中性子星連星合体
シミュレーションに向けた
モンテカルロニュートリノ輻射
流体コードの開発

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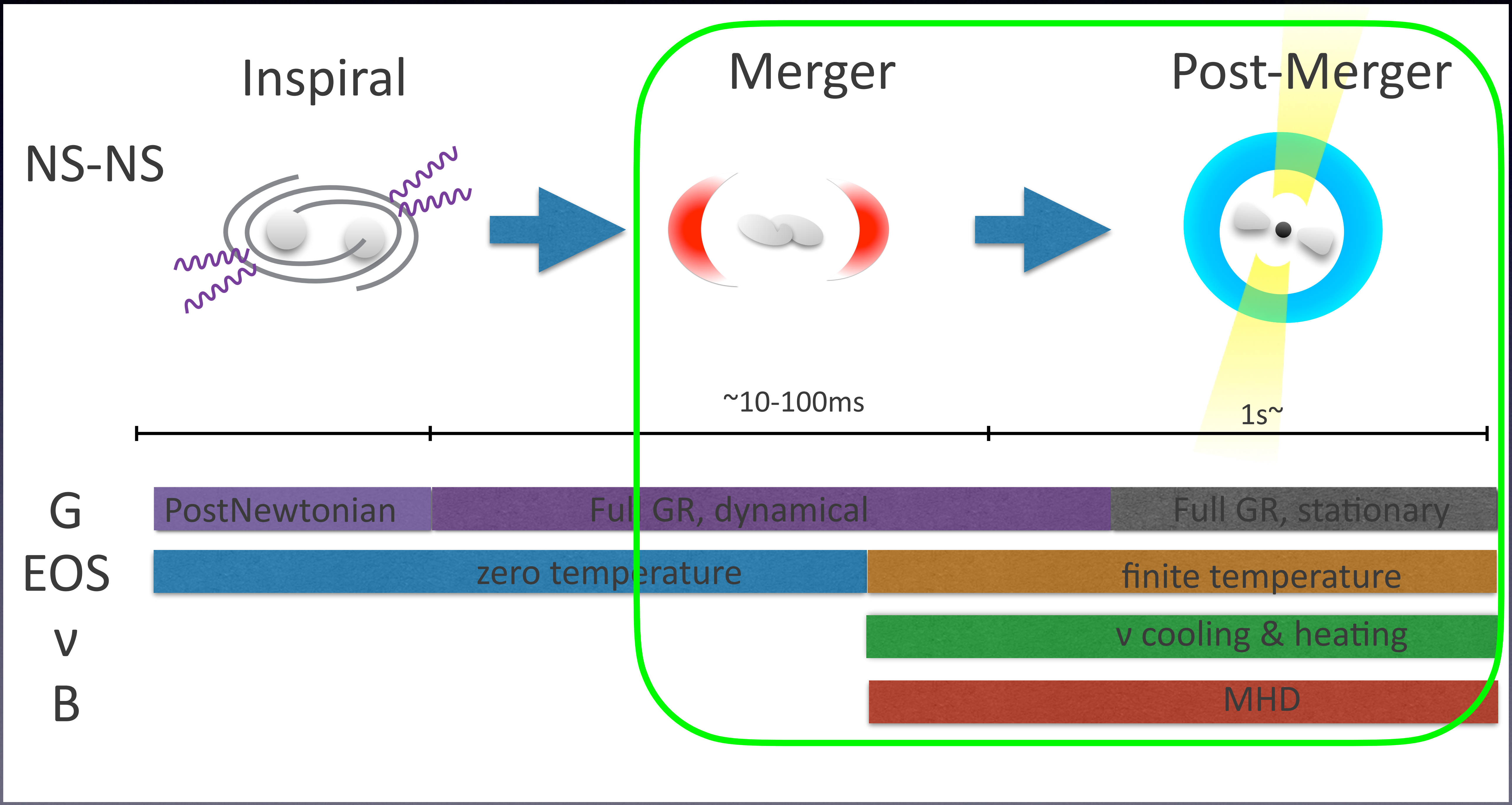
研究会「高エネルギー現象で探る宇宙の多様性 I 」

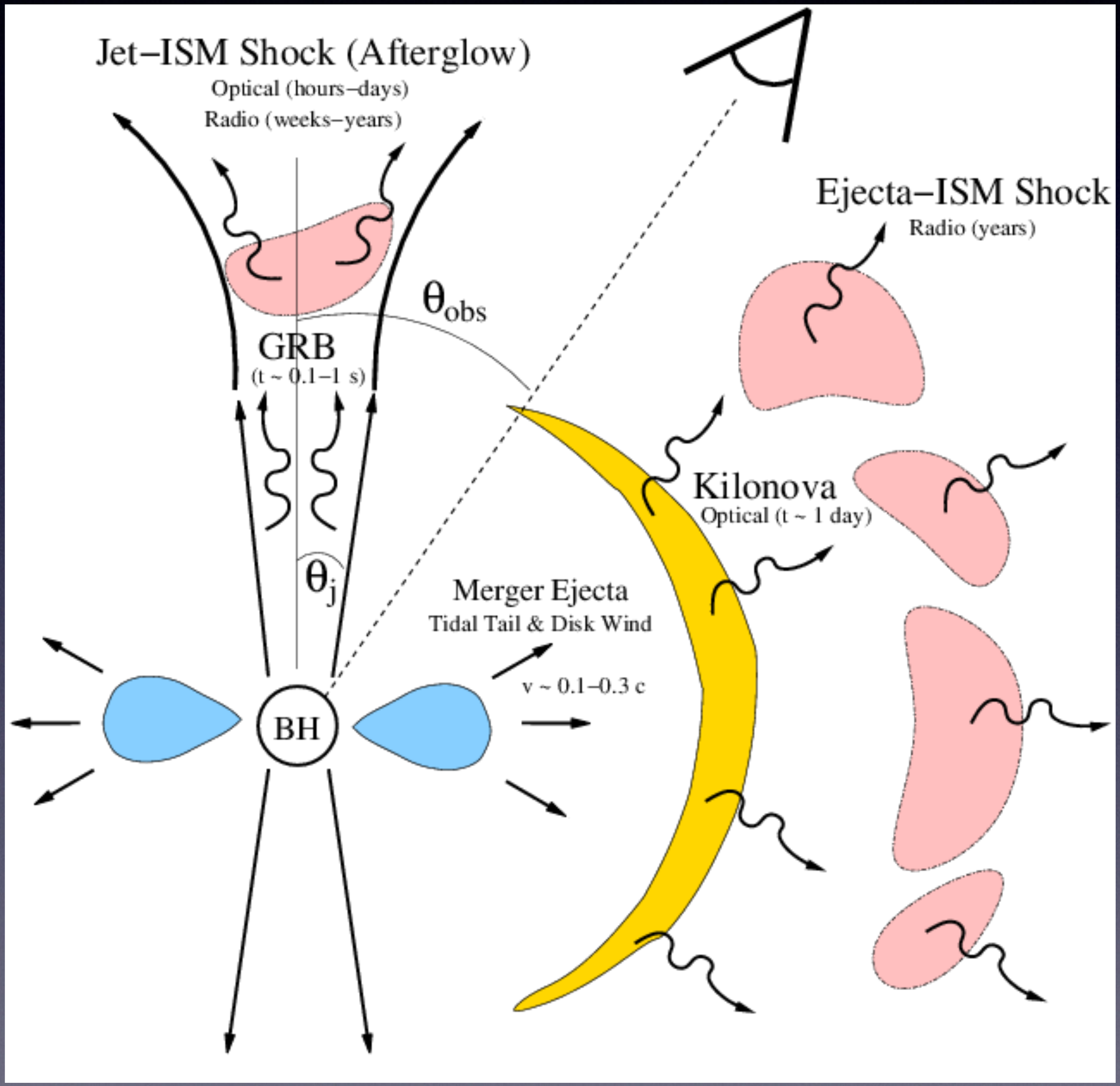
Neutron star mergers



Remnants / outflow formed in the merger/post-merger phase will be the source of the electromagnetic counterparts

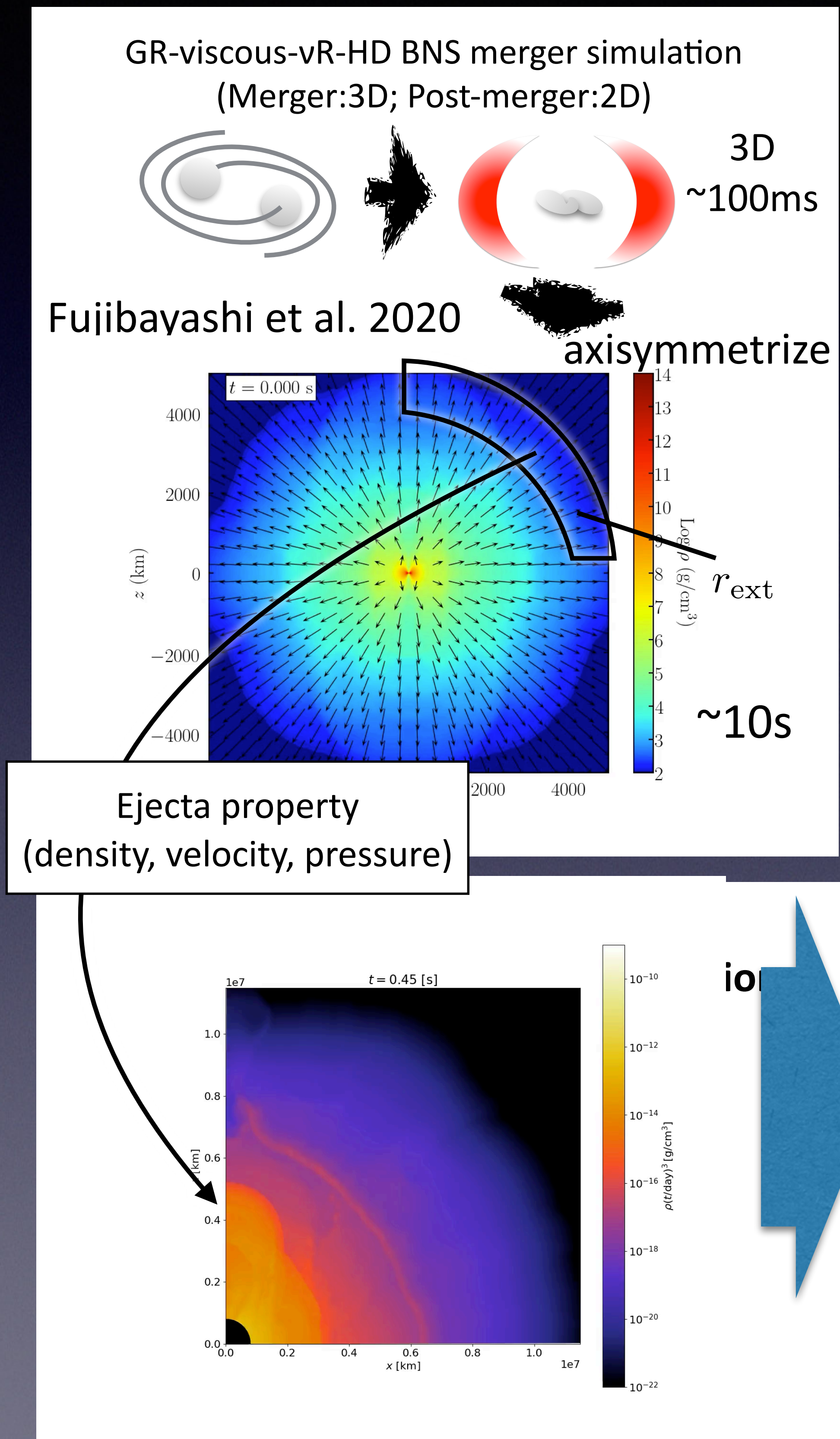
Electromagnetic Counterparts to NS binary mergers

