\theta_{\odot}$, Δm^2_{atm}, θ_{atm}.

- Searching for possible manifestations, other than ν_l-oscillations, of the non-conservation of L_l, $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.

• Understanding at fundamental level the mechanism giving rise to the $\nu-$ masses and mixing and to the L_l-non-conservation. Includes understanding

– the origin of the observed patterns of ν-mixing and ν-masses ;

– the physical origin of CPV phases in U_{PMNS} ;

– Are the observed patterns of ν-mixing and of $\Delta m^2_{21,31}$ related to the existence of a new symmetry?

– Is there any relations between $q-$mixing and $\nu-$ mixing? Is $\theta_{12} + \theta_c = \pi/4$?

– Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?

– Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS}?

• Progress in the theory of ν-mixing might lead to a better understanding of the origin of the BAU.

– Can the Majorana and/or Dirac CPVP in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

HOW?

- $\nu_\odot-$, $\nu_{\rm atm}-$ experiments

  SK ($\nu_{\rm atm}$);

  INO ($\nu_{\rm atm}$); MEMPHYS (projects)

  MINOS ($\nu_\mu^{\rm atm}$); ATLAS, CMS ($\nu_\mu^{\rm atm}$) (?)

  SNO (2006)

  SAGE

  BOREXINO

  LowNu (XMASS, LENS,...) projects

- Experiments with Reactor $\bar{\nu}_e$, $\sim (1 - 180)$ km (SKGd)

- Accelerator Experiments

  MINOS 732 km

  CNGS (OPERA) 732 km

- Super Beams

  T2K, SK (HK) 295 km

  NOνA $\sim$800 km

  SPL+$\beta-$beams, MEMPHYS (0.5 megaton):
  CERN-Frejus $\sim$140 km

  $\nu-$Factories $\sim$ 3000, 7000 km

- $(\beta\beta)_{0\nu}-$Decay, ^{3}H $\beta-$Decay
- Astrophysics, Cosmology

$$\Delta m^2_{\odot} = \Delta m^2_{21},\ \theta_{\odot} = \theta_{12}$$

Data from $\nu_{\odot}$- experiments

- **SNO**: $A_{\mathrm{D-N}} < 4.3\%$

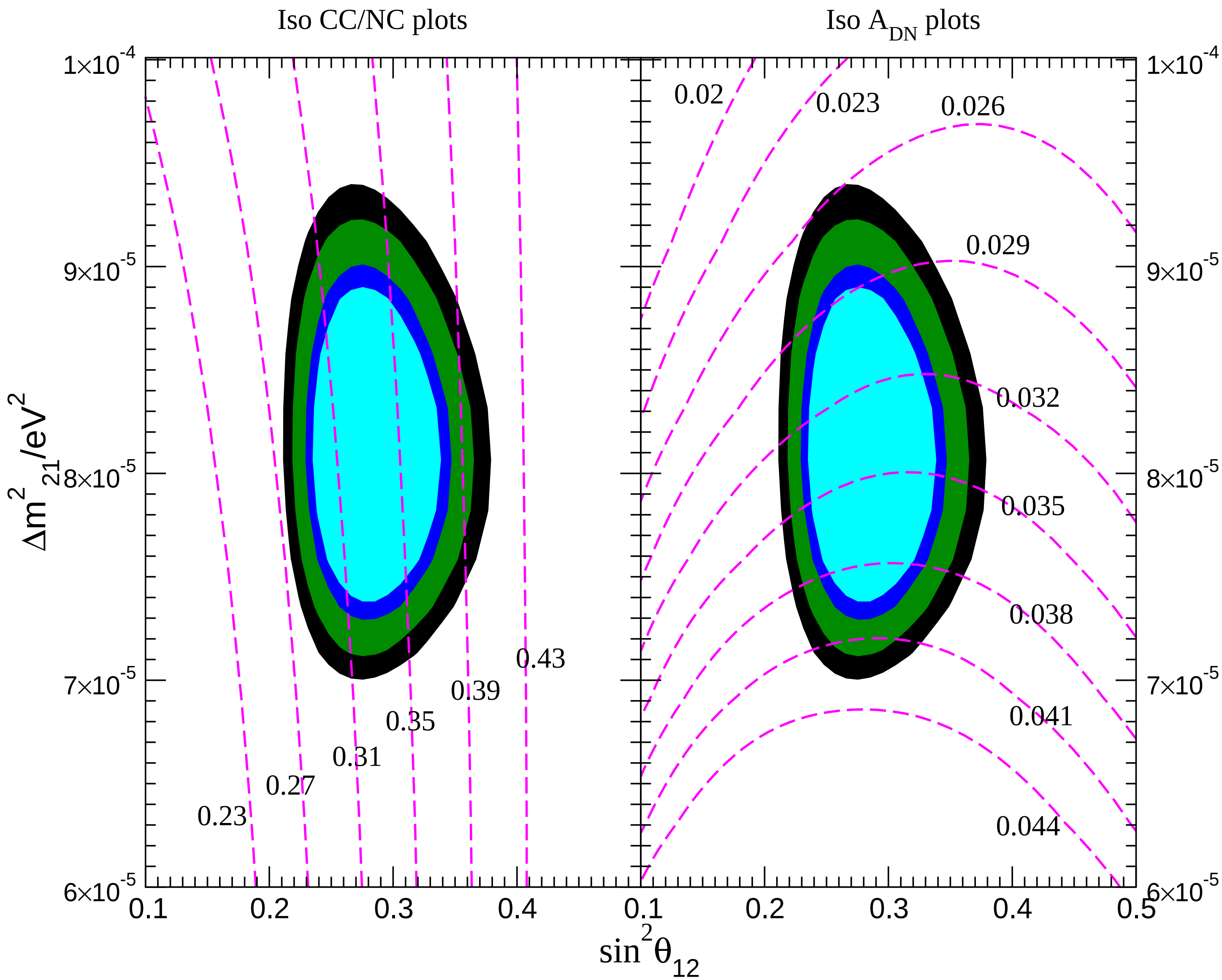
  would restrict further Δm^2_{21} from below

  $R_{\mathrm{CC/NC}} = 0.306 \pm 0.035$, **reducing the error**

  would restrict further

  the range of $\sin^2\theta_{12}$

- **BOREXINO**
- **LowNu (pp neutrinos) - LENS, XMASS**: $\sin^2 2\theta_{12}$



A. Bandyopadhyay et al, hep-ph/0406328

LowNu: generic $\nu - e^-$ ES experiment

pp: $E_\nu \leq 0.42$ **MeV**, $\bar{E}_\nu = 0.286$ **MeV**

Assume $T_e \geq 50$ **keV**

$R_{pp} \cong \bar{P} + r_{pp}(1 - \bar{P})$, $\bar{P} \cong \cos^4\theta_{13}(1 - \frac{1}{2}\sin^2 2\theta_{12})$, $r_{pp} \cong 0.3$

$R_{CC/NC}(SNO) \cong \sin^2\theta_{12}\cos^4\theta_{13}$

$\Delta(\sin^2\theta_{12}) \sim 0.5\Delta(R_{pp})/(\cos 2\theta_{12}(1 - r_{pp}))$

$\Delta(R_{pp}) < \Delta(R_{CC/NC})$ **to reduce** $\Delta(\sin^2\theta_{12})$**; SNO3:** $\sim 6\%$

BP04: $R_{pp} \cong 0.71$ **(**3σ**: 0.67 - 0.76)**

With $\Delta(R_{pp}) = 2\%$, $\Delta(\sin^2\theta_{12}) \gtrsim 15\%$ **at** 3σ

Dedicated reactor experiment with $L \sim 60$ **km:**

$\Delta(\sin^2\theta_{12}) = (6 - 9)\%$ **at** 3σ

A. Bandyopadhyay et al., hep-ph/0302243 and hep-ph/0410283;

H. Minakata et al., hep-ph/0407326

Reactor Experiments

Future more precise KamLAND data: Δm^2_{21} **with higher precision**

$\sin^2\theta_{12}$ **cannot be determined with a high precision**

("wrong distance")

even with SHIKA-2 reactor when operative

("right distance", L = 88 km, but signal too weak (3.926 GW))

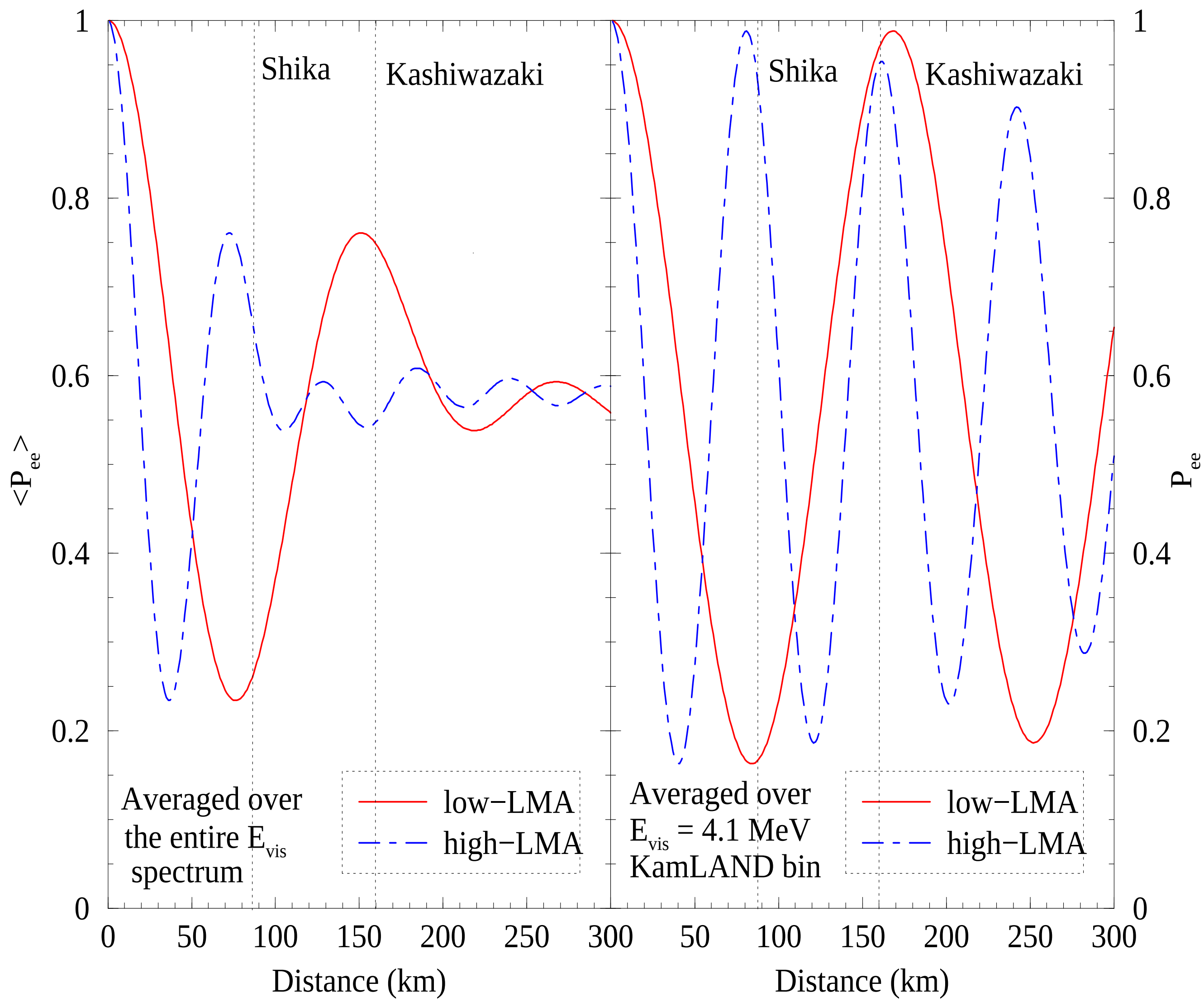
$$P^{3\nu}_{\rm KL} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2(\frac{\Delta m^2_{\odot}}{4E}L)\right]$$

$\sin^2(\frac{\Delta m^2_{\odot}}{4E}L) \cong 0$ **(SPMAX; KamLAND):**

strong sensitivity to $\Delta m^2_{\odot} \equiv \Delta m^2_{21}$**, weak sensitivity to** $\sin^2\theta_{12}$

$\sin^2(\frac{\Delta m^2_{\odot}}{4E}L) \cong 1$ **(SPMIN): E = 4 MeV,** $L \cong 60$ **km,**

strong sensitivity to $\sin^2\theta_{12}$



Shika
Kashiwazaki
Averaged over the entire E_{vis} spectrum
low−LMA
high−LMA
Averaged over E_{vis} = 4.1 MeV KamLAND bin
$<P_{ee}>$
P_{ee}
Distance (km)
Distance (km)

SK + 0.1% Gd

J.F. Beacom and M.R. Vagins, hep-ph/0309300

- **SK-Gd reactor $\bar{\nu}_e$ rate $\sim$ 43 times KamLAND rate**

3 years statistics in SK-Gd, 99% C.L.:

$$\Delta m^2_{21} = (8.01 - 8.61) \times 10^{-5}\text{eV}^2; \quad \text{spread} = 3.6\%$$
$$\sin^2\theta_{12} = (0.22 - 0.34); \quad \text{spread} = 21\%$$

5 years statistics in SK-Gd, 99% C.L.:

$$\Delta m^2_{21} = (8.07 - 8.53) \times 10^{-5}\text{eV}^2; \quad \text{spread} = 2.8\%$$
$$\sin^2\theta_{12} = (0.22 - 0.32); \quad \text{spread} = 18\%$$

$$\text{spread} = \frac{a_{max} - a_{min}}{a_{max} + a_{min}}, \;\; \mathbf{a} \equiv \mathbf{\Delta m^2_{21}} \;\text{or}\; \sin^2\boldsymbol{\theta_{12}}$$

Comment: SK-Gd data simulated at $\Delta m^2_{21} = 8.3 \times 10^{-5}$ eV^2, $\sin^2\theta_{12} = 0.27$ (the "old" global best-fit point). The precision on Δm^2_{21} and $\sin^2\theta_{12}$ for a given statistics remains approximately the same for $\Delta m^2_{21} = 7.6 \times 10^{-5}$ eV^2, $\sin^2\theta_{12} = 0.30$ (the new global best-fit point).

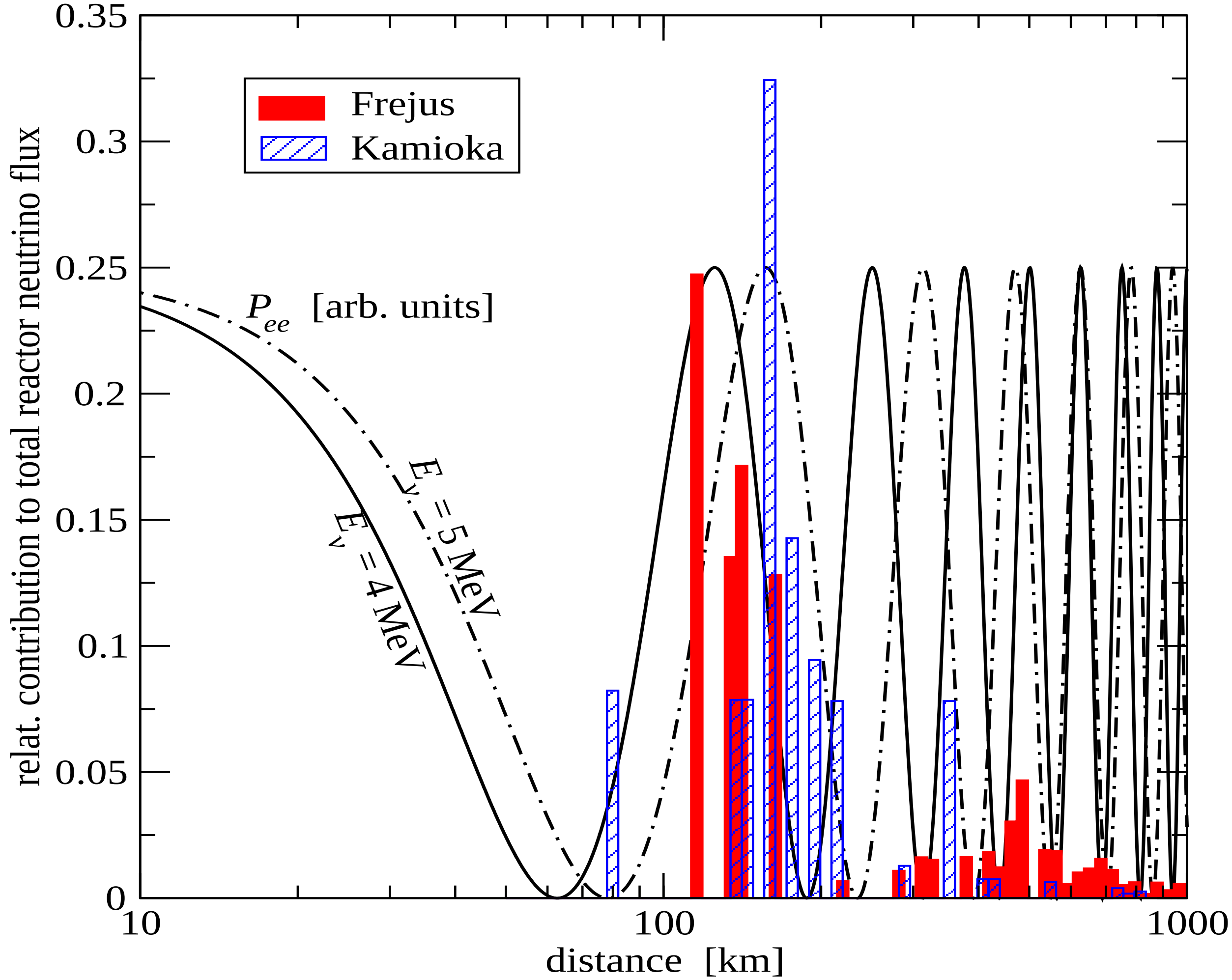
Sensitivity to Δm^2_{21} and $\sin^2\theta_{12}$

Data set used	99% CL range of $\Delta m^2_{21} \times 10^{-5}\text{eV}^2$	99% CL spread of Δm^2_{21}	99% CL range of $\sin^2\theta_{12}$	99% CL spread in $\sin^2\theta_{12}$
only solar	3.2 - 14.9	65%	0.22 – 0.37	25%
solar with future SNO	3.3 – 11.9	57%	2.2 – 0.34	21%
solar+1 kTy KL(low-LMA)	6.5 - 8.0	10%	0.23 – 0.37	23%
solar+2.6 kTy KL(low-LMA)	6.7 – 7.7	7%	0.23 – 0.36	22%
solar with future SNO+1.3 kTy KL(low-LMA)	6.7 – 7.8	8%	0.24 – 0.34	17%
3 yrs SK-Gd	7.0 - 7.4	3%	0.25 – 0.37	19%
5 yrs SK-Gd	7.0 – 7.3	2%	0.26 – 0.35	15%
solar+3 yrs SK-Gd(low-LMA)	7.0 – 7.4	3%	0.25 – 0.34	15%
solar with future SNO+3 yrs SK-Gd(low-LMA)	7.0 – 7.4	3%	0.25 – 0.335	14%
7 yrs SK-Gd with *only* Shika-2 “up”	7.0 – 7.3	2%	0.28 – 0.32	6.7%

