

# **Nonlinear Explosive Magnetic Reconnection in Collisionless System**

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# Magneto-luminescence:

Blandford+ Space Sci. Rev. 2017

## Magnetic Energy Dissipation in Astrophysics

### ◎ Pulsars and their nebulae:

$\gamma$ -ray flares in Crab Nebula (AGILE/Fermi observation, e.g., Tavani+11, Abdo+11)  
 $\sigma$ -problem ( $\sigma \gg 1$  in magnetosphere, whereas  $\sigma \ll 1$  in nebula)

### ◎ Blazars:

jets from a massive black hole ( $10^6$ - $10^{10}M_{\text{sun}}$ ), bulk Lorentz factor  $\Gamma \sim 10$   
energy supply by gas accretion or BH spin-up  
rapid time variable GeV/TeV flares (e.g., Albert+07, Aharonian+ 07, Aleksic+ 11)

### ◎ Gamma Ray Bursts: (e.g., Kouveliotou+12 for a review)

short bursts (neutron star binary), long bursts (core-collapse supernovae)  
relativistic jets with bulk Lorentz factor  $\Gamma \sim 100$ -1000

### ◎ Magnetars: (e.g., Kaspi & Beloborodov 17 for a review)

a minority of neutron stars/born with millisecond pulsars  
 $B \sim 10$ -100 GT @ surface, flare observations (e.g., Evans+ 80)

# Time Scale of Magnetic Reconnection

## Collisionless tearing-mode instability

$$\gamma_L \tau_A \approx (r_g / \lambda)^{\frac{3}{2}} \left(1 + \frac{T_i}{T_e}\right) (1 - k^2 \lambda^2)$$

$\tau_A = \lambda / V_A$ : Alfvén transit time     $r_g$ : gyro-radius

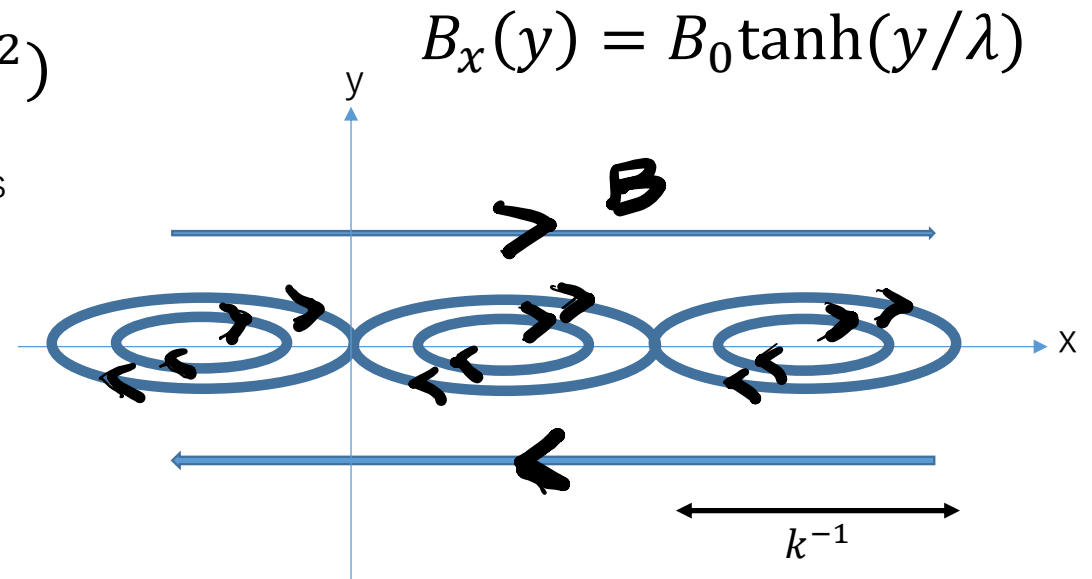
Coppi+ 1966, Hoh 1966

Open question:  
very slow growth rate for  $r_g \ll \lambda$

**This Talk=>**

## Nonlinear tearing-mode instability

$$\gamma \tau_A \approx O(0.1) - O(1)$$



cf. MHD tearing-mode instability

$$\gamma_{FKR} \tau_A \approx R_m^{-3/5} \left( \frac{1 - k^2 \lambda^2}{k \lambda} \right)^{4/5}$$

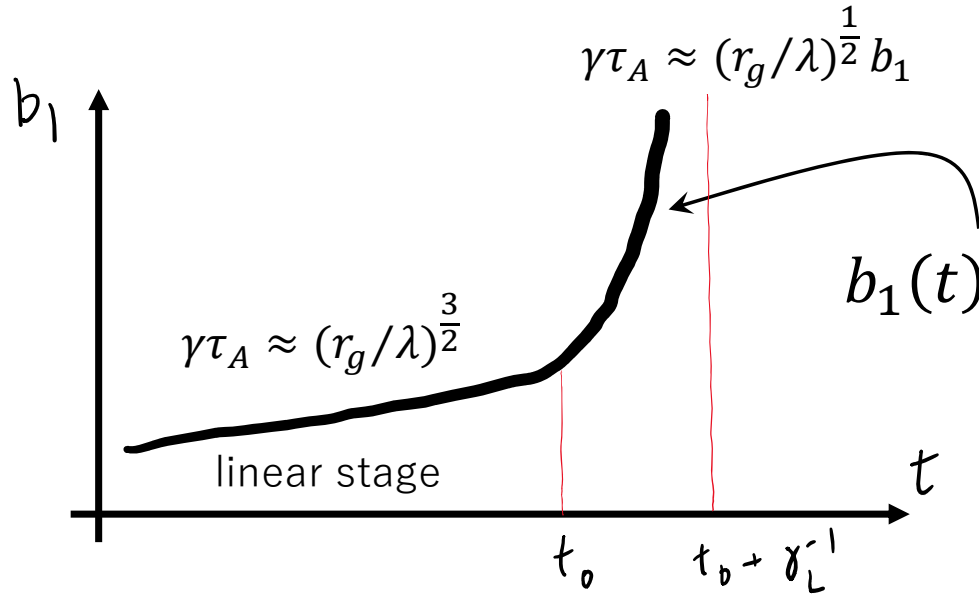
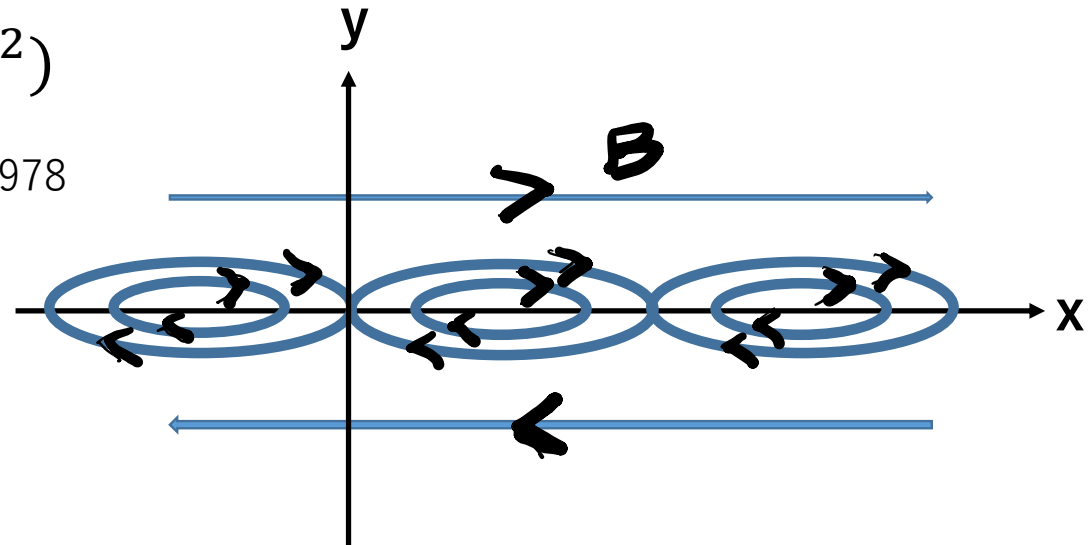
Furth, Killeen & Rosenbluth 1963

