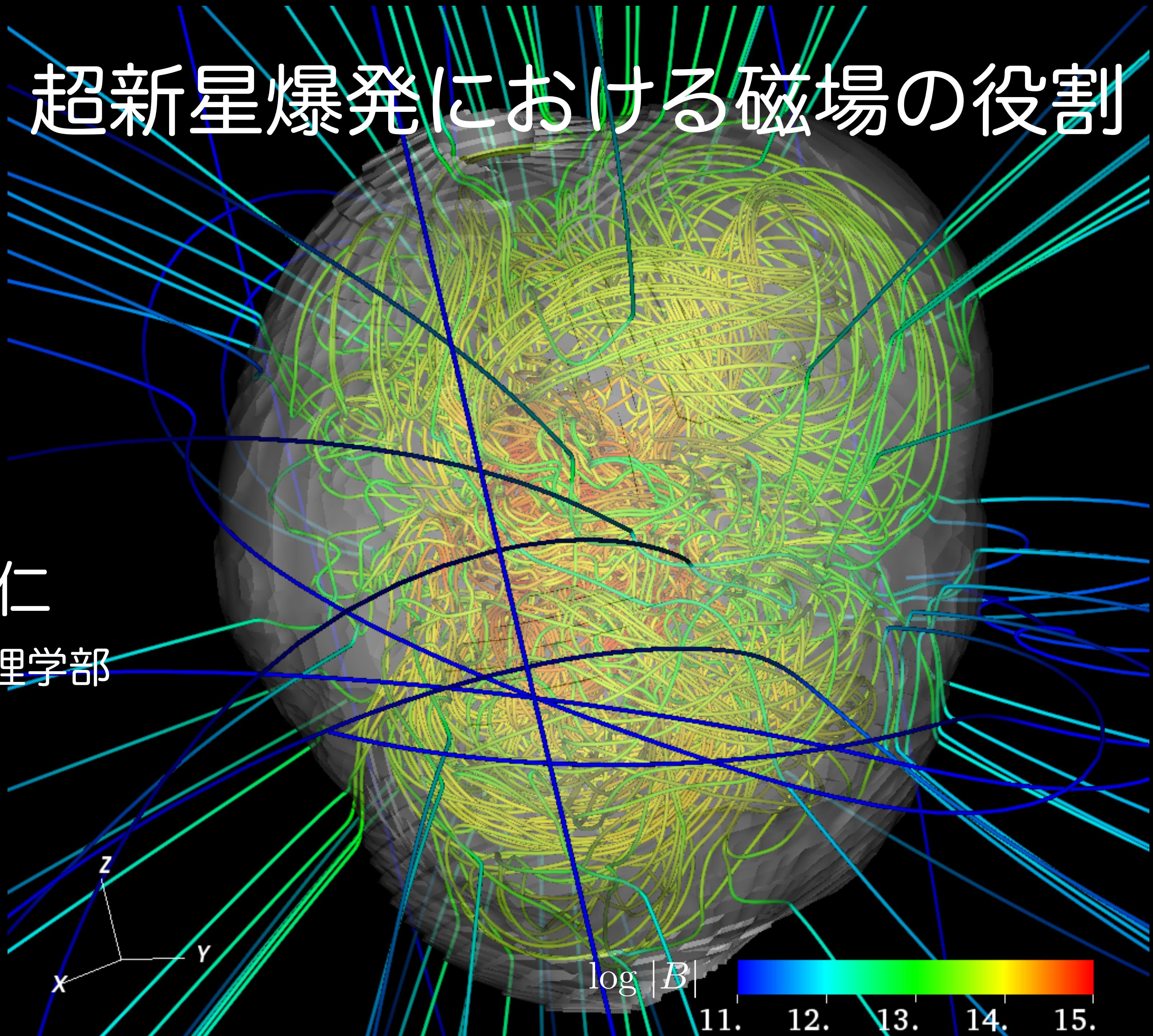


超新星爆発における磁場の役割

松本仁

福岡大学理学部

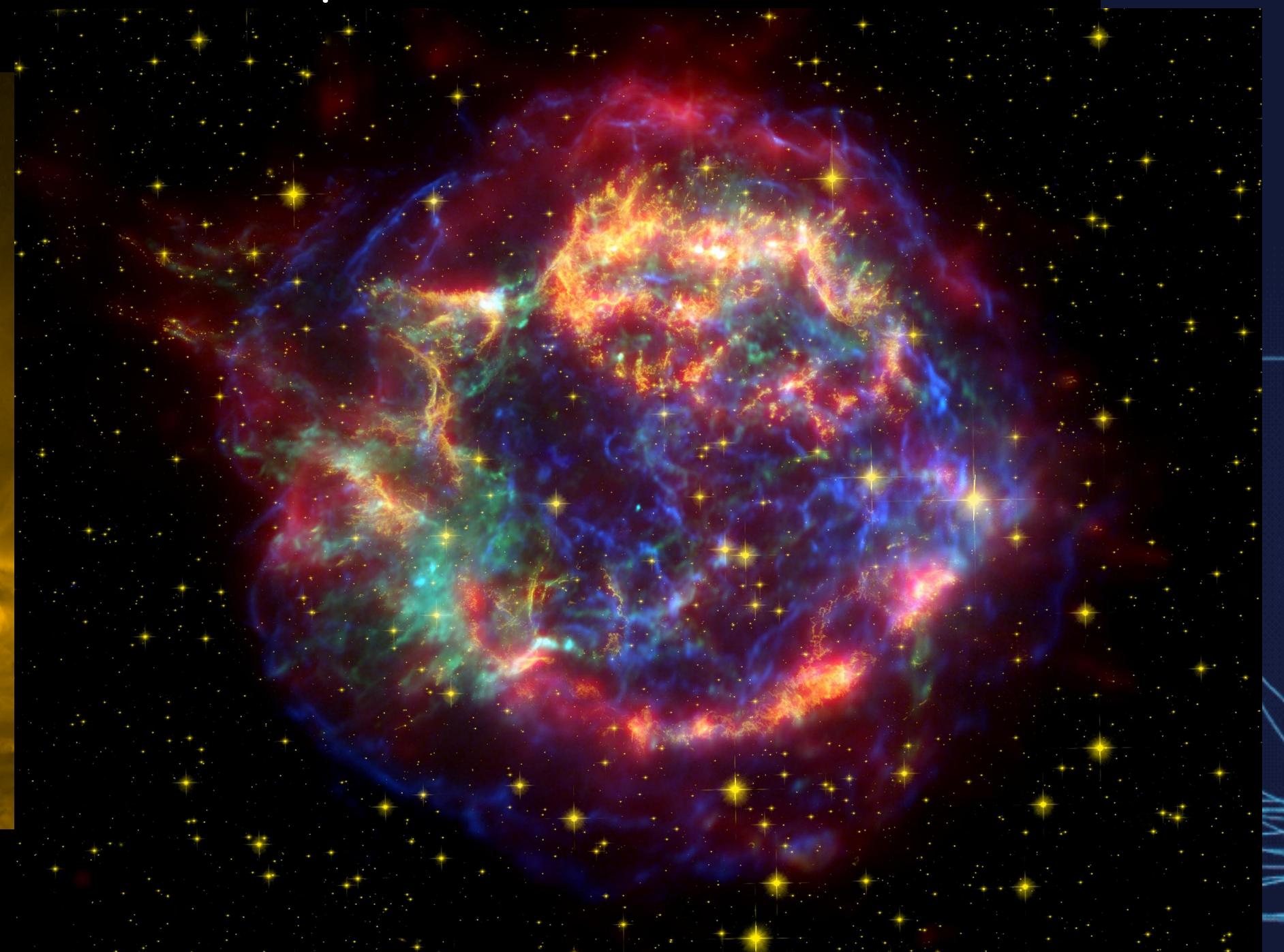


ミッシングリンクとしての超新星爆発フェーズ



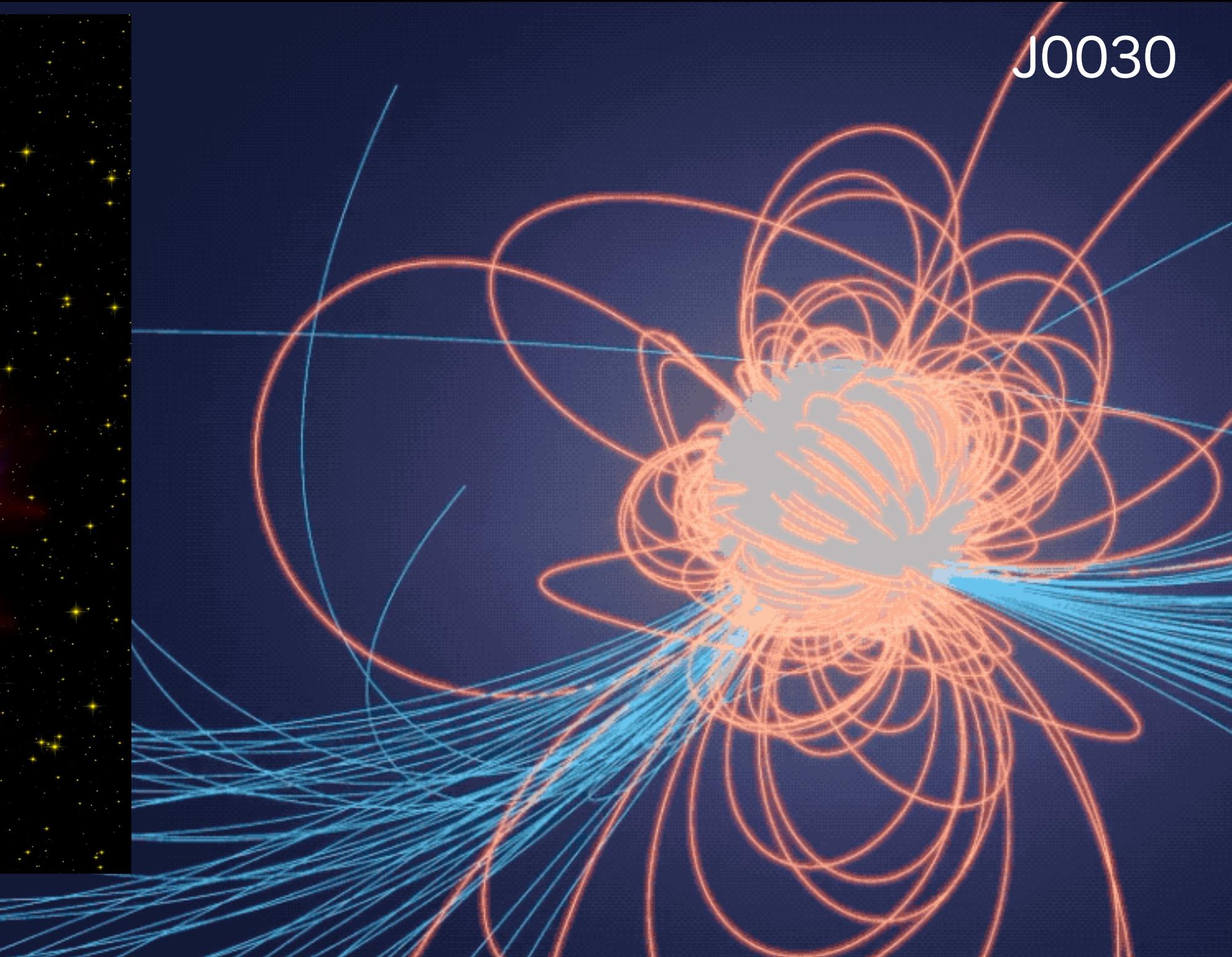
NASA/SDO

Cassiopeia A



恒星には磁場がある！ - ->

?

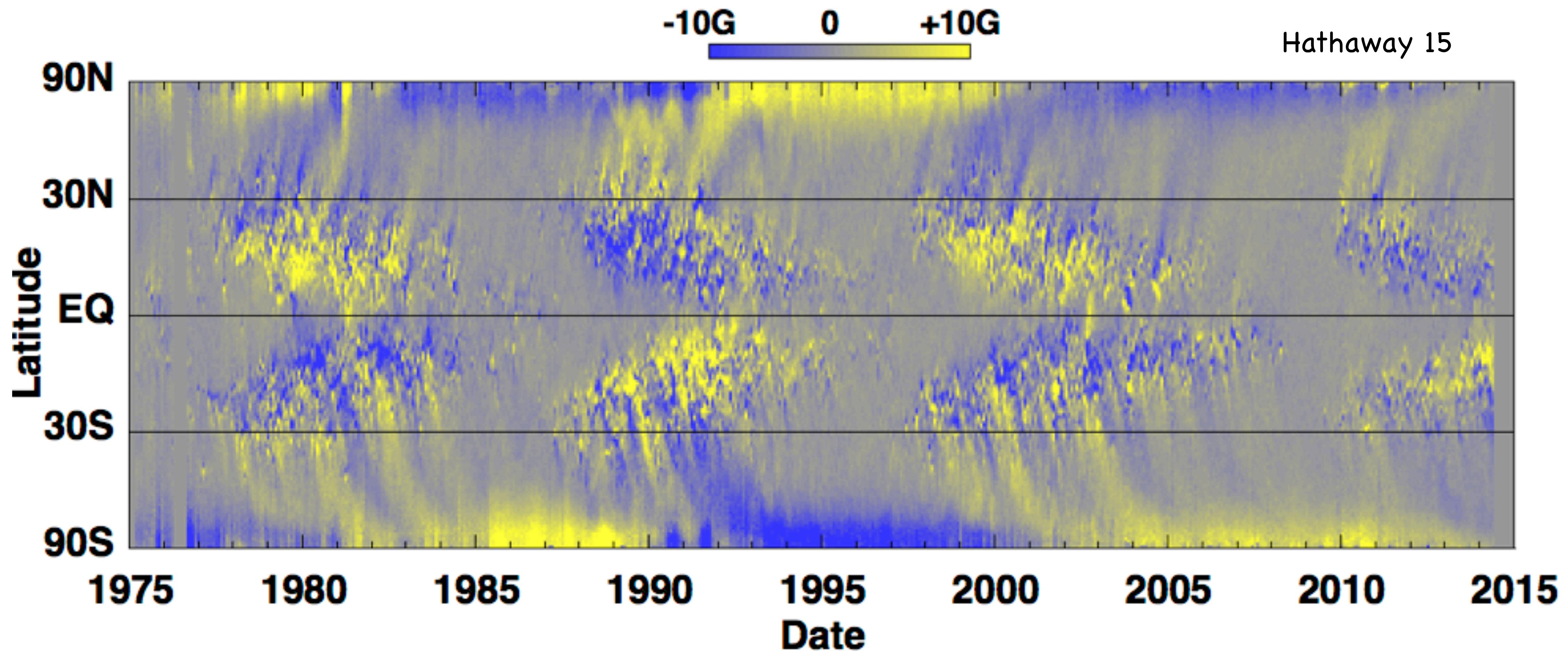


NASA/NICER

- -> 中性子星にも磁場がある！

Polar field reversals in the Sun

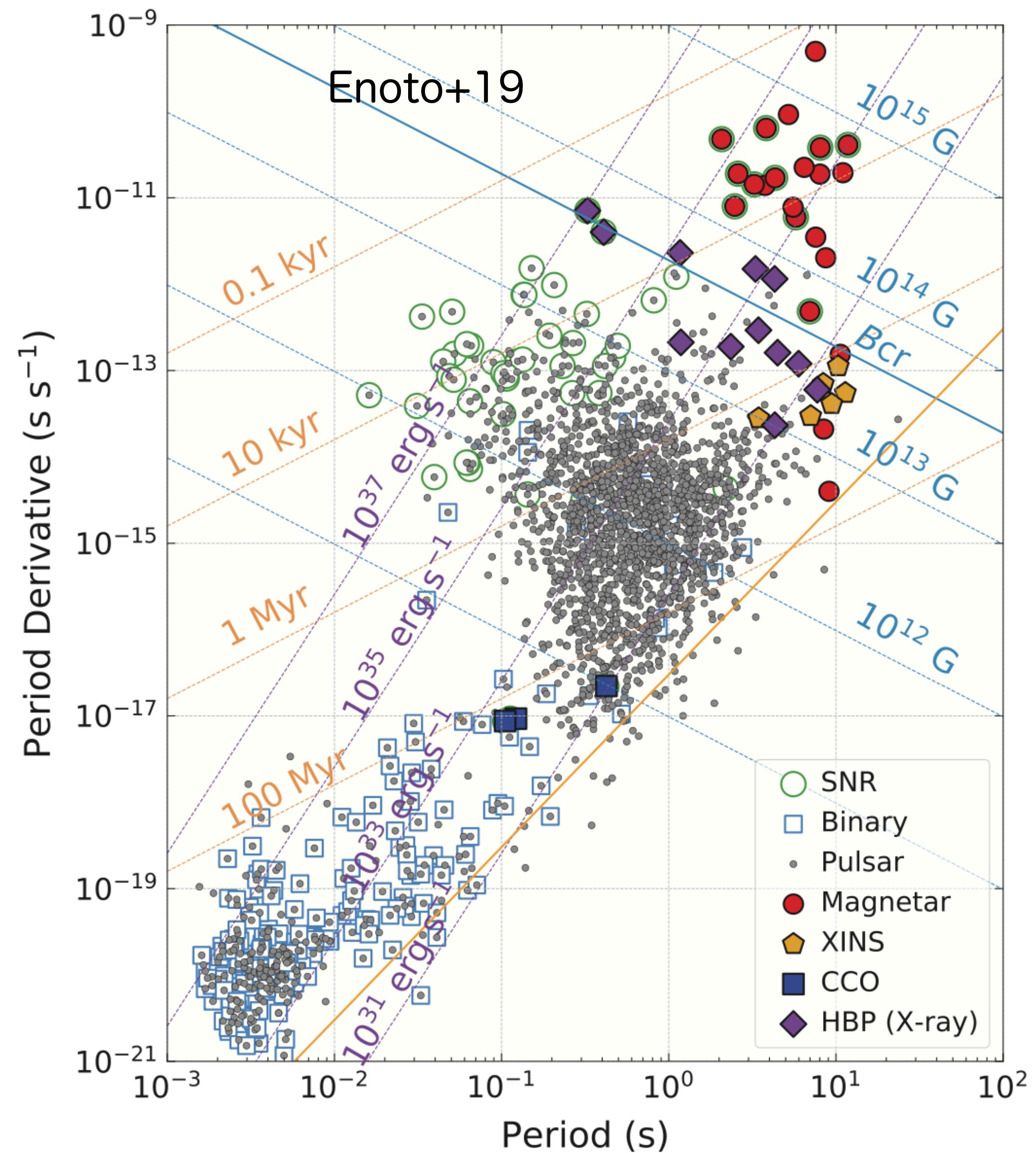
Magnetic Butterfly Diagram



longitudinally averaged radial magnetic field obtained from instruments on Kitt Peak and SOHO

11-year cycle for the polarity of the magnetic field in the sun

磁場強度の多様性



- 多様性：
磁場、回転
- ダイナミクス：
磁場と回転の親和性

マグネター

FORMATION OF VERY STRONGLY MAGNETIZED NEUTRON STARS: IMPLICATIONS FOR GAMMA-RAY BURSTS

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Received 1991 December 23; accepted 1992 March 2

ABSTRACT

Neutron stars with unusually strong magnetic dipole fields, $B_{\text{dipole}} \sim 10^{14}\text{--}10^{15}$ G, can form when conditions for efficient helical dynamo action are met during the first few seconds after gravitational collapse. Such high-field neutron stars, "magnetars," initially rotate with short periods ~ 1 ms, but quickly lose most of their rotational energy via magnetic braking, giving a large energy boost to the associated supernova explosion. Several mechanisms unique to magnetars can plausibly generate large (~ 1000 km s $^{-1}$) recoil velocities. These include magnetically-induced anisotropic neutrino emission, core rotational instability and fragmentation, and/or anisotropic magnetic winds.

Magnetars are relatively difficult to detect because they drop below the radio death line faster than ordinary pulsars, and because they probably do not remain bound in binary systems. We conjecture that their main observational signature is gamma-ray bursts powered by their vast reservoirs of magnetic energy. If they acquire large recoils, most magnetars are unbound from the Galaxy or reside in an extended, weakly bound Galactic corona. There is evidence that the soft gamma repeaters are young magnetars.

Finally, we note that a convective dynamo can also generate a very strong dipole field after the merger of a neutron star binary, but only if the merged star survives for as long as $\sim 10\text{--}100$ ms.

Subject headings: gamma rays: bursts — magnetic fields — pulsars: general — stars: neutron

Millisecond pulsars with extremely strong magnetic fields as a cosmological source of γ -ray bursts

V. V. Usov

Physics Department, Weizmann Institute of Science,
Rehovot 76100, Israel

THE spatial and luminosity distribution of γ -ray bursts as observed by the BATSE instrument on the Compton Gamma Ray Observatory^{1,2} provides support for the revival of the idea^{3,4} that the burst sources are at cosmological distances⁵. I present here a new model for γ -ray bursts at cosmological distances, based on the formation of rapidly rotating neutron stars with surface magnetic fields of the order of 10^{15} G. Such objects could form by the gravitational collapse of accreting white dwarfs with anomalously high magnetic fields in binaries, as in magnetic cataclysmic binaries. Once formed, such rapidly rotating and strongly magnetized neutron stars would lose their rotational kinetic energy catastrophically, on a timescale of seconds or less: rotation of the magnetic field creates a strong electric field, and hence an electron–positron plasma, which I show to be optically thick and in quasi-thermodynamic equilibrium. This plasma flows away from the neutron star at relativistic speeds, and X-ray and γ -ray emission at the photosphere of this relativistic wind may then reproduce the observational characteristics of a γ -ray burst.

The rotational kinetic energy of a neutron star with moment of inertia $I \approx 10^{45}$ g cm² is

$$E_{\text{kin}} = \frac{1}{2} I \Omega^2 \approx 5 \times 10^{52} \text{ erg} \text{ for } \Omega \approx 10^4 \text{ s}^{-1} \quad (1)$$

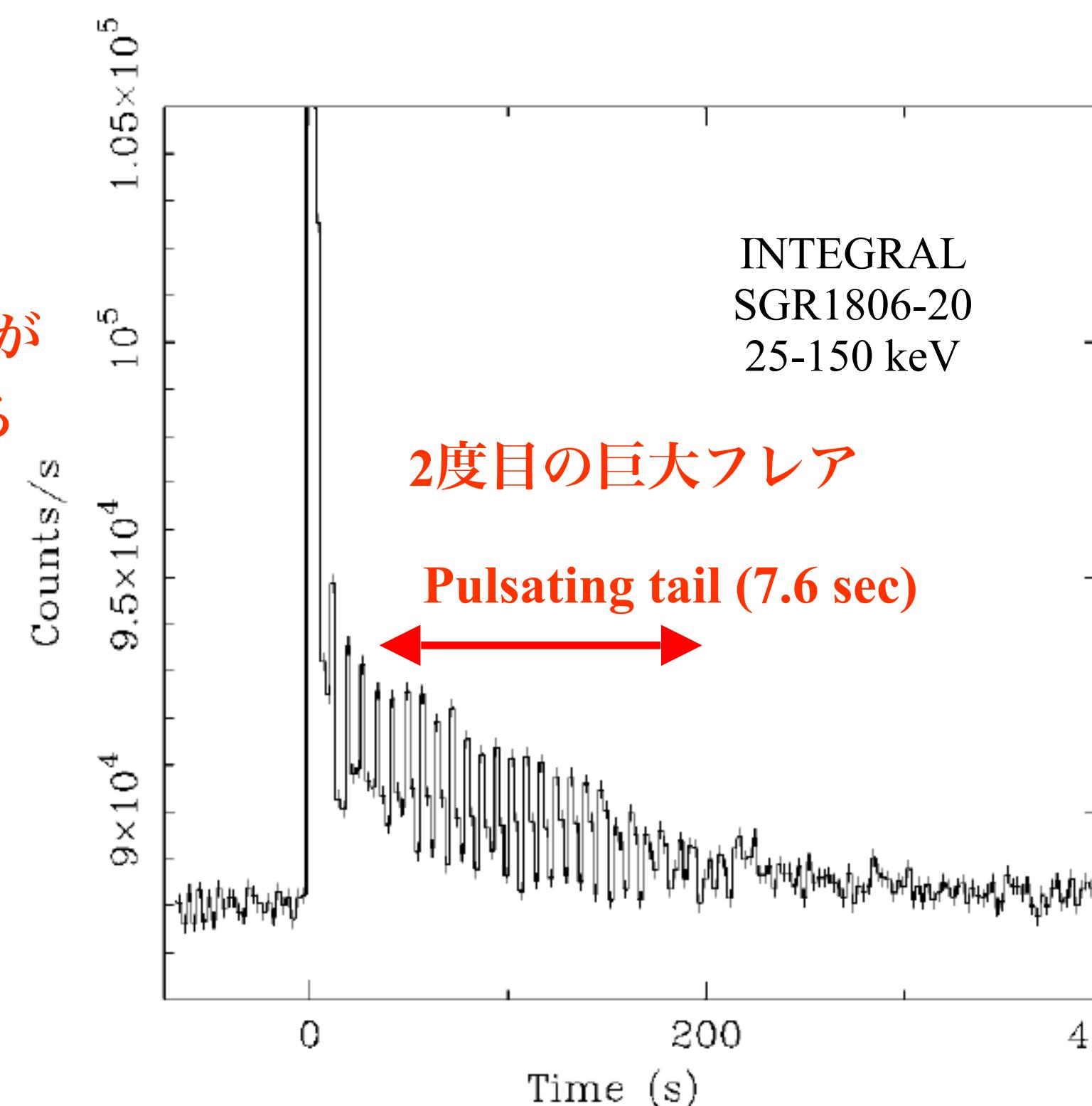
which is enough to explain the total energy released during the burst if the γ -ray bursts are of cosmological origin^{3–5}.

マグネター巨大フレア:Soft Gamma-ray Repeater (SGR)

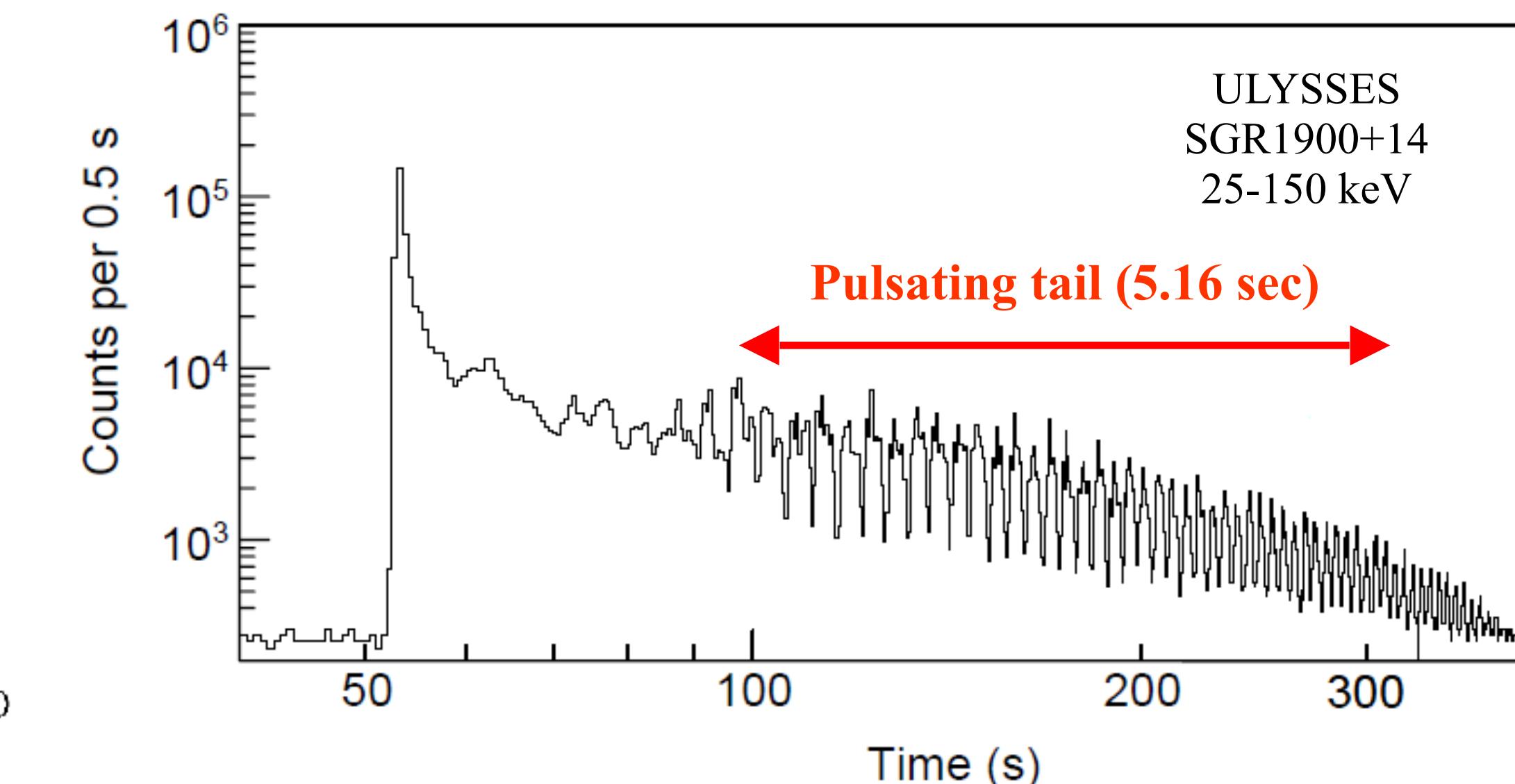
- 爆発のタイムスケール：数100 ms (初期のスパイク)
- 爆発のエネルギー： $\sim 10^{44} - 10^{46}$ erg

→磁場のエネルギーの解放

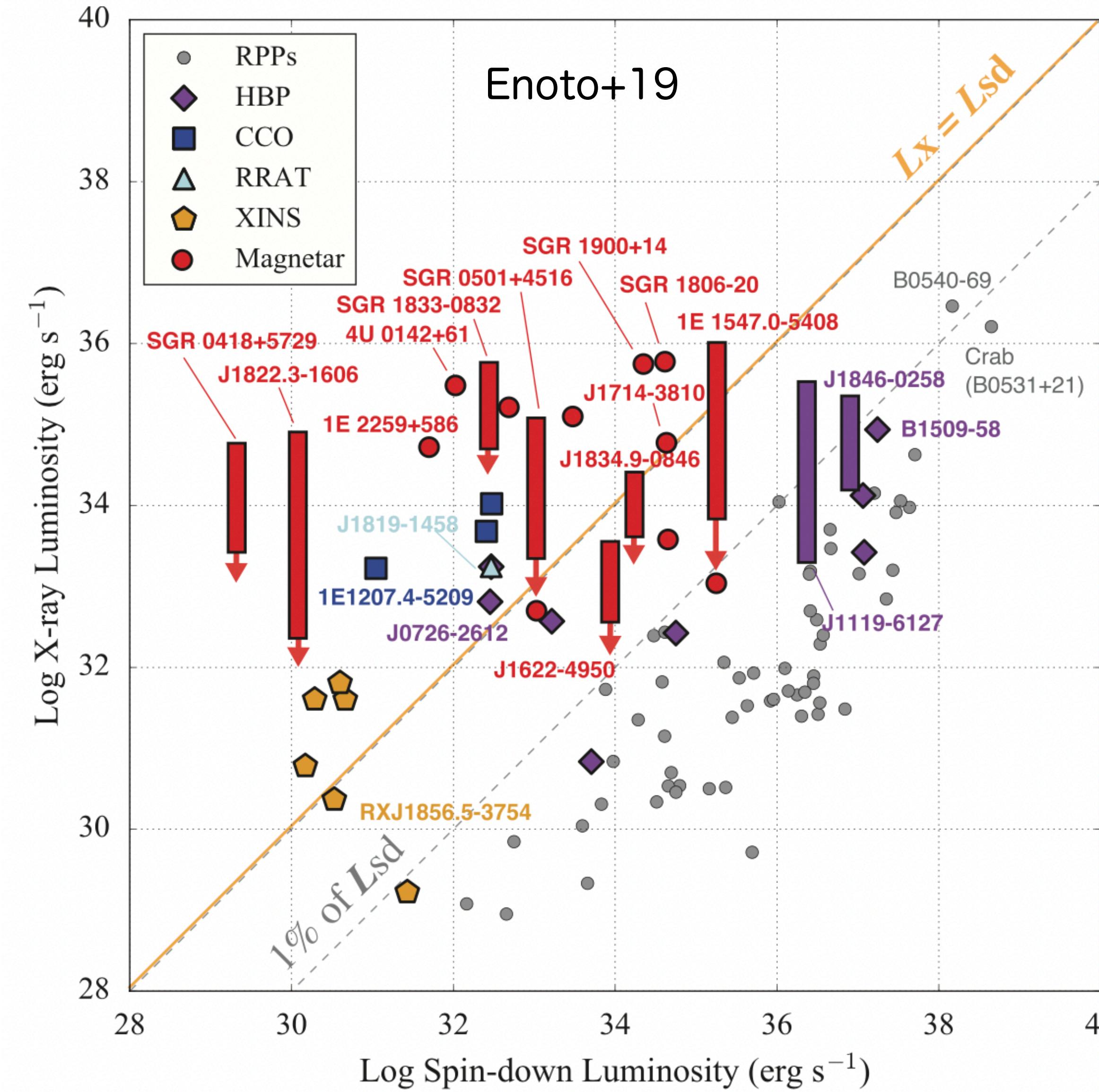
マグネターの研究が
一気に盛り上がる



- Pulse tail phase:
pulseの周期と回転周期が同じ



Anomalous X-ray Pulsar (AXP)



Rotational Energy:

$$E_{\text{rot}} = \frac{1}{2} I \Omega^2$$

Spin-Down Luminosity:

$$L_{\text{sd}} = \dot{E}_{\text{rot}} = I \Omega \dot{\Omega} = I \frac{(2\pi)^2}{P^3} \dot{P}$$

$\sim 10^{33} \text{ erg/s} < L_x$

$$\Omega = \frac{2\pi}{P}, \quad \dot{\Omega} = -\frac{2\pi}{P^2} \dot{P}, \quad |\Omega \dot{\Omega}| = \frac{(2\pi)^2}{P^3} \dot{P}$$

Spin-Down Luminosityでは説明できない

-> Anomalous X-ray Pulsar

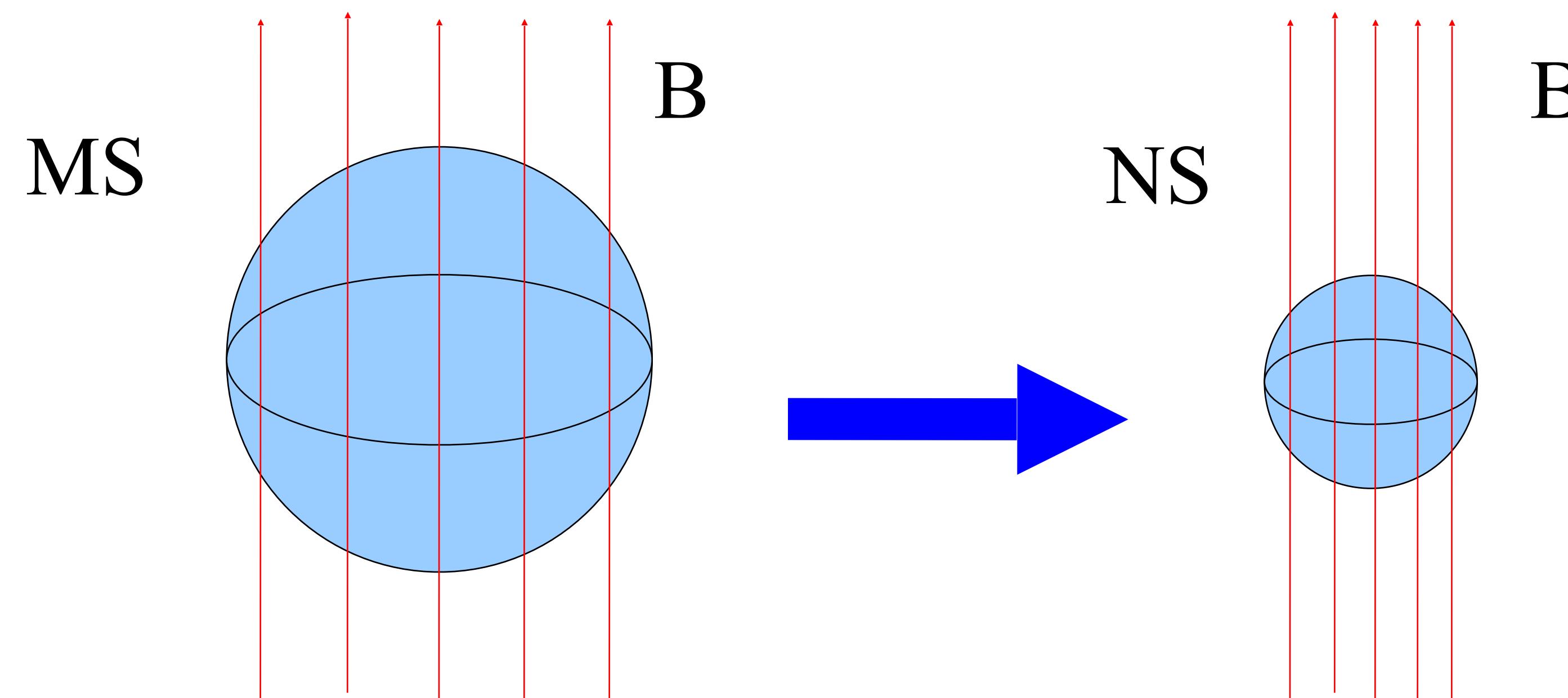
AXPでSGRのようなバーストを起こすものが見つかるようになった

マグネターの形成シナリオ

possible formation scenarios of magnetar

- turbulent dynamo amplification in a rapidly rotating proto-neutron star
(Thompson+93)
- fossil field hypothesis (magnetic flux conservation) (Ferrario+06)
- Chiral Plasma Instability (Yamamoto 16)
 ← - magnetic helicity

Fossil fields



$$B_{\text{PNS}} \sim 10^{15} \text{ G} \left(\frac{B_{0,r=1000\text{km}}}{10^{12}\text{G}} \right) \left(\frac{30\text{km}}{r_{\text{PNS}}} \right)$$

B

コアの磁場が強いなら
magnetic brakingが効いて
角運動量が抜かれる可能性

- 貫く磁束が大質量星とNSで等しいとおいて
NSの磁場を見積もると

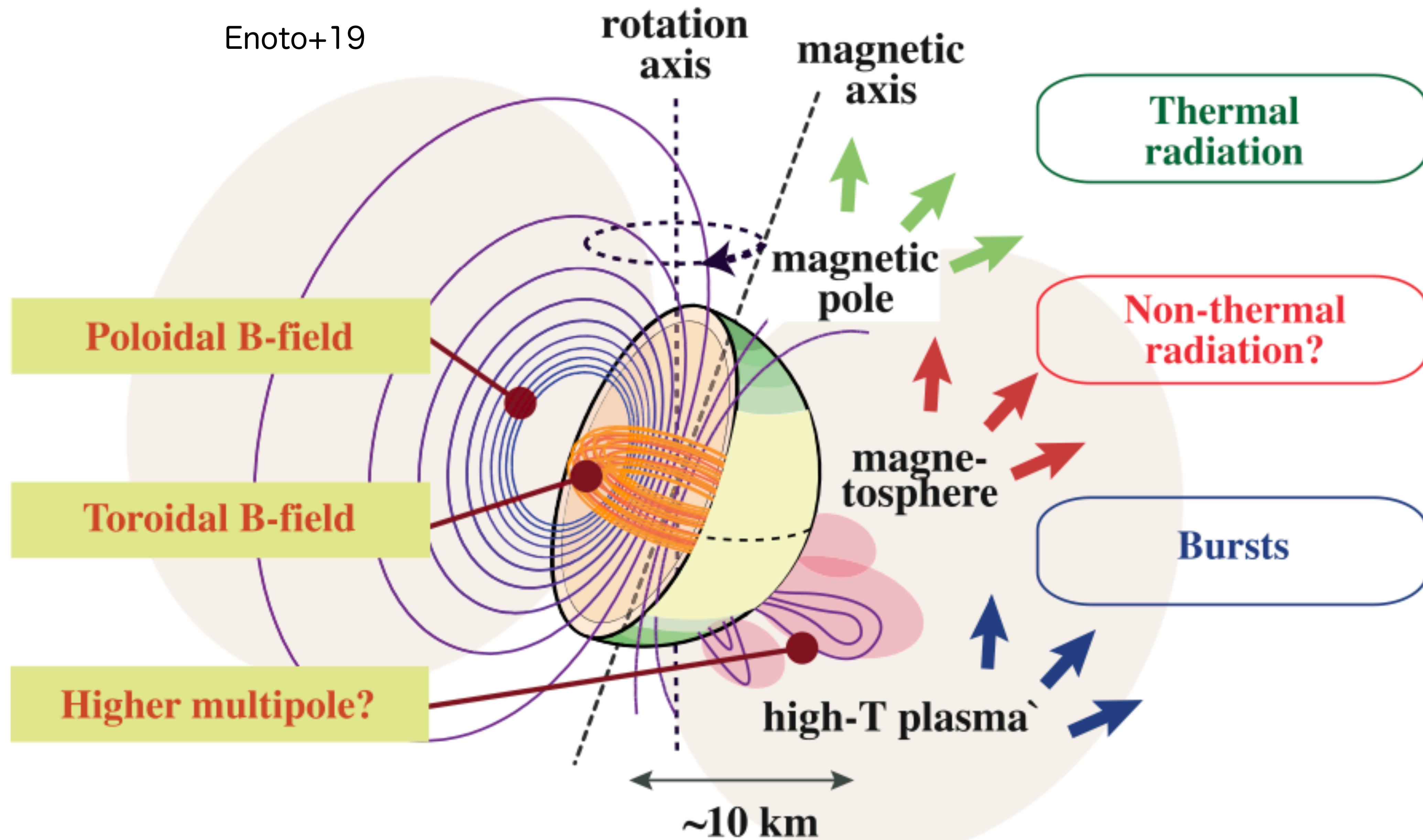
$$R = 4 \times 10^{11} \text{ cm}, B = 10^4 \text{ G}, R_{\text{NS}} = 10^6 \text{ cm}$$

$$\Phi = B \times \pi R^2 = 5 \times 10^{27} \text{ G cm}^2 = B_{\text{NS}} \times \pi R_{\text{NS}}^2$$

