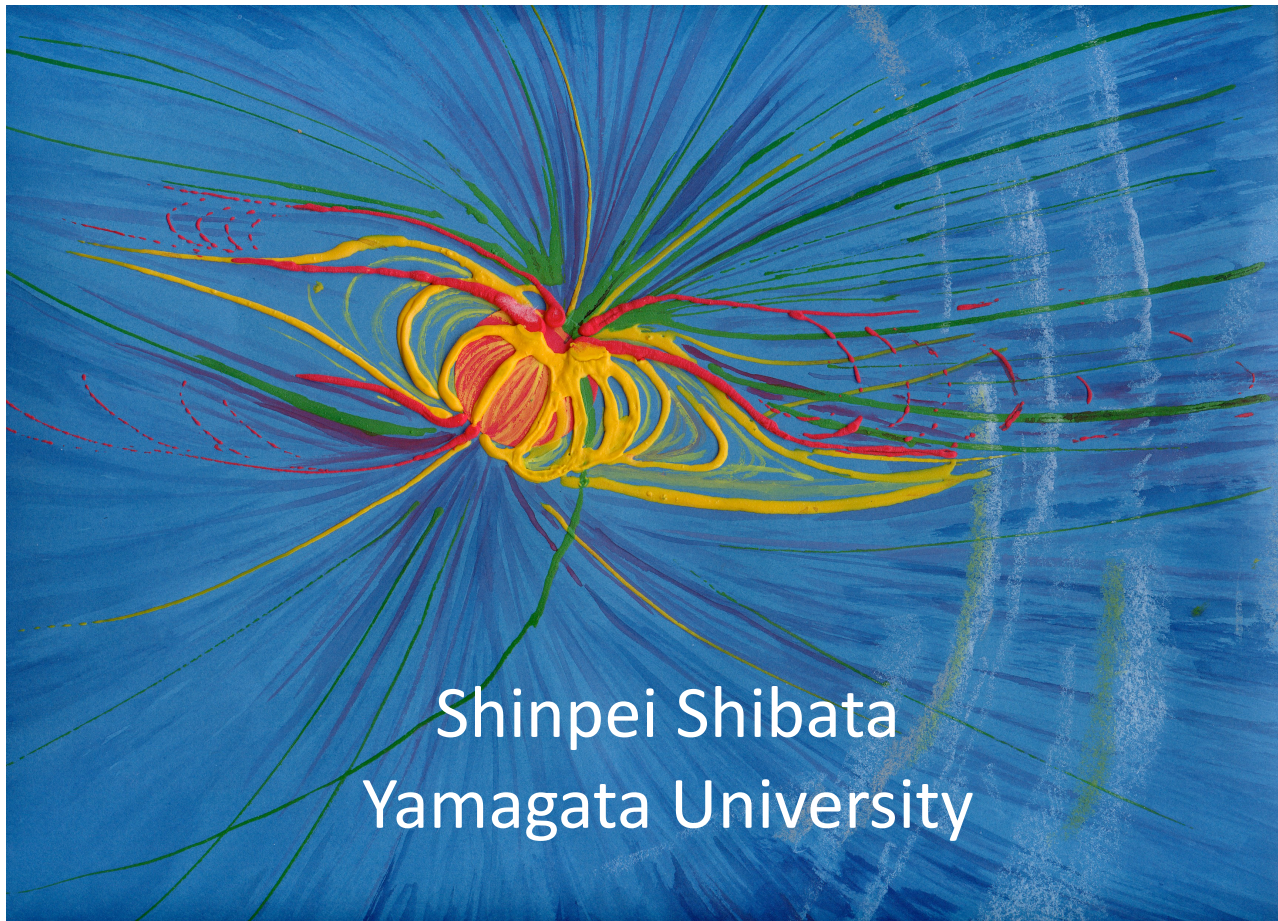


パルサーのトルク問題と 中性子星内部電流の決定



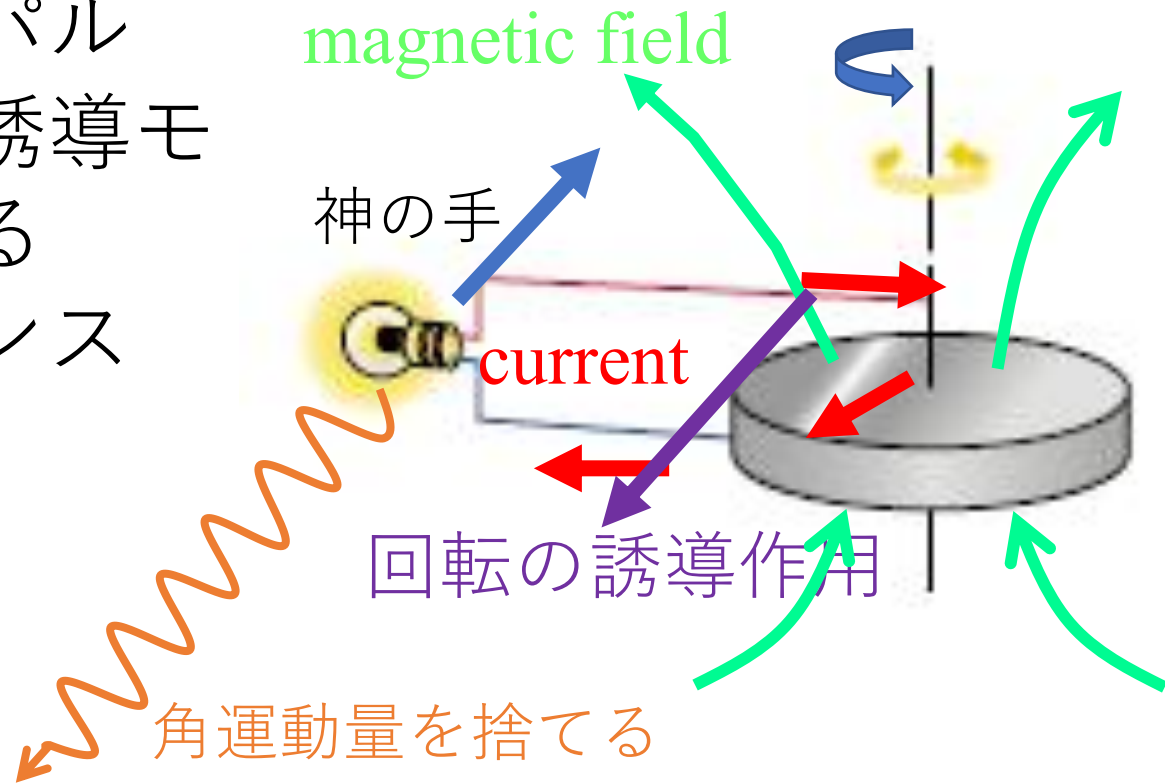
中性子星種族
PCモデル

道草
トルク問題

Shinpei Shibata
Yamagata University

回転駆動型パルサーの単極誘導モデルにおけるトルクバランス

磁石は回転
回路は静止



トルクバランスの問題は角運動量の輸送問題である。

星の

自転のエネルギー $E = \frac{1}{2} \mathfrak{I} \Omega^2$ $\dot{E} = \mathfrak{I} \dot{\Omega} \Omega$

energy loss rate /
angular momentum
loss rate



角運動量 $L = \mathfrak{I} \Omega$ $\dot{L} = \mathfrak{I} \dot{\Omega}$

$$\dot{E} / \dot{L} = \Omega$$

“エネルギー・角運動量関係”

角運動量を適切に捨てるために必要な腕の長さは光半径 c/Ω

photons

$$\frac{\sum_{i=1}^N h\nu_i}{\sum_{i=1}^N (h\nu_i/c)\varpi_i} = \Omega$$

relativistic particles

$$\frac{\sum_{i=1}^N \gamma_i m_i c^2}{\sum_{i=1}^N \gamma_i m_i c \varpi_i} = \Omega$$