# Explosive Reconnection by Galeev

$$\gamma_G \tau_A \approx (r_g/\lambda)^{\frac{1}{2}} b_1 \left(1 + \frac{T_i}{T_e}\right) (1 - k^2 \lambda^2)$$

Galeev + 1978

$b_1 \equiv \delta B_y / B_0$   
amplitude of reconnecting B



$$b_1(t) = b_1(t_0) / (1 - (t - t_0)\gamma_L)$$

$$\gamma_G \equiv d \log(b_1) / dt$$

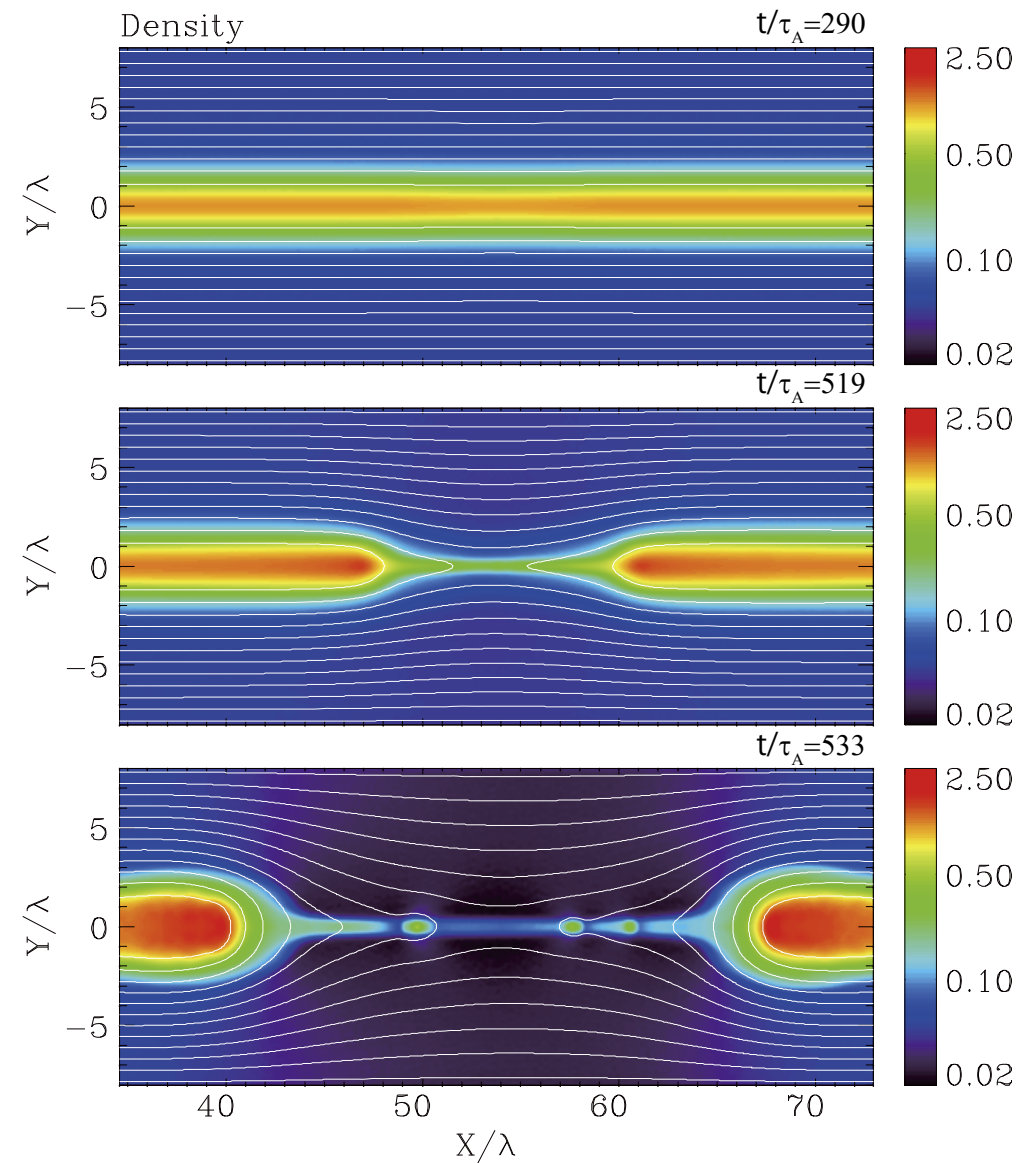