Future SNO: 5% on R_{CC}, 6% on R_{NC}

S.T.P. and S. Choubey, hep-ph/0404103

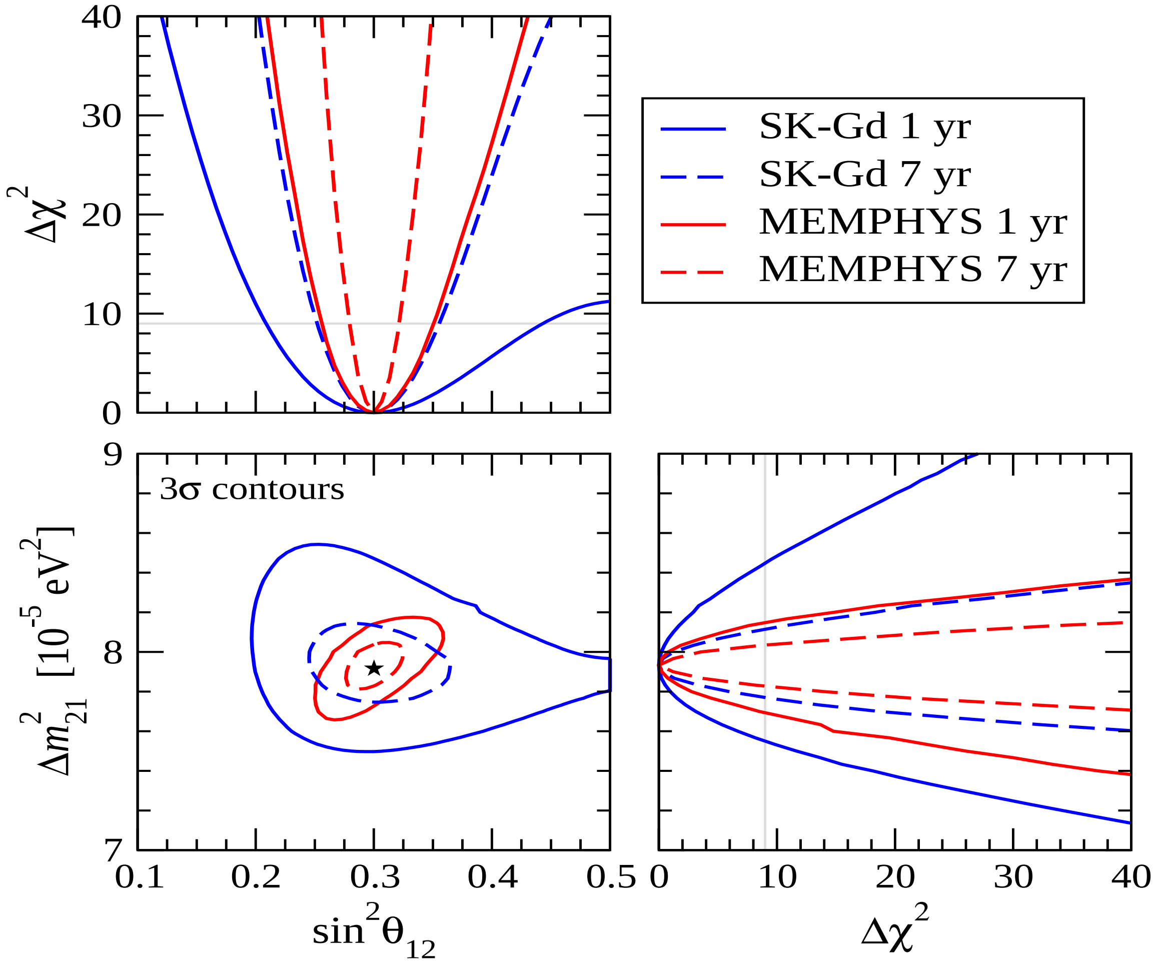
MEMPHYS (Frejus) + 0.1% Gd



MEMPHYS (Frejus): 147 kt water-Čerenkov detector, ~6.5×SK

56 reactors within 1000 km; 65% of the flux from reactors within 160 km

MEMPHYSGd vs SKGd



1 year MEMGd ≅ 7 years SKGd: $3\sigma(\Delta m^2_{21}) \cong 3\%$, $3\sigma(\sin^2_{21}) \cong 20\%$

7 years MEMPHYSGd: $3\sigma(\Delta m^2_{21}) \cong 1.4\%$, $3\sigma(\sin^2_{21}) \cong 13\%$

S.T.P. and T. Schwetz, hep-ph/0607155

Dedicated Reactor Experiment on $\sin^2 2\theta_{12}$

$$P_{KL}^{3\nu} \cong \sin^4\theta_{13} + \cos^4\theta_{13}\left[1 - \sin^2 2\theta_{12}\sin^2\left(\frac{\Delta m^2_{\odot}}{4E}L\right)\right]$$

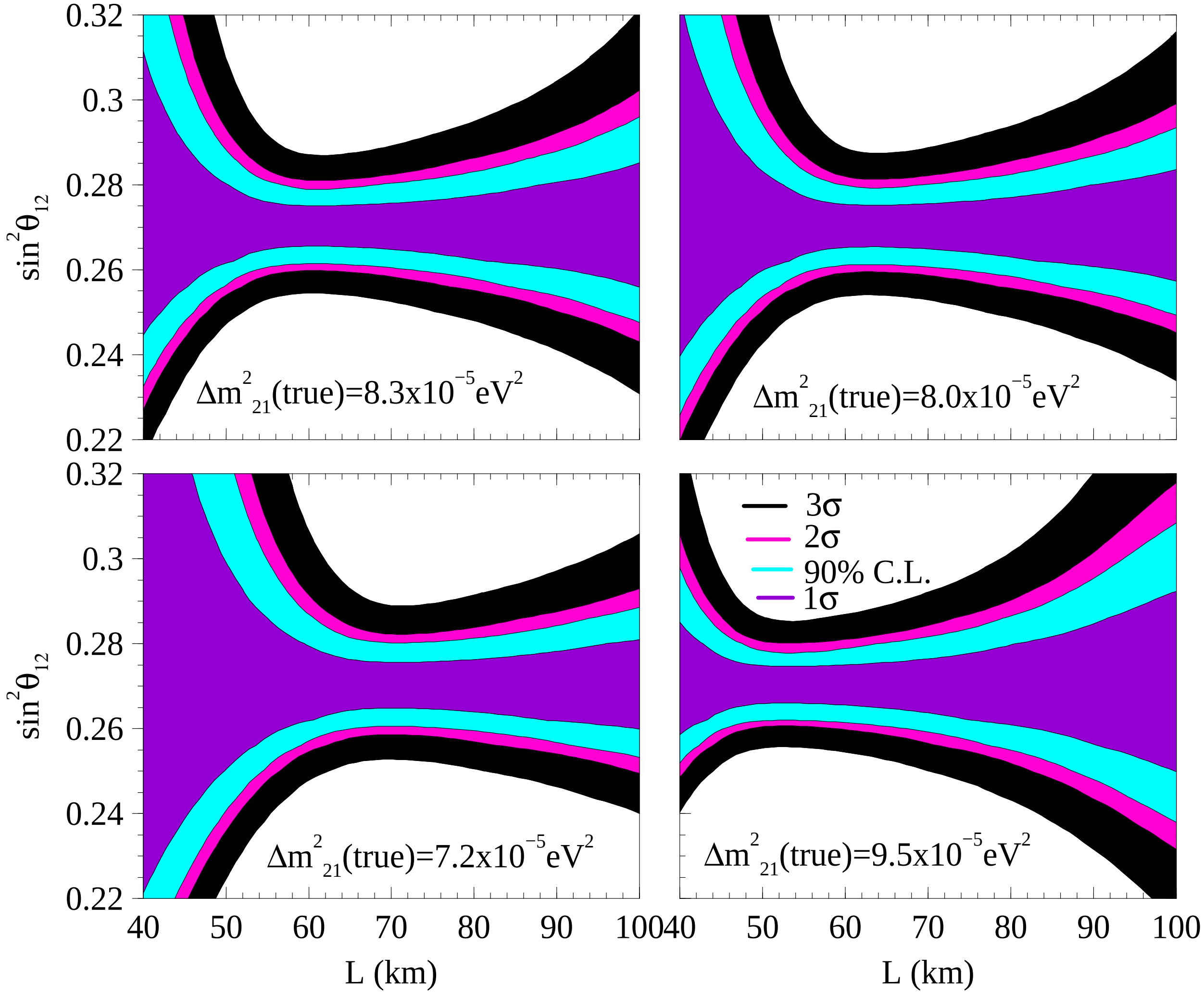
SPMIN: $L \sim 60$ **km:** $\sin^2 2\theta_{12}$

$\Delta(\sin^2\theta_{12}) = (6-9)\%$ **at** 3σ

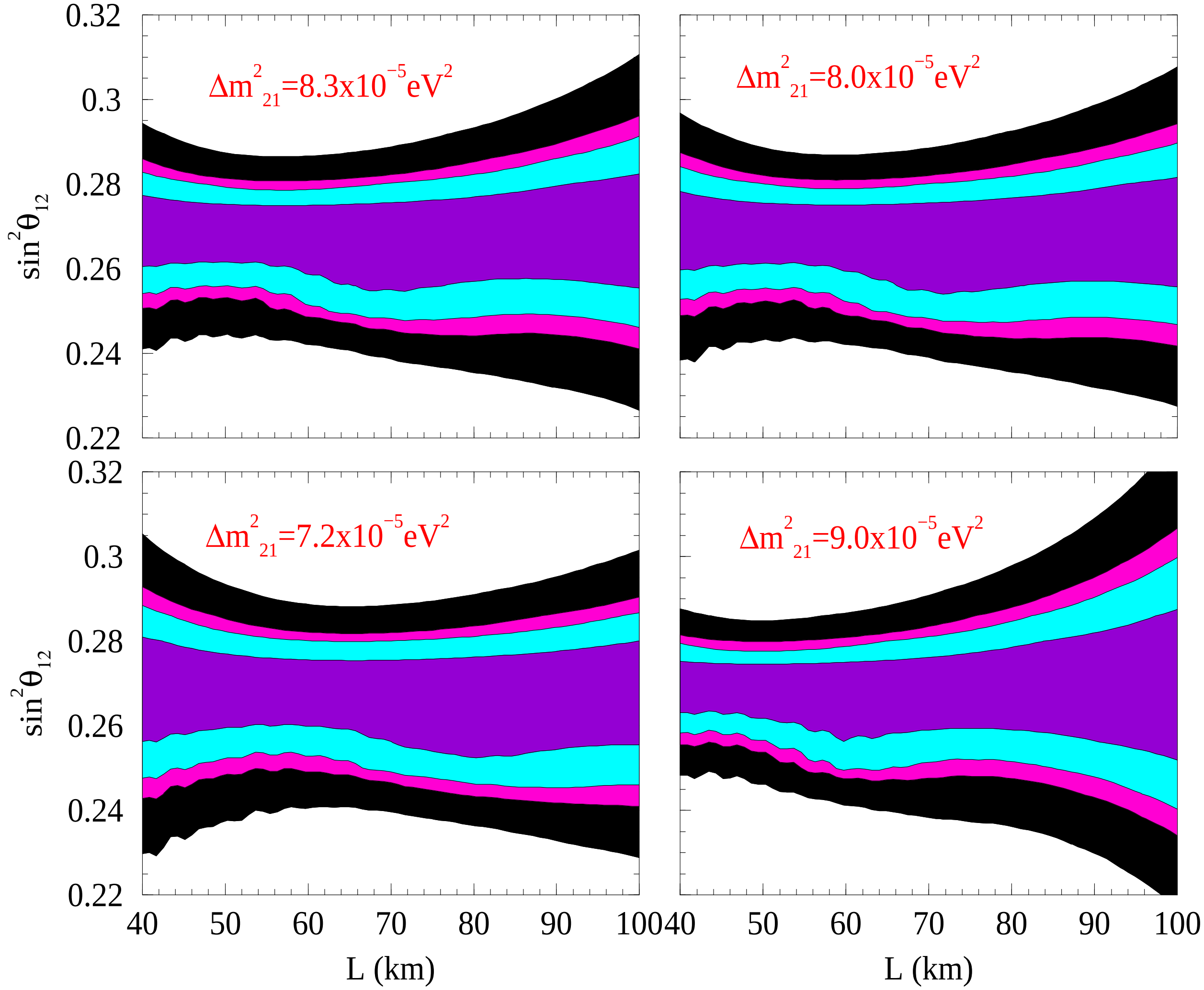
A. Bandyopadhyay, S. Choubey, S. Goswami, hep-ph/0302243;

A. Bandyopadhyay et al., hep-ph/0410283;

H. Minakata et al., hep-ph/0407326



Systematic uncertainty 2%; statistics 73 GWkTy; KamLAND-like detector



$\sin^2\theta_{13}$ - **free within the 3σ allowed range**

SPMIN: $\delta(\sin^2 2\theta_{12}) \approx 2\Delta P_{ee}\sin^2\theta_{13} + 2\ \cos^2 2\theta_{12}\ \Delta(\sin^2\theta_{13})$

Oscillation Parameters

$$\Delta m^2_\odot = 8.0\ (7.6) \times 10^{-5}\ \text{eV}^2\ ,\quad 3\sigma(\Delta m^2_\odot) = 9\%\ ,$$

$$\sin^2\theta_\odot = 0.30\ ,\quad 3\sigma(\sin^2\theta_\odot) = 24\%\ ,$$

$$|\Delta m^2_{\text{atm}}| = 2.5 \times 10^{-3}\ \text{eV}\ ,\quad 3\sigma(|\Delta m^2_{\text{atm}}|) = 18\%.$$

Future:

SNO III: $3\sigma(\sin^2\theta_\odot) = 21\%$ **;**

3 kTy KamLAND: $3\sigma(\Delta m^2_\odot) = 7\%\ ,\quad 3\sigma(\sin^2\theta_\odot) = 18\%$ **;**

SK-Gd (0.1% Gd: 43×(KL $\bar{\nu}_e$ rate)), 3y: $3\sigma(\Delta m^2_\odot) \cong 4\%$

KL type reactor $\bar{\nu}_e$ detector, $L \sim 60$ **km,** ~ 60 **GW kTy:**

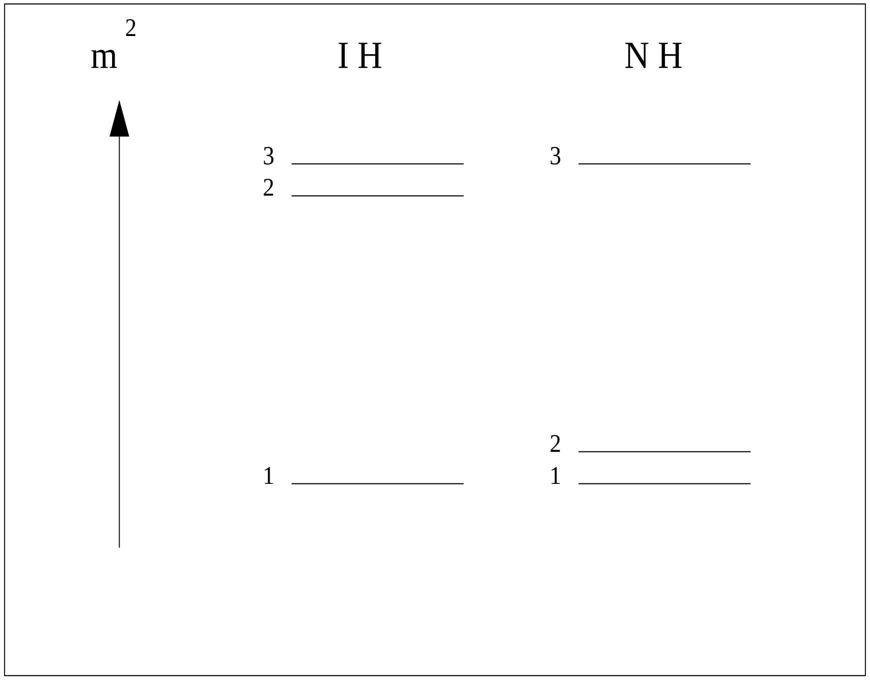
$3\sigma(\sin^2\theta_\odot) \cong 6\%\ (9\%)$ **for 2% (5%) syst. error;** $+\ \delta(\sin^2\theta_{13}) : 9\%\ (11\%)$

A. Bandyopadhyay, et al., hep-ph/0410283

T2K (SK): $3\sigma(|\Delta m^2_{\text{atm}}|) \cong 12\%$

P. Huber et al., hep-ph/0403068

Determining the $\nu-$Mass Hierarchy (sgn(Δm^2_{atm})



- Reactor $\bar{\nu}_e$ Oscillations in vacuum.
- Atmospheric ν experiments: subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations (matter effects).
- LBL $\nu-$oscillation experiments (T2KK, NOνA); $\nu-$factory.
- ^{3}H β-decay Experiments (sensitivity to 5×10^{-2} eV).
- $(\beta\beta)_{0\nu}-$Decay Experiments (ν_j- Majorana particles).

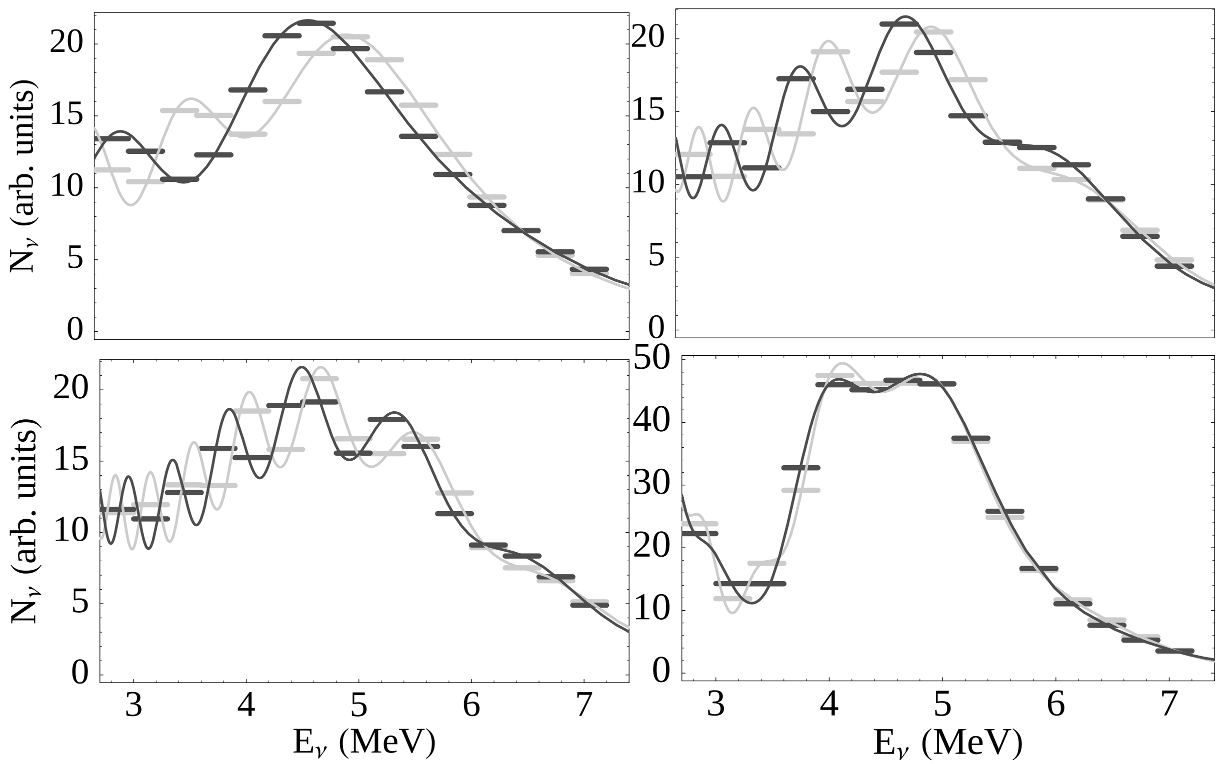
Reactor $\bar{\nu}_e$ Oscillations in vacuum

$$P_{\mathrm{NH}}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{\mathrm{A}} L}{2E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_\odot\left(1 - \cos\frac{\Delta m^2_\odot L}{2E_\nu}\right)$$
$$+ \sin^2 2\theta_{13}\sin^2\theta_\odot \sin\frac{\Delta m^2_\odot L}{4E_\nu}\sin\left(\frac{\Delta m^2_{\mathrm{A}} L}{2E_\nu} - \frac{\Delta m^2_\odot L}{4E_\nu}\right),$$

$$P_{\mathrm{IH}}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \cos\frac{\Delta m^2_{\mathrm{A}} L}{2E_\nu}\right) - \frac{1}{2}\cos^4\theta_{13}\sin^2 2\theta_\odot\left(1 - \cos\frac{\Delta m^2_\odot L}{2E_\nu}\right)$$
$$+ \sin^2 2\theta_{13}\cos^2\theta_\odot \sin\frac{\Delta m^2_\odot L}{4E_\nu}\sin\left(\frac{\Delta m^2_{\mathrm{A}} L}{2E_\nu} - \frac{\Delta m^2_\odot L}{4E_\nu}\right),$$

$$\theta_\odot = \theta_{12},\ \Delta m^2_\odot = \Delta m^2_{21} > 0;\quad \sin^2\theta_{12} = 0.30\ (b.f.); \sin^2\theta_{12} \leq 0.38 \text{ at } 3\sigma;$$
$$\Delta m^2_{\mathrm{A}} = \Delta m^2_{31} > 0,\ \text{NH spectrum},$$
$$\Delta m^2_{\mathrm{A}} = \Delta m^2_{23} > 0,\ \text{IH spectrum}$$

S.M. Bilenky, D. Nicolo, S.T.P., hep-ph/0112216;
M. Piai, S.T.P., hep-ph/0112074;