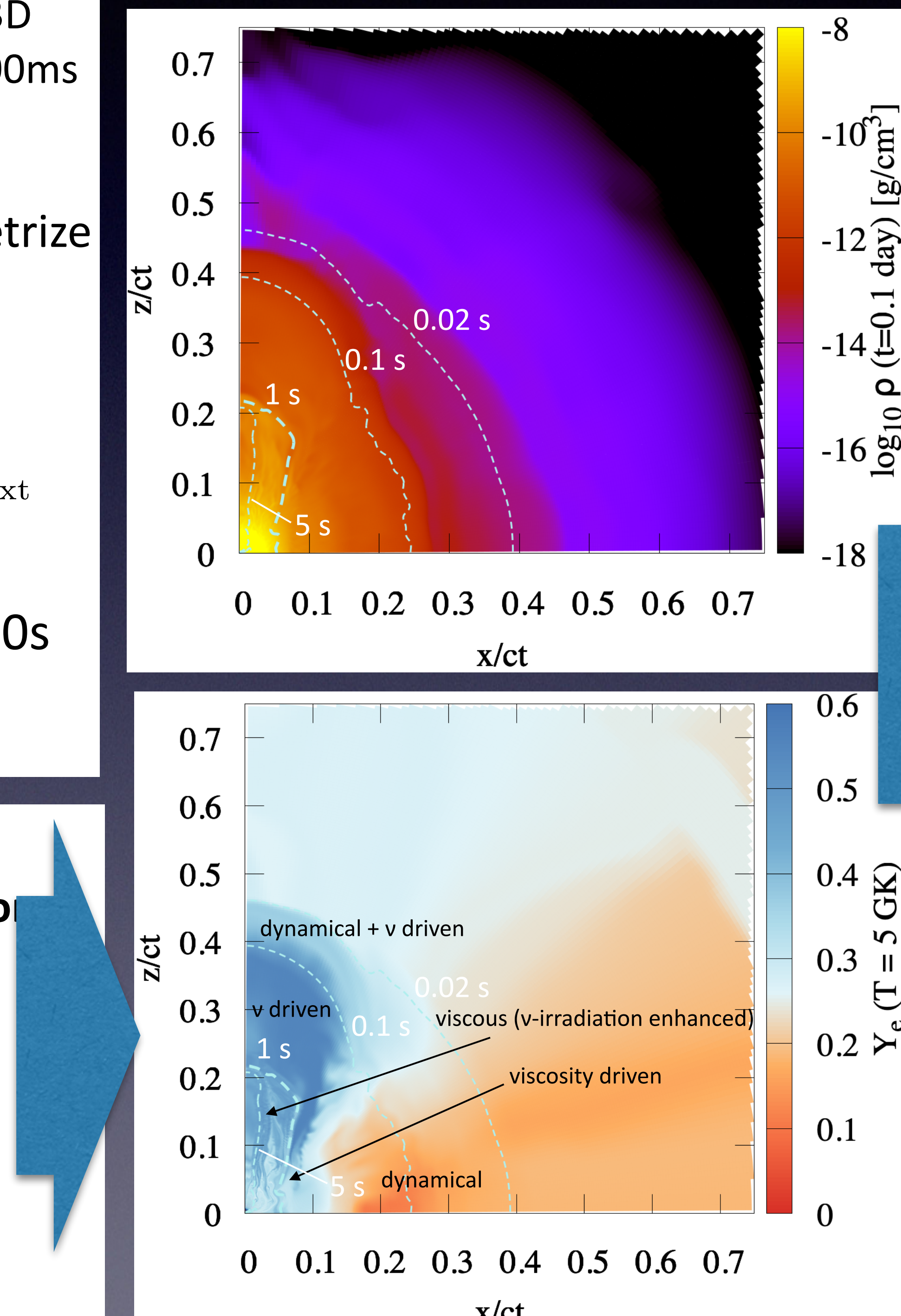
- Various transient EM counterparts are proposed for NS binary mergers
- for example,
  - short-hard gamma-ray-burst
  - Afterglow
  - cocoon emission
  - kilonovae/macronovae
  - radio flare, etc.
- Host galaxy identification, remnant properties, environment
- Possible synthesis site of r-process nuclei

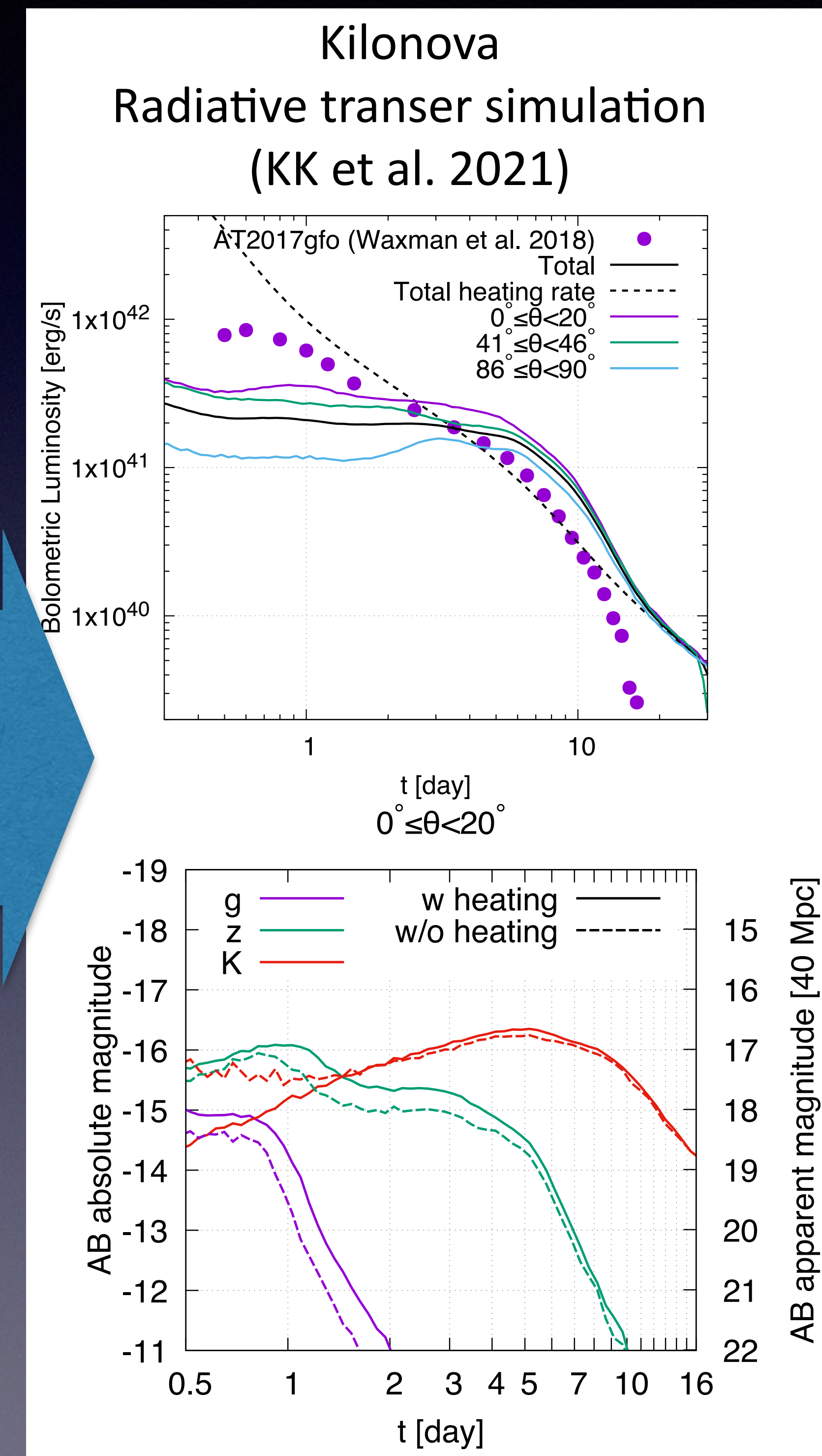


Ref: B. Metzger and E. Berger 2012

Kilonova lightcurve prediction







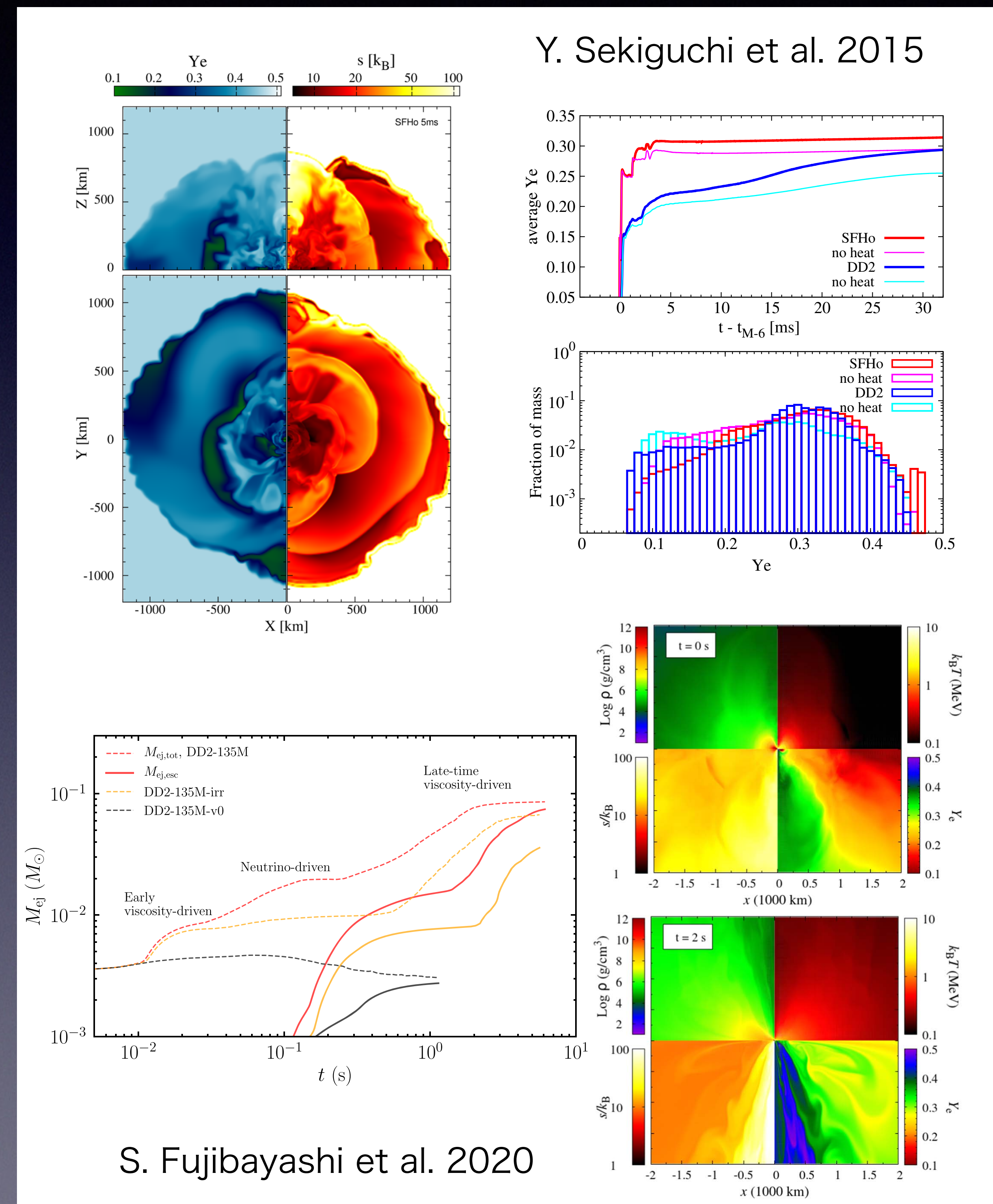
Neutrino-matter interaction

Neutrino-matter interaction plays an important role in the merger/post-merger phase of a BNS merger:

- Determines the thermodynamical property of the remnant NS and disk
- Controls the nucleosynthesis in the outflow
- Possible mechanism for launching a relativistic outflow / jet (pair-annihilation)

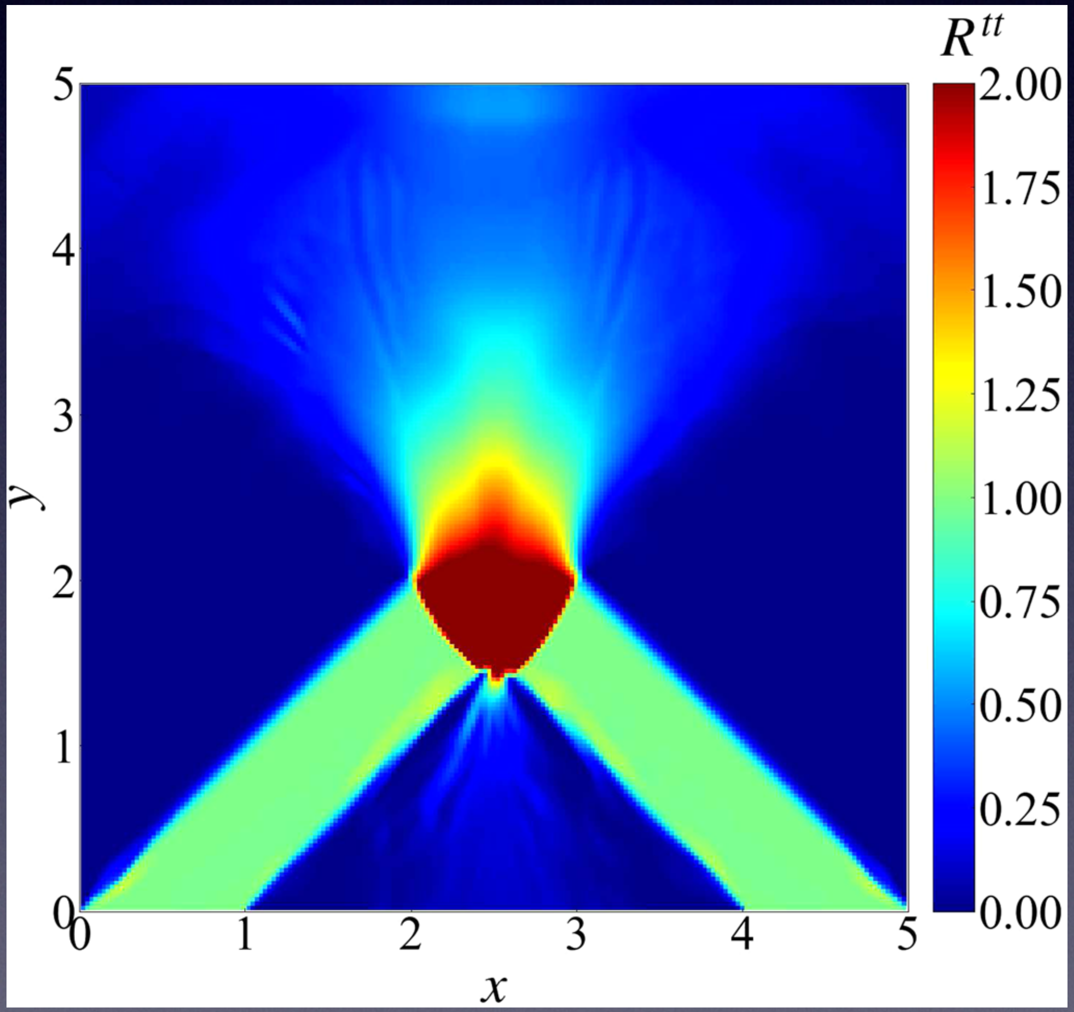
The moment formalism M1(M0) method is often used for the latest merger simulations to take the effect of neutrino transport into account