Simulation study: Terasawa 1981

# PIC Simulations (particle-in-cell)

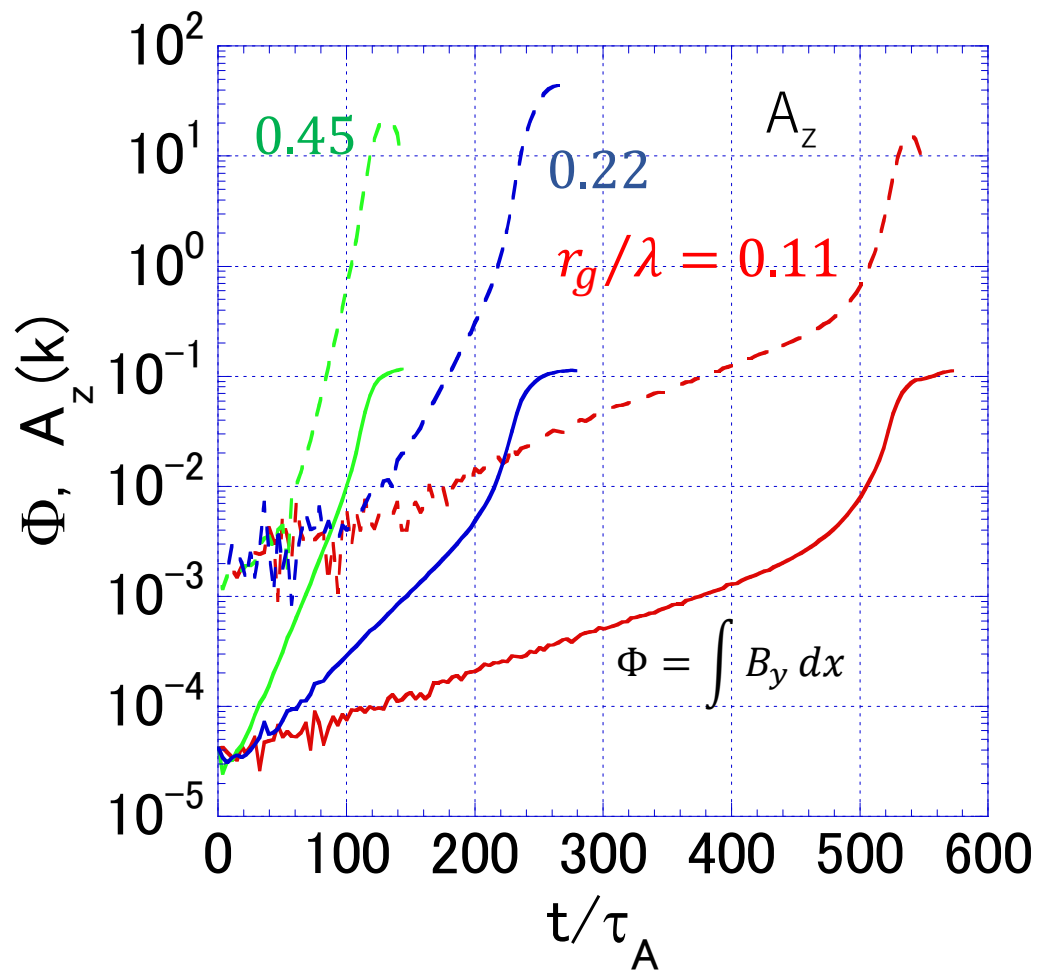
linear stage:  
Harris equilibrium,  
for simplicity, pair plasma

early nonlinear stage:  
shrinkage of current  
sheet @ maximum  
growth rate

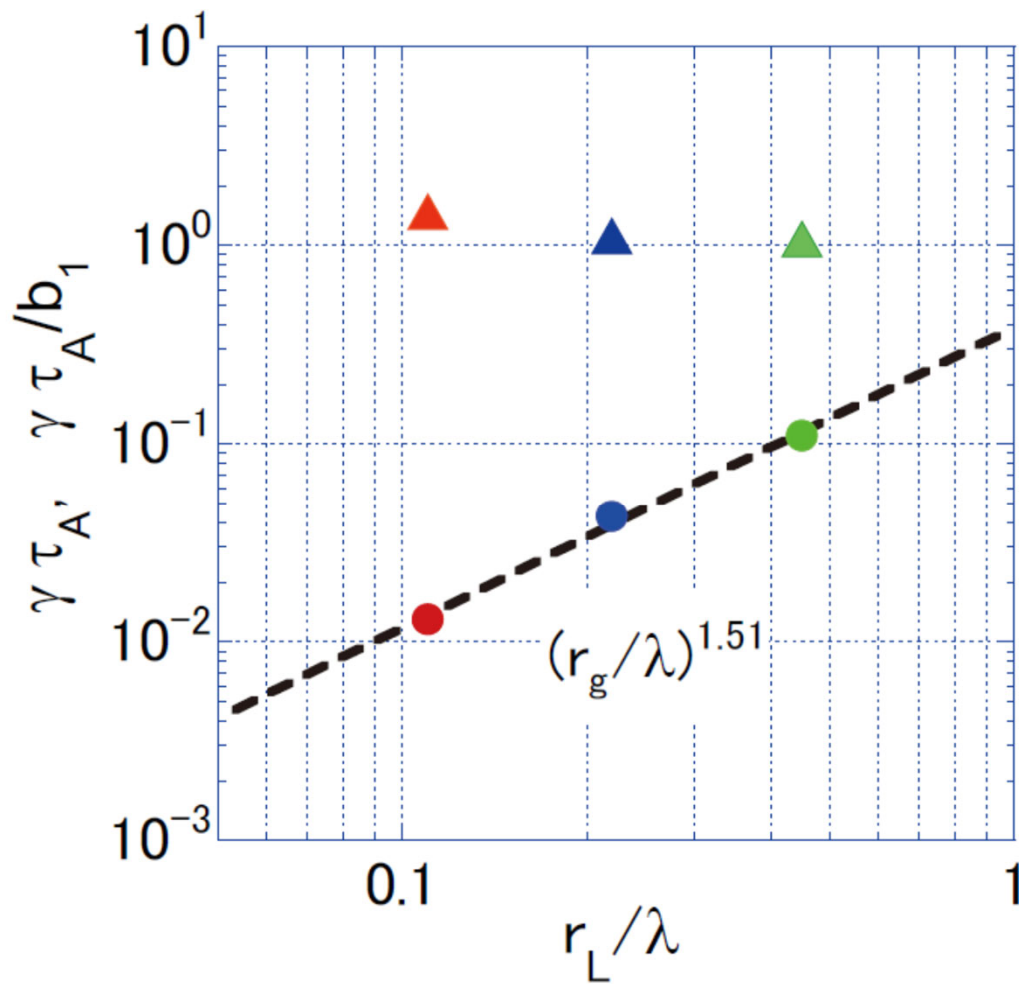
late nonlinear stage:  
elongated current  
sheet with several  
plasmoids/turbulence