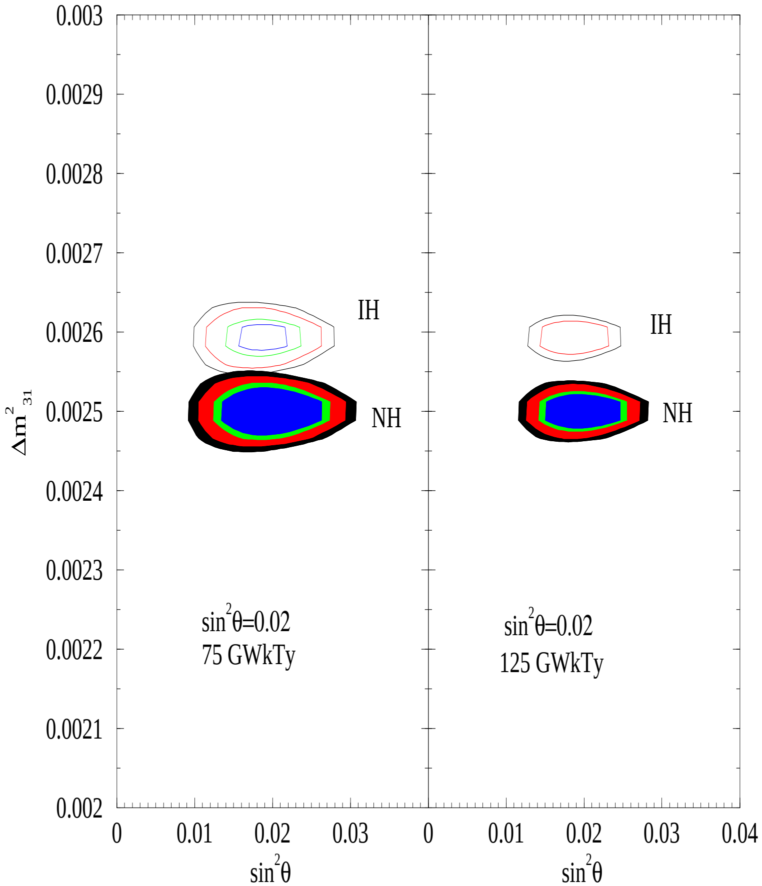


M. Piai, S.T.P., 2001

$\sin^2\theta_{13} = 0.05$, $\Delta m^2_{21} = 2\times10^{-4}$ **eV**^{2}; $\Delta m^2_{\rm A} = 1.3;\ 2.5;\ 3.5\times10^{-3}$ **eV**^{2}

$L = 20$ **km**; $\Delta E_\nu = 0.3$ **MeV**

NH − light grey; IH − dark grey



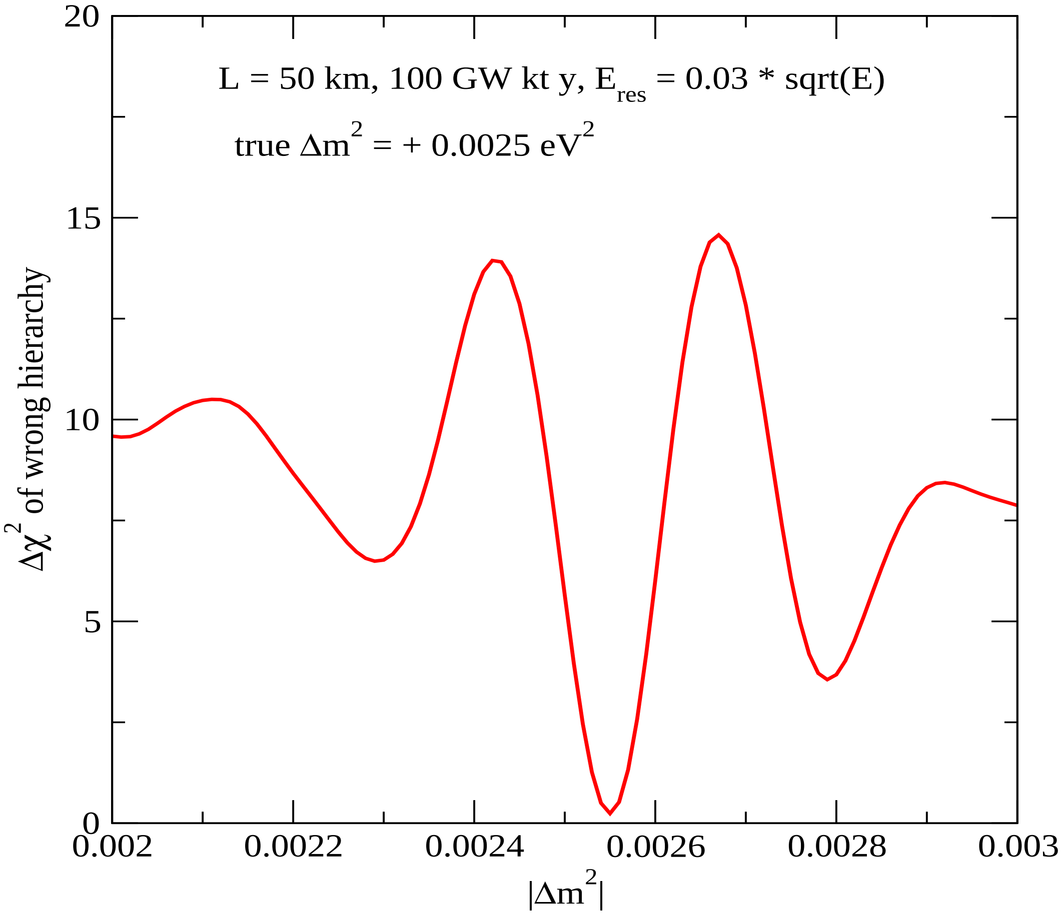
S. Choubey, S.T.P., 2003

$\sin^2\theta_\odot = 0.30$, $\Delta m^2_{21} = 1.5 \times 10^{-4}$ **eV^2**, $\Delta m^2_{\mathrm{A}} = 2.5 \times 10^{-3}$ **eV^2**

$L = 20$ **km**; $\Delta E_\nu = 0.1$ **MeV; syst. error 2%**

"True": NH; 90%, 95%, 99% and 99.73% solution regions

J. Learned et al., 2006 (Hanohano project)



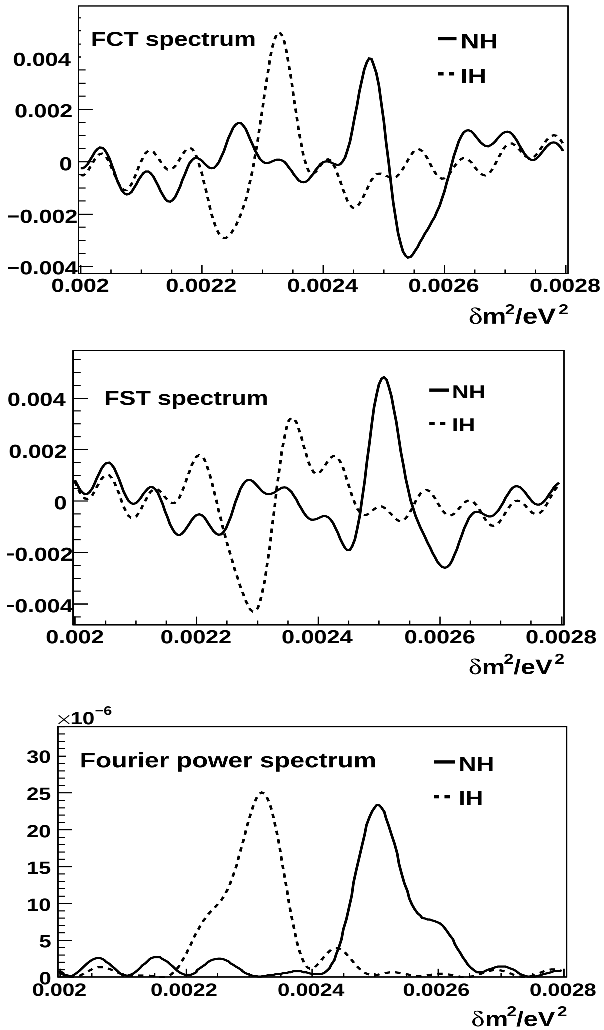
T. Schwetz, September 2006

$\sin^2\theta_\odot = 0.30$, $\Delta m^2_{21} = 8 \times 10^{-5}$ **eV^2; "true"** $\Delta m^2_{\mathrm{A}} = 2.50 \times 10^{-3}$ **eV^2 (NH)**

Minimum at $\Delta m^2_{\mathrm{A}} = -\ 2.55 \times 10^{-3}$ **eV^2 (IH)**

Precision of $\sim$ **1% on** $|\Delta m^2_{\mathrm{A}}|$ **required**

J. Learned et al., 2006 (Hanohano project): can achieve it.



L. Zhan, Y. Wang, J. Cao, L. Wen, August 2008

Estimated sensitivity to the hierarchy for $\sin^2 2\theta_{13} > 0.005$

Atmospheric ν experiments

Subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations in the Earth.

$$P_{3\nu}(\nu_e \rightarrow \nu_\mu) \cong P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong s^2_{23}\, P_{2\nu}\,, P_{3\nu}(\nu_e \rightarrow \nu_\tau) \cong c^2_{23}\, P_{2\nu}\,,$$

$$P_{3\nu}(\nu_\mu \rightarrow \nu_\mu) \cong 1 - s^4_{23}\ P_{2\nu} - 2c^2_{23}s^2_{23}\ \left[1 - Re\ (e^{-i\kappa}A_{2\nu}(\nu_\tau \rightarrow \nu_\tau))\right]\ ,$$

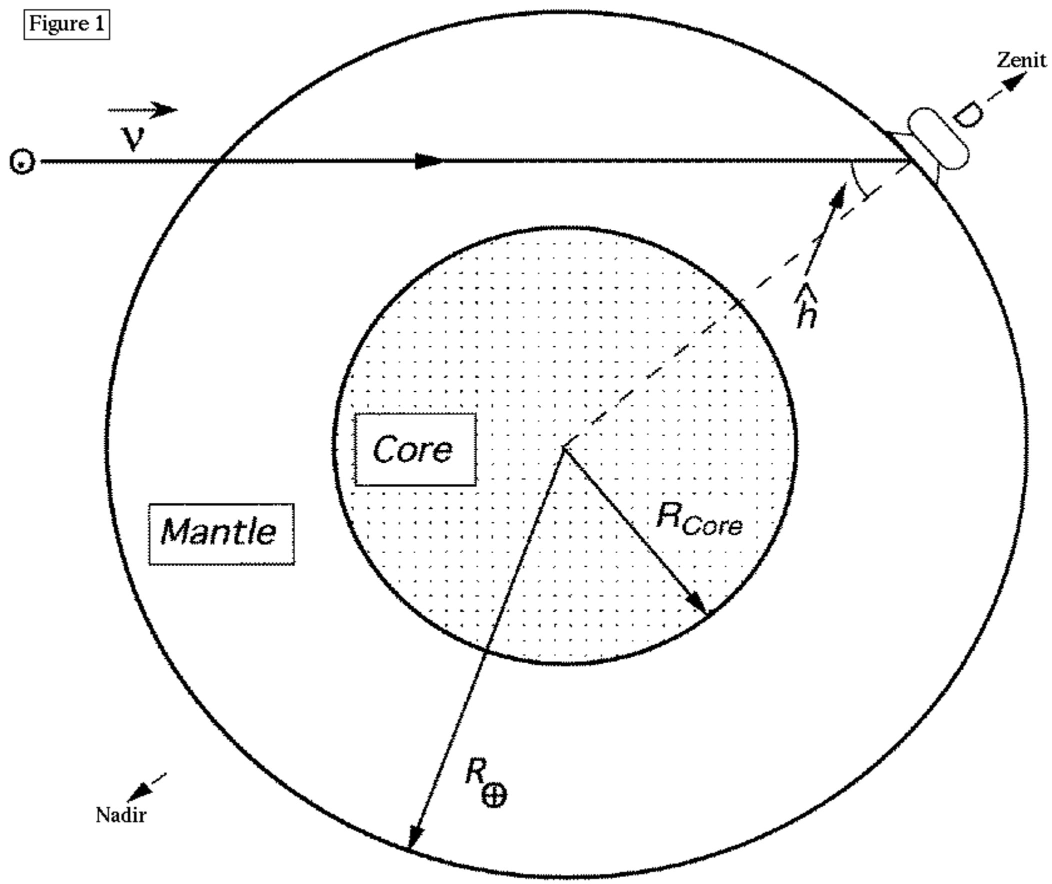
$P_{2\nu} \equiv P_{2\nu}(\Delta m^2_{31}, \theta_{13}; E, \theta_n; N_e)$: **2-$\nu$ $\nu_e \rightarrow \nu'_\tau$ oscillations in the Earth,**
$\nu'_\tau = s_{23}\,\nu_\mu + c_{23}\,\nu_\tau$;

κ **and** $A_{2\nu}(\nu_\tau \rightarrow \nu_\tau) \equiv A_{2\nu}$ **are known phase and 2-ν amplitude.**

NH: $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ **matter enhanced,** $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ **- suppressed**

IH: $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ **matter enhanced,** $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$**−suppressed**

The Earth



Earth: $R_{core} = 3446$ **km,** $R_{man} = 2885$ **km;**

Neutrino trajectories crossing the Earth core: Nadir angle $\theta_n \leq 33.17^0$**;**

Earth: $\bar{N}_e^{man} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 6.0\ N_A\ cm^{-3}$

The Earth

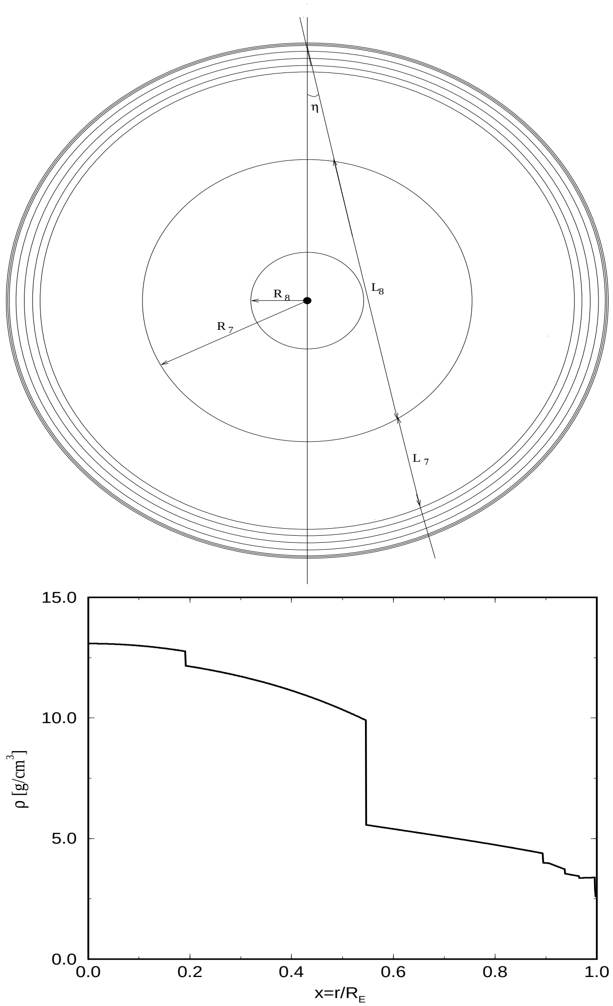
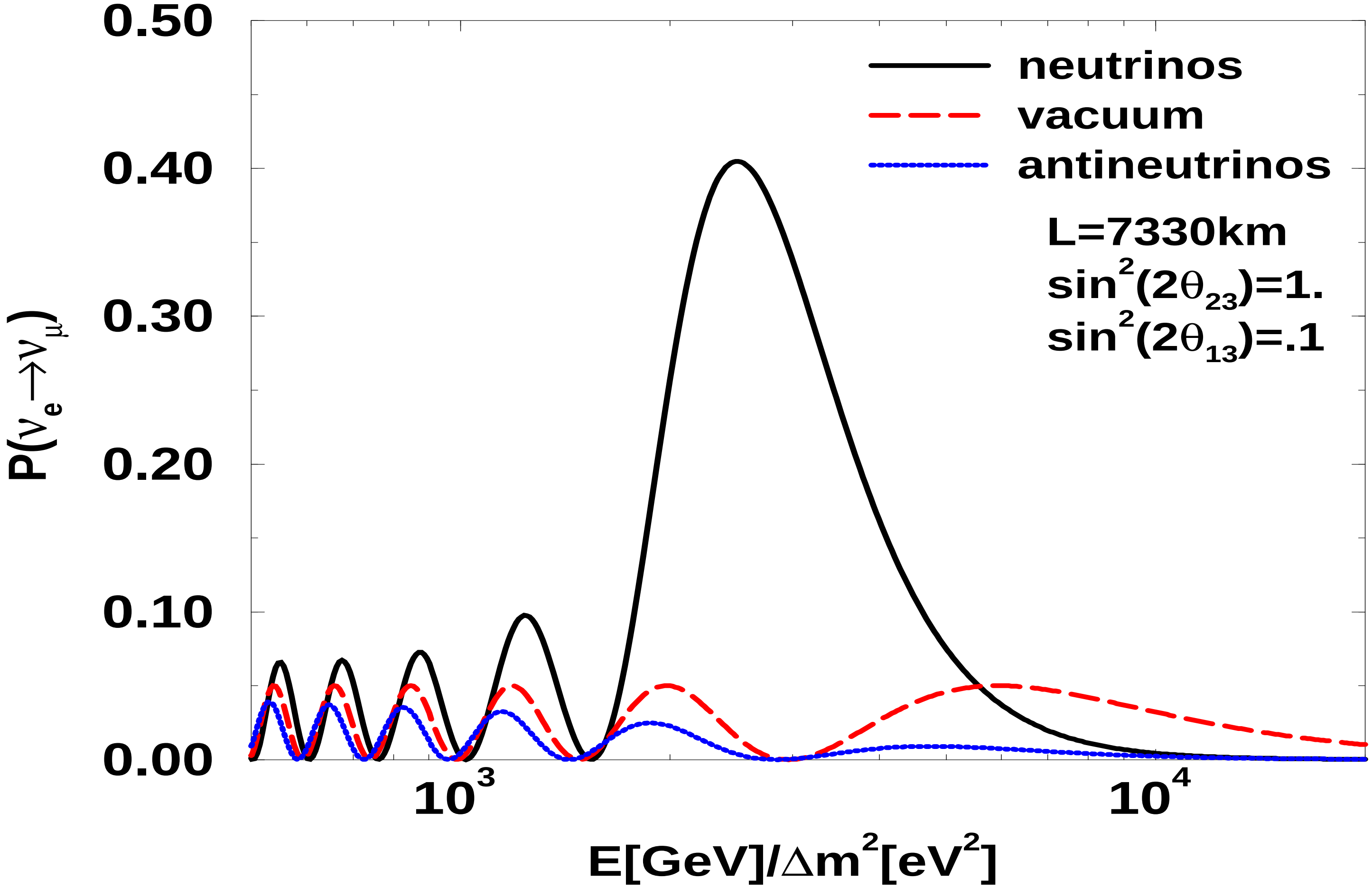


FIG. 1. Density profile of the Earth.