$$\rightarrow B_{\text{NS}} = 5 \times 10^{15} \text{ G}$$

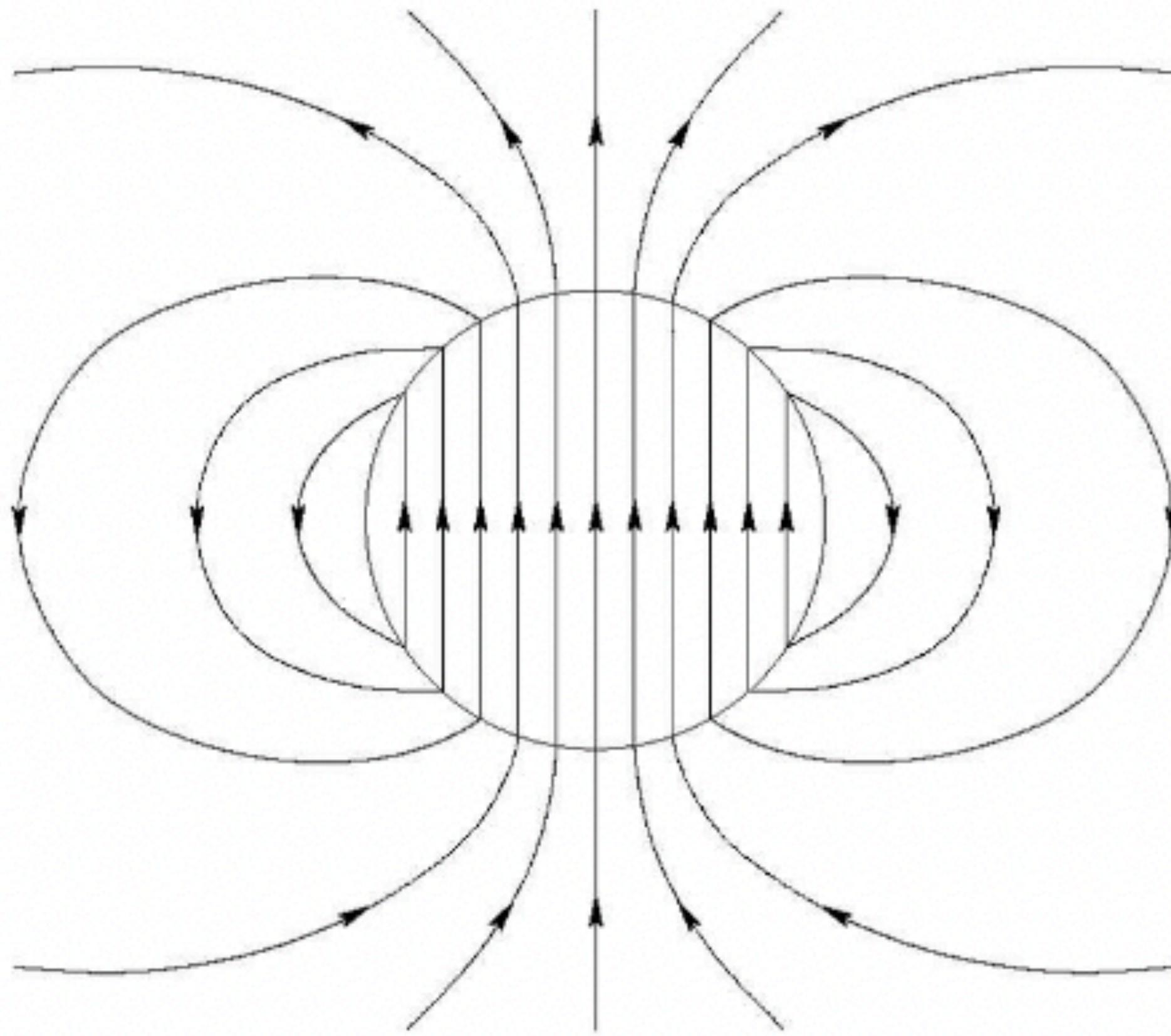
magnetic helicityの必要性

Enoto+19

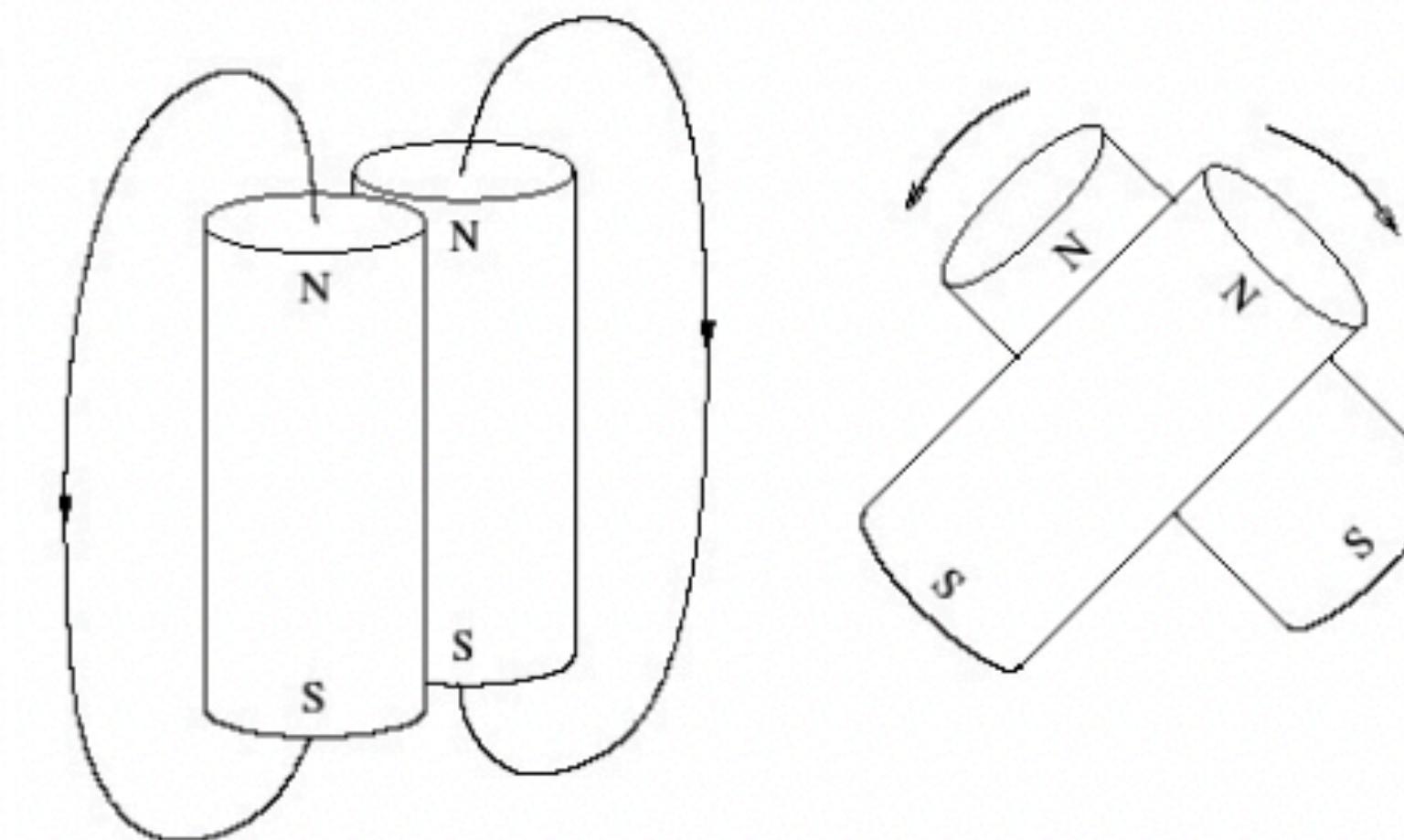


magnetic helicityの必要性

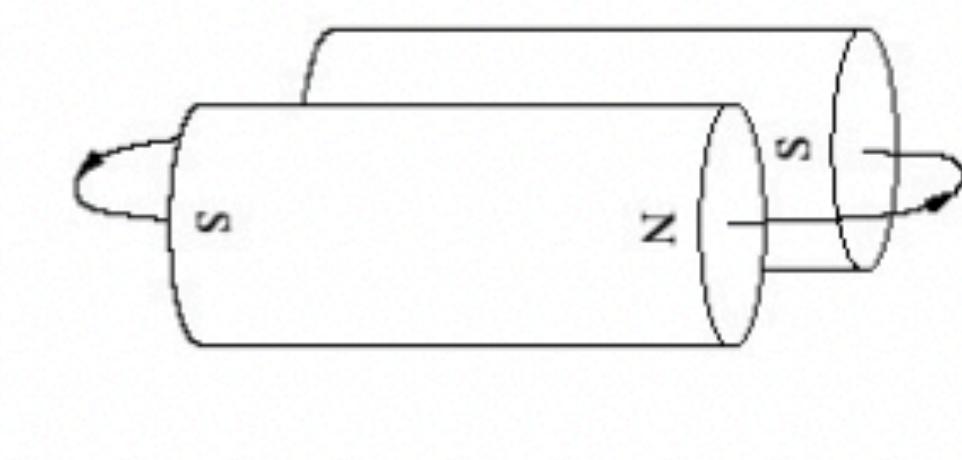
Flowers 77, Spruit 08



pure poloidalは不安定



エネルギー的に高い

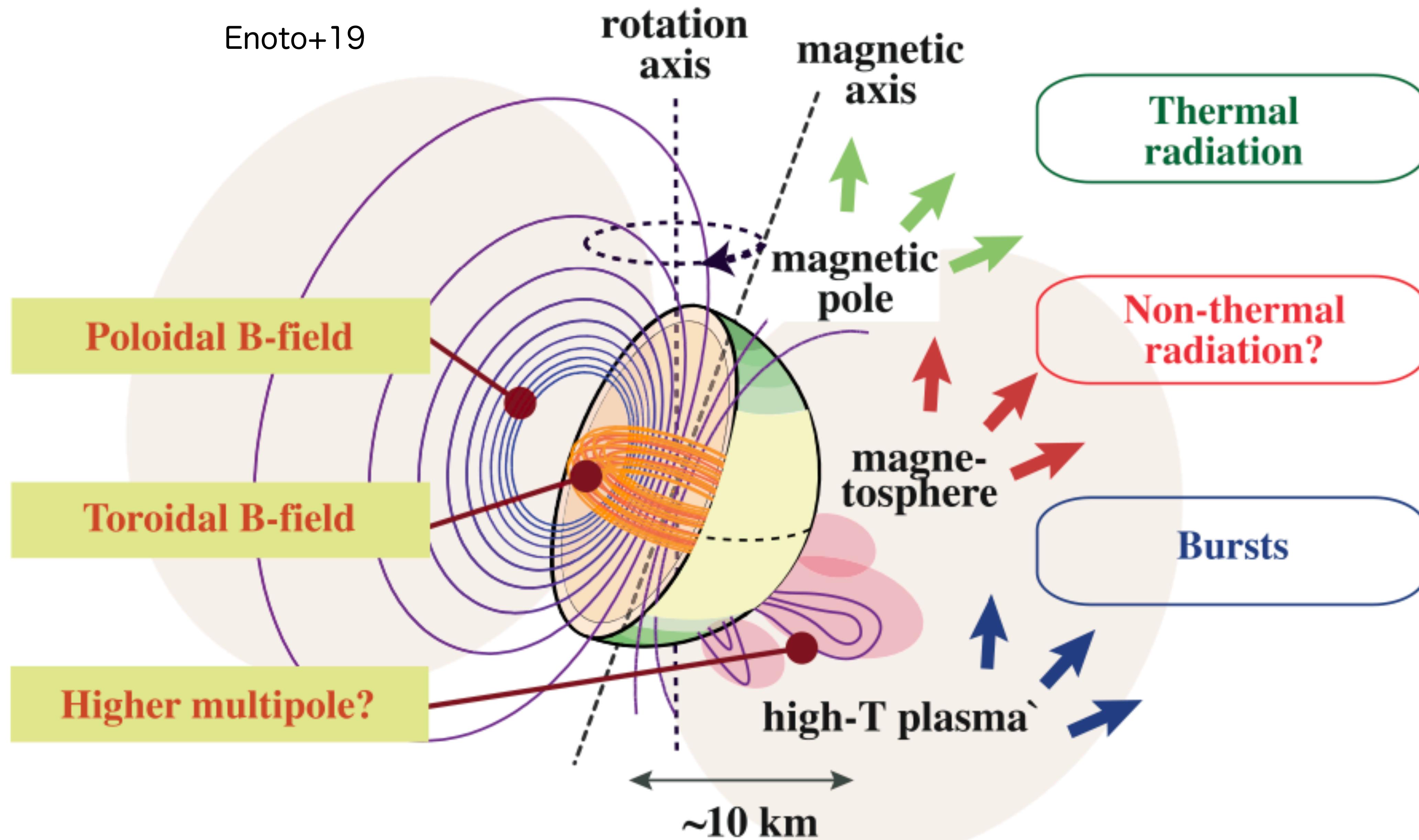


エネルギー的に低い

- -> 不安定

magnetic helicityの必要性

Enoto+19



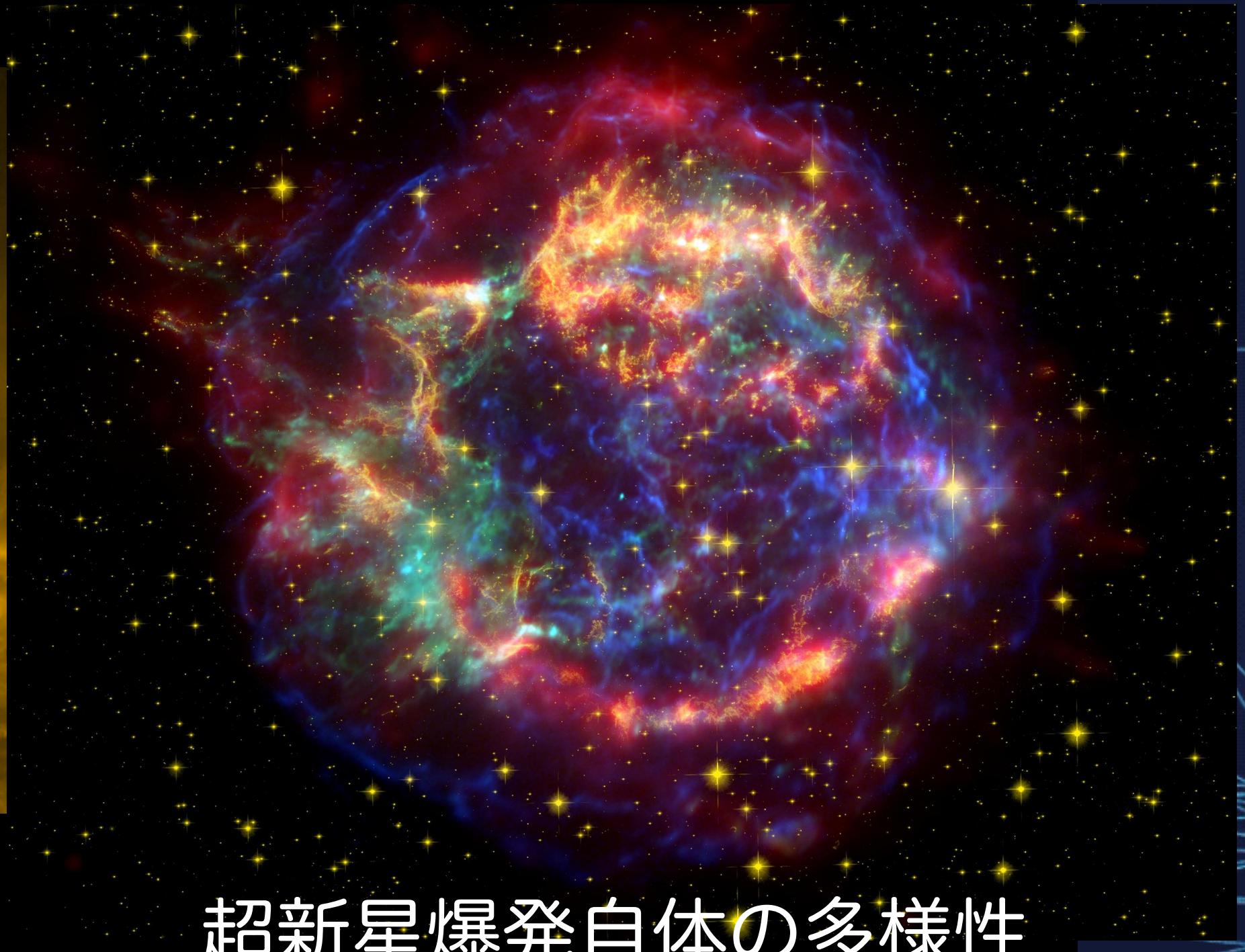
ミッシングリンクとしての超新星爆発フェーズ

Sun

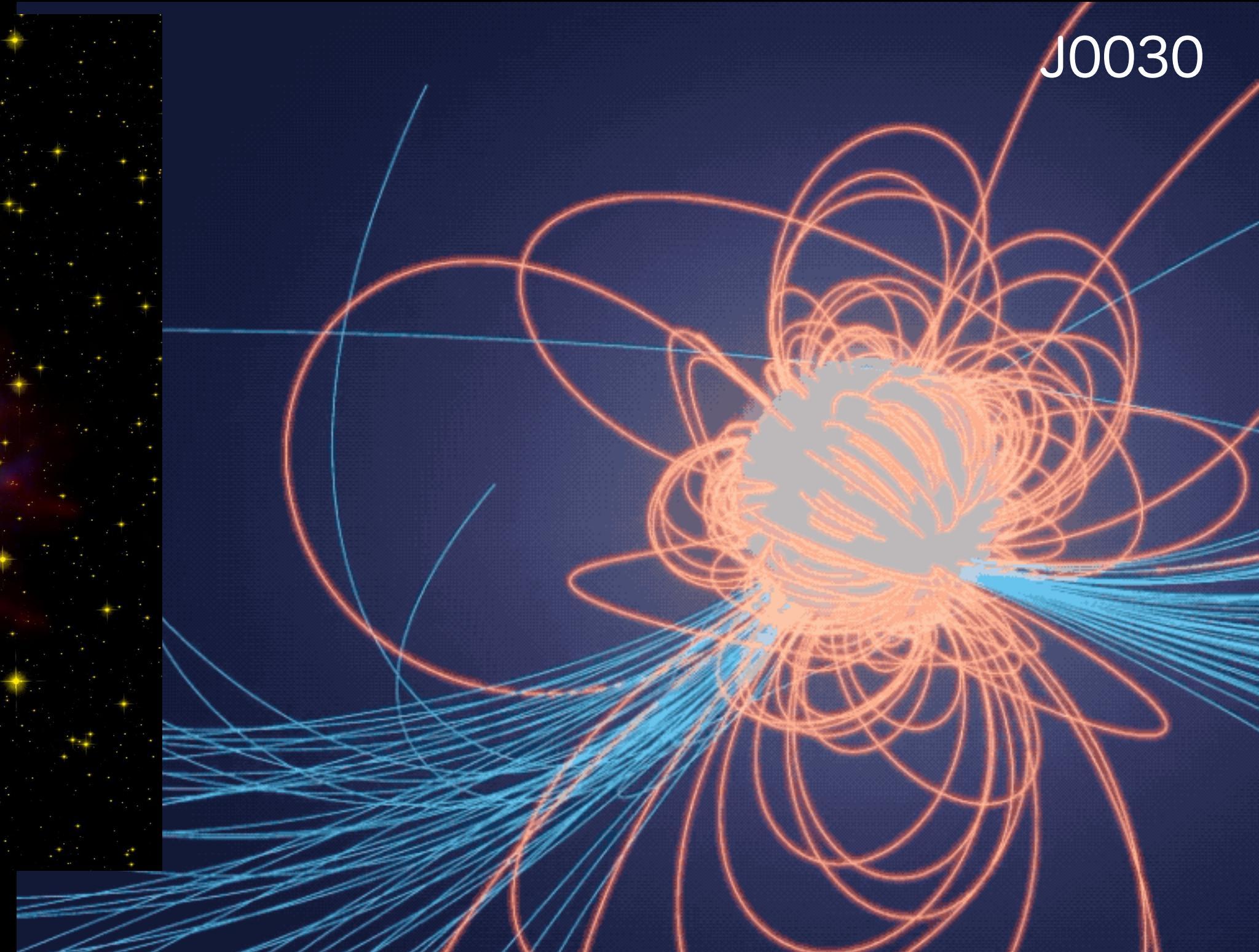


NASA/SDO

Cassiopeia A



超新星爆発自体の多様性



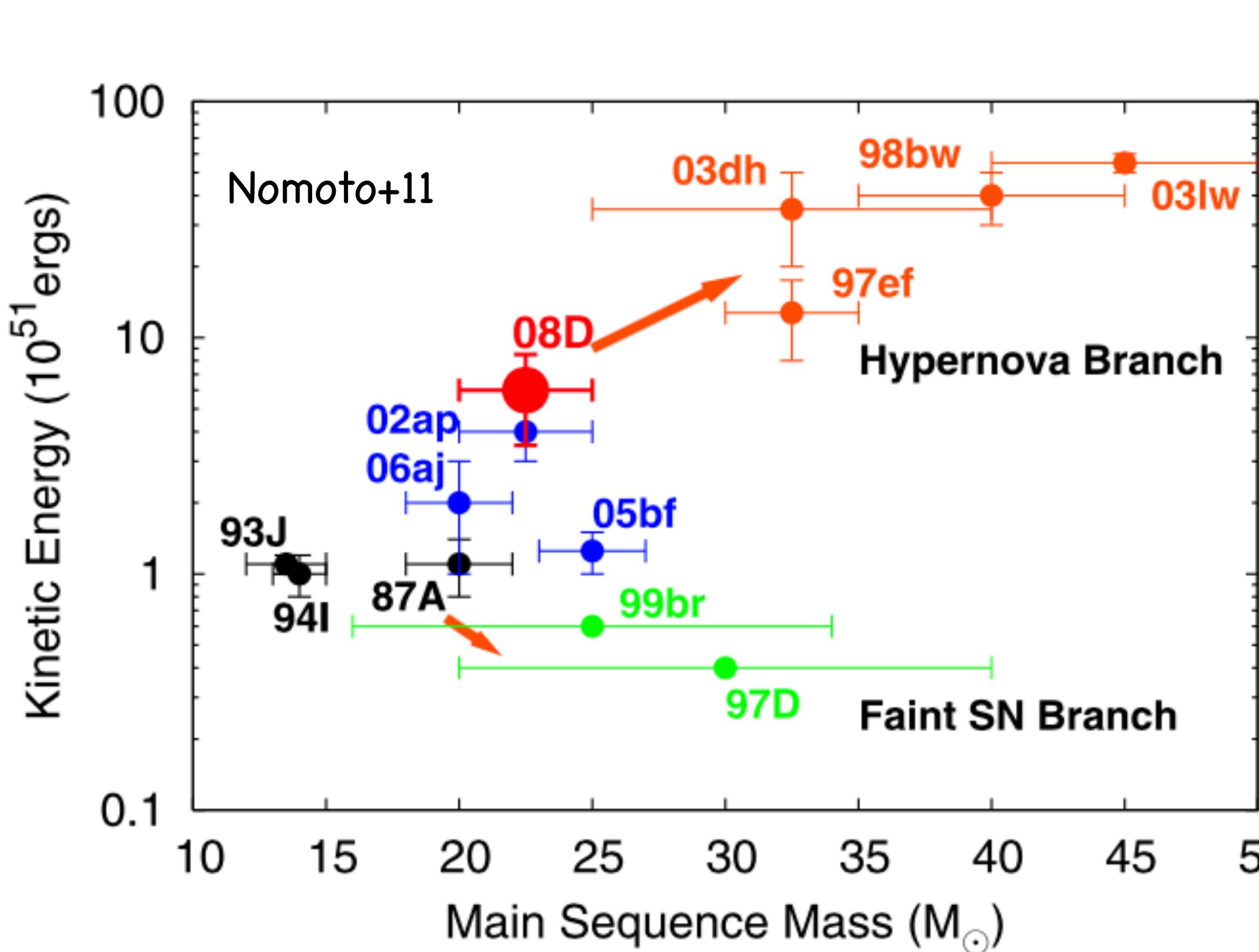
NASA/NICER

恒星には磁場がある！ - ->
十分な理解に至っていない！！

?

- -> 中性子星にも磁場がある！
多様性

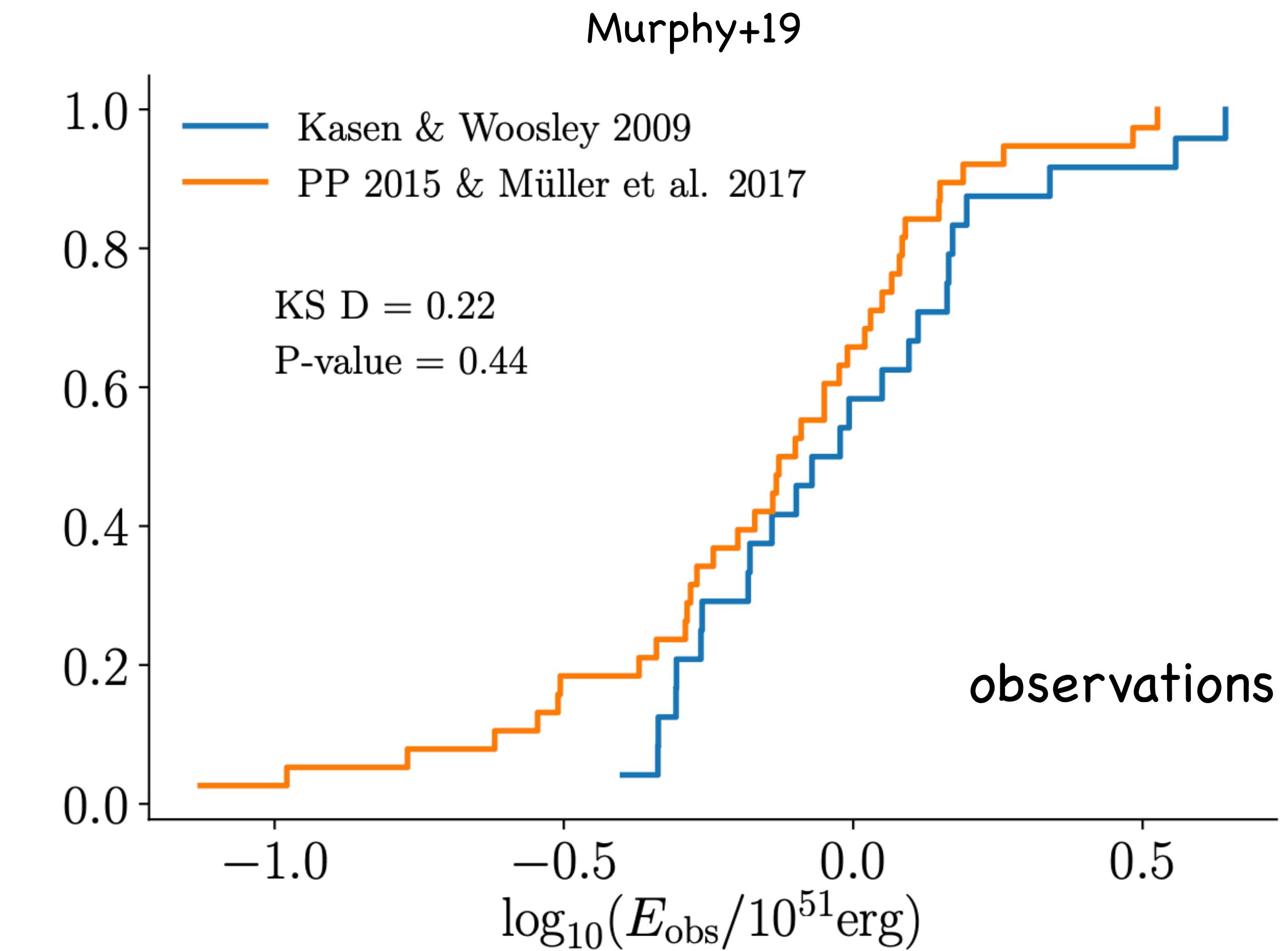
Energetic subclass of supernovae



Hypernovae:

Kinetic energy is 10 times large than
That of canonical CCSNe.

Does B-field support the explosion?



重力崩壊型超新星爆発 (Core-collapse supernova)

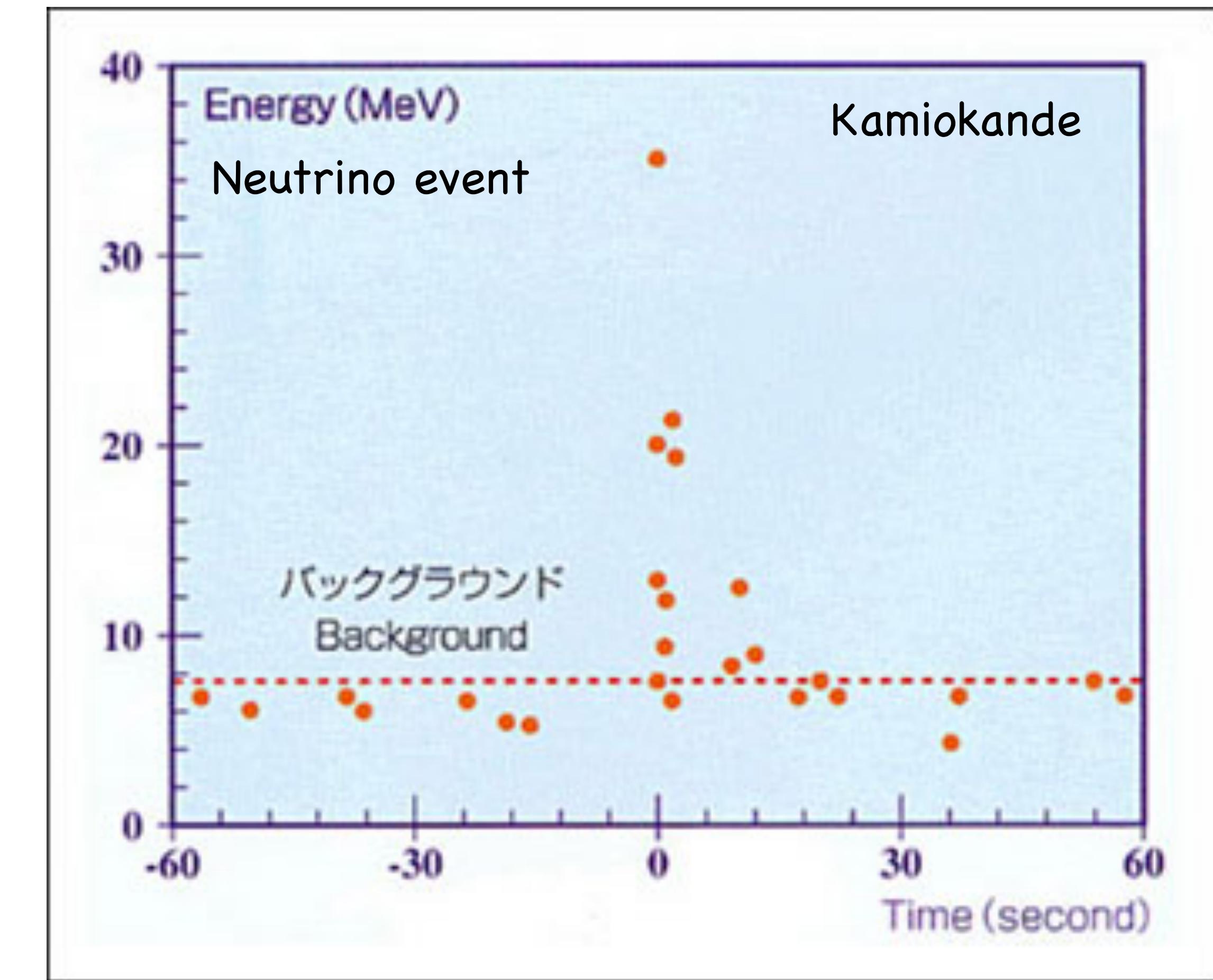
爆発エネルギー : 10^{51} erg

爆発メカニズム : 未解明



David Malin / Australian Astronomical Observatory

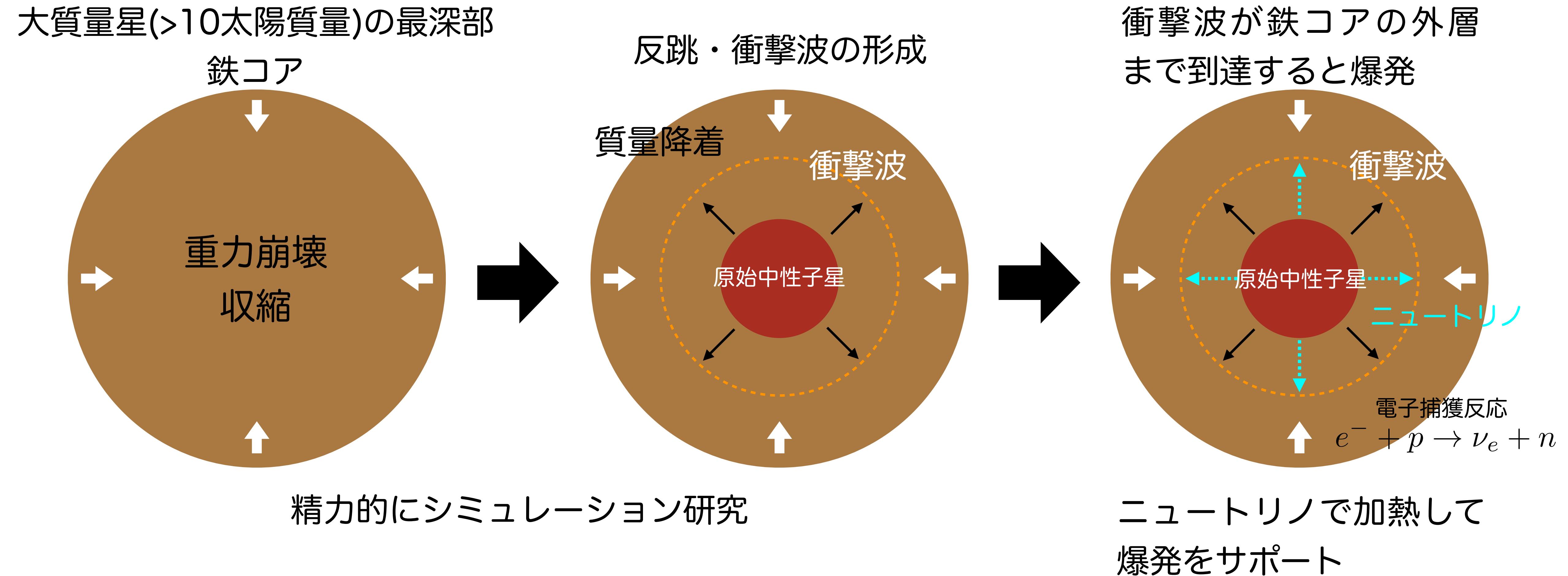
- SN 1987A
Large Magellanic Cloud (49 kpc $\sim 16 \times 10^4$ light years)
- Betelgeuse (possibly in 10^5 years, 168 pc)



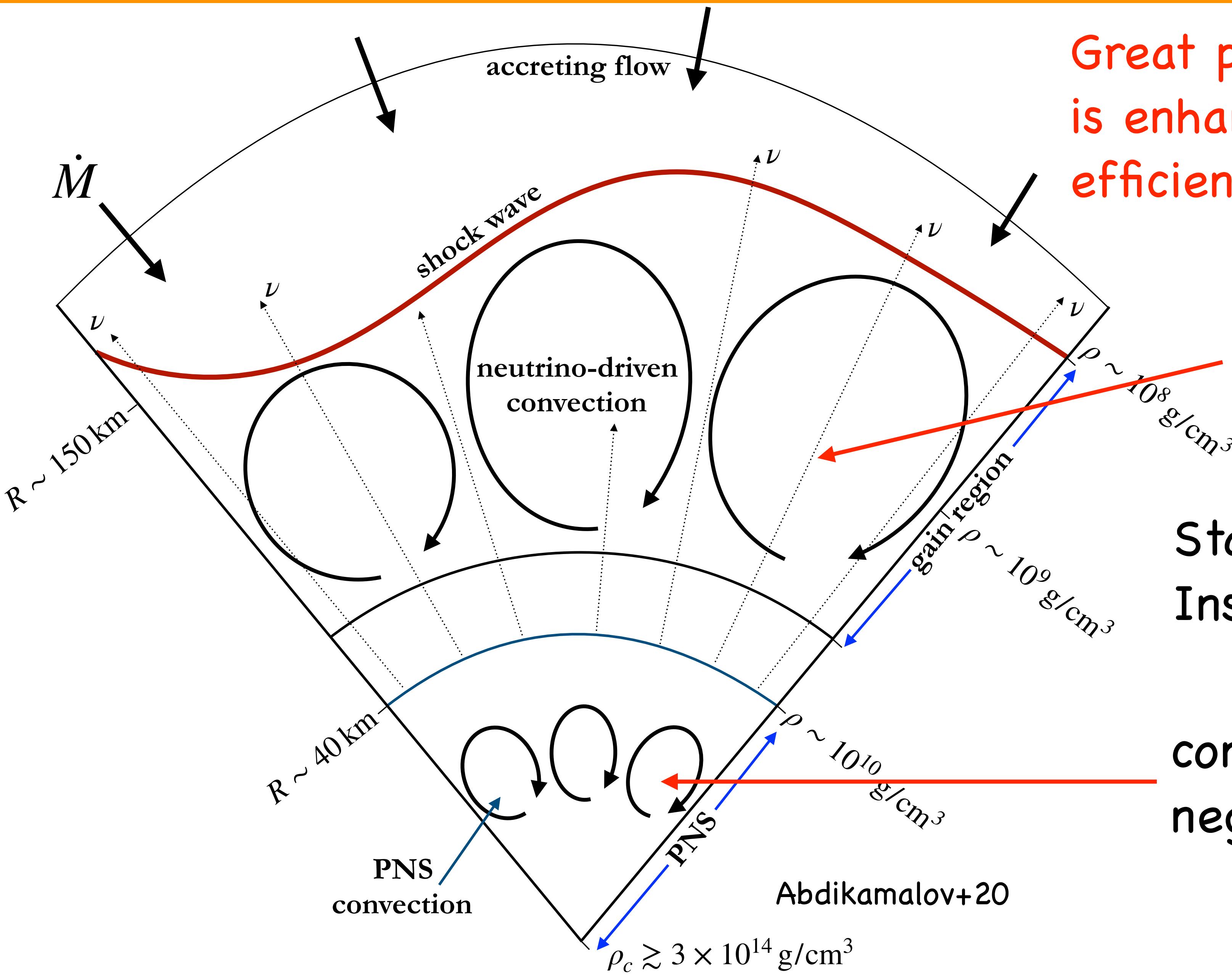
http://www-sk.icrr.u-tokyo.ac.jp/sk/_images/photo/sk/shinsei_gazou02.jpg

- First detection of neutrinos coming from outside our Galaxy
- a possible candidate for detection of GWs

超新星爆発におけるニュートリノ加熱



Multi-D effect: convection and hydrodynamic instability



Great progress of CCSN simulations
is enhancement of neutrino-heating
efficiency by non-radial flows.

convection due to negative
entropy gradient

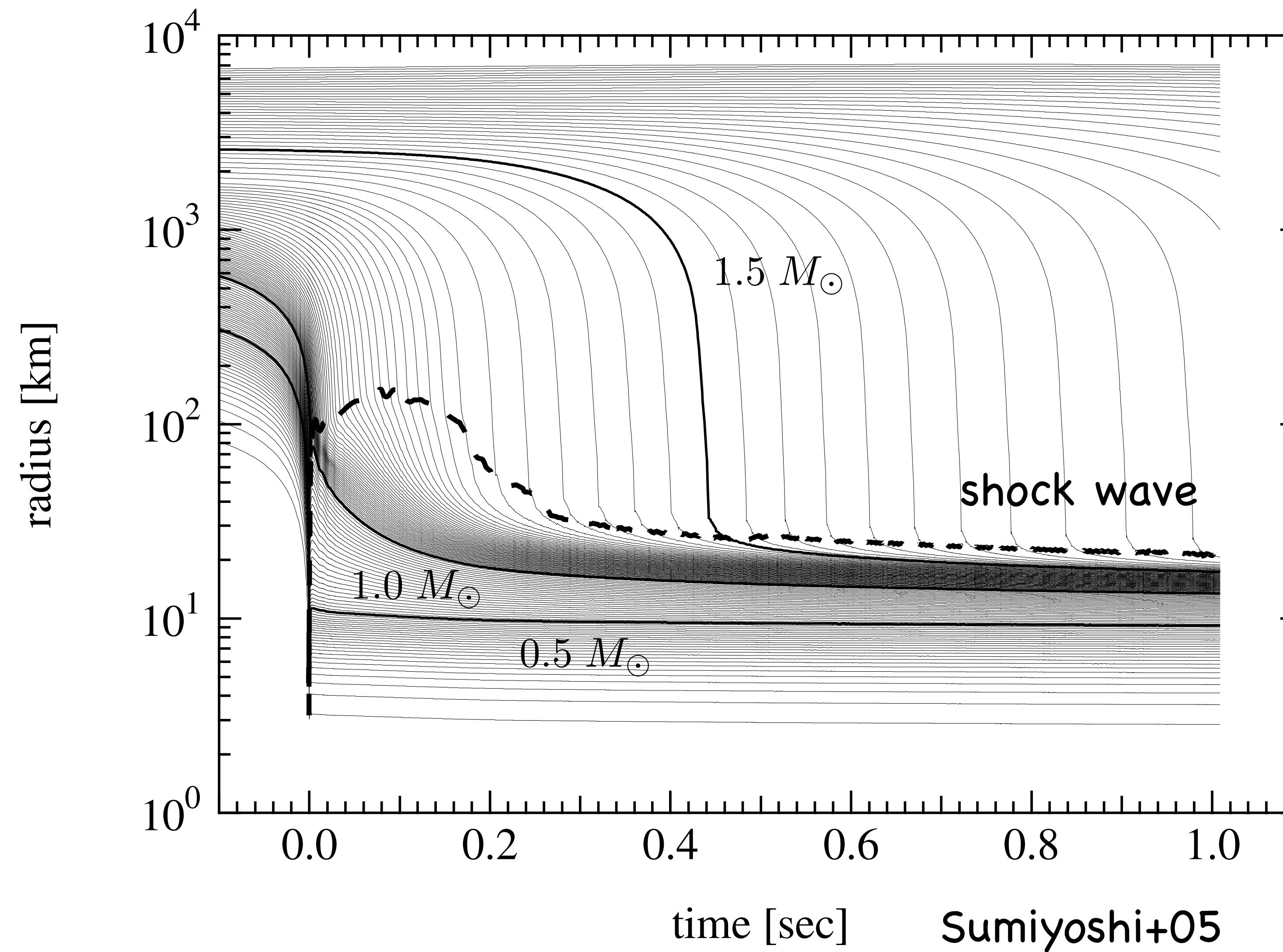
Standing Accretion shock
Instability (SASI, Blondin+03)

convection due to
negative lepton gradient

Abdikamalov+20

1D calculation

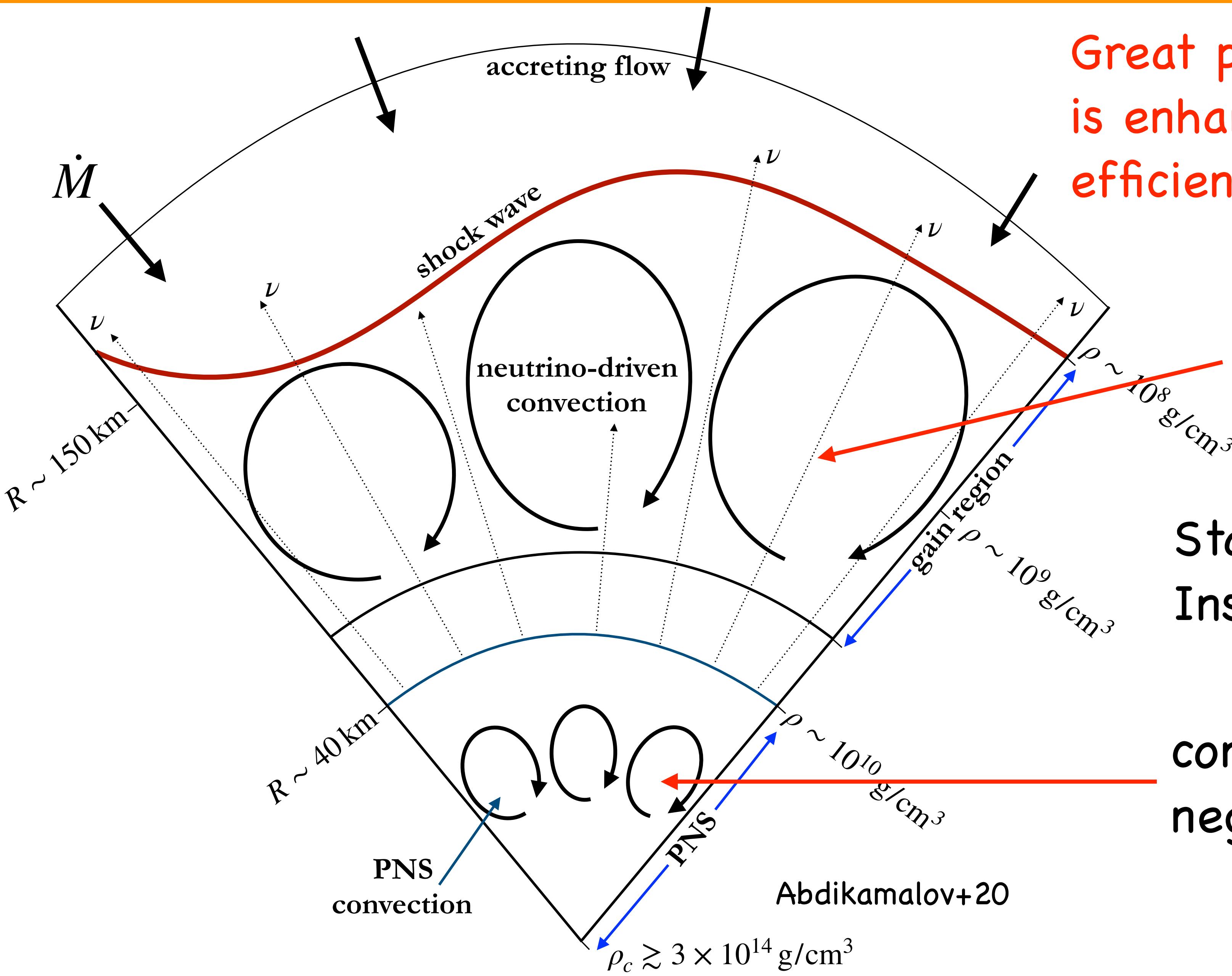
Time evolution of mass shells and shock wave



time [sec]

Sumiyoshi+05

Multi-D effect: convection and hydrodynamic instability



Great progress of CCSN simulations
is enhancement of neutrino-heating
efficiency by non-radial flows.

convection due to negative
entropy gradient

Standing Accretion shock
Instability (SASI, Blondin+03)

convection due to
negative lepton gradient

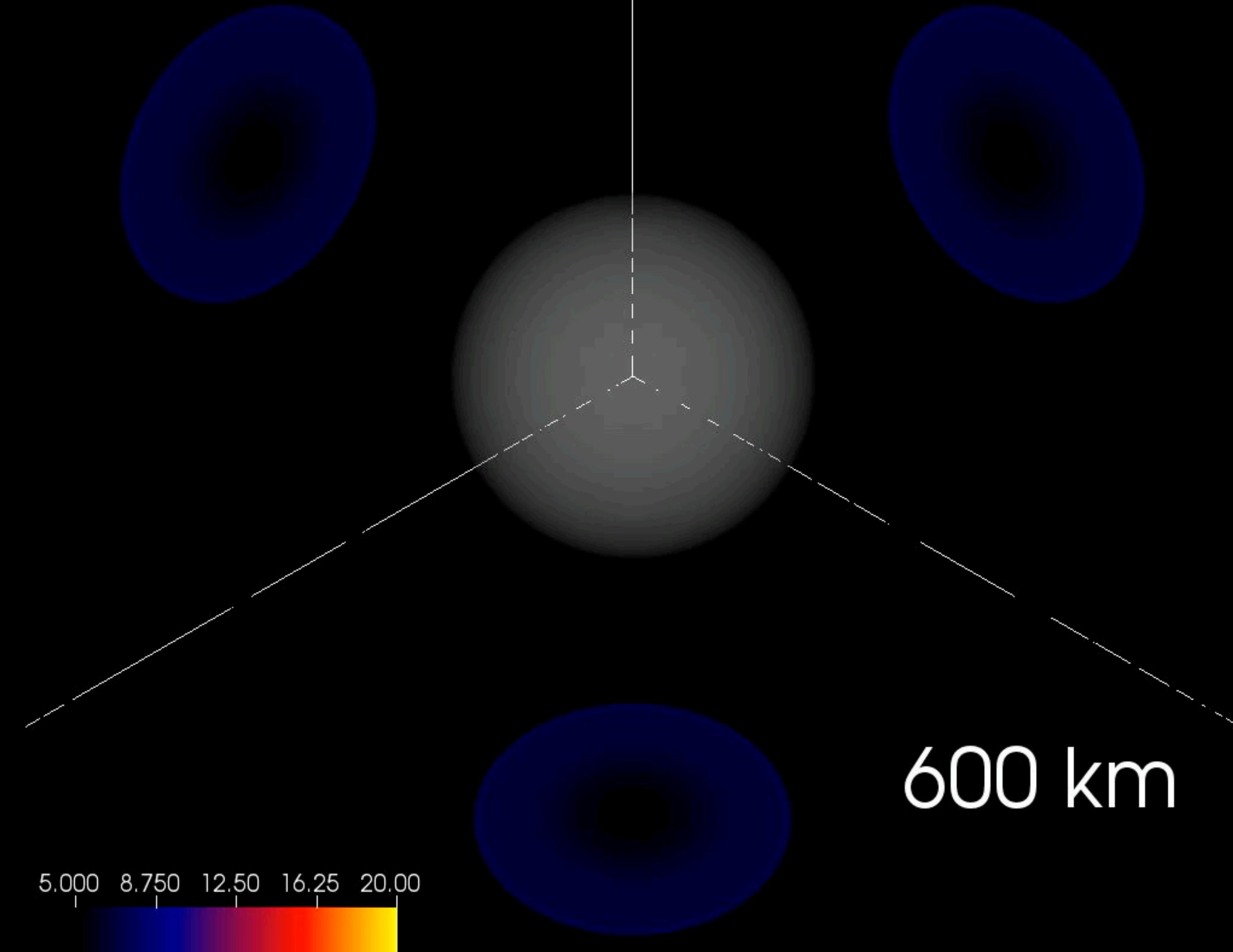
Abdikamalov+20

s27.0B12PPM5

12 ms

三次元超新星爆発
シミュレーション

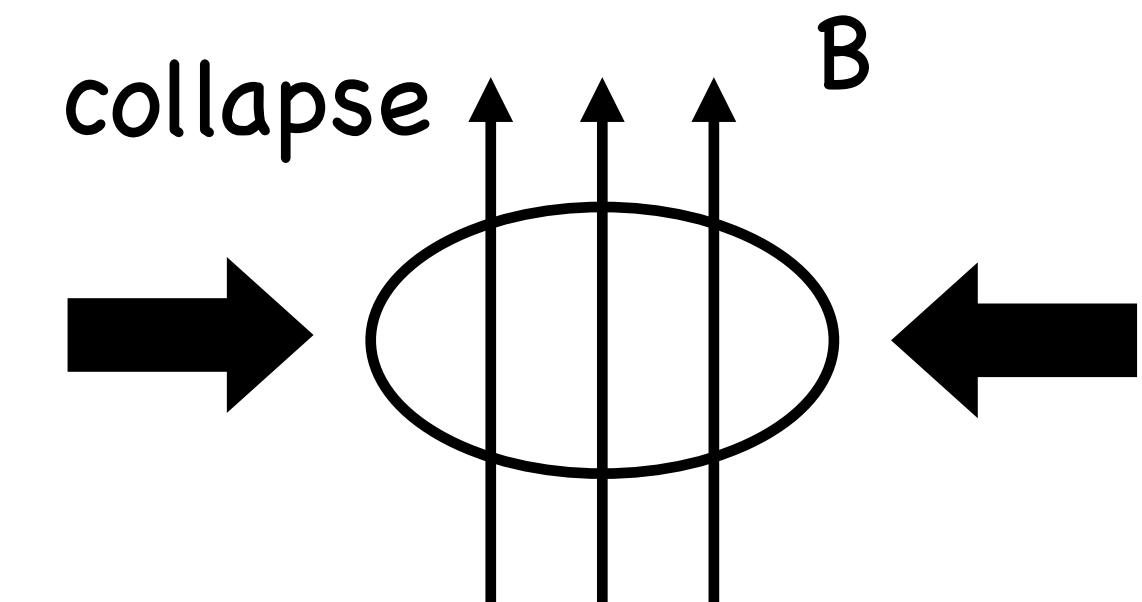
JM+22



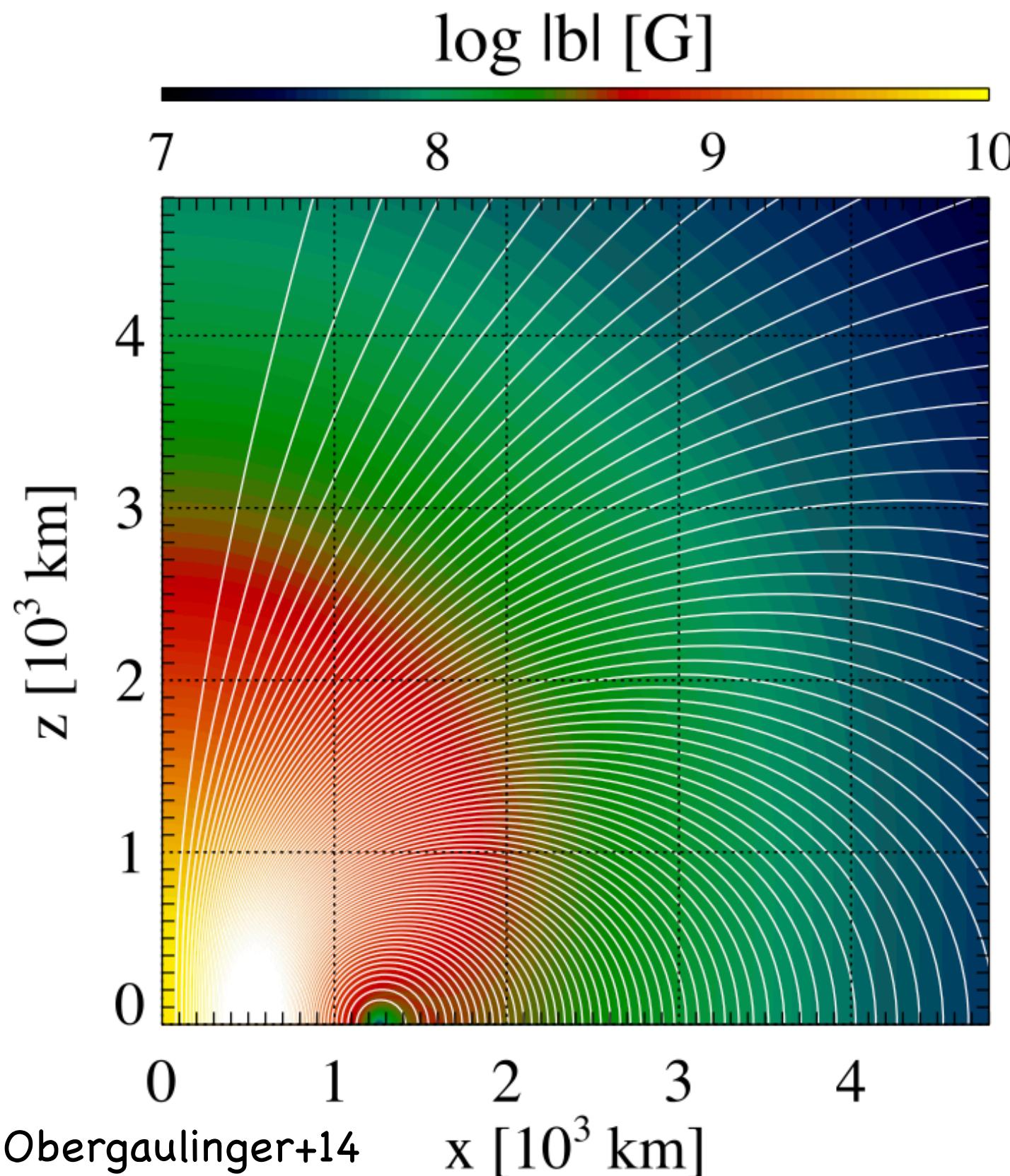
Initial condition of B-field

possible formation scenarios of magnetar

- turbulent dynamo amplification in a rapidly rotating PNS (Thompson+93)
- fossil field hypothesis (magnetic flux conservation) (Ferrario+06)



- initial B-field: 10^{10} (weak field model), 10^{12} (strong field model) G



magnetic flux conservation: $B_{\text{PNS}} \sim 10^{15} \text{ G} \left(\frac{B_{0,r=1000\text{km}}}{10^{12}\text{G}} \right) \left(\frac{30\text{km}}{r_{\text{PNS}}} \right)^2$

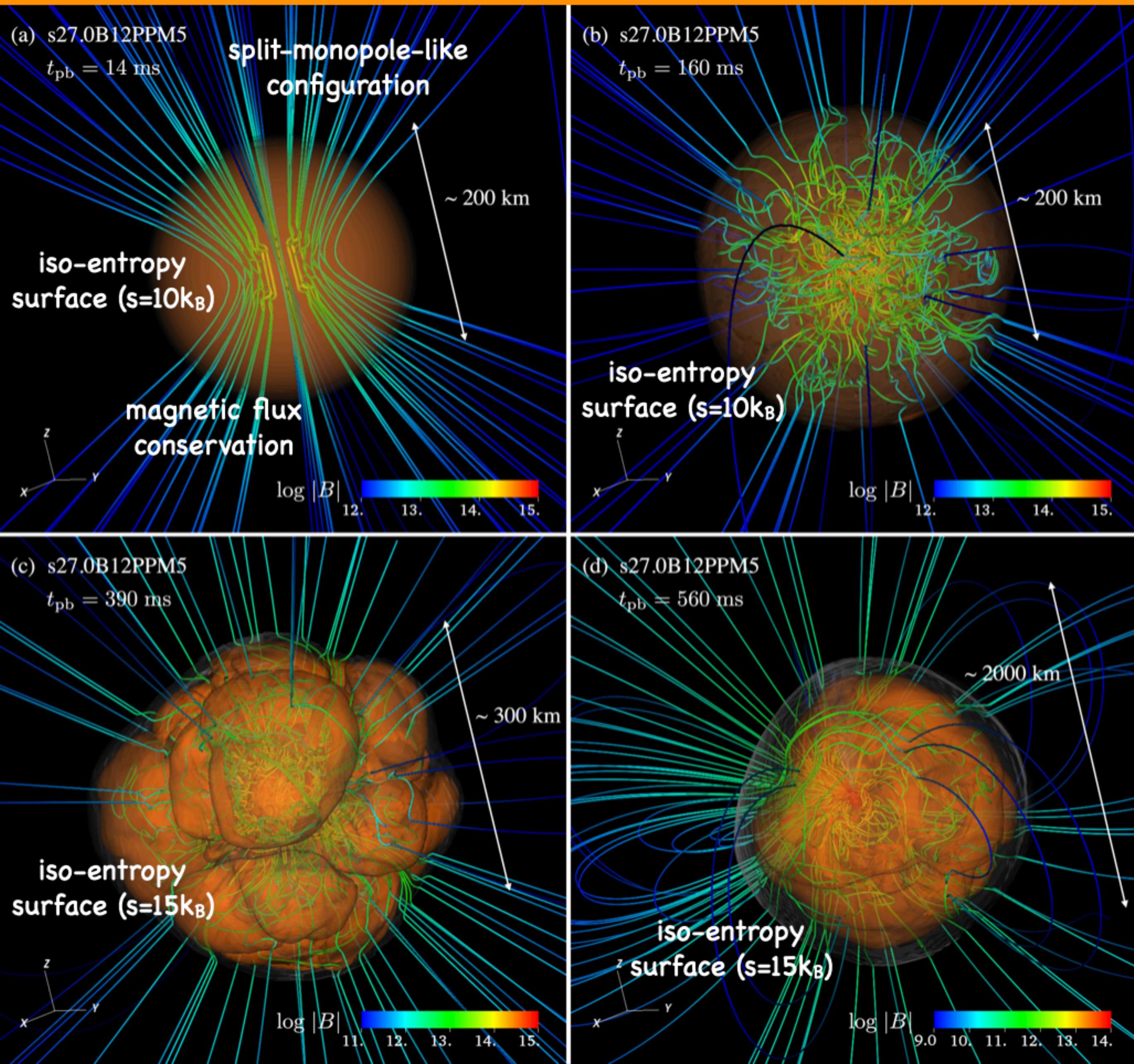
10^{12} G (strong field model): $\rightarrow 10^{15}$ G ($r < 30\text{km}$) $<--$ magnetar class

10^{10} G (weak field model): $\rightarrow 10^{13}$ G ($r < 30\text{km}$)

→ The impact of the magnetic field on the explosion
in our weak field model is passive.