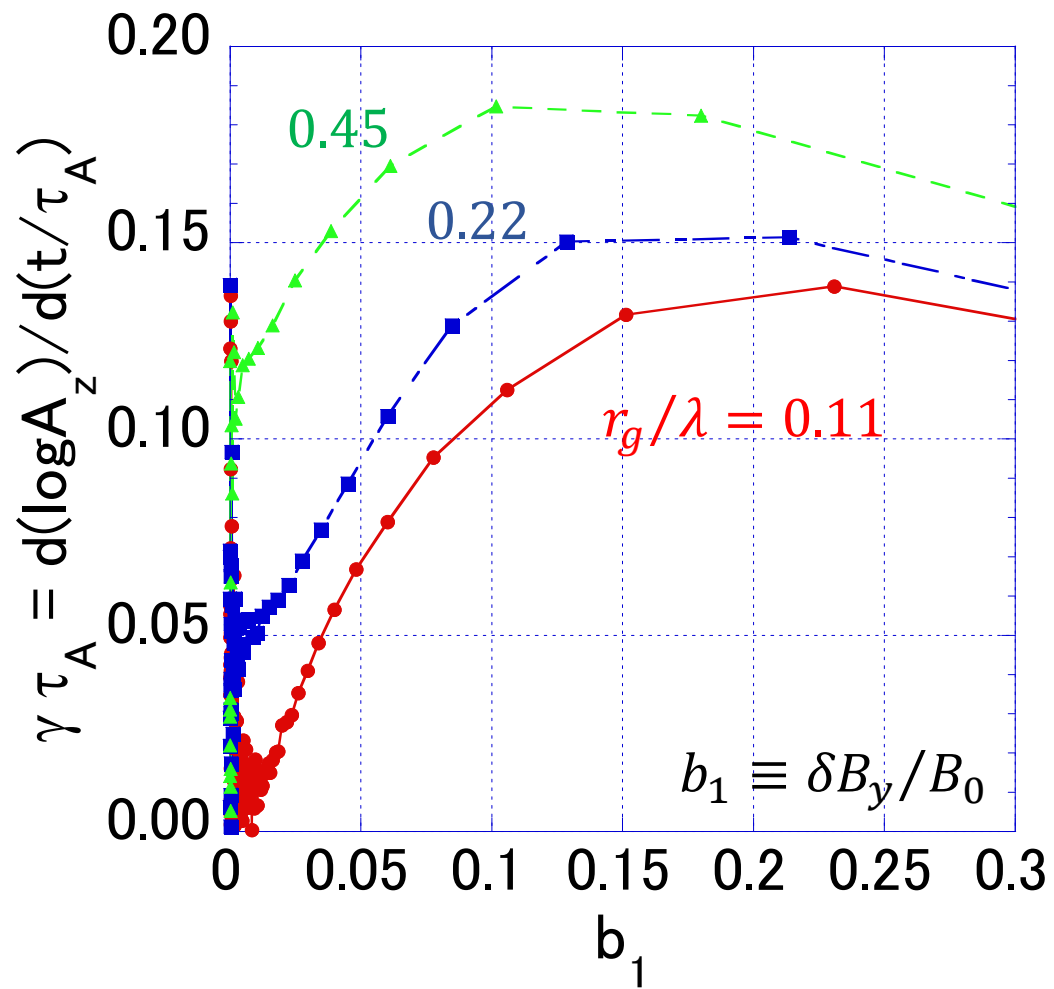
time history: growth curve



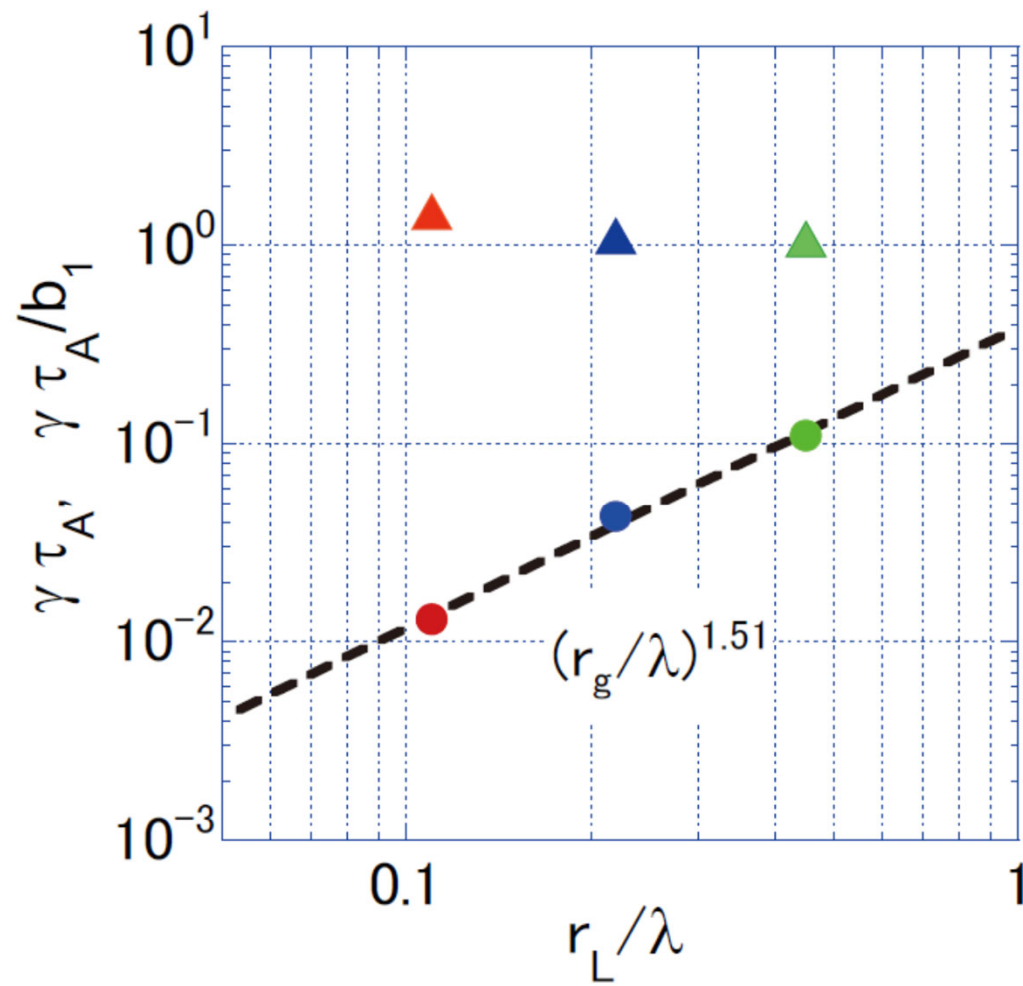
growth rates for linear & nonlinear



“instantaneous” growth rates



growth rates for linear & nonlinear



# Theory of tearing instability (1/3)

$$\frac{1}{8\pi} \frac{\partial}{\partial t} (\vec{B}^2 + \vec{E}^2) = -\frac{c}{4\pi} \nabla \cdot (\vec{E} \times \vec{B}) - \vec{E} \cdot \vec{J}$$

$$\vec{B} = \nabla \times A \vec{e}_z \quad A(x, y) = A_0(y) + A_1(y) \cos(kx)$$

$$\frac{\partial}{\partial t} \iint \frac{B_1^2}{8\pi} dx dy + \iint E_1 \cdot J_1^{ad} dx dy = - \iint E_1 \cdot J_1^{res} dx dy$$