$$\langle \varpi \rangle = c/\Omega \equiv R_L$$

平均のemission siteは光円柱

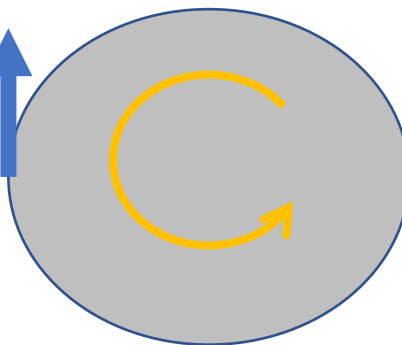
reaction
by emission



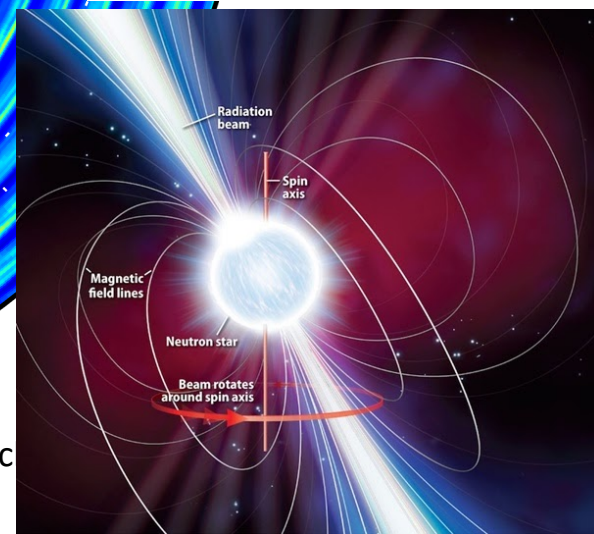
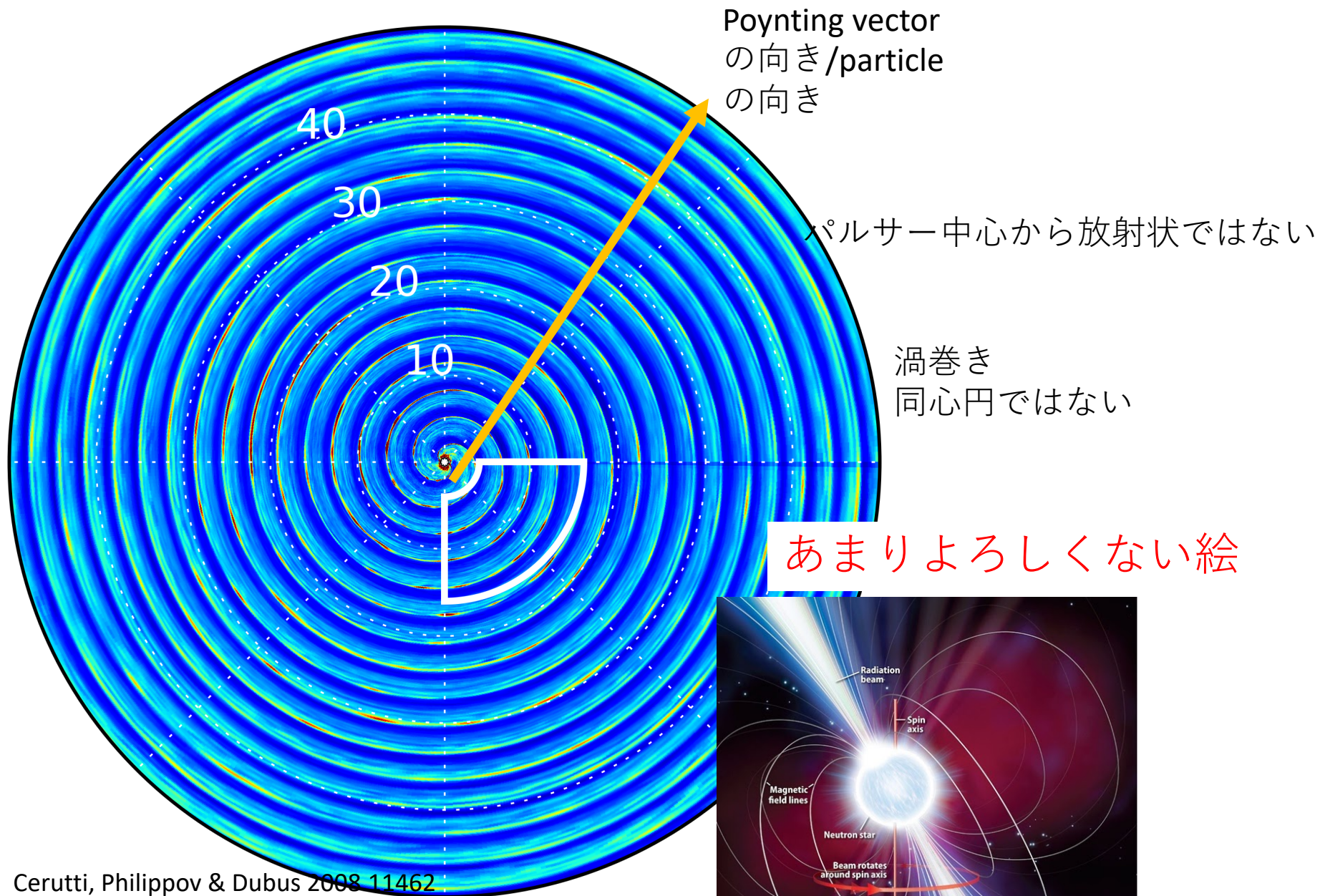
braking torque on the star



enforce corotation
(electromagnetic force)



問題がself-consistent に解かれていれば、エネルギー・角運動量関係は自動的に満たされるので気にしなくても良い。だれも気にしていない。



Cerutti, Philippov & Dubus 2008 11462

“Dissipation of the striped pulsar wind and non-thermal particle

<http://arxiv.org/abs/2008.11462>

In what way $\dot{E} = \mathfrak{I}\dot{\Omega}\Omega$ is released

Pulsar wind

gamma-ray pulses from
outer magnetosphere

thermal X-ray from NS

particles, non-thermal radiation
(unseen?)

radio emission from Polar cap



$$\dot{E} = \mathfrak{I}\dot{\Omega}\Omega$$

system全体をself-consistent に扱っているひとは誰もいない。どの理論もfull global でない。エネルギー・角運動量関係は気にしたくても気にしようがない。でも、やっぱり、エネルギー・角運動量関係は気にした方が良い場面がある。

考えた方が良い場合は少なくとも2つある：

1. Polar Cap model では回転エネルギーを粒子や光子で持ち出すけど、光半径のずっと内側なので、角運動量を持ち出さない。
2. 磁気圏電流が星内部(表面?)を流って星にブレーキをかけるとき、流れた電流にOhmic loss があると、回転エネルギーが熱になって逃げるが、角運動量は持ち出さない。

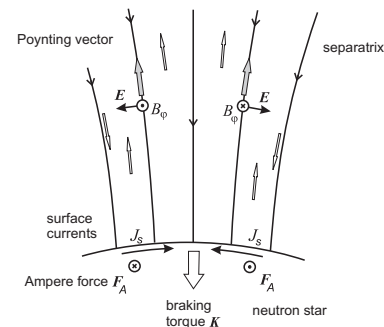
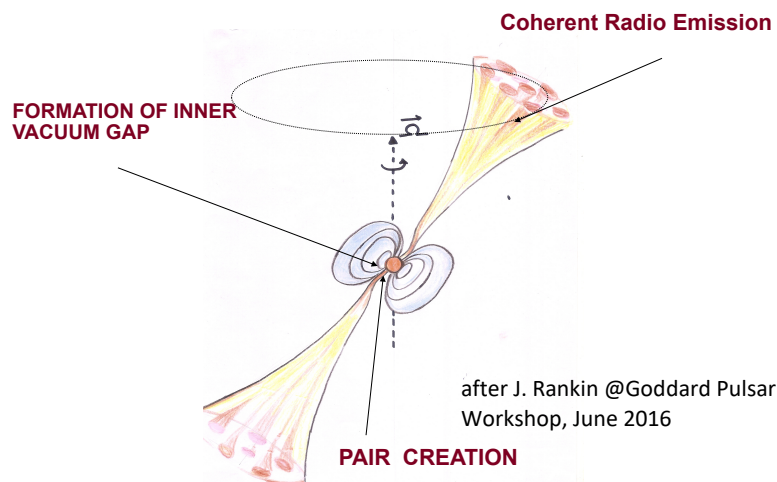


Fig. 5 Schematic view of the axisymmetric polar cap showing magnetospheric current density (open arrows), surface currents, Ampère force on surface currents, and braking torque. Here only the symmetric current i_s is present. Taken from Beskin (1999).

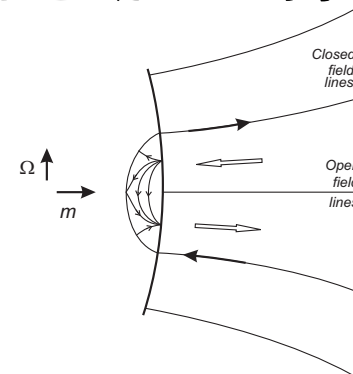
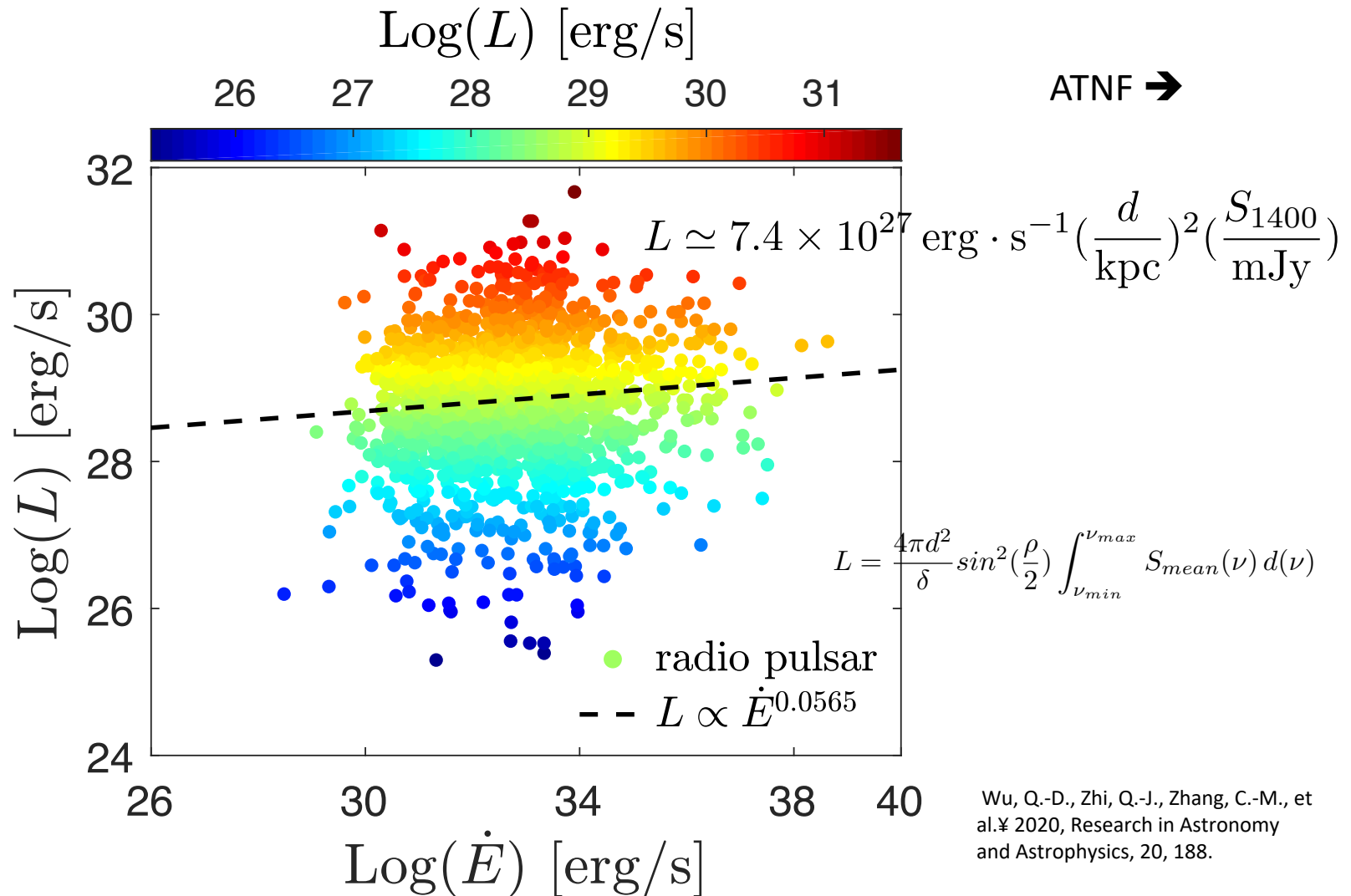
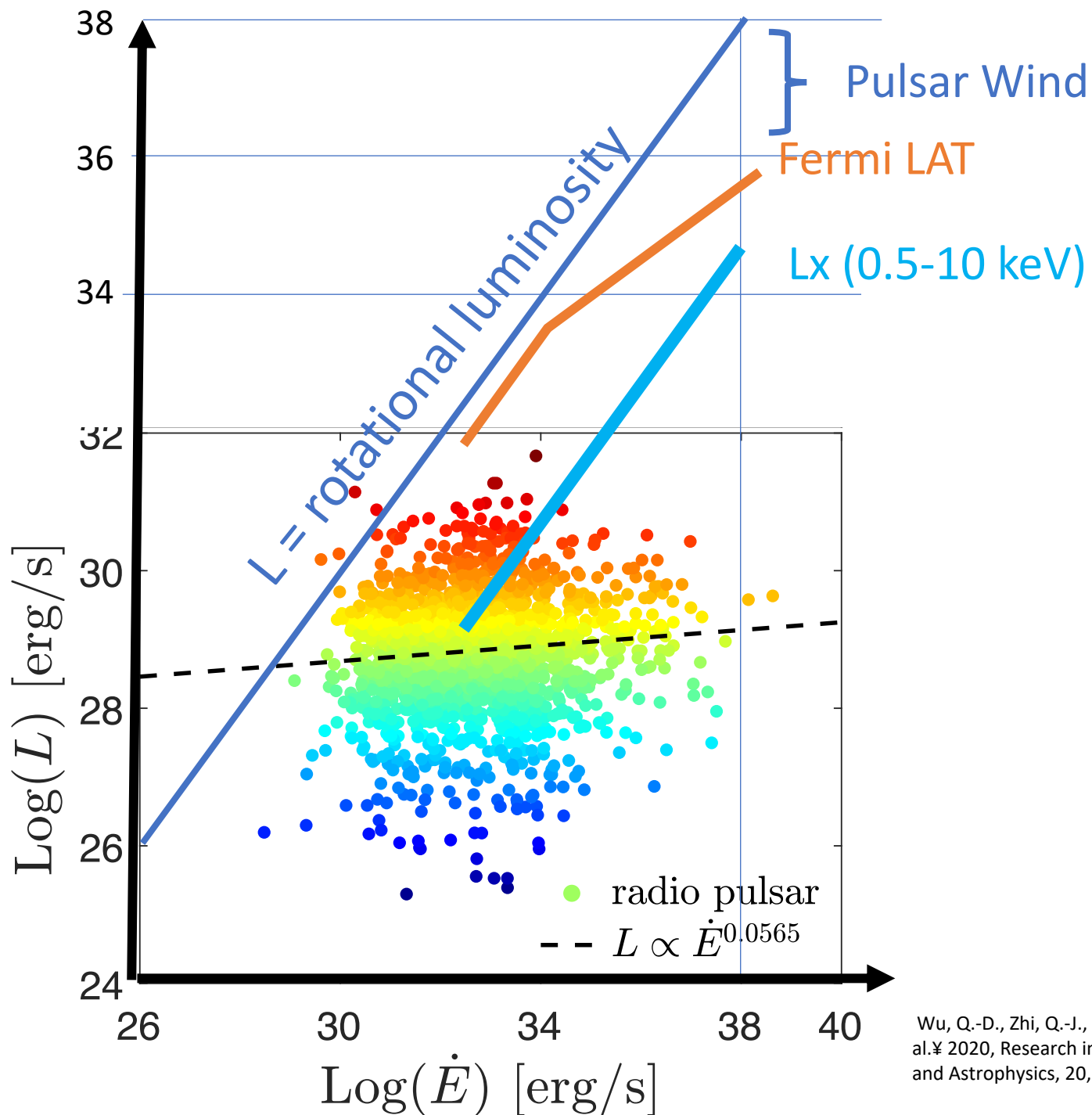