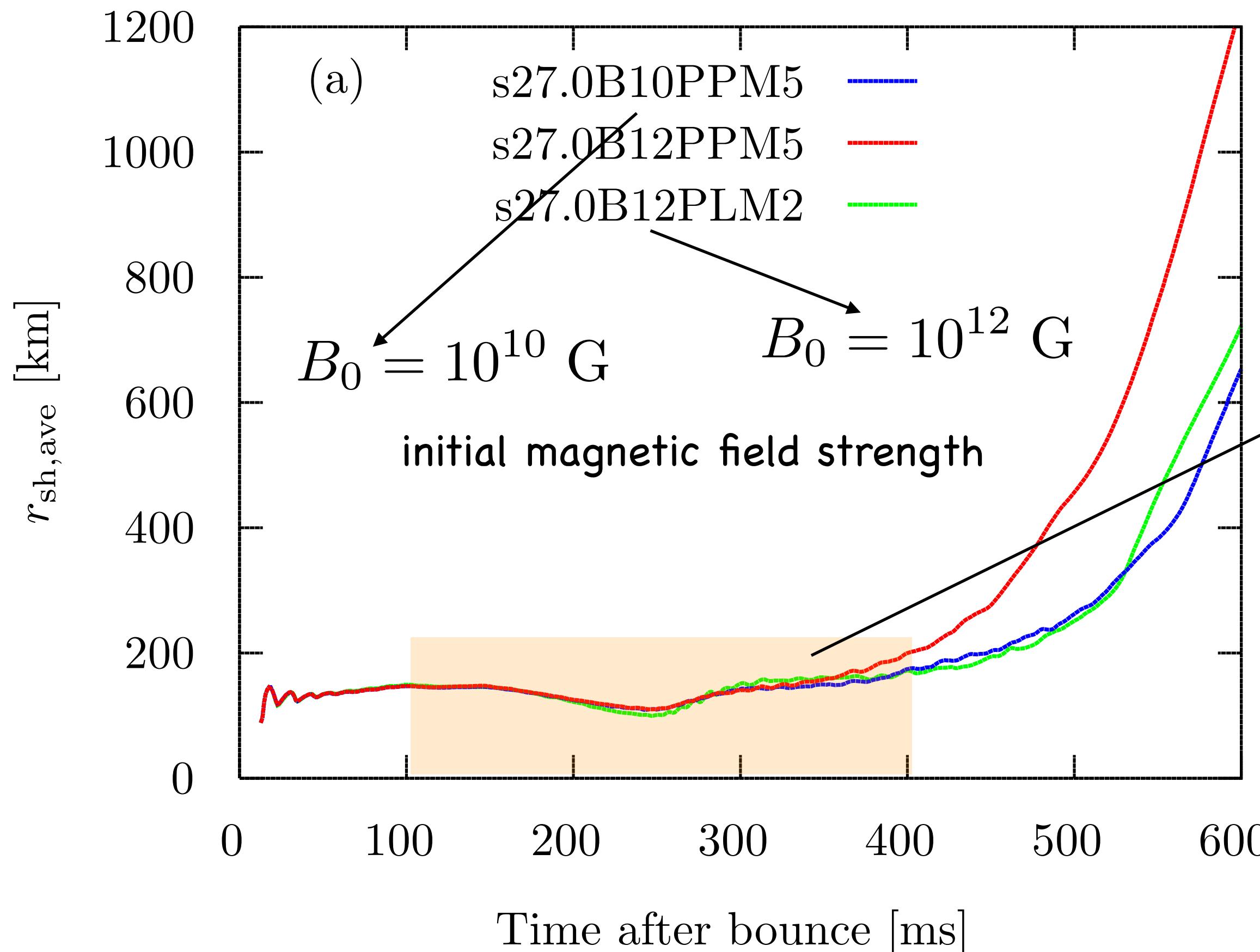
Future work: realistic configuration and field strength

Overall evolution: focusing on B-field

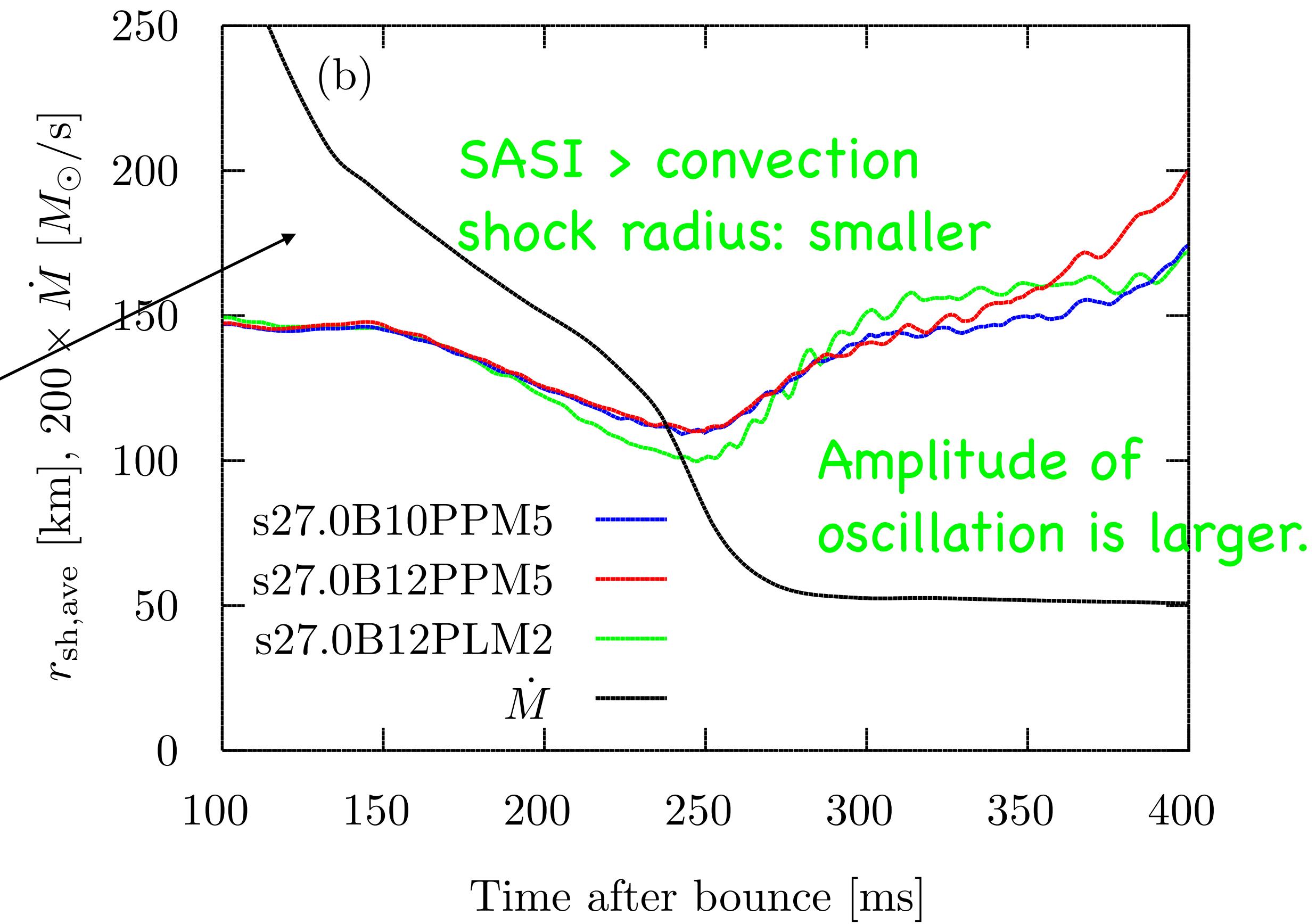


- magnetic field lines: a split-monopole-like configuration ($t_{pb}=14\text{ms}$)
- non-radial component of B-field due to the convection ($t_{pb}=160\text{ms}$)
- field amplification in the post shock region due to compression and stretching the magnetic field
- onset of the shock revival at $t_{pb}=250\text{ms}$
- B-field lines: trajectory of fluid motion that forms large hot bubble ($t_{pb}=390\text{ms}$)
- accumulation of the B-field lines around down flow region between bubbles
- magnetic loops: remnant of the initial configuration ($t_{pb}=560\text{ms}$)
- neutrino-heating driven explosion

Time evolution of shock radius

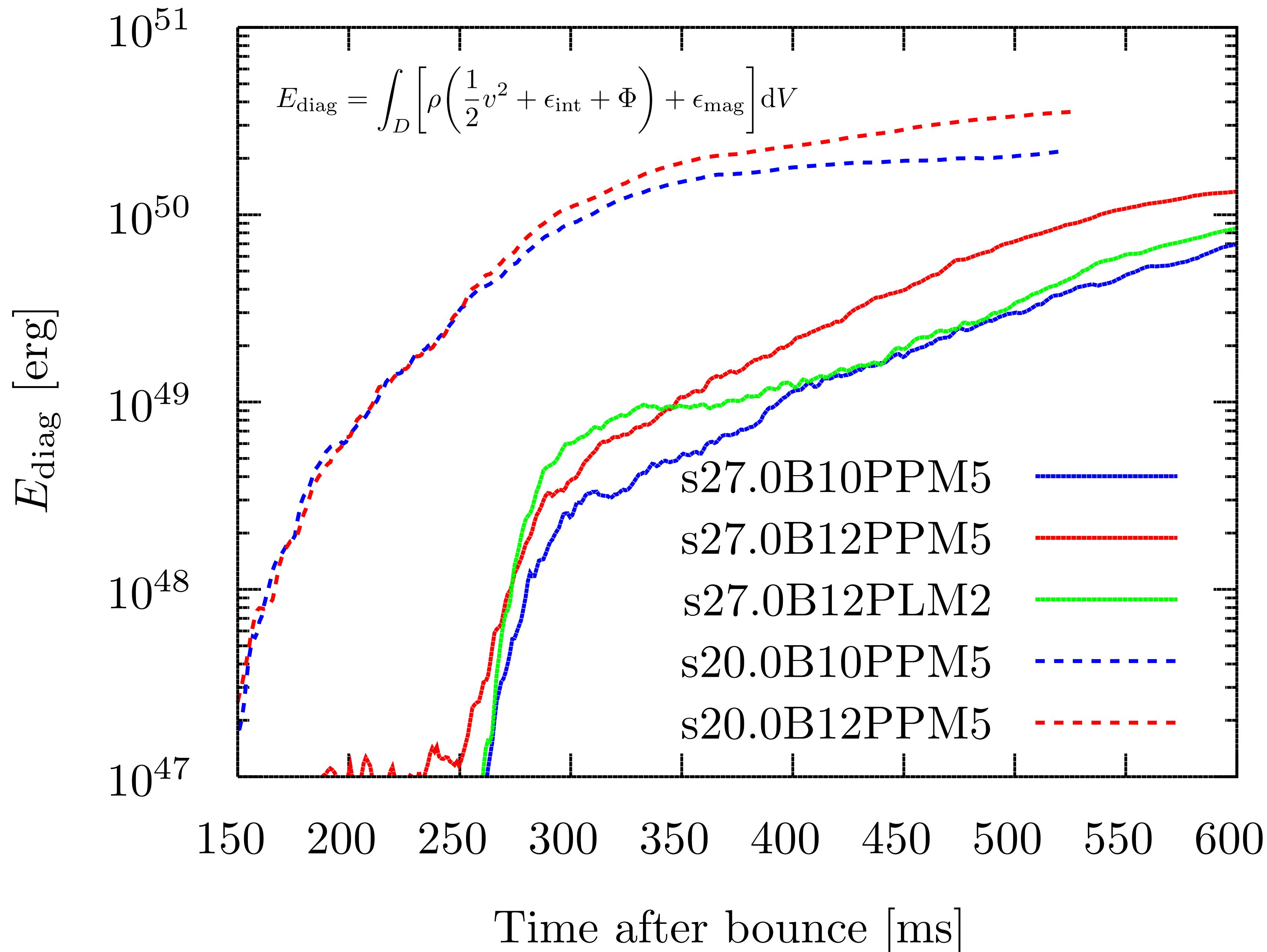


Shock wave evolves fast in strong magnetic field model compared to weak magnetic field model.

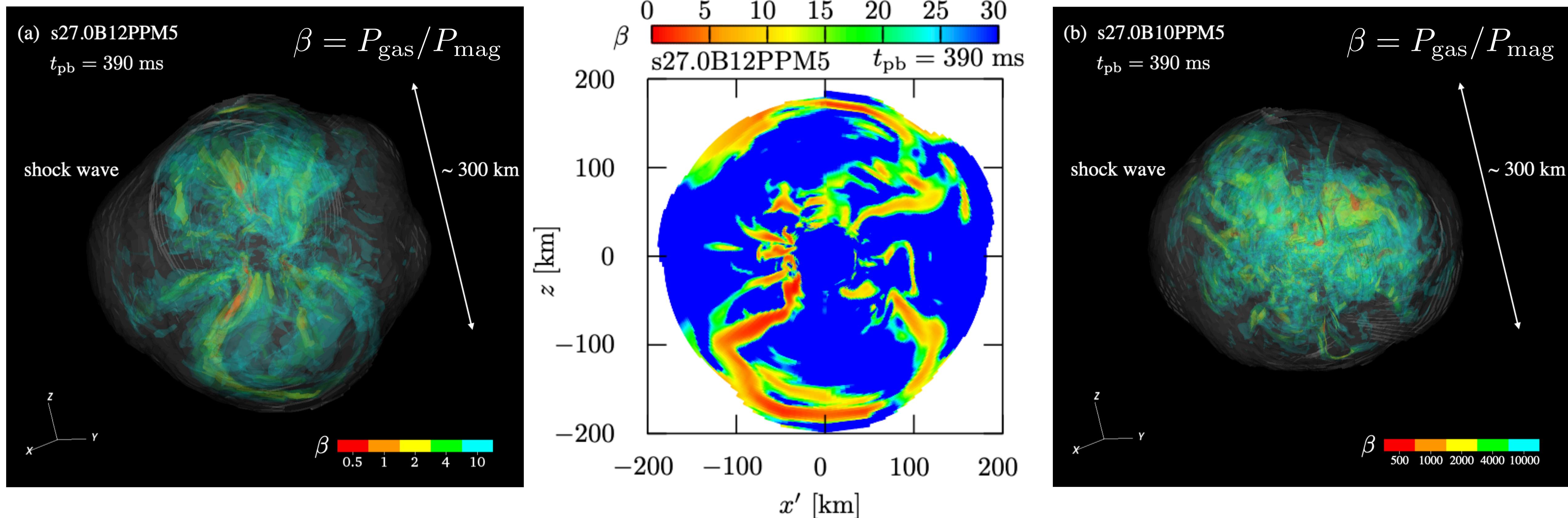


\dot{M} decreases at around $t_{pb}=250$ ms.
Since the ram pressure for the shock surface in the upper stream also decreases, it is reasonable that the shock revival occurs around the sudden drop of the mass accretion rate.

Diagnostic explosion energy



3D distribution of plasma beta

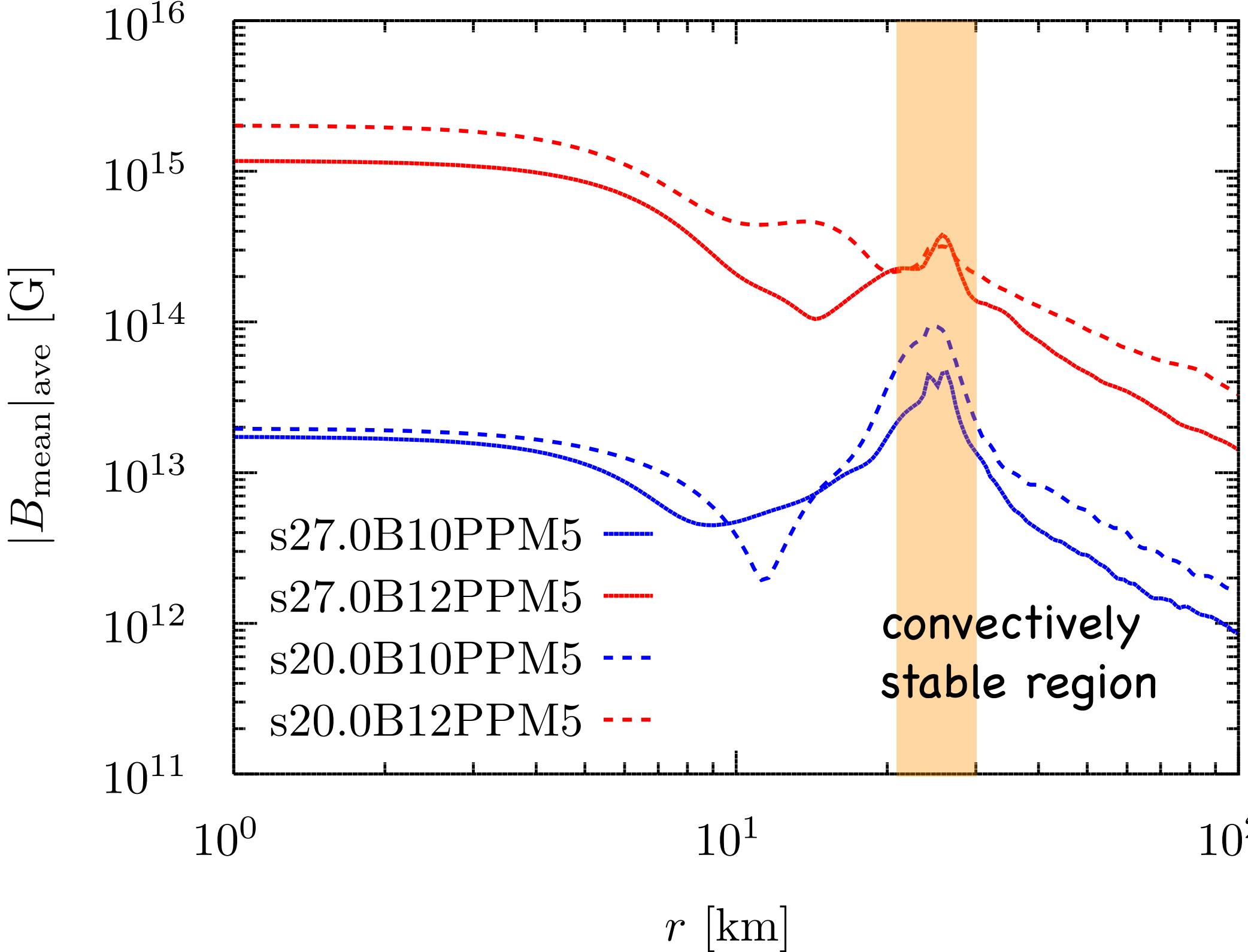
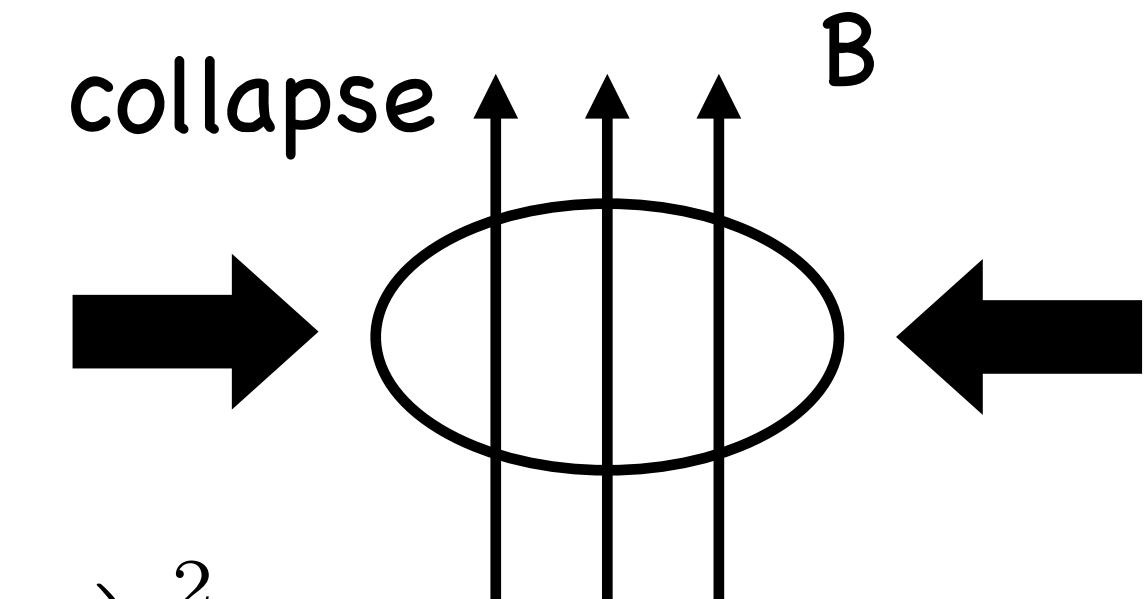


- plasma $\beta \sim 10$ behind the shock in strong field model
 - > Magnetic pressure/tension can partially contribute to the shock evolution in the strong field model.
- low $\beta < 1$: down flow region between bubbles
 - > converging flow under gravity
 - > accumulation and amplification of the magnetic field

Magnetic energy in convective zone

possible formation scenarios of magnetar

- turbulent dynamo amplification in a rapidly rotating PNS (Thompson+93)
- fossil field hypothesis (magnetic flux conservation) (Ferrario+06)



$$B_{\text{PNS}} \sim 10^{15} \text{ G} \left(\frac{B_{0,r=1000\text{km}}}{10^{12} \text{ G}} \right) \left(\frac{30\text{km}}{r_{\text{PNS}}} \right)^2$$

supernova remnants + magnetars in our galaxy

CTB 109 (AXP 1E 2259+586)

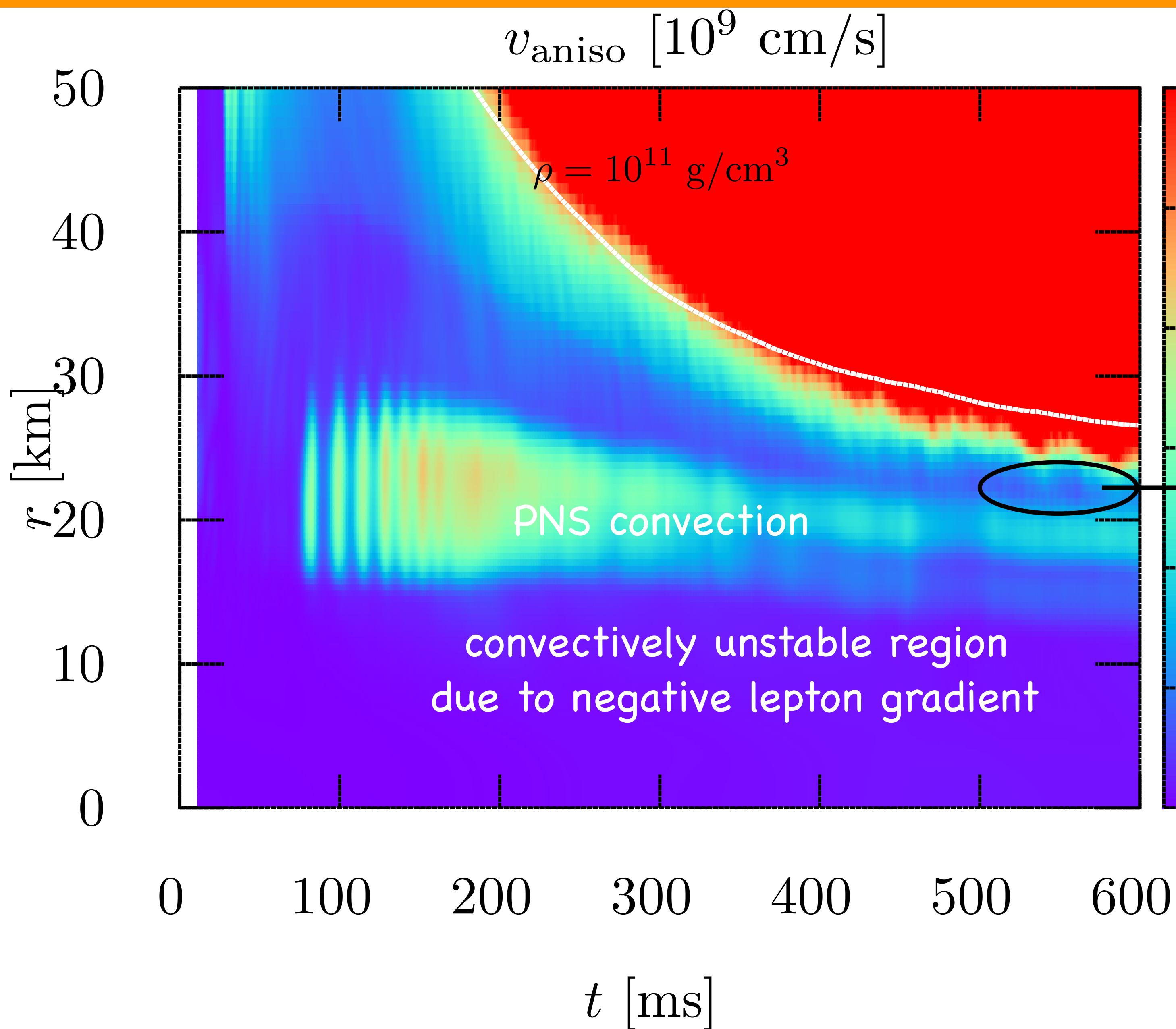
Kes 73 (AXP 1E 1841-045)

N49 (SGR 0526-66)

- typical explosion energy (10^{51} erg)
- slowly rotating ($P=10$ s) (Vink+06, Nakano+17)

Magnetars may not require rapid rotators with highly aspherical and energetic jets, but simply the normal neutrino-driven explosion as the central engine.

Anisotropic velocity



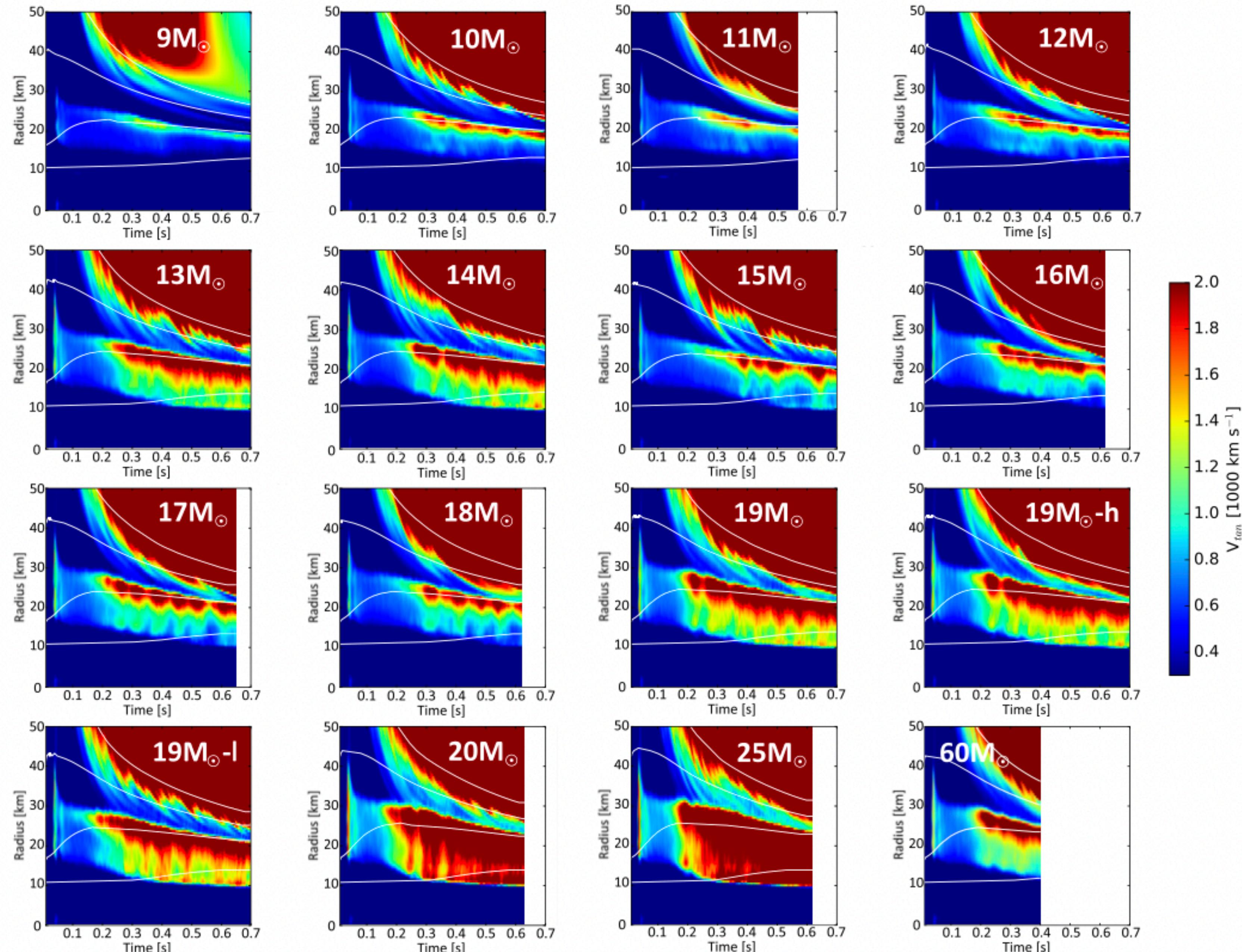
$$v_{\text{aniso}} = \sqrt{\frac{\langle \rho[(v_r - \langle v_r \rangle)^2 + v_\theta^2 + v_\phi^2] \rangle}{\langle \rho \rangle}}$$
$$\langle A \rangle = \frac{\int A \sin \theta d\theta d\phi}{4\pi}$$

Takiwaki+12

convectively stable region

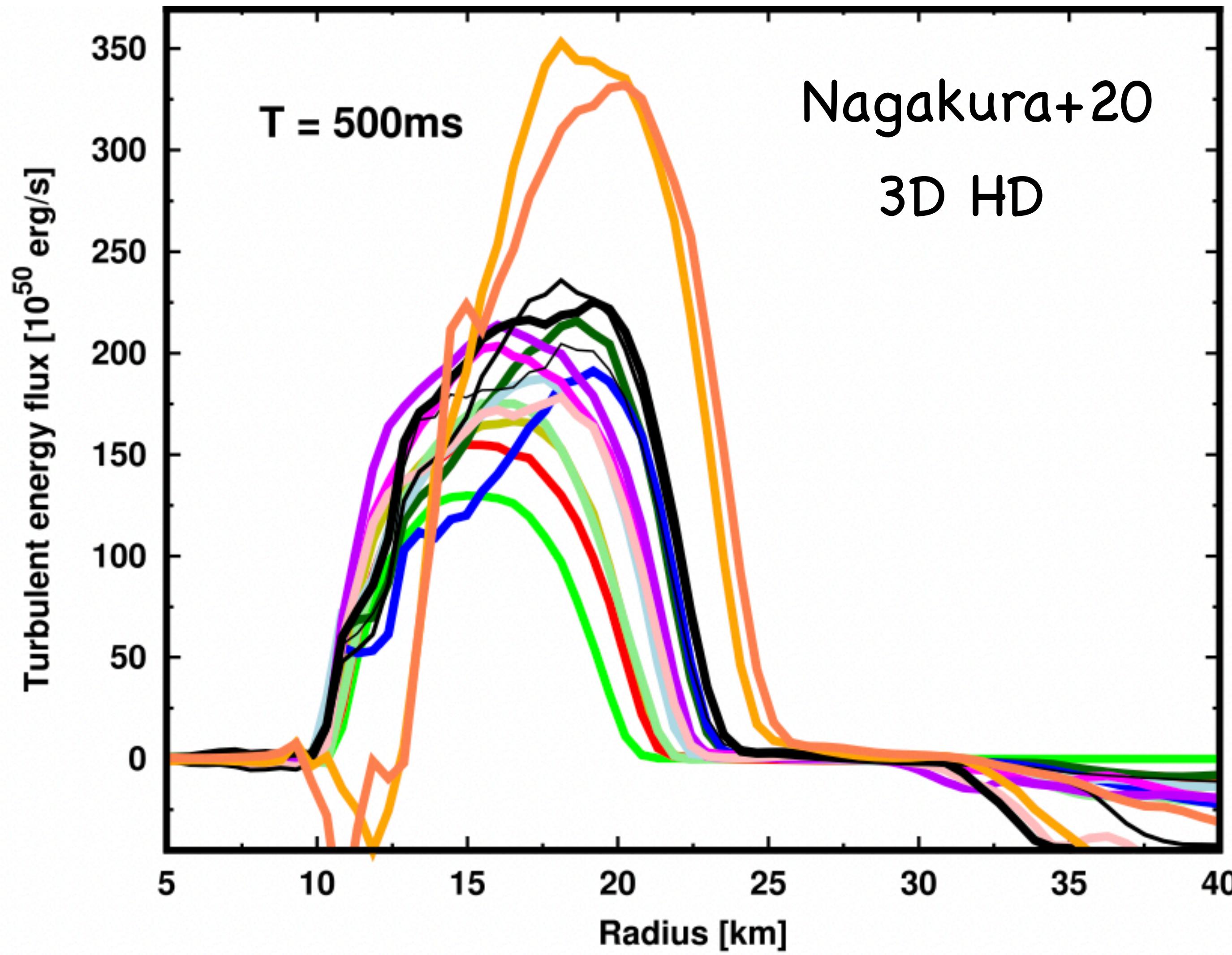
observed in HD models
(e.g. Nagakura+20)

Anisotropic velocity



convectively stable region
observed in HD models
(e.g. Nagakura+20)

Estimated B-field



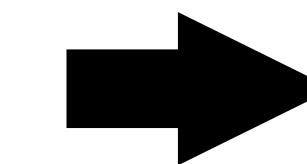
$$E_{\text{turb}} \sim 10^{49} \text{ erg}$$

$$R^3 \sim 10^{19} \text{ cm}^3$$

$$\rho \sim 10^{14} \text{ g/cm}^3$$

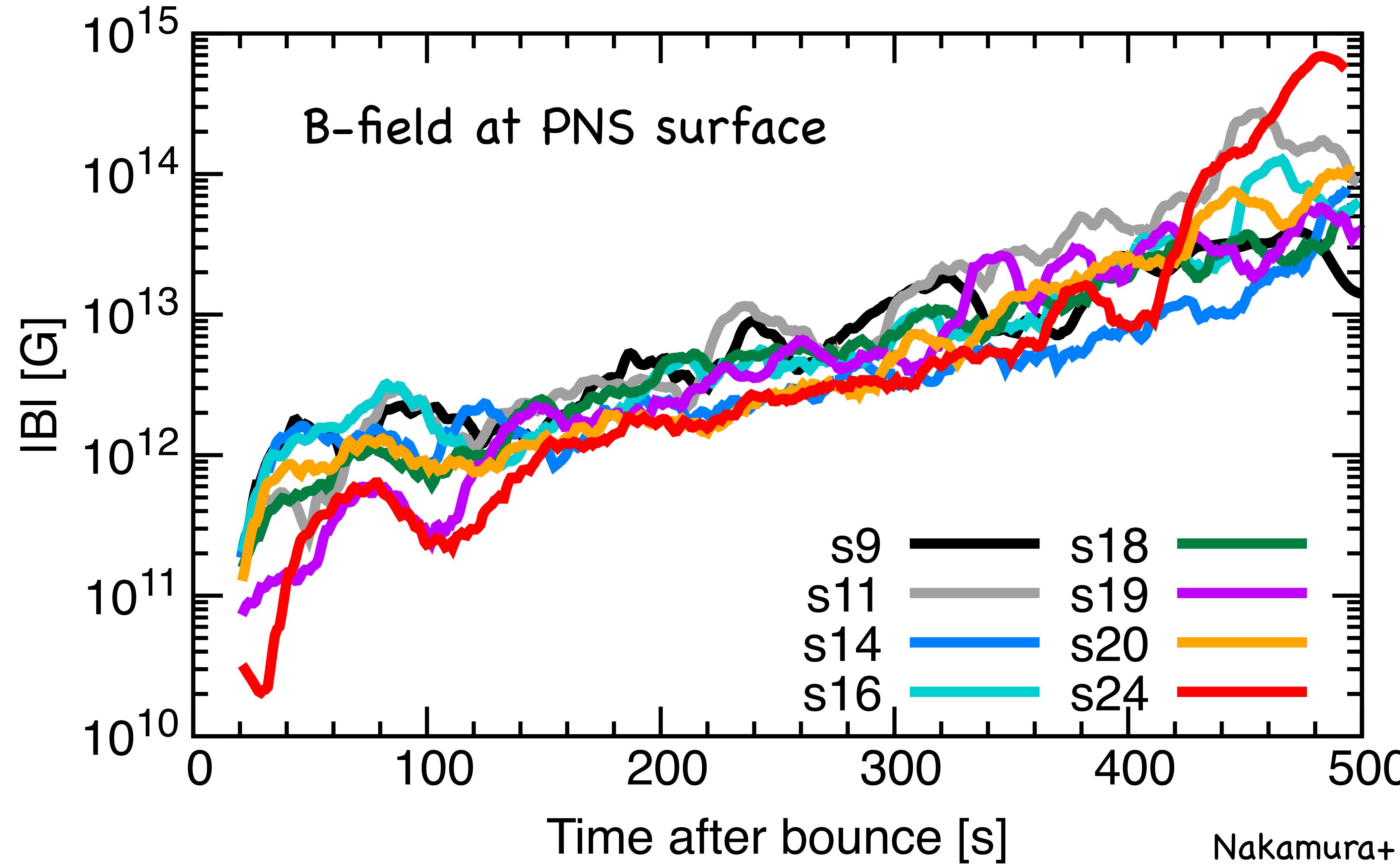
$$v_{\text{turb}} \sim 10^8 \text{ cm/s}$$

$$\rho v_{\text{turb}}^2 = B^2 \sim 10^{30} \text{ erg/cm}^3$$



$$B \sim 10^{15} \text{ G}$$

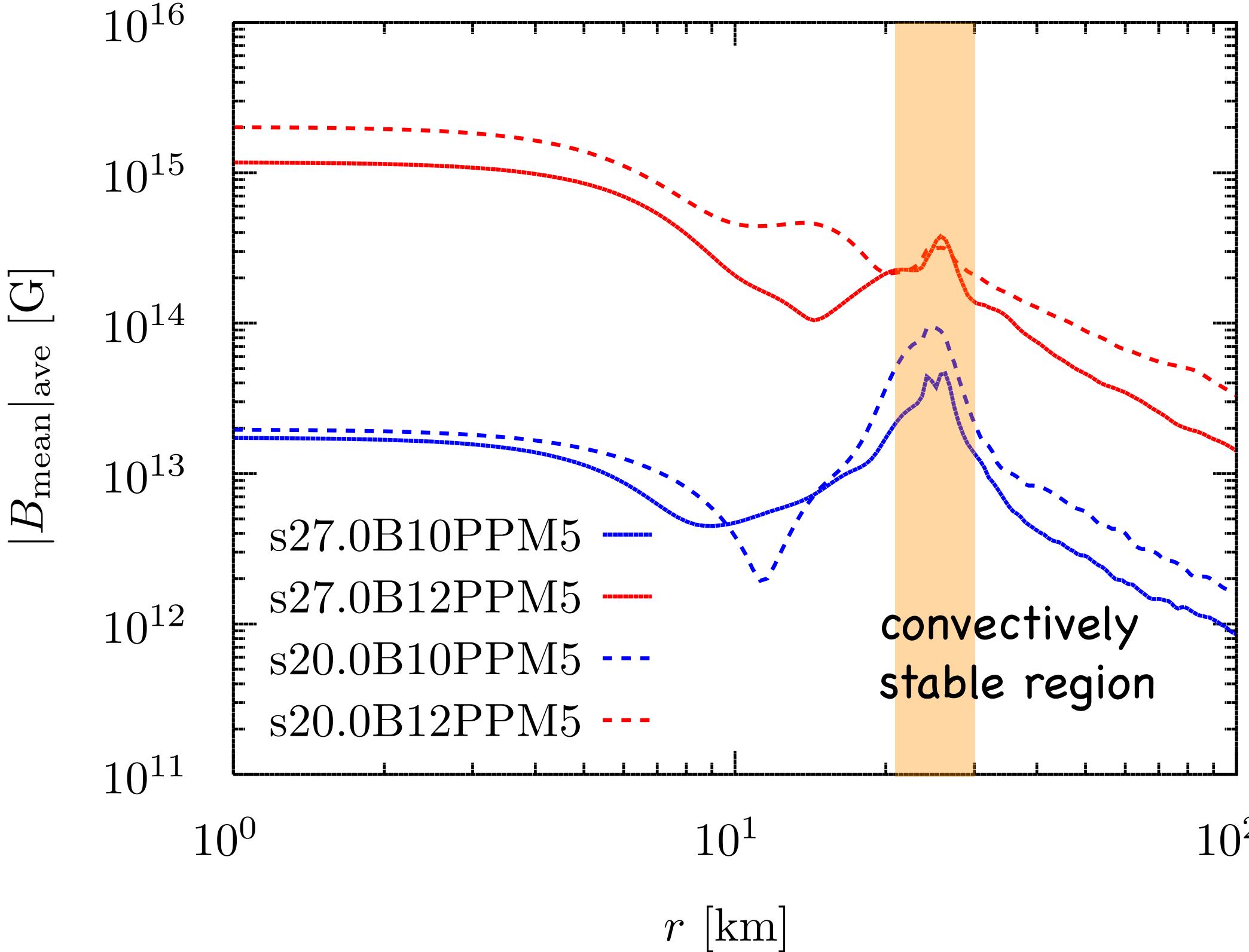
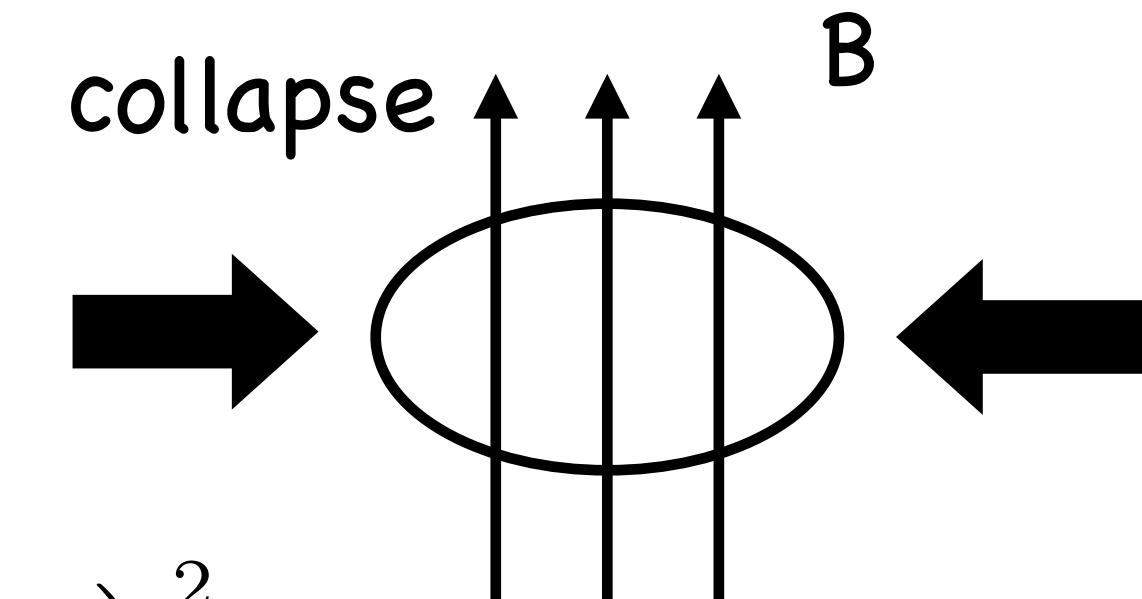
Systematic 3D MHD simulations



Magnetic energy in convective zone

possible formation scenarios of magnetar

- turbulent dynamo amplification in a rapidly rotating PNS (Thompson+93)
- fossil field hypothesis (magnetic flux conservation) (Ferrario+06)



$$B_{\text{PNS}} \sim 10^{15} \text{ G} \left(\frac{B_{0,r=1000\text{km}}}{10^{12} \text{ G}} \right) \left(\frac{30\text{km}}{r_{\text{PNS}}} \right)^2$$

supernova remnants + magnetars in our galaxy

CTB 109 (AXP 1E 2259+586)

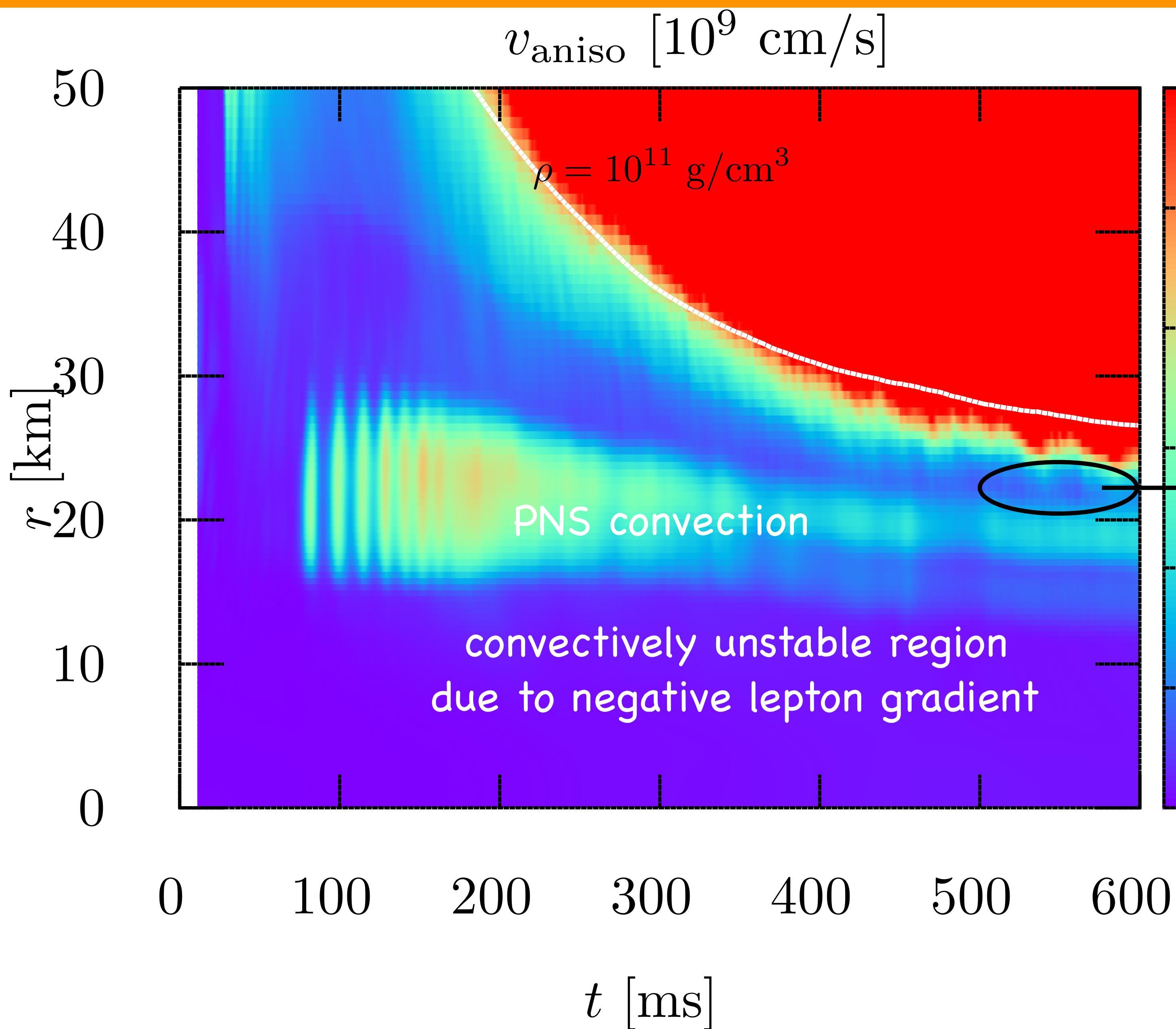
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Anisotropic velocity



