Blandford-Znajek process
& propagation of Alfvén waves

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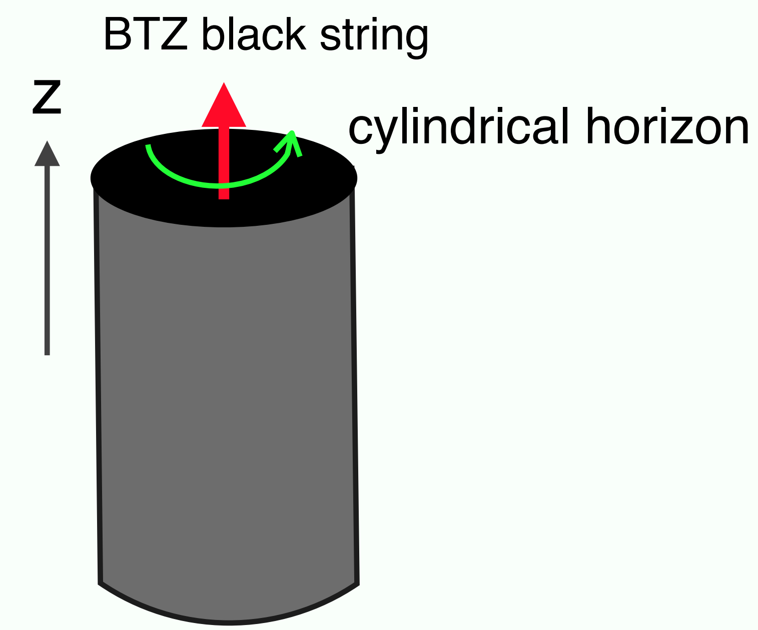
Reference

S.N *etal.*, Phys. Rev. D **101** 023003 (2020) ⇒ BTZ string (each z-const = BTZ)

S.N, T. Tsukamoto, Y. Nambu, and M. Takahashi in prep ⇒ Kerr (this talk)



2021年1月18日
Black Hole Astrophysics with VLBI: Multi-Wavelength and Multi-Messenger Era

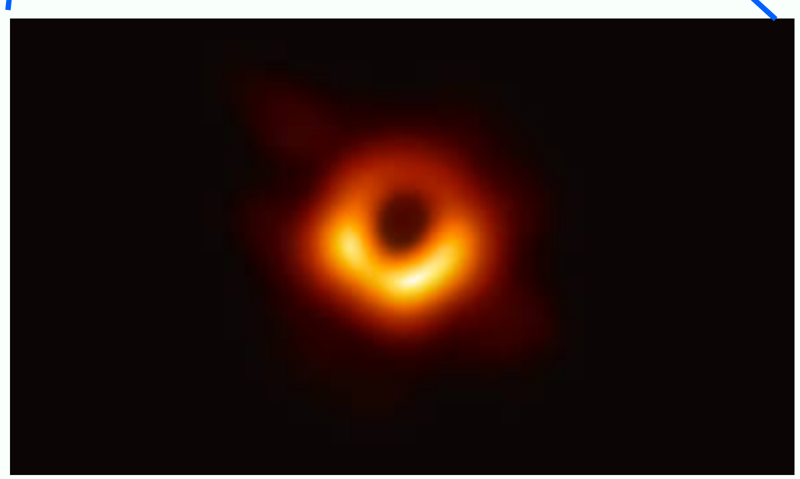
Relativistic jet & BH

M87

Credits: NASA and the Hubble Heritage Team (STScI/AURA)

Collimated stream of plasma

speed ~ c

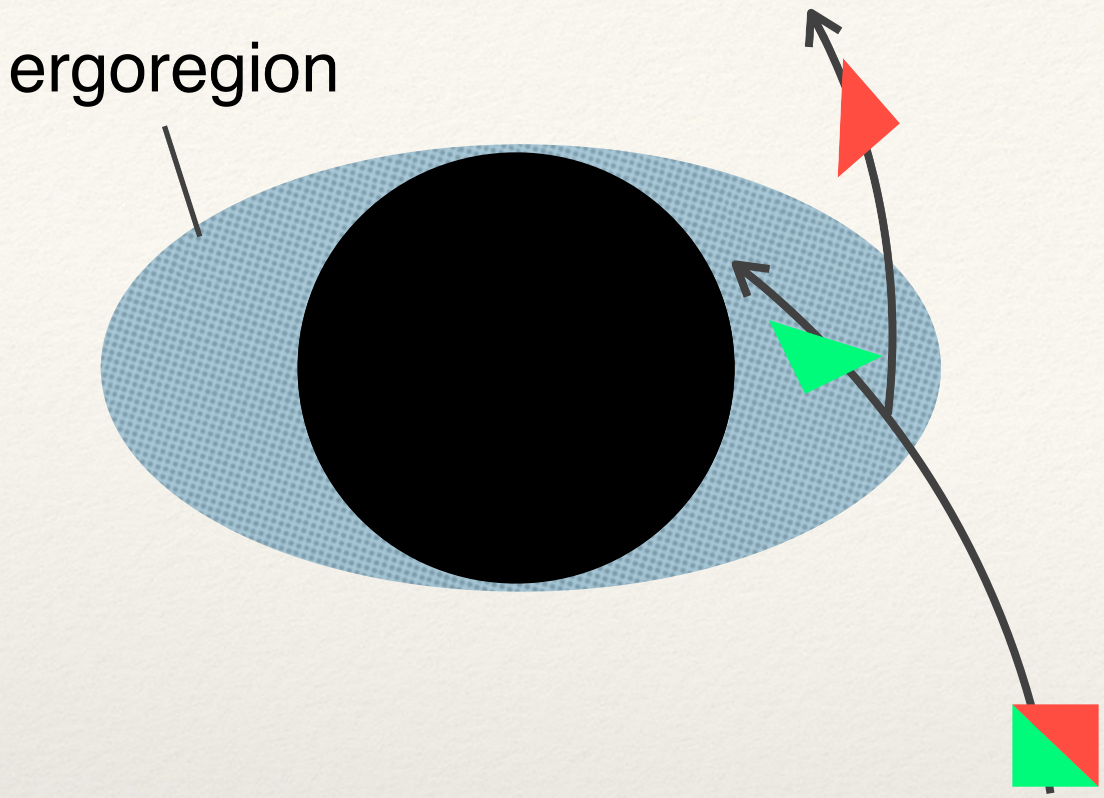


BH Shadow (EHTC)

Energy source of jets/gamma ray bursts ⇒ **BH's rotational energy**

⟶ **Energy extraction processes**

Penrose Process



Energy (Killing energy)

$$E := -p_\mu \xi^\mu_{(t)}$$

Energy conservation

$$E = E + E$$

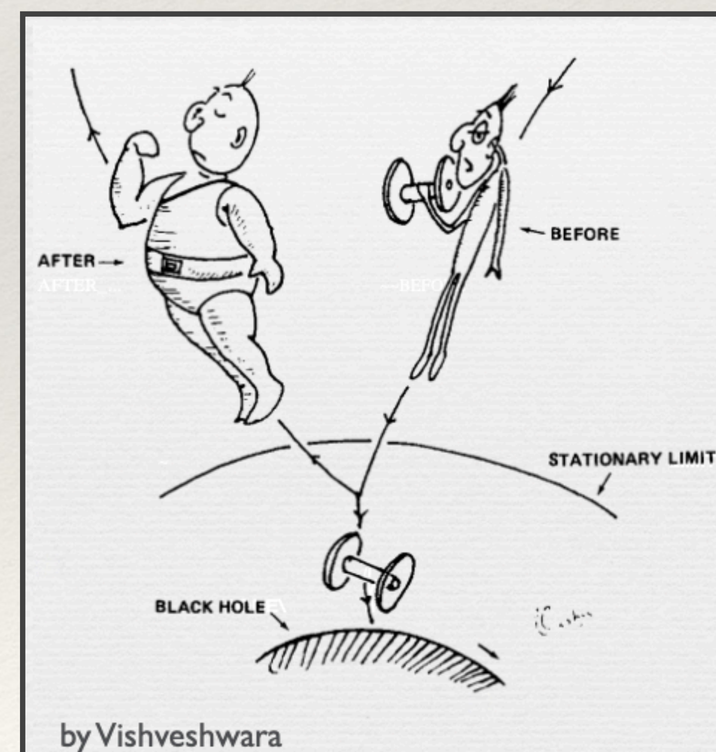
Timelike Killing vector is spacelike in the ergoregion