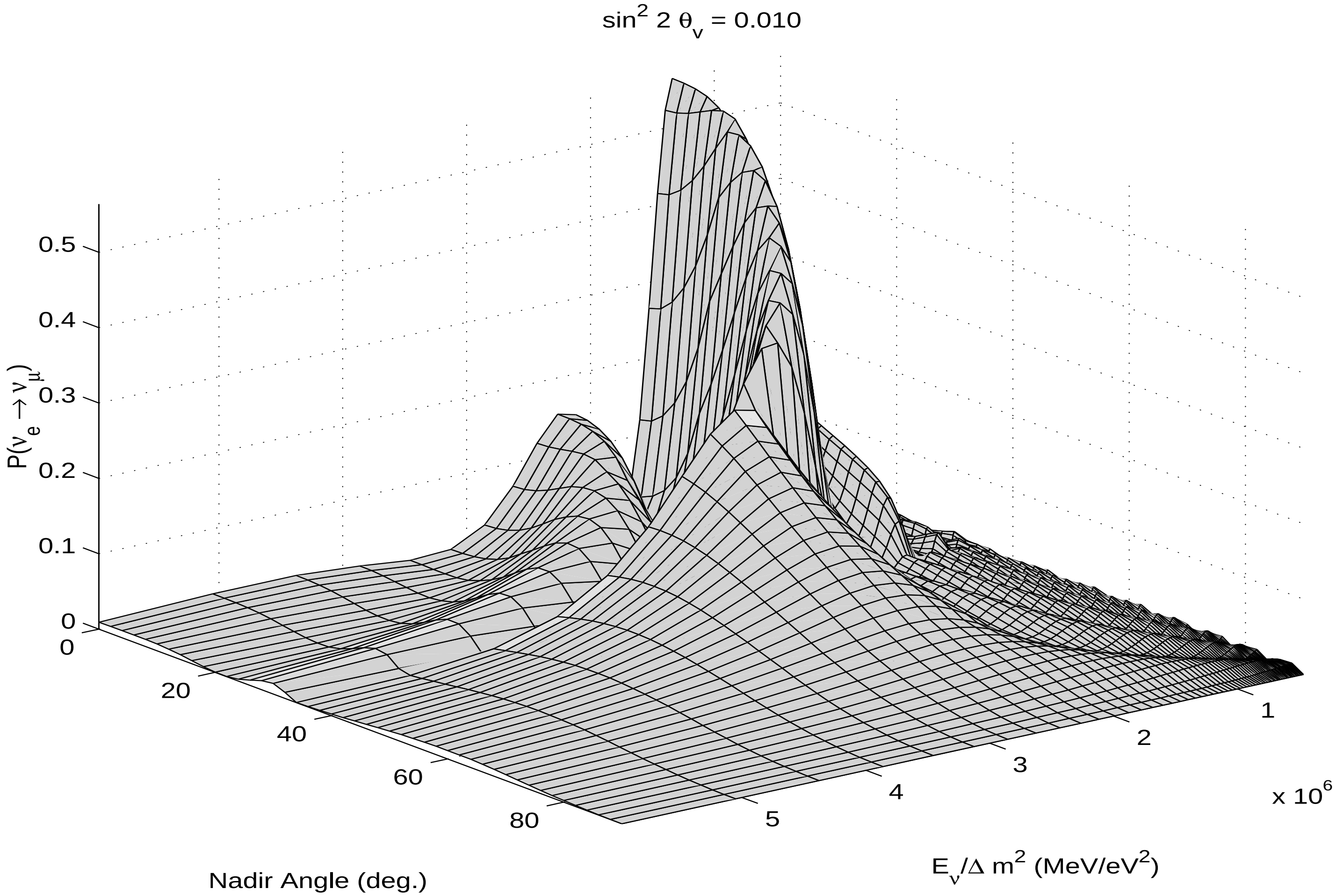
$R_{core} = 3446$ **km,** $R_{man} = 2886$ **km;** $\bar{N}_e^{man} \sim 2.3\ N_A\ cm^{-3}$, $\bar{N}_e^{core} \sim 6.0\ N_A\ cm^{-3}$

Earth matter effect in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (MSW)



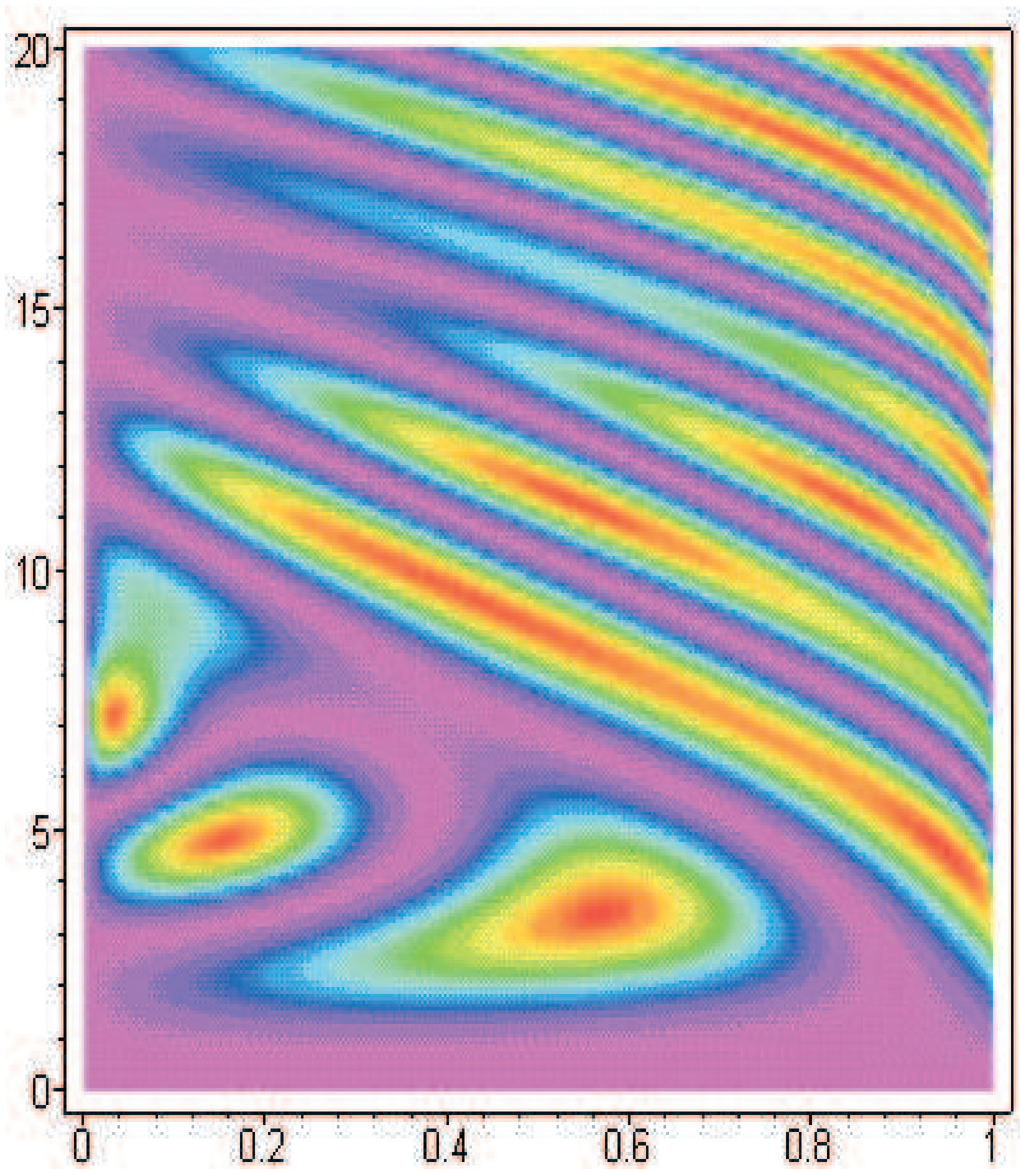
I. Mocioiu, R. Shrock, 2000

Earth matter effects in $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$ (NOLR)



S.T.P., 1998;
M. Chizhov, M. Maris, S.T.P., 1998; M. Chizhov, S.T.P., 1999

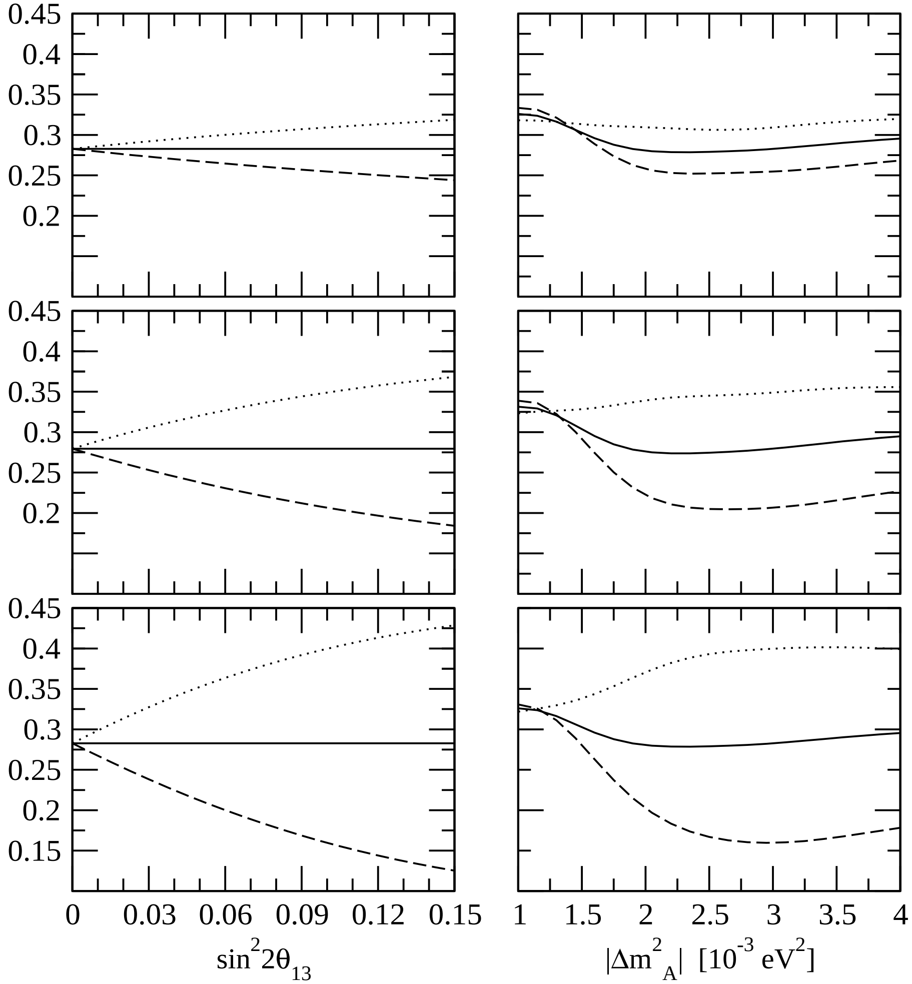
$P(\nu_e \to \nu_\mu) \equiv P_{2\nu} \equiv (s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)})$, $\theta_v \equiv \theta_{13}$, $\Delta m^2 \equiv \Delta m^2_{atm}$;
Absolute maximum: Neutrino Oscillation Length Resonance (NOLR);
Local maxima: MSW effect in the Earth mantle or core.



$(s_{23})^{-2} P_{3\nu}(\nu_{e(\mu)} \to \nu_{\mu(e)}) \equiv P_{2\nu}$; **NOLR: "Dark Red Spots"**, $P_{2\nu} = 1$;

Vertical axis: $\Delta m^2/E\ [10^{-7} eV^2/MeV]$; **horizontal axis:** $\sin^2 2\theta_{13}$; $\theta_n = 0$

M. Chizhov, S.T.P., 1999 (hep-ph/9903399,9903424)

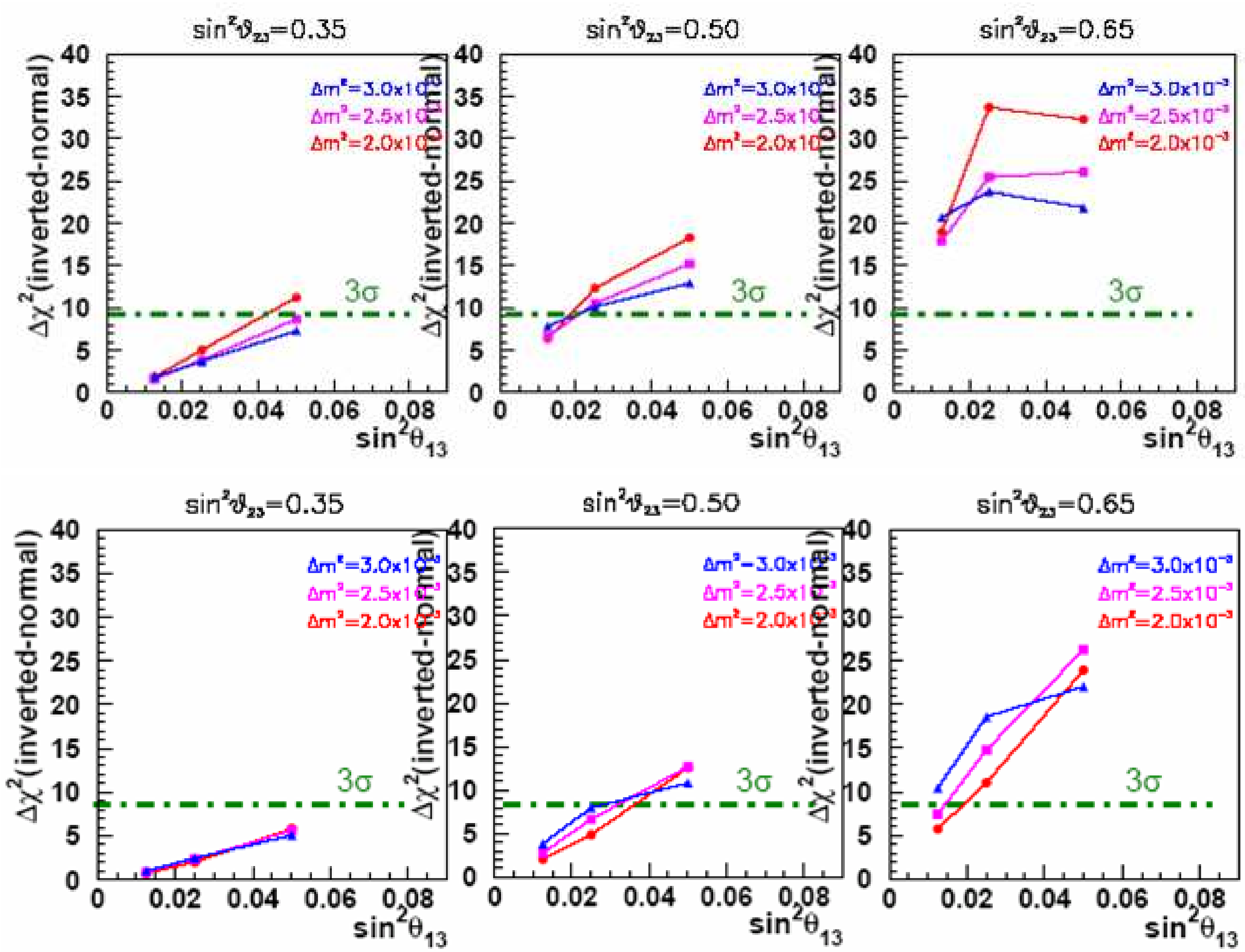


Iron Magnetized Detectors (MINOS, INO): multi-GeV μ^- and μ^+ event rates, N_{μ^-} and N_{μ^+}; $\cos\theta_n = (0.30 - 0.84)$ **mantle bin, E = [5,20] GeV**

$A \equiv \frac{U-D}{U+D}$ in the θ_n- dependence of $\frac{N_{\mu^-}}{N_{\mu^+}}$

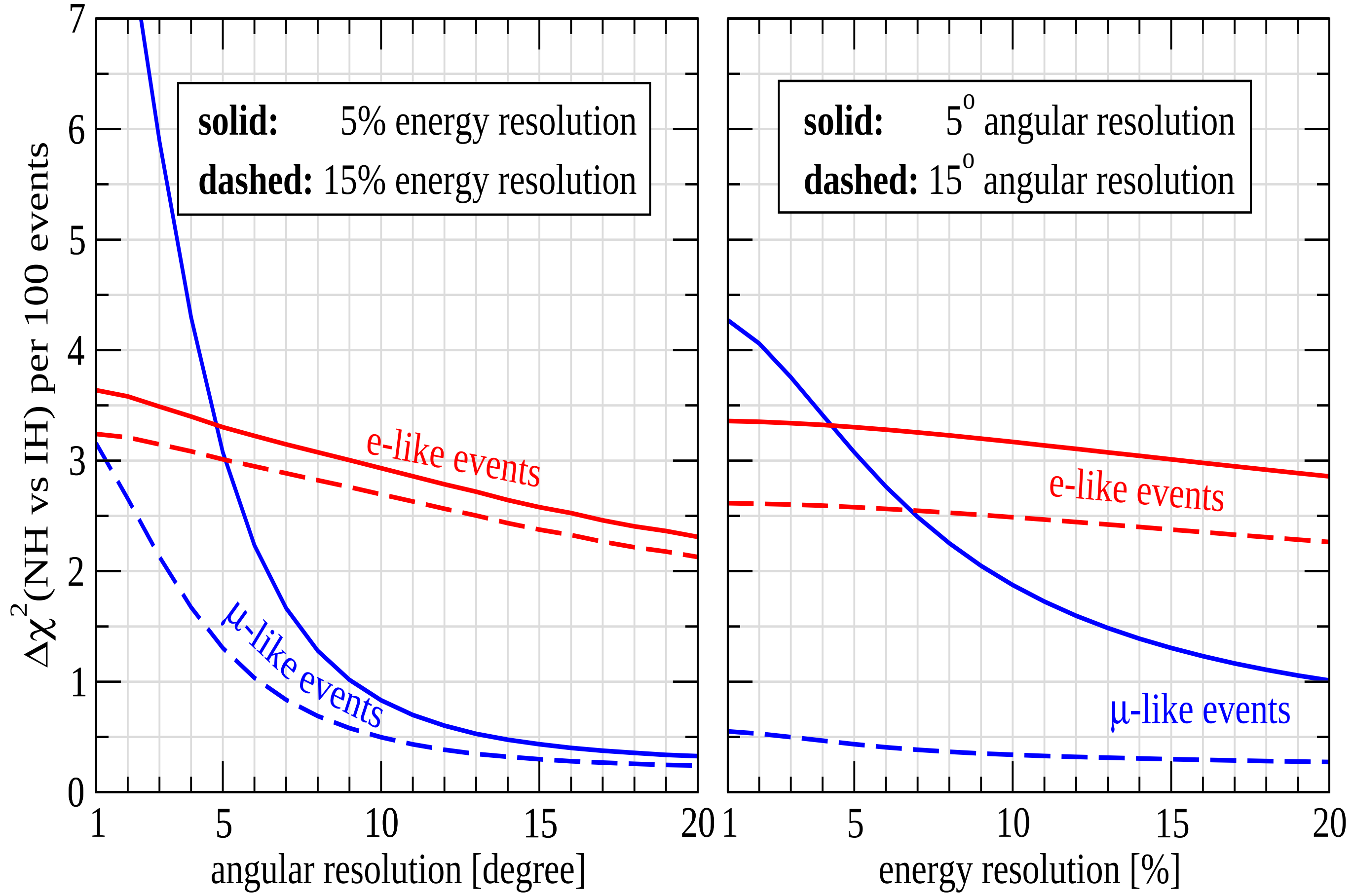
- $|\Delta m^2_{31}| = 3 \times 10^{-3}$ eV^2, $\sin^2\theta_{23}$ =**0.36, 0.50, 0.64**
- $\Delta m^2_{31} > 0$-**NH** (dashed), $\Delta m^2_{31} < 0$-**IH** (dotted), 2$-\nu$ (solid)

S.T.P., S. Palomares-Ruiz, hep-ph/0406



Water-Cerenkov detector, 1.8 MTy

T. Kajita et al., 2004



INO; ATLAS, CMS (?)

T. Schwetz, S.T.P., 2005

$\sin^2 2\theta_{13} = 0.10$, $\sin^2 \theta_{23} = 0.50$, $|\Delta m^2_{\text{A}}| = 2.4 \times 10^{-3}$ **eV**^{2}

$E_\nu = (2 - 10)$ **GeV**; $0.1 \leq \cos\theta_n \leq 1.0$

^{3}H β-decay : 3H $\to^3$ He + e$^-$ + $\bar{\nu}_e$

$$\frac{d\,\Gamma}{d\,E_e} = \sum_i |U_{ei}|^2 \ \frac{d\,\Gamma(m_i)}{d\,E_e}\ ,$$

$$\frac{d\,\Gamma(m_i)}{d\,E_e} = C\,p_e\,(E_e + m_e)\,(E_0 - E_e)\,\sqrt{(E_0 - E_e)^2 - m_i^2}\,F(E_e)\,\theta(E_0 - E_e - m_i)\ .$$

NH: $m_1 << m_2 < m_3$, $m_2 \cong \sqrt{\Delta m^2_{21}} \cong 9 \times 10^{-3}$ **eV**, $m_3 \cong \sqrt{\Delta m^2_{31}} \cong 5 \times 10^{-2}$ eV

IH: $m_3 << m_1 \cong m_2$, $m_{1,2} \cong \sqrt{\Delta m^2_{23}} \cong 5 \times 10^{-2}$ eV

Assume sensitivity to 5×10^{-2} eV.