Fig. 6 For the orthogonal rotator only antisymmetric current i_a (i.e., the current having different direction in the north and south parts of the polar cap) takes place. The structure of the surface currents within the polar cap and along the separatrix is also shown. Taken from Beskin et al. (2013).

- ほとんどのケースで電波はPolar Cap 起源
- Polar Cap Luminosity は Rotation Power に比べて無視できない(at least for old pulsars)





Polar Cap は結構な回転エネルギーを放出しているのに
角運動量は捨てていない。どうなっているか。

How inefficiency of the angular momentum loss
can be compensated?

CASE: Polar Cap

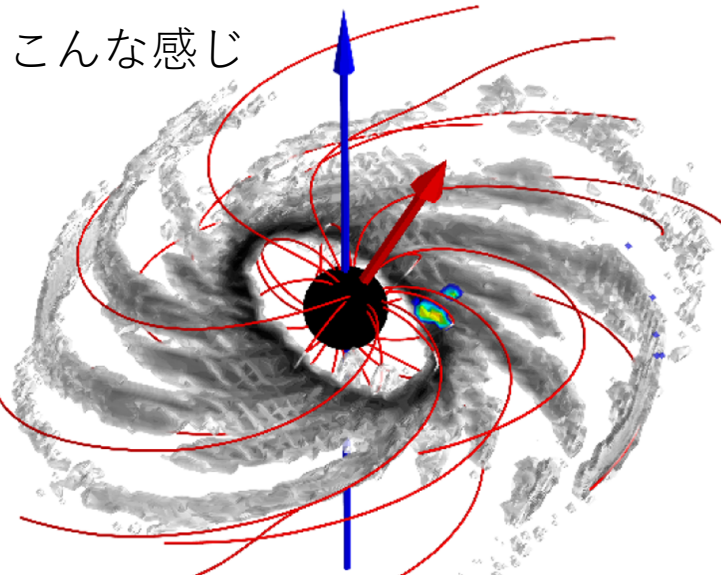
モデル： 仮定：斜め回転で定常 $\boxed{u_c} = \Omega_* \times r$ co-rotation velocity

ファラデーの誘導法則より、
一般性を失うことなく電場は

$$\mathbf{E} = -\underbrace{\frac{1}{c} \mathbf{u}_c \times \mathbf{B}}_{\text{共回転電場}} - \underbrace{\nabla \Phi}_{\text{非共回転電場}}$$