$\longrightarrow$ E can be negative. (It depends on p_μ)

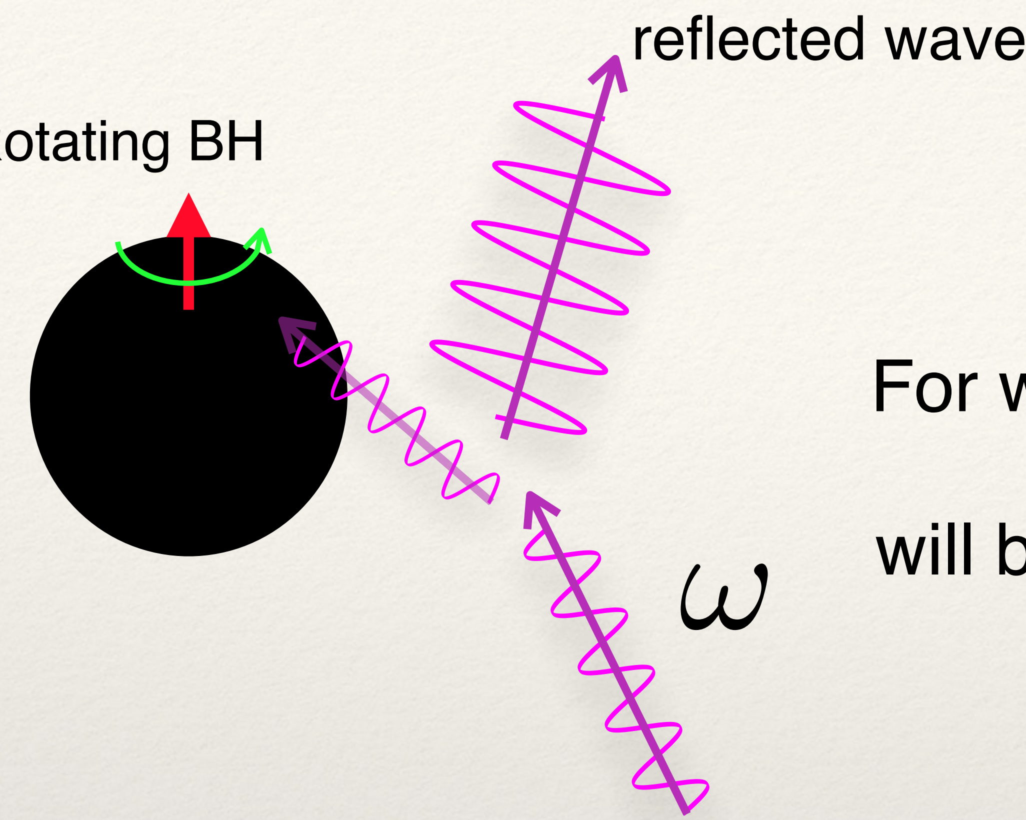
From the energy conservation law,

$$E = E - \underset{<0}{\underline{E}} \longrightarrow E > E$$

energy gain



Superradiant wave scattering (superradiance)



$$\Phi = \int d\omega \; e^{-i\omega t} e^{im\varphi} f(r,\theta)$$

For wave modes satisfying $\omega < m\Omega_{\rm H}$

will be amplified by scattering.

angular velocity of the horizon

Superradiant wave scattering is possible for

scalar wave — Y. B. Zel'dovich, Zh. Eksp. Teor. Fiz. Pis'ma Red. **14**, 270, (1971)

EM, gravitational waves — A.A. Starobinsky, Zh. Eksp. Teor. Phyz. **64**, 48 (1973); A.A. Starobinsky and S.M. Churilov, Zh. Eksp. Teor. Phyz. **65**, 3 (1973)

acoustic wave in fluids — M. Visser , Class.Quant. Grav, **15** 1767 (1998)

magnetosonic fast wave — S.N *etal.,* Phys. Rev. D **95** 104055 (2017)

Alfvén wave — S.N *etal.,* Phys. Rev. D **101** 023003 (2020)

Blandford-Znajek process

BH's rotational energy ⇒ jet ?

Blandford & Znajek (1977)

1. Kerr BH
2. Stationary axisymmetric rotating magnetosphere (plasma + electromagnetic field)
3. Force-free magnetosphere (magnetically dominated)

$$\Omega_{\rm H} := -\left.\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right|_{r=r_{\rm H}}$$

Poynting flux

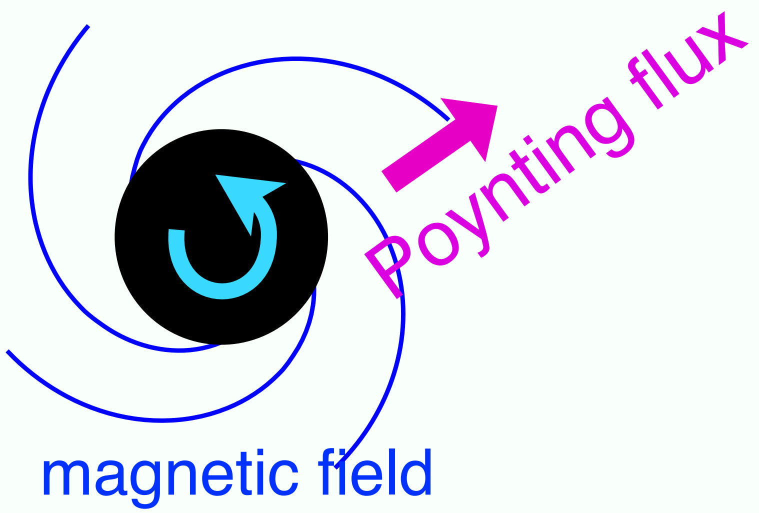
$$P^r \propto \Omega_F(\Omega_{\rm H} - \Omega_F)$$

If $0 < \Omega_F < \Omega_{\rm H}$, nonzero outward flux!!

⇒ Rotational energy is extracted by magnetosphere

BZ power

$$P_{\rm BZ} \approx 10^{45}{\rm erg/s}\left(\frac{a}{M}\right)^2\left(\frac{B_0}{10^4{\rm G}}\right)^2\left(\frac{M}{10^9 M_\odot}\right)^2$$