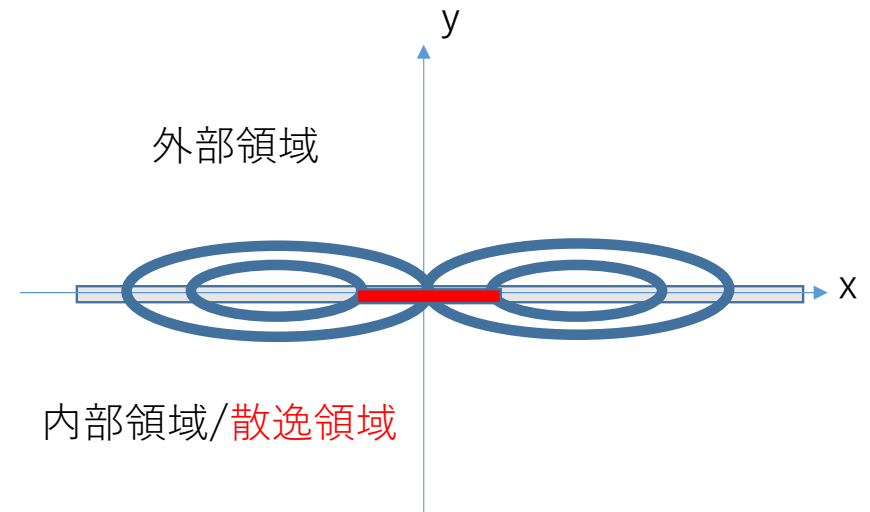
内部領域の電流

$$J_1^{res} = \sigma E_1 \quad \sigma = \left( \frac{ne^2}{m} \right) \left( \frac{d_x}{v_{th}} \right)$$

外部領域の電流・ベクトルポテンシャル A

$$B_{0x}(y) = B_0 \tanh(y/\lambda)$$

$$A_1(y) = A_1(0) \left( 1 + \frac{\tanh(|y|/\lambda)}{k\lambda} \right) \exp(-k|y|)$$





# Theory of tearing instability (2/3)

外部領域

$$\frac{\partial}{\partial t} \iint \frac{B_1^2}{8\pi} dx dy + \iint E_1 \cdot J_1^{ad} dx dy = \frac{\partial}{\partial t} \left[ \frac{1}{4\pi} \frac{(k^2 \lambda^2 - 1)}{k^2 \lambda^2} |A_1(0)|^2 \right]$$

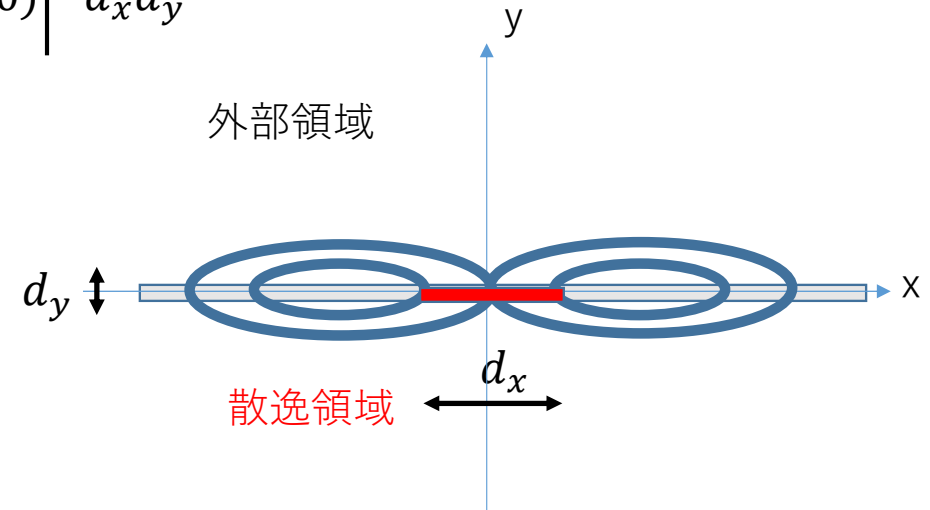
内部領域/散逸領域/Ohm拡散領域

$$- \iint E_1 \cdot J_1^{res} dx dy = -2\sigma |E_1|^2 dx dy = -\frac{1}{2\pi} \frac{\omega_p^2}{c^2} \frac{d_x}{v_{th}} \left| \frac{\partial}{\partial t} A_1(0) \right|^2 dx dy$$

$$J_1^{res} = \sigma E_1 \quad E_1 = -\frac{1}{c} \frac{\partial}{\partial t} A_1 \quad \sigma = \left( \frac{ne^2}{m} \right) \left( \frac{d_x}{v_{th}} \right)$$

(外部領域のエネルギー) = (散逸領域のエネルギー)

$$\frac{\pi (1 - k^2 \lambda^2)}{k} \frac{1}{k \lambda^2} = \gamma \frac{\omega_p^2}{c^2} \frac{d_x}{v_{th}} dx dy$$



# Theory of tearing instability (3/3)

Master Equation (外部領域のエネルギー) = (散逸領域のエネルギー)

$$\frac{\pi}{k} \frac{(1 - k^2 \lambda^2)}{k \lambda^2} = \gamma \frac{\omega_p^2}{c^2} \frac{d_x}{v_{th}} d_x d_y$$