共回転電場

非共回転電場



Cerutti, B.,
Philippov, A.~A., &
Spitkovsky, A., 2016,
mnras, 457

$$E_{\parallel} = \hat{B} \cdot \nabla \Phi$$

沿磁力線電場

星本体は完全導体で、剛体回転とすると

電場は共回転電場で

$$\Phi = \text{constant} = 0$$

モデル

outer magnetosphere

inner magnetosphere

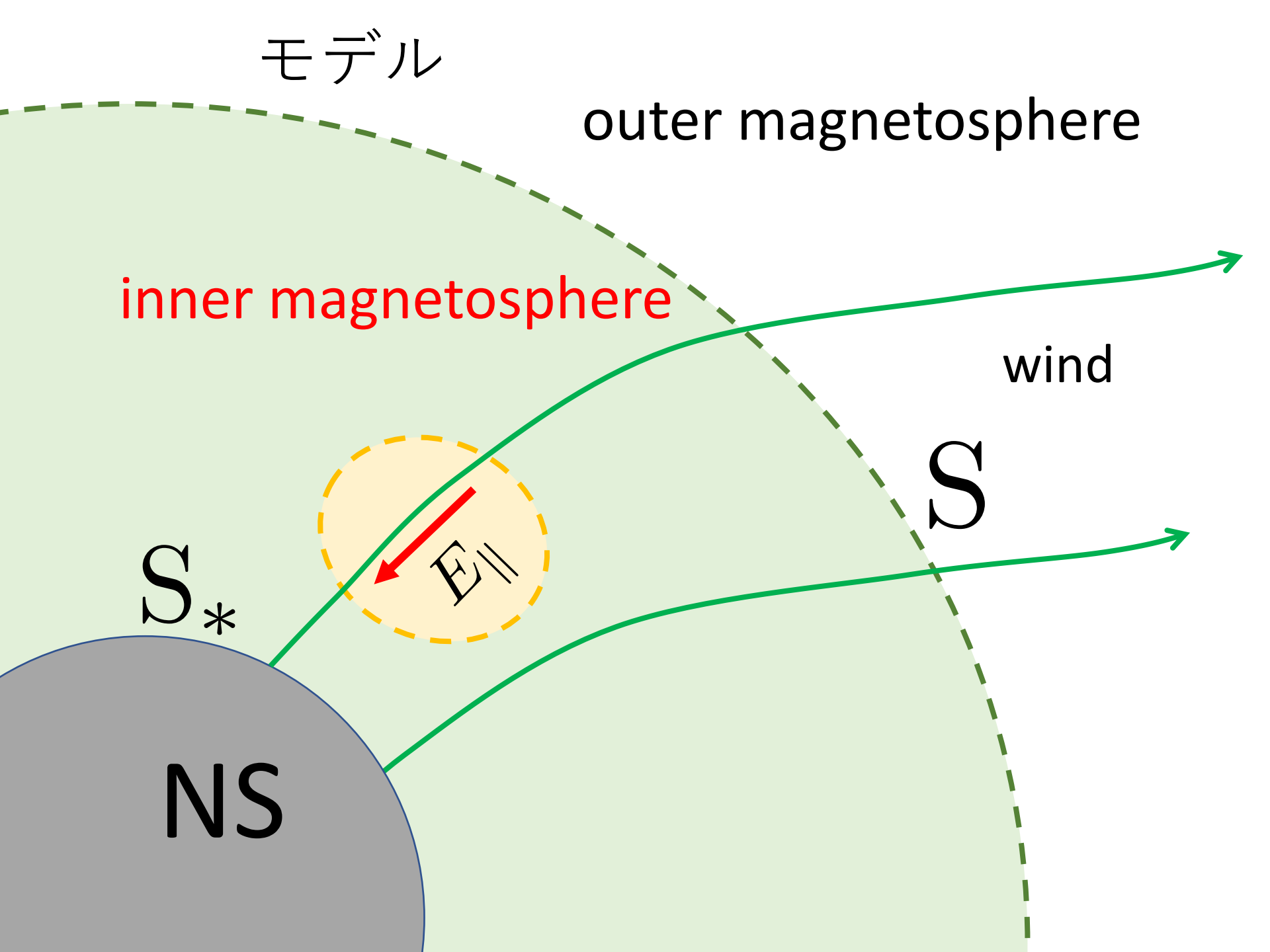
wind

S

S_*

$E_{\parallel}$

NS



outer magnetosphere

inner magnetosphere

$$\mathbf{B} \cdot \nabla \Phi = 0, \quad \Phi = 0$$

S

S_*

$\Phi = 0$

NS

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} / c = 0,$$

$$(\mathbf{v} - \mathbf{u}_c) \times \mathbf{B} = \nabla \Phi$$

$$\mathbf{v} = \mathbf{u}_c + \kappa \mathbf{B}$$

outer magnetosphere

inner magnetosphere

$$E_{\parallel} = \hat{\mathbf{B}} \cdot \nabla \Phi$$

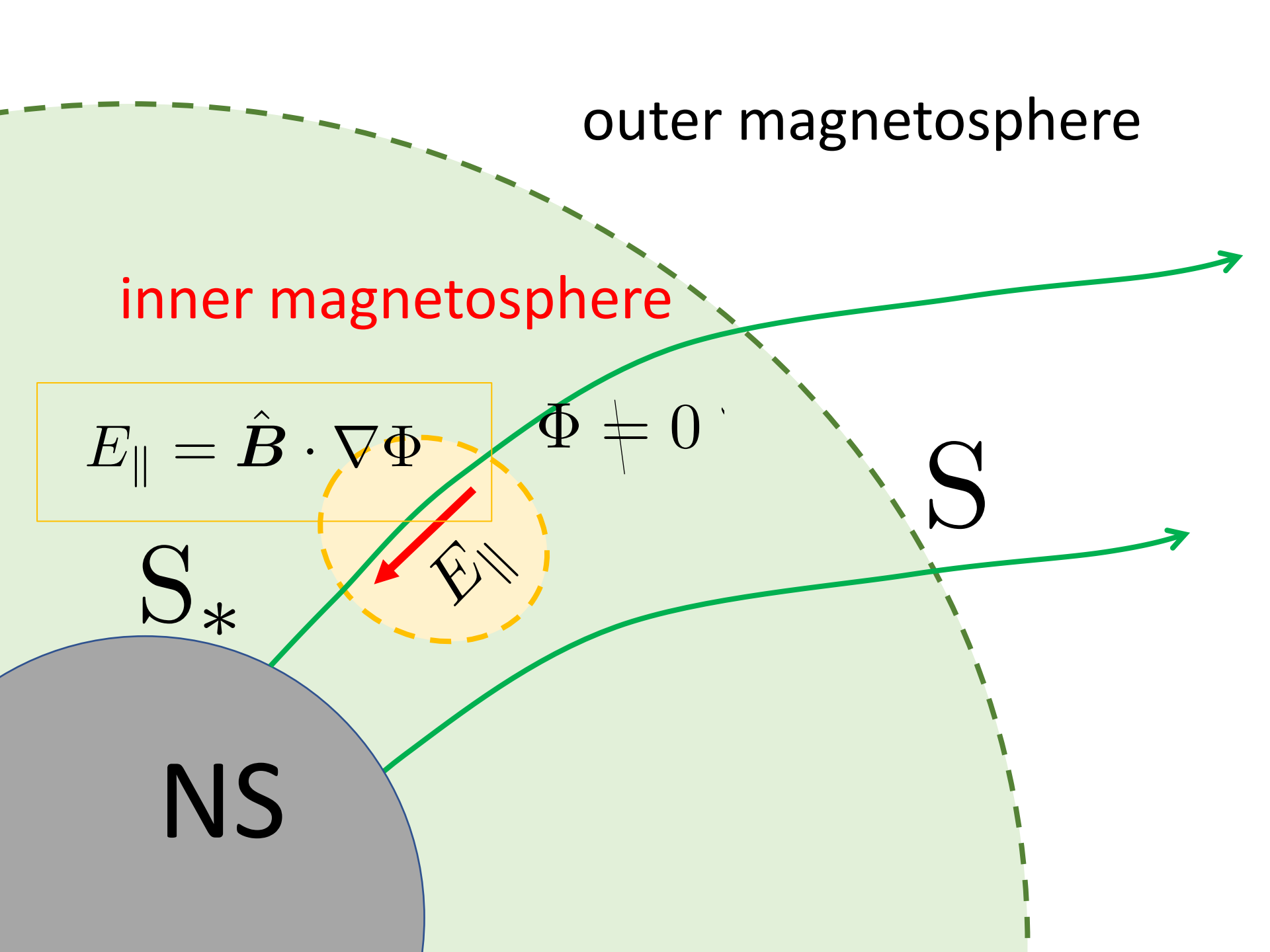
$$\Phi \neq 0$$

S

S_{*}

NS

$E_{\parallel}$



The diagram illustrates a magnetosphere with a central grey sphere labeled 'NS' (Neutron Star). Two green magnetic field lines with arrows pointing away from the star are shown. The region between the star and the innermost field line is labeled 'inner magnetosphere' in red. The region between the inner and outer field lines is labeled 'outer magnetosphere'. A dashed green line represents the outer boundary. A specific field line is labeled 'S' at its outer end and 'S*' near the star. A yellow dashed circle highlights a point on this field line, with a red arrow pointing towards the star labeled 'E_||'. A yellow box contains the equation E_|| = B-hat · ∇Φ, and the text Φ ≠ 0 is placed nearby.

Inner magnetosphere

$$\frac{1}{c}e_s n_s \mathbf{v}_s \times \mathbf{B} + e_s n_s \mathbf{E} = n_s \left(\frac{\partial}{\partial t} + \mathbf{v}_s \cdot \nabla \right) (m_s \gamma_s \mathbf{v}_s) + \mathbf{F}_{ext}$$

$$\mathbf{F}_{ext} = \frac{n_s \mathcal{P}_s}{c} \frac{\mathbf{v}_s}{c}$$

$$\mathcal{P}_s = (2e_s^2/3c^3)\gamma_s^4 |\mathbf{v}_s \times (\nabla \times \mathbf{v}_s)|^2 = (2e_s^2 c/3R_c)\gamma_s^4$$

$$\mathbf{v} \cdot \boxed{\phantom{\mathbf{v} \cdot \mathbf{E}}}$$

Poynting energy converted into particles and photons

$$\dot{E}_{in} = \int_{in} \sum_s \mathbf{v}_s \cdot (e_s n_s \mathbf{E}) dV = \int_{in} \mathbf{j} \cdot \mathbf{E} dV$$

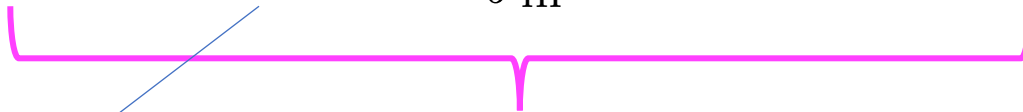
$$(\mathbf{u}_c / \Omega_*) \cdot \boxed{\phantom{\mathbf{u}_c / \Omega_*}}$$

angular momentum goes to particle and radiation

$$\begin{aligned} \dot{L}_{\text{in}} &= \frac{1}{\Omega_*} \int_{\text{in}} \sum_s \mathbf{u}_c \cdot \left[\frac{1}{c} e_s n_s \mathbf{v}_s \times \mathbf{B} + e_s n_s \mathbf{E} \right] dV \\ &= \frac{1}{\Omega_*} \int_{\text{in}} (\mathbf{j} \cdot \mathbf{E} + \mathbf{j} \cdot \nabla \Phi - q \mathbf{u}_c \cdot \nabla \Phi) dV \end{aligned}$$

Energy-angular-momentum balance

$$\Delta \dot{E}_{\text{in}} \equiv \dot{E}_{\text{in}} - \Omega_* \dot{L}_{\text{in}} = - \int_{\text{in}} (\mathbf{j} - q \mathbf{u}_c) \cdot \nabla \Phi dV \approx \int j_{\parallel} E_{\parallel} dV \neq 0$$



ほとんどゼロ



outer magnetosphere

inner magnetosphere

wind

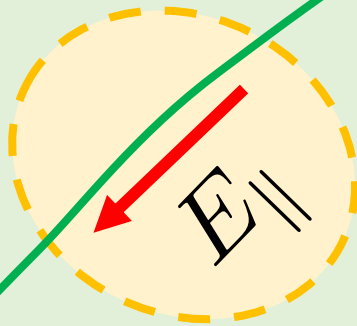
1流体モデルで

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} / c = 0,$$

S

S_*

NS



Outer magnetosphere: ideal-MHD (one fluid model)

$$\frac{1}{c} \mathbf{j} \times \mathbf{B} + q\mathbf{E} = n \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) (m\gamma \mathbf{v}) \quad (17.3.9)$$

$$= n \left[\frac{\partial m\gamma \mathbf{v}}{\partial t} + \nabla (mc^2 \gamma) - \mathbf{v} \times (\nabla \times m\gamma \mathbf{v}) \right] \quad (17.3.10)$$

(17.3.9) と $\mathbf{v}$ の内積、 (17.3.9) と $\mathbf{u}_c/\Omega_*$ の内積

$$\nabla \cdot \mathbf{F}_E = 0, \quad \nabla \cdot \mathbf{L}_L = 0$$

flow of electromagnetic energy
and angular momentum

$$\mathbf{F}_E = mn\gamma c^2 \mathbf{w} + \mathbf{S} - U \mathbf{u}_c$$

$$\mathbf{F}_L = nm\gamma [(\mathbf{u}_c/\Omega_*) \cdot \mathbf{v}] \mathbf{w} + (\mathbf{S} - U \mathbf{u}_c)/\Omega_* - (\mathbf{j} - q\mathbf{u}_c)(\Phi/\Omega_*)$$

Thus the energy and angular momentum come into the outer magnetosphere through the surface S are



$$\dot{E}_{\text{out}} = \int_S (\mathbf{S} - U\mathbf{u}_c) \cdot d\mathbf{a}$$

帳尻を合わせる項

$$\dot{L}_{\text{out}} = \frac{1}{\Omega_*} \left[\int_S (\mathbf{S} - U\mathbf{u}_c) \cdot d\mathbf{a} + \int_S [(\mathbf{j} - q\mathbf{u}_c)(-\Phi)] \cdot d\mathbf{a} \right]$$

Some part of this will be used to accelerate particles, and the rest of it reaches the termination shock in a pulsar wind.



$$\dot{E}_{\text{in}} + \dot{E}_{\text{out}} = \Omega_* (\dot{L}_{\text{in}} + \dot{L}_{\text{out}})$$

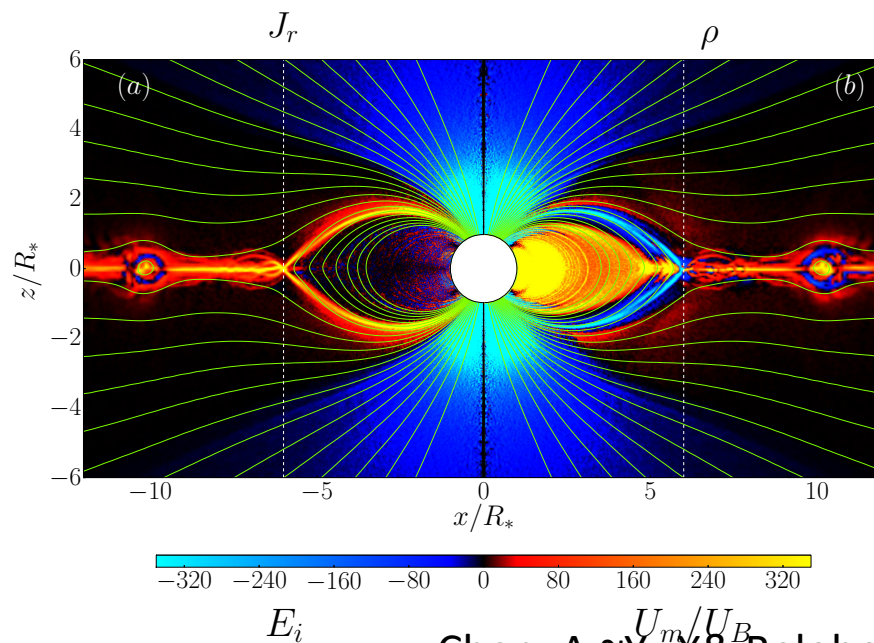
Less efficient angular momentum loss of the inner magnetosphere is compensated by efficient loss of the outer magnetosphere.