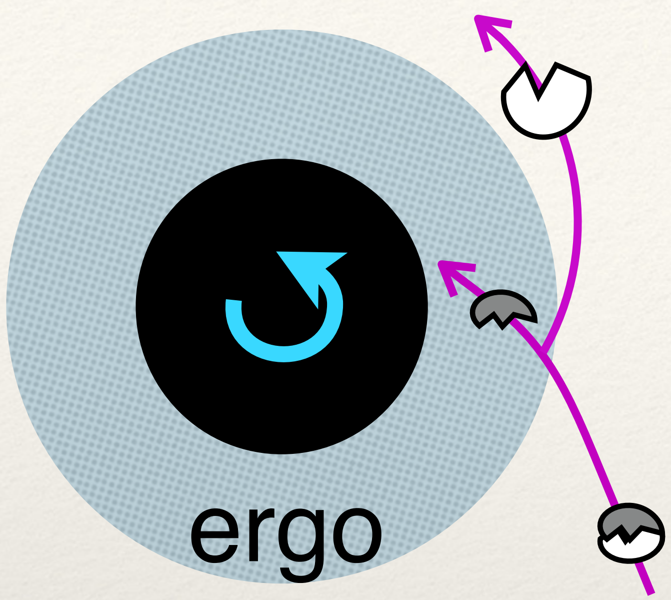
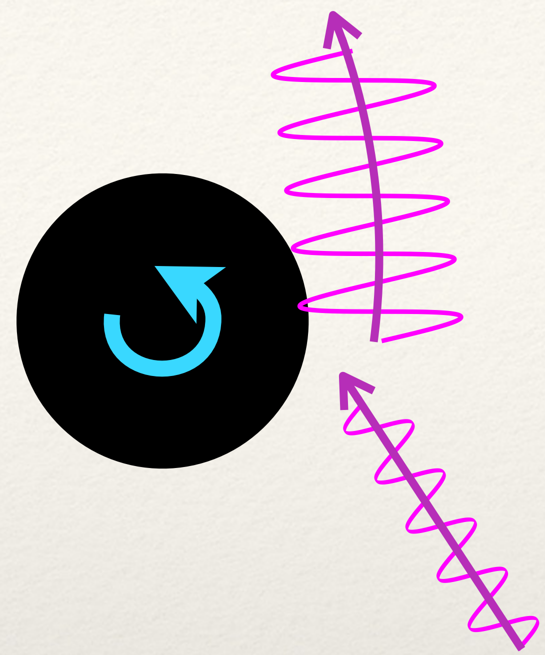
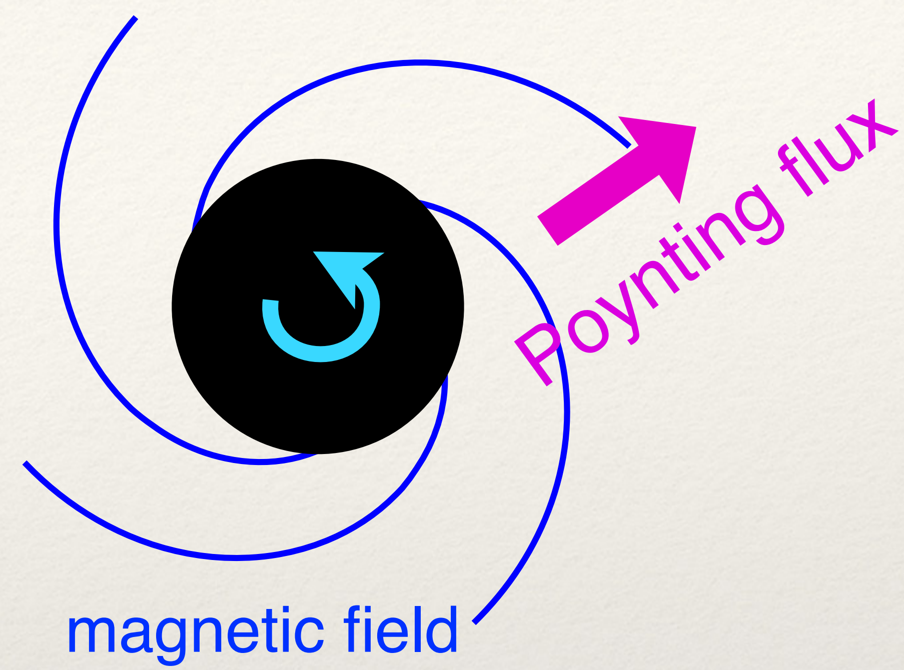


Questions

- Non-stationary magnetosphere case ??
- Wave propagation ??
- Essence of the BZ process ??

Extraction Mechanisms

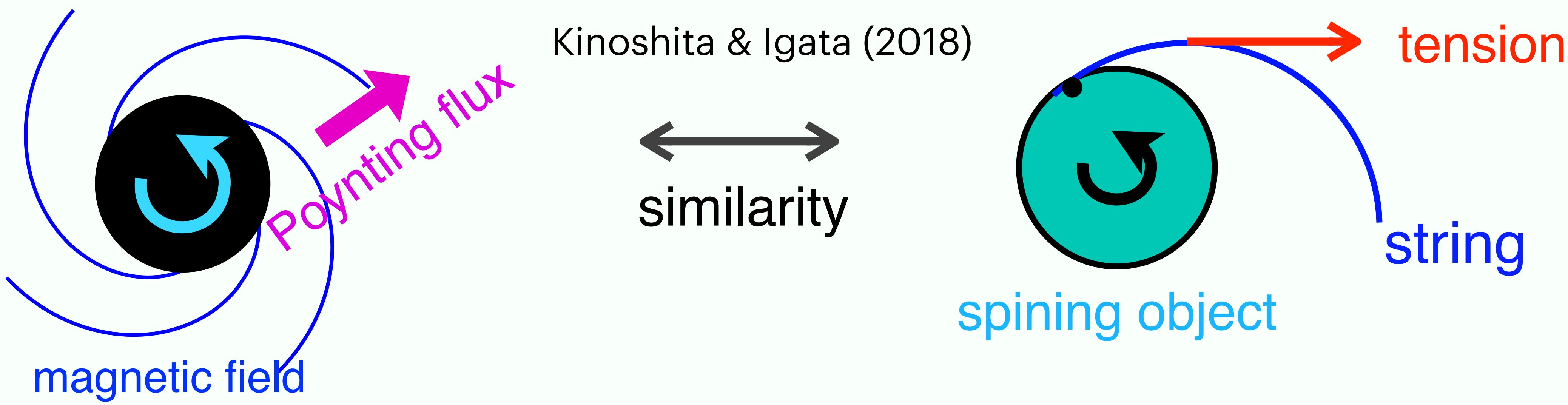
any relation ??

Penrose process	Superradiance	BZ process
		
particle	waves	EM field
particle decay	wave scattering	magnetic torque
energy of particle	amplitude of wave	Poynting flux

This is not
particle/wave phenomenon

Essence of the Blandford-Znajek process ?

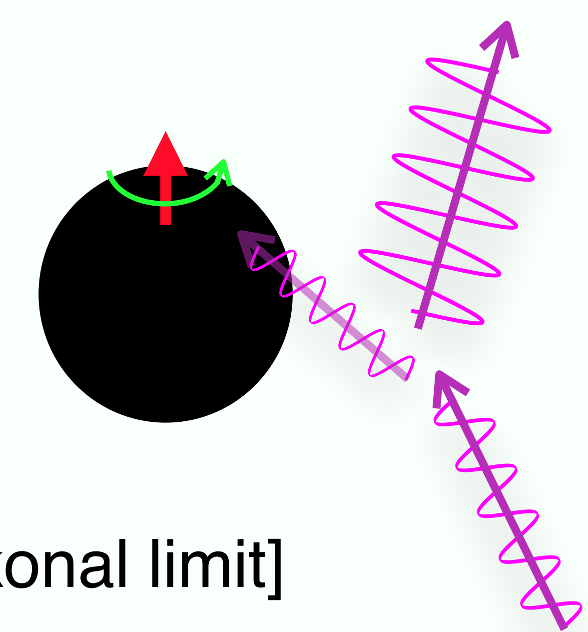
Magnetic tension plays crucial role in the BZ process



Our Ideas

- Wave regarding the magnetic tension in magnetosphere ?
- Its superradiant scattering may be related to the BZ process.

Alfvén waves



- Propagation due to the magnetic tension
- Superradiance is not possible. [Uchida (1997) based on Eikonal limit]
- It's possible! S.N *etal.*, Phys. Rev. D **101** 023003 (2020) ⇒ BTZ string spacetime

In this talk...

1. Force-free magnetosphere around a Kerr BH

2. Perturbation to the background magnetosphere

3. Mode decoupling (fast magnetosonic wave & Alfvén wave)

4. Alfvén wave equation & wave scattering problem

5. Reflection rate & Poynting flux

⇒ BZ process ~ Alfvénic superradiance

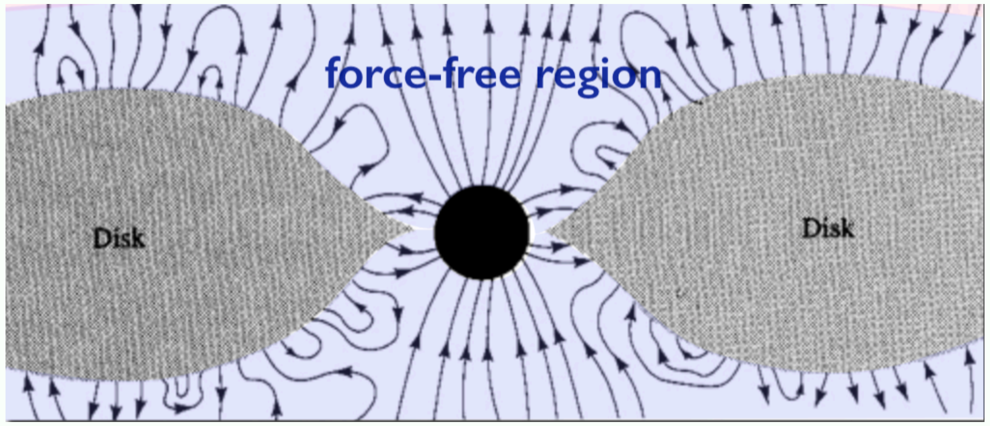
(superradiance for Alfvén wave)