(K. Thorne 1981, M. Shibata et al. 2011, Y. Sekiguchi et al. 2015, 2016
, F. Foucart et al. 2015, D. Radice et al. 2016,
see also McKinney et al. 2014, Sadowski et al. 2014, Takahashi et al. 2016
for GR-RMHD)

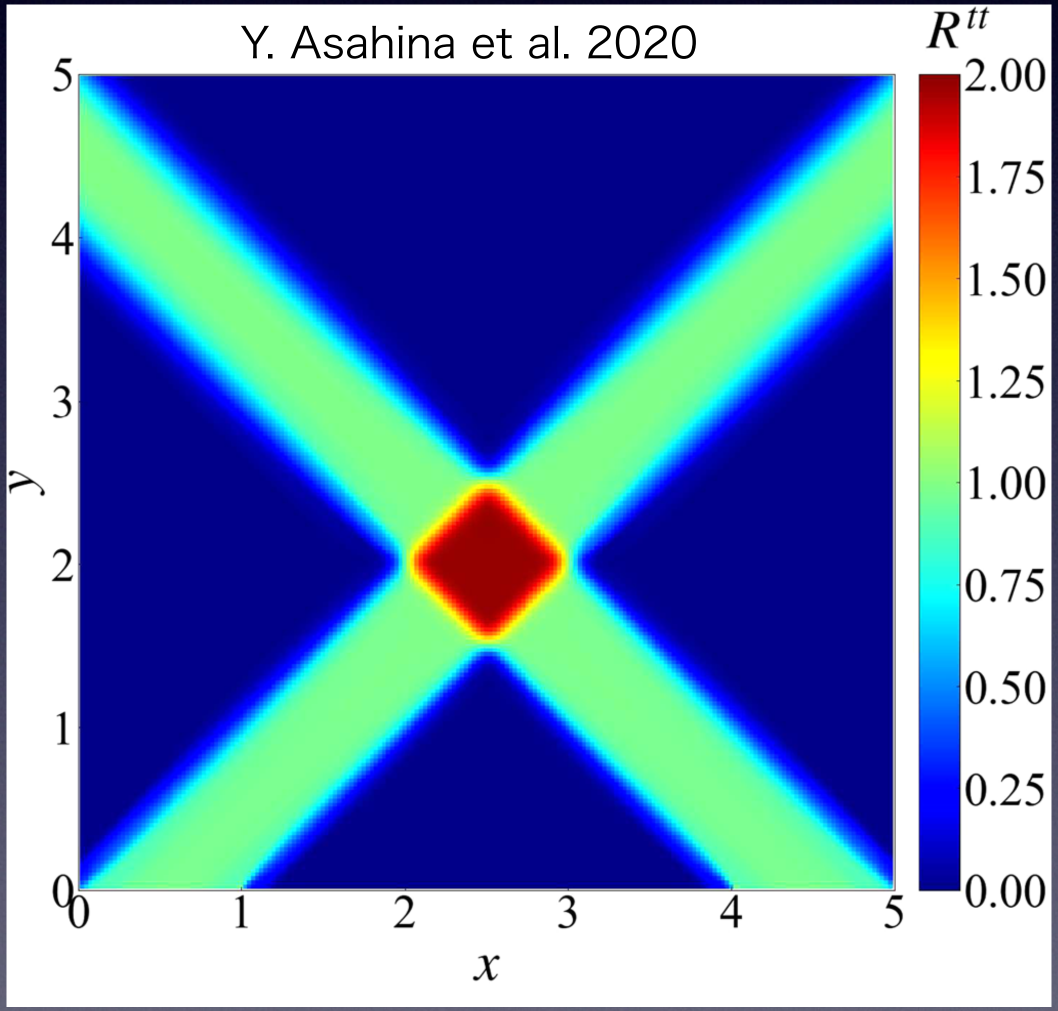


Limitation of M1 method (truncated-moment formalism)

M1-method



Full Boltzmann (grid-based)



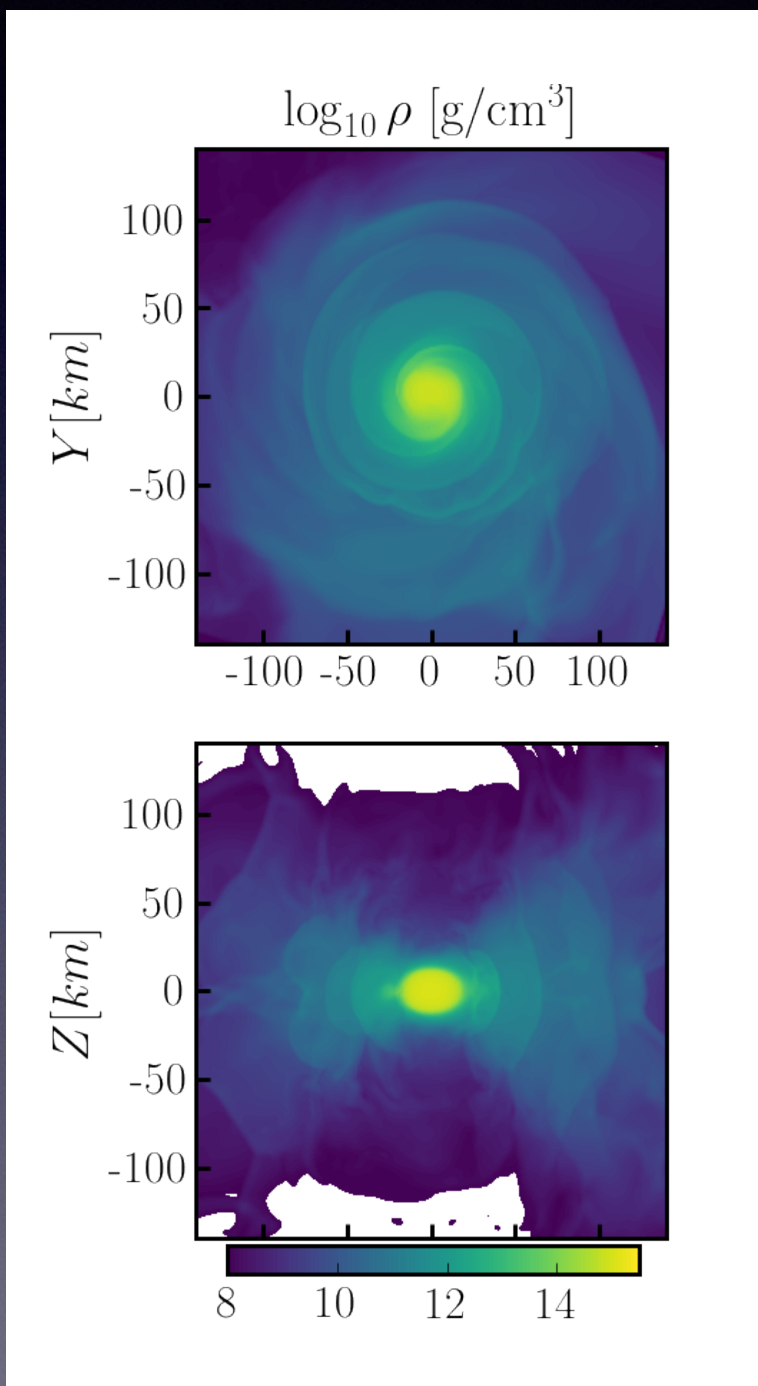
M1 method is not always guaranteed to provide physically correct results.

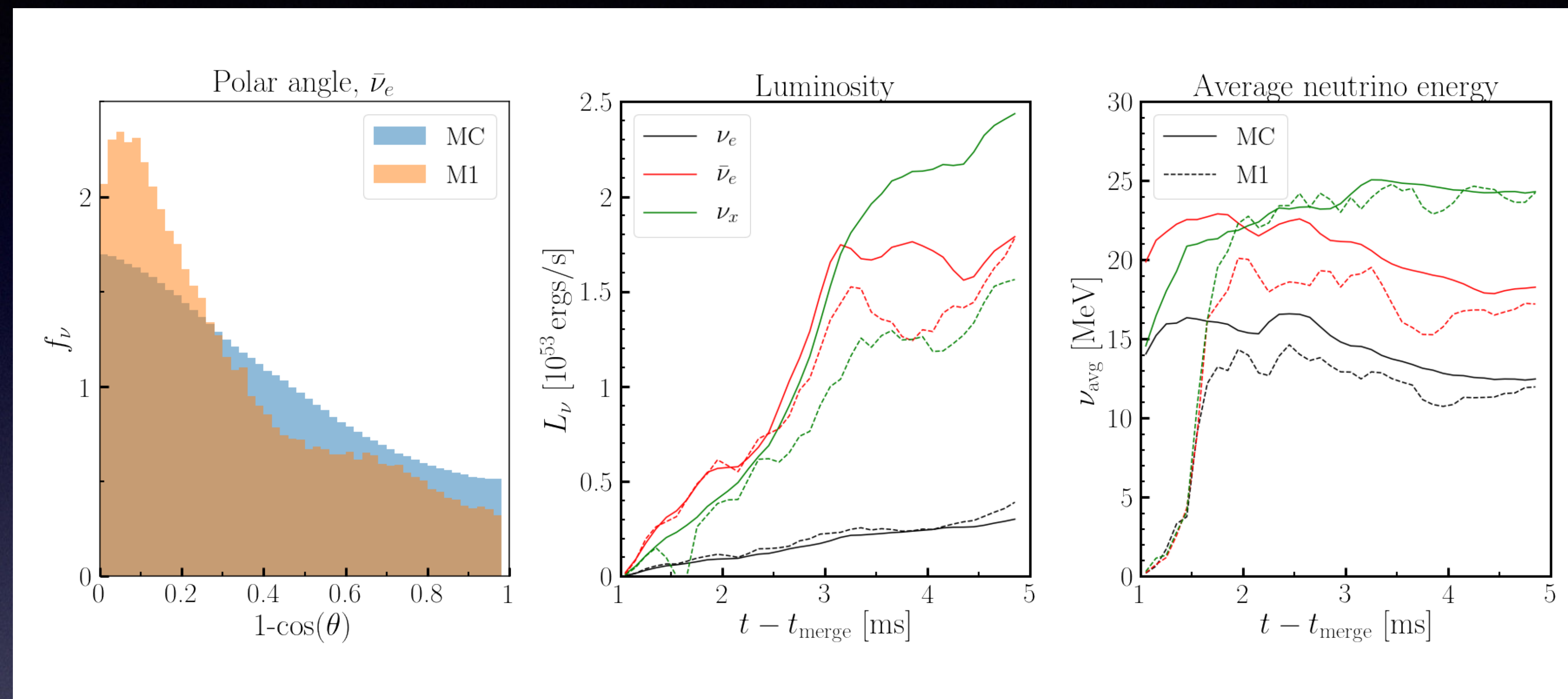
(see, e.g., H. Nagakura et al. 2017 & Y. Asahina et al. 2020 for grid-based full-Boltzmann method in GR)

Monte Carlo methods

Neutron star merger simulation
(GRHD+MCRadiation)

F. Foucart et al. 2020



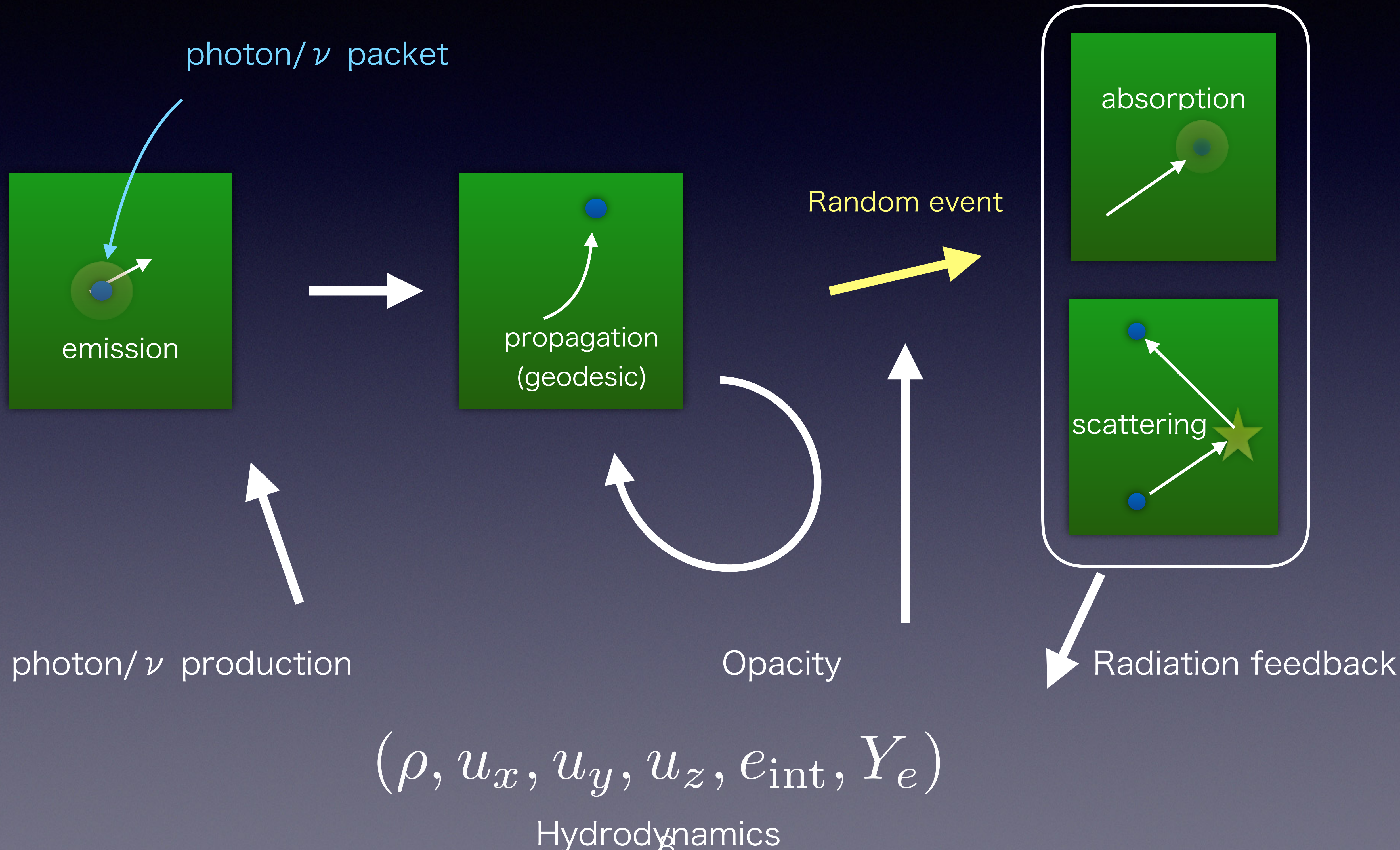


GR Monte-Carlo RHD:
- N. Roth & D. Kasen 2015
- Ryan et al. 2015
- Miller et al. 2019, 2020
- F. Foucart et al. 2017, 2018, 2020