- **NH:** m_1, m_2 - **below the sensitivity; the effect of** m_3 - **unobservable, suppressed by** $\sin^2\theta_{13}$:

$$\frac{d\,\Gamma}{d\,E_e} \cong \frac{d\,\Gamma(m_i = 0)}{d\,E_e}$$

- **IH:** m_3 - **below the sensitivity;** $m_2 - m_1 \cong 1.6 \times 10^{-3}$ **eV - unobservable:**

$$\frac{d\,\Gamma}{d\,E_e} \cong \frac{d\,\Gamma(m_{1,2})}{d\,E_e} \cong \frac{d\,\Gamma(\sqrt{\Delta m^2_{23}})}{d\,E_e}$$

No e^-spectrum deformation observed: NH spectrum.

Deformations observed:

1) spectrum with inverted neutrino mass ordering, $\Delta m^2_{23} < 0$,
a) inverted hierarchical (IH), $m_3 \ll m_1 < m_2$, or
b) partial inverted hierarchy, $m_3 < m_1 < m_2$;

2) spectrum with normal neutrino mass ordering, $\Delta m^2_{23} > 0$, but with partial neutrino mass hierarchy, $m_1 < m_2 < m_3$.

Example (hypothetical) of the possibility 2): $m_1 = 5.0 \cdot 10^{-2}$ eV,

$m_2 = \sqrt{m_1^2 + \Delta m^2_{12}} \cong 5.1 \cdot 10^{-2}$ eV, $m_3 = \sqrt{m_1^2 + \Delta m^2_{13}} \cong 6.9 \cdot 10^{-2}$ eV

$m_1 + m_2 + m_3 \cong 0.17$ eV

$$\frac{d\,\Gamma}{d\,E_e} \cong (1 - |U_{e3}|^2)\,\frac{d\,\Gamma(m_{1,2})}{d\,E_e} + |U_{e3}|^2\,\frac{d\,\Gamma(m_3)}{d\,E_e} \cong \frac{d\,\Gamma(m_{1,2})}{d\,E_e}$$

S.M. Bilenky, M. Mateyev, S.T.P., 2006

Dirac CP-Nonconservation: δ in $U_{\rm PMNS}$

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}\ , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}\ , \qquad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP $\alpha_{21},\ \alpha_{31}$

CP-invariance:

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P.,1980;
V. Barger et al.,1980.

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_l \to \bar{\nu}_{l'})\ , \quad l \neq l' = e, \mu, \tau$$

CPT-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\bar{\nu}_{l'} \to \bar{\nu}_l)$$

$$l = l':\quad P(\nu_l \to \nu_l) = P(\bar{\nu}_l \to \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \to \nu_{l'}) = P(\nu_{l'} \to \nu_l),\ l \neq l'$$

3ν—mixing:

$$A_{\rm CP}^{(l,l')} \equiv P(\nu_l \to \nu_{l'}) - P(\bar{\nu}_l \to \bar{\nu}_{l'})\ , \quad l \neq l' = e, \mu, \tau$$

$$A_{\rm T}^{(l,l')} \equiv P(\nu_l \to \nu_{l'}) - P(\nu_{l'} \to \nu_l),\ l \neq l'$$

$$A_{\rm T}^{(e,\mu)} = A_{\rm T}^{(\mu,\tau)} = -A_{\rm T}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988

In vacuum: $A^{(e,\mu)}_{\mathrm{CP(T)}} = J_{\mathrm{CP}} F^{vac}_{osc}$

$$J_{CP} = \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^{*}_{e2}\, U^{*}_{\mu 1}\right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13} \sin\delta$$

$$F^{vac}_{osc} = \sin\left(\frac{\Delta m^2_{21}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{32}}{2E} L\right) + \sin\left(\frac{\Delta m^2_{13}}{2E} L\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

$$\mathrm{CP}: \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$$

$$\mathrm{CPT}: \quad P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

P. Langacker et al., 1987

Can conserve the T-invariance (Earth)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \;\; l \neq l'$$

In matter with constant density: $\mathrm{A}^{(e,\mu)}_{\mathrm{T}} = \mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} \mathrm{F}^{\mathrm{mat}}_{\mathrm{osc}}$

$$\mathrm{J}^{\mathrm{mat}}_{\mathrm{CP}} = \mathrm{J}^{\mathrm{vac}}_{\mathrm{CP}} \; \mathrm{R}_{\mathrm{CP}}$$

R_{CP} **does not depend on** θ_{23} **and** δ**;** $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

HOW?

- **Reactor Experiments at $L \sim 1$ km: Duoble CHOOZ, Daya Bay,...;**

MINOS, CNGS (OPERA), $L \sim 730$ km:

$\sin^2\theta_{13}$

- **Super Beams: θ_{13}, δ, ...**

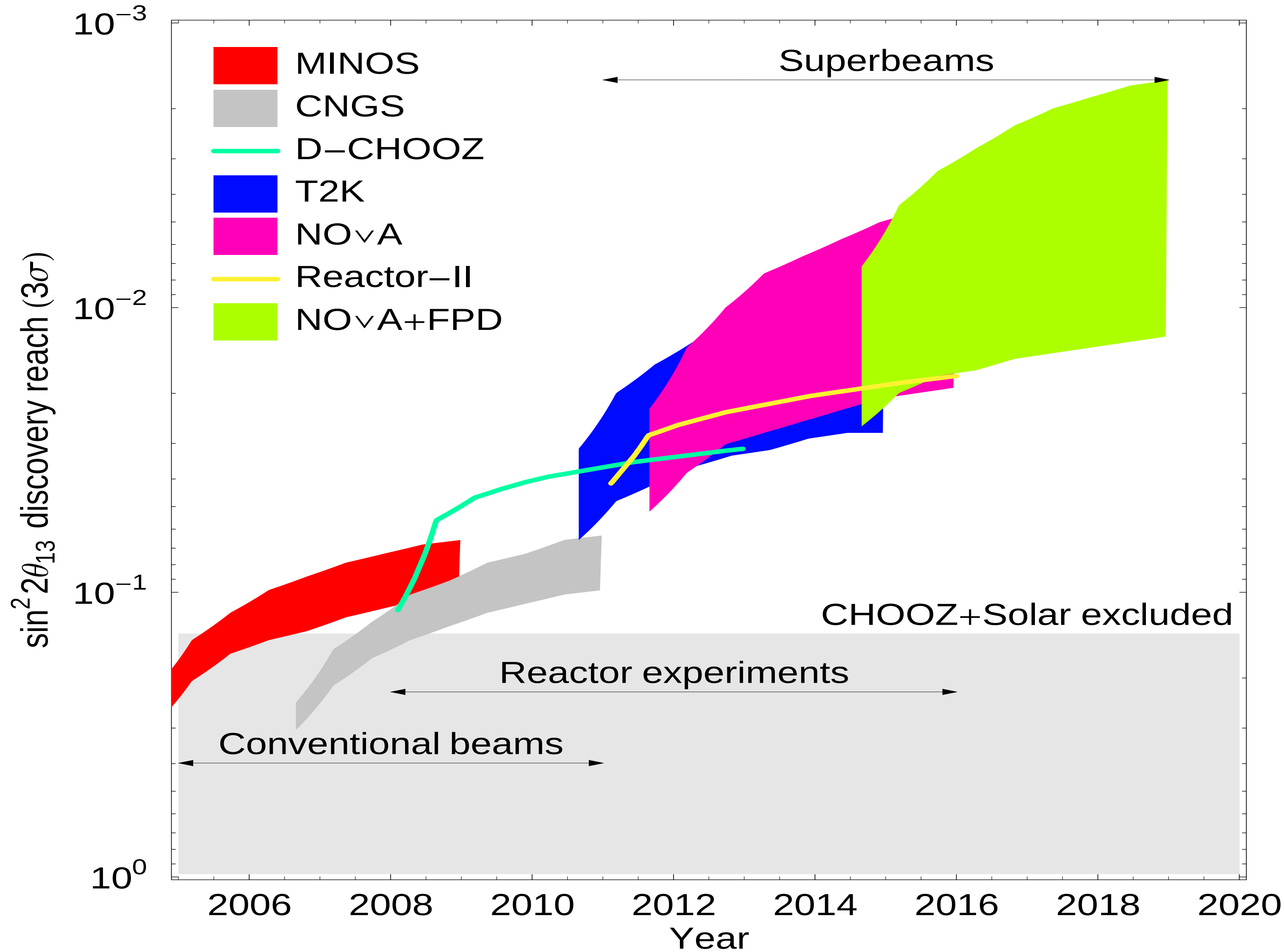
T2K, SK (HK) 295 km

NOνA $\sim$800 km

SPL+$\beta-$beams, MEMPHYS (0.5 megaton):
CERN-Frejus $\sim$140 km

$\nu-$Factories $\sim$ 3000, 7000 km


Sendai
Super Kamiokande
295km
JAERI
(Tokai)
Japan
KEK
Tokyo
Gifu
Nagoya
Kyoto
Osaka
Kawasaki
Yokohama
(c) 2000 ESRI
~ 420.0 mi / 675.8 km across



MINOS
CNGS
D−CHOOZ
T2K
NOνA
Reactor−II
NOνA+FPD
Superbeams
CHOOZ+Solar excluded
Reactor experiments
Conventional beams
$\sin^2 2\theta_{13}$ discovery reach (3σ)
10^{-3}
10^{-2}
10^{-1}
10^{0}
2006
2008
2010
2012
2014
2016
2018
2020
Year

M_ν from the See-Saw Mechanism

P. Minkowski, 1977.
M. Gell-Mann, P. Ramond, R. Slansky, 1979;
T. Yanagida, 1979;
R. Mohapatra, G. Senjanovic, 1980.

- **Explains the smallness of $\nu-$masses.**

- **Through leptogenesis theory links the $\nu-$mass generation to the generation of baryon asymmetry of the Universe Y_B.**

S. Fukugita, T. Yanagida, 1986.

- **In SUSY GUT's with see-saw mechanism of $\nu-$mass generation, the LFV decays**

$$\mu \rightarrow e + \gamma, \quad \tau \rightarrow \mu + \gamma, \quad \tau \rightarrow e + \gamma \ , \ \text{etc.}$$

are predicted to take place with rates within the reach of present and future experiments.

F. Borzumati, A. Masiero, 1986.

- The ν_j are **Majorana particles**; $(\beta\beta)_{0\nu}-$decay is allowed.

See-Saw: Dirac ν-mass m_D + Majorana mass M_R for N_R

The See-Saw Lagrangian

$$\mathcal{L}^{\mathrm{lep}}(x) \;=\; \mathcal{L}_{\mathrm{CC}}(x) + \mathcal{L}_{\mathrm{Y}}(x) + \mathcal{L}^{\mathrm{N}}_{\mathrm{M}}(x)\,,$$

$$\mathcal{L}_{\mathrm{CC}} \;=\; -\,\frac{g}{\sqrt{2}}\,\overline{l_L}(x)\,\gamma_\alpha\,\nu_{lL}(x)\,W^{\alpha\dagger}(x) + h.c.\,,$$

$$\mathcal{L}_{\mathrm{Y}}(x) \;=\; \lambda_{il}\,\overline{N_{iR}}(x)\,H^\dagger(x)\,\psi_{lL}(x) + Y_l\,H^c(x)\,\overline{l_R}(x)\,\psi_{lL}(x) + \mathbf{h.c.}\,,$$

$$\mathcal{L}^{\mathrm{N}}_{\mathrm{M}}(x) \;=\; -\,\frac{1}{2}\,M_i\,\overline{N_i}(x)\,N_i(x)\,.$$

ψ_{lL} - **LH doublet,** $\psi^{\mathsf{T}}_{lL} = (\nu_{lL}\;\; l_L)$, l_R - **RH singlet,** H - **Higgs doublet.**
Basis: $M_R = (M_1, M_2, M_3)$; $D_N \equiv \mathrm{diag}(M_1, M_2, M_3)$, $D_\nu \equiv \mathrm{diag}(m_1, m_2, m_3)$**.**
m_D **generated by the Yukawa interaction:**

$$-\mathcal{L}^{\nu}_{\mathrm{Y}} = \lambda_{il}\overline{N_{iR}}H^\dagger(x)\psi_{lL}(x),\;\; v = 174\ \mathrm{GeV},\;\; v\,\lambda = m_D - \mathrm{complex}$$

For M_R **- sufficiently large,**

$$m_\nu \simeq v^2\;\lambda^T\,M_R^{-1}\,\lambda = U^*_{\mathrm{PMNS}}\;m_\nu^{\mathrm{diag}}\;U^\dagger_{\mathrm{PMNS}}\;.$$

$Y_\nu \equiv \lambda = \sqrt{D_N}\;\; R\;\sqrt{D_\nu}\;\; (U_{\mathrm{PMNS}})^\dagger / v_u$, **all at** M_R **;** R**-complex,** $R^T R = 1$**.**

J.A. Casas and A. Ibarra, 2001

In GUTs, $M_R < M_X$, $M_X \sim 10^{16}$ **GeV;**

in GUTs, e.g., $M_R = (10^9, 10^{12}, 10^{15})$ **GeV,** $m_D \sim 1$ **GeV.**

The CP-Invarinace Constraints

Assume: $C(\overline{\nu}_j)^T = \nu_j$, $C(\overline{N}_k)^T = N_k$, $j,k = 1,2,3$.

The CP-symmetry transformation:

$$\begin{aligned} U_{\rm CP}\, N_j(x)\, U^{\dagger}_{\rm CP} &= \eta_j^{NCP}\, \gamma_0\, N_j(x')\,, \quad \eta_j^{NCP} = i\rho^N_j = \pm i\,, \\ U_{\rm CP}\, \nu_k(x)\, U^{\dagger}_{\rm CP} &= \eta_k^{\nu CP}\, \gamma_0\, \nu_k(x')\,, \quad \eta_k^{\nu CP} = i\rho^{\nu}_k = \pm i\,. \end{aligned}$$

CP-invariance:

$$\lambda^*_{jl} = \lambda_{jl}\, (\eta_j^{NCP})^*\, \eta^l\, \eta^{H*}\,, \quad j = 1,2,3,\ l = e,\mu,\tau,$$

Convenient choice: $\eta^l = i$, $\eta^H = 1$ $(\eta^W = 1)$:

$$\begin{aligned} \lambda^*_{jl} &= \lambda_{jl}\, \rho^N_j\,, \ \ \rho^N_j = \pm 1\,, \\ U^*_{lj} &= U_{lj}\, \rho^{\nu}_j\,, \ \ \rho^{\nu}_j = \pm 1\,, \\ R^*_{jk} &= R_{jk}\, \rho^N_j\, \rho^{\nu}_k, \quad j,k = 1,2,3,\ l = e,\mu,\tau, \end{aligned}$$

λ_{jl}, U_{lj}, R_{jk} - **either real or purely imaginary.**

Relevant quantity:

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,, \ \ k \neq m\,,$$

$$CP: \quad P^*_{jkml} = P_{jkml}\, (\rho^N_j)^2\, (\rho^{\nu}_k)^2\, (\rho^{\nu}_m)^2 = P_{jkml}\,, \quad {\rm Im}(P_{jkml}) = 0\,.$$

$$P_{jkml} \equiv R_{jk}\, R_{jm}\, U^*_{lk}\, U_{lm}\,, \;\; k \neq m\,,$$

$$CP: \quad P^*_{jkml} = P_{jkml}\, (\rho^N_j)^2\, (\rho^\nu_k)^2\, (\rho^\nu_m)^2 = P_{jkml}\,, \quad \mathrm{Im}(P_{jkml}) = 0\,.$$

Consider NH N_j, NH ν_k: $\quad P_{123\tau} = R_{12}\, R_{13}\, U^*_{\tau 2}\, U_{\tau 3}$

Suppose, CP-invrainace holds at low E: $\delta = 0$, $\alpha_{21} = \pi$, $\alpha_{31} = 0$.

Thus, $U^*_{\tau 2}\, U_{\tau 3}$ - purely imaginary.

Then real $R_{12}\, R_{13}$ corresponds to CP-violation at “high” E.

Leptogenesis

$Y_B = \frac{n_B - n_{\bar{B}}}{S} \sim 8.6 \times 10^{-11}$ (n_γ: $\sim 6.3 \times 10^{-10}$)

$Y_B \cong -10^{-2} \;\; \varepsilon\, \kappa$

W. Buchmüller, M. Plümacher, 1998;
W. Buchmüller, P. Di Bari, M. Plümacher, 2004

$\kappa-$ efficiency factor; $\kappa \sim 10^{-1} - 10^{-3}$: $\varepsilon \gtrsim 10^{-7}$.

ε: $CP-$, $L-$ violating asymmetry generated in out of equilibrium $N_{Rj}-$decays in the early Universe,

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$

M.A. Luty, 1992;
L. Covi, E. Roulet and F. Vissani, 1996;
M. Flanz *et al.*, 1996;
M. Plümacher, 1997;
A. Pilaftsis, 1997.

$\kappa = \kappa(\widetilde{m})$, $\widetilde{m}$ - determines the rate of wash-out processes:

$\Phi^+ + \ell^- \to N_1$, $\ell^- + \Phi^+ \to \Phi^- + \ell^+$, etc.

W. Buchmuller, P. Di Bari and M. Plumacher, 2002;
G. F. Giudice *et al.*, 2004

Low Energy Leptonic CPV and Leptogenesis

Assume: $M_1 \ll M_2 \ll M_3$

Individual asymmetries:

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{lj} U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}\,, \qquad v = 174 \text{ GeV}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2\, v^2}{M_1} \;=\; \left|\sum_k R_{1k}\, m_k^{1/2}\, U^*_{lk}\right|^2\,, \quad l = e, \mu, \tau\,.$$

The "One-Flavor" Regime: $M_1 \sim T > 10^{12}$ **GeV;** $\mathrm{Y}_{e,\mu,\tau}$ **- "small"**

Boltzmann eqn. for $n(N_1)$ **and** $\Delta L = \Delta(L_e + L_\mu + L_\tau)$**.**

$Y_l \;\; H^c(x)\overline{l_R}(x)\psi_{lL}$**- out of equilibrium at** $T \sim M_1$**.**

One-flavor approximation: $M_1 \sim T > 10^{12}$ **GeV**

$$\varepsilon_1 \;=\; \sum_l \varepsilon_{1l} = -\frac{3M_1}{16\pi v^2}\,\frac{\mathrm{Im}\left(\sum_{j,k} m_j^2 R_{1j}^2\right)}{\sum_k m_k \left|R_{1k}\right|^2}\,,$$

$$\widetilde{m_1} \;=\; \sum_l \widetilde{m_l} = \sum_k m_k \left|R_{1k}\right|^2\,.$$

Two-Flavour Regime: 10^9 GeV $\lesssim M_1 \sim T \lesssim 10^{12}$ **GeV**

At $M_1 \sim T \lesssim 10^{12}$ **GeV:** Y_τ **- in equilibrium,** $Y_{e,\mu}$ **- not;**

wash-out dynamics changes: τ_R^-, τ_L^+

$N_1 \rightarrow (\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+$; $(\lambda_{1e}\, e_L^- + \lambda_{1\mu}\, \mu_L^- + \lambda_{1\tau}\, \tau_L^-) + \Phi^+ \rightarrow N_1$;

$\tau_L^- + \Phi^0 \rightarrow \tau_R^-$, $\tau_L^- + \tau_L^+ \rightarrow N_1 + \nu_L$, **etc.**

$\varepsilon_{1\tau}$ **and** $(\varepsilon_{1e} + \varepsilon_{1\mu}) \equiv \varepsilon_2$ **evolve independently.**

Thus, at $M_1 \sim 10^9 - 10^{12}$ **GeV:** L_τ, ΔL_τ **- distinguishable;**

$L_e + L_\mu$, $\Delta(L_e + L_\mu)$ **- distinguishable;**

L_e, L_μ, ΔL_e, ΔL_μ **- individually not distinguishable.**

Three-Flavour Regime: $M_1 \sim T < 10^9$ **GeV**

At $M_1 \sim T \sim 10^9$ **GeV:** Y_τ, Y_μ **- in equilibrium,** Y_e **- not.**

$\varepsilon_{1\tau}$, ε_{1e} **and** $\varepsilon_{1\mu}$ **evolve independently.**

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Individual asymmetries:

Assume: $M_1 \ll M_2 \ll M_3$, $10^9 \lesssim M_1 \ (\sim T) \lesssim 10^{12}$ **GeV,**

$$\varepsilon_{1l} = -\frac{3M_1}{16\pi v^2} \frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U_{lj}^* U_{lk} R_{1j} R_{1k}\right)}{\sum_j m_j \left|R_{1j}\right|^2}$$

$$\widetilde{m_l} \equiv \frac{|\lambda_{1l}|^2 v^2}{M_1} = \left|\sum_k R_{1k} m_k^{1/2} U_{lk}^*\right|^2, \quad l = e, \mu, \tau \,.$$