Force-Free Electromagnetic Field

Maxwell eq.

$$\nabla_{[a}F_{bc]} = 0 \qquad \nabla_b T^{ab}_{(\mathrm{EM})} = -F^{ab}j_b$$

$$\nabla_b F^{ab} = 4\pi j^a \quad \nabla_b \left(T^{ab}_{(\mathrm{EM})} + \cancel{T^{ab}_{\mathrm{plasma}}}\right) = 0$$



If the EM fields are dominant , $\nabla_b T^{ab}_{(\mathrm{EM})} \approx 0 \quad \rightarrow \quad F^{ab}j_b \approx 0.$ (Force-free approximation)

$$\nabla_{[a}F_{bc]} = 0 \ , \quad F_{ab}\nabla_c F^{bc} = 0 \ , \quad j^a \propto \nabla_b F^{ab} \neq 0$$

$$\Rightarrow \quad \boxed{F = d\phi_1 \wedge d\phi_2}$$

ϕ_1 , ϕ_2 : Euler potential

Carter (1979)
Uchida (1997)
Gralla & Jacobson (2014)

Force-Free EM field eq.

$$\partial_a \underline{\phi_1} \partial_b \left[\sqrt{-g}\left(\partial^a\phi_1\partial^b\phi_2 - \partial^b\phi_1\partial^a\phi_2\right)\right] = 0$$

$$\partial_a \underline{\phi_2} \partial_b \left[\sqrt{-g}\left(\partial^a\phi_1\partial^b\phi_2 - \partial^b\phi_1\partial^a\phi_2\right)\right] = 0$$

Euler potential & FF-Magnetosphere

For stationary and axisymmetric (**rotating**) force-free magnetosphere

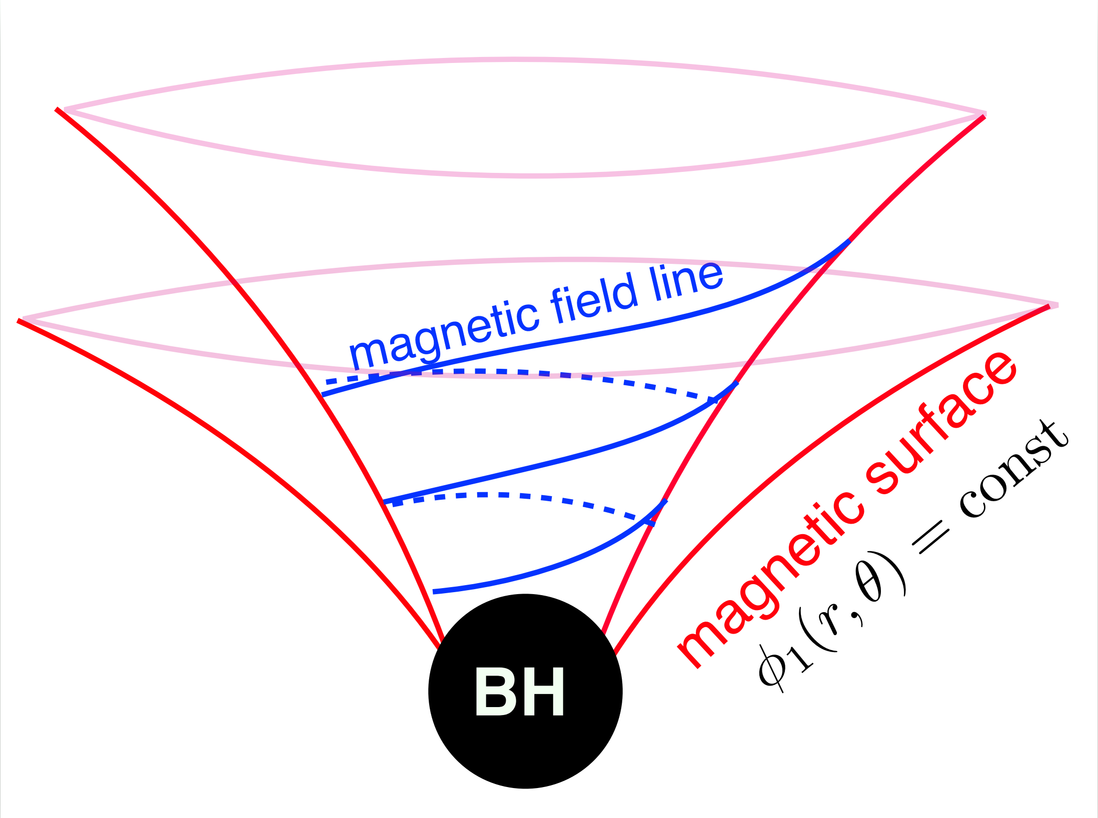
$$\phi_1 = \Psi_1(r,\theta)$$
$$\phi_2 = \varphi - \Omega_F(\Psi_1)t + \Psi_2(r,\theta)$$

This can be derived with Killing vector [e.g. Uchida (1997)]

ϕ_1 = const. gives magnetic surface

Axisymmetric magnetosphere is **foliated** by magnetic surfaces

ϕ_2 = const. magnetic field line on a magnetic surface



wave modes

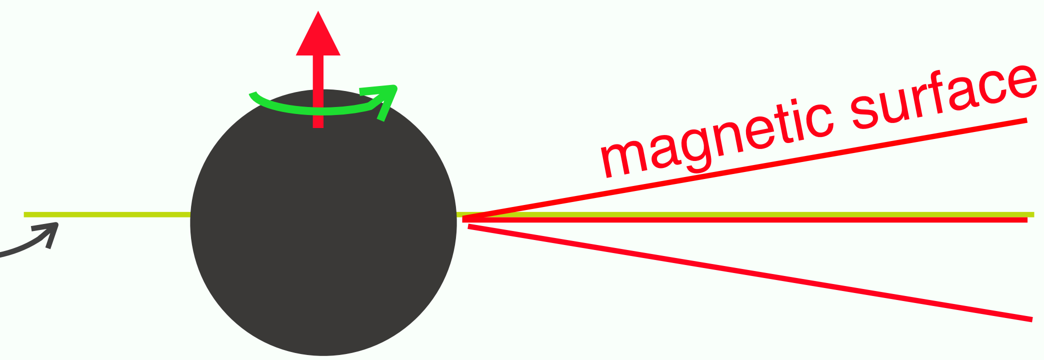
Alfvén waves propagate on the magnetic surface

Fast magnetosonic wave can across the surfaces

To obtain global magnetosphere in the Kerr spacetime is very difficult (Grad-Shafranov eq)

Background magnetosphere

Magnetosphere near the equatorial plane.



Ansatz $\phi_1 = \phi_1(\underline{\theta})\,,\quad \phi_2 = \varphi - \Omega_F t + \Psi(\underline{r})$

Maxwell equation $\Rightarrow \quad \partial_\theta\left(\phi_1 \sin\theta\right) = 0\,,\quad \partial_r\left(\partial^r \phi_2\right) = 0$

$$\Rightarrow \quad \phi_1 = B_0 \log\left(\tan\frac{\theta}{2}\right) \qquad \phi_2 = \varphi - \Omega_F t + C\int \frac{r^2}{\Delta}$$

Regularity at horizon

$$C = \frac{r_{\mathrm{H}}^2 + a^2}{r_{\mathrm{H}}^2}(\Omega_{\mathrm{H}} - \Omega_F)$$

Electromagnetic field

$$E^\theta = -\frac{B_0 \Omega_F}{r^2}\,,\quad B^r = -\frac{B_0}{r^2\sin^2\theta}(g_{tt} + \Omega_F g_{t\varphi}) > 0\,,\quad B^\varphi = B_0 C \frac{g_{tt}}{\Delta\sin\theta} < 0$$