Linear Tearing Instability e.g. Coppi+ 1966, Hoh 1966

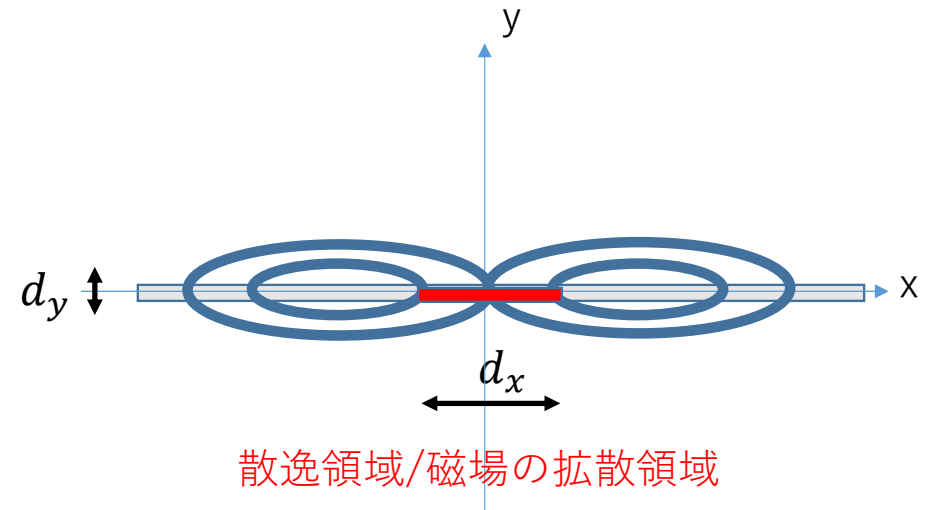
$$d_x = k^{-1}, \quad d_y = \sqrt{r_g \lambda}$$

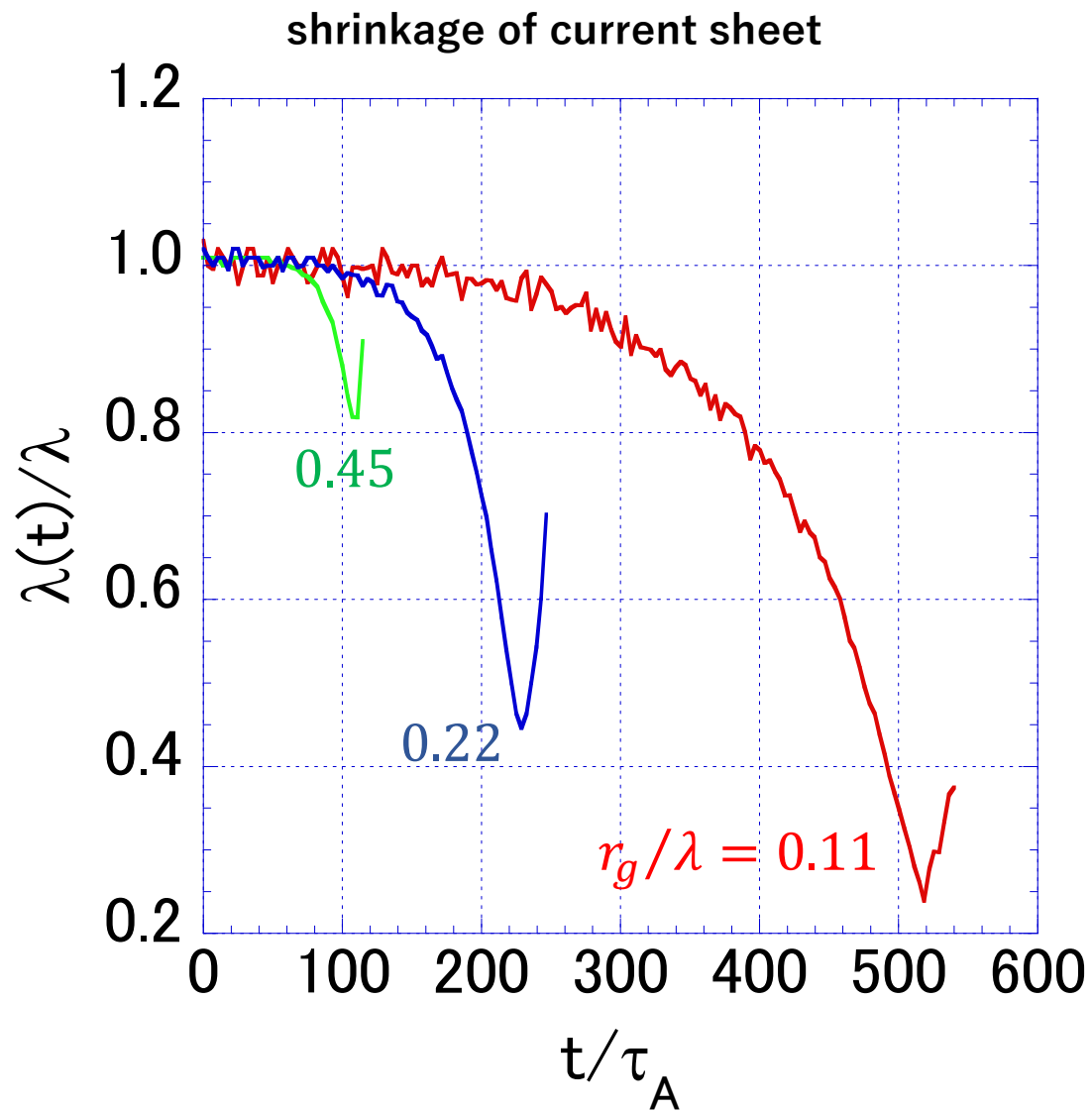
$$\Rightarrow \gamma \tau_A \approx (r_g / \lambda)^{\frac{3}{2}}$$