angular velocity of the outer magnetosphere may be represented by

$\Omega_{out} = \dot{E}_{out} / \dot{L}_{out}$, for which we have

$$\Omega_{out} = \Omega_* \frac{1}{1 + \Delta \dot{E}_{in} / \dot{E}_{out}} < \Omega_*$$

effective light radius c/Ω_{out} は R_L



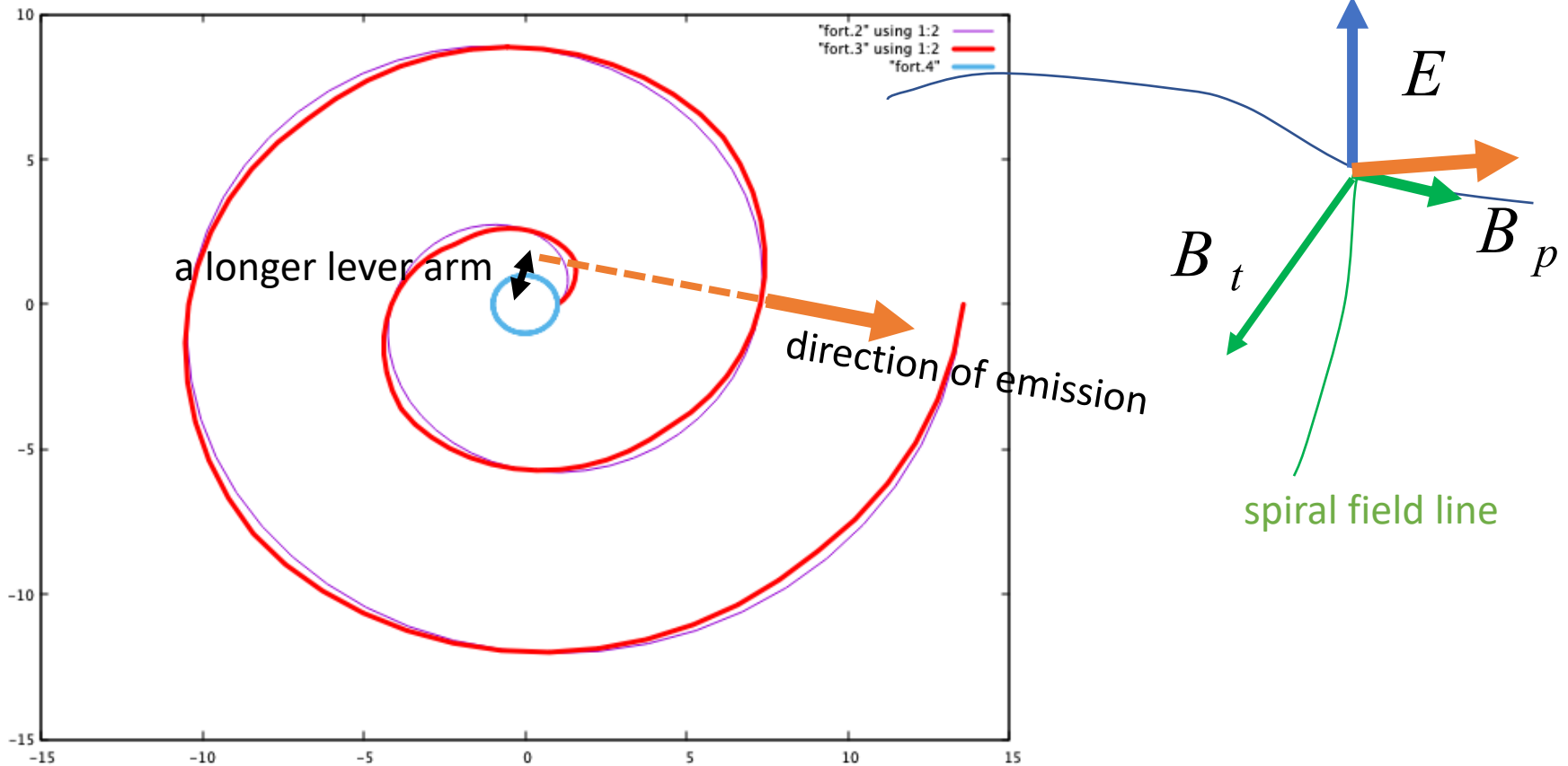
このような計算はどのように変更をうけるか？

光円柱が膨張する。ゆっくり回転。

- 巻きつく？星の角速度で

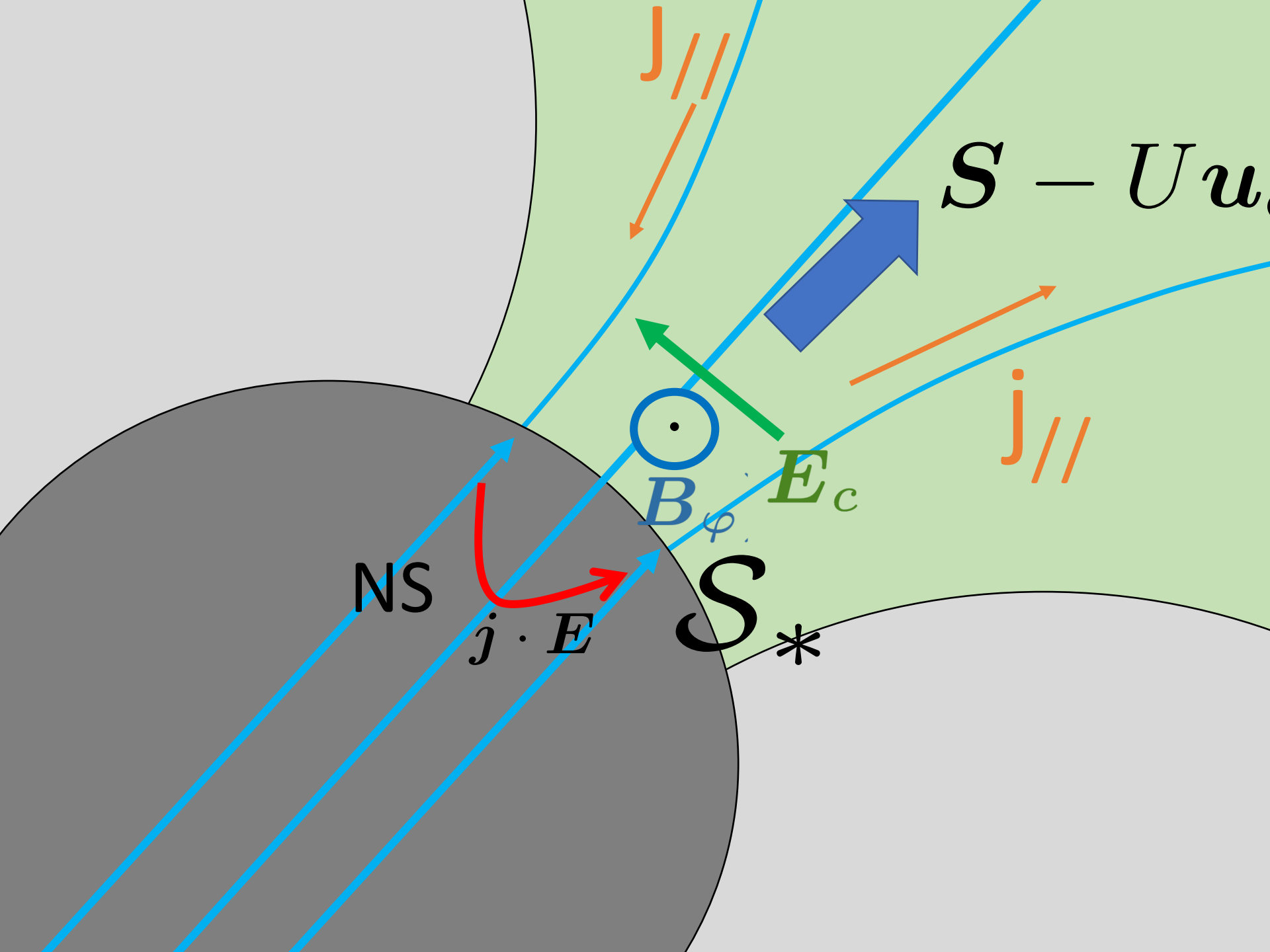
- パターンは回る？

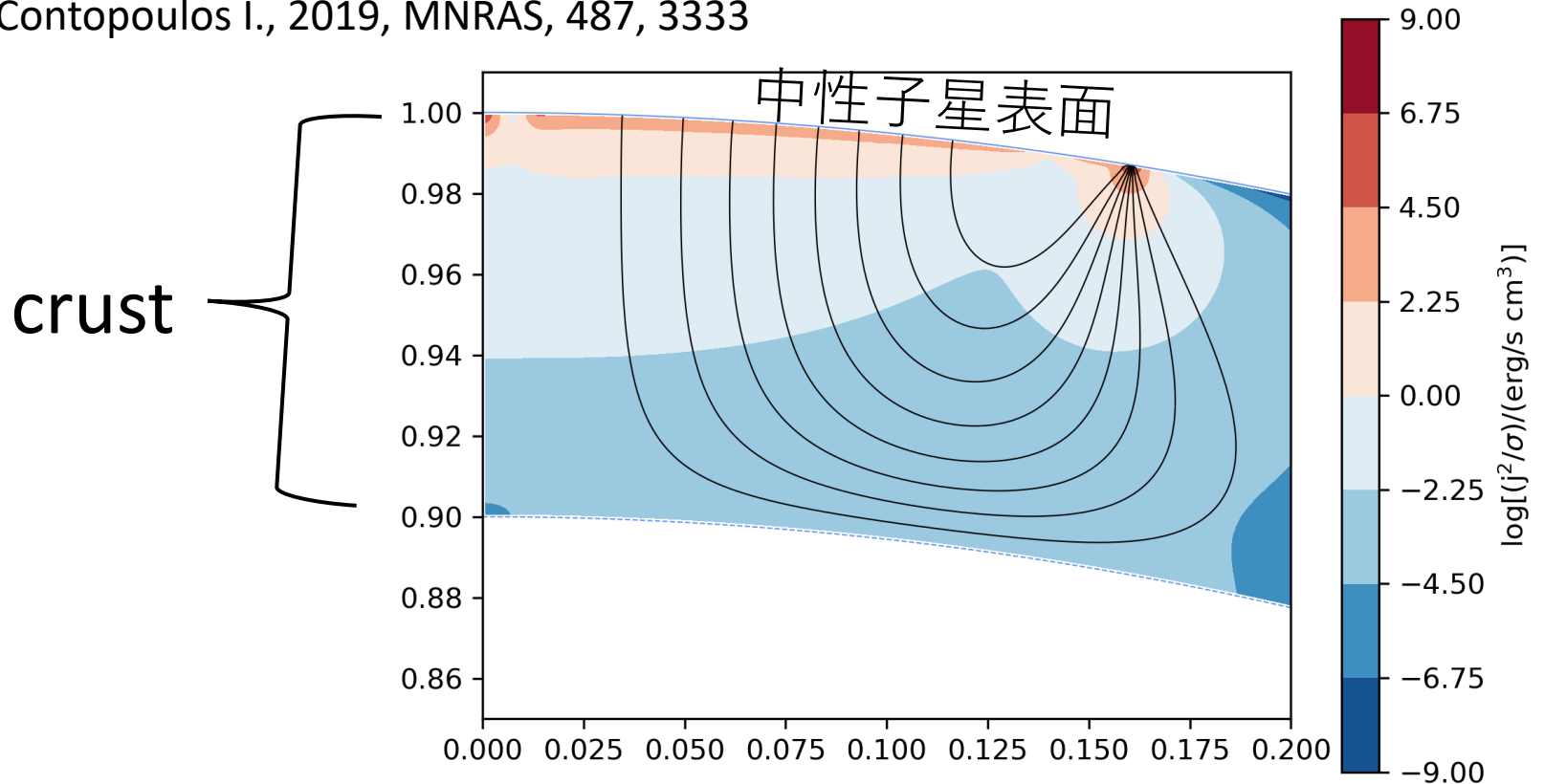
emission region の巻き込みが緩い、
平均の巻き込みは Ω_*



otherwise 巻き込みと緩和を間欠的に行う。

Braking torque on the star





$$P_{\text{Ohm}} \equiv \int_V \mathbf{j} \cdot \mathbf{E} \, dV = \int_V \frac{j^2}{\sigma} \, dV = \left(\frac{c}{4\pi} \right)^2 \int_V \frac{(\nabla \times \mathbf{B})^2}{\sigma} \, dV .$$

that P_{Ohm} is minimized , we obtain the condition

$$\frac{c}{4\pi} \nabla \times \left(\frac{\nabla \times \mathbf{B}}{\sigma} \right) \equiv \frac{c}{4\pi} \nabla \times \left(\frac{\mathbf{j}}{\sigma} \right) = 0$$

星の回転のことは考えていない。

How the Ohmic heating is possible even though it release no angular momentum but energy

相互作用する二流体モデルで考察

$$\mathbf{I}_p = en_p \mathbf{E} + \frac{1}{c} en_p \mathbf{v}_p \times \mathbf{B} + \mathbf{F}_f$$

$$\mathbf{I}_e = -en_e \mathbf{E} - \frac{1}{c} en_e \mathbf{v}_e \times \mathbf{B} - \mathbf{F}_f$$

$$\mathbf{F}_f = -m_e n_e (\mathbf{v}_p - \mathbf{v}_e) \nu_e$$

$$\mathbf{I}_p + \mathbf{I}_e = q \mathbf{E} + \frac{1}{c} \mathbf{j} \times \mathbf{B}$$

$$\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \approx \frac{\mathbf{j}}{\sigma}$$



How the Ohmic heating is possible even though it release no angular momentum but energy

$$\mathbf{I}_p = en_p \mathbf{E} + \frac{1}{c} en_p \mathbf{v}_p \times \mathbf{B} + \mathbf{F}_f$$

$$\mathbf{I}_e = -en_e \mathbf{E} - \frac{1}{c} en_e \mathbf{v}_e \times \mathbf{B} - \mathbf{F}_f$$

速度との内積をとってエネルギーの収支を確認

$$\mathbf{I}_p \cdot \mathbf{v}_p + \mathbf{I}_e \cdot \mathbf{v}_e = \mathbf{j} \cdot \mathbf{E} - m_e n_e \nu_e (v_p^2 + v_e^2) = \mathbf{j} \cdot \mathbf{E} - j^2 / \sigma$$

$$\mathbf{v}_p = m_p n_p \Omega \varpi \mathbf{e}_\varphi \quad (\text{differential rotation} \neq \Omega_* \text{ allowed})$$

$$; \Omega \dot{\Omega} m_p n_p \varpi^2 = \mathbf{j} \cdot \mathbf{E} - j^2 / \sigma$$

$$\langle \Omega \dot{\Omega} \rangle \mathfrak{V} = \int_{V_*} \mathbf{j} \cdot \mathbf{E} dV - \int_{V_*} (j^2 / \sigma) dV$$

Rotation energy goes to the magnetosphere and Ohmic heat

goes to the magnetosphere

$$\dot{\mathcal{E}}_{S_*} = \Omega_* \dot{\mathcal{L}}_{S_*} = \int_{S_*} (\mathbf{S} - U \mathbf{u}_c) \cdot d\mathbf{a}$$

$$\mathbf{I}_p = en_p \mathbf{E} + \frac{1}{c} en_p \mathbf{v}_p \times \mathbf{B} + \mathbf{F}_f$$

$$\mathbf{I}_e = -en_e \mathbf{E} - \frac{1}{c} en_e \mathbf{v}_e \times \mathbf{B} - \mathbf{F}_f$$