The BZ process for this solution

energy flux vector: $P^\mu = -T^\mu{}_\nu \xi^\nu_{(t)}$

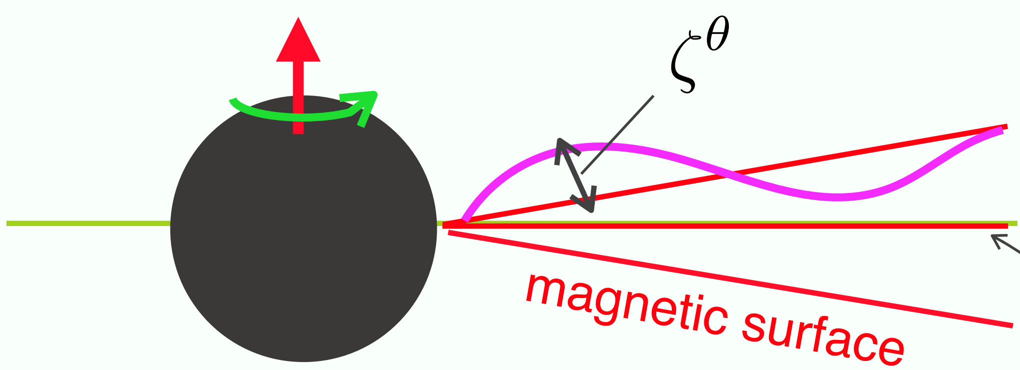
timelike Killing vector $\xi^\nu_{(t)} := (\partial_t)^\nu$

Poynting flux

$$P^r = \Omega_F(\Omega_{\mathrm{H}} - \Omega_F)\frac{r_{\mathrm{H}}^2 + a^2}{r_{\mathrm{H}}^2}\frac{B_0^2}{r^2}$$

$0 < \Omega_F < \Omega_{\mathrm{H}} \Rightarrow$ outward flux

Perturbation & Mode decoupling

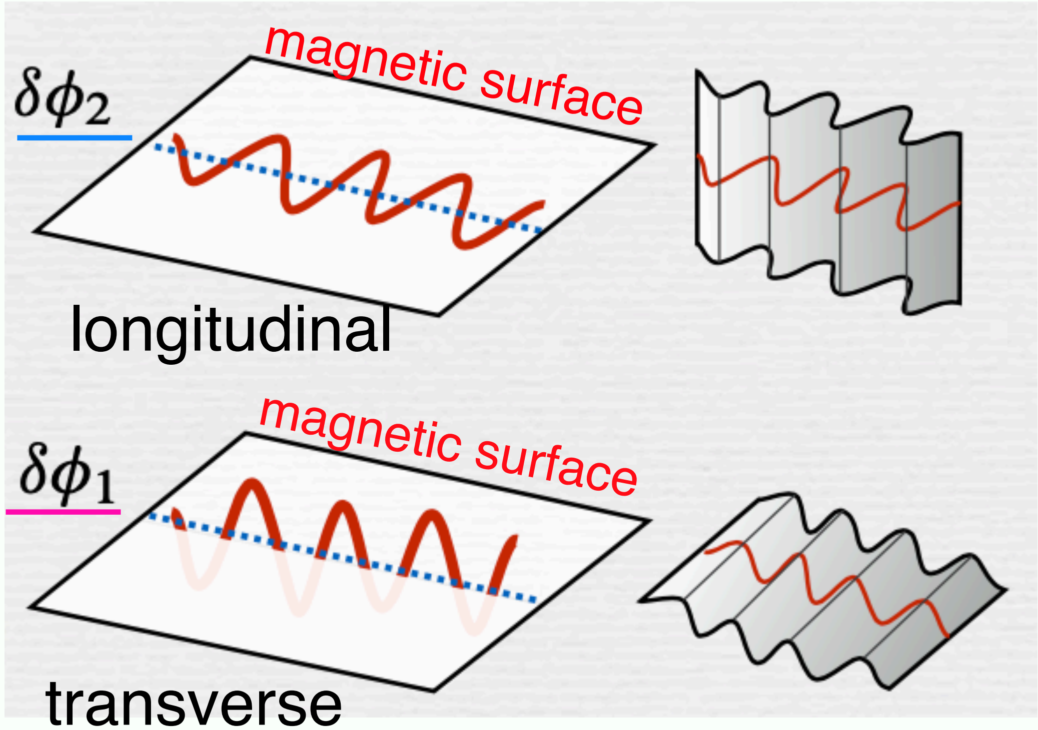


Definition of the perturbation

$$\delta\phi_i = \zeta^\mu \partial_\mu \phi_i$$

displacement vector

On the equatorial plane magnetic surface…



Magnetosonic wave

$$\partial_j \left(\sqrt{-g}\,\partial^j \delta\phi_2\right) = 0$$

Alfvén wave

$$\partial_\nu \phi_2 \partial_\mu \left(\sqrt{-g}\,\partial^{[\mu} \delta\phi_1 \partial^{\nu]} \phi_2\right) = 0$$

propagate along magnetic surface

Wave equation for Alfvén wave

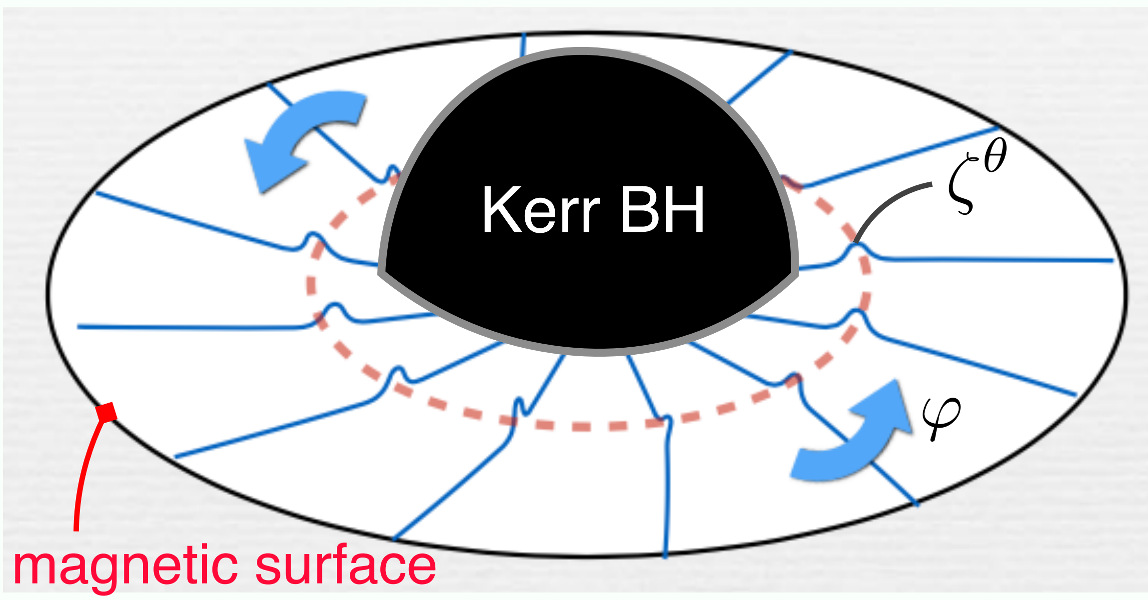
$$\partial_\nu \phi_2 \partial_\mu \left(\sqrt{-g} \partial^{[\mu} \delta\phi_1 \partial^{\nu]} \phi_2 \right) = 0$$

separation of variables

$$\delta\phi_1 = A(\rho)\psi(t,r) \qquad \rho = \varphi - \Omega_F t + \int dr \frac{r^2}{\Delta}$$

$\rho = \mathrm{const}$ gives a magnetic field line.