The baryon asymmetry is

$$Y_B \simeq -\frac{12}{37 g_*}\left(\epsilon_2 \, \eta\left(\frac{417}{589}\widetilde{m_2}\right) + \epsilon_\tau \, \eta\left(\frac{390}{589}\widetilde{m_\tau}\right)\right),$$

$$\eta\left(\widetilde{m_l}\right) \simeq \left(\left(\frac{\widetilde{m_l}}{8.25 \times 10^{-3}\,\mathrm{eV}}\right)^{-1} + \left(\frac{0.2 \times 10^{-3}\,\mathrm{eV}}{\widetilde{m_l}}\right)^{-1.16}\right)^{-1}.$$

$Y_{\mathcal{B}} = -(12/37)\,(Y_2 + Y_\tau)$,

$Y_2 = Y_{e+\mu}$, $\varepsilon_2 = \varepsilon_{1e} + \varepsilon_{1\mu}$, $\widetilde{m_2} = \widetilde{m_{1e}} + \widetilde{m_{1\mu}}$

A. Abada et al., 2006; E. Nardi et al., 2006

A. Abada et al., 2006

Real (Purely Imaginary) R: $\varepsilon_{1l} \neq 0$, CPV from U

$\varepsilon_{1e} + \varepsilon_{1\mu} + \varepsilon_{1\tau} = \varepsilon_2 + \varepsilon_{1\tau} = 0$,

$$\begin{aligned}
\varepsilon_{1\tau} &= -\frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}\left(\sum_{j,k} m_j^{1/2} m_k^{3/2} U^*_{\tau j} U_{\tau k} R_{1j} R_{1k}\right)}{\sum_j m_j \, |R_{1j}|^2} \\
&= -\frac{3M_1}{16\pi v^2}\frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k - m_j)\, R_{1j} R_{1k} \mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \, |R_{1j}|^2}\,, R_{1j}R_{1k} = \pm\, |R_{1j}R_{1k}|\,, \\
&= \mp\frac{3M_1}{16\pi v^2}\frac{\sum_{j,k>j} m_j^{1/2} m_k^{1/2} (m_k + m_j)\, |R_{1j} R_{1k}|\, \mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right)}{\sum_j m_j \, |R_{1j}|^2}\,, R_{1j}R_{1k} = \pm i |R_{1j}R_{1k}|
\end{aligned}$$

S. Pascoli, S.T.P., A. Riotto, 2006.

CP-Violation: $\mathrm{Im}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$, $\mathrm{Re}\left(U^*_{\tau j} U_{\tau k}\right) \neq 0$;

$$Y_B = -\frac{12}{37}\,\frac{\varepsilon_{1\tau}}{g_*}\left(\eta\left(\frac{390}{589}\widetilde{m_\tau}\right) - \eta\left(\frac{417}{589}\widetilde{m_2}\right)\right)$$

$m_1 \ll m_2 \ll m_3$, $M_1 \ll M_{2,3}$; $R_{12}R_{13}$ **– real;** $m_1 \cong 0$, $R_{11} \cong 0$ **(**N_3 **decoupling)**

$$\varepsilon_{1\tau} = -\frac{3M_1\sqrt{\Delta m^2_{31}}}{16\pi v^2}\left(\frac{\Delta m^2_\odot}{\Delta m^2_{31}}\right)^{\frac{1}{4}} \frac{|R_{12}R_{13}|}{\left(\frac{\Delta m^2_\odot}{\Delta m^2_{31}}\right)^{\frac{1}{2}} |R_{12}|^2 + |R_{13}|^2}$$

$$\times \left(1 - \frac{\sqrt{\Delta m^2_\odot}}{\sqrt{\Delta m^2_{31}}}\right) \text{Im}\,(U^*_{\tau 2} U_{\tau 3})$$

$$\text{Im}\,(U^*_{\tau 2} U_{\tau 3}) = -c_{13}\left[c_{23}s_{23}c_{12}\sin\left(\frac{\alpha_{32}}{2}\right) - c^2_{23}s_{12}s_{13}\sin\left(\delta - \frac{\alpha_{32}}{2}\right)\right]$$

$\alpha_{32} = \pi$, $\delta = 0$: $\text{Re}(U^*_{\tau 2} U_{\tau 3}) = 0$, **CPV due to** R

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ **(NH)**

Dirac CP-violation

$\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = \pi\ (0)$; $\beta_{23} \equiv \beta_{12} + \beta_{13} \equiv \arg(R_{12}R_{13})$.

$|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - **maximise** $|\epsilon_\tau|$ **and** $|Y_B|$:

$$|Y_B| \;\cong 2.8 \times 10^{-13}\, |\sin\delta| \left(\frac{s_{13}}{0.2}\right)\left(\frac{M_1}{10^9\ \text{GeV}}\right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV **imply**

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.11\,, \qquad \sin\theta_{13} \gtrsim 0.11\,.$$

The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 2.4 \times 10^{-2}$$

FOR $\alpha_{32} = 0\ (2\pi)$, $\beta_{23} = 0\ (\pi)$:

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \quad |J_{\text{CP}}| \gtrsim 2.0 \times 10^{-2}$$

$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$ **(NH)**

Majorana CP-violation

$\delta = 0$, **real** R_{12}, R_{13} **(**$\beta_{23} = \pi$ (0)**);**

$\alpha_{32} \cong \pi/2$, $|R_{12}|^2 \cong 0.85$, $|R_{13}|^2 = 1 - |R_{12}|^2 \cong 0.15$ - **maximise** $|\epsilon_\tau|$ **and** $|Y_B|$**:**

$$|Y_B| \cong 2 \times 10^{-12} \left(\frac{\sqrt{\Delta m^2_{31}}}{0.05 \text{ eV}}\right)\left(\frac{M_1}{10^9 \text{ GeV}}\right).$$

We get $|Y_B| \gtrsim 8 \times 10^{-11}$, **for** $M_1 \gtrsim 3.6 \times 10^{10}$ GeV, **or** $|sin\alpha_{32}/2| \gtrsim 0.15$

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ **(IH)**

$m_3 \cong 0$, $R_{13} \cong 0$ **(N_3 decoupling): impossible to reproduce Y_B^{obs} for real $R_{11}R_{12}$;**

$|Y_B|$ **suppressed by the additional factor** $\Delta m^2_\odot / || \cong 0.03$.

Purely imaginary $R_{11}R_{12}$: no (additional) suppression

Dirac CP-violation

$\alpha_{21} = \pi$; $R_{11}R_{12} = i\kappa |R_{11}R_{12}|$, $\kappa = 1$;

$|R_{11}| \cong 1.07$, $|R_{12}|^2 = |R_{11}|^2 - 1$, $|R_{12}| \cong 0.38$ - **maximise** $|\epsilon_\tau|$ **and** $|Y_B|$:

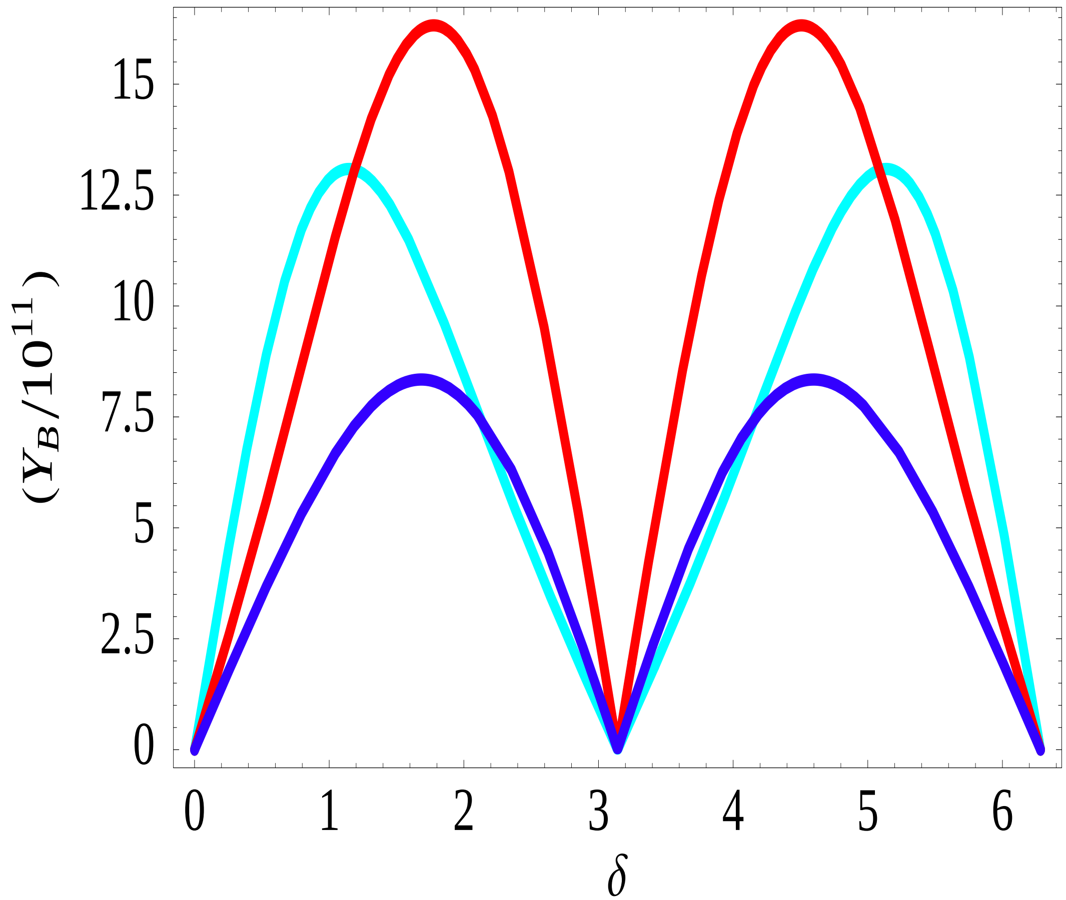
$$|Y_B| \cong 8.1 \times 10^{-12} \, |s_{13} \sin\delta| \left(\frac{M_1}{10^9 \text{ GeV}} \right).$$

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV **imply**

$$|\sin\theta_{13} \sin\delta| \gtrsim 0.02 \,, \qquad \sin\theta_{13} \gtrsim 0.02 \,.$$

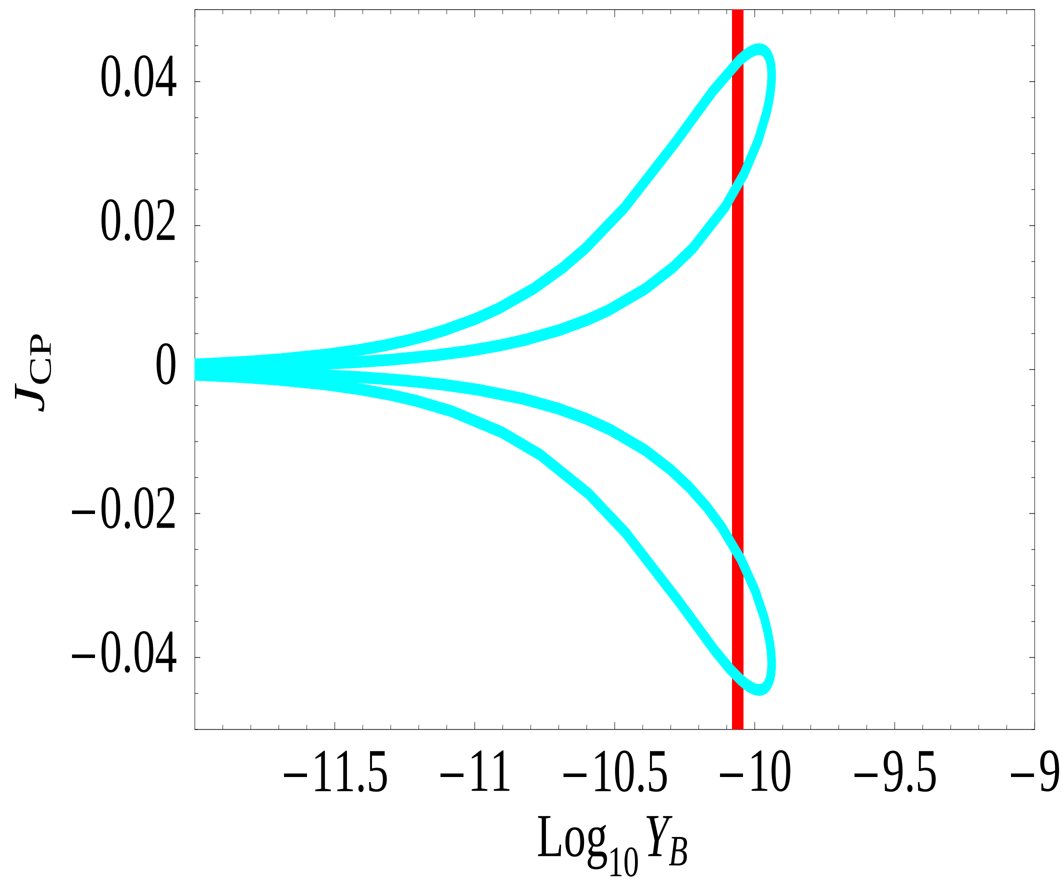
The lower limit corresponds to

$$|J_{\text{CP}}| \gtrsim 4.6 \times 10^{-3}$$



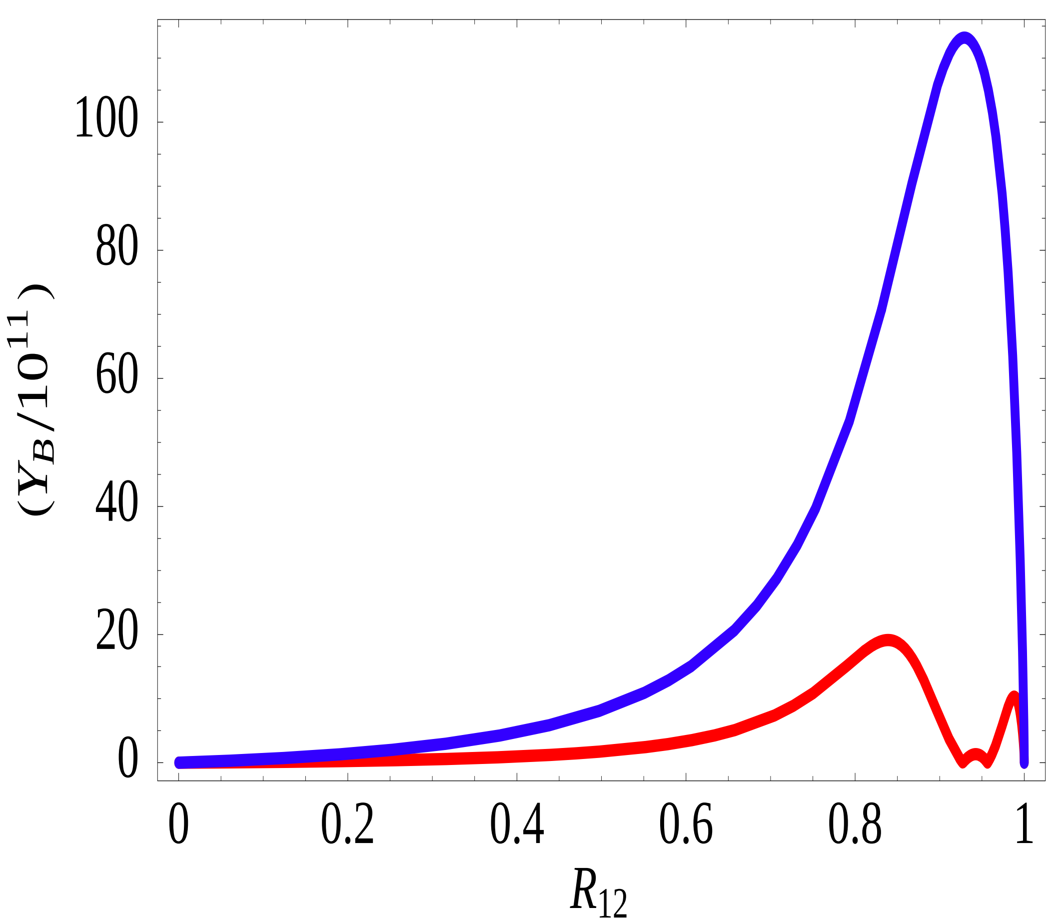
$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; **Dirac CP-violation,** $\alpha_{32} = 0;\ 2\pi$;
real R_{12}, R_{13}, $|R_{12}|^2 + |R_{13}|^2 = 1$, $|R_{12}| = 0.86$, $|R_{13}| = 0.51$, $\text{sign}\,(R_{12}R_{13}) = +1$;
i) $\alpha_{32} = 0$ **(**$\kappa' = +1$**),** $s_{13} = 0.2$ **(red line) and** $s_{13} = 0.1$ **(dark blue line);**
ii) $\alpha_{32} = 2\pi$ **(**$\kappa' = -1$**),** $s_{13} = 0.2$ **(light blue line);**
$M_1 = 5 \times 10^{11}$ **GeV.**

S. Pascoli, S.T.P., A. Riotto, 2006.

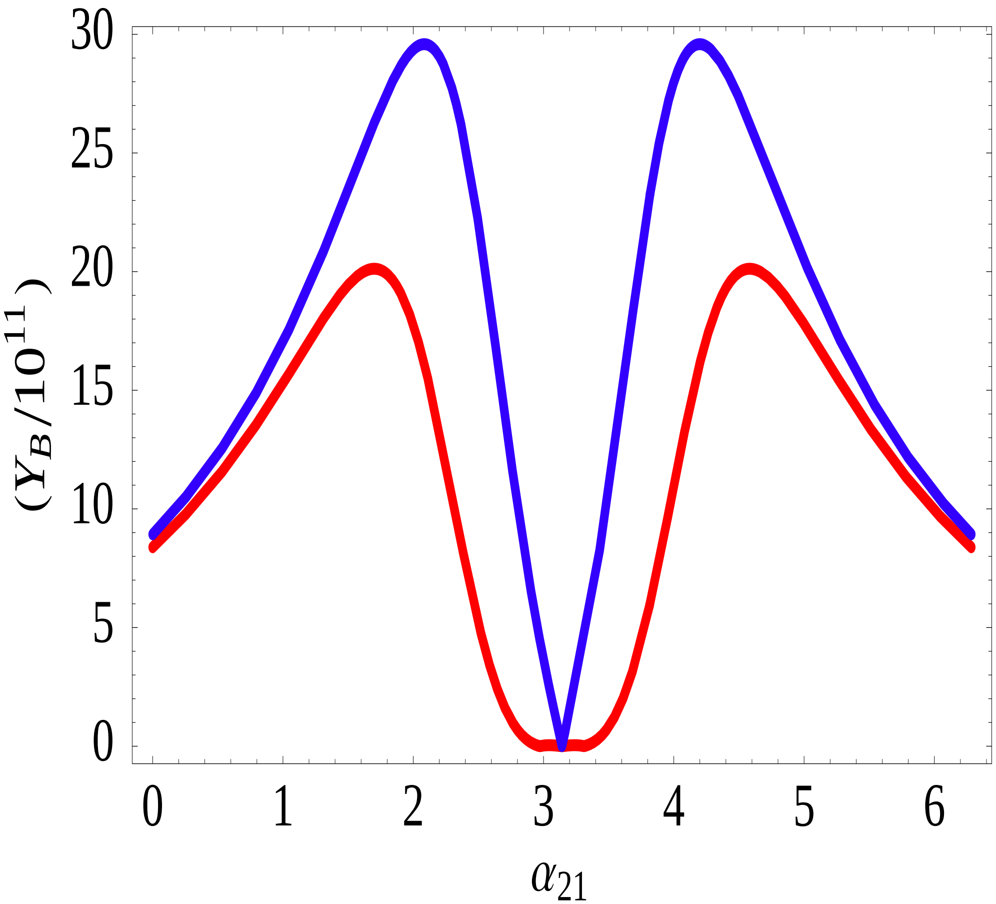


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ **GeV;**
Dirac CP-violation, $\alpha_{32} = 0\ (2\pi)$;
$|R_{12}| = 0.86$, $|R_{13}| = 0.51$, $\mathrm{sign}\,(R_{12}R_{13}) = +1\ (-1)$ **(**$\beta_{23} = 0\ (\pi)$, $\kappa' = +1$**);**
The red region denotes the 2σ **allowed range of** Y_{B}**.**