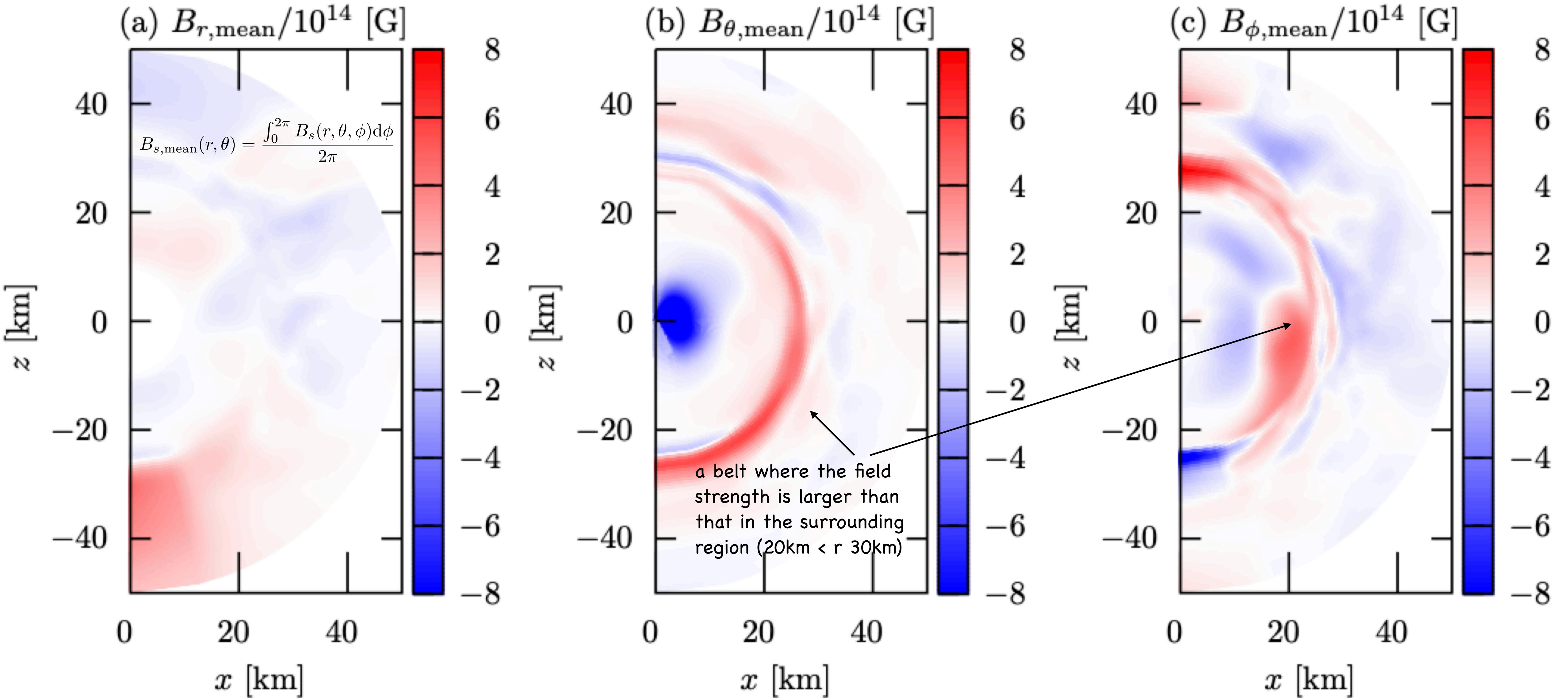
$$v_{\text{aniso}} = \sqrt{\frac{\langle \rho[(v_r - \langle v_r \rangle)^2 + v_\theta^2 + v_\phi^2] \rangle}{\langle \rho \rangle}}$$
$$\langle A \rangle = \frac{\int A \sin \theta d\theta d\phi}{4\pi}$$

Takiwaki+12

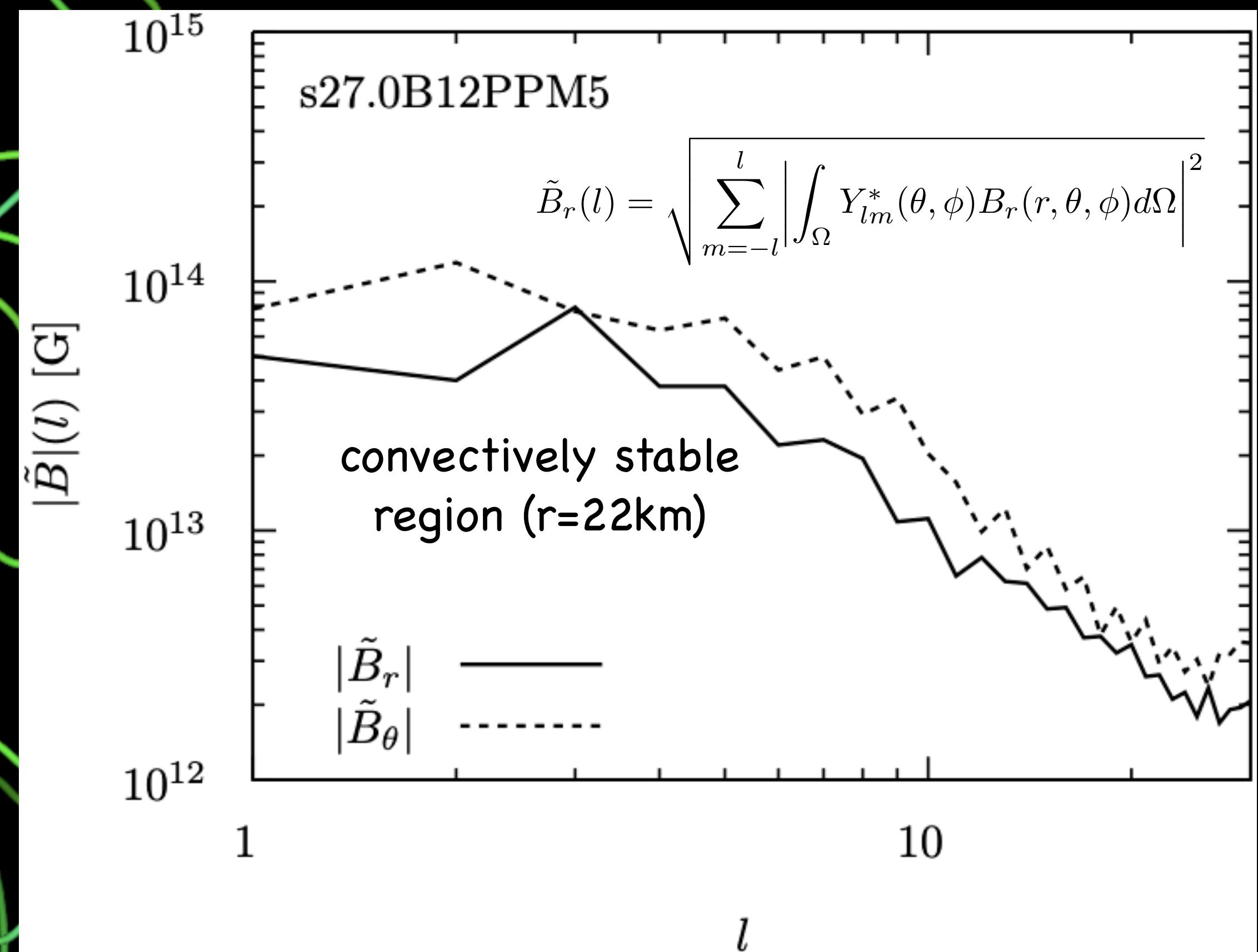
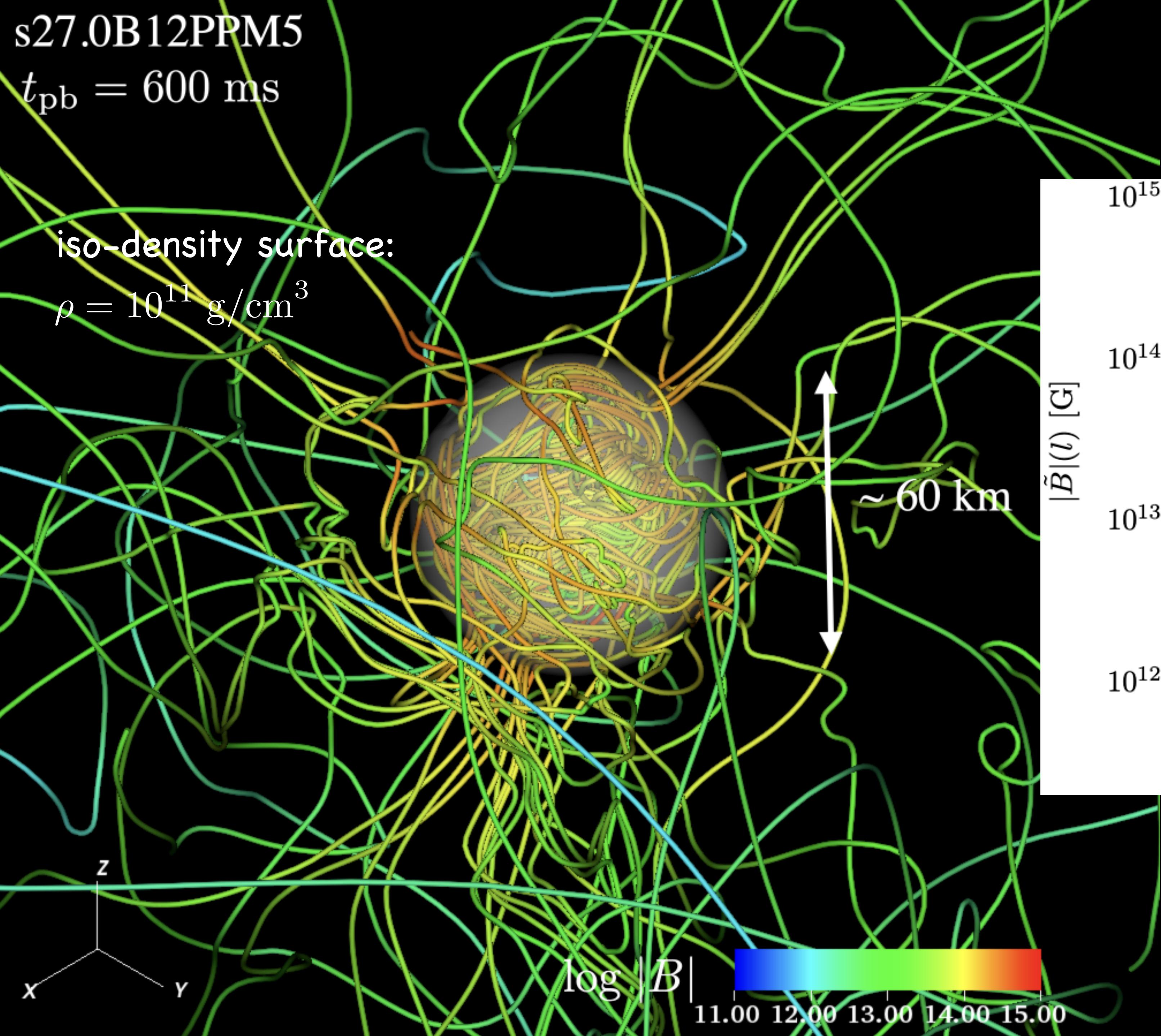
convectively stable region

observed in HD models
(e.g. Nagakura+20)

Meridional distributions of mean components of the magnetic field



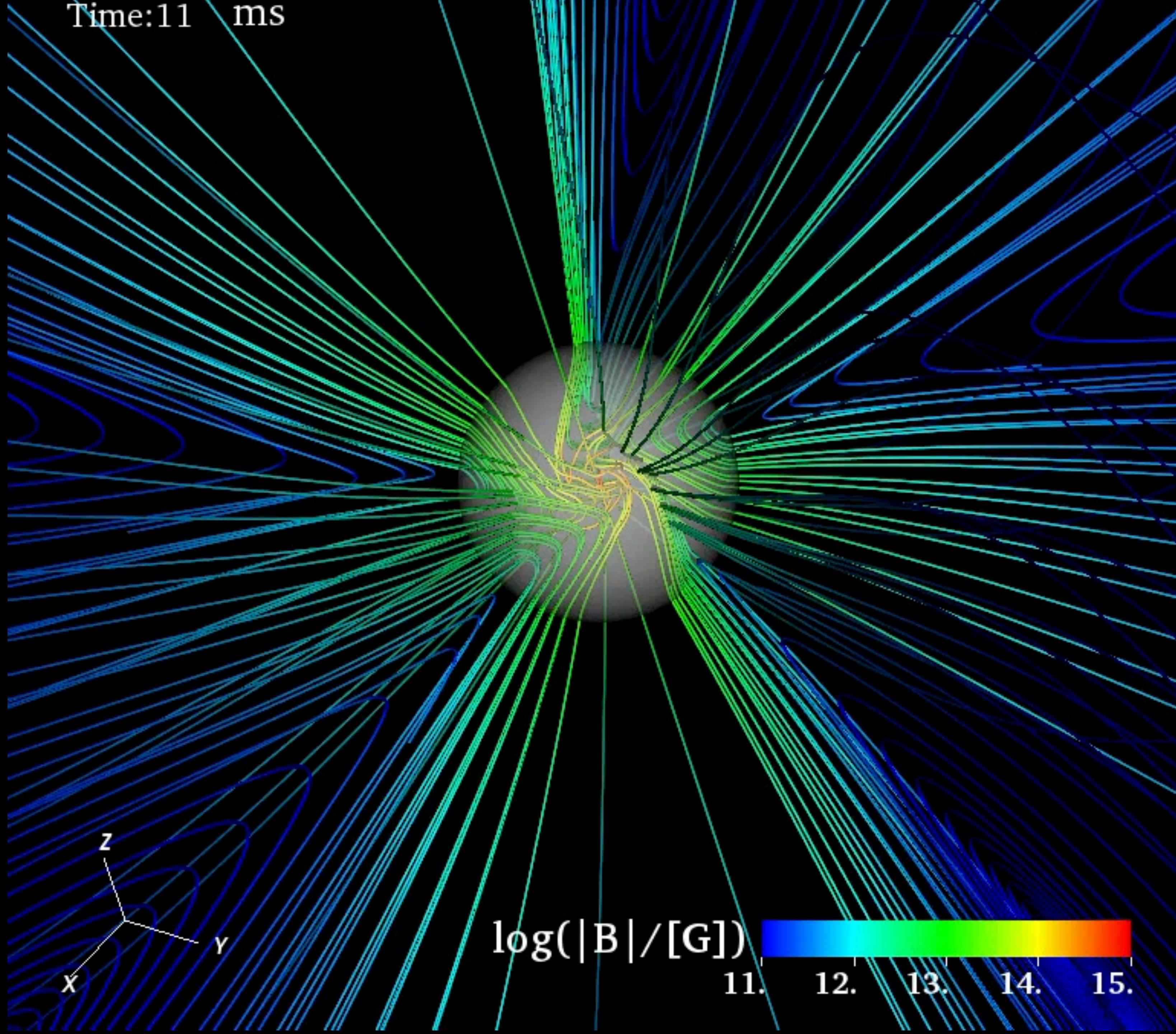
The position of the belt overlaps with the convectively stable region beneath the PNS surface.



Large scale modes become dominant compared to the small scale modes.

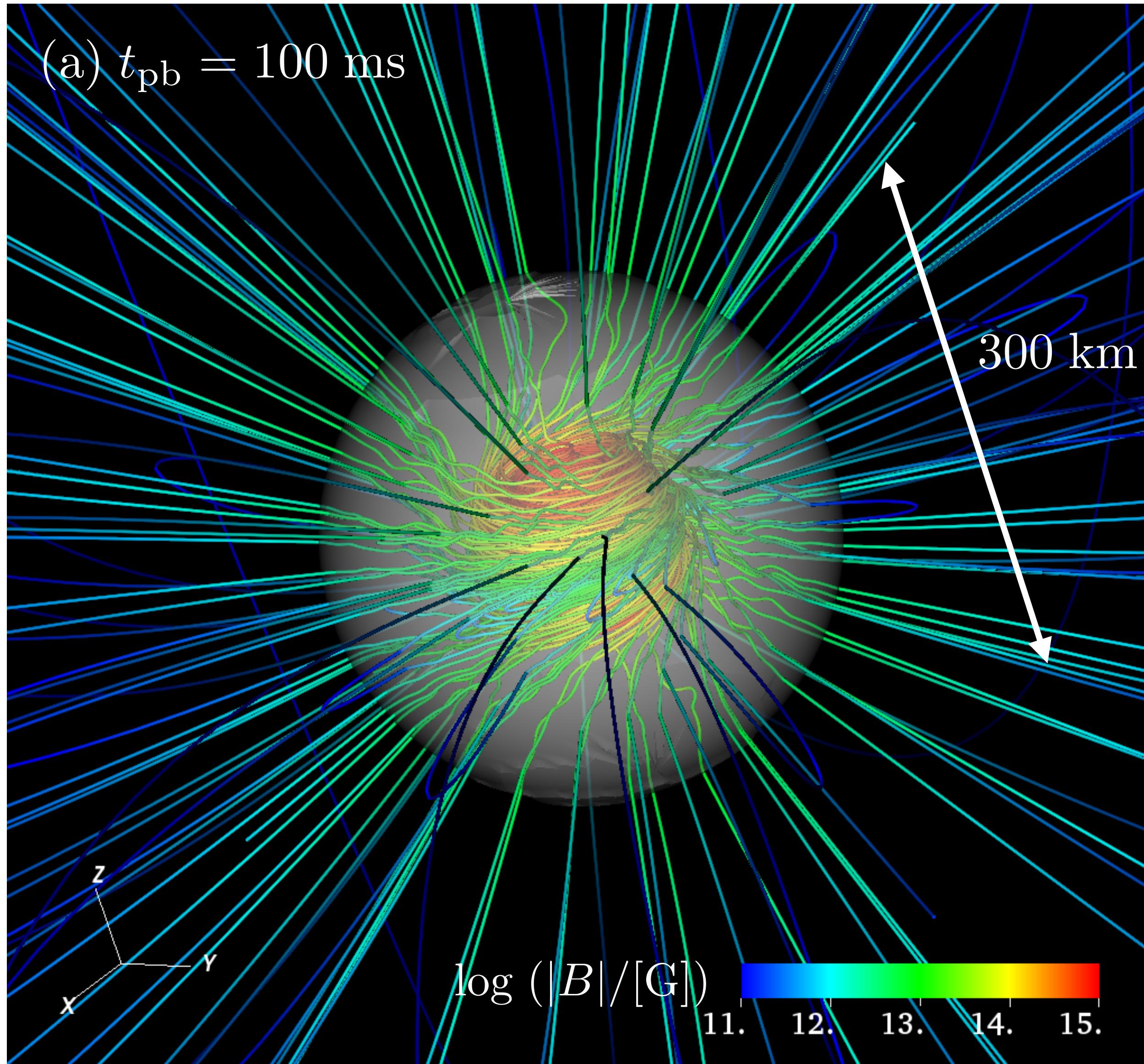
Time:11 ms

rotating model

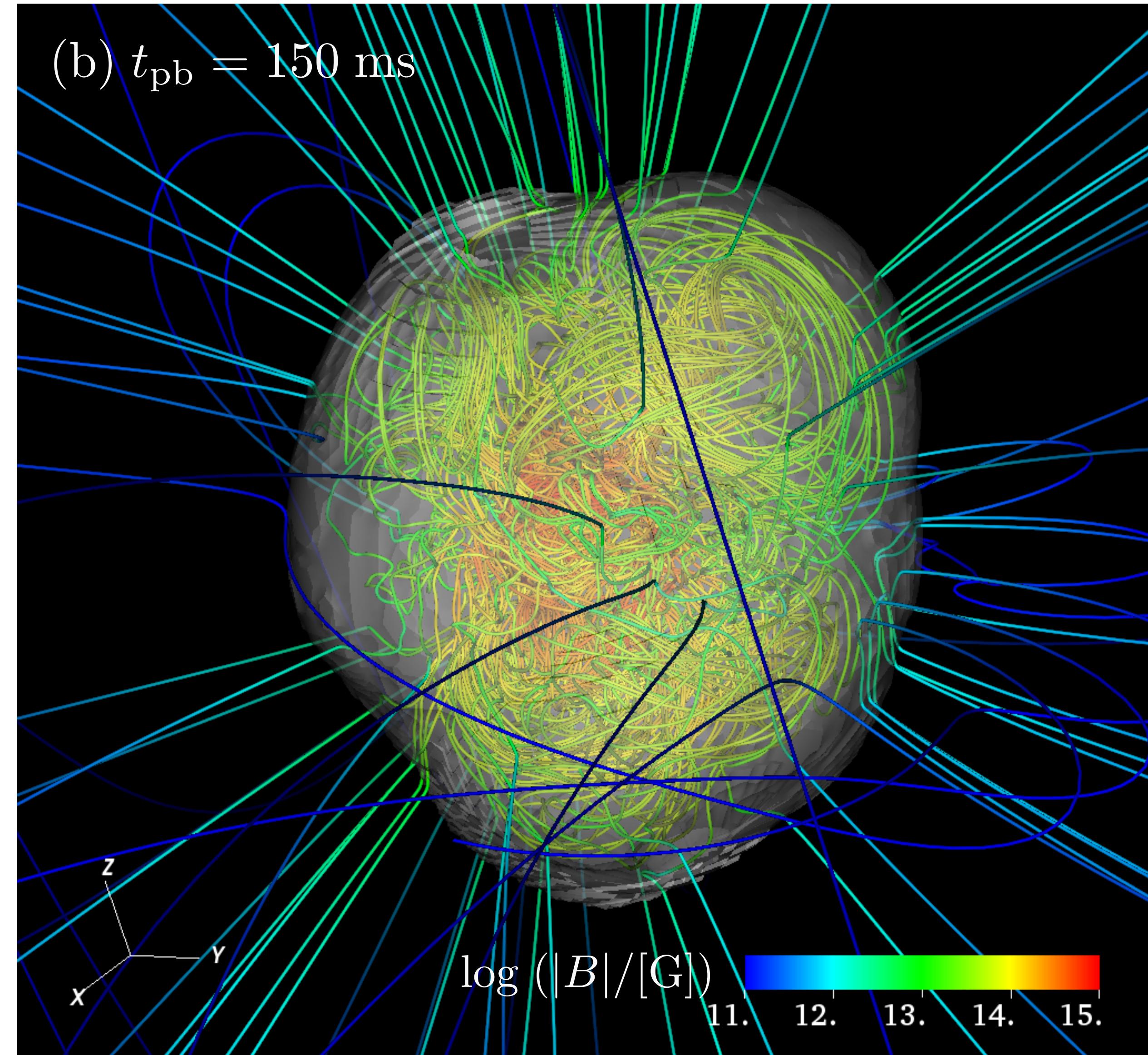


Distribution of B-field: slowly-rotating model

onset of neutrino-driven convection

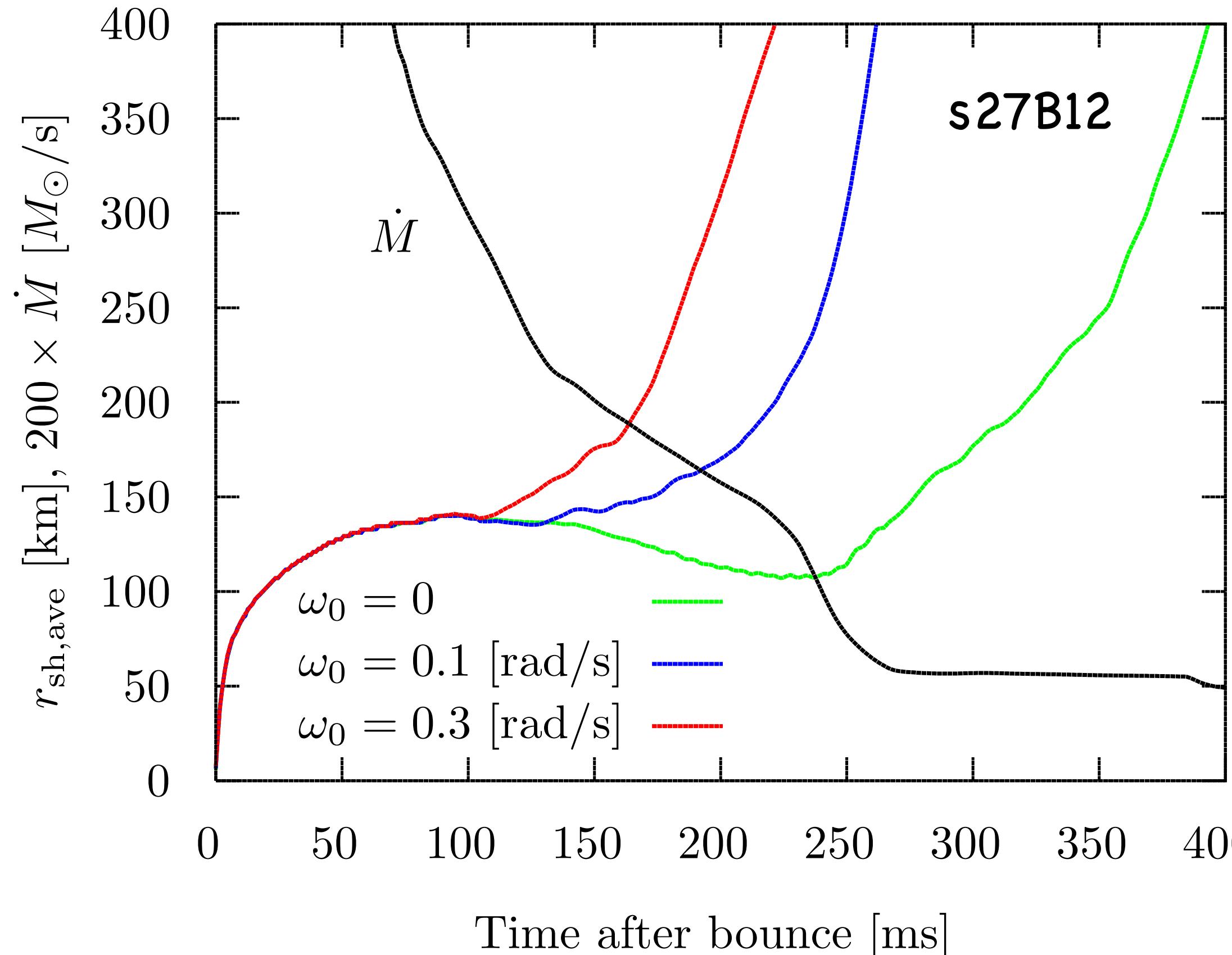


after shock revival

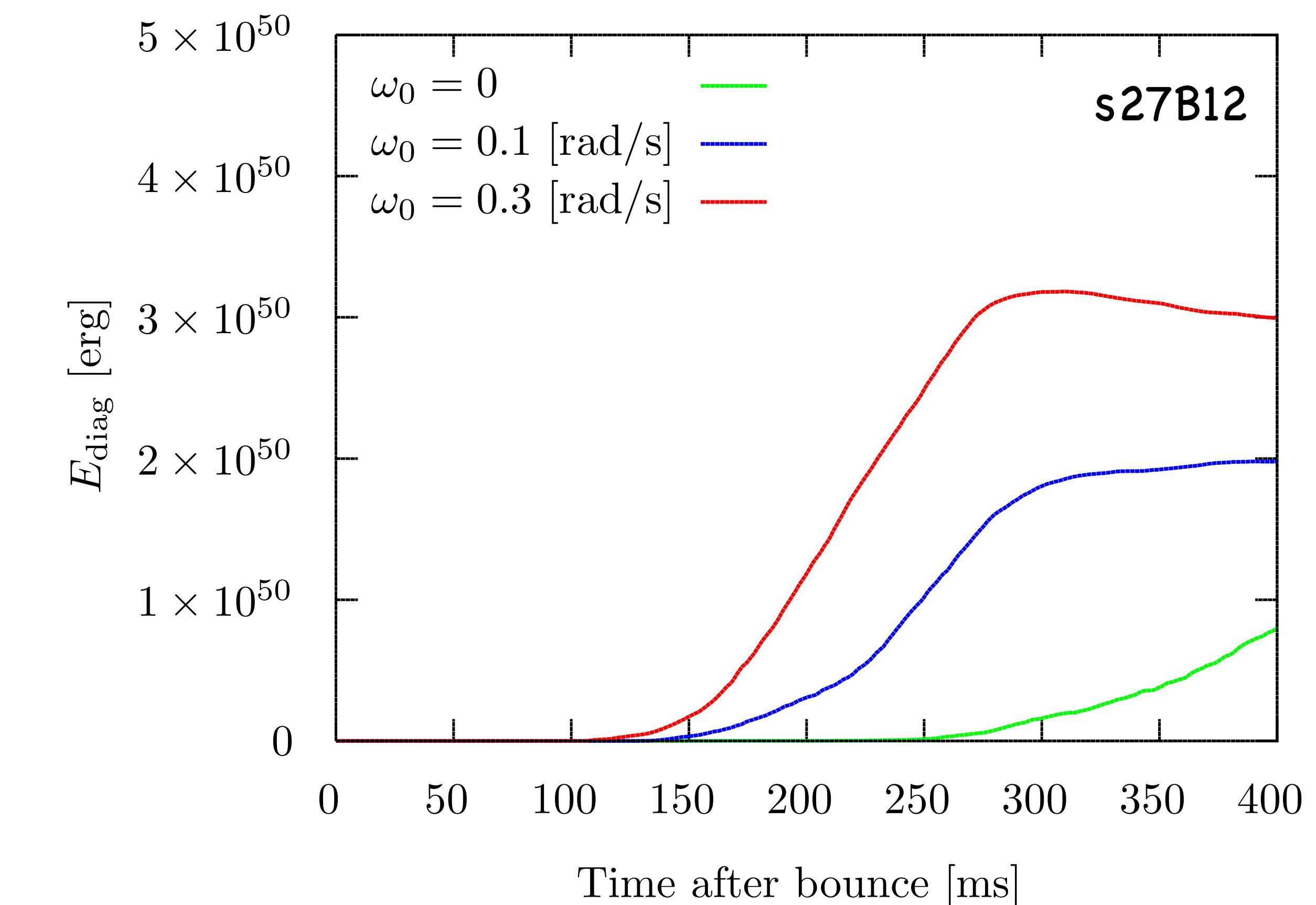


Dependence of the rotation

shock evolution



evolution of explosion energy



Magnetic pressure driven explosion occurs in rotating models. The magnetic field is fully amplified due to the effect of turbulence.

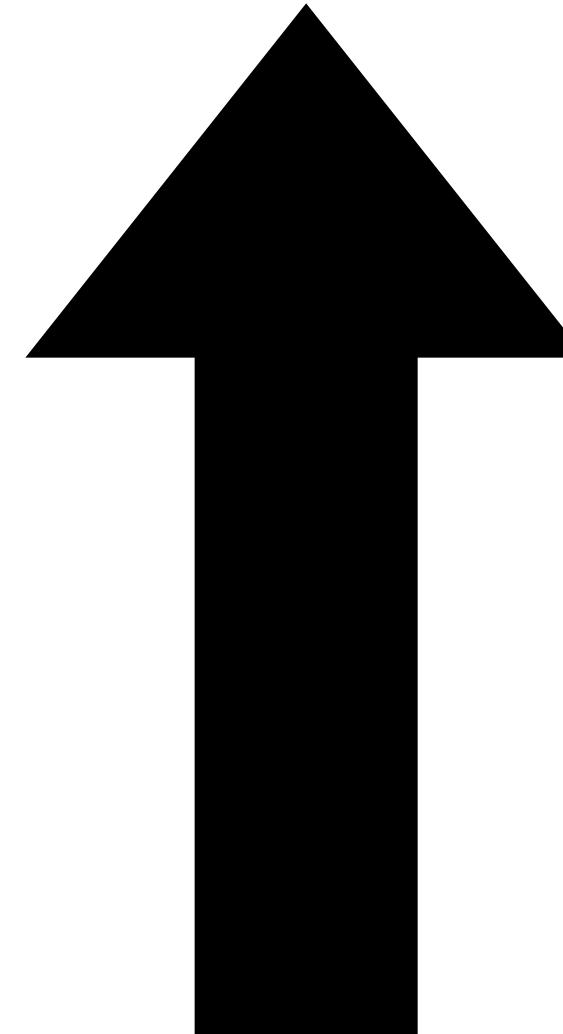
Explosion energy in faster explosion model is larger.

回転しているシステムでの対流にともなうらせん運動

対流にともなう上昇流（下降流）は（回転座標系で考えると）コリオリ力を受けらせん運動となる

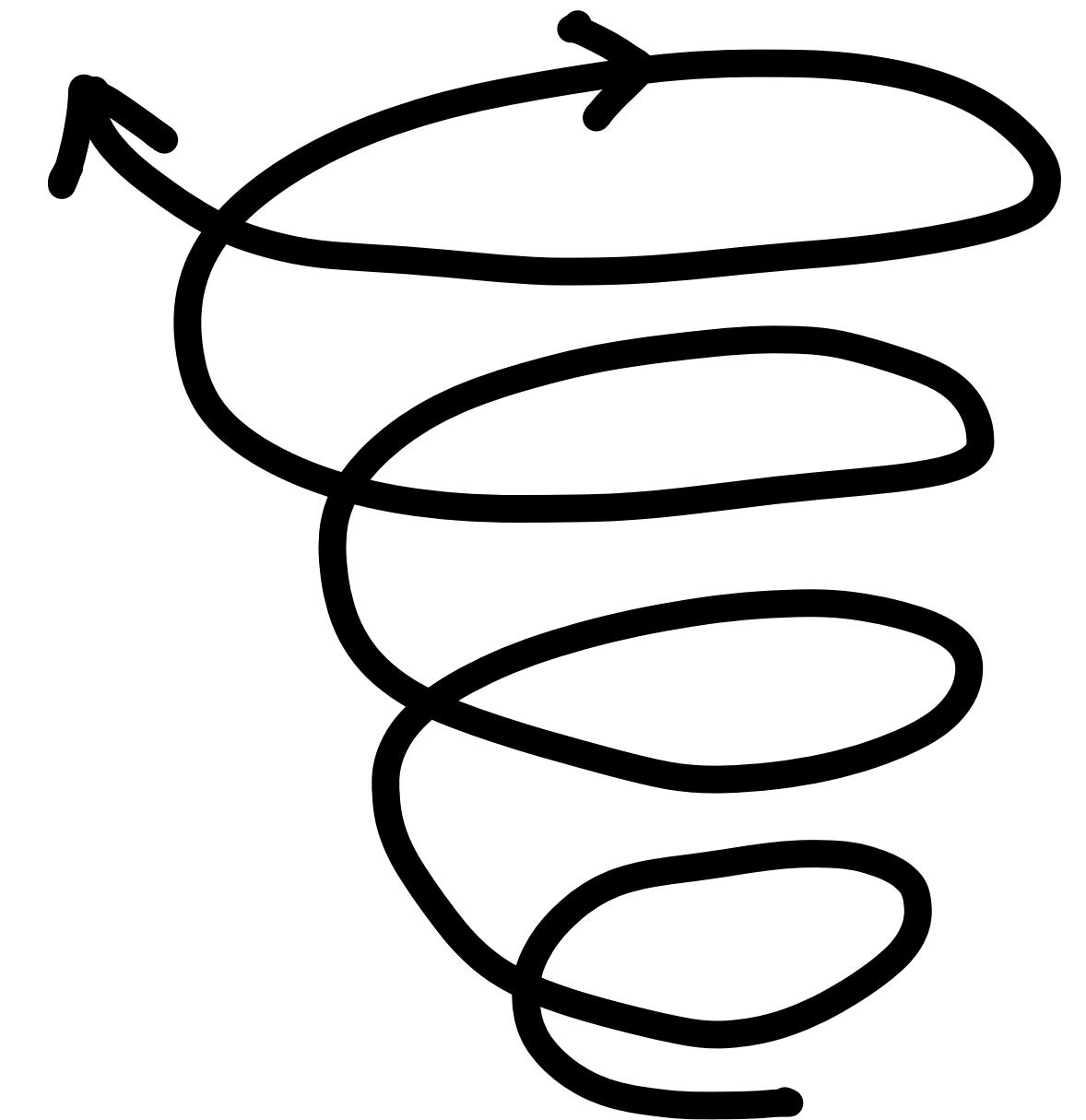
らせん運動

kinetic helicityの生成



対流での上昇流

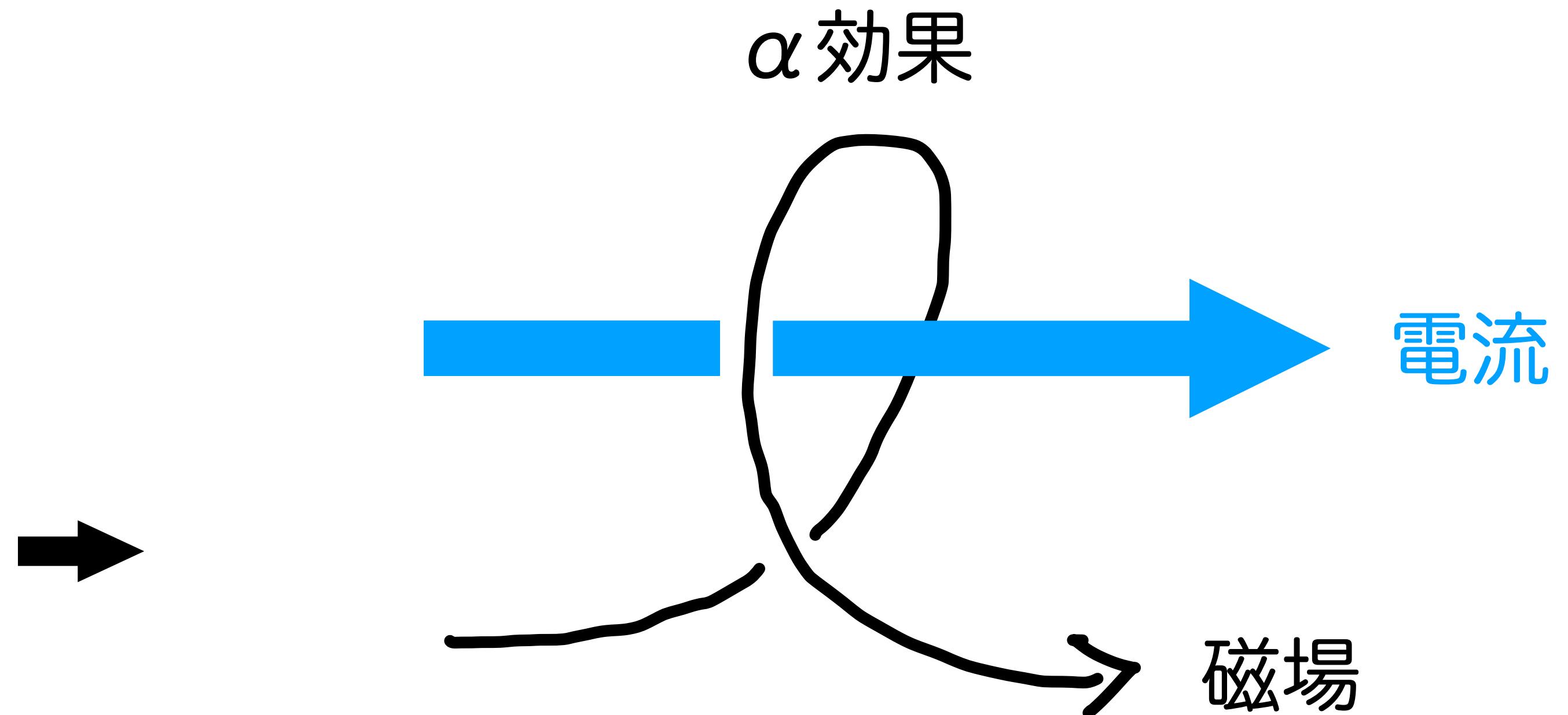
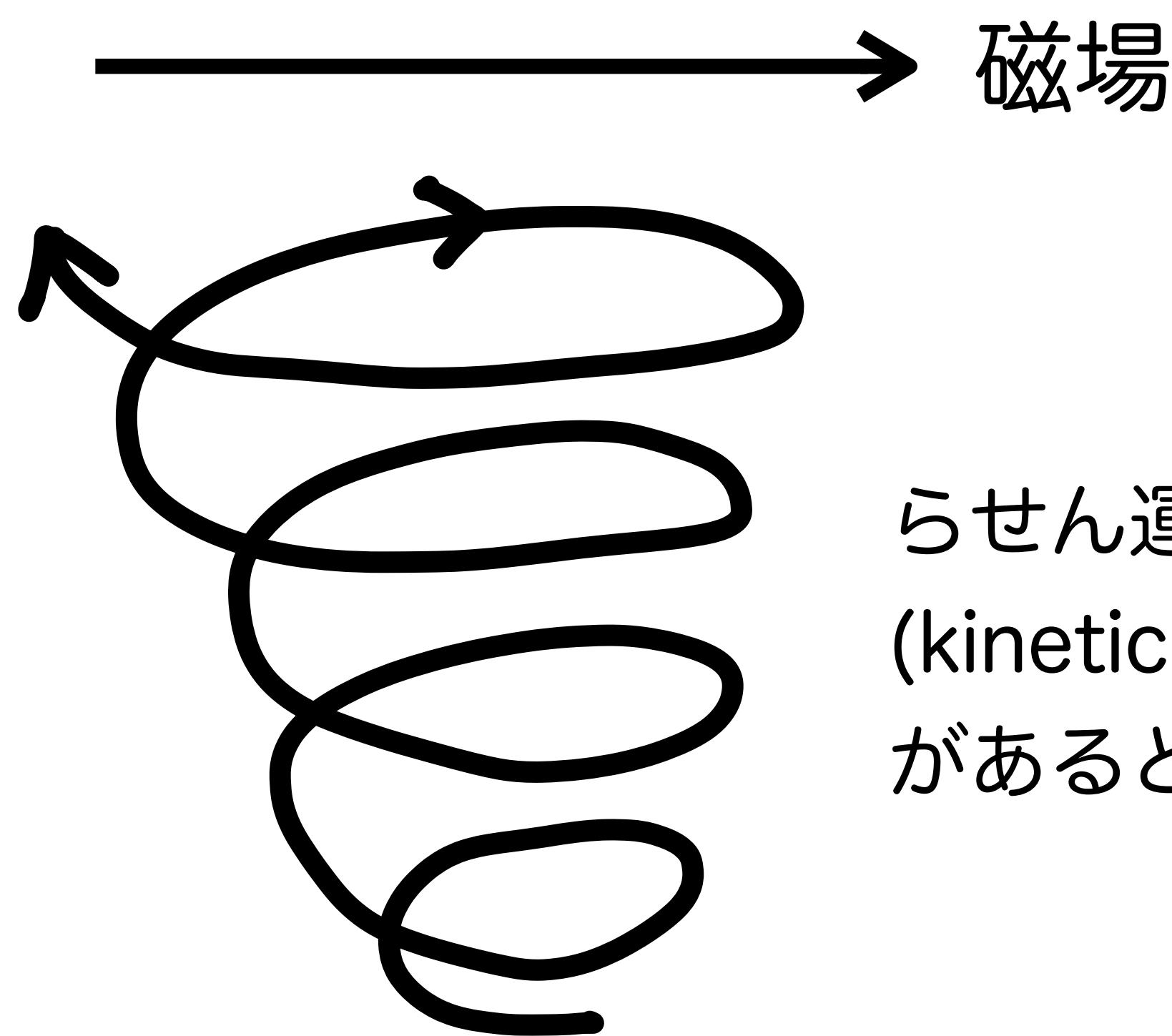
無回転のシステム