} 3D: physically accurate, but computationally expensive

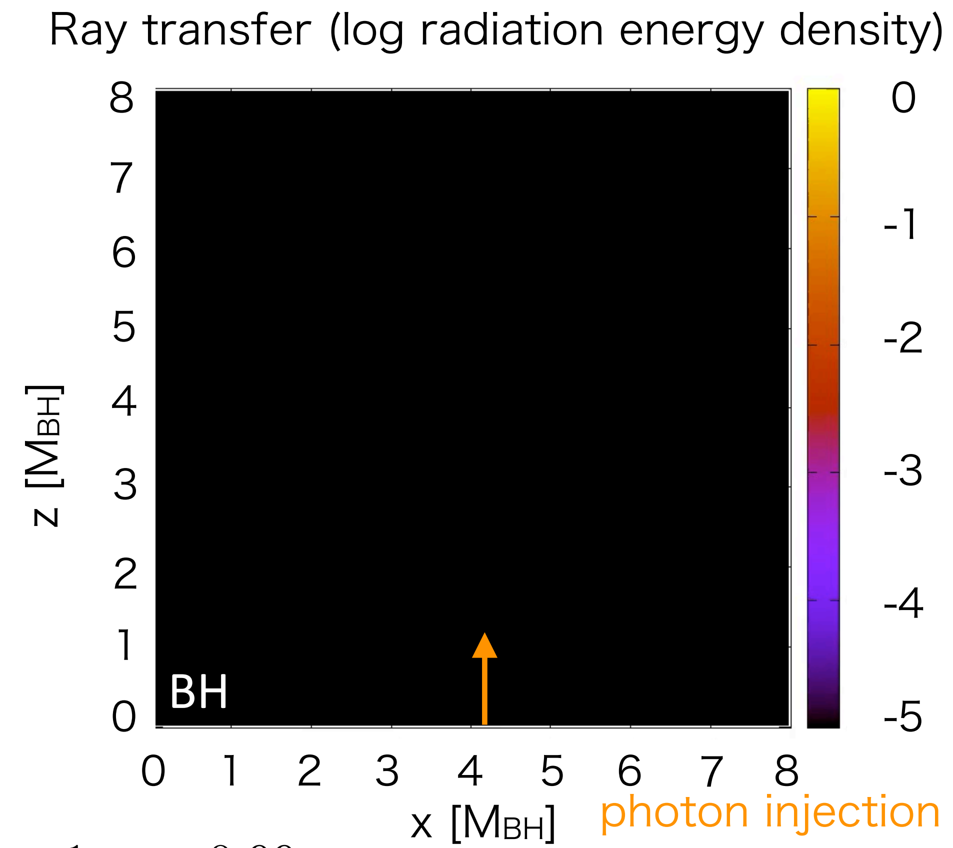
Developing an axisymmetric (2d) code would be useful for longterm simulations and systematic studies!

Monte-Carlo method: Procedure

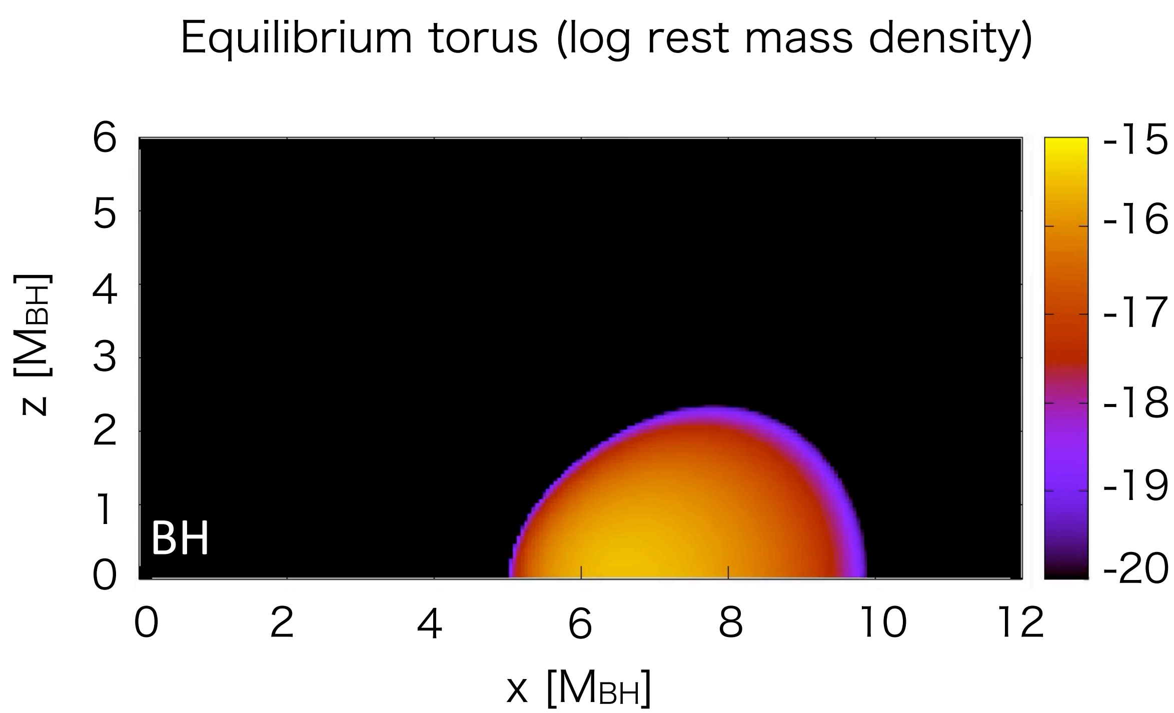


Axisymmetric GR-MCRHD code

- Geodesic:
  4th order spatial interpolation
- Hydrodynamics:
  GR hydro (fixed metric)
  3rd Order MUSCL
  + Kurganov-Tadmor (central) scheme
- Time integration:
  SSP-RK3 (3rd order)
  for hydro & geodesic solver
- isotropic scattering
  (as a first step)



$M_{\rm BH} = 1, \chi = 0.99$



Thermalization test

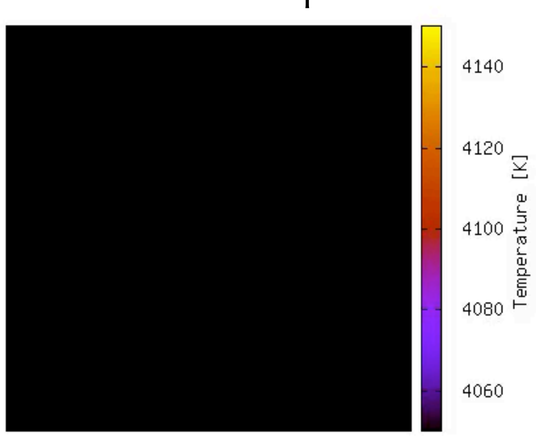
$$\kappa_{\rm abs} = 1\,{\rm g/cm^3}$$

$$\frac{1}{\kappa_{\rm abs}\rho c} = 1\,{\rm s}$$

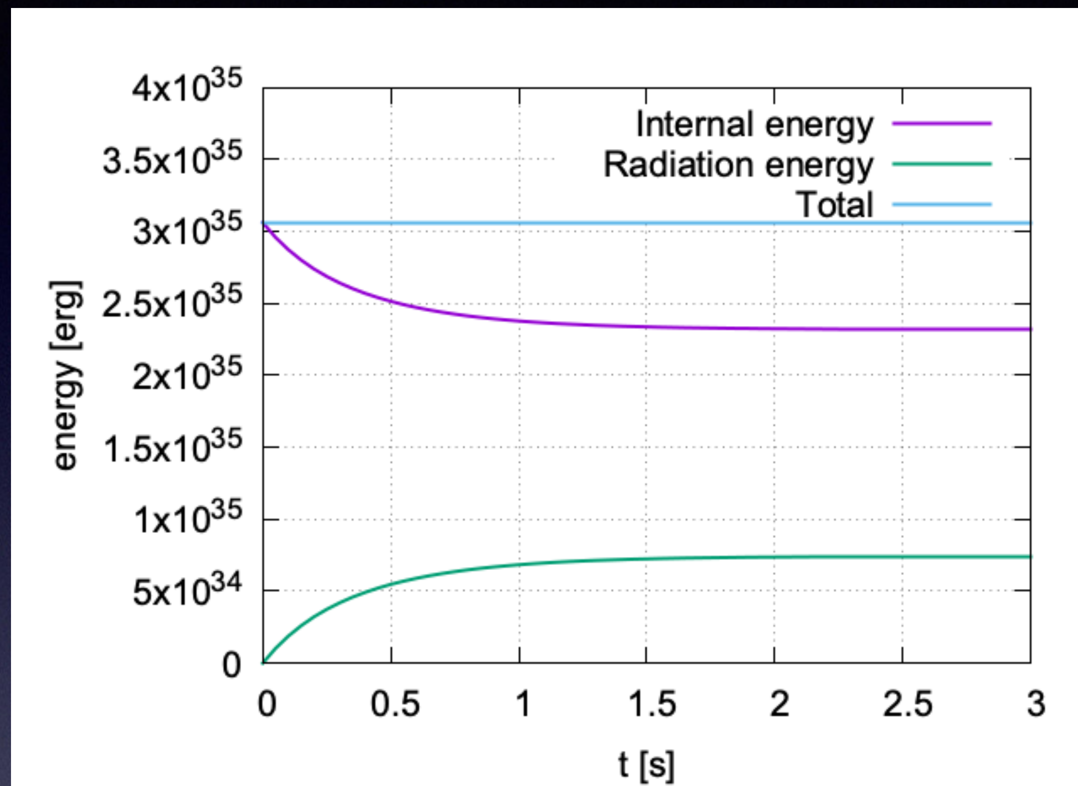
$$P_{\rm gas} = (\Gamma - 1)\rho\epsilon$$

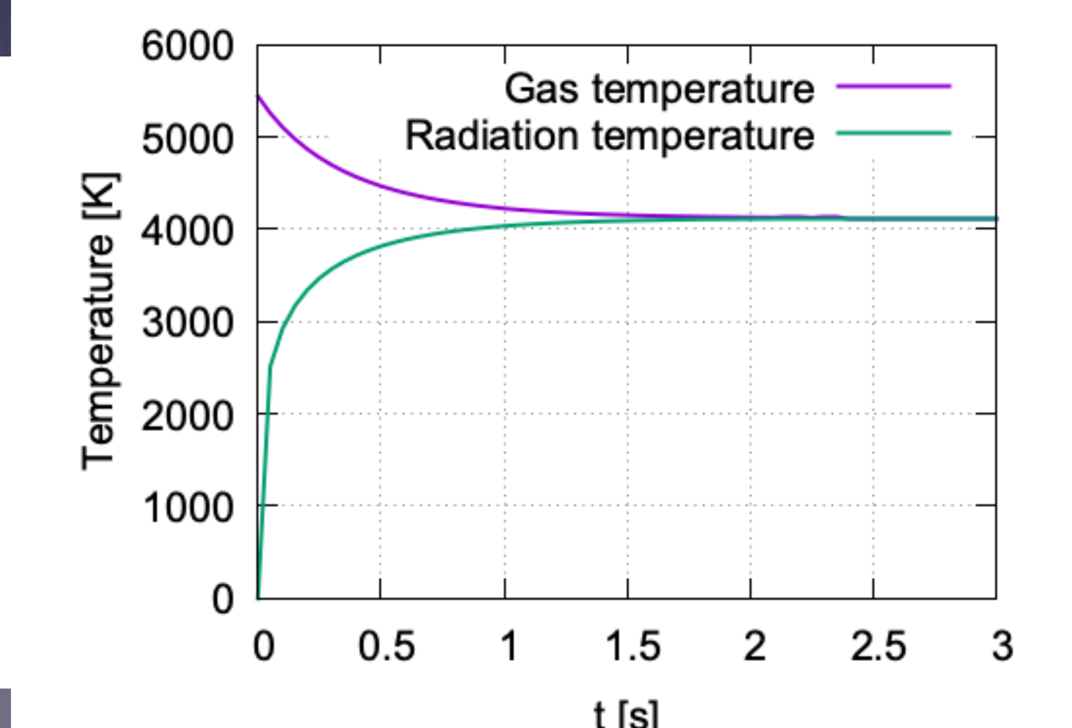
$$T_{\rm gas} = \frac{(\Gamma - 1)m_p}{k_{\rm B}}\epsilon$$