Choice of A(ρ)



$A(\rho)$: configuration of the perturbation in φ direction

$A(\rho) = 1 \quad \Rightarrow$ axisymmetric

$A(\rho) = \delta(\rho - \rho_0) \quad \Rightarrow$ only $\rho = \rho_0$ is perturbed

Wave equation

$$\left[\Gamma \left(\partial_r - \frac{Cr^2 g_{\varphi\varphi}}{\Gamma\Delta}(\Omega - \Omega_F)\partial_t \right) \psi \right]_r - \frac{Cr^2 g_{\varphi\varphi}}{\Delta}(\Omega - \Omega_F)\psi_{tr} + \frac{C^2 r^2 - \Gamma}{\Delta}\psi$$
$$+ \frac{r^2 g_{\varphi\varphi}}{\Delta^2}\left[C^2 r^2 - \Gamma + g_{\varphi\varphi}(\Omega - \Omega_F)^2 \right] \psi_{tt} = 0,$$

Schrödinger type

$$\frac{d^2 R}{dx^2} - V_{\mathrm{eff}} R = 0,$$

$\Gamma := g_{\mu\nu}\chi^\mu\chi^\nu$, $\quad \chi^\mu = \xi^\mu_{(t)} + \Omega_F \xi^\mu_{(\varphi)}$: four vector of corotating observer with magnetic field lines

Coordinate transformation

$$r = X, \quad t = T - C\int dX \frac{X^2 g_{\varphi\varphi}}{\Gamma\Delta}(\Omega - \Omega_F),$$

New coordinate for eliminating the cross term in the wave eq.

$$\partial_X\left(\Gamma\partial_X\psi\right) + \frac{X^2 g_{\varphi\varphi}}{\Delta^2}(c^2X^2 - \Gamma)\left[1 - \frac{g_{\varphi\varphi}(\Omega-\Omega_F)^2}{\Gamma}\right]\psi_{TT} + \frac{c^2X^2-\Gamma}{\Delta}\psi = 0.$$

(※) Γ=0: Light surfaces (inner LS and outer LS)

- Light surfaces are one way boundaries for Alfvén waves (← shown by ray motion)
- The inner LS is an effective horizon for Alfvén waves (purely ingoing / coordinate singularity)

Wave eq in tortoise coordinate

$$\frac{dx}{dX} = -\frac{1}{\Gamma}$$

r	$r_{\rm in}$	.....	$r_{\rm out}$
x	$-\infty$	.....	$+\infty$

$$\psi = e^{-i\omega T}R$$ stationary scattering

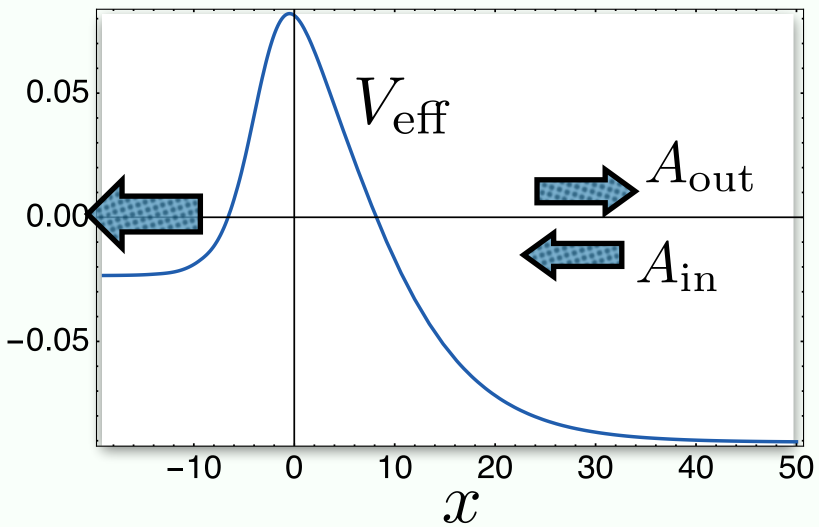
$$\frac{d^2R}{dx^2} - V_{\rm eff}R = 0,$$

$$V_{\rm eff} = -\frac{C^2X^2-\Gamma}{\Delta}\left[\Gamma + \frac{\omega^2X^2g_{\varphi\varphi}}{\Delta}\left[g_{\varphi\varphi}(\Omega-\Omega_F)^2 - \Gamma\right]\right].$$

Alfvénic superradiance

Asymptotic solutions

$$\frac{d^2R}{dx^2} - V_{\rm eff}R = 0,$$



$$R = \begin{cases} \exp\left[-iC\omega\int \frac{dx}{\Delta}X^2 g_{\varphi\varphi}(\Omega-\Omega_F)\right]. & \text{for} \quad x\to-\infty. \\ A_{\rm in}\exp\left[-iC\omega\int \frac{dx}{\Delta}X^2 g_{\varphi\varphi}(\Omega-\Omega_F)\right] + A_{\rm out}\exp\left[ic\omega\int \frac{dx}{\Delta}X^2 g_{\varphi\varphi}(\Omega-\Omega_F)\right] & \text{for} \quad x\to+\infty. \end{cases}$$

purely ingoing at inner LS

Superradiant condition

Boundary Condition should be given at the inner LS for energy extraction

not BH horizon !!

Wronskian = const.

$$\left|\frac{A_{\rm out}}{A_{\rm in}}\right|^2 = 1 - \frac{f_{\rm in}}{f_{\rm out}}\frac{\Omega|_{r_{\rm in}}-\Omega_F}{\Omega|_{r_{\rm out}}-\Omega_F}\frac{1}{|A_{\rm in}|^2}$$

reflection rate

$$f_{\rm in/out} = r^2 g_{\varphi\varphi}/\Delta|_{r_{\rm in/out}}$$

$$\Omega|_{r_{\rm out}} < \Omega_F < \Omega|_{r_{\rm in}}$$

⇒ reflection rate exceeds 1

angular velocity of magnetic field line

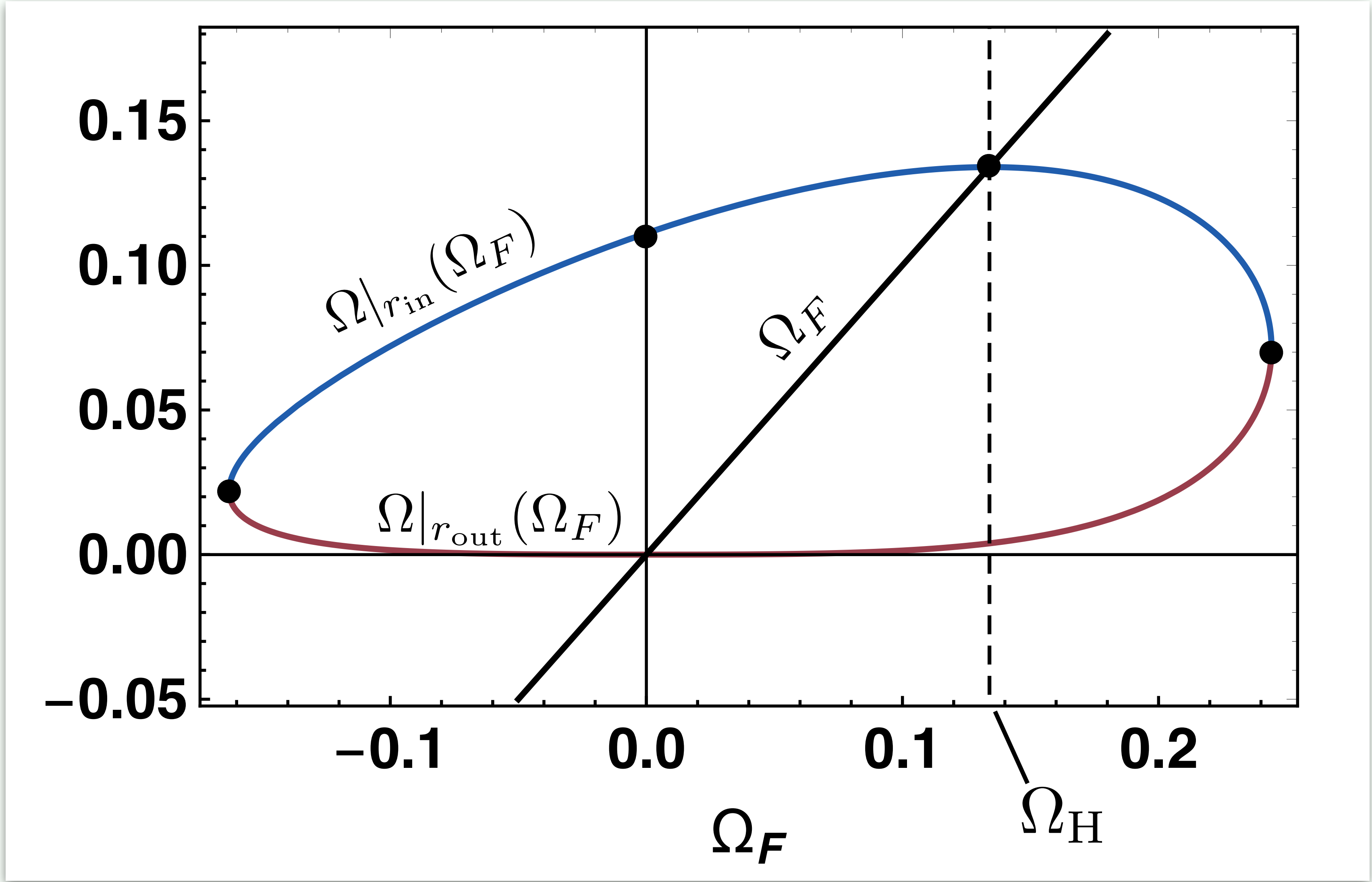
$$\Omega|_{r_{\rm in/out}} = -\left.\frac{g_{t\varphi}}{g_{\varphi\varphi}}\right|_{r=r_{\rm in/out}}$$