対流での上昇流

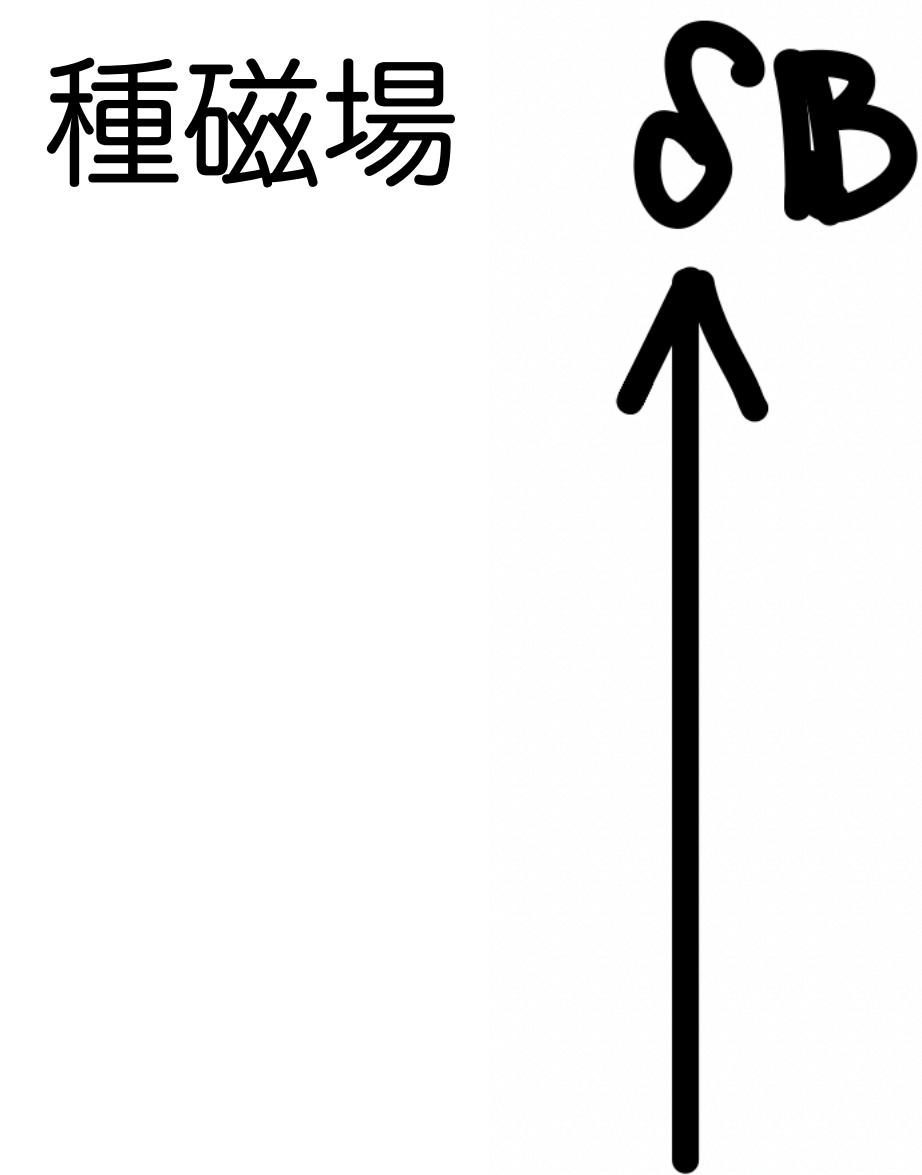
回転しているシステム

α 効果にともなう磁場に比例する電流



はじめの磁場と垂直な磁場が生じるとともに
はじめの磁場と平行な電流が流れる

α 効果にともなう磁場の指数関数的な增幅



α 効果にともなう磁場の指数関数的な増幅

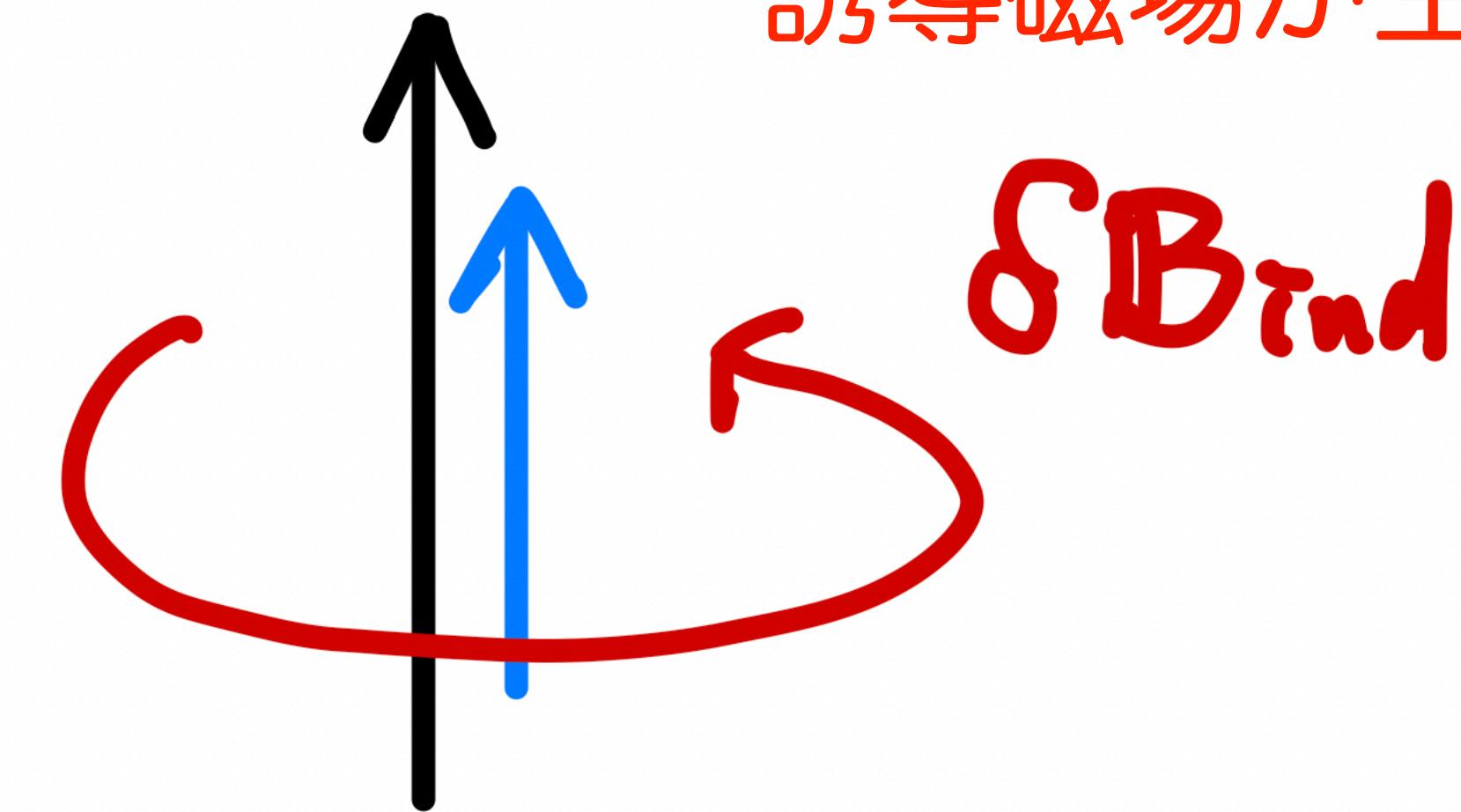
α 効果で磁場に比例した
電流が流れる



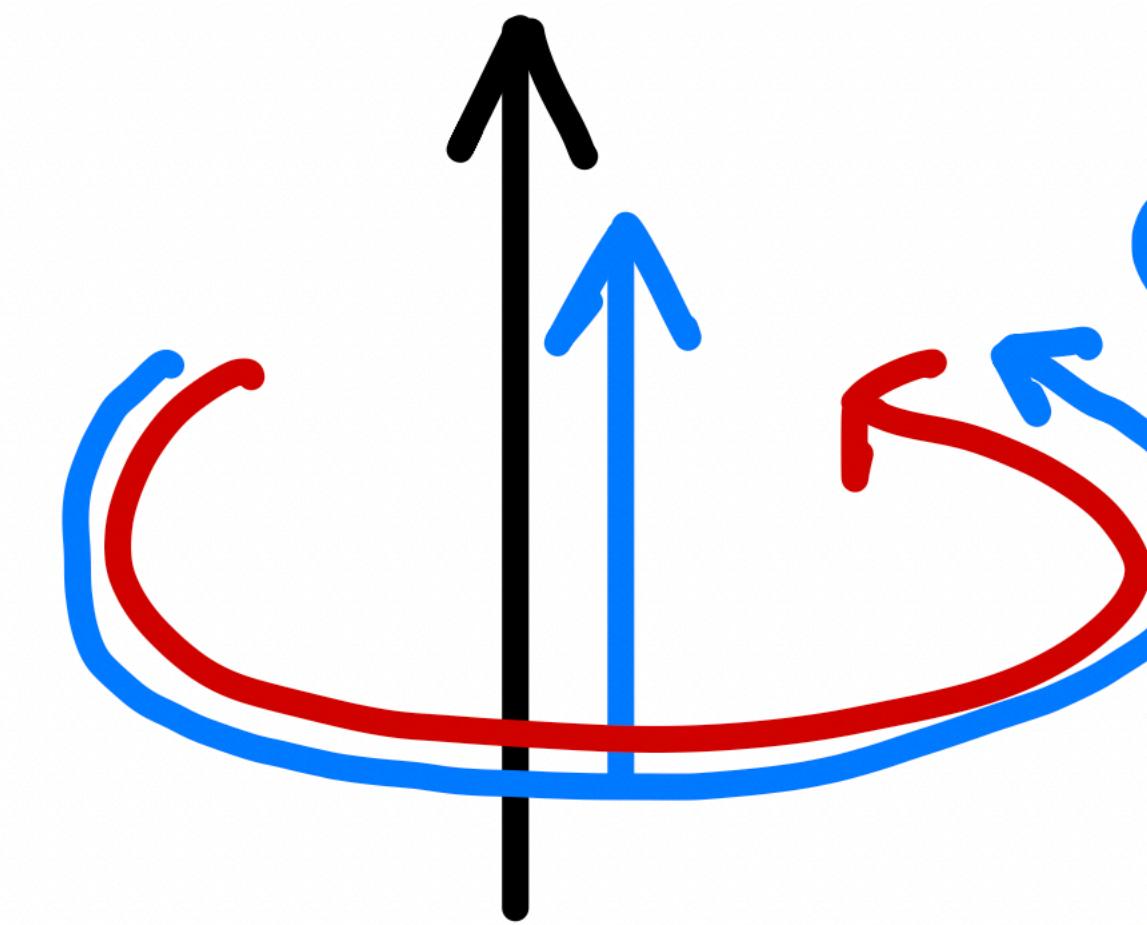
$$\delta J \propto \delta B$$

α 効果にともなう磁場の指数関数的な増幅

アンペールの法則により
誘導磁場が生成



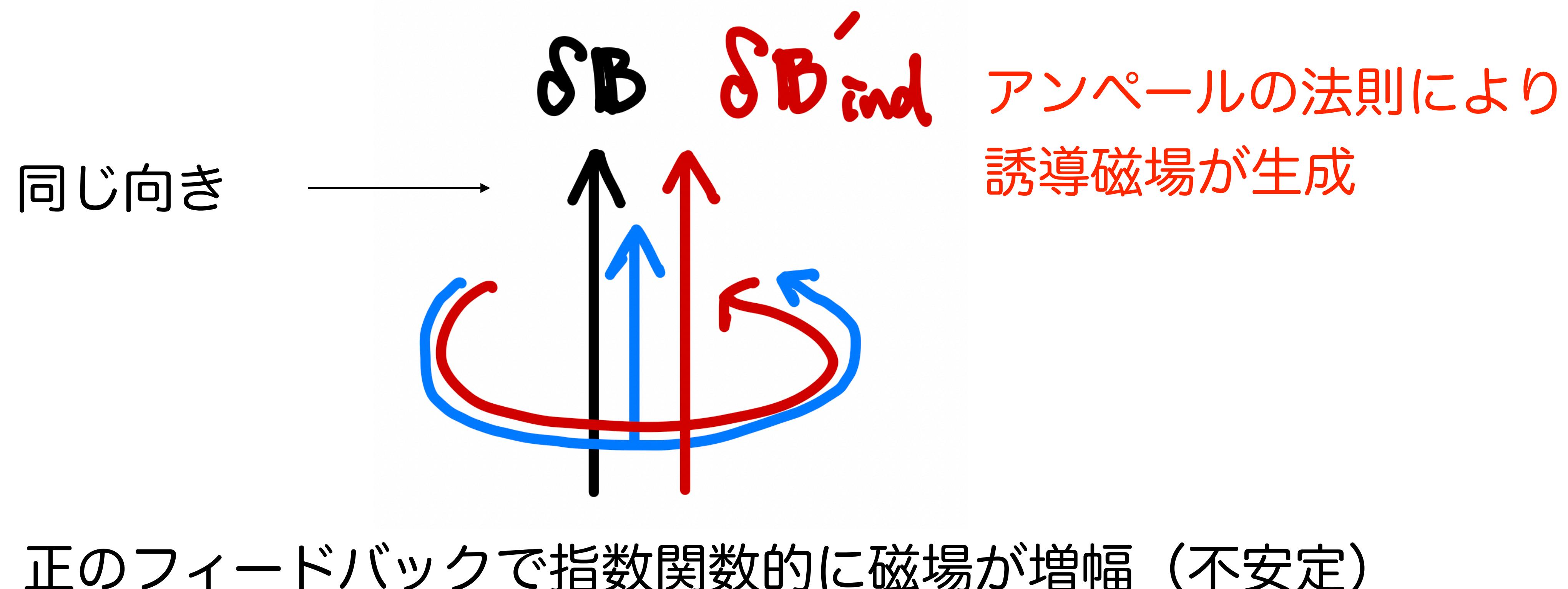
α 効果にともなう磁場の指数関数的な増幅



$$\delta J_{\text{ind}} \propto \delta B_{\text{ind}}$$

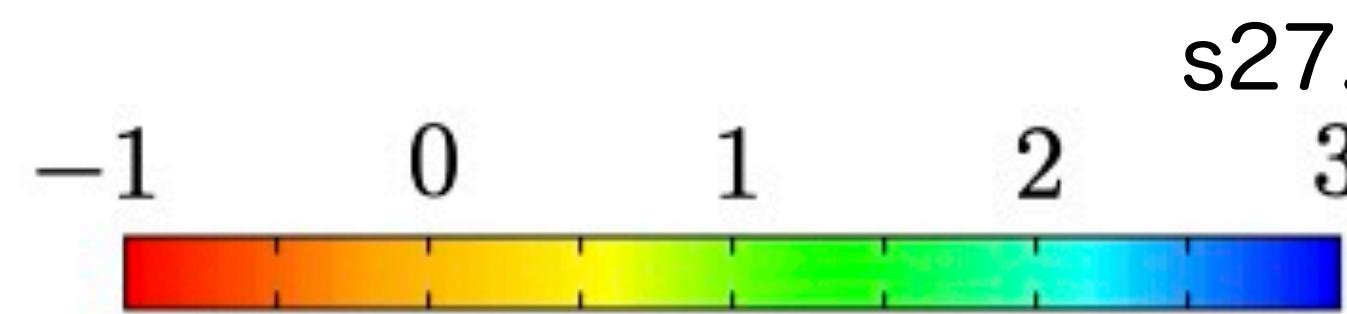
α 効果で磁場に比例した
電流が流れる

α 効果にともなう磁場の指数関数的な増幅

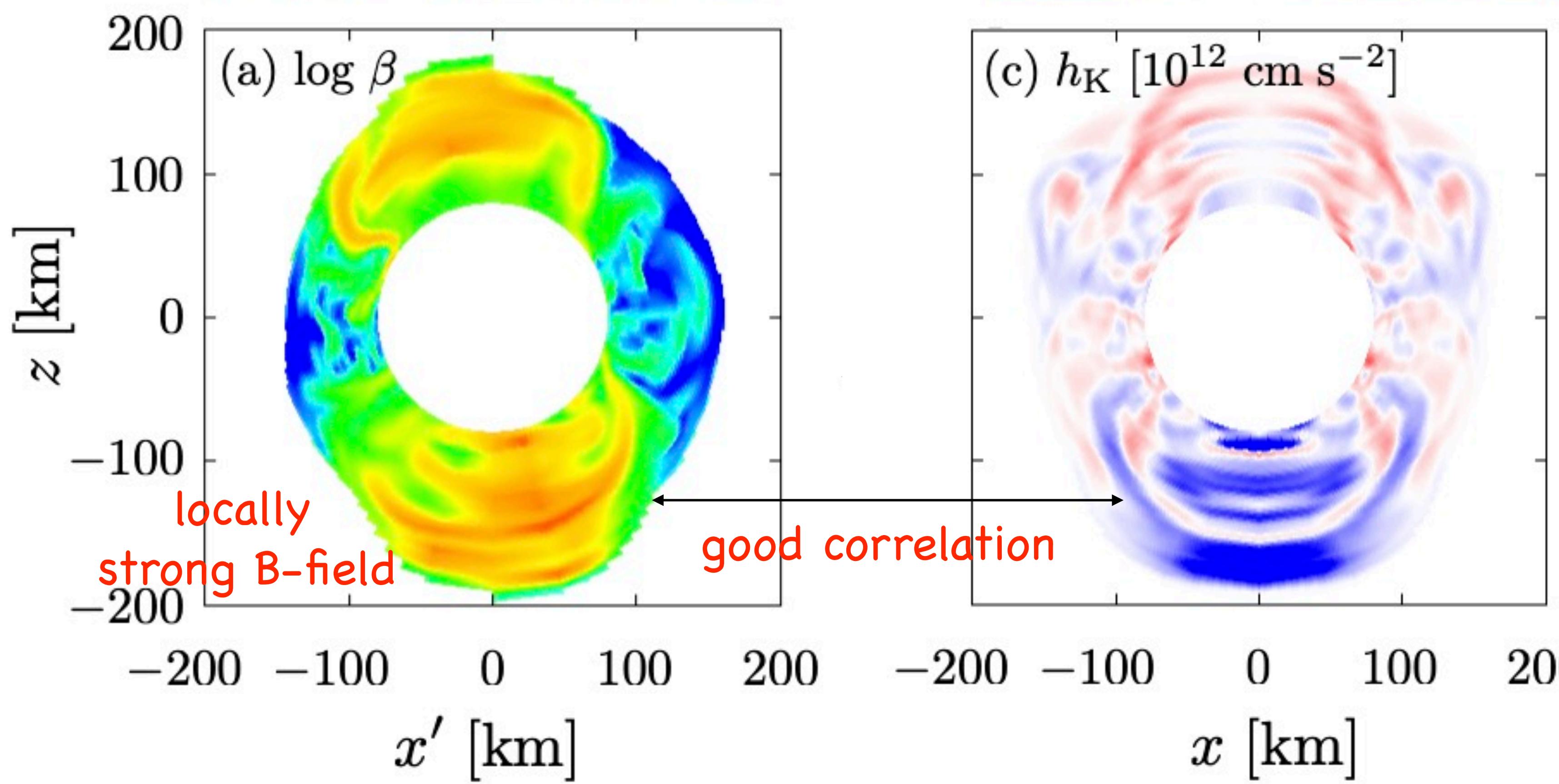
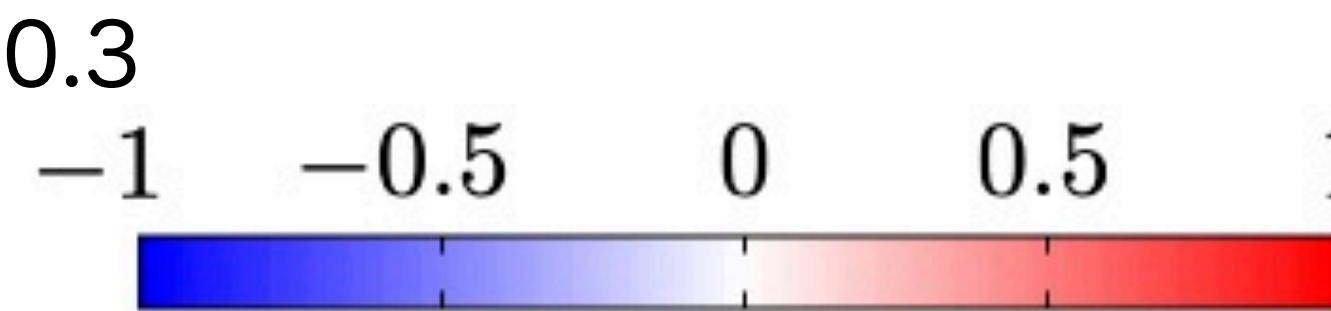


Amplification of the magnetic field

plasma $\beta \equiv P_{\text{gas}}/P_{\text{mag}}$



kinetic helicity $H_K = \langle \mathbf{v}' \cdot \boldsymbol{\omega}' \rangle_\phi$



Magnetic pressure driven explosion

mean field theory

$$\mathbf{v}(r, \theta, \phi) = \langle \mathbf{v} \rangle(r, \theta) + \mathbf{v}'(r, \theta, \phi),$$

$$\mathbf{B}(r, \theta, \phi) = \langle \mathbf{B} \rangle(r, \theta) + \mathbf{B}'(r, \theta, \phi).$$

induction equation:

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{v} \rangle \times \langle \mathbf{B} \rangle - \eta \nabla \times \langle \mathbf{B} \rangle + \epsilon)$$

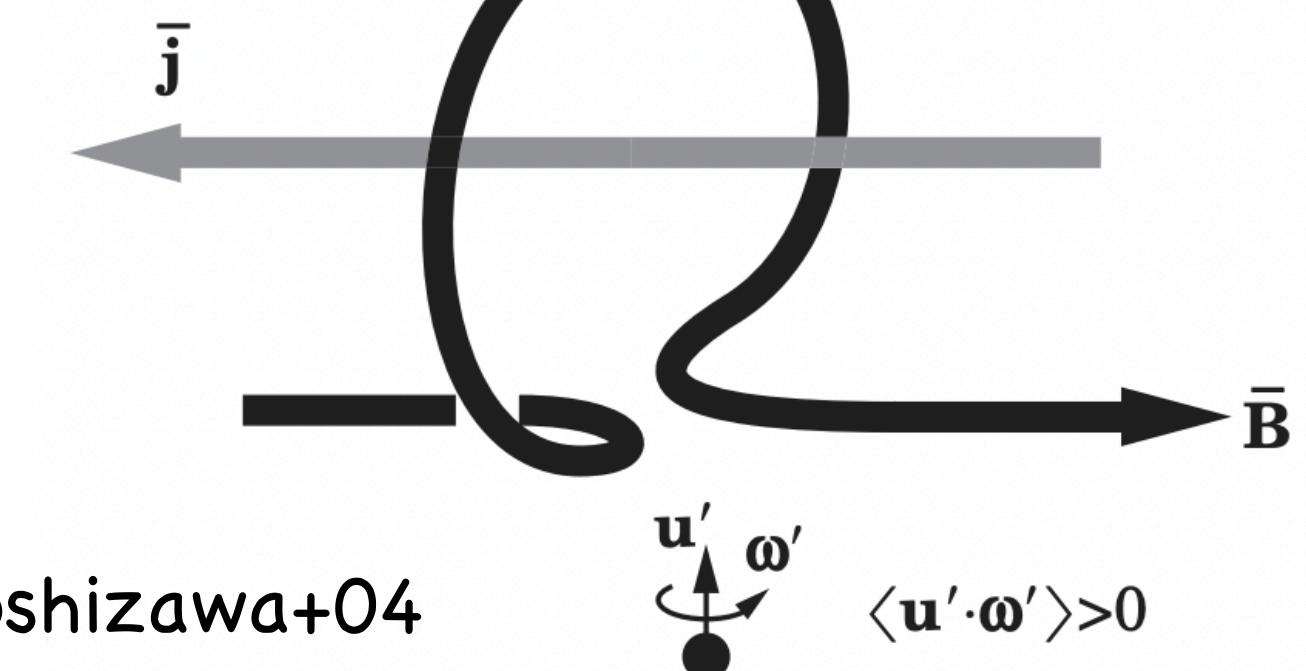
$$\epsilon \equiv \alpha \langle \mathbf{B} \rangle - \eta_t \nabla \times \langle \mathbf{B} \rangle$$

$$\alpha \equiv -\frac{1}{3} \tau_{\text{cor}} h_K$$

$$\eta_t \equiv \frac{1}{3} \tau_{\text{cor}} \langle v'^2 \rangle$$

Brandenburg+05

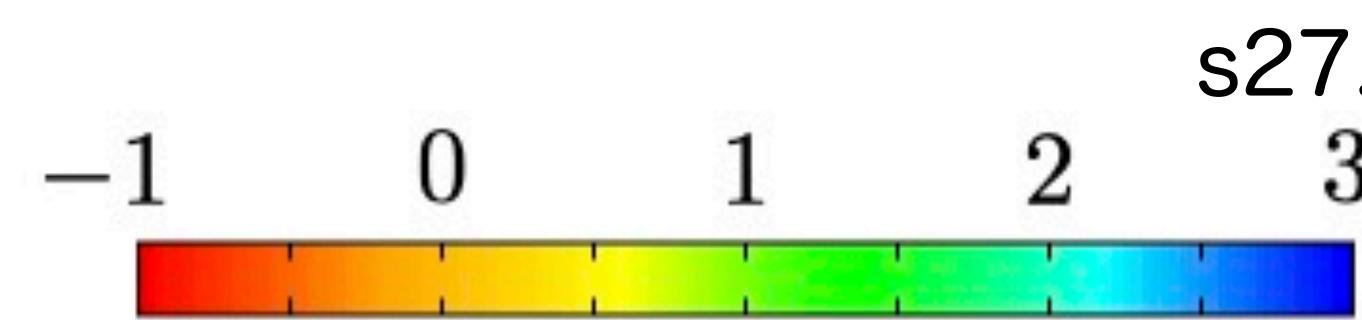
α -effect



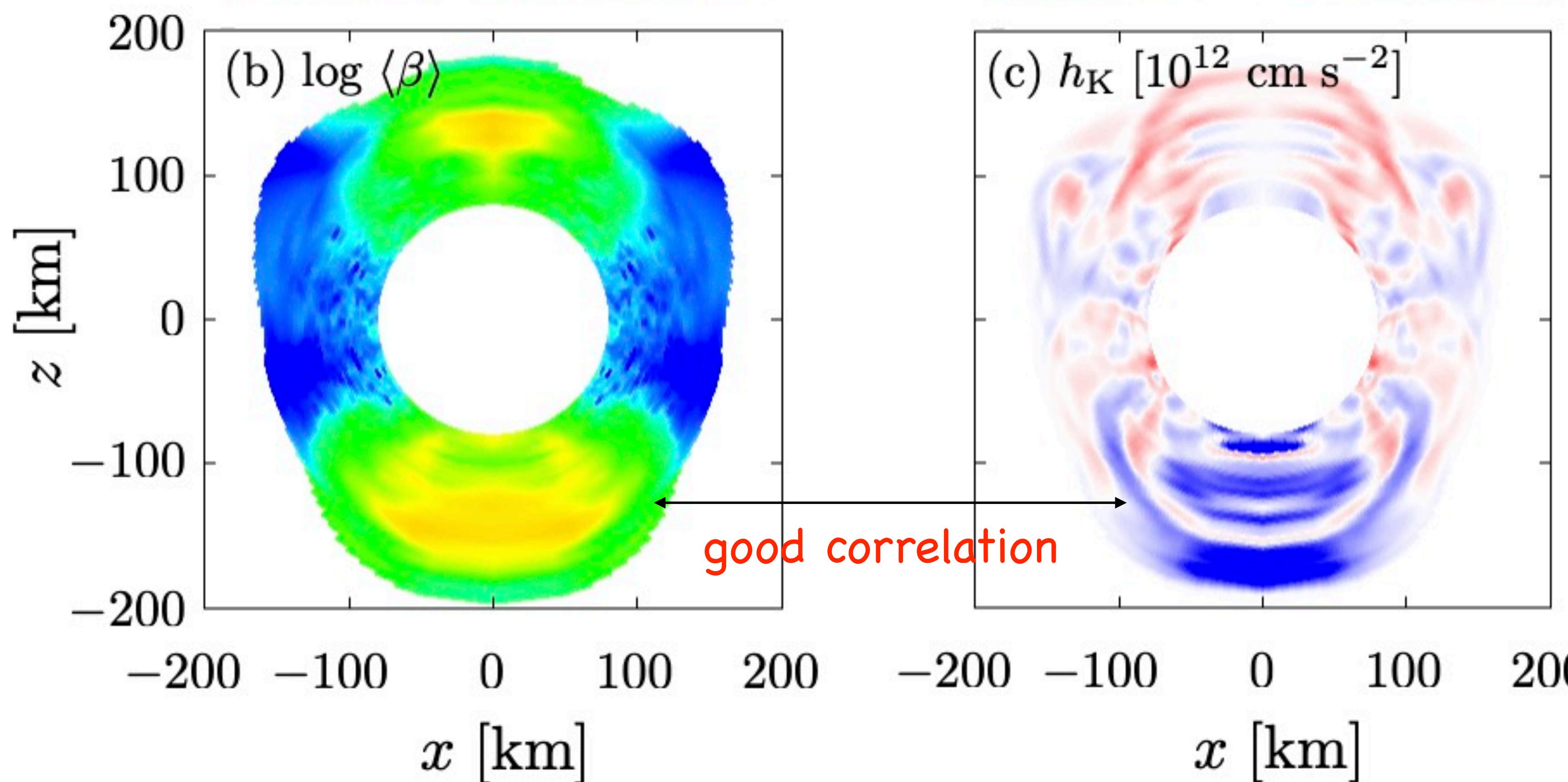
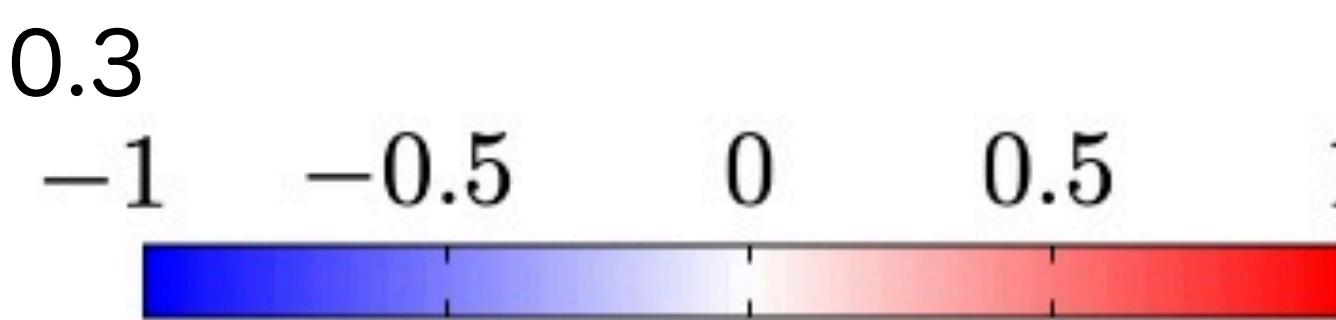
Yoshizawa+04

Amplification of the magnetic field

averaged plasma $\beta \equiv P_{\text{gas}}/P_{\text{mag}}$



kinetic helicity $H_K = \langle \mathbf{v}' \cdot \boldsymbol{\omega}' \rangle_\phi$



Total thermal energy in gain region
is larger than magnetic energy.

mean field theory

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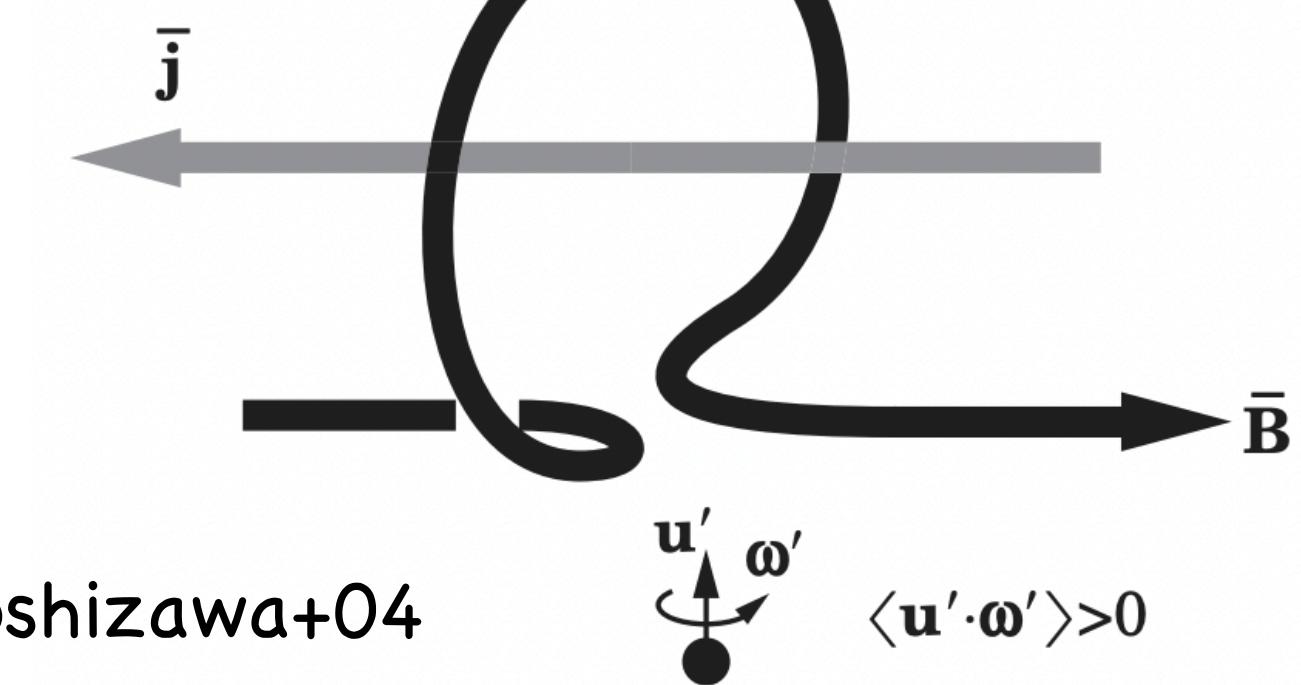
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$$\alpha \equiv -\frac{1}{3} \tau_{\text{cor}} h_K$$

$$\eta_t \equiv \frac{1}{3} \tau_{\text{cor}} \langle v'^2 \rangle$$

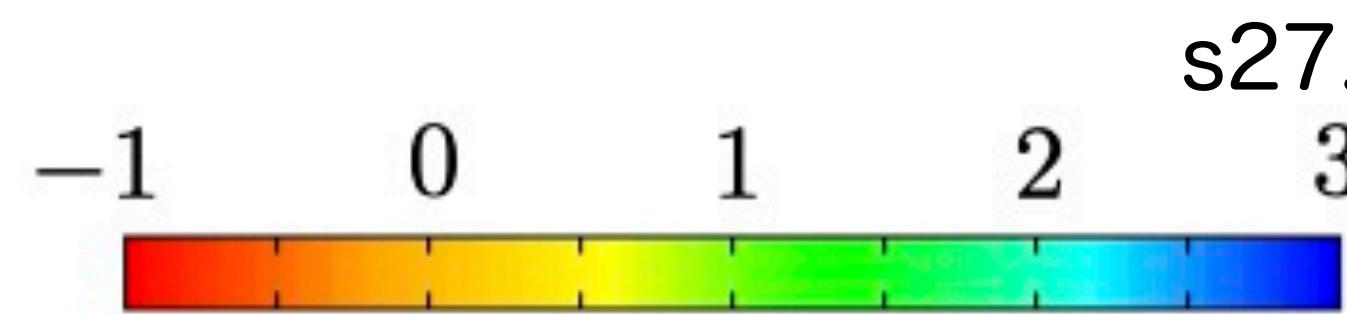
Brandenburg+05

α -effect

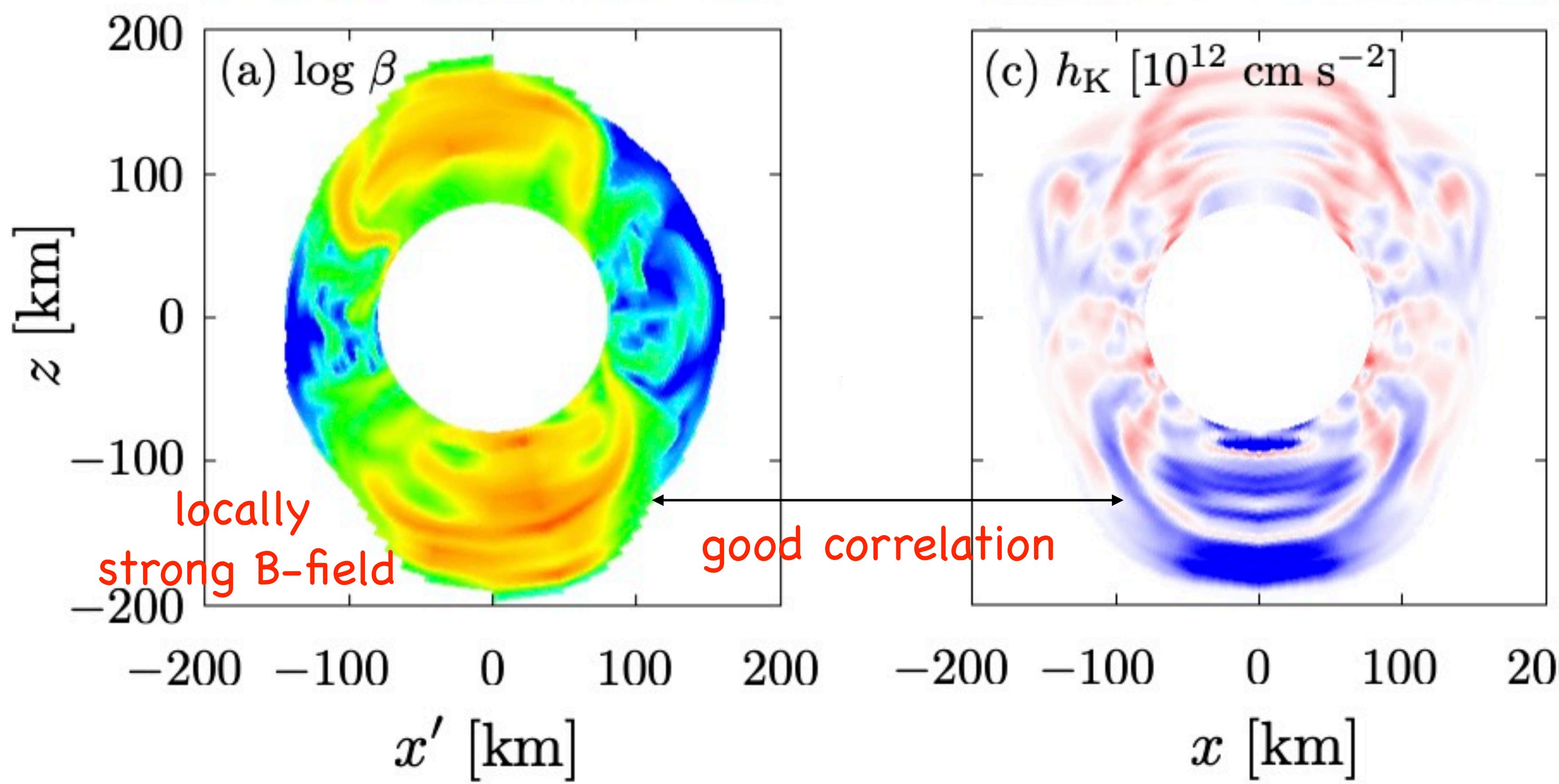
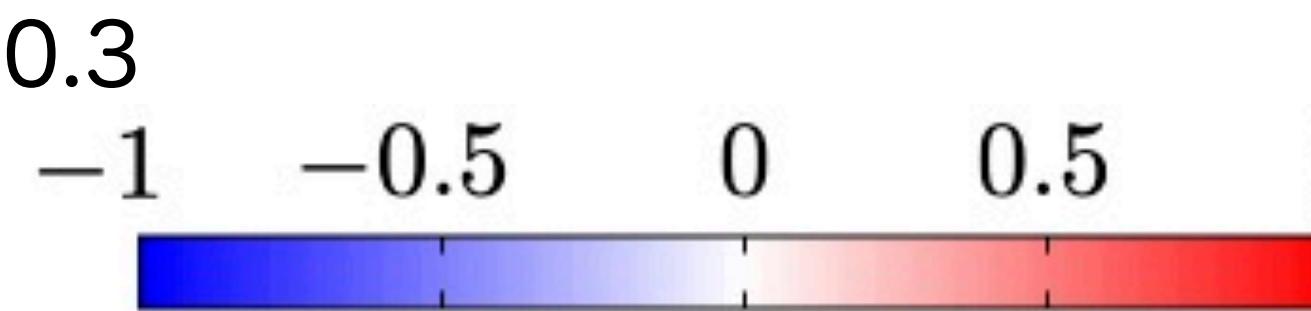


Amplification of the magnetic field

plasma $\beta \equiv P_{\text{gas}}/P_{\text{mag}}$



kinetic helicity $H_K = \langle \mathbf{v}' \cdot \boldsymbol{\omega}' \rangle_\phi$



Magnetic pressure driven explosion

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induction equation:

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{v} \rangle \times \langle \mathbf{B} \rangle - \eta_t \nabla \times \langle \mathbf{B} \rangle) + \epsilon$$

ideal

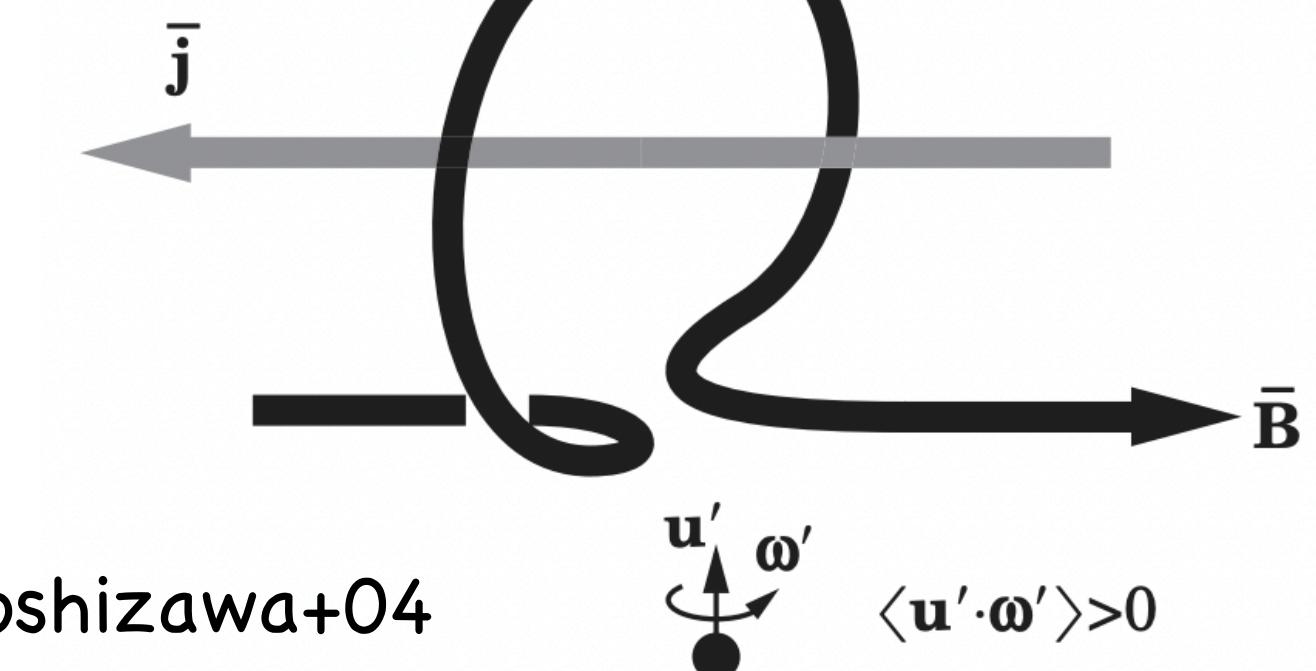
$$\epsilon \equiv \alpha \langle \mathbf{B} \rangle - \eta_t \nabla \times \langle \mathbf{B} \rangle$$

$$\alpha \equiv -\frac{1}{3} \tau_{\text{cor}} h_K$$

$$\eta_t \equiv \frac{1}{3} \tau_{\text{cor}} \langle v'^2 \rangle$$

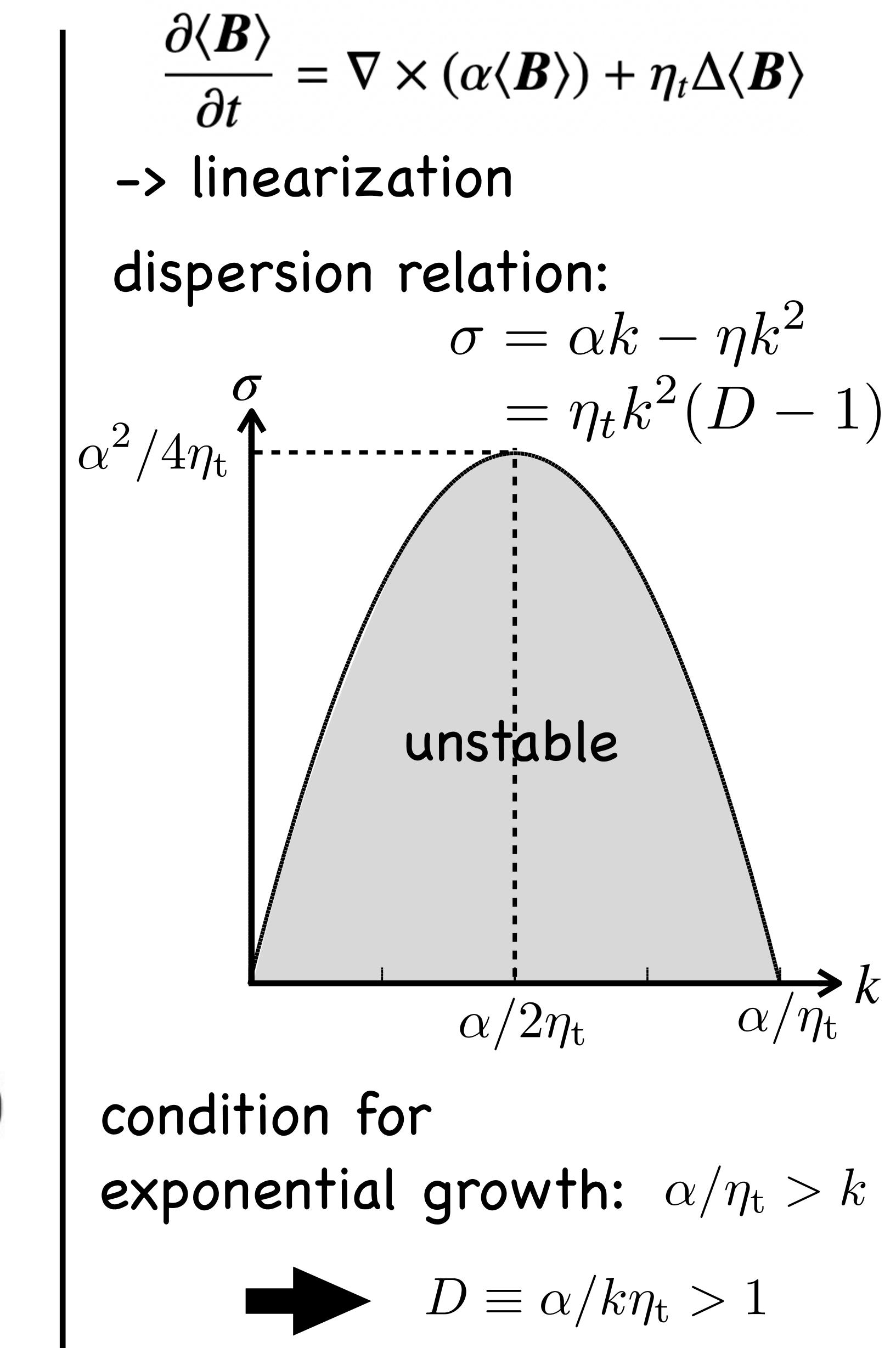
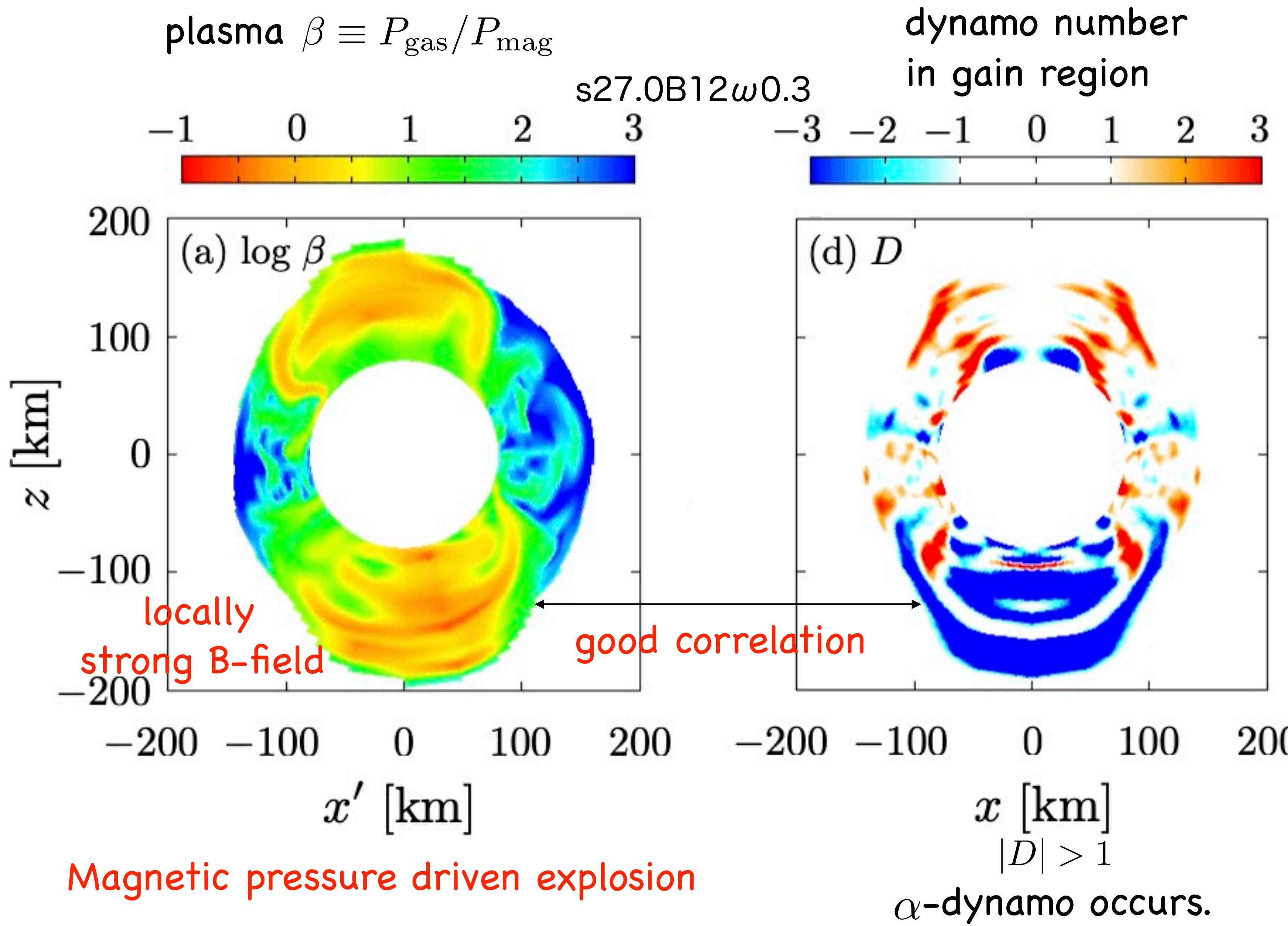
Brandenburg+05

α -effect

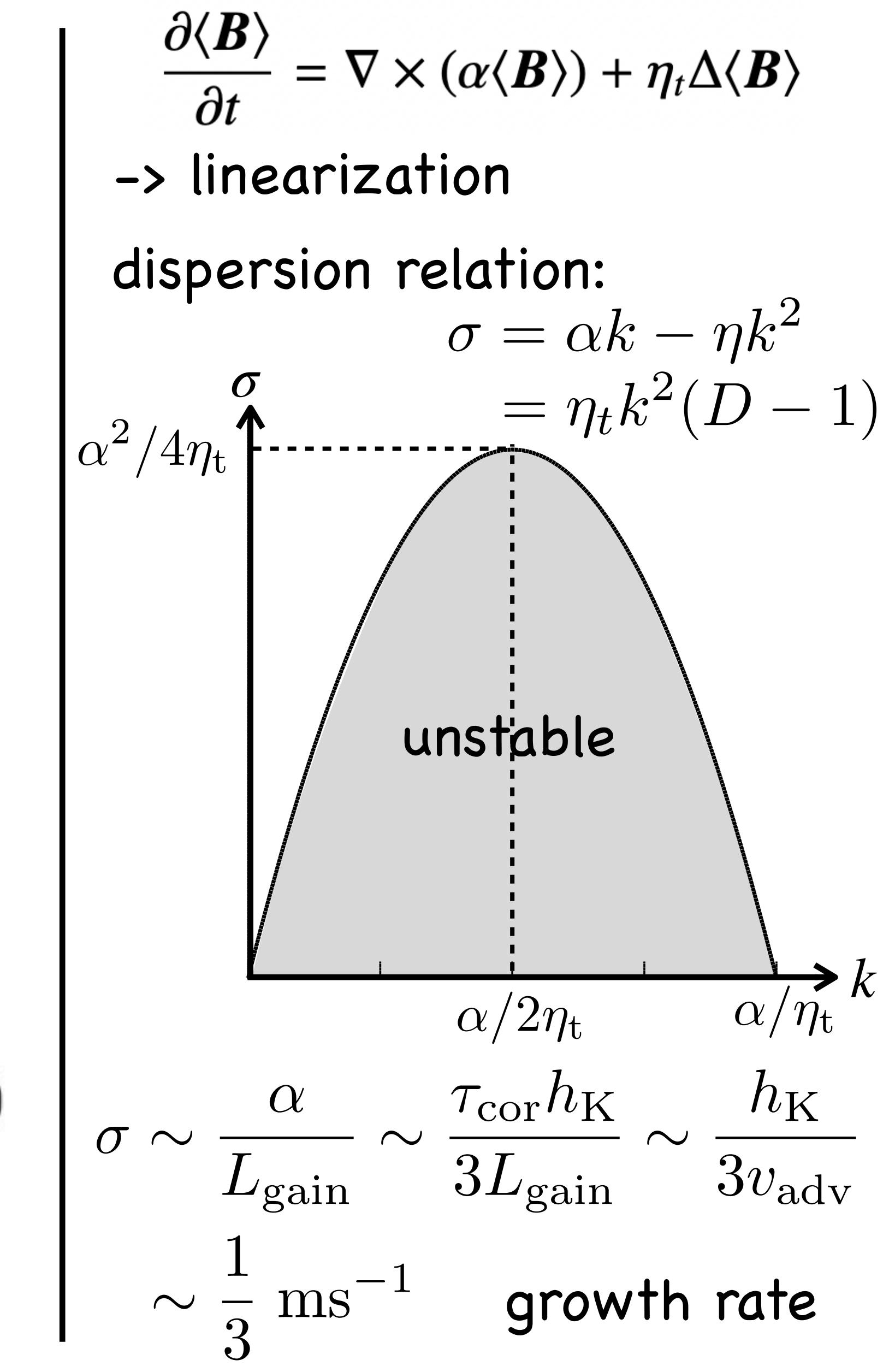
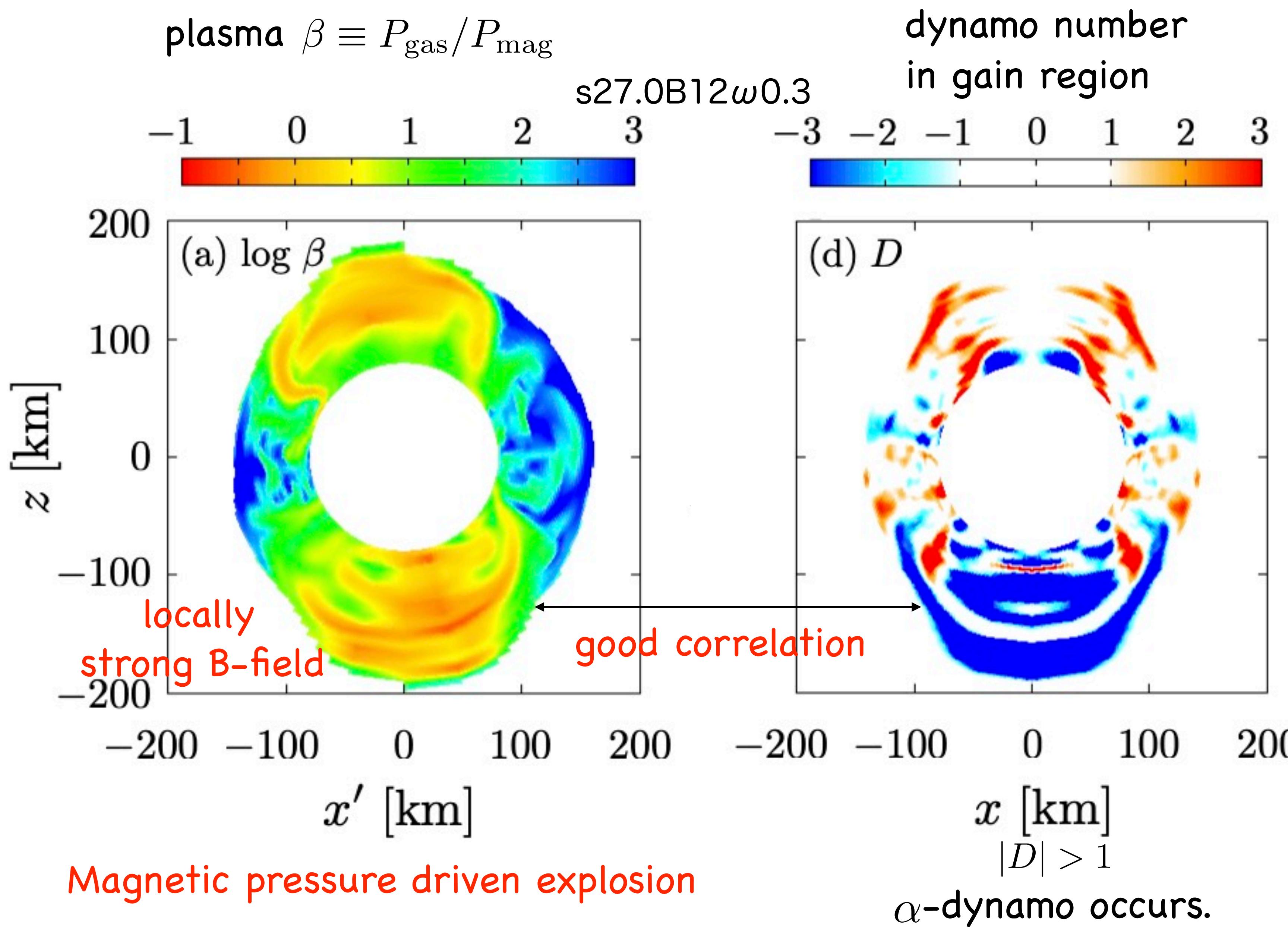


Yoshizawa+04

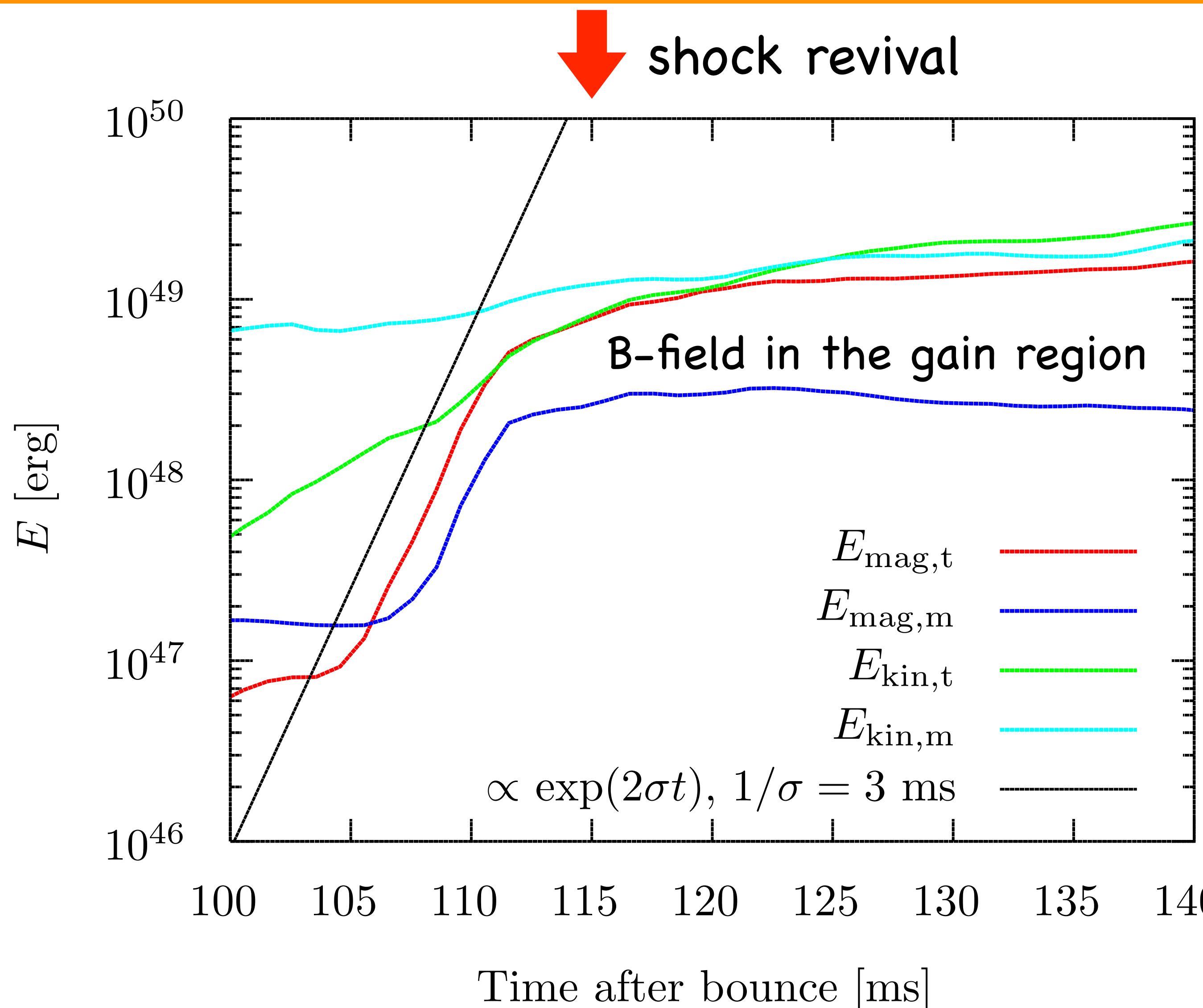
Amplification of the magnetic field



Amplification of the magnetic field



Growth rate of the magnetic energy



Magnetic pressure amplified due to α -effect is responsible for fast explosion in our rotating model.