Radiation temperature

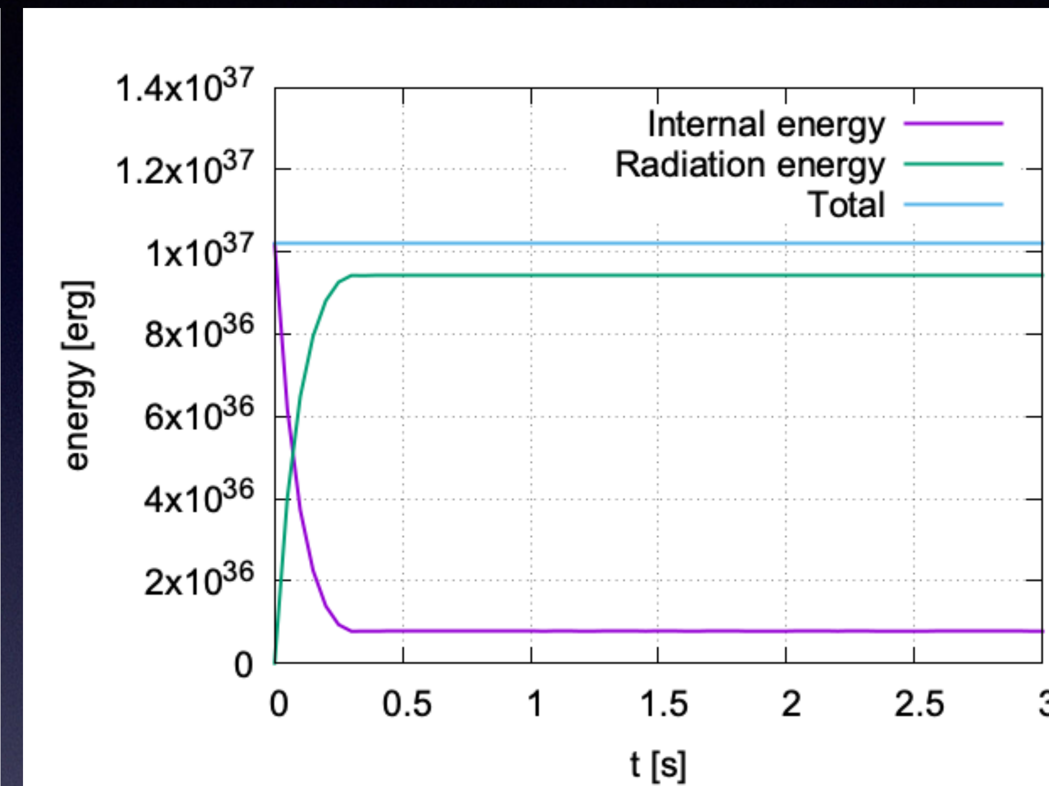


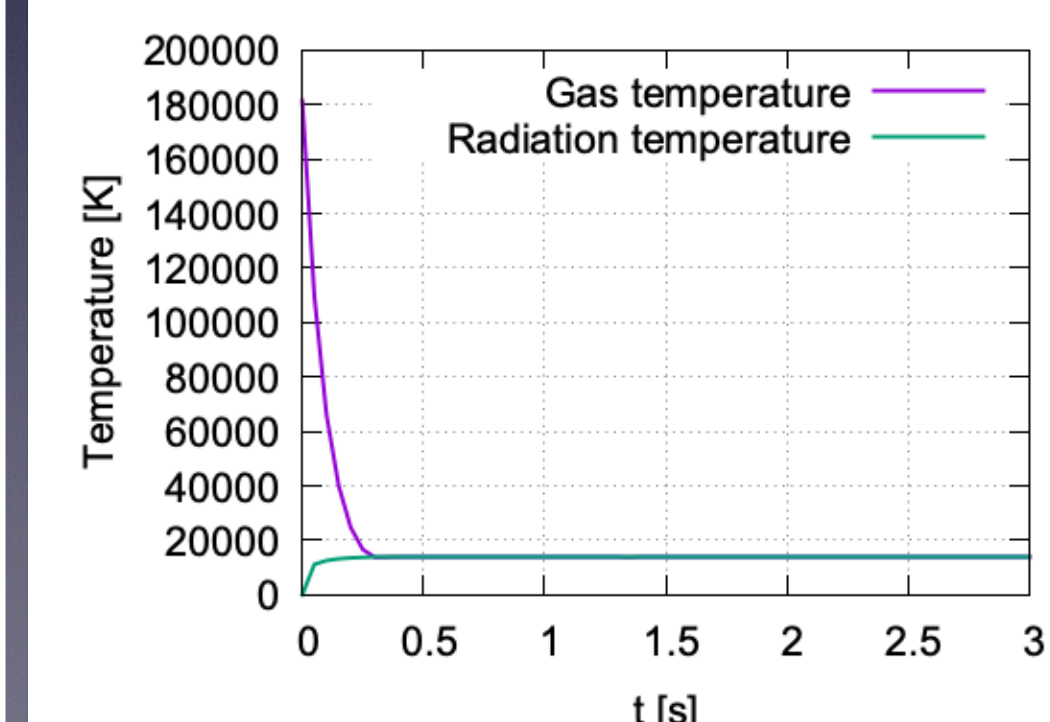
Case 1 (gas dom.)





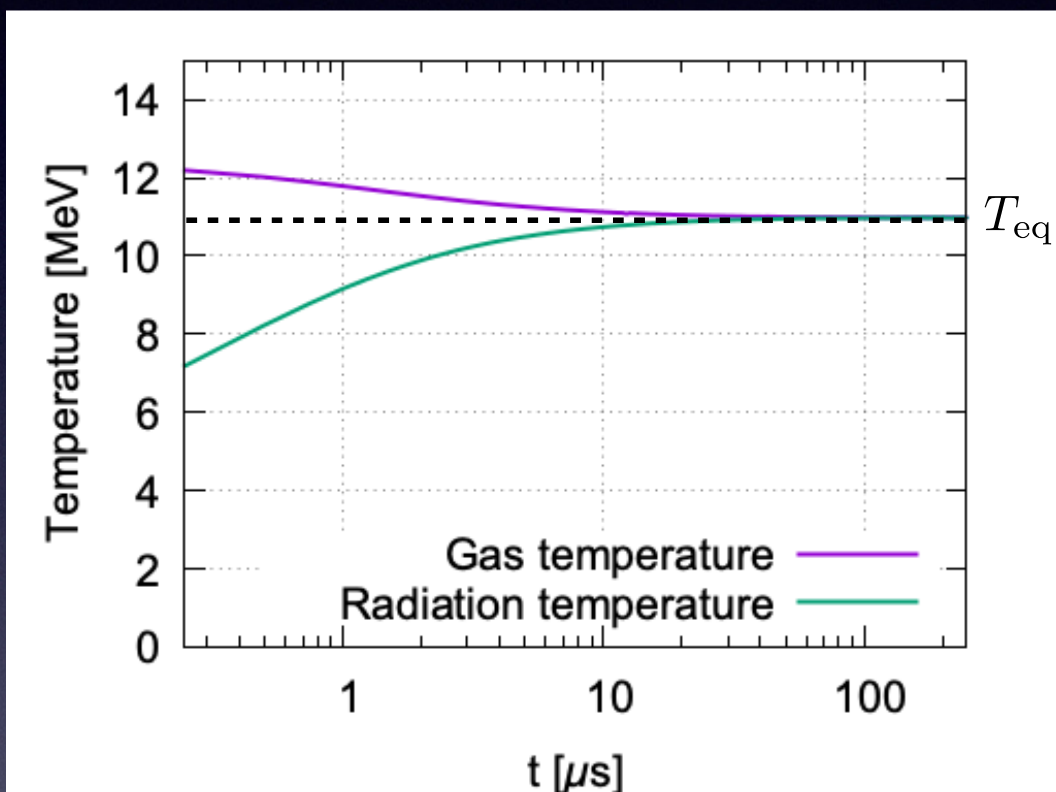
Case 2 (rad dom.)

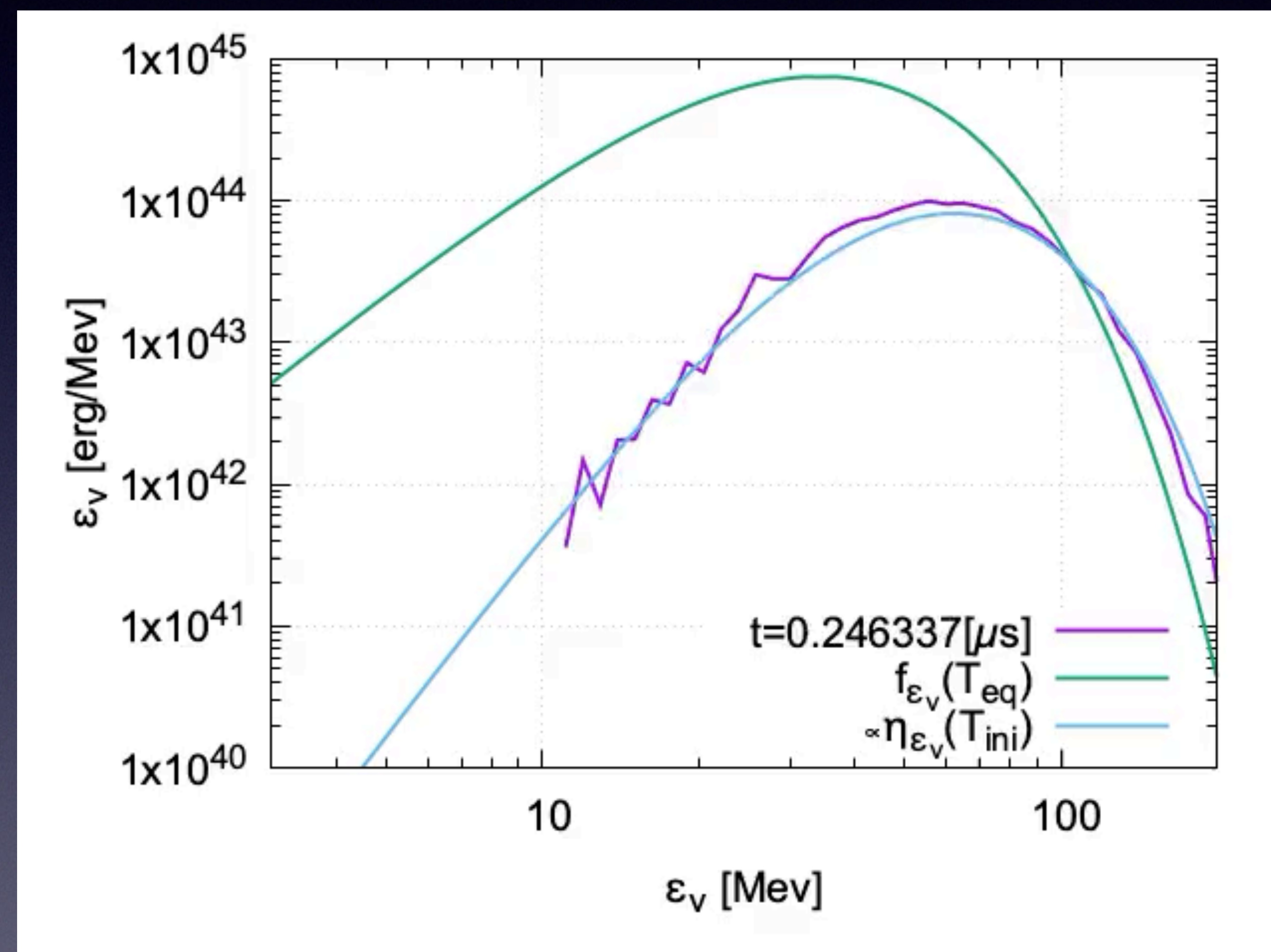




Multi-energy (neutrino)

$$\rho = 10^{11}\,\mathrm{g/cm^3} \qquad T_{\mathrm{gas,ini}} = 12.4\,\mathrm{MeV} \qquad T_{\mathrm{eq}} = 11.0\,\mathrm{MeV}$$





$$P = P_{e^-} + P_{e^+} + P_{\mathrm{rad}} = \frac{11}{12} a T^4$$

$$\kappa_{\mathrm{abs}} \approx \frac{(1+3g_{\mathrm{A}}^2)G_{\mathrm{F}}^2}{\pi/m_p}\epsilon_\nu^2$$

$$\approx 5.7\times 10^{-20}\mathrm{cm^2/g}\left(\frac{\epsilon_\nu}{1\mathrm{MeV}}\right)^2$$

$$f_{\epsilon_\nu}(T) \propto \frac{\epsilon_\nu^3}{\exp\left(\epsilon_\nu/k_{\mathrm{B}}T\right)+1}$$

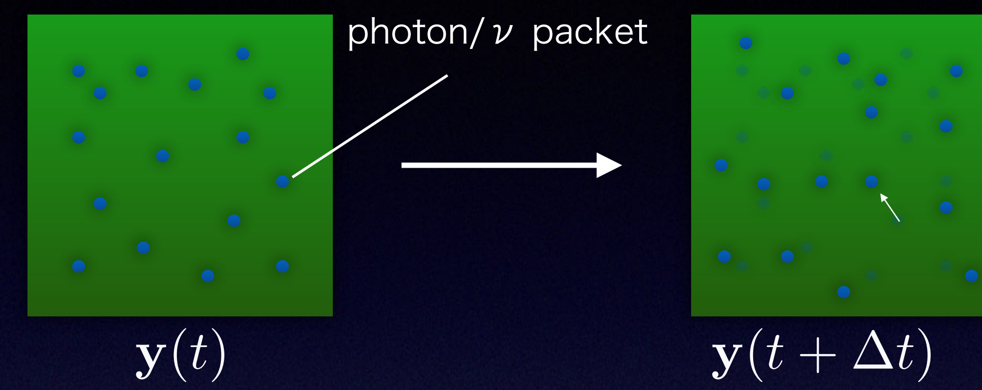
$$\eta_{\epsilon_\nu}(T) \propto \frac{\epsilon_\nu^5}{\exp\left(\epsilon_\nu/k_{\mathrm{B}}T\right)+1}$$