Galeev's tearing instability Galeev + 1978

$$d_x = \sqrt{r_g / (b_1 k)}, \quad d_y = \sqrt{r_g \lambda}$$

$$\Rightarrow \gamma \tau_A \approx (r_g / \lambda)^{\frac{1}{2}} b_1$$





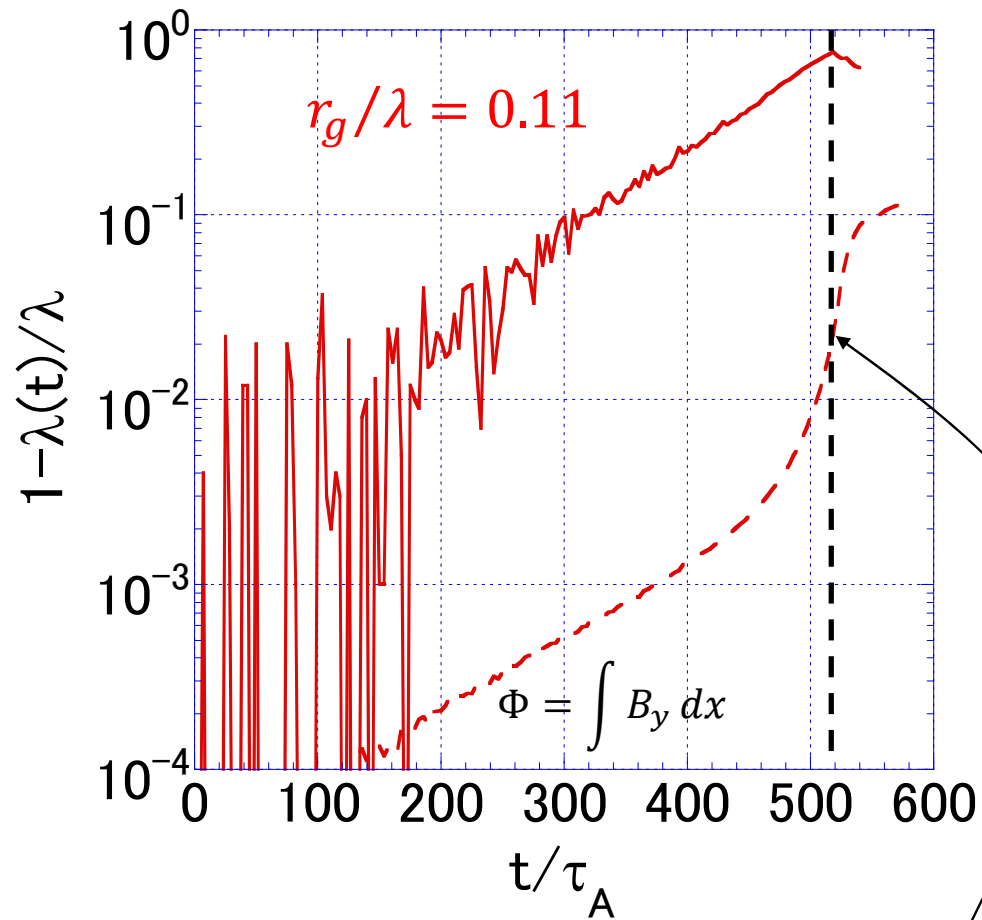
fitting of current sheet

$$J_z(y) \propto \text{sech}^2(y/\lambda(t))$$

thickness of shrinkage

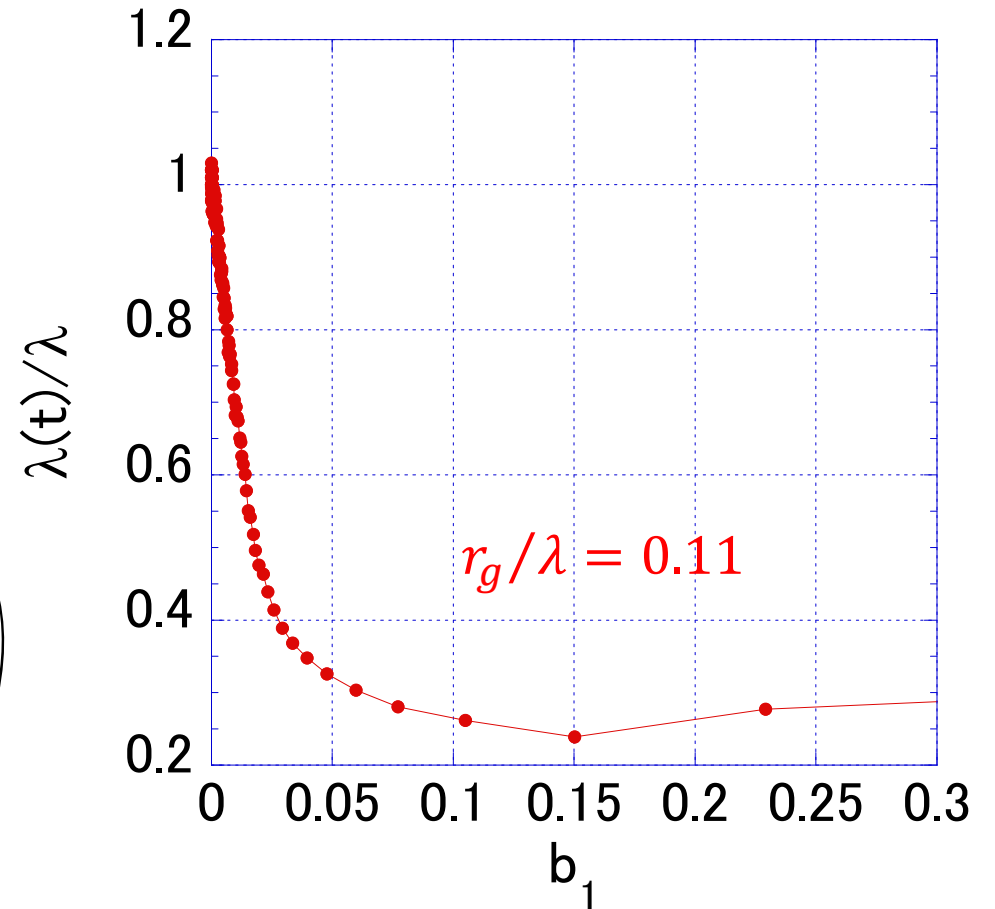
$$\lambda_{min} \approx 2r_g \text{ for any } r_g/\lambda$$

time history of current sheet & reconnecting flux  $\Phi$



minimum thickness =  
maximum growth rate

shrinkage of current sheet as function  $b_1$



# Nonlinear explosive magnetic reconnection

Master Equation (外部領域のエネルギー) = (散逸領域のエネルギー)

$$\frac{\pi}{k} \frac{(1 - k^2 \lambda^2)}{k \lambda^2} = \gamma \frac{\omega_p^2}{c^2} \frac{d_x}{v_{th}} d_x d_y$$