Dragging effect at light surfaces

vs

Angular velocity of magnetic field line

Superradiant condition and the BZ process



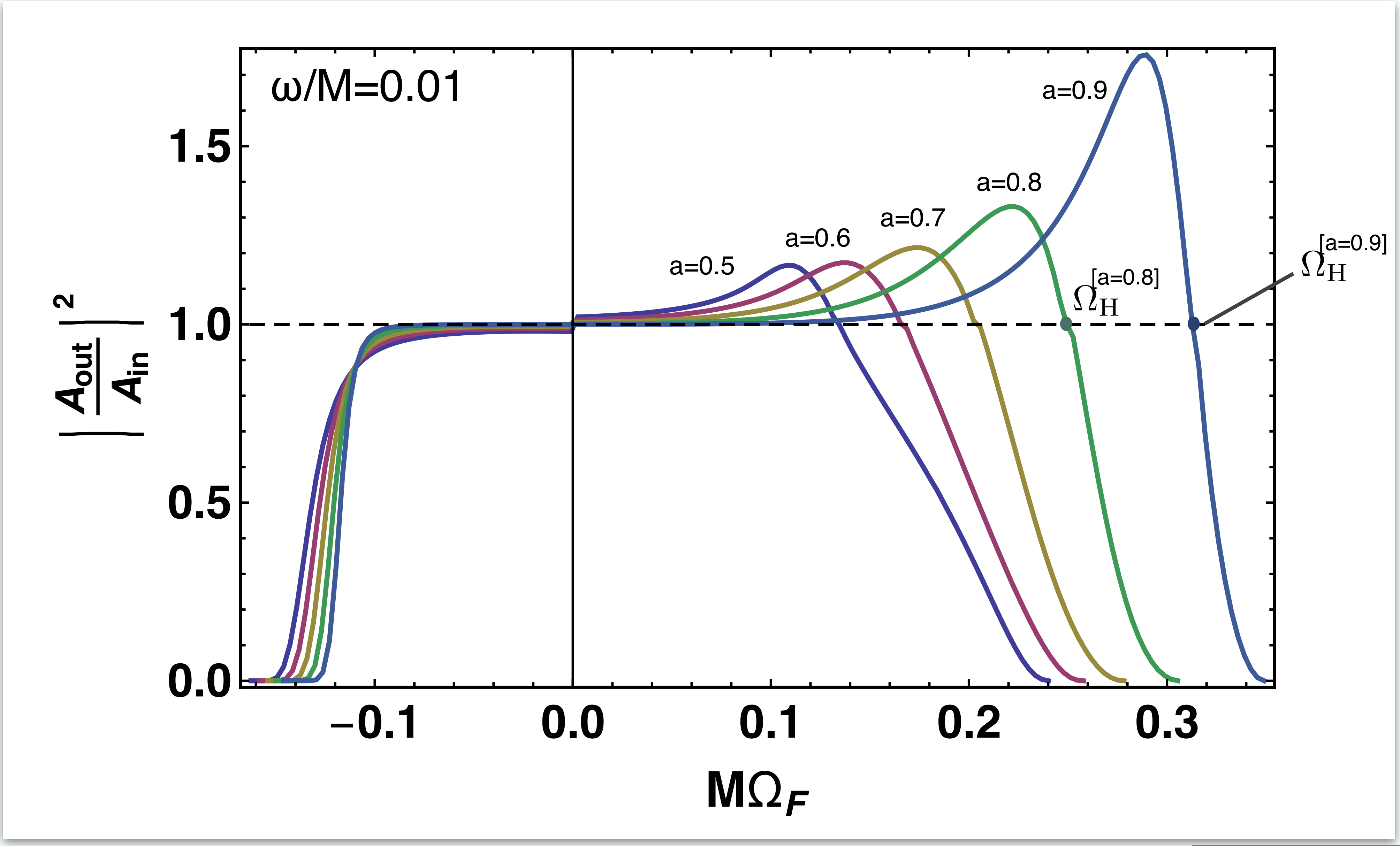
Superradiant condition

$$\Omega|_{r_{\rm out}(\Omega_F)} < \Omega_F < \Omega|_{r_{\rm in}(\Omega_F)}$$

Condition for the BZ process!!!!

$$0 < \Omega_F < \Omega_{\rm H}$$

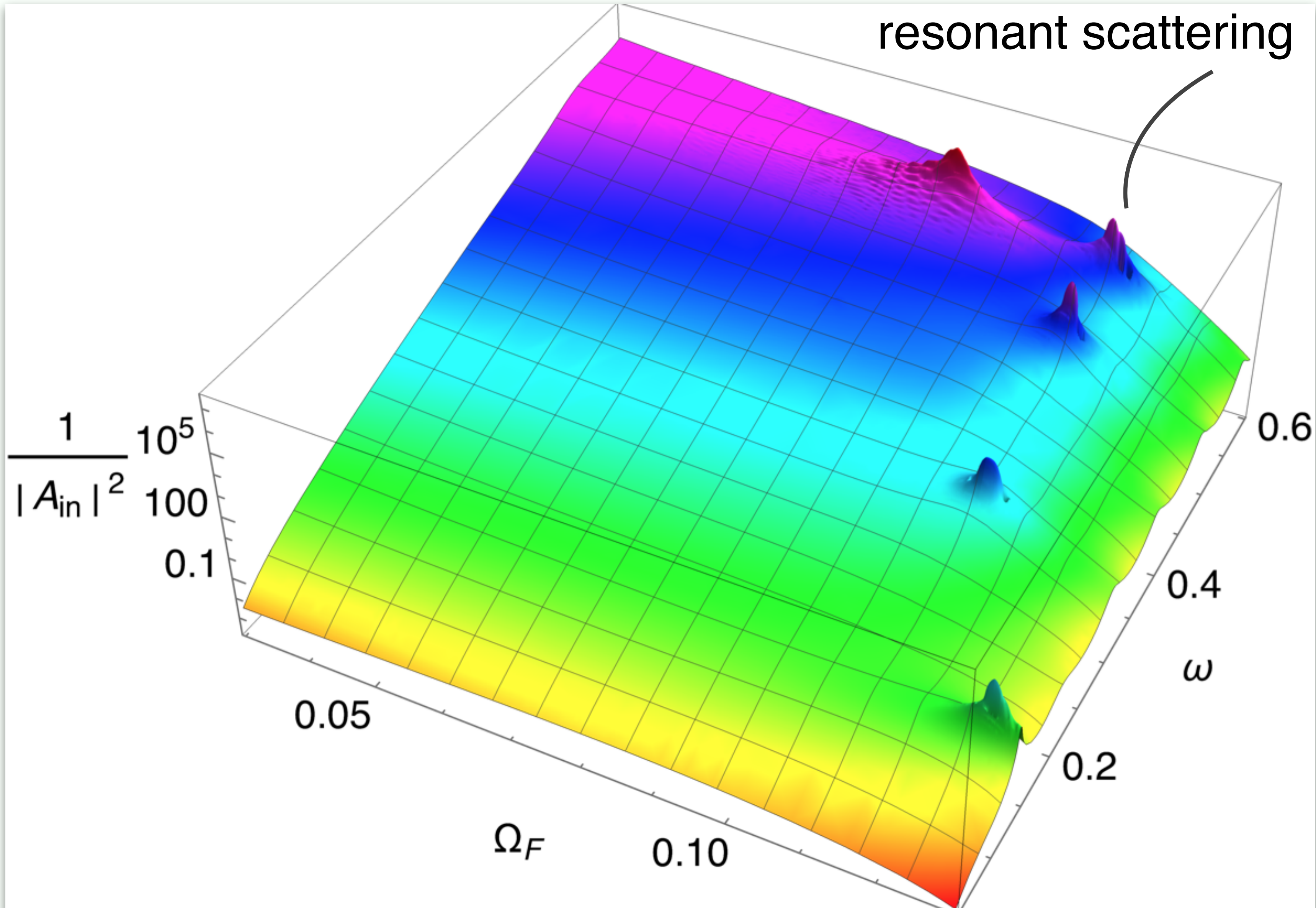
Reflection rate (Numerical calculation)



$$0 < \Omega_F < \Omega_H$$

Reflection rates exceed unity when the superradiant condition (condition for the BZ process) is satisfied.