S. Pascoli, S.T.P., A. Riotto, 2006.

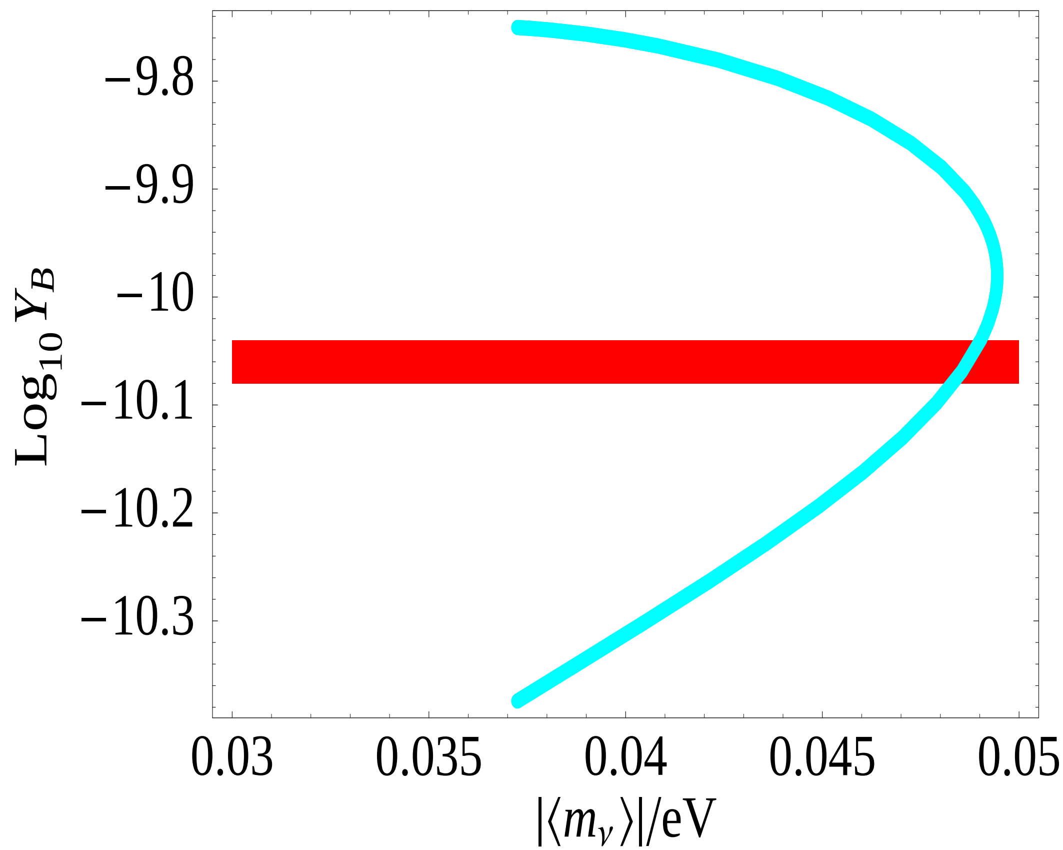


$M_1 \ll M_2 \ll M_3$, $m_1 \ll m_2 \ll m_3$; $M_1 = 5 \times 10^{11}$ **GeV;**
real R_{12}, R_{13}, sign$\,(R_{12}R_{13}) = +1$, $R_{12}^2 + R_{13}^2 = 1$, $s_{13} = 0.20$;
a) Majorana CP-violation (blue line), $\delta = 0$ **and** $\alpha_{32} = \pi/2$ **(**$\kappa = +1$**);**
b) Dirac CP-violation (red line), $\delta = \pi/2$ **and** $\alpha_{32} = 0$ **(**$\kappa' = +1$**);**
$\Delta m^2_{\odot}$, $\sin^2\theta_{12}$, Δm^2_{31}, $\sin^2 2\theta_{23}$ **- fixed at their best fit values.**



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ **GeV;**
Majorana CP-violation, $\delta = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = -1$, $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.2$;
$s_{13} = 0$ **(blue line) and** 0.2 **(red line).**

S. Pascoli, S.T.P., A. Riotto, 2006.



$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$; $M_1 = 2 \times 10^{11}$ **GeV;**
Majorana CP-violation, $\delta = 0$, $s_{13} = 0$;
purely imaginary $R_{11}R_{12} = i\kappa|R_{11}R_{12}|$, $\kappa = +1$ $|R_{11}|^2 - |R_{12}|^2 = 1$, $|R_{11}| = 1.05$.
The Majorana phase α_{21} **is varied in the interval** $[-\pi/2, \pi/2]$.

S. Pascoli, S.T.P., A. Riotto, 2006.

$M_1 \ll M_2 \ll M_3$, $m_3 \ll m_1 < m_2$ **(IH)**

Majorana or Dirac CP-violation

$m_3 \neq 0$, $R_{13} \neq 0$, $R_{11}(R_{12}) = 0$: **possible to reproduce** Y_B^{obs} **for real** $R_{12(11)} R_{13} \neq 0$

Requires $m_3 \cong (10^{-5} - 10^{-2})$ **eV; non-trivial dependence of** $|Y_B|$ **on** m_3

Majorana CPV, $\delta = 0$ (π)**: requires** $M_1 \gtrsim 3.5 \times 10^{10}$ GeV

Dirac CPV, $\alpha_{32(31)} = 0$**: typically requires** $M_1 \gtrsim 10^{11}$ GeV

$|Y_B| \gtrsim 8 \times 10^{-11}$, $M_1 \lesssim 5 \times 10^{11}$ GeV **imply**

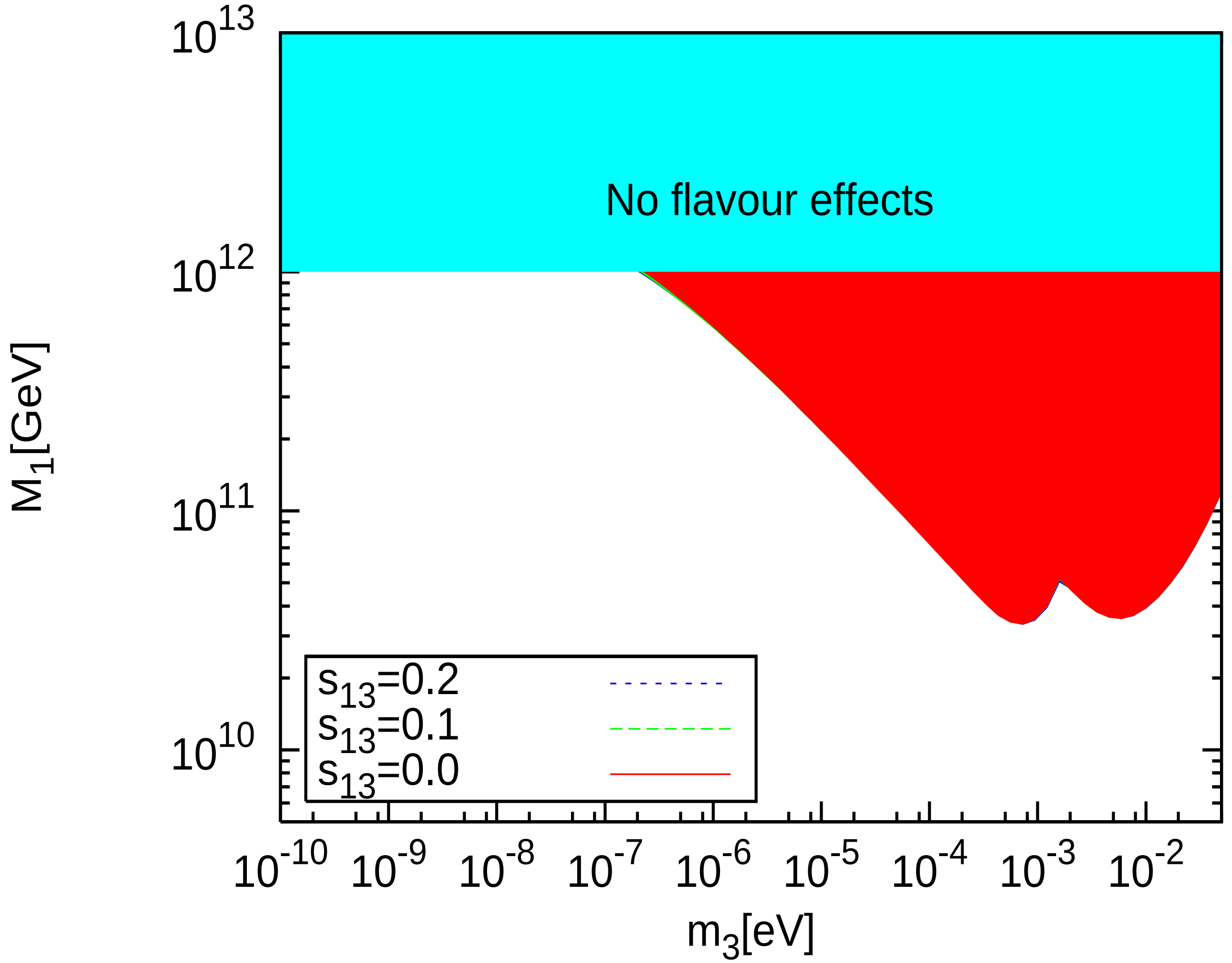
$$|\sin\theta_{13} \sin\delta|, \;\; \sin\theta_{13} \gtrsim (0.04 - 0.09)\,.$$

The lower limit corresponds to

$$|J_{CP}| \gtrsim (0.009 - 0.02)$$

NO (NH) spectrum, $m_1 <$ $(\ll)$ $m_2 < m_3$**: similar dependence of** $|Y_B|$ **on** m_1 **if** $R_{12} = 0$, $R_{11} R_{13} \neq 0$**; non-trivial effects for** $m_1 \cong (10^{-4} - 5 \times 10^{-2})$ **eV.**

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



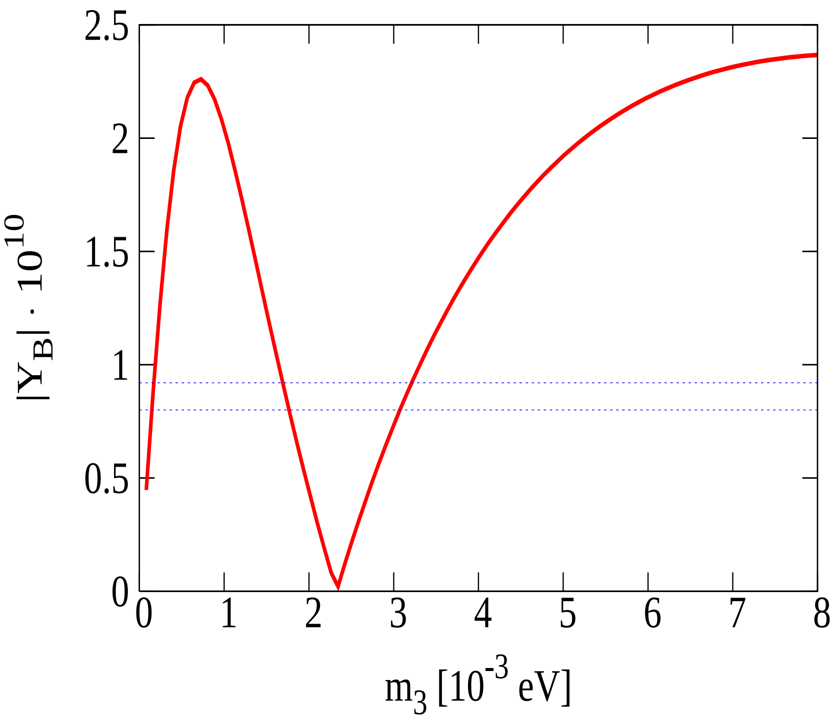
$m_3 < m_1 < m_2$, $M_1 \ll M_2 \ll M_3$, **real** R_{1j}; $M_1 = (10^9 - 10^{12})$ **GeV**, $s_{13} = 0.2; 0.1; 0$;

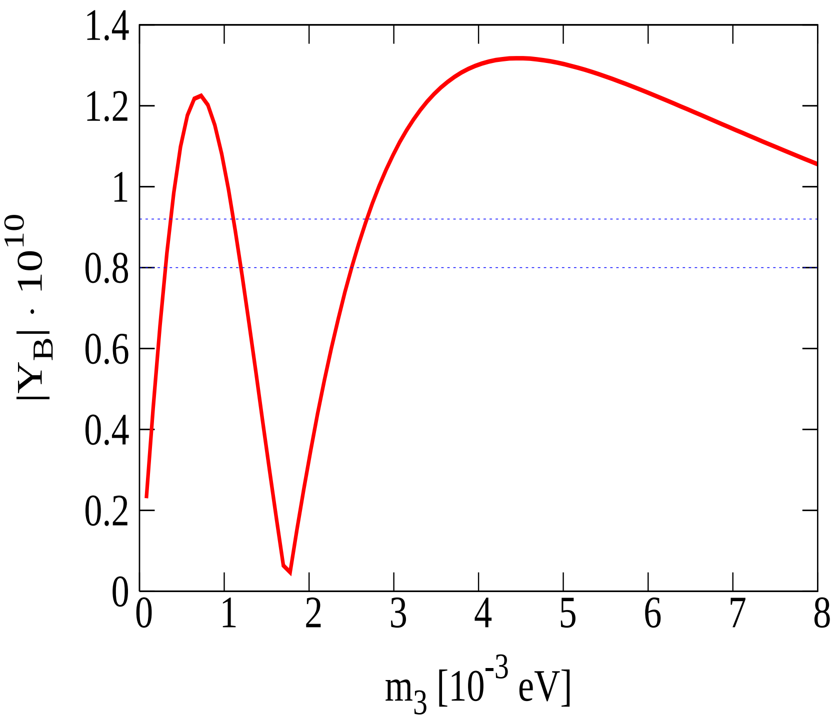
R_{1j} **varied within** $|R_{13}|^2 + |R_{12}|^2 + |R_{13}|^2 = 1$; $\alpha_{21}, \alpha_{31}, \delta$ **varied in [0,**2π**];**

$\min(M_1)$ **for given** m_3: $|Y_B| = 8.6 \times 10^{-11}$; **absolute minima of** M_1:

$m_3 \cong 5.5 \times 10^{-4}$; 5.9×10^{-3} **eV**, $\alpha_{32} \cong \pi/2$, $M_1 = 3.4\ (3.5) \times 10^{10}$ **GeV.**

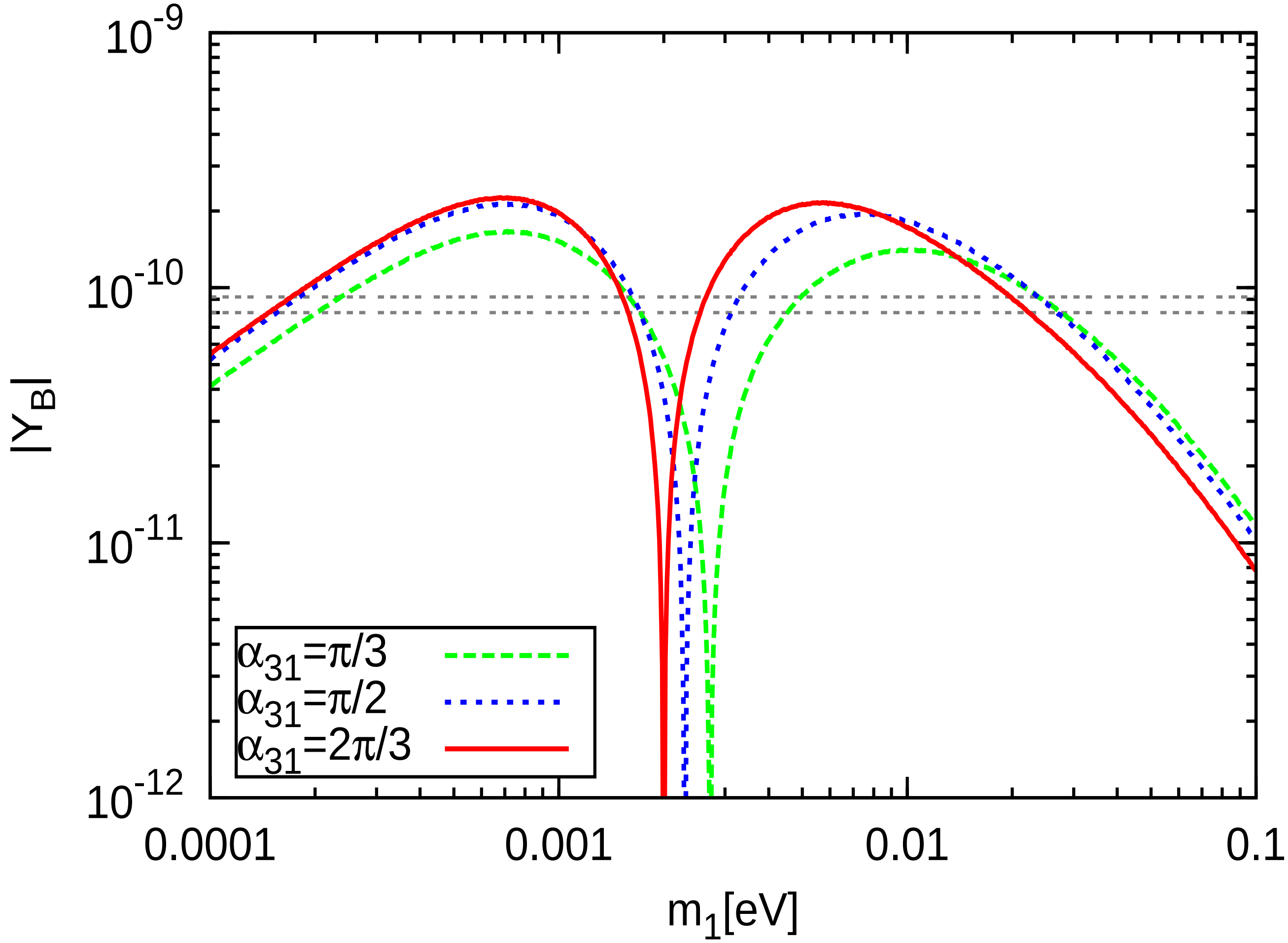
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007





$m_3 \ll m_1 \ll m_2$ **(IH)**, $R_{11} = 0$, **real** $R_{12}R_{13}$, **Majorana CPV;**
$\alpha_{32} = \pi/2$, $s_{13} = 0$, $M_1 = 10^{11}$ **GeV;** $R_{12}^2/R_{13}^2 = m_3/m_2$: **maximises** $|\epsilon_\tau|$;
i) sgn$(R_{12}R_{13}) = +1$; **ii)** sgn$(R_{12}R_{13}) = -1$.