Time integration

- Operator splitting method is often employed for the coupling between radiation field and fluid part:
  $\rightarrow$ time integration is 1st order for entire simulation
  N. Roth & D. Kasen 2015, Ryan et al. 2015, Miller et al. 2019, 2020,
  F. Foucart et al. 2017, 2018, 2020

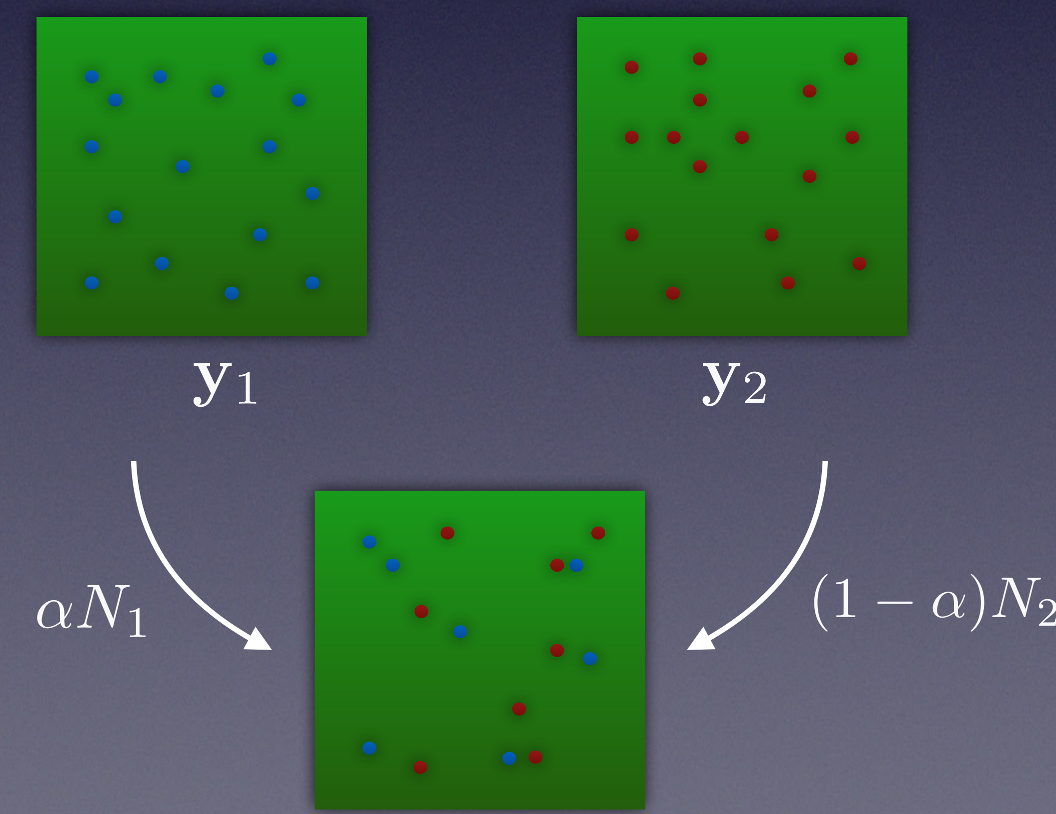
- How can we implement higher-order time integration scheme?
  - Usual iterative higher-order time integration schemes are not applicable for radiation flied described by MC packets
  - Algebraic addition of radiation field can be defined by appropriate thinning and joint of MC packets

Radiation field



$$\Delta \mathbf{y} = \mathbf{y}(t+\Delta t) - \mathbf{y}(t)? \quad \text{X}$$

“Addition” of radiation field



$$\mathbf{y}_* = \alpha \mathbf{y}_1 + (1-\alpha)\mathbf{y}_2 \quad (\alpha \in [0,1])$$

Higher-order scheme

$\mathbf{u}_n, \mathbf{y}_n$: matter and radiation field at n-th time step

$$\mathbf{u}_1 = \mathbf{u}_n + \Delta\mathcal{F}\left(\mathbf{u}_n, \mathbf{y}_n\right)$$
$$\mathbf{y}_1 = \mathcal{G}\left(\mathbf{y}_n, \mathbf{u}_p\right)$$

$$\mathbf{u}_2 = \mathbf{u}_n + \Delta\mathcal{F}\left(\mathbf{u}_1, \mathbf{y}_n\right)$$
$$\mathbf{y}_2 = \mathcal{G}\left(\mathbf{y}_n, \mathbf{u}_p\right)$$

$$\mathbf{u}_3 = \mathbf{u}_n + \Delta\mathcal{F}\left(\mathbf{u}_p, \mathbf{y}_n\right)$$
$$\mathbf{y}_3 = \mathcal{G}\left(\mathbf{y}_n, \mathbf{u}_p\right)$$
$$\mathbf{u}_p = \frac{1}{2}\mathbf{u}_n + \frac{1}{4}\left(\mathbf{u}_1 + \mathbf{u}_2\right)$$

$$\mathbf{y}(t+\Delta t)|_{\mathbf{u}} = \mathcal{G}\left[\mathbf{y}(t), \mathbf{u}\right]$$

$$\frac{d\mathbf{u}}{dt}\Delta t = \Delta\mathcal{F}\left[\mathbf{u}, \mathbf{y}(t)\right]$$

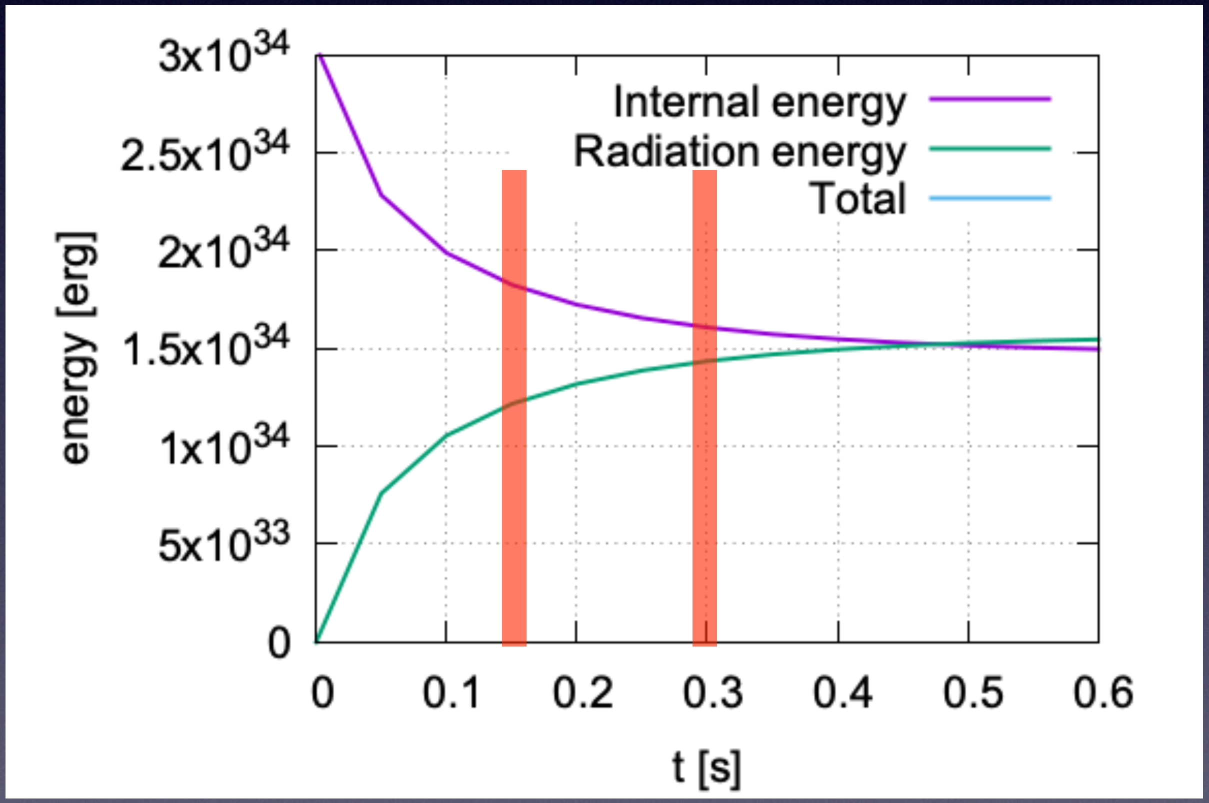
*including feed back from
radiation field during t~t+Δt

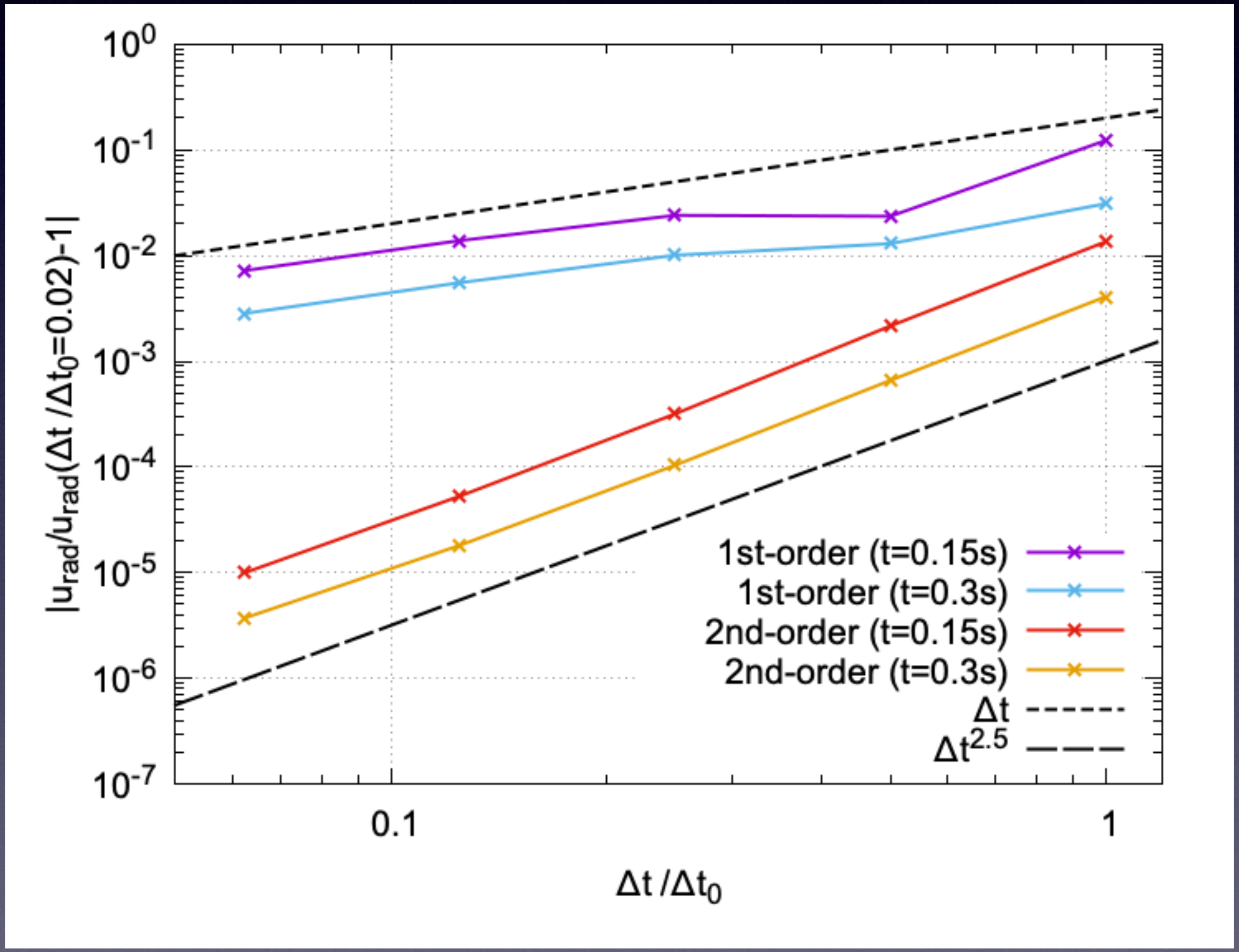
$$\mathbf{u}_{n+1} = \frac{1}{6}\mathbf{u}_1 + \frac{1}{6}\mathbf{u}_2 + \frac{2}{3}\mathbf{u}_3$$
$$\mathbf{y}_{n+1} = \frac{1}{6}\mathbf{y}_1 + \frac{1}{6}\mathbf{y}_2 + \frac{2}{3}\mathbf{y}_3$$