Resonance?



- Resonant scattering can induce burst-like emission in magnetosphere ??
- Plasma effect should be taken into account to discuss it.

Poynting flux

wave part

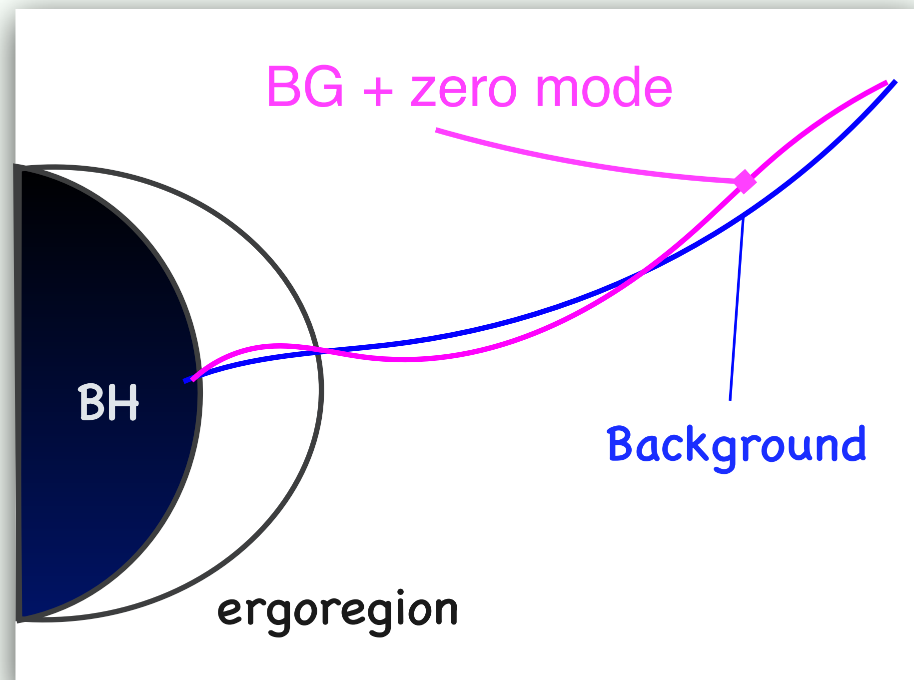
$$P^r_{\rm BZ} + P^r_{\rm Alfvén} = \frac{B_0^2}{r^2}\Omega_F(\Omega_{\rm H} - \Omega_F)\left[\,1\; + \;(\zeta^\theta)^2\; + \;\omega^2\mathcal{F}\,\right]$$

background — zero mode

$\omega \to 0$ long wavelength

$$\frac{B_0^2}{r^2}\Omega_F(\Omega_{\rm H} - \Omega_F)\left[\,1\; + \;(\zeta^\theta)^2\,\right] = P^r_{\rm BZ\ with\ deformation}$$

deformed (new) background



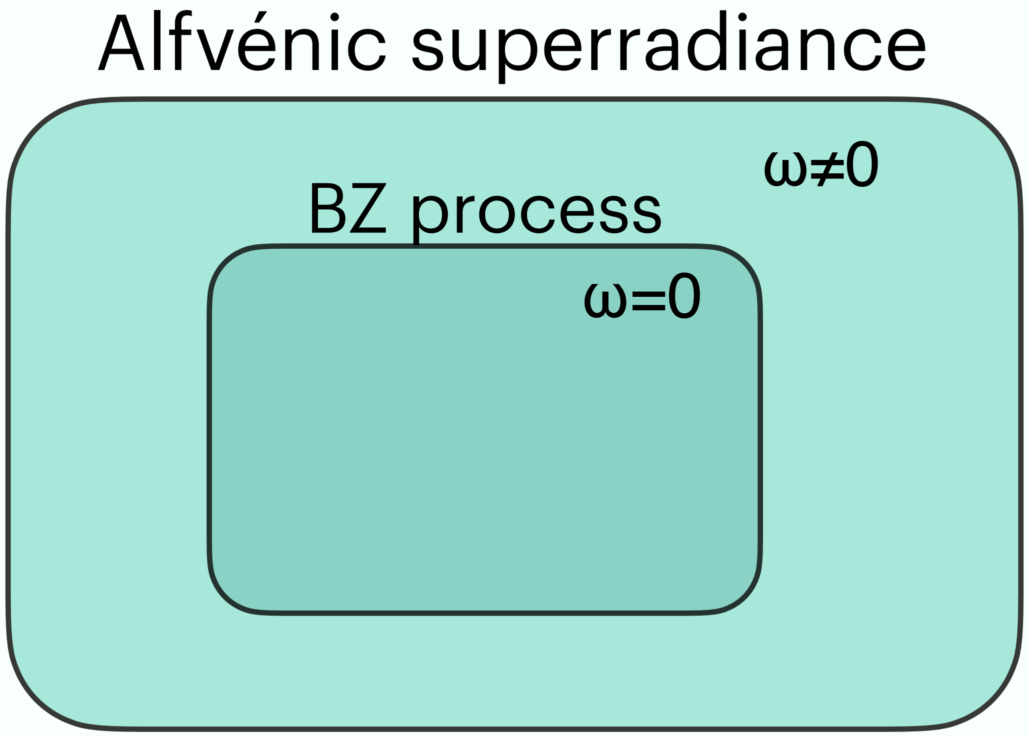
The zero mode is incorporated into the background!

Poynting flux of the zero mode of Alfvénic superradance

⇔ Poynting flux of the BZ process with deformed background.

BZ process is the long wavelength limit of Alfvénic superradiance

Summary



Superradiant condition & BZ process

$$\Omega|_{r_{\rm out}} < \Omega_F < \Omega|_{r_{\rm in}} \quad \Leftrightarrow \quad 0 < \Omega_F < \Omega_{\rm H}$$

Poynting flux

$$P^r_{\rm BZ} + P^r_{\rm Alfvén} \xrightarrow{\omega \to 0} P^r_{\rm BZ\ with\ deviation}$$

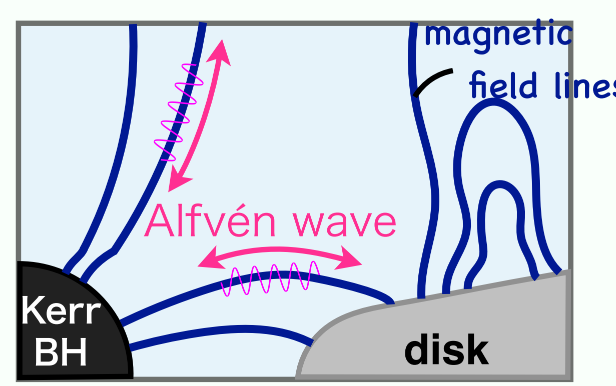
BZ process is the long wavelength limit of Alfvénic superradiance

What's next ?

Plasma effect (Magnetohydrodynamics) $\nabla_b \left(T^{ab}_{\rm (EM)} + T^{ab}_{\rm plasma} \right) = 0$

Extracted energy by Alfvén waves → Kinetic energy of plasma

⇒ Jet



Confinement of Alfvén wave?

(Alfvénic BH Bomb)