$$\varpi \mathbf{e}_\varphi \cdot (\mathbf{I}_p + \mathbf{I}_e) = \frac{1}{\Omega_*} \left[q \mathbf{u}_c \cdot \mathbf{E} + \frac{1}{c} (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{u}_c \right]$$

$$\mathbf{E} = -\mathbf{u}_c \times \mathbf{B}/c - \nabla \Phi \quad \nabla \cdot (\mathbf{j} - q \mathbf{u}_c) = 0$$

$$\varpi \mathbf{e}_\varphi \cdot (\mathbf{I}_p + \mathbf{I}_e) = \frac{1}{\Omega_*} [\mathbf{j} \cdot \mathbf{E} + \nabla \cdot \{(\mathbf{j} - \rho_e \mathbf{u}_c) \Phi\}]$$

$$\langle \dot{\Omega} \rangle \mathfrak{V} = \frac{1}{\Omega_*} \int_{V_*} \mathbf{j} \cdot \mathbf{E} dV$$

goes to the
magnetosphere

$$\dot{\mathcal{E}}_{S_*} = \Omega_* \dot{\mathcal{L}}_{S_*} = \int_{S_*} (\mathbf{S} - U \mathbf{u}_c) \cdot d\mathbf{a}$$

$$\langle \Omega \rangle \equiv \frac{-\Im \langle \Omega \dot{\Omega} \rangle}{-\Im \langle \dot{\Omega} \rangle} = \Omega_* \left(1 + \frac{\int_{V_*} (j^2 / \sigma) dV}{\int_{S_*} (\mathbf{S} - U \mathbf{u}_c) \cdot d\mathbf{A}} \right) > \Omega_*$$

When there is Ohmic heating, the star internal matter rotate faster than the stellar surface.

Previous works which attempt to determine the current closure inside star do not care about the rotational structure inside **are ill-conceived**.

Demonstrate how the current inside the star is determined.

Actually Ohmic heating is not significant, so ideal-MHD is a good approximation.
No Ohmic dissipation, no differential rotation

Basic Equations: NS: elastic body

$$\rho \frac{D\mathbf{v}}{Dt} = \frac{1}{c} \mathbf{j} \times \mathbf{B} - \nabla \Pi - \nabla \Phi_g$$

$$\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} = 0 \quad \text{ideal-MHD (no differential rotation)}$$

$$\mathbf{v} = \Omega(t) \mathbf{e}_\varphi$$

$$\mathbf{j} \times \mathbf{B} = \mathbf{j}_0 \times \mathbf{B}_0 + \mathbf{j}_1 \times \mathbf{B}_0 + \mathbf{j}_0 \times \mathbf{B}_1 + \mathbf{j}_1 \times \mathbf{B}_1$$

Any electromagnetic force by the magnetospheric current circulation is balanced by the grad-stress force. The current distribution becomes minimize the elastic energy.

eq. mo. in azimuthal direction

$$\frac{\partial^2 u_\varphi}{\partial r^2} + \frac{1}{r} \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r^2} + \frac{\partial^2 u_\varphi}{\partial x^2} + \frac{F_\varphi - \rho \dot{\Omega} r}{G} = 0$$

azimuthal displacement
shear modulus

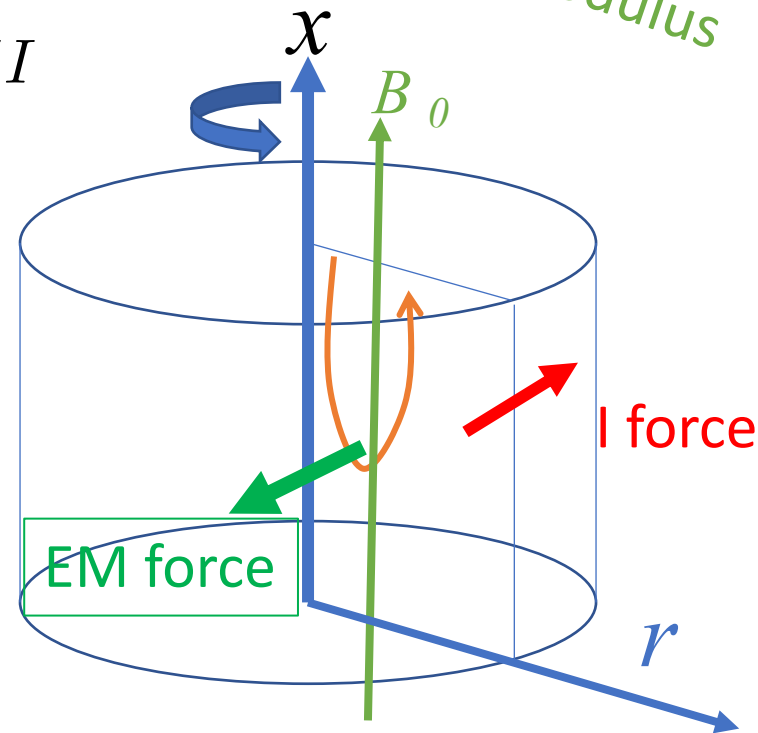
$$F_\varphi = \frac{1}{c} (\mathbf{j}_p \times \mathbf{B}_0) \cdot \mathbf{e}_\varphi = \frac{1}{2\pi c r} \mathbf{B}_0 \cdot \nabla I$$

BC : $I(s) \rightarrow \dot{\Omega}$

BC: no shear stress on SN surface

assume a distribution of $I \rightarrow$
obtain the deformation $\rightarrow$ obtain
elastic energy