Guarantees 2nd order accuracy for time integration

*hydro scheme reduces to SSP-RK3 for the case that radiation field is negligible

Convergence test

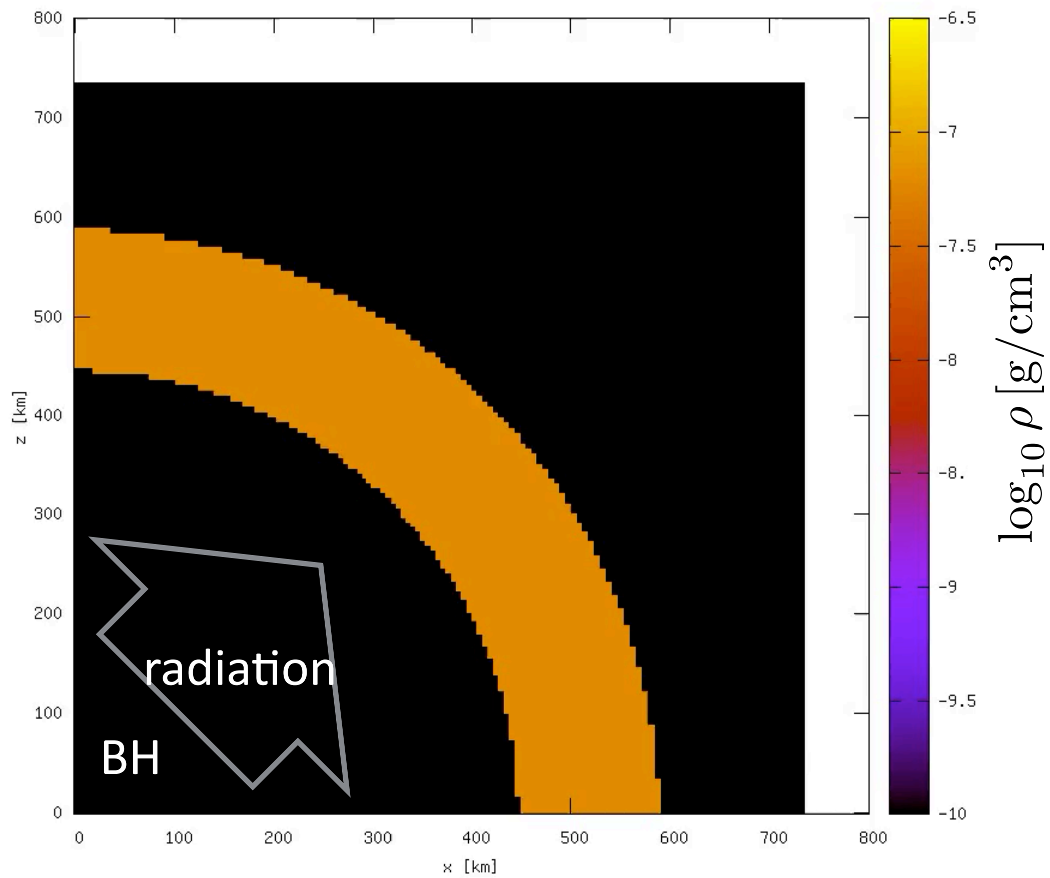




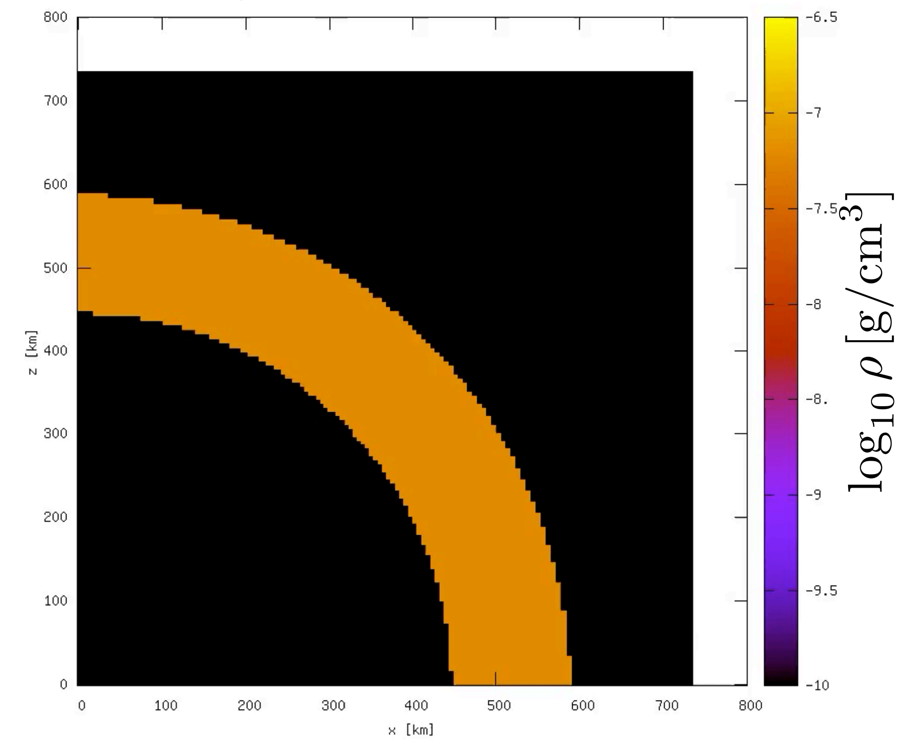
Eddington Limit

$$L_{\rm Edd} = \frac{4\pi G M_{\rm BH} c}{\kappa}$$

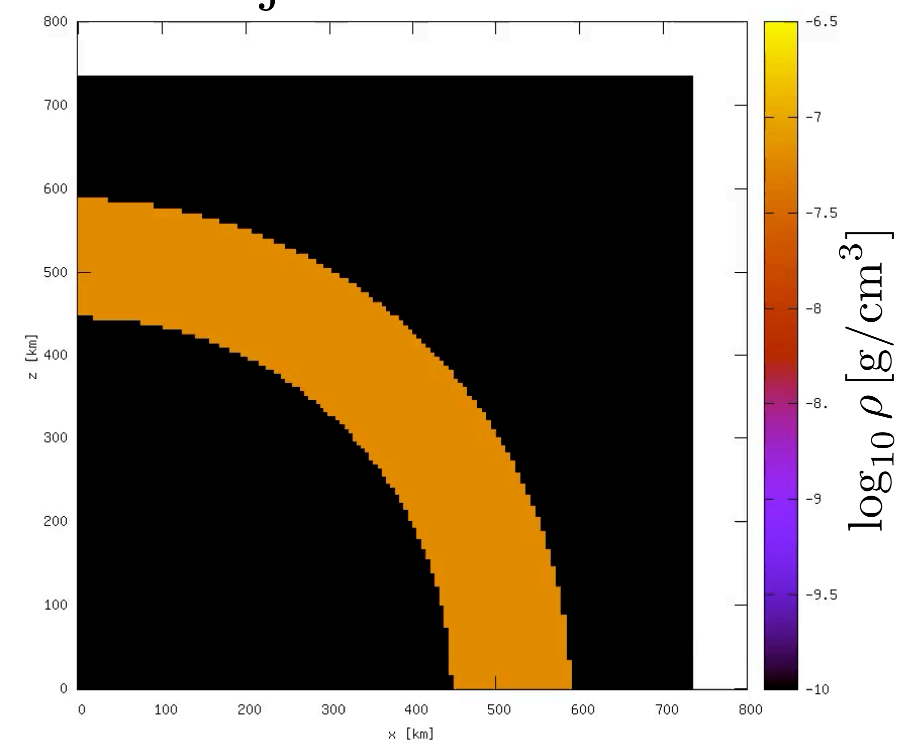
$L_{\rm inj} = L_{\rm Edd}$



$L_{\rm inj} = 1.25 L_{\rm Edd}$



$L_{\rm inj} = 0.75 L_{\rm Edd}$



$M_{\rm BH} = 1\,M_{\odot}, \chi = 0$

$\kappa_{\rm abs} = 1\,{\rm cm^2/g}$

Prescription for optically thick region

Dynamical timescale
(resolution in the simulation)

$$\Delta t_{\rm LC} = \frac{\Delta x}{c}$$

Thermal timescale

$$\Delta t_{\rm ems} = \frac{u_{\rm gas}}{\Delta u_{\rm ems}} = \frac{u_{\rm gas}}{\Delta u_{\rm ems}} = \frac{1}{\kappa \rho c} \frac{u_{\rm gas}}{u_{\rm rad}}$$

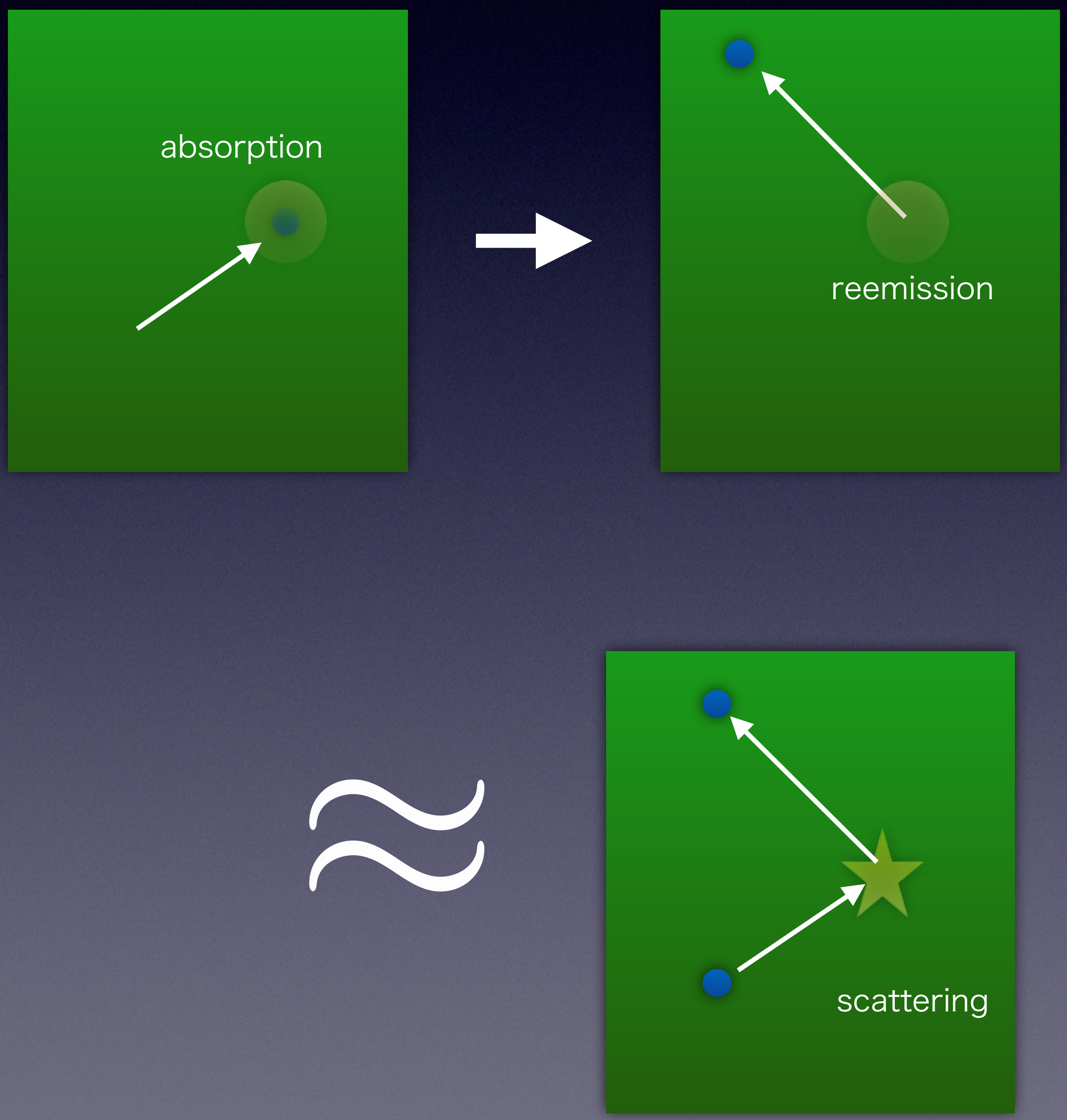
$$\frac{\Delta t_{\rm ems}}{\Delta t_{\rm LC}} = \frac{1}{\kappa_{\rm abs} \rho \Delta x} \frac{u_{\rm gas}}{u_{\rm rad}} = \frac{1}{\Delta \tau} \frac{u_{\rm gas}}{u_{\rm rad}} \ll 1 \; (\text{for } \Delta \tau \gg 1)$$

Prescription:
(Foucart et al. 2020, Fleck & Cummings 1971)

$$\kappa_{\rm abs} \rightarrow \kappa'_{\rm abs} = (1 - \lambda) \kappa_{\rm abs}$$
$$\kappa_{\rm sct} \rightarrow \kappa'_{\rm sct} = \kappa_{\rm sct} + \lambda \kappa_{\rm abs}$$

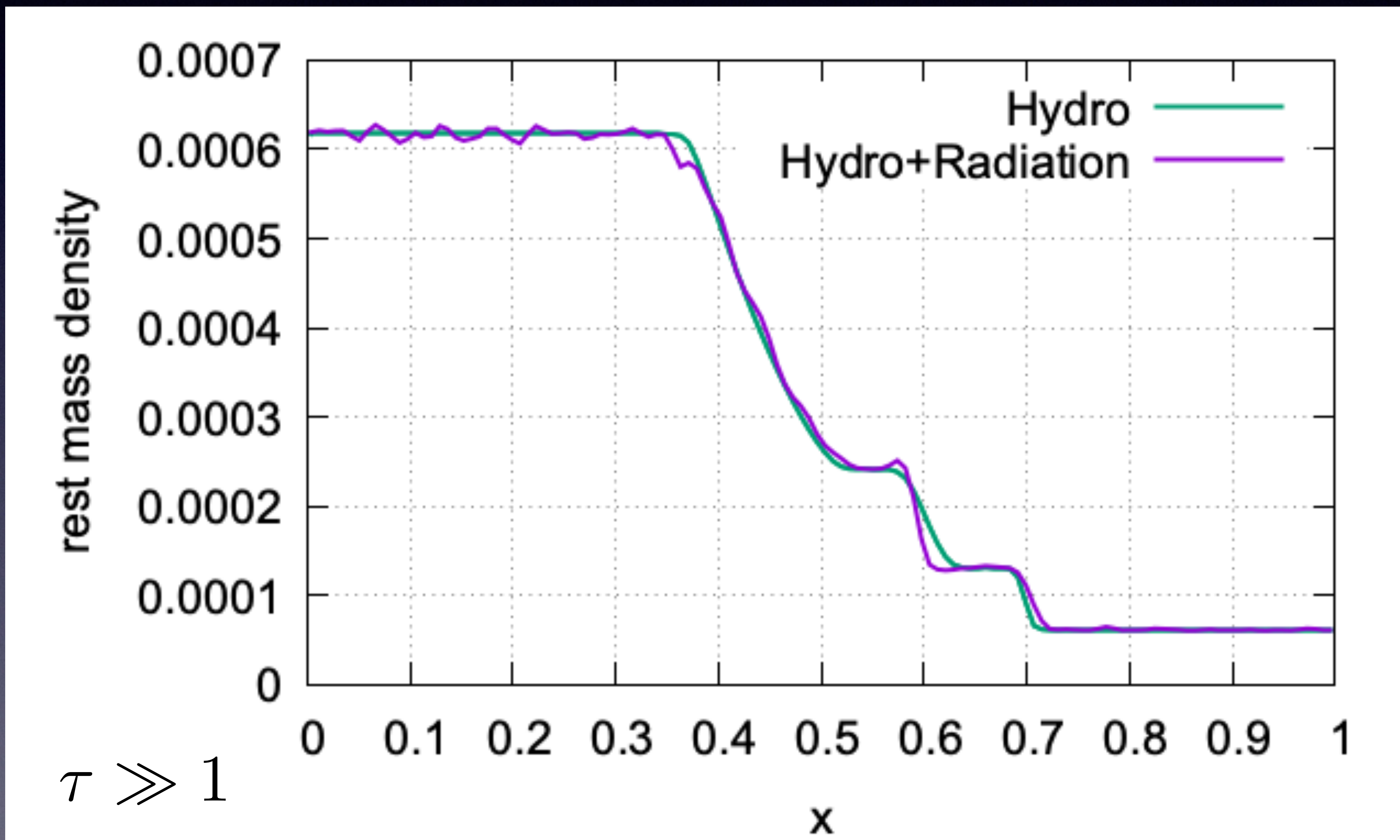
Justified if the state in the cell is
close to thermal equilibrium