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007



$m_1 < m_2 < m_3$ **(NO(NH)),** $R_{12} = 0$**, real** $R_{11}R_{13}$**, Majorana CPV,** $s_{13} = 0$**;**
$\text{sgn}(R_{11}R_{13}) = -1$, $\sin^2\theta_{23} = 0.50$, $M_1 = 1.5 \times \times 10^{11}$ **GeV;**
$\alpha_{32} = 2\pi/3; \pi/2; \pi/3$ **(red, blue, green lines).**

E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007

Complex R: $\varepsilon_{1l} \neq 0$, CPV from U and R

$m_1 \ll m_2 < m_3$ **(NH)**, $M_1 \ll M_{2,3}$; $m_1 \cong 0$, $R_{11} \cong 0$ **(N_3 decoupling)**

$$R_{12}^2 + R_{13}^2 = |R_{12}|^2\, e^{i2\varphi_{12}} + |R_{13}|^2\, e^{i2\varphi_{13}} = 1\,,$$

$$|R_{12}|^2 \sin 2\varphi_{12} + |R_{13}|^2 \sin 2\varphi_{13} = 0: \ \mathrm{sgn}(\sin 2\varphi_{12}) = -\mathrm{sgn}(\sin 2\varphi_{13})\,.$$

$$\cos 2\varphi_{12} = \frac{1+|R_{12}|^4-|R_{13}|^4}{2|R_{12}|^2}\,, \quad \sin 2\varphi_{12} = \pm\sqrt{1-\cos^2 2\varphi_{12}}\,,$$

$$\cos 2\varphi_{13} = \frac{1-|R_{12}|^4+|R_{13}|^4}{2|R_{13}|^2}\,, \quad \sin 2\varphi_{13} = \mp\sqrt{1-\cos^2 2\varphi_{13}}\,.$$

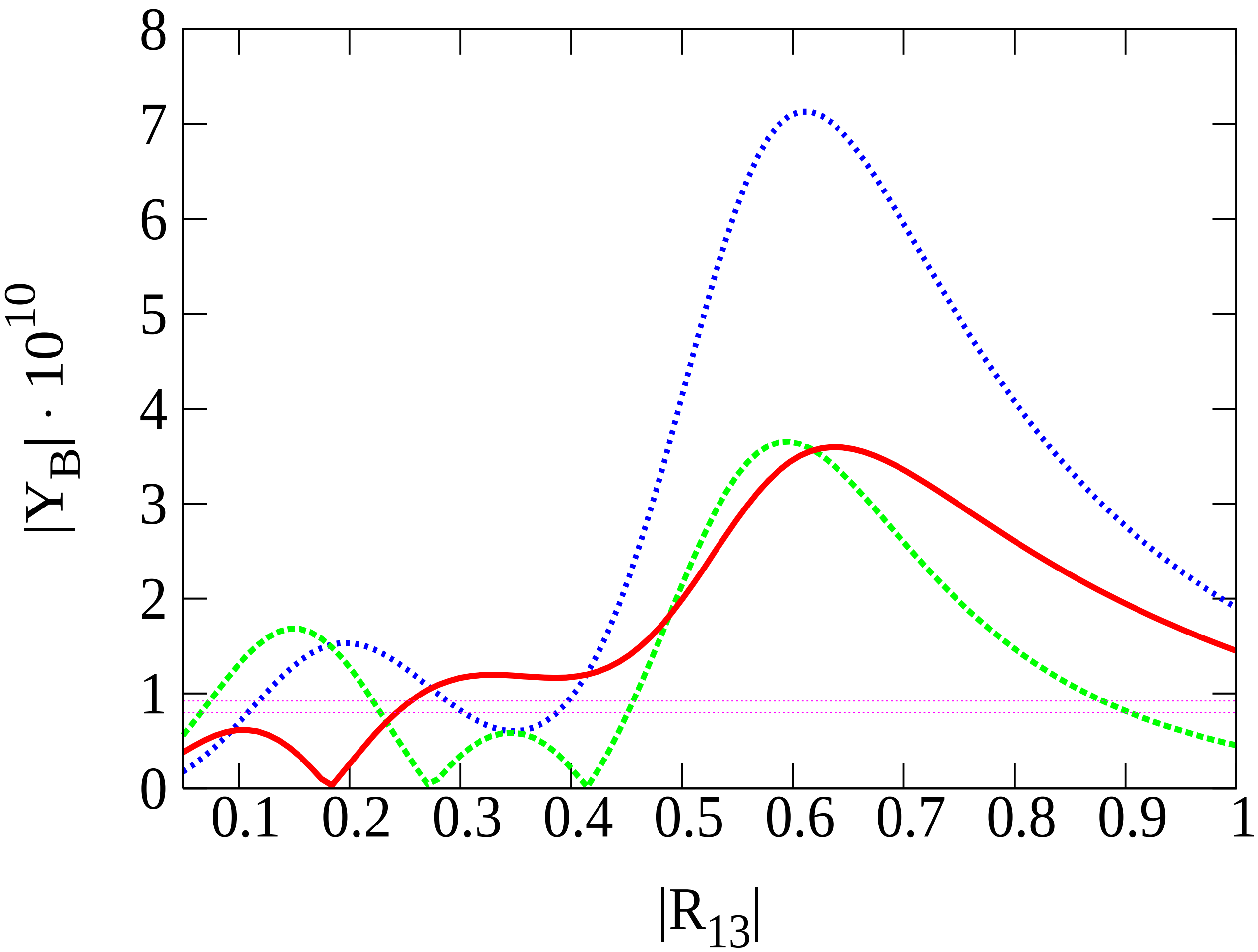
$m_3 \ll m_1 < m_2$ **(IH)**, $M_1 \ll M_{2,3}$; $m_3 \cong 0$, $R_{13} \cong 0$ **(N_3 decoupling)**

$$R_{11}^2 + R_{12}^2 = |R_{11}|^2\, e^{i2\varphi_{11}} + R_{12}^2\, e^{i2\varphi_{12}} = 1\,,$$

$$|R_{11}|^2 \sin 2\varphi_{11} + |R_{12}|^2 \sin 2\varphi_{12} = 0\,.$$

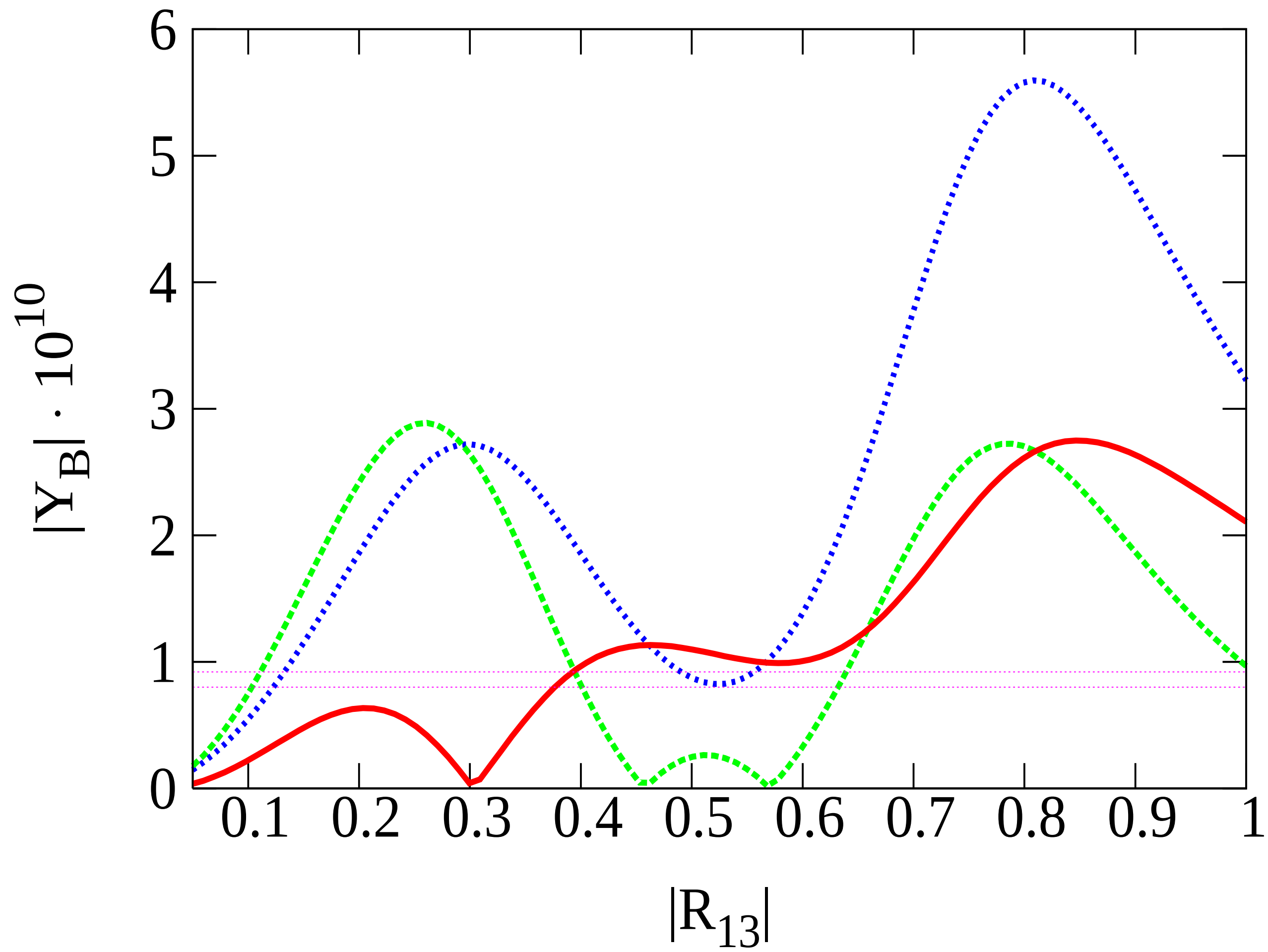
$|Y_B^0 A_{\mathrm{HE}}| \propto |R_{11}|^2 \sin(2\varphi_{11})\,(|U_{\tau 1}|^2 - |U_{\tau 2}|^2)$ - **can be suppressed:**

$$|U_{\tau 1}|^2 - |U_{\tau 2}|^2 \cong (s_{12}^2 - c_{12}^2)s_{23}^2 - 4 s_{12}c_{12}s_{23}c_{23}s_{13}\cos\delta \cong -0.20 - 0.92\, s_{13}\cos\delta\,.$$



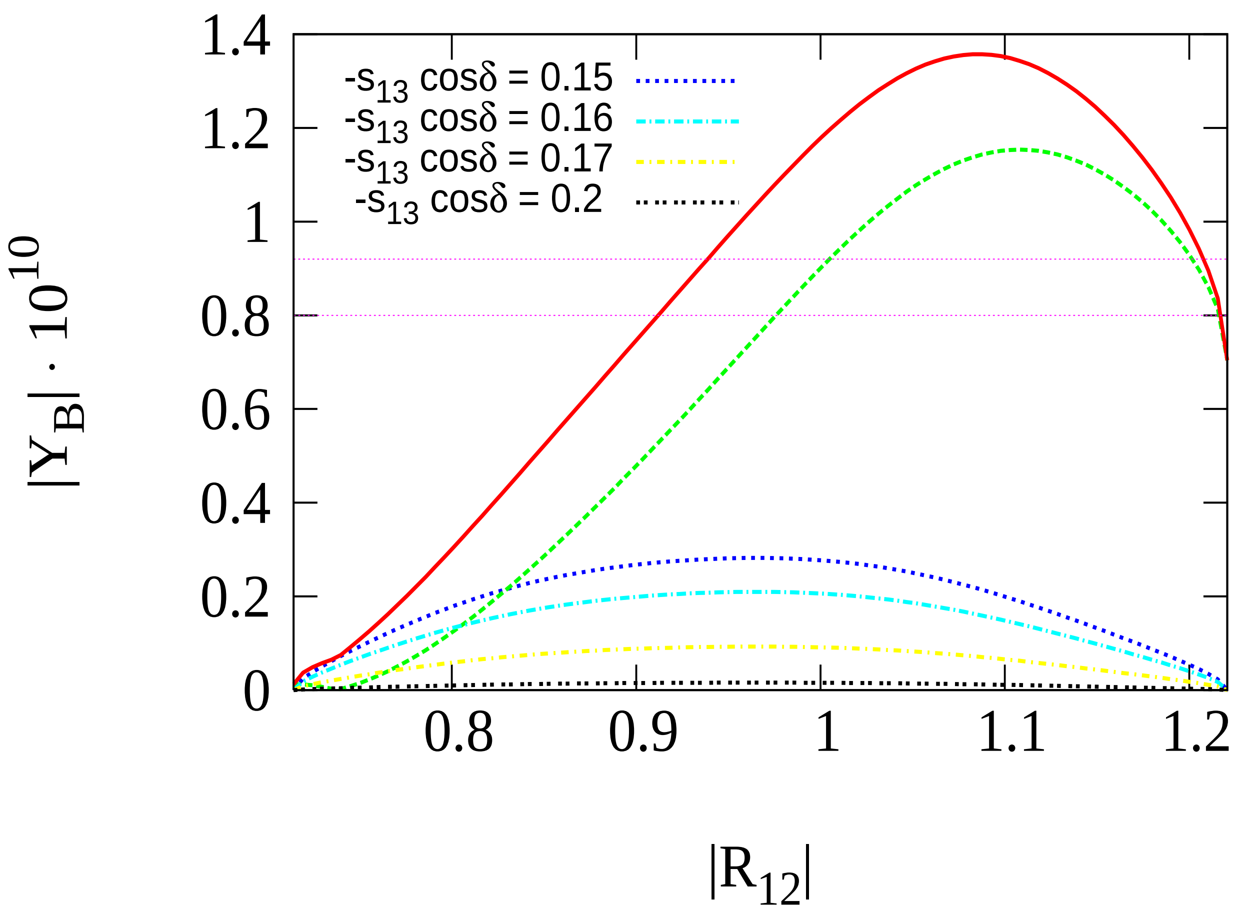
$m_1 < m_2 < m_3$ **(NO(NH))**, $R_{11} = 0$, **CPV due to** R **and** U,
$\alpha_{32} = \pi/2$, $s_{13} = 0$, $\sin^2\theta_{23} = 0.50$, $M_1 = 10^{11}$ **GeV;**
$|Y_B^0 A_{\mathrm{HE}}|$ **(**R **CPV, blue),** $|Y_B^0 A_{\mathrm{MIX}}|$ **(**U **CPV, green), total** $|Y_B|$ **(red line)**

E. Molinaro, S.T.P., 2008



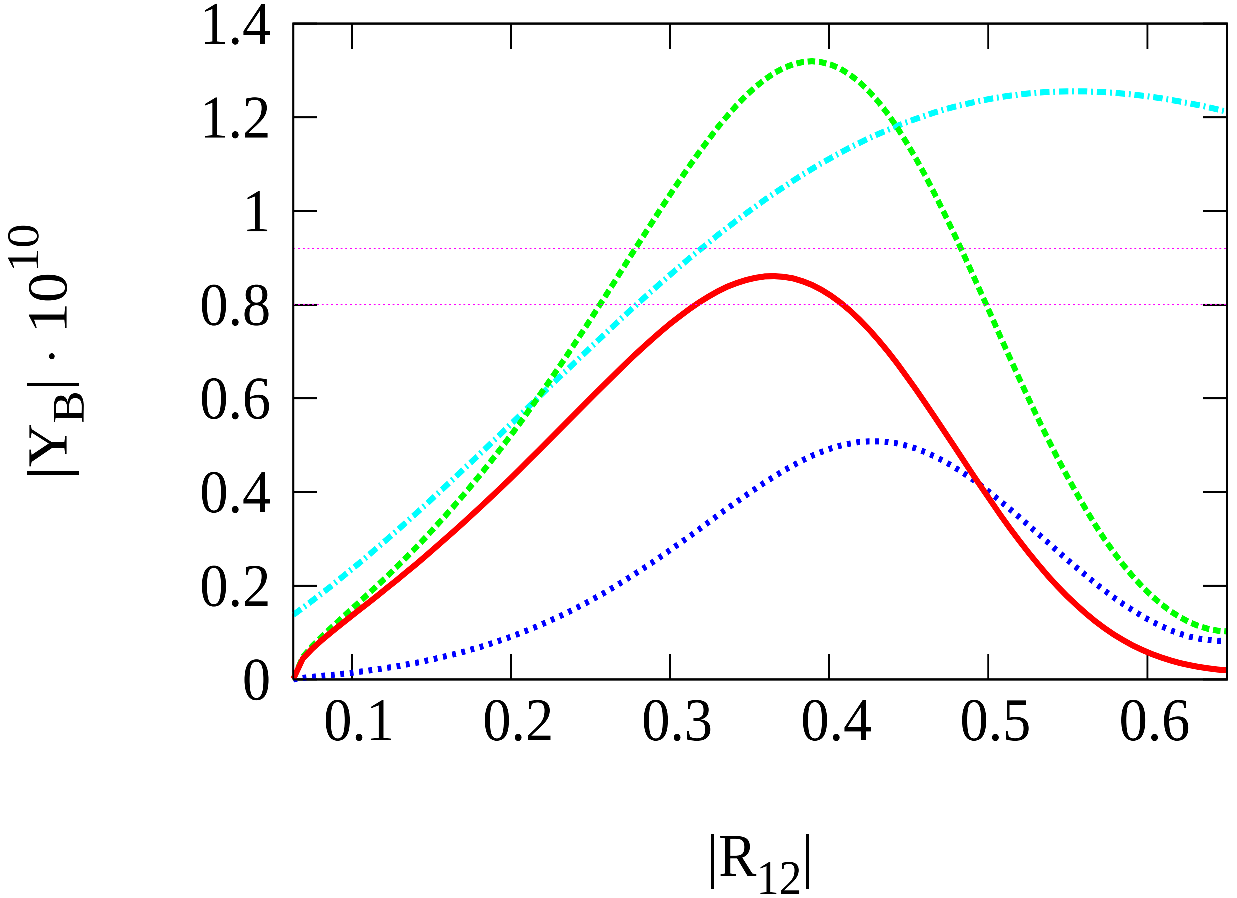
$m_1 < m_2 < m_3$ **(NO(NH))**, $R_{11} = 0$, **CPV due to** R **and** U,
$\alpha_{32} = \pi/2$ **,** $s_{13} = 0$, $\sin^2\theta_{23} = 0.64$, $M_1 = 10^{11}$ **GeV;**
$|Y_B^0 A_{\mathrm{HE}}|$ **(**R **CPV, blue),** $|Y_B^0 A_{\mathrm{MIX}}|$ **(**U **CPV, green), total** $|Y_B|$ **(red line)**

E. Molinaro, S.T.P., 2008



$m_3 \ll m_1 < m_2$ **(IH))**, $R_{13} = 0$, **Majorana and** R**-matrix CPV** ,
$\alpha_{21} = \pi/2$, $(-s_{13}\cos\delta) = 0.15$, $|R_{11}| = 1.2$, $M_1 = 10^{11}$ **GeV**;
$|Y_B^0 A_{\rm HE}|$ **(**R **CPV, blue)**, $|Y_B^0 A_{\rm MIX}|$ **(**U **CPV, green), total** $|Y_B|$ **(red line)** .

E. Molinaro, S.T.P., 2008



$m_3 \ll m_1 < m_2$ **(IH))**, $R_{13} = 0$, **Majorana and** R**-matrix CPV** ,
$\alpha_{21} = \pi/2$, $s_{13} = 0$, $|R_{11}| \cong 1$, $M_1 = 10^{11}$ **GeV**;
$|Y_B^0 A_{\mathrm{HE}}|$ **(**R **CPV, blue)**, $|Y_B^0 A_{\mathrm{MIX}}|$ **(**U **CPV, green), total** $|Y_B|$ **(red line) .**
Light-blue line: CP-conserving R, $R_{11}R_{12} \equiv ik|R_{11}R_{12}|$, $k = -1$ $|R_{11}|^2 - |R_{12}|^2 = 1$.

E. Molinaro, S.T.P., 2008

Low Energy Leptonic CPV and Leptogenesis: Summary

Leptogenesis: see-saw mechanism; N_j **- heavy RH** ν**'s;**
N_j, ν_k **- Majorana particles**

N_j: $M_1 \ll M_2 \ll M_3$

The observed value of the baryon asymmetry of the Universe can be generated

A. CP-violation due to the Dirac phase δ **in** U_{PMNS}**, no other sources of CPV (Majorana phases in** U_{PMNS} **equal to 0, etc.); requires** $M_1 \gtrsim 10^{11}$ **GeV.**

$m_1 \ll m_2 \ll m_3$ **(NH):**

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.09\,, \qquad \sin\theta_{13} \gtrsim 0.09\,; \qquad |J_{\text{CP}}| \gtrsim 2.0\times 10^{-2}$$

$m_3 \ll m_1 < m_2$ **(IH):**

$$|\sin\theta_{13}\sin\delta| \gtrsim 0.02\,, \qquad \sin\theta_{13} \gtrsim 0.02\,; \qquad |J_{\text{CP}}| \gtrsim 4.6\times 10^{-3}$$

B. CP-violation due to the Majorana phases in U_{PMNS}**, no other sources of CPV (Dirac phase in** U_{PMNS} **equal to 0, etc.); requires** $M_1 \gtrsim 3.5\times 10^{10}$ **GeV.**

C. CP-violation due to both Dirac and Majorana phases in U_{PMNS}**.**

D. Y_B **can depend non-trivially on** $\min(m_j) \sim (10^{-5} - 10^{-2})$ **eV.**

S. Pascoli, S.T.P., A. Riotto, 2006 (A-C);
E. Molinaro, S.T.P., T. Shindou, Y. Takanishi, 2007 (D).

Conclusions

Determining the nature - Dirac or Majorana, of massive neutrinos is of fundamental importance for understanding the origin of neutrino masses.

The see-saw mechanism provides a link between ν-mass generation and BAU.

Majorana CPV phases in U_{PMNS}: $(\beta\beta)_{0\nu}$-decay, Y_{B}.

Any of the CPV phases in U_{PMNS} can be the leptogenesis CPV parameters.

Obtaining information on Dirac and Majorana CPV is a remarkably challenging problem.

Dirac and Majorana CPV may have the same source.

Low energy leptonic CPV can be directly related to the existence of BAU.

Understanding the status of the CP-symmetry in the lepton sector is of fundamental importance.

These results underline further the importance of the experiments aiming to measure the CHOOZ angle θ_{13} and of the experimental searches for Dirac and/or Majorana leptonic CP-violation at low energies.