Mean magnetic field is amplified by α -effect.

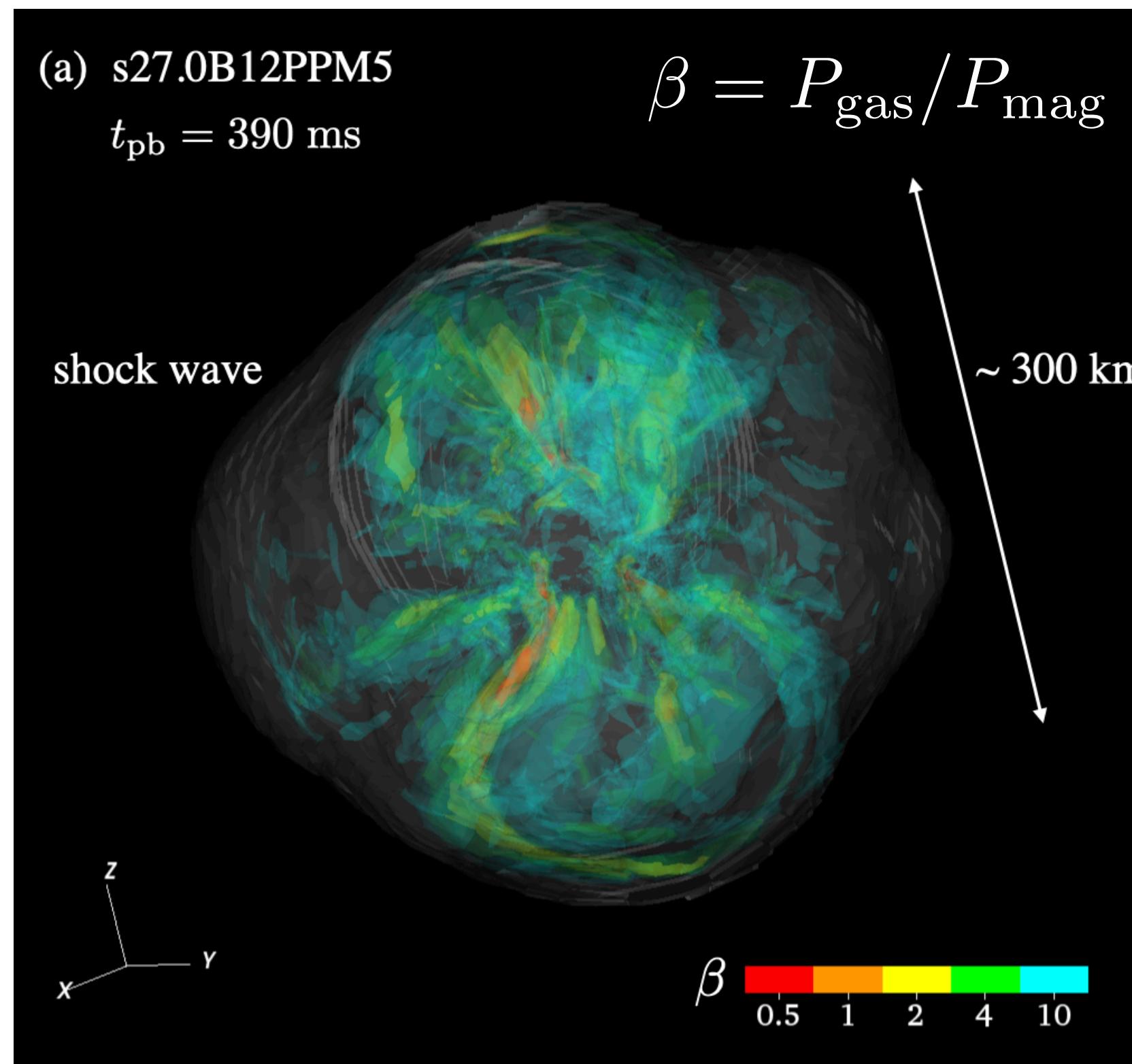
In addition, turbulent magnetic field is also amplified via α -dynamo action of mean magnetic field.

Induction equation for turbulent magnetic field:

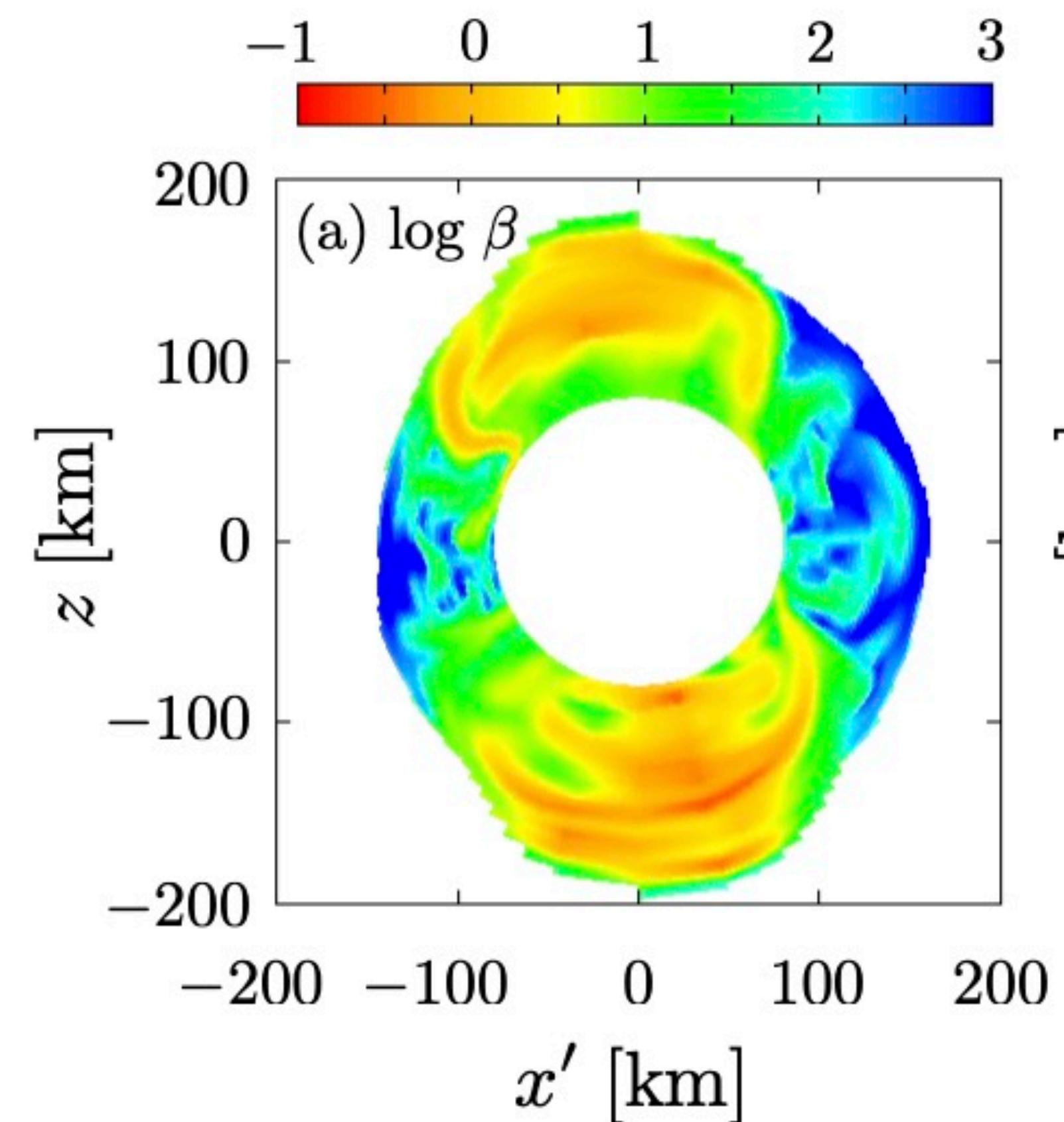
$$\frac{\partial \mathbf{B}'}{\partial t} = \nabla \times (\mathbf{v}' \times \underline{\langle \mathbf{B} \rangle})$$

mean magnetic field

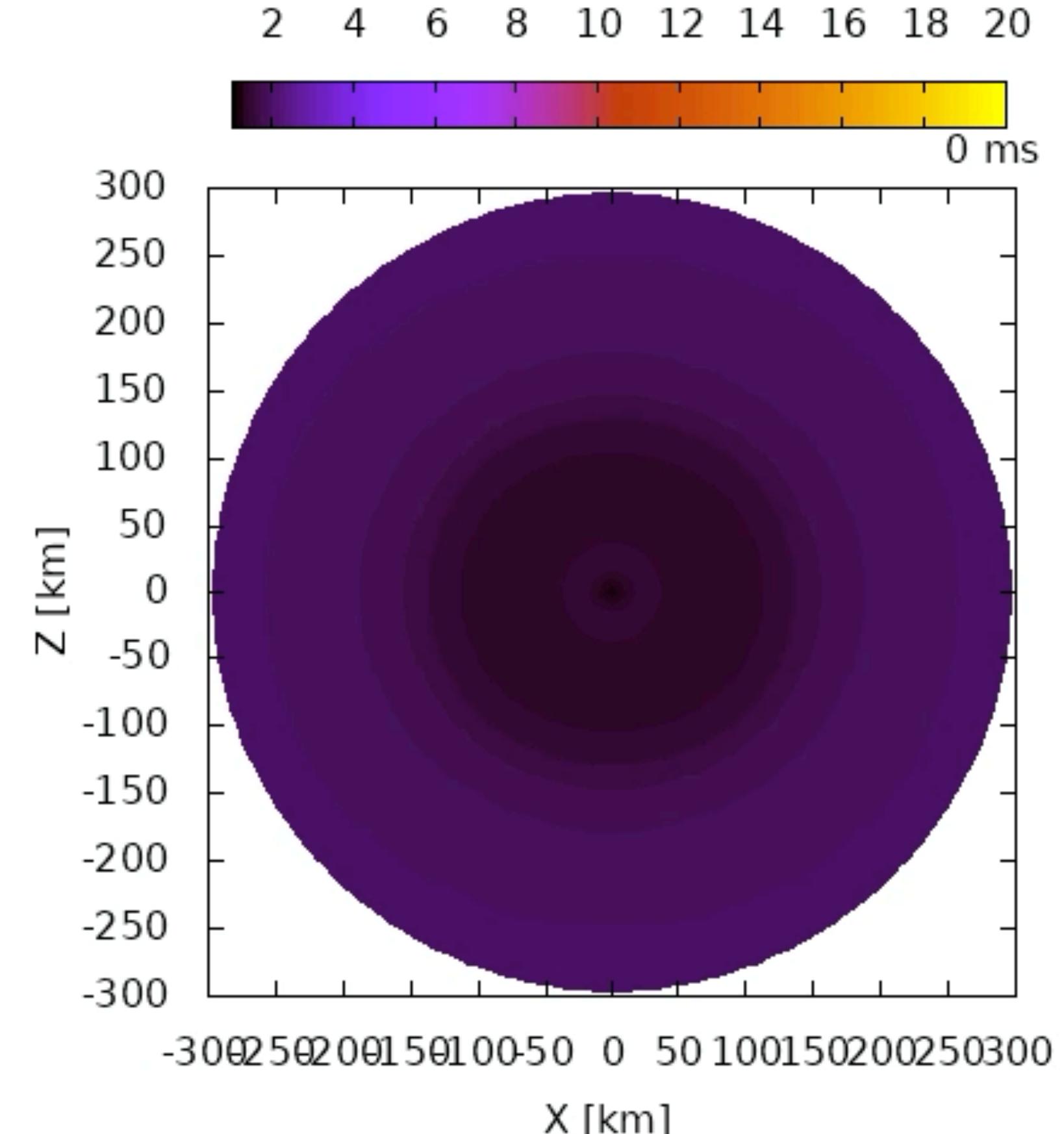
Strong field models



non-rotating



mildly-rotating ($\omega=0.3 \text{ rad/s}$)
 α -dynamo occurs.



rapidly-rotating ($\omega=1 \text{ rad/s}$)
jet

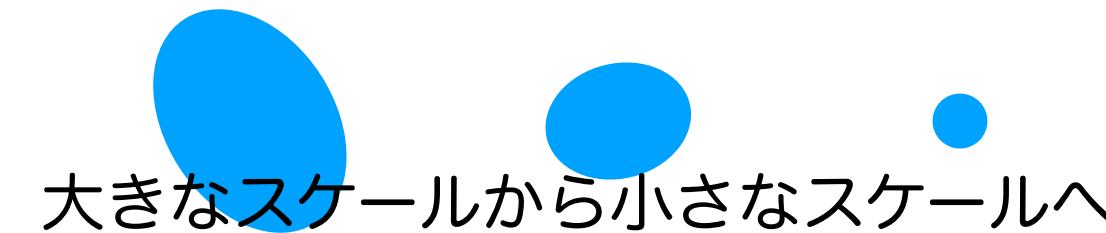
Obergaulinger+ 21, 22,
Bugli+21, 23,

カイラル効果を導入するモチベーション

ニュートリノはカイラル対称性を破った粒子

1. ニュートリノ加熱メカニズムにおけるキープロセス：乱流 <- 加熱効率の向上

- 乱流のdirect cascade



→ 爆発にネガティブ

- もし乱流がinverse cascadeを起こすなら？



→ 爆発にポジティブ

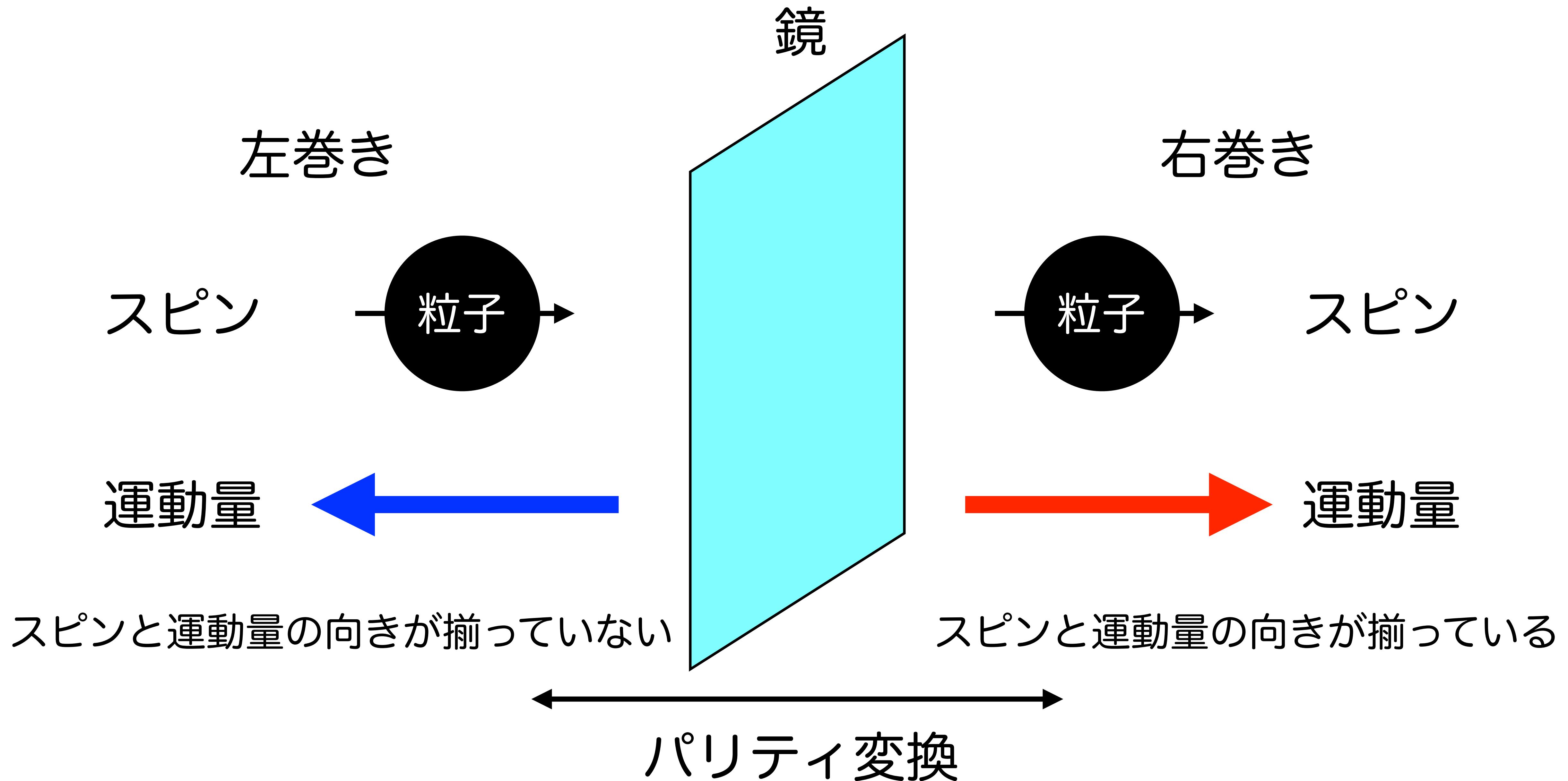
← カイラル乱流（磁気ヘリシティ）

ローカルボックス
シミュレーション
でカイラルプラズマ不安定性の基礎的な性質を調べる

2. カイラルプラズマ不安定性の成長により強磁場の生成

→ ダイナミクスを大きく変える可能性

右巻き左巻きの概念



右巻き左巻きの概念

左手



右手



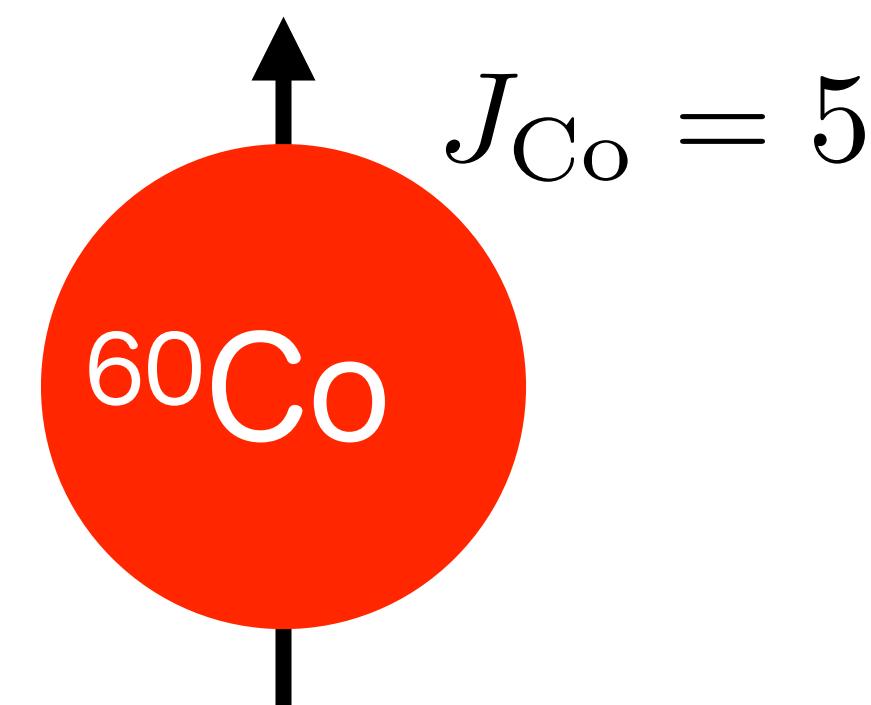
右巻き左巻きの概念



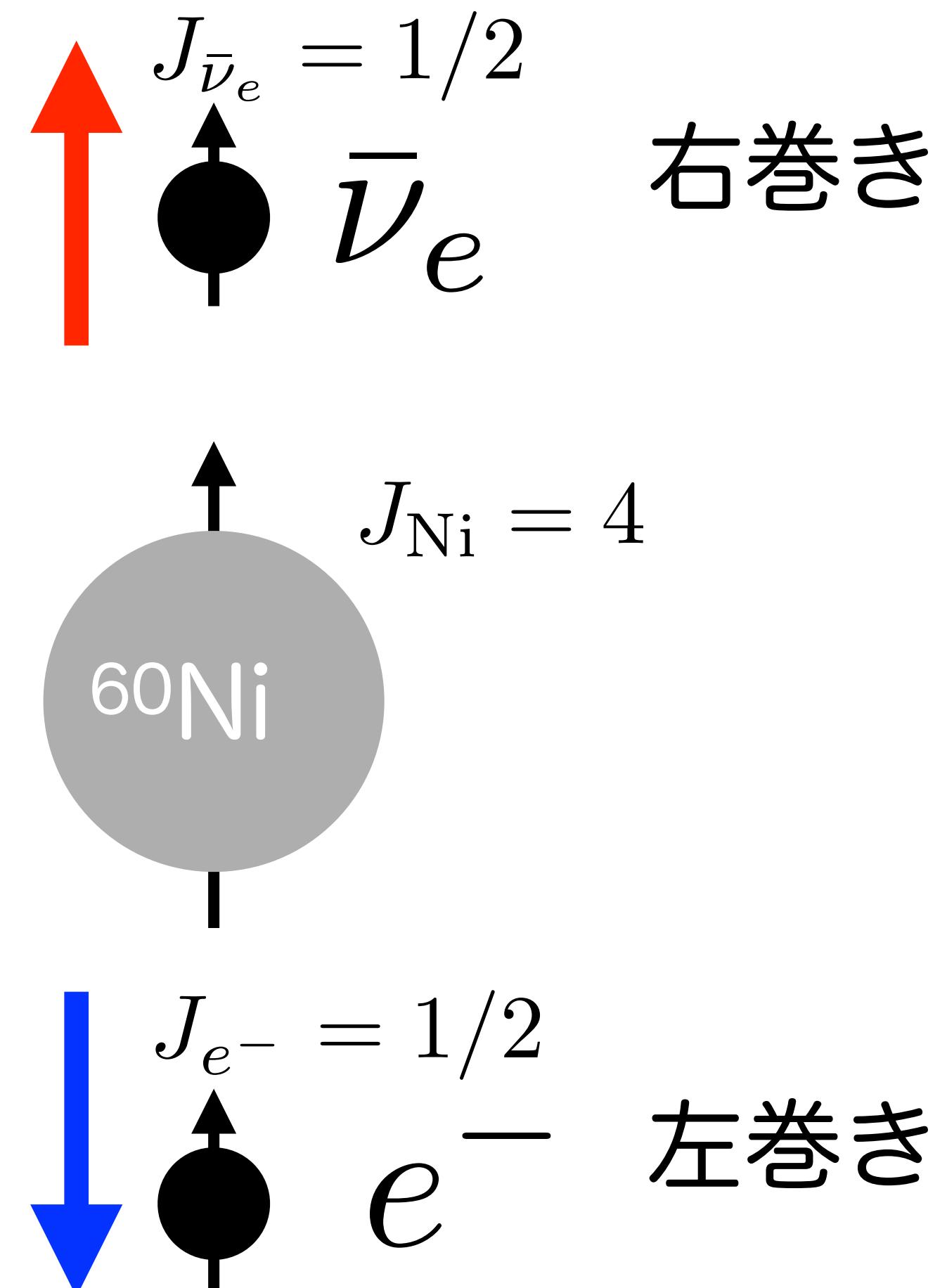
ニュートリノは左巻き

Wuの実験(1956)：コバルト60の崩壊

磁場



磁場



実験結果：電子は下向きのみ

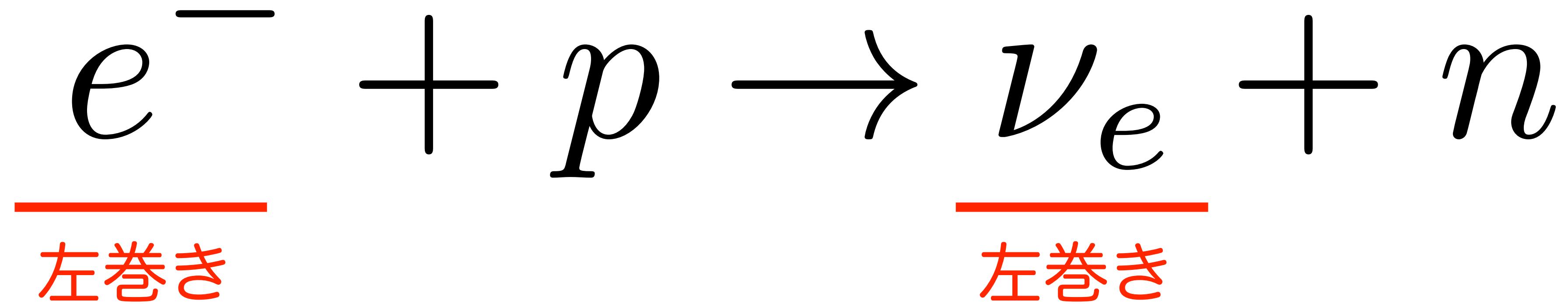
左巻きの反ニュートリノがあるなら電子は
上向きに出てもよい

反ニュートリノは右巻きのみ

ニュートリノは左巻きのみ

超新星におけるカイラル効果

電子捕獲反応



選択的に左巻き電子が捕獲されるため右巻き電子過剰状態が生じる

-> カイラリティインバランスが生じる

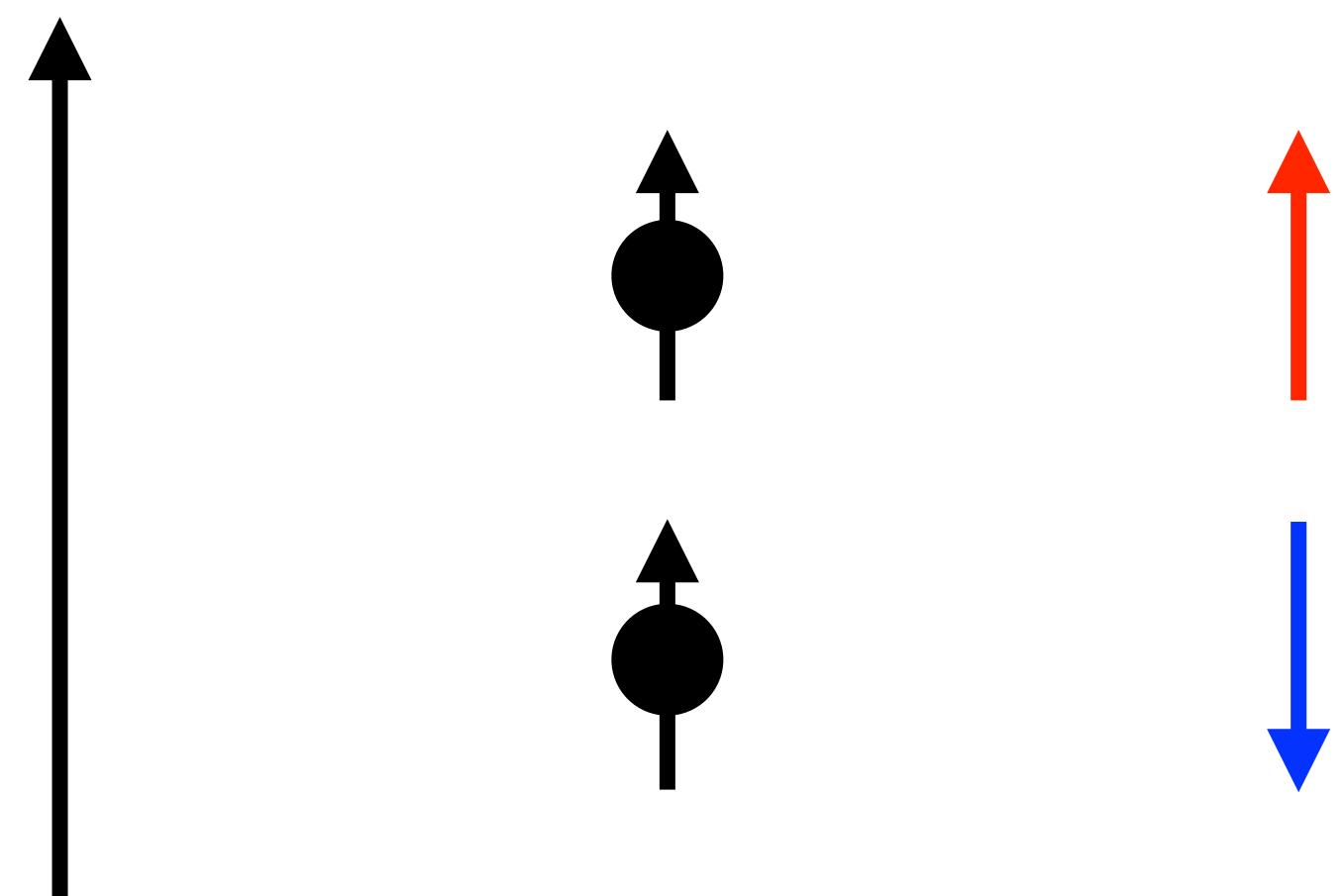
カイラル効果が超新星のダイナミクスに影響を与える？

カイラル磁気効果

磁場B スピン

運動量

Vilenken 80, Nielsen & Ninomiya 83, Fukushima+08



右巻き電子

左巻き電子

右巻き電子のカレント : $J_R = \frac{\mu_R}{4\pi^2} B$

左巻き電子のカレント : $J_L = -\frac{\mu_L}{4\pi^2} B$

右巻き電子の化学ポテンシャルと
左巻き電子の化学ポテンシャルに
差があると磁場に比例した電流が
流れれる

カイラル磁気効果 (CME)

$$J_R + J_L = \frac{1}{4\pi^2} (\mu_R - \mu_L) B$$
$$\xi_B$$

自然単位系

$$c = e = \hbar = 1$$

“通常物質”の電流のパリティ対称性

オームの法則：

電流（極性ベクトル）

電場（極性ベクトル）

$$J = \sigma E$$

電気伝導度（定数）

パリティ変換： $J \rightarrow -J, \quad E \rightarrow -E, \quad \sigma \rightarrow \sigma$

この変換のもとでオームの法則はパリティ対称性がある

磁場に比例する電流？

オームの法則のアナロジー

電流（極性ベクトル）

磁場（軸性ベクトル）

$$J = \sigma_m B$$

定数

パリティ変換： $J \rightarrow -J, B \rightarrow B, \sigma_m \rightarrow \sigma_m = 0$

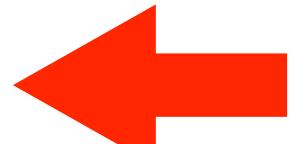
通常の物質 <- 要請

右巻き粒子数と左巻き粒子数に差があるような

(カイラリティインバランスがある) 物質 (カイラル物質)

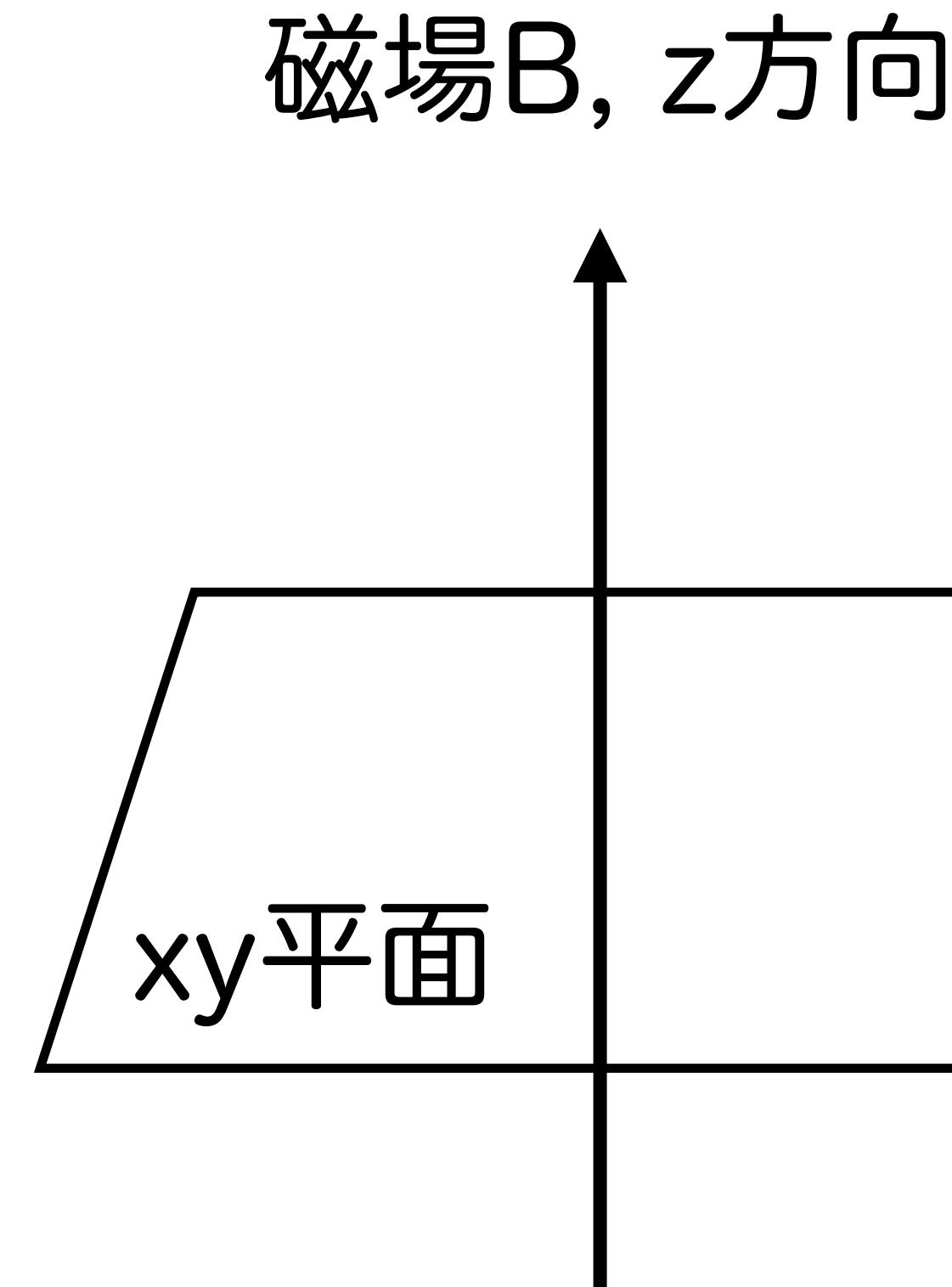
$\sigma_m \rightarrow -\sigma_m$

パリティ対称性を破る様な物質



準備：右巻きと左巻きの粒子数差の発展方程式

Nielsen & Ninomiya 83

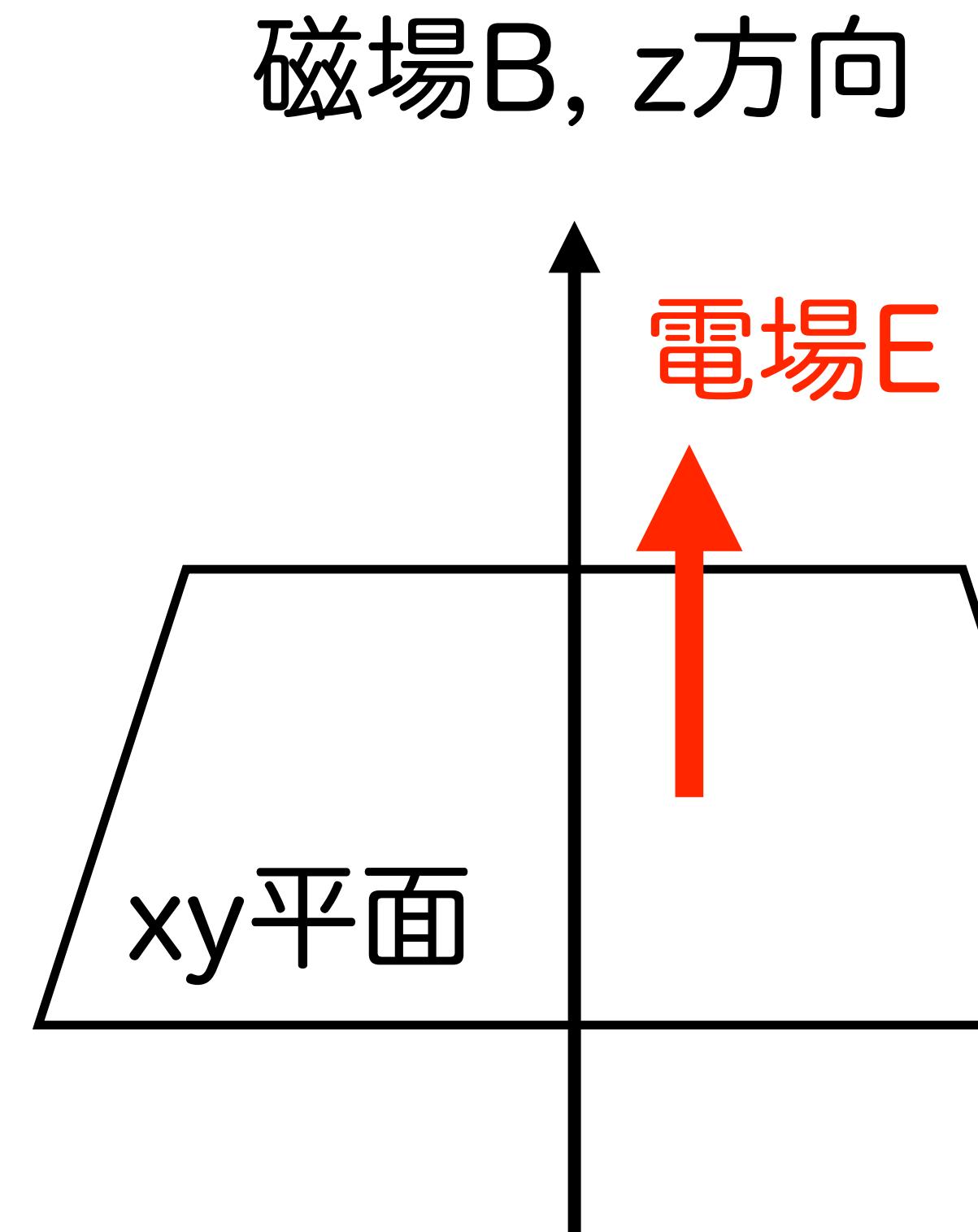


- z方向の運動量は連続
- xy方向には量子化 (Landau準位)
- Zeeman効果
- 分散関係：
 - $E_n^2 = p_z^2 + (2n + 1)B - 2BS_z,$
 - $n = (0, 1, 2, \dots), S_z = \pm 1/2$
- 最低Landau準位：

$$E_0 = \pm p_z$$

準備：右巻きと左巻きの粒子数差の発展方程式

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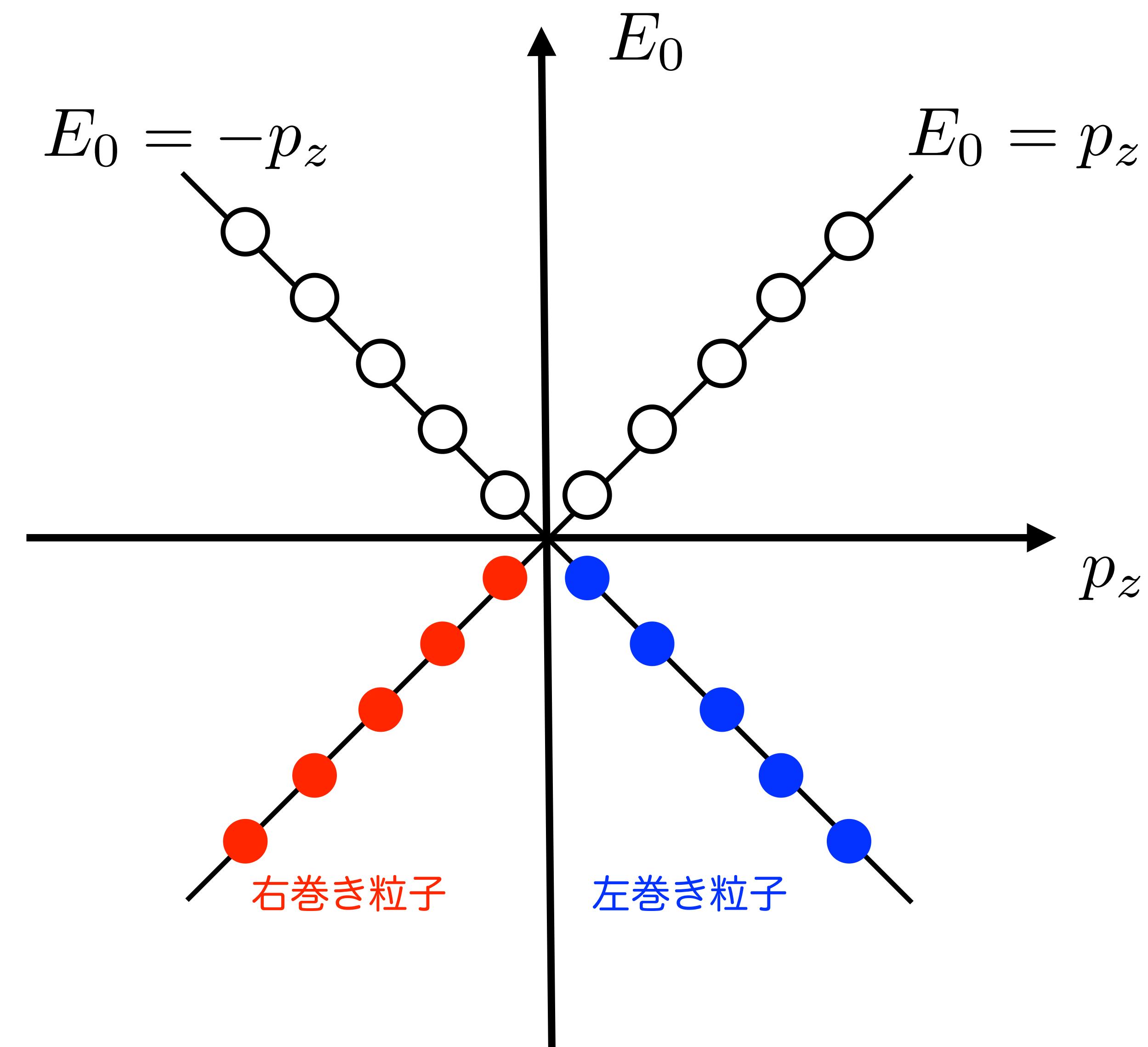
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準備：右巻きと左巻きの粒子数差の発展方程式

Nielsen & Ninomiya 83



準備：右巻きと左巻きの粒子数差の発展方程式

- z 方向に長さ L の周期境界条件

- 運動量を離散化

- 運動量の間隔: $2\pi/L$

- 右巻き粒子数の増加分 :

$$\Delta N_R = \frac{\Delta p_z}{2\pi/L} \int \left[\frac{B}{2\pi} \right] dx dy$$

磁場と垂直な方向の単位面積
あたりのLandau縮重重度

- 左巻き粒子数の増加分 :

$$\Delta N_L = -\frac{\Delta p_z}{2\pi/L} \int \left[\frac{B}{2\pi} \right] dx dy$$