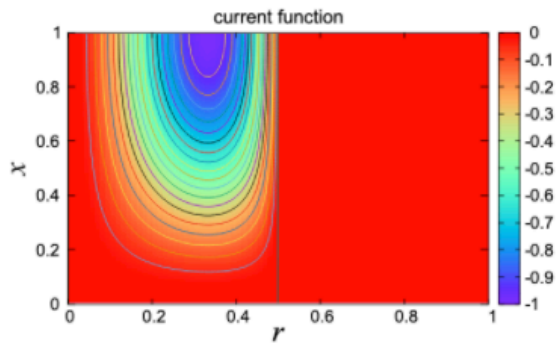
minimize the elastic energy



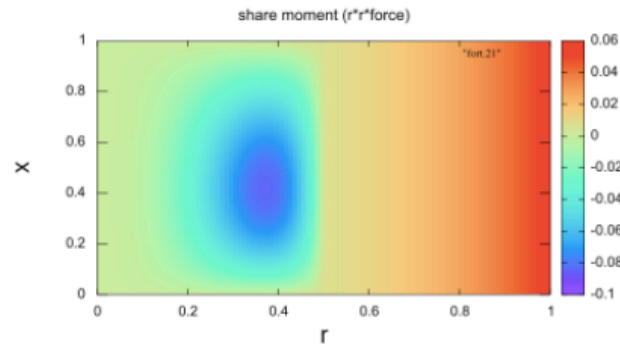
moderately extended circuit within polar cap magnetic flux

Ele=3.2748862500955418E-003

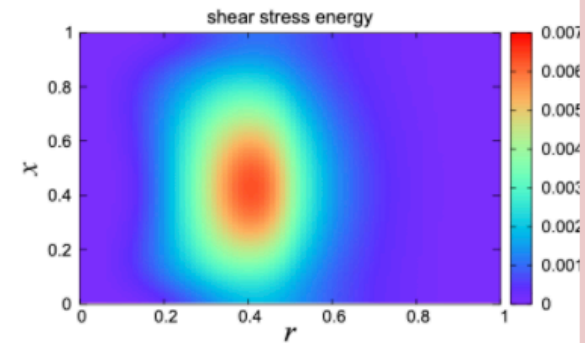
Current



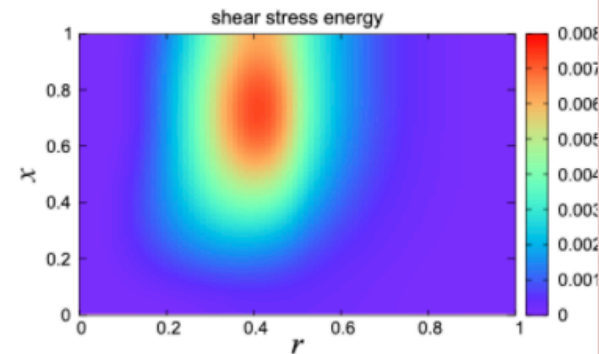
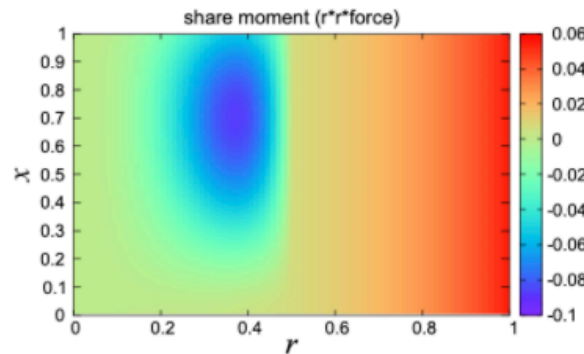
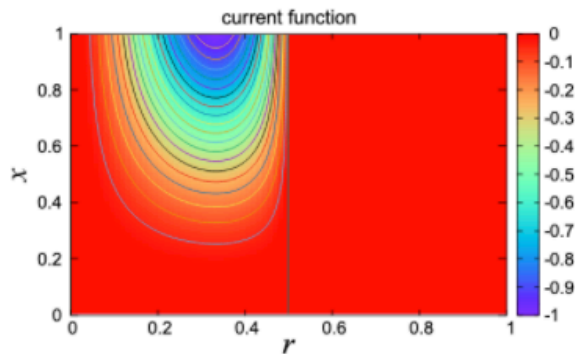
share moment ($r^*r^*F\phi$)



stress energy



Ele=3.6586679768049546E-003

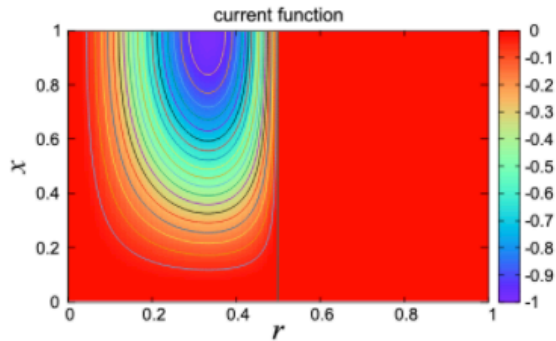


short circuited

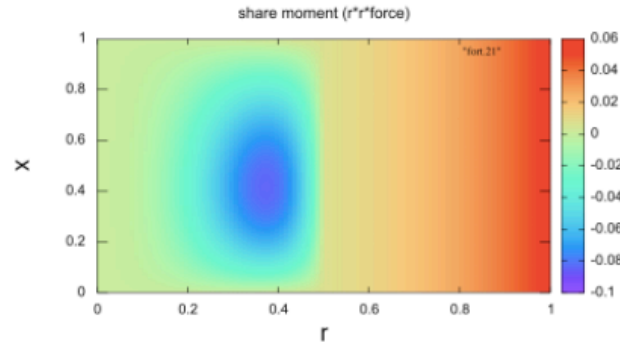
moderately extended circuit within polar cap magnetic flux

Ele=3.2748862500955418E-003

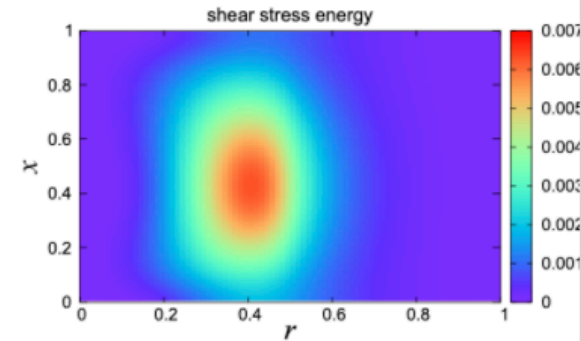
Current



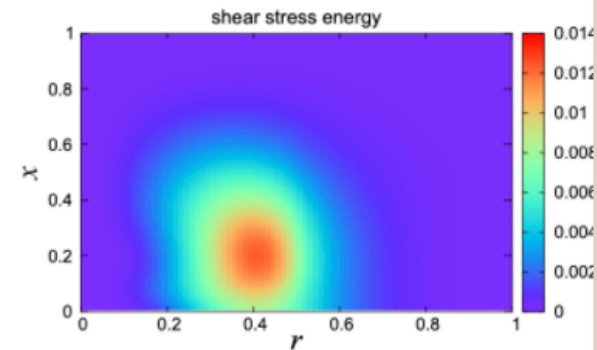
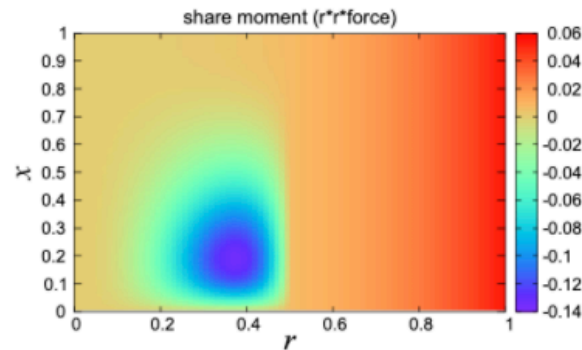
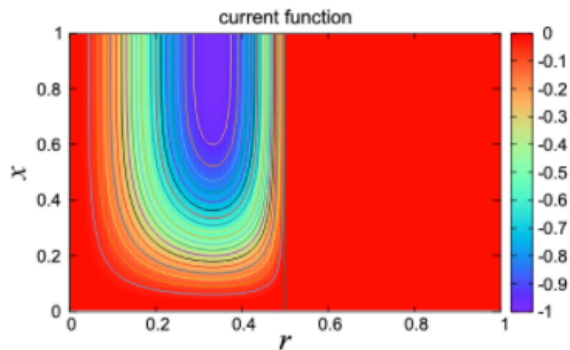
share moment ($r^*r^*F\phi$)



stress energy



Ele=4.6752470054759309E-003

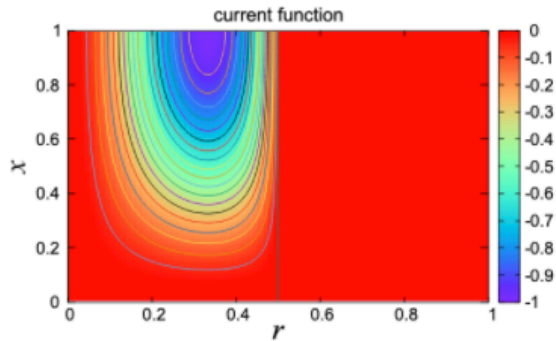


extended circuit

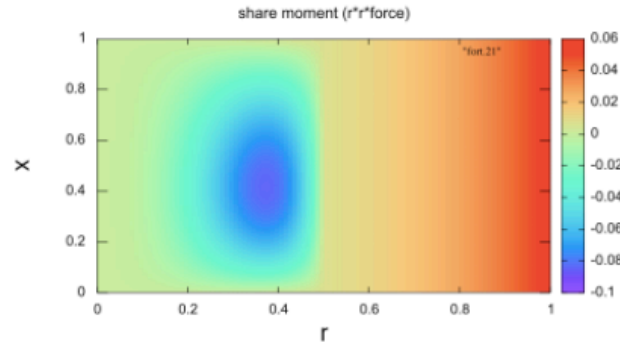
moderately extended circuit within polar cap magnetic flux

Ele=3.2748862500955418E-003

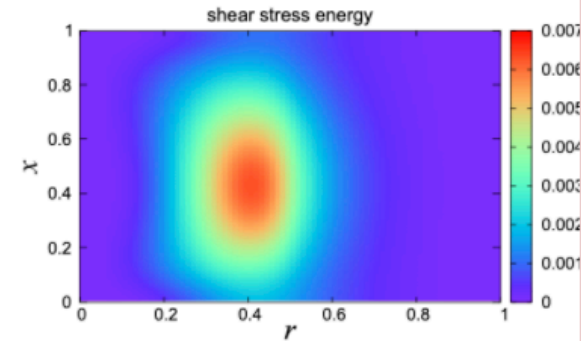
Current



share moment ($r^*r^*F\phi$)