$\Delta t_{\rm ems} \ll \Delta t$



Shock tube

$e_{\rm fl} = aT_{\rm fl}^4$: radiation pressure dom. gas

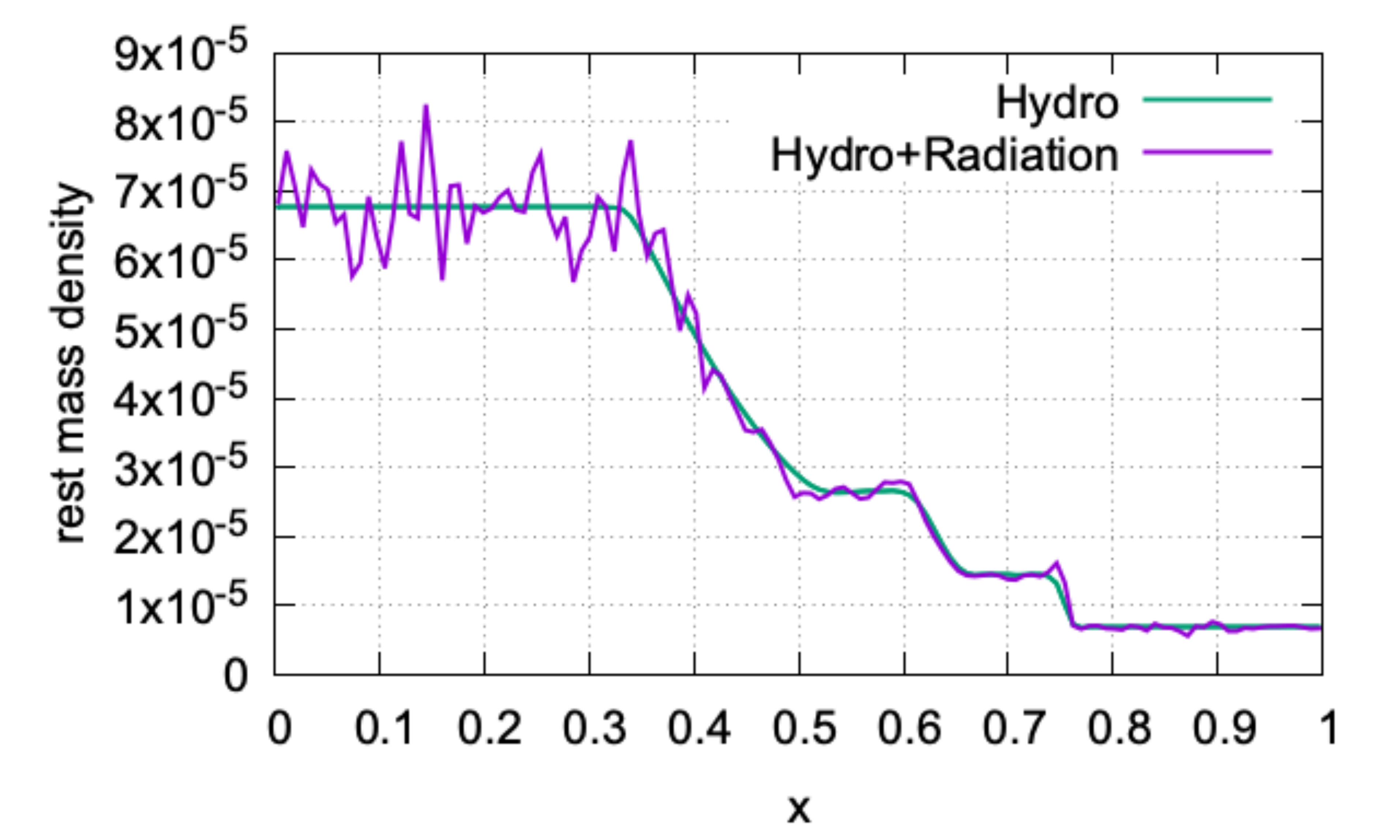


$e_{\rm fl} \approx e_{\rm rad}$ @ thermal equilibrium

Shock tube

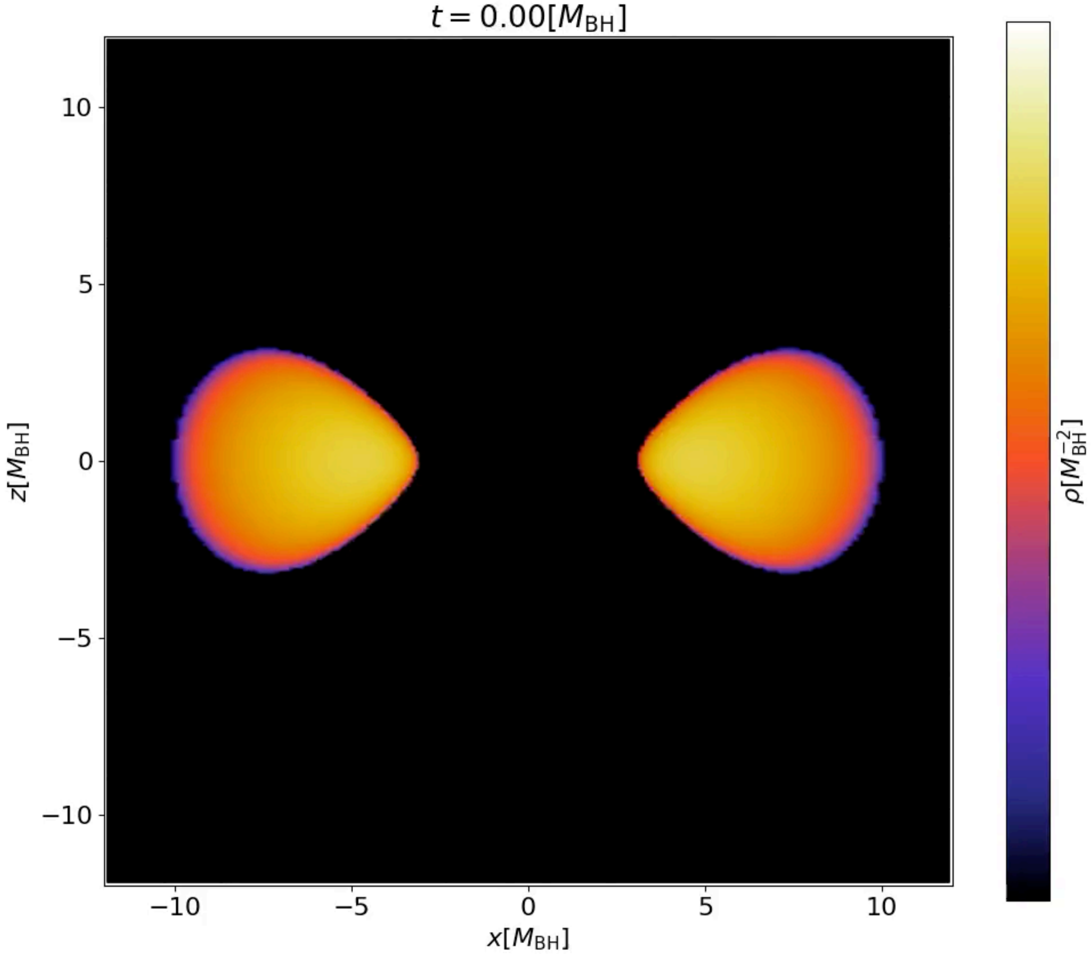
radiation pressure dom. gas

(the packet number was not sufficient…)

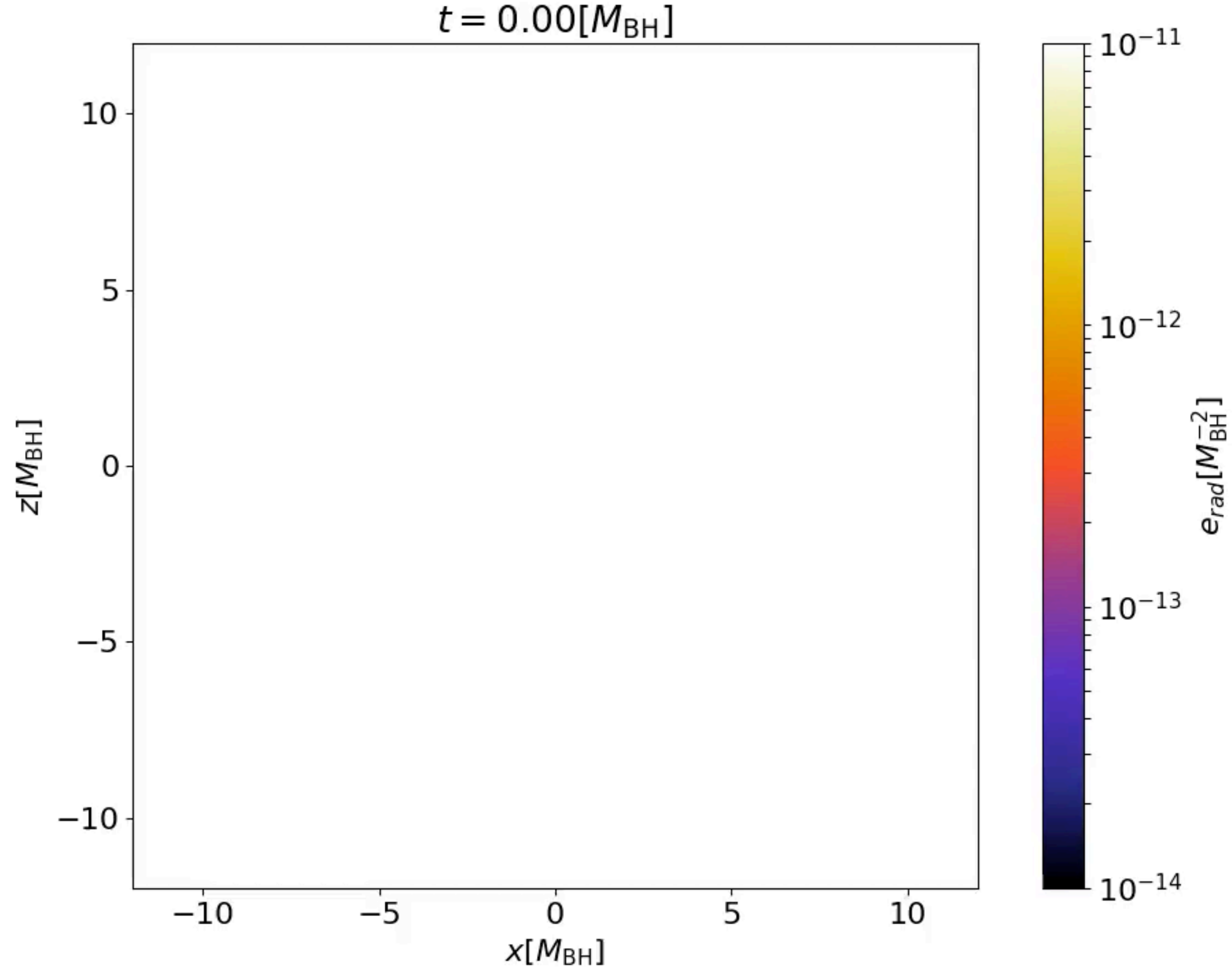


Next steps/tasks

Rest-mass density



Radiation energy density



Summary

- We develop **an axisymmetric (2d) GR-MCRHD** code
  as a tool to study NS binary mergers in the the post-merger phase
- Most of the infrastructures are implemented:
  - photon/ν packet transfer, hydrodynamics
    , appropriate matter-radiation interaction
  - **Higher-order time integration MC scheme**
    for matter-radiation interaction is implemented and demonstrated
    (for the first time, as far as I know)
  - prescription for optically thick region (needs further tests)
- Tasks and problems to be solved:
  - more simple/robust treatment of optically thick / thermalized region
  - more realistic microphysical process
    (Compton scattering, energy-dependent cross-section)
  - implementing dissipation/ang. mom. transport process in hydrodynamics
    (viscosity, magnetic fields)
  - parallelization / optimization