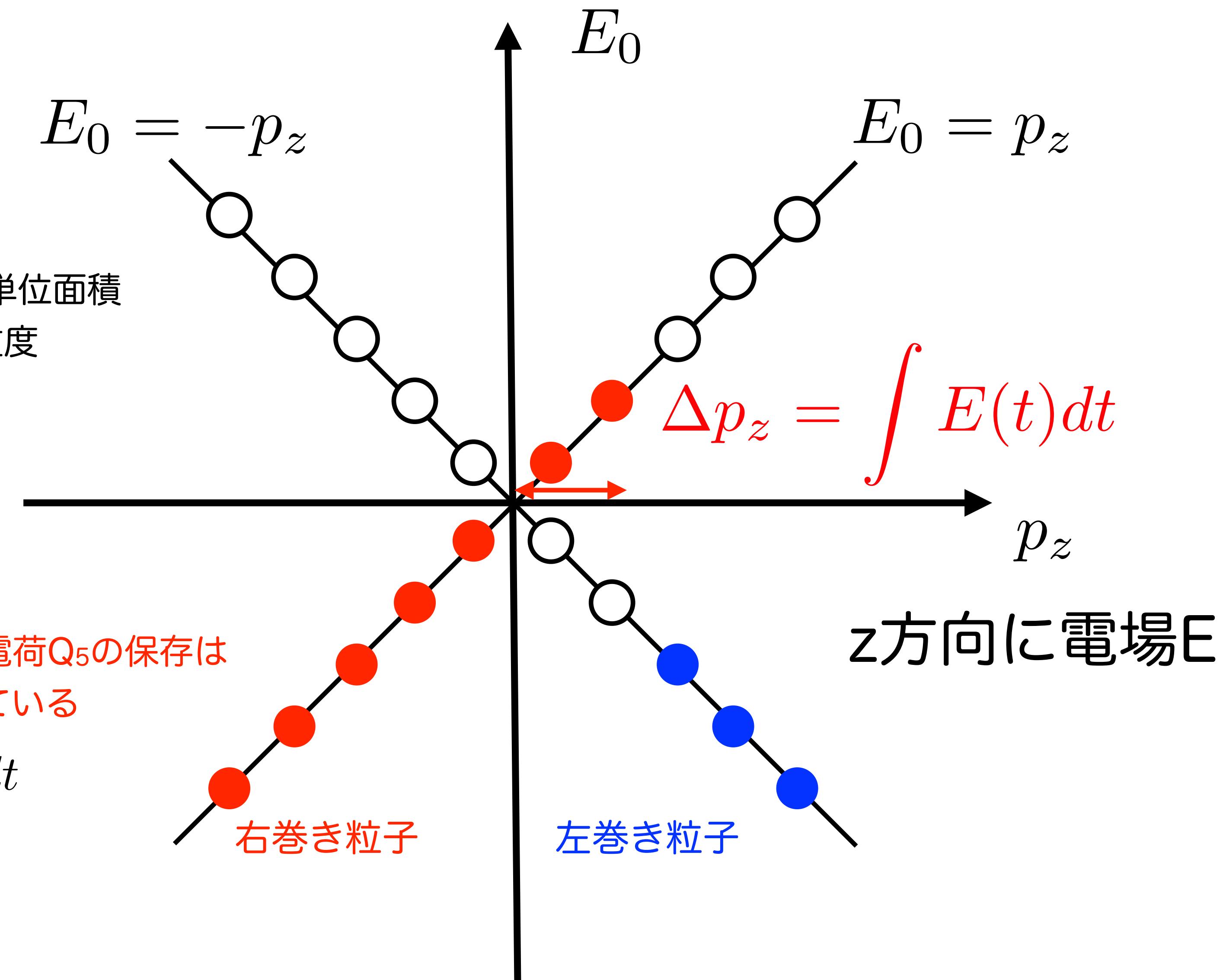
- 右巻き左巻き粒子数の増加分の差 :

$$\Delta Q_5 = \Delta N_R - \Delta N_L = \frac{1}{2\pi^2} \int E B dx dy dz dt$$

- 右巻き左巻き粒子数の増加分の和 :

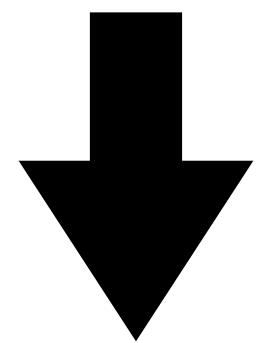
$$\Delta Q = \Delta N_R + \Delta N_L = 0$$

Nielsen & Ninomiya 83



準備：右巻きと左巻きの粒子数差の発展方程式

anomaly equation: $\partial_\mu j_5^\mu = \frac{1}{2\pi^2} \mathbf{B} \cdot \mathbf{E}$



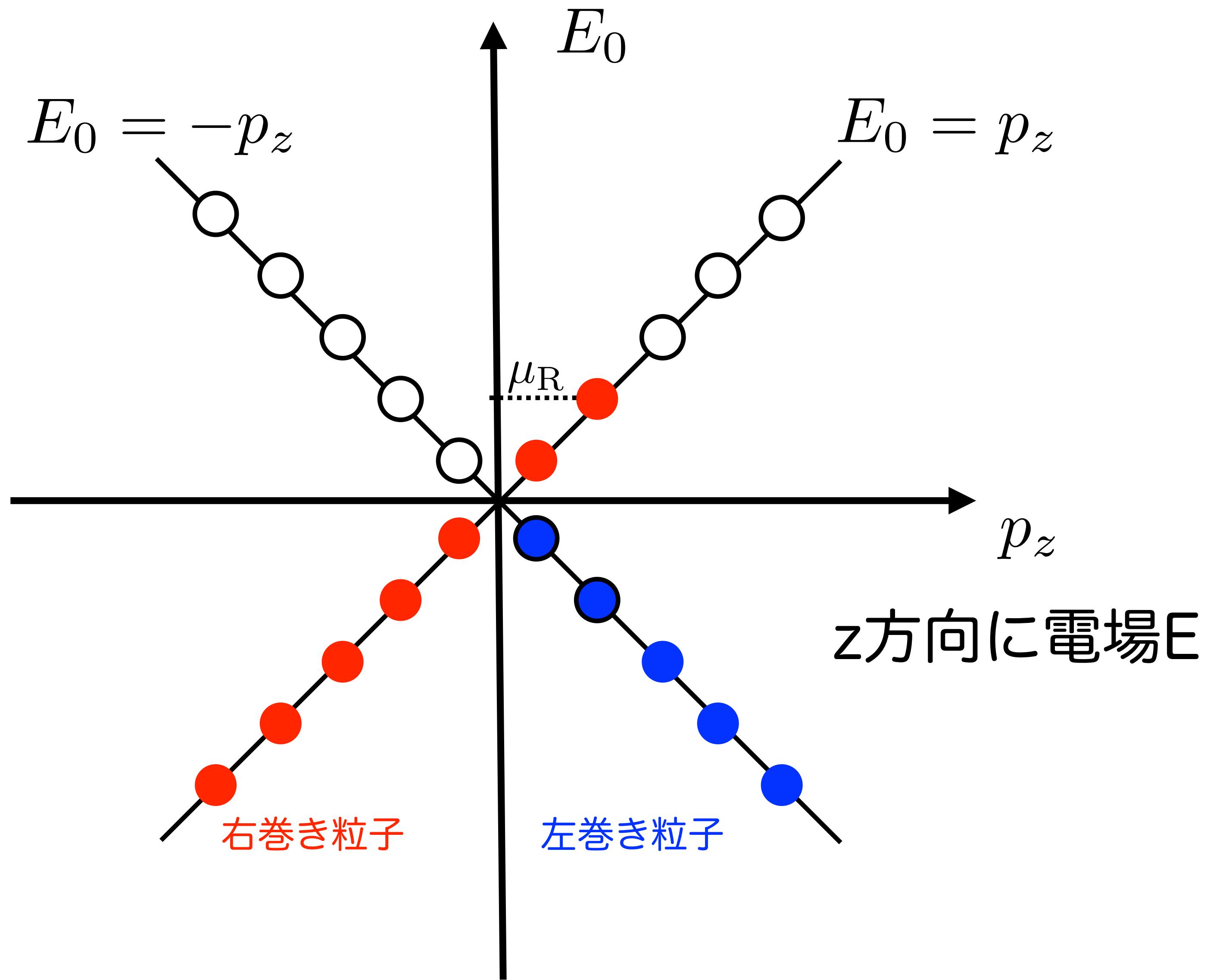
ヘリシティの観点

全ヘリシティの保存: $\frac{d}{dt} \left(Q_5 + \frac{H_{\text{mag}}}{4\pi^2} \right) = 0 \quad H_{\text{mag}} \equiv \int d^3x \mathbf{A} \cdot \mathbf{B}$

磁気ヘリシティ

カイラル磁気効果

Nielsen & Ninomiya 83

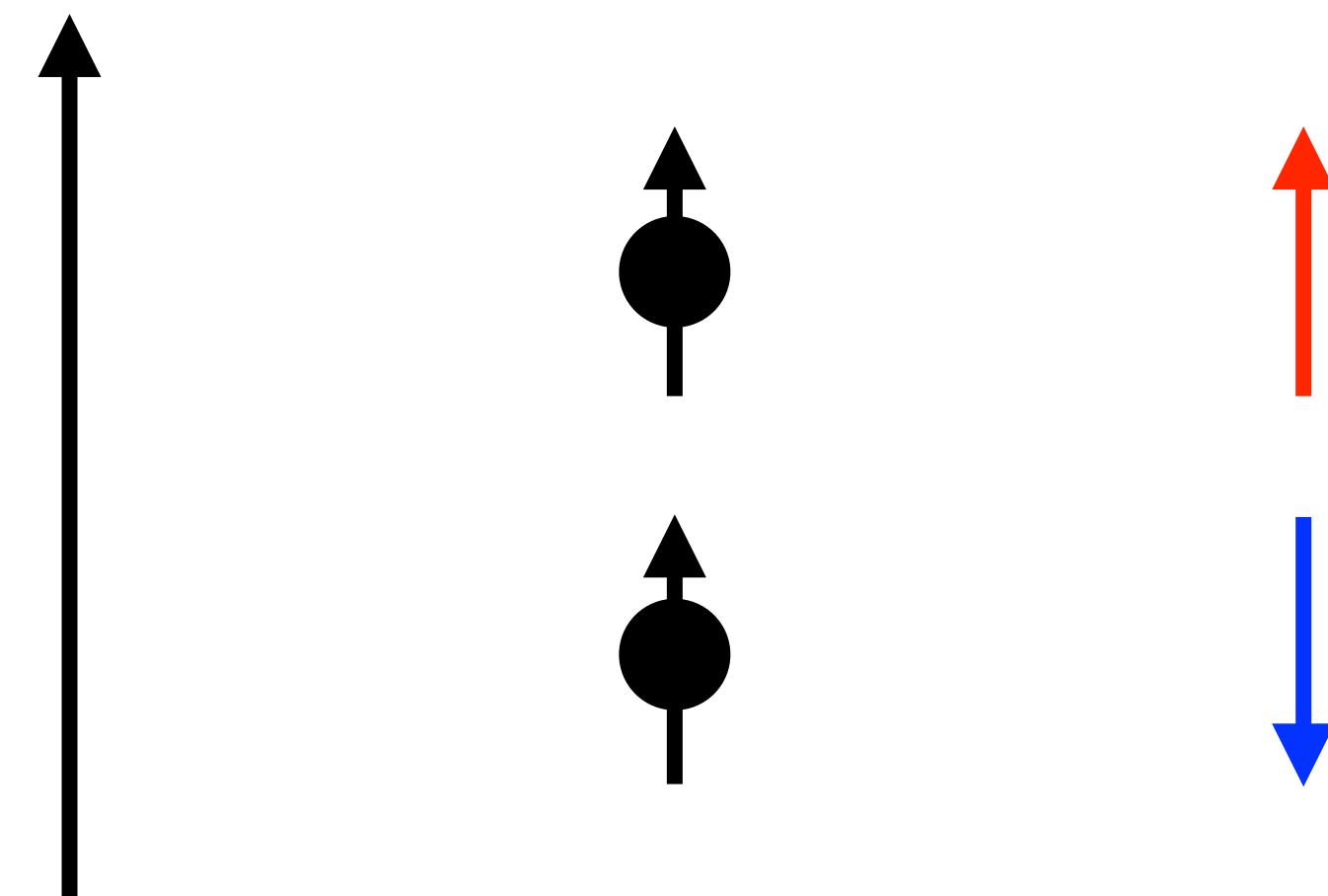


磁場と垂直な方向の単位面積
あたりのLandau縮重重度

$$j_R^z = \frac{B}{2\pi} \int_0^{\mu_R} \frac{dp_z}{2\pi} = \frac{\mu_R}{4\pi^2} B$$

カイラル効果：カイラル磁気効果

磁場B スピン 運動量



Vilenken 80, Nielsen & Ninomiya 83, Fukushima+08

右巻き電子 右巻き電子のカレント : $J_R = \frac{\mu_R}{4\pi^2} B$
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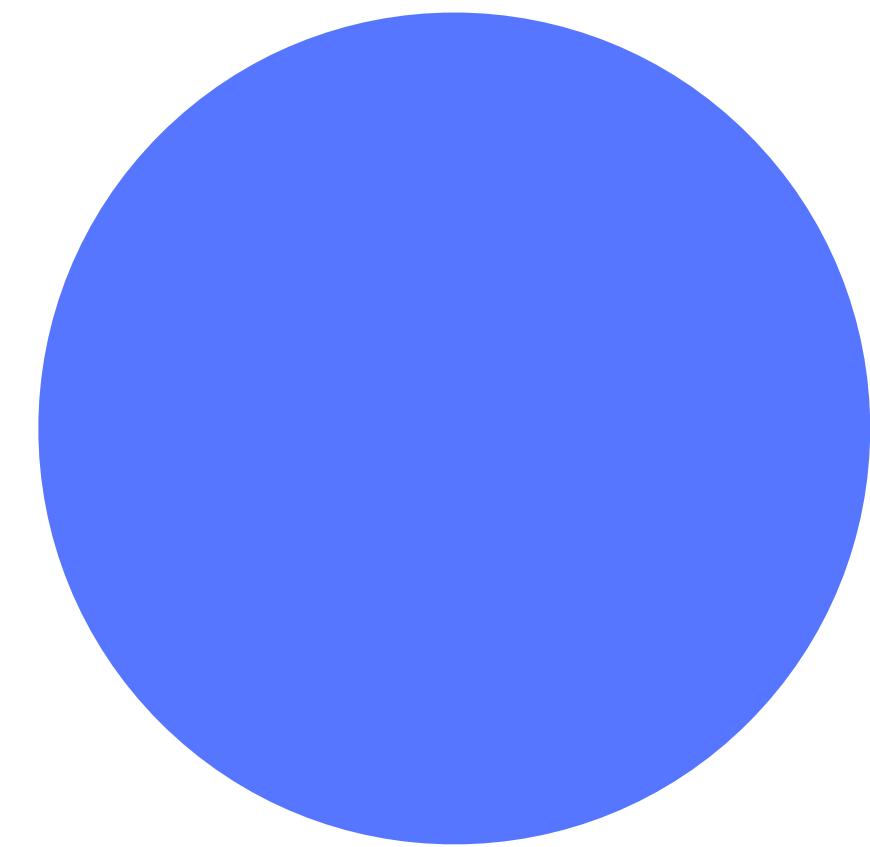
$$J_R + J_L = \frac{1}{4\pi^2} (\mu_R - \mu_L) B$$
$$\xi_B$$

自然単位系
 $c = e = \hbar = 1$

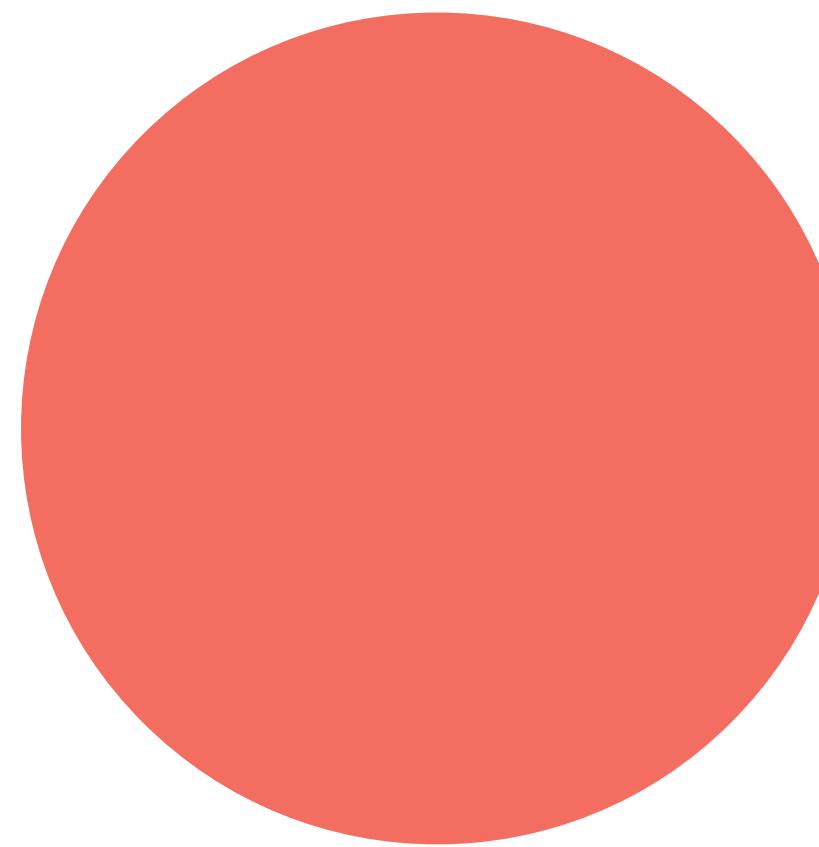
エネルギー収支の観点から再考

Nielsen & Ninomiya 83

フェルミ球



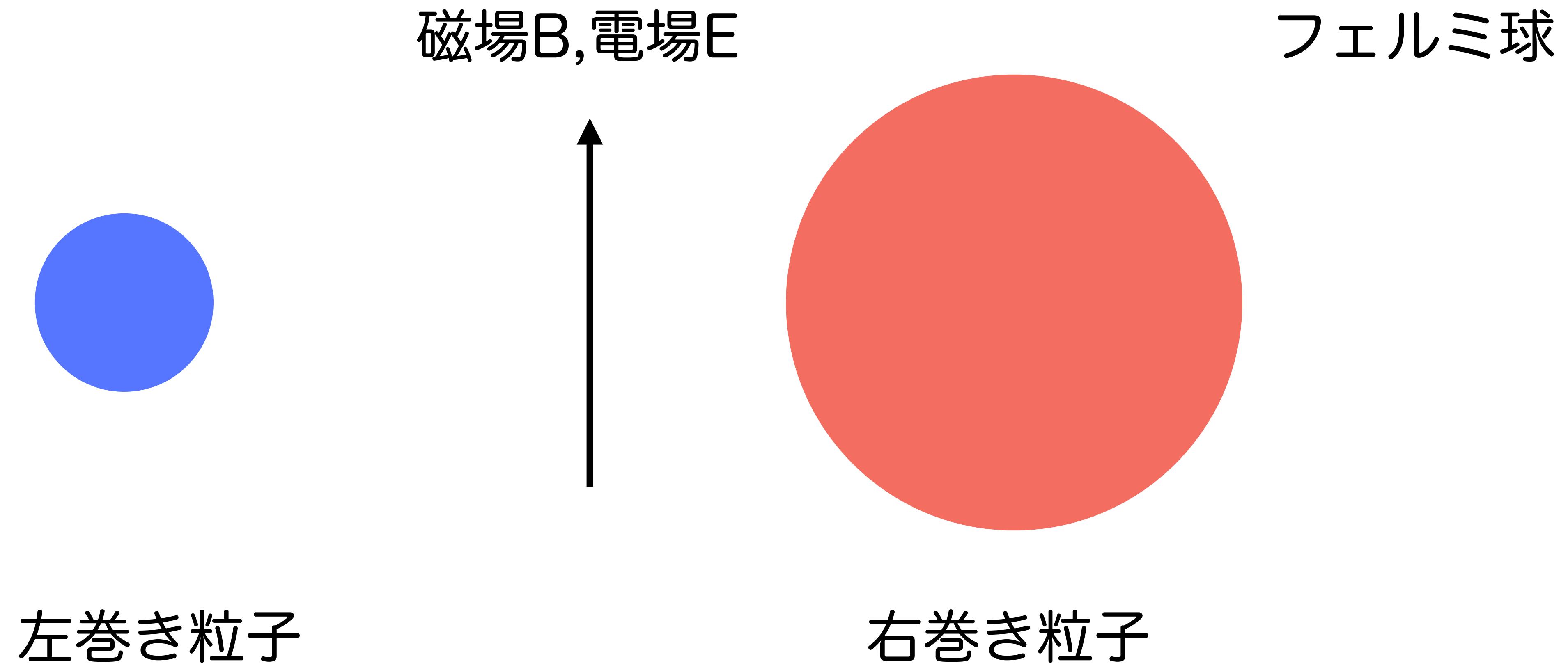
左巻き粒子



右巻き粒子

エネルギー収支の観点から再考

Nielsen & Ninomiya 83



$$\Delta Q_5 = \frac{1}{2\pi^2} \mathbf{B} \cdot \mathbf{E}$$

エネルギー収支の観点から再考

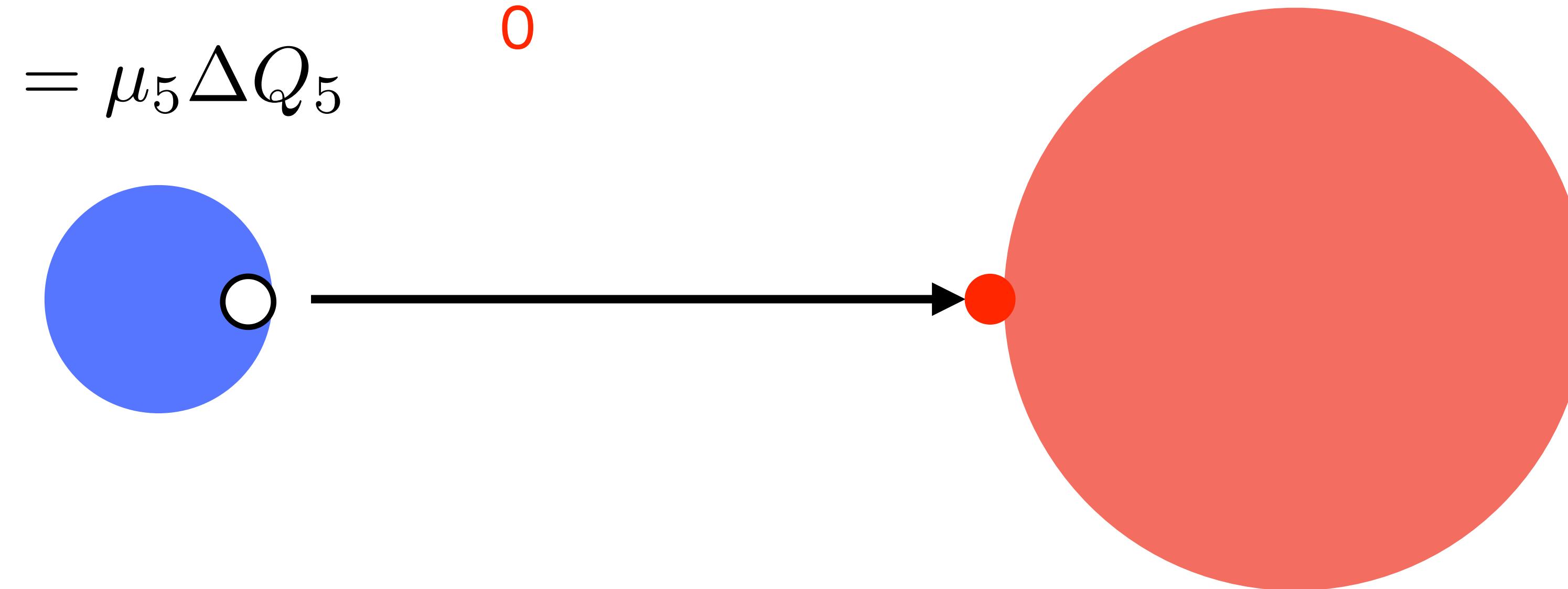
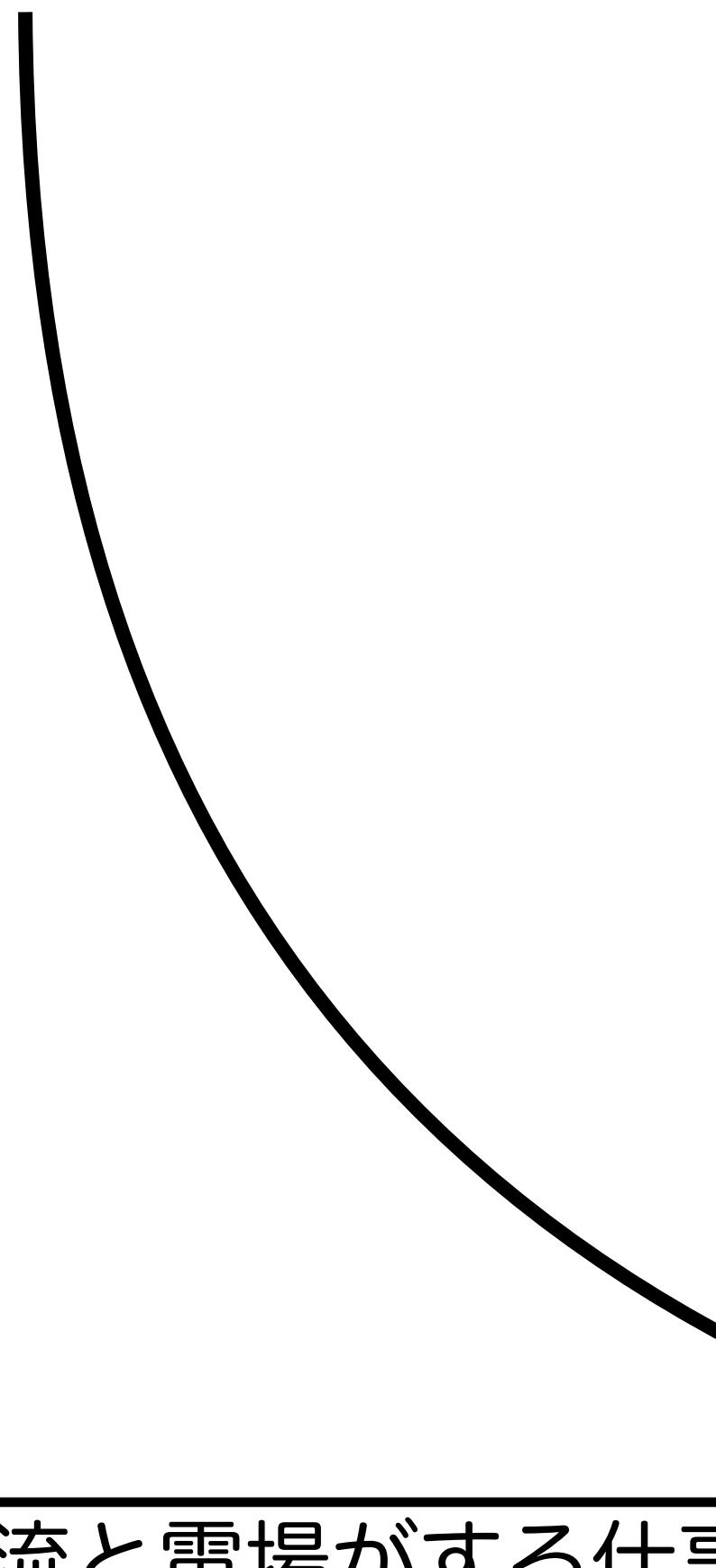
$$\mu_R \Delta n_R + \mu_L \Delta n_L = \mu_R \Delta(n_e + Q_5)/2 + \mu_L \Delta(n_e - Q_5)/2$$

Nielsen & Ninomiya 83

粒子数の増減に関する
エネルギーの増減

$$= (\mu_R + \mu_L) \frac{\Delta n_e}{0} / 2 + (\mu_R - \mu_L) \Delta Q_5 / 2$$
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左巻き粒子

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カイラル磁気効果

電流と電場がする仕事に相当

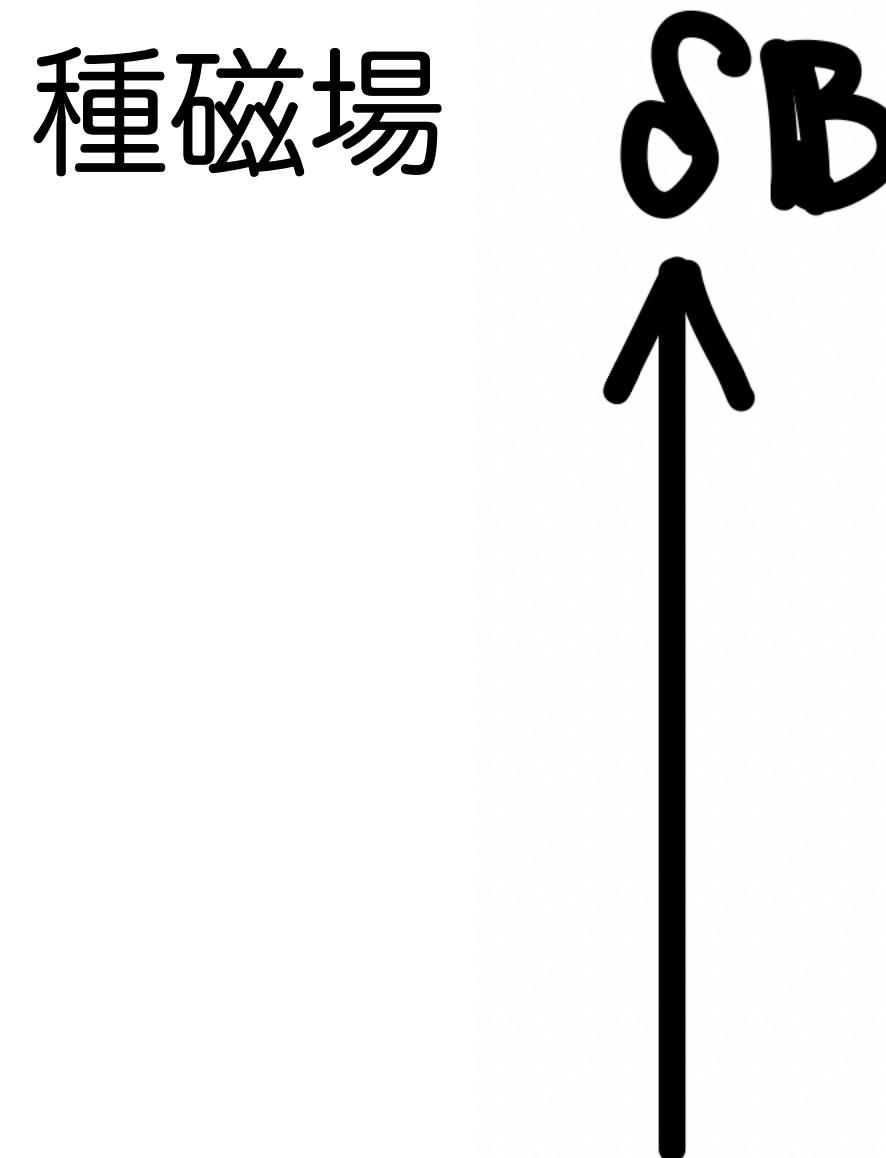
$$\mu_5 \Delta Q_5 = \mathbf{j} \cdot \mathbf{E} \quad \rightarrow$$

$$\mathbf{j} = \frac{\mu_5}{2\pi^2} \mathbf{B}$$

カイラルプラズマ不安定性

カイラル磁気効果 (Chiral Magnetic Effect) の自然な帰結として
カイラルプラズマ不安定性 (Akamatsu+13) を内在

磁場の誘導方程式： $\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \Delta \mathbf{B} + \underline{\eta \nabla \times (\xi_B \mathbf{B})}$



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カイラル磁気効果で磁場に
比例した電流が流れる

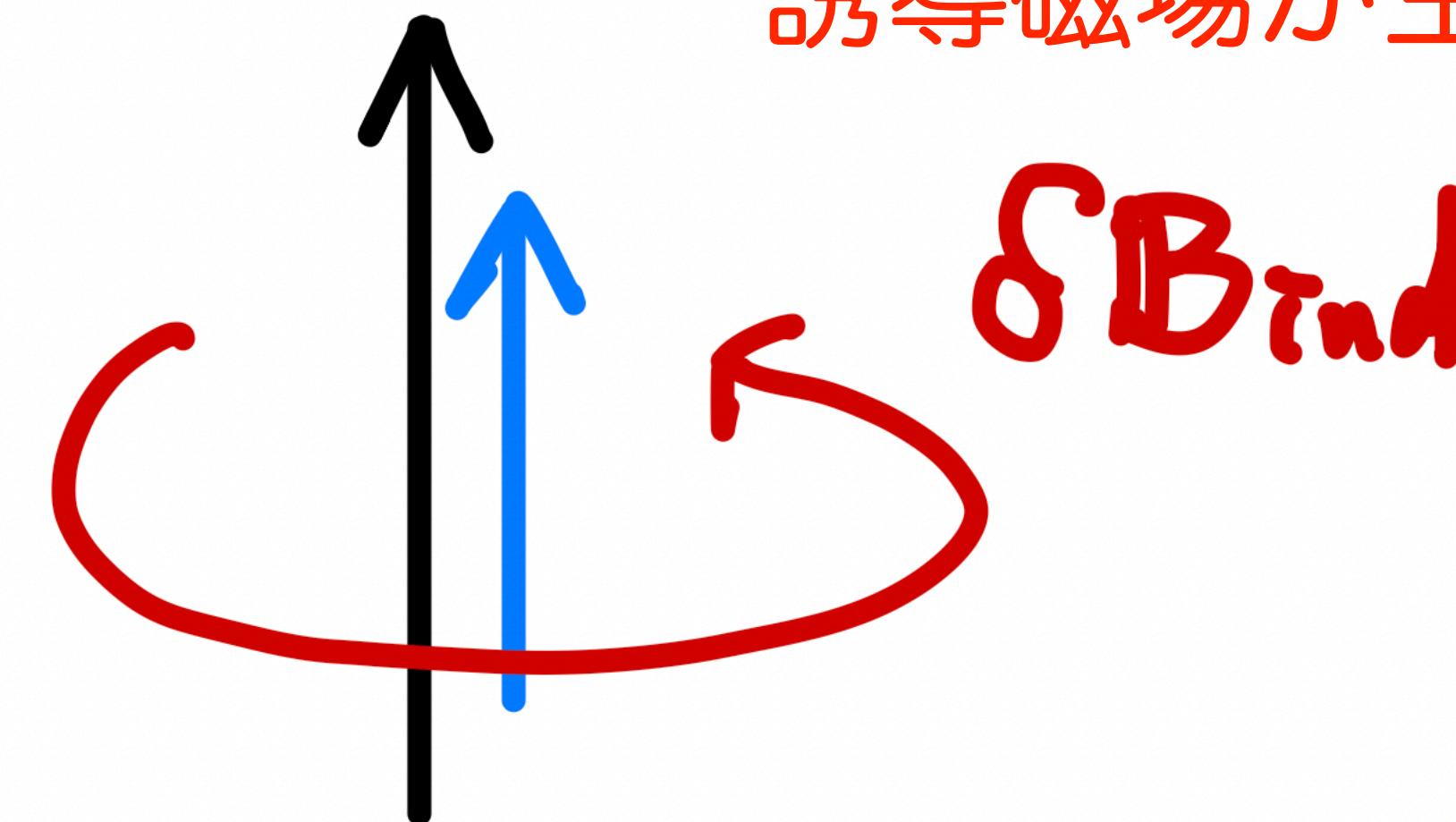
$$\delta J \propto \delta B$$

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アンペールの法則により
誘導磁場が生成



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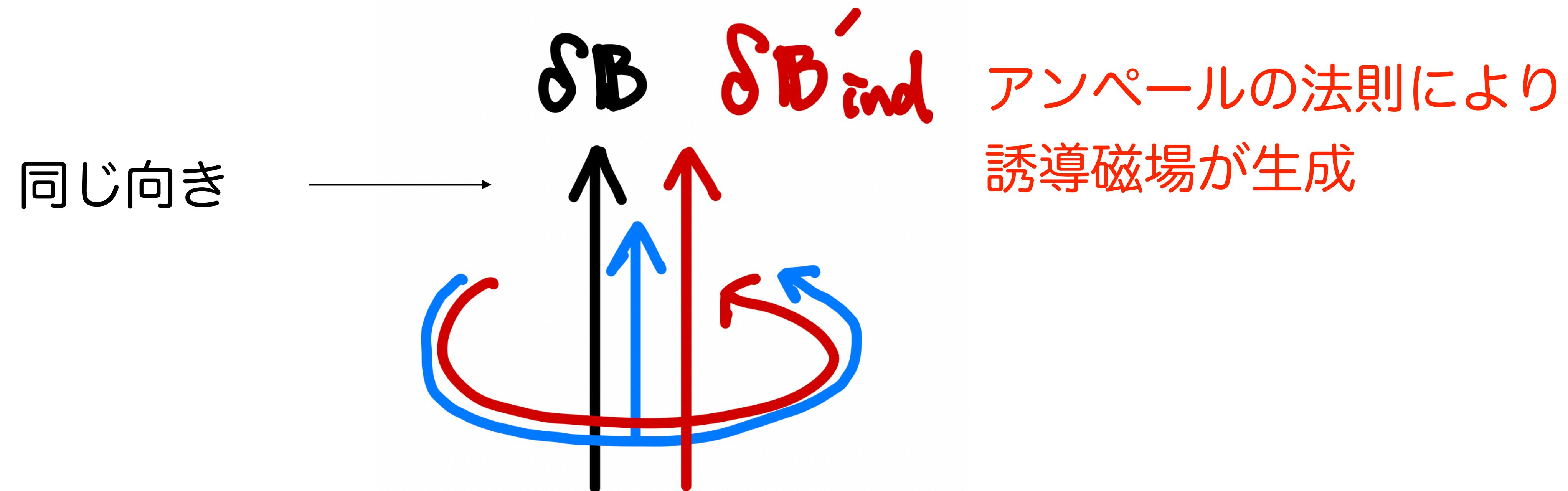


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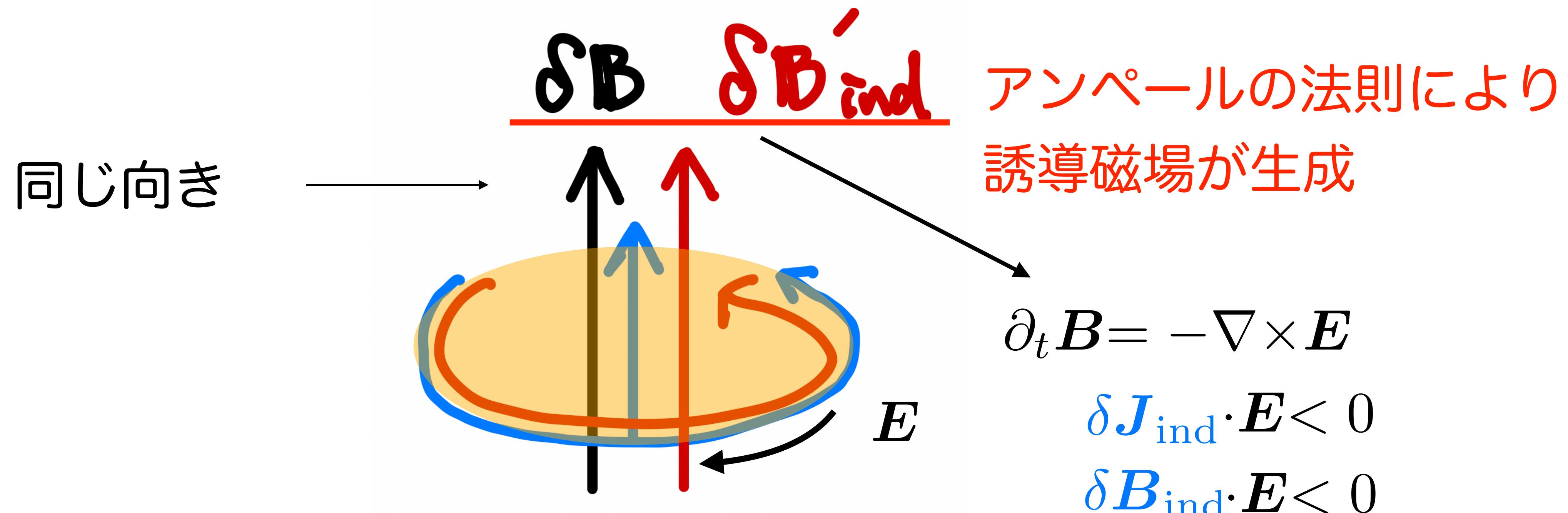
アンペールの法則により
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正のフィードバックで指数関数的に磁場が増幅 (不安定)

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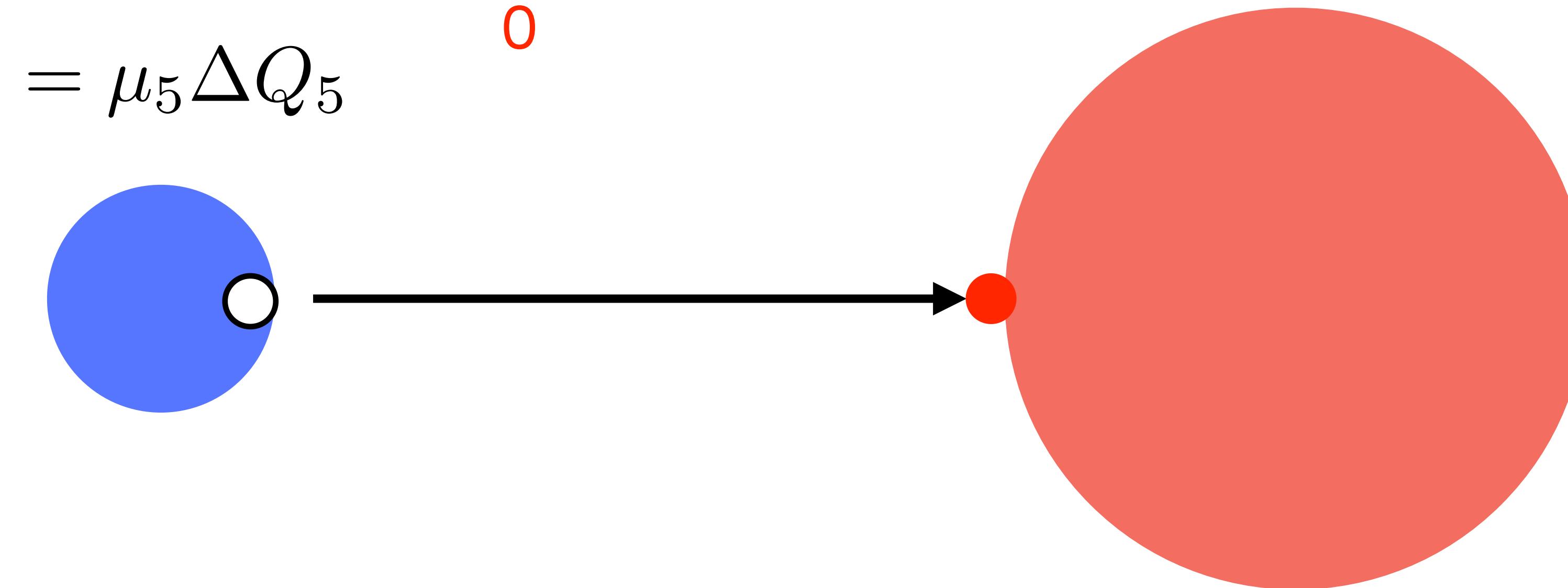
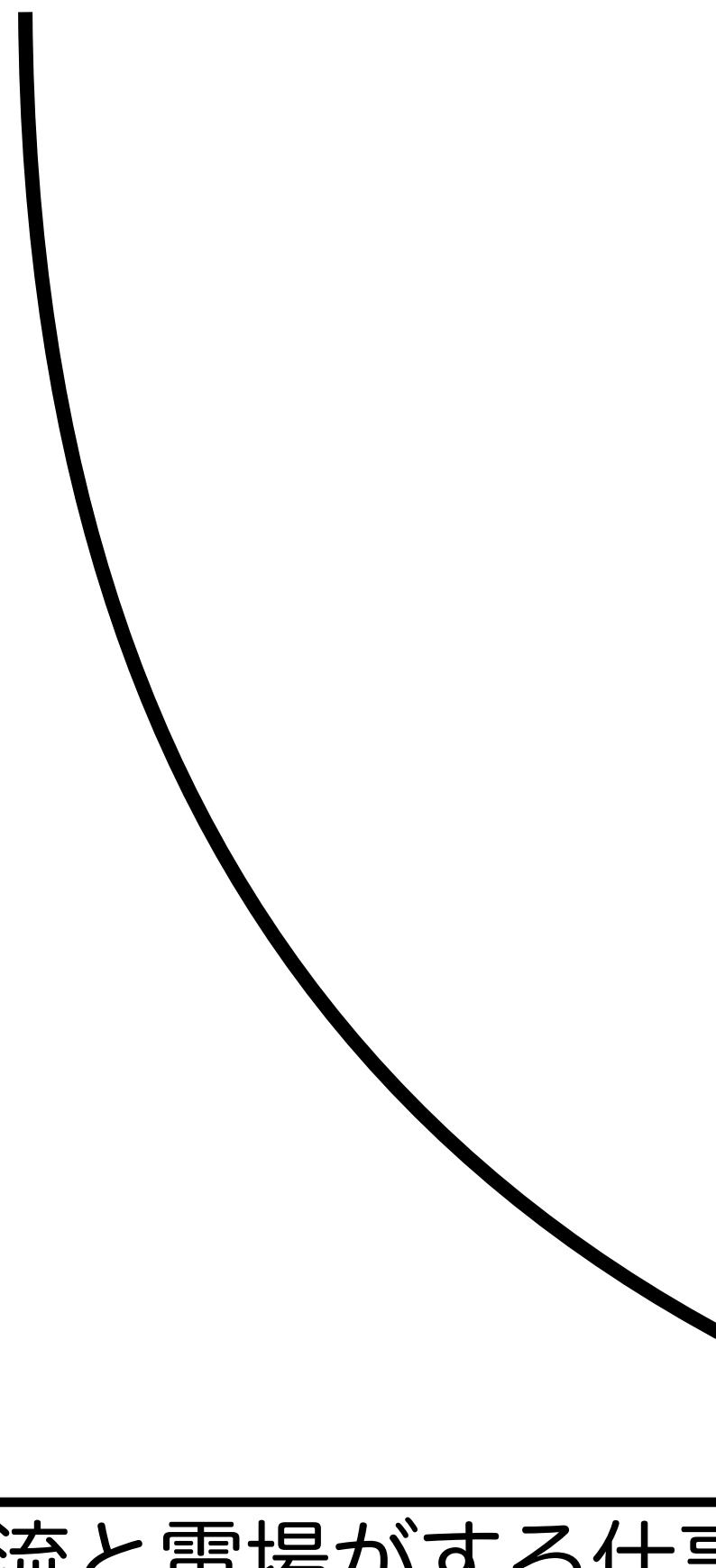
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フェルミ球



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線形解析

$$\partial_t \mathbf{B} = \eta \nabla \times (\xi_B \mathbf{B}) + \eta \Delta \mathbf{B}$$

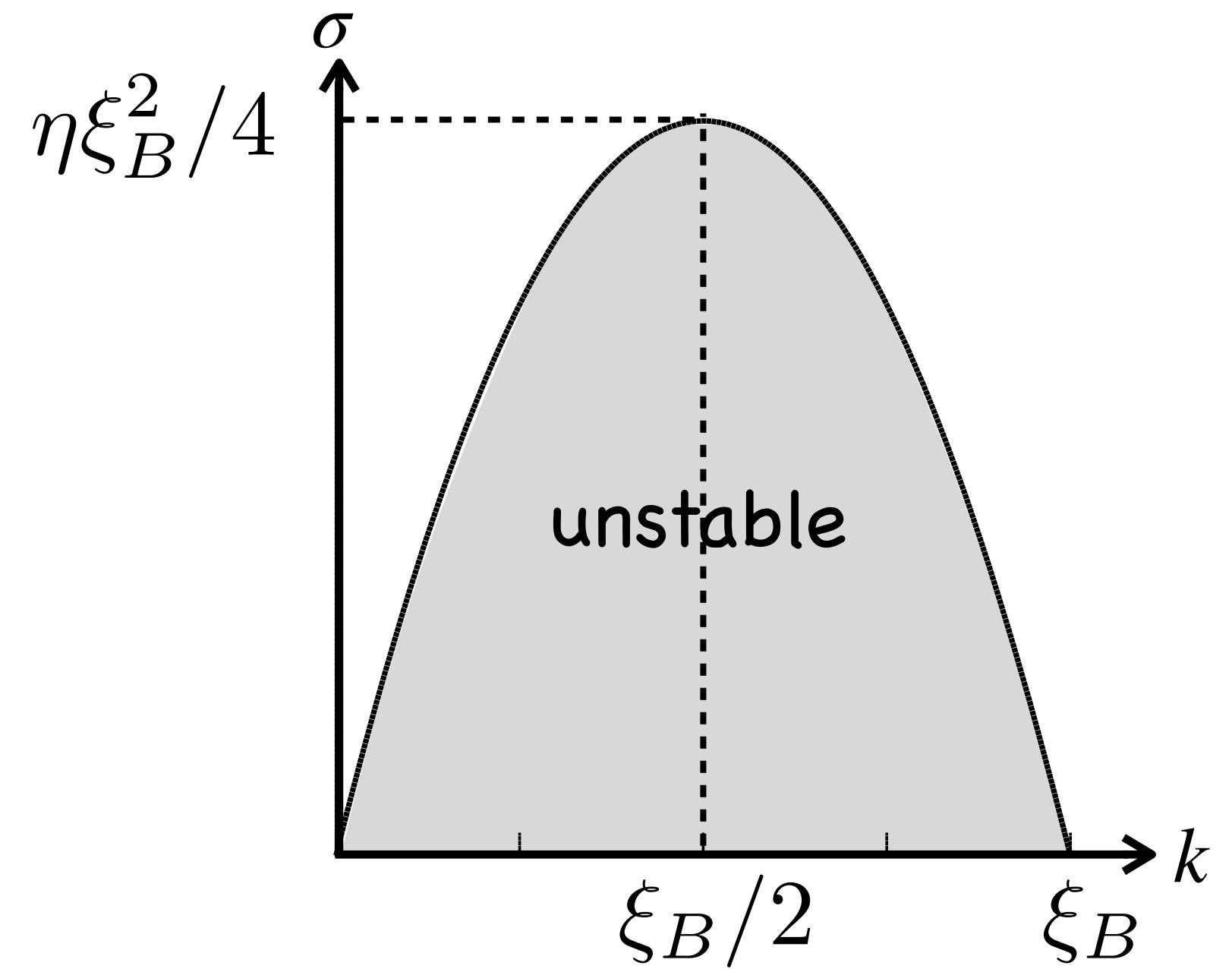
linear analysis: $\delta \mathbf{B} = \exp[i\mathbf{k} \cdot \mathbf{r} + \sigma t]$