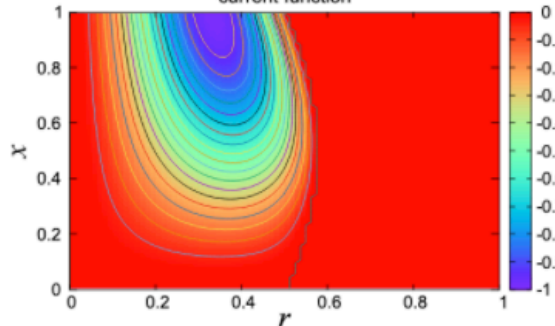


stress energy

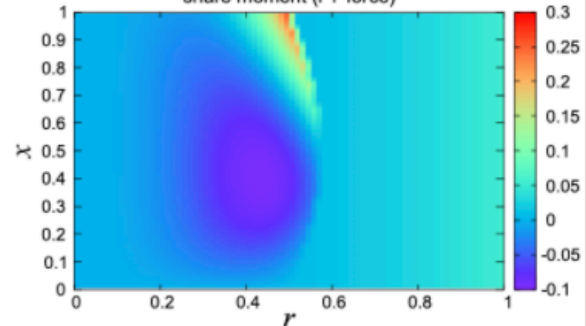


Ele=3.7001186317131493E-003

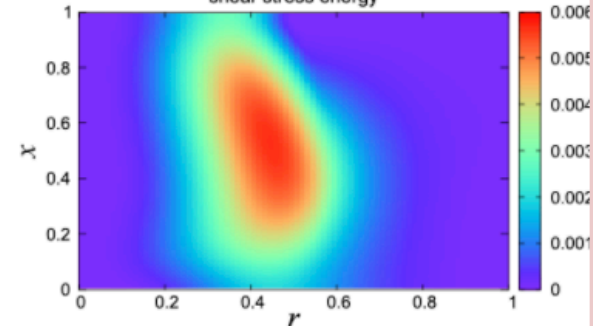
current function



share moment ($r^*r^*F\phi$)



shear stress energy



cross and extend the polar cap flux

Result:
(not surprisingly)

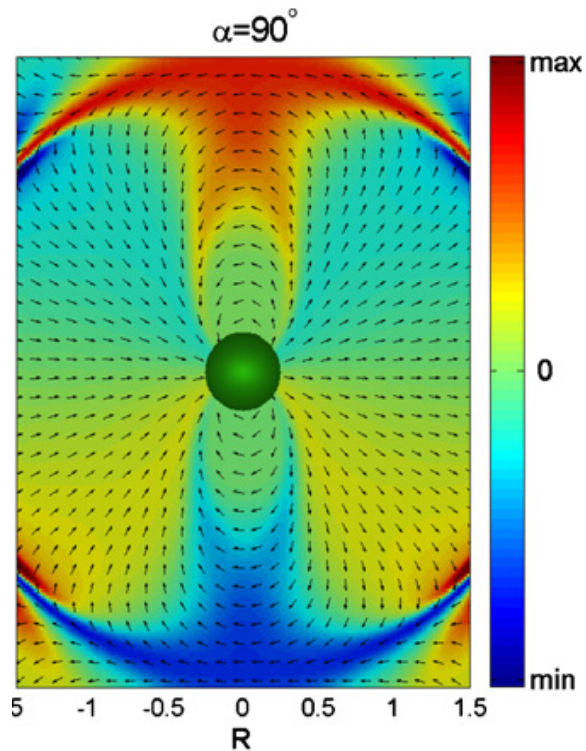
moderately extended circuit within polar cap magnetic flux

Ohm's law で電流を決めるアルゴリズムは間違い。
星内部の角運動量輸送と連立しないとけない。

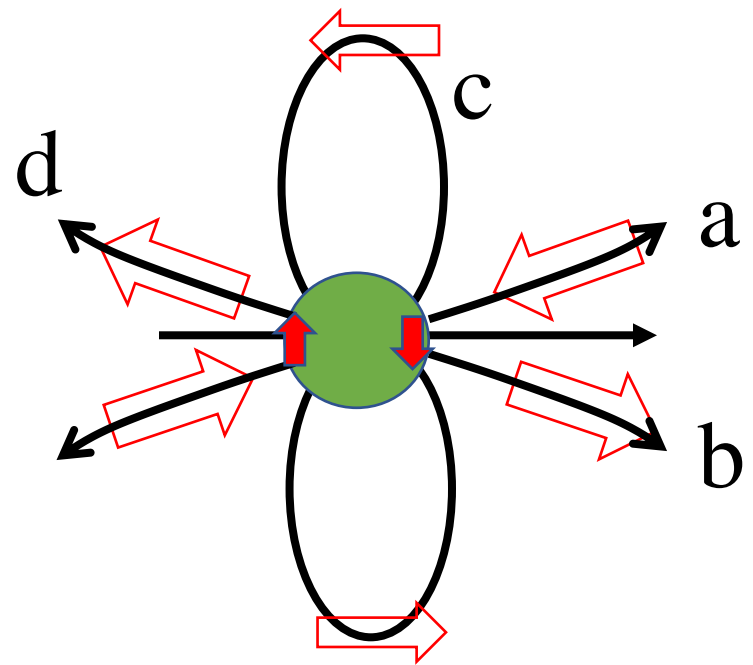
電気抵抗は小さいので。
電流の閉じ方を決める物理は中性子星の弾性変形でないだろうか。

トルクをよく考えていない理論は他にもある。

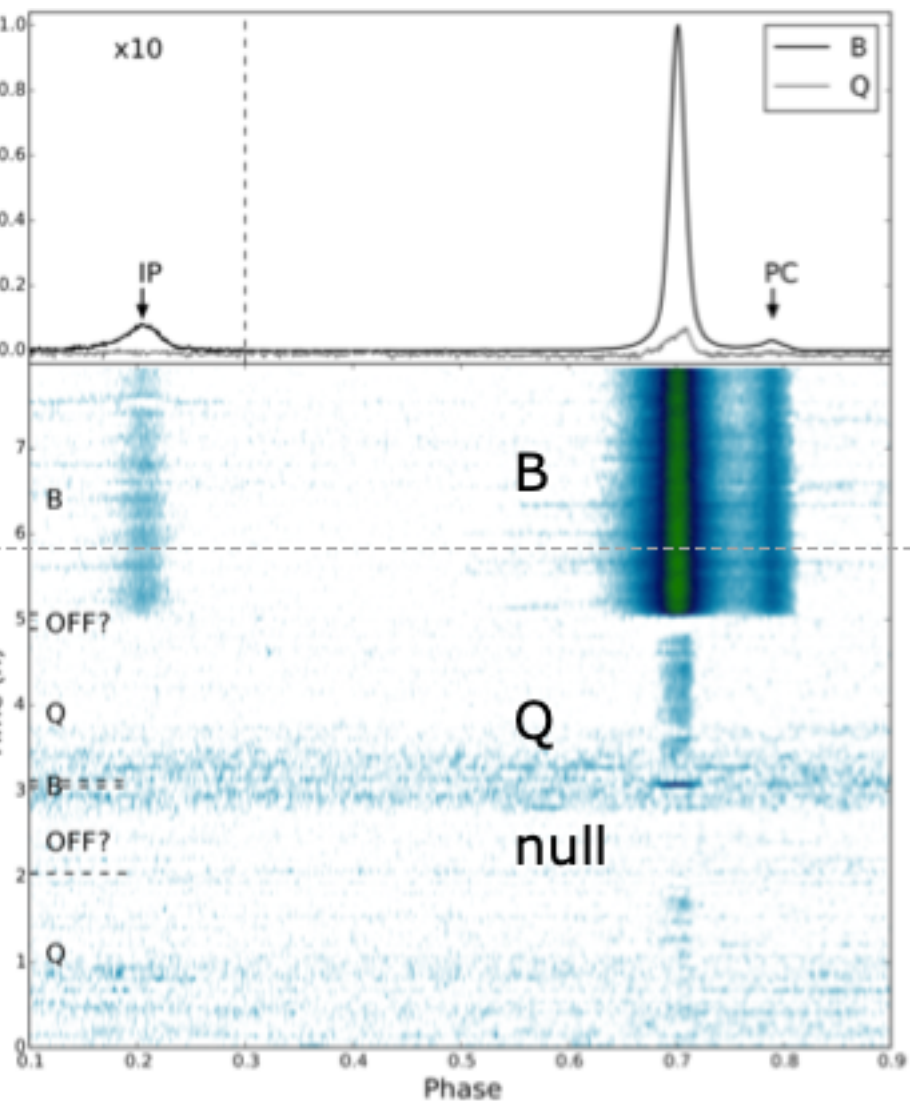
λ map



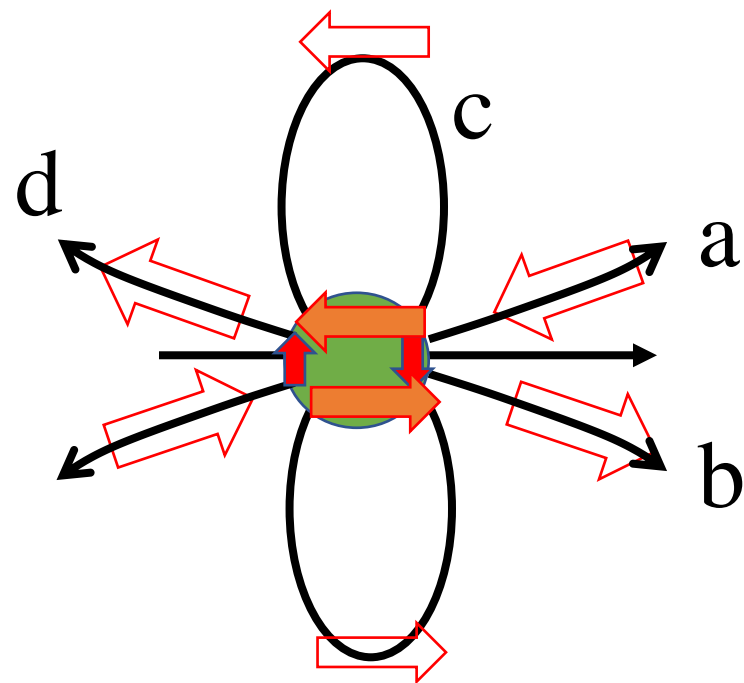
磁気圏のFFも完全でないのかも
BCに注意



$$\lambda \mathbf{B} = \mathbf{j} - \rho(\boldsymbol{\Omega} \times \mathbf{r}) \equiv \mathbf{j}^C,$$



Sobey et al. 2015; LOFAR



いろいろな電流の可能性も
今後の検討課題