Linear Tearing Instability

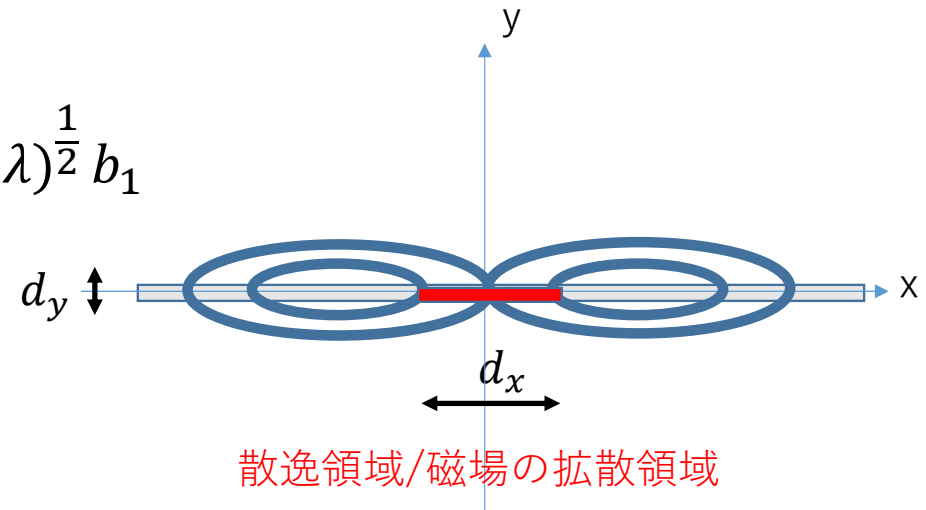
$$d_x = k^{-1}, \quad d_y = \sqrt{r_g \lambda} \quad \Rightarrow \quad \gamma \tau_A \approx (r_g / \lambda)^{\frac{3}{2}}$$

Galeev's Nonlinear Tearing Instability

$$d_x = \sqrt{r_g / (b_1 k)}, \quad d_y = \sqrt{r_g \lambda} \quad \Rightarrow \quad \gamma \tau_A \approx (r_g / \lambda)^{\frac{1}{2}} b_1$$

Nonlinear Explosive Tearing Instability

$$d_x = \sqrt{r_g / (b_1 k)}, \quad d_y = 2r_g \quad \Rightarrow \quad \gamma \tau_A \approx b_1$$

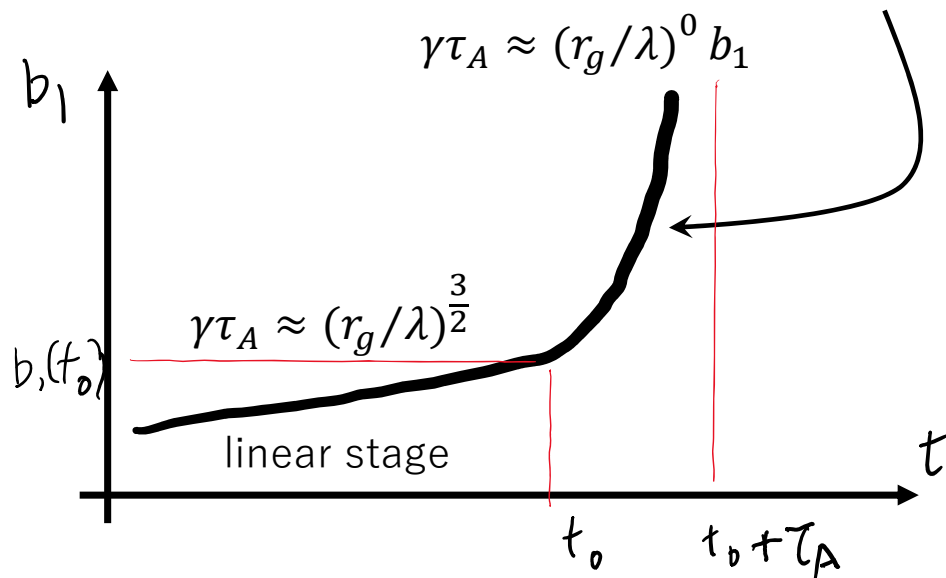


# Summary

## nonlinear explosive reconnection

$$\gamma\tau_A \approx b_1$$

$$b_1(t) = b_1(t_0) / \left(1 - (t - t_0) \frac{\pi b_1(t_0)}{k\lambda\tau_A}\right)$$



Onset of nonlinear explosive growth

$$b_1(t_0) = k r_g$$

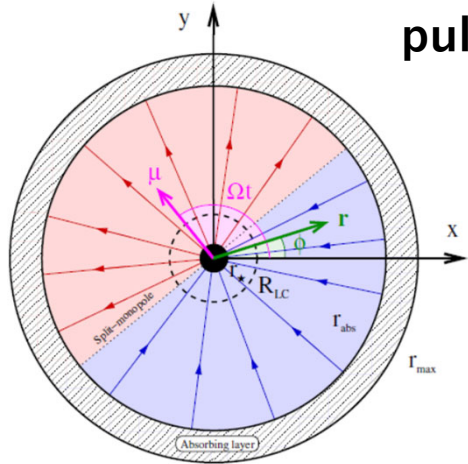
$$k\lambda = O(10^{-1})$$

$$\Rightarrow b_1(t_0) = O(10^{-1})(r_g/\lambda)$$

onset appears at very early stage for a thick current sheet with  $r_g \ll \lambda$

# Final Remark

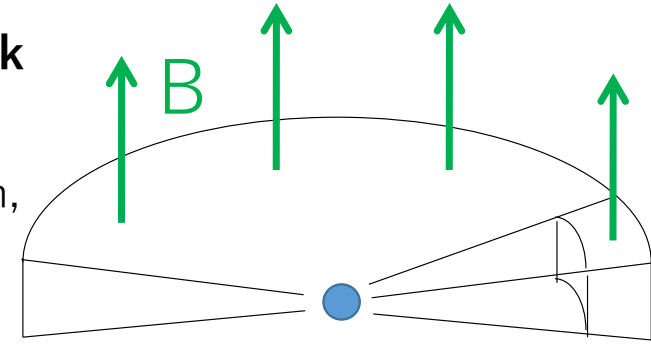
## pulsar wind evolution



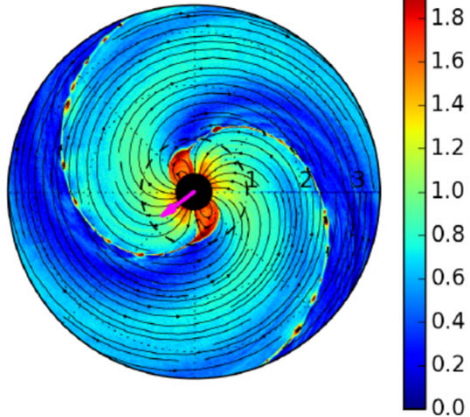
Split-monopole,  
2d & 3d-PIC simulation,  
 $\sigma$  problem by reconnection  
Cerutti+, 17,20

## MRI in accretion disk

2d & 3d-PIC simulation,  
particle acceleration  
by reconnection,  
Hoshino, 13, 15



Density



$B_\phi$