- -> dispersion relation:

$$\sigma = \eta \xi_B k - \eta k^2$$

$$= -\eta [k - \xi_B/2]^2 + \eta \xi_B^2/4$$

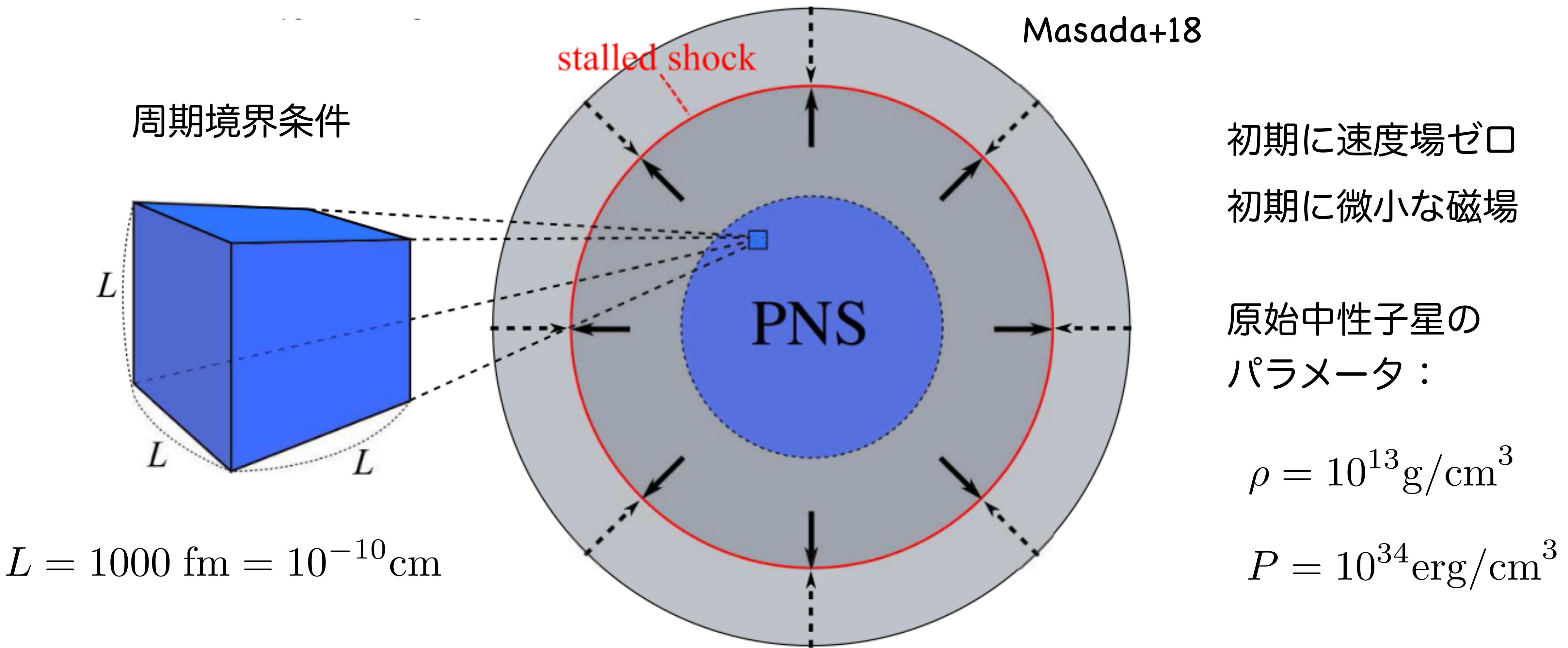
parabolic equation of k



$$\lambda_{\text{CPI}} \equiv 2\pi/k_{\text{CPI}} = 4\pi/\xi_B$$

カイラル電磁流体力学シミュレーションの設定

Chiral MHD simulations in the context of CCSN (Masada+18, JM22)



カイラル電磁流体力学の基礎方程式

保存系：

JM+22

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 ,$$

非保存系

(Masada+18)

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot \left[\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B} + \left(P + \frac{B^2}{2} \right) \mathbf{I} \right] = \mathbf{S} ,$$

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho v^2 + \frac{1}{\Gamma - 1} P + \frac{B^2}{2} \right) + \nabla \cdot \left[\left(\frac{1}{2} \rho v^2 + \frac{\Gamma}{\Gamma - 1} P \right) \mathbf{v} + \mathbf{E} \times \mathbf{B} \right] = \mathbf{S} \cdot \mathbf{v} - \underline{\mathbf{J}_{\text{CME}} \cdot \mathbf{E}} ,$$

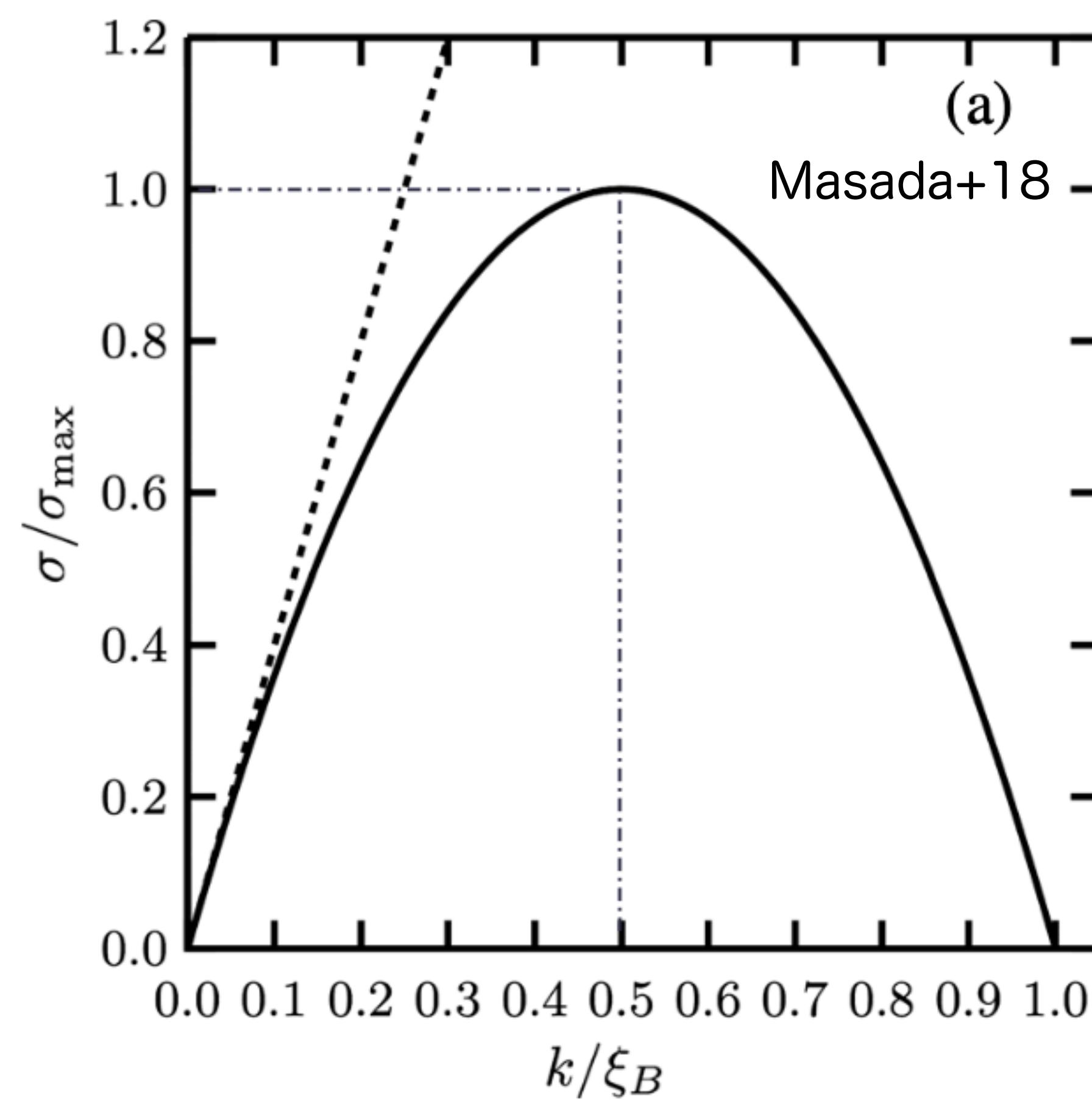
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \Delta \mathbf{B} + \underline{\eta \nabla \times (\xi_B \mathbf{B})} ,$$

$$\underline{\frac{\partial n_5}{\partial t} = \frac{1}{2\pi^2} \mathbf{E} \cdot \mathbf{B}} ,$$

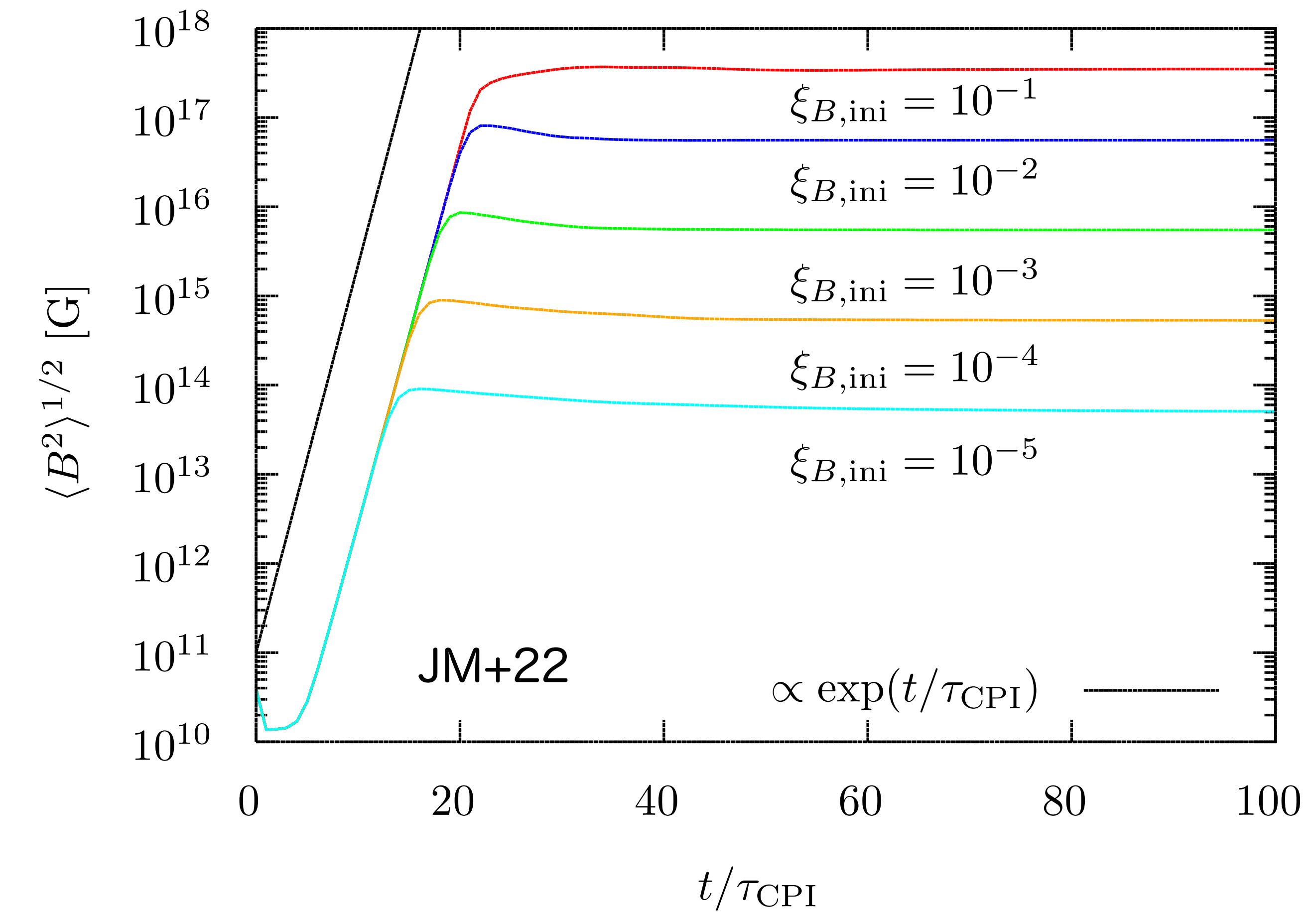
全ヘリシティの保存： $\frac{d}{dt} \left(Q_5 + \frac{H_{\text{mag}}}{4\pi^2} \right) = 0 \quad H_{\text{mag}} \equiv \int d^3x A \cdot B$

カイラルプラズマ不安定性

カイラルプラズマ不安定性の
分散関係式 $\sigma = \eta k(\xi_B - k)$

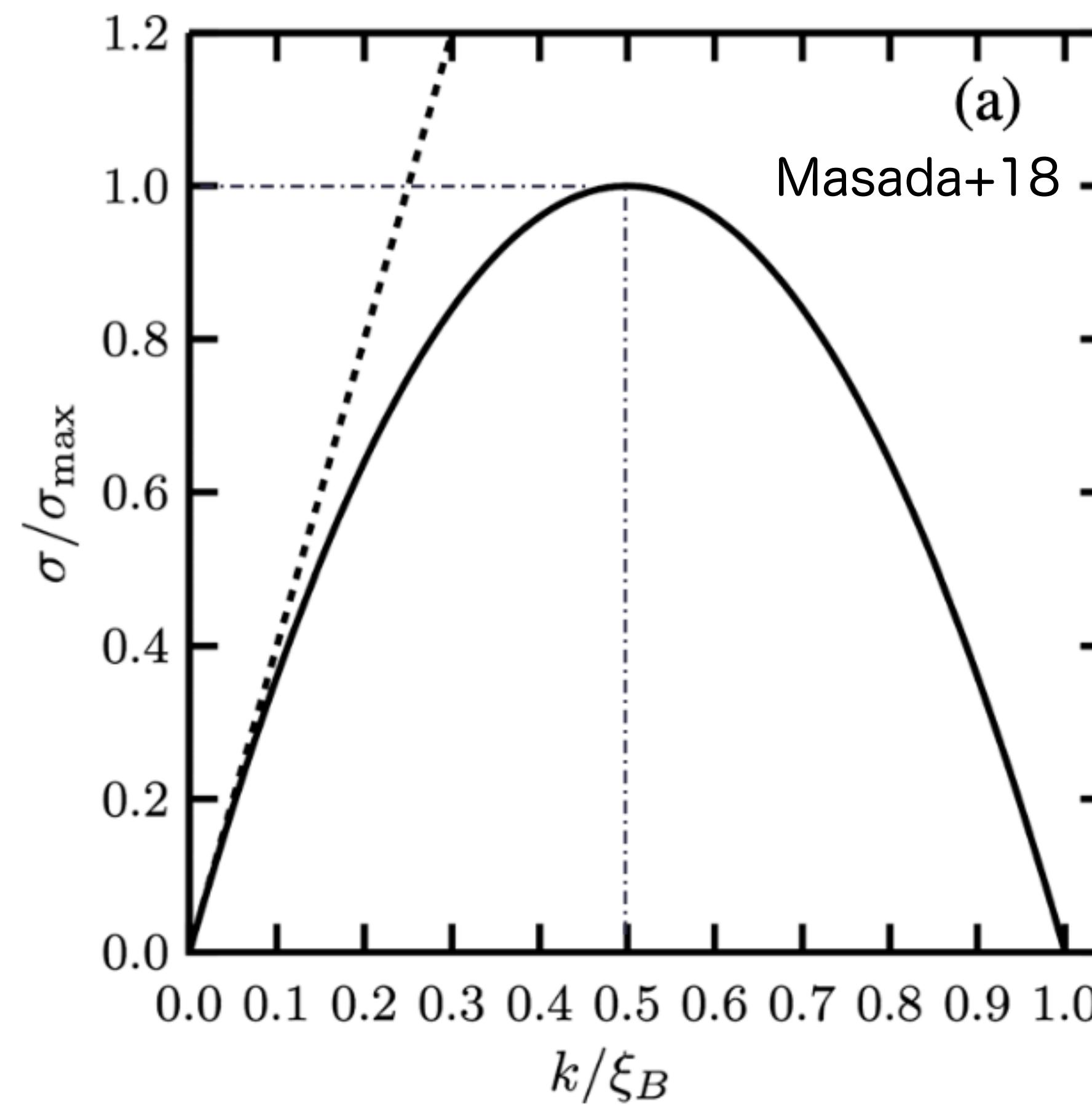


カイラルプラズマ不安定性により
マグネター級の磁場の生成

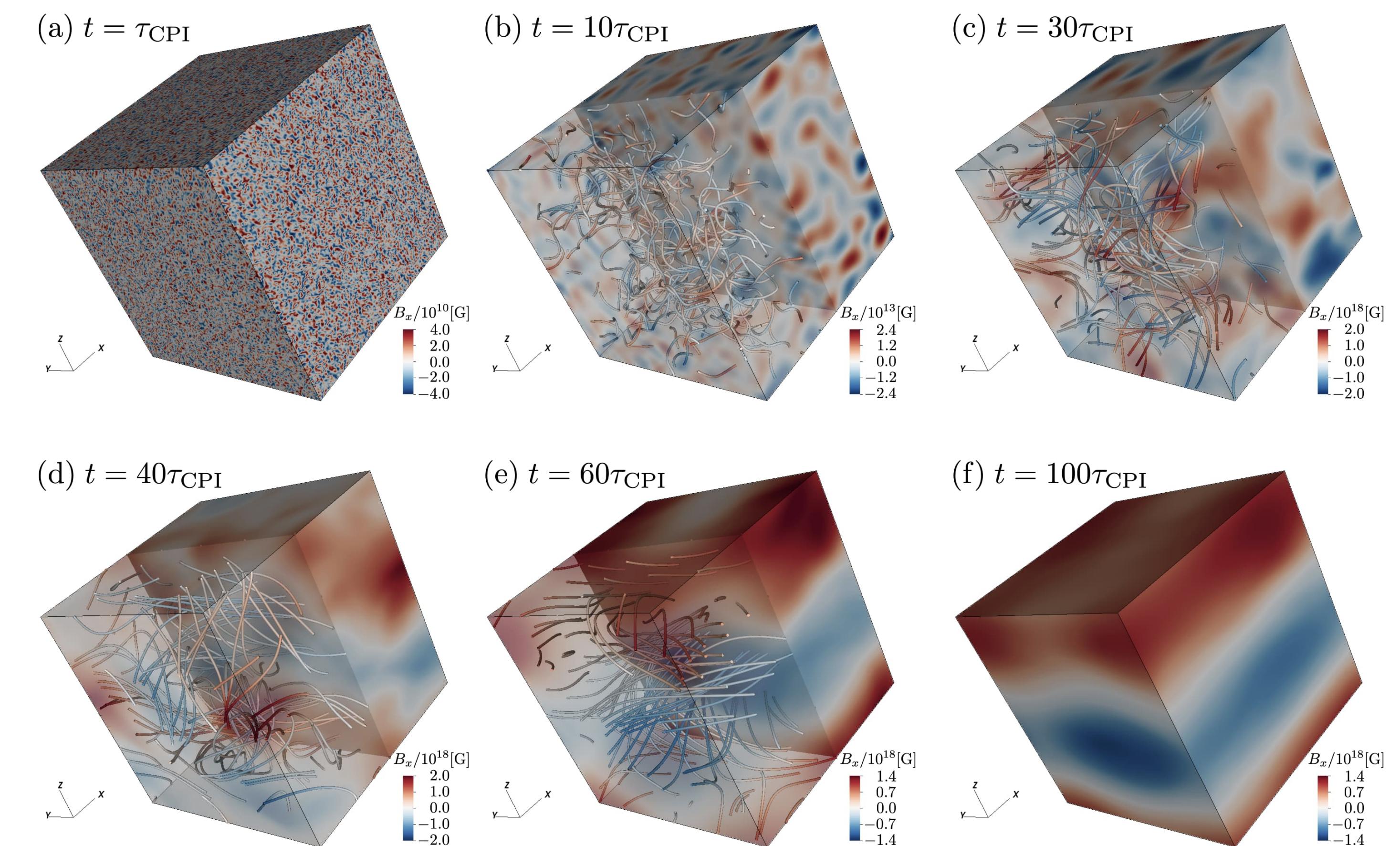


カイラルプラズマ不安定性

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磁場の相関長のサイズアップ

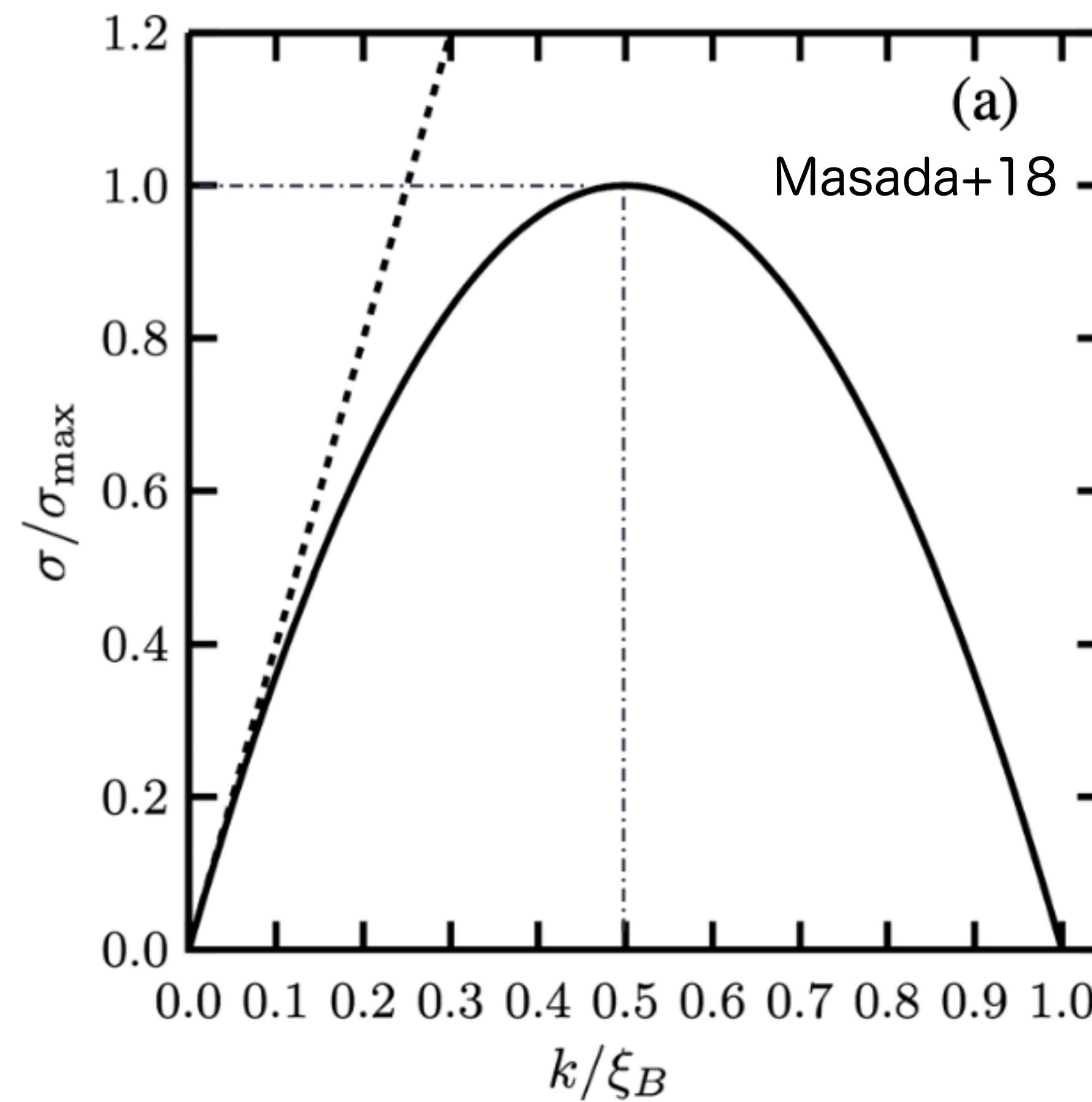


Masada+18とコンシスティント

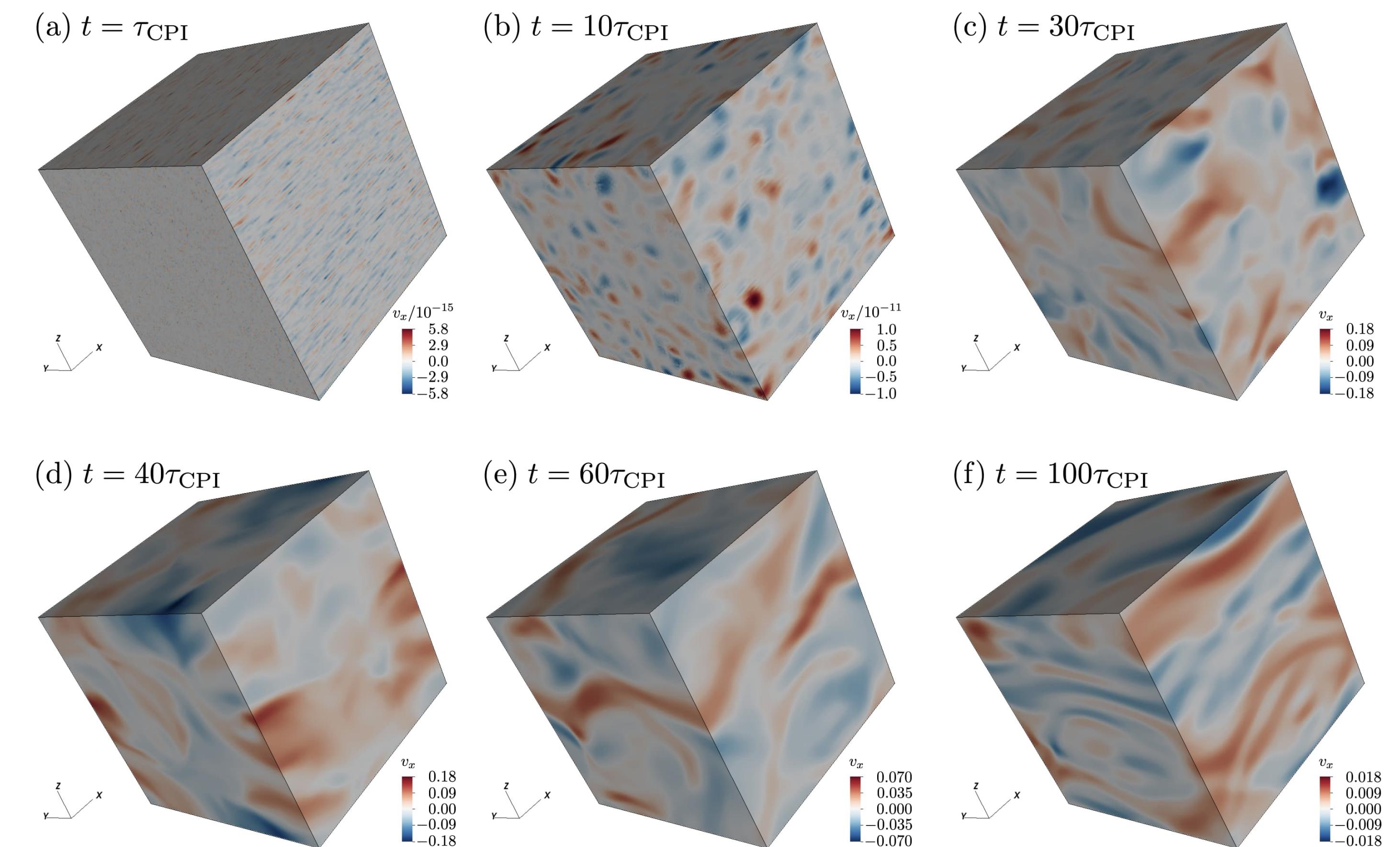
JM+22

カイラルプラズマ不安定性

カイラルプラズマ不安定性の
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速度場のインバースカスケード



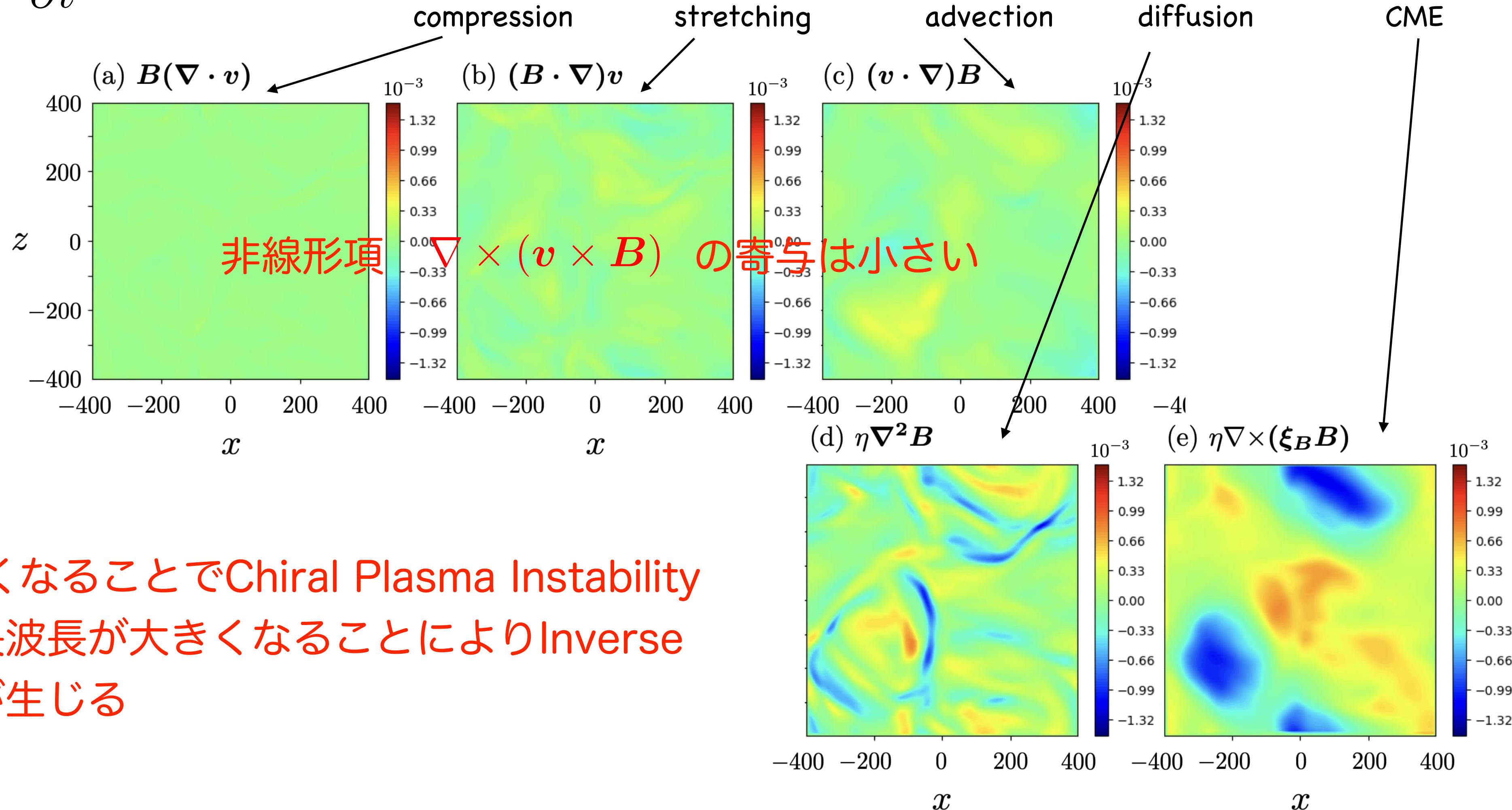
Masada+18とコンシスティント

JM+22

磁場の誘導方程式における寄与の大小

$$\frac{\partial \mathbf{B}}{\partial t} = \mathbf{v}(\cancel{\nabla \times \mathbf{B}}) - \mathbf{B}(\nabla \cdot \mathbf{v}) + (\mathbf{B} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{B} + \eta \Delta \mathbf{B} + \eta \text{rot}(\xi_B \mathbf{B})$$

JM+22



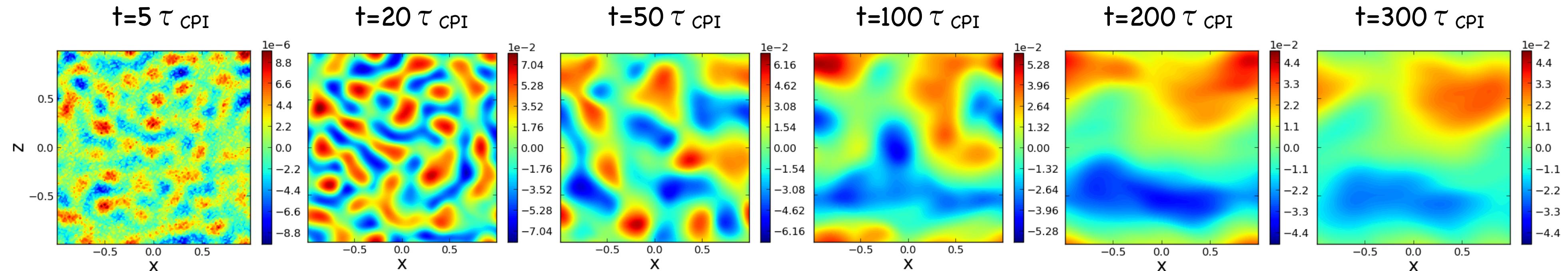
induction equation: $\partial_t \mathbf{B} = \cancel{\nabla \times (\mathbf{v} \times \mathbf{B})} + \eta \Delta \mathbf{B} + \eta \nabla \times (\xi_B \mathbf{B})$

インバースカスケードのメカニズムの検証

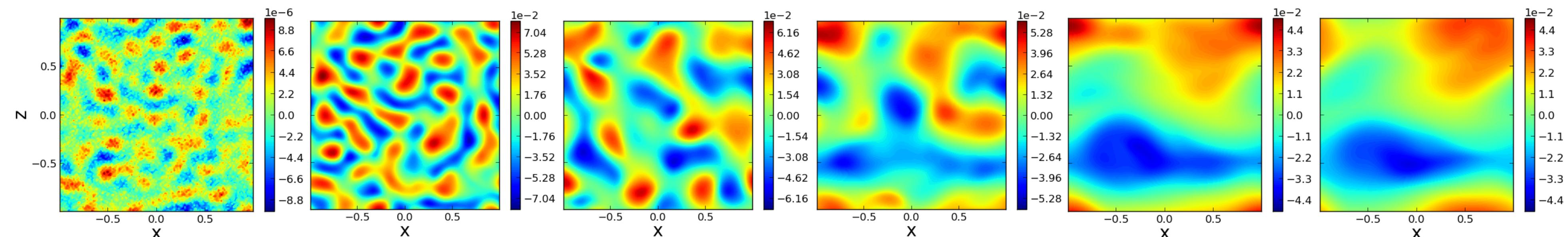
- 誘導方程式から $\nabla \times (\mathbf{v} \times \mathbf{B})$ を削除しシミュレーションを実行

$$\partial_t \mathbf{B} = \eta \Delta \mathbf{B} + \eta \nabla \times (\xi_B \mathbf{B}) \text{ を解く}$$

full induction equation



without $\nabla \times (\mathbf{v} \times \mathbf{B})$



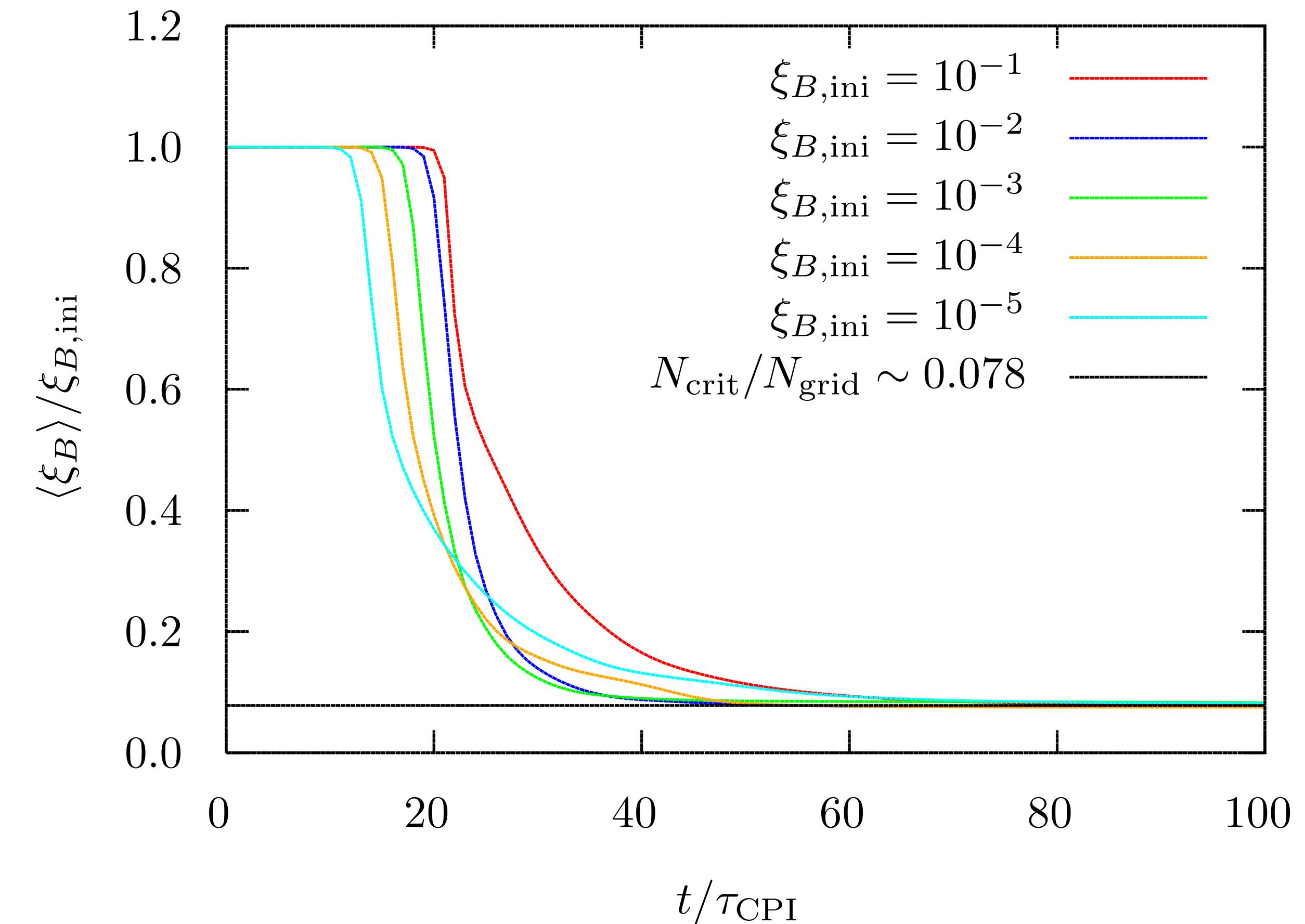
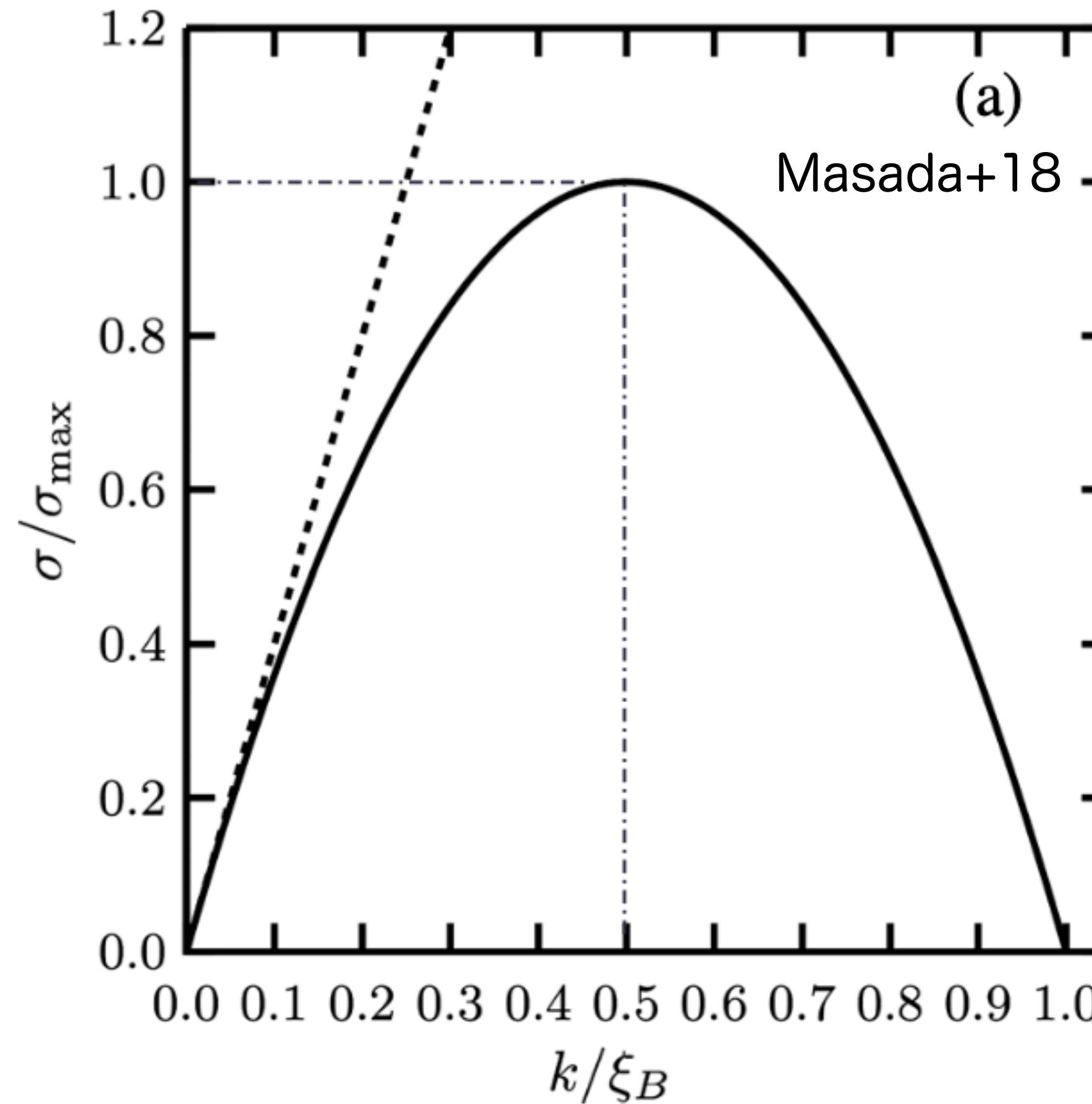
非線形項 $\nabla \times (\mathbf{v} \times \mathbf{B})$ の寄与は小さいことがわかる

非線形項が効かない条件

$$|\mathbf{v}| \ll \eta |\xi_B|$$

ξ_B の時間変化

カイラルプラズマ不安定性の
分散関係式

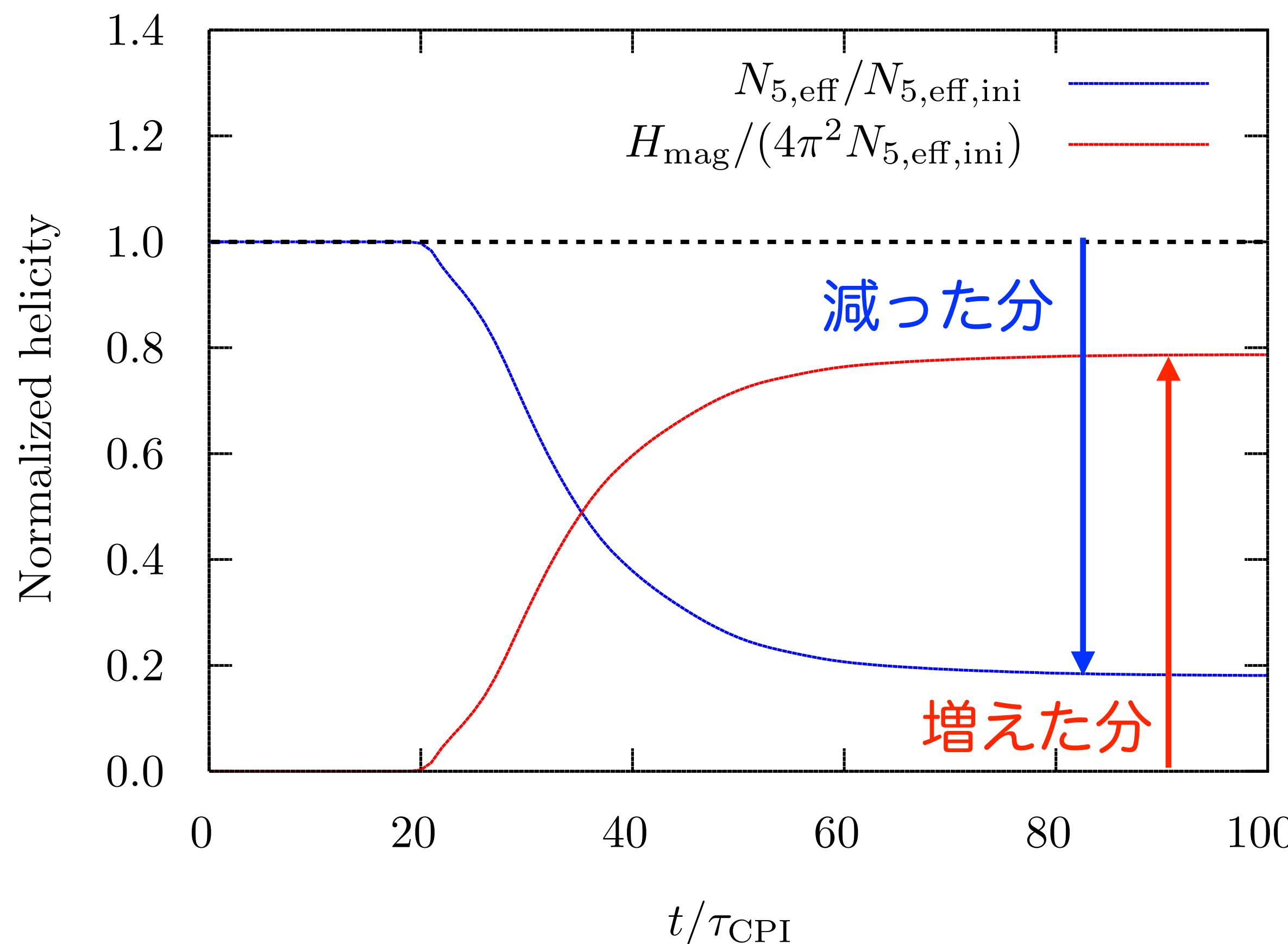


ξ_B 小さくなることで Chiral Plasma Instability
の最大成長波長が大きくなることにより
Inverse cascadeが生じる

議論：磁気ヘリシティをもった磁場の生成

全ヘリシティの保存： $\frac{d}{dt} \left(Q_5 + \frac{H_{\text{mag}}}{4\pi^2} \right) = 0$

$$H_{\text{mag}} \equiv \int d^3x A \cdot B$$



Axial chargeが減った分だけ磁気ヘリシティをもった磁場が生成される

- -> 乱流がインバースカスケード
- -> ニュートリノ加熱の効率アップ
- -> 爆発にポジティブ

ミッシングリンクとしての超新星爆発フェーズ

Sun

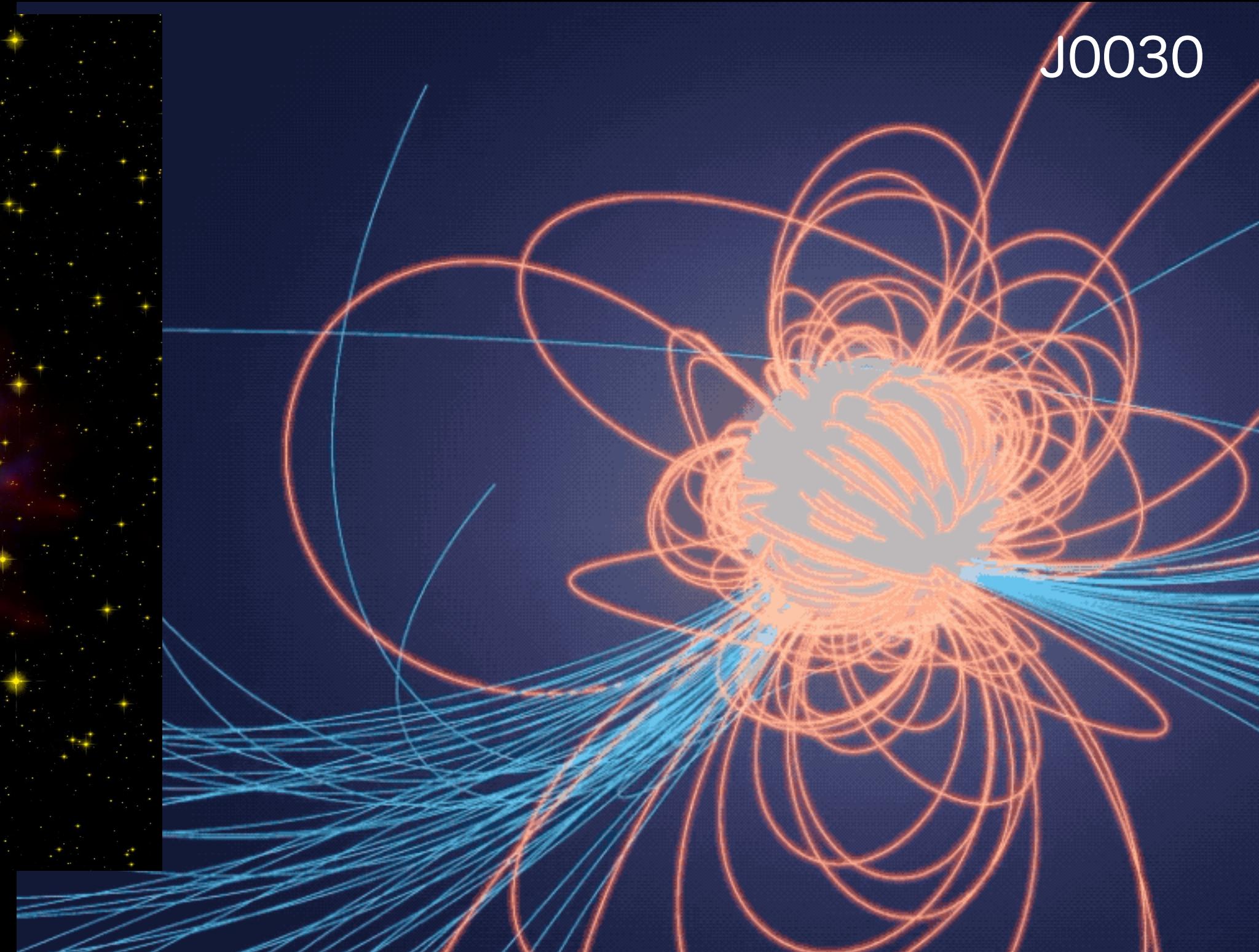


NASA/SDO

Cassiopeia A



超新星爆発自体の多様性



NASA/NICER

恒星には磁場がある！ - ->
十分な理解に至っていない！！

?

- -> 中性子星にも磁場がある！
多様性