一般相対論的磁気流体コードの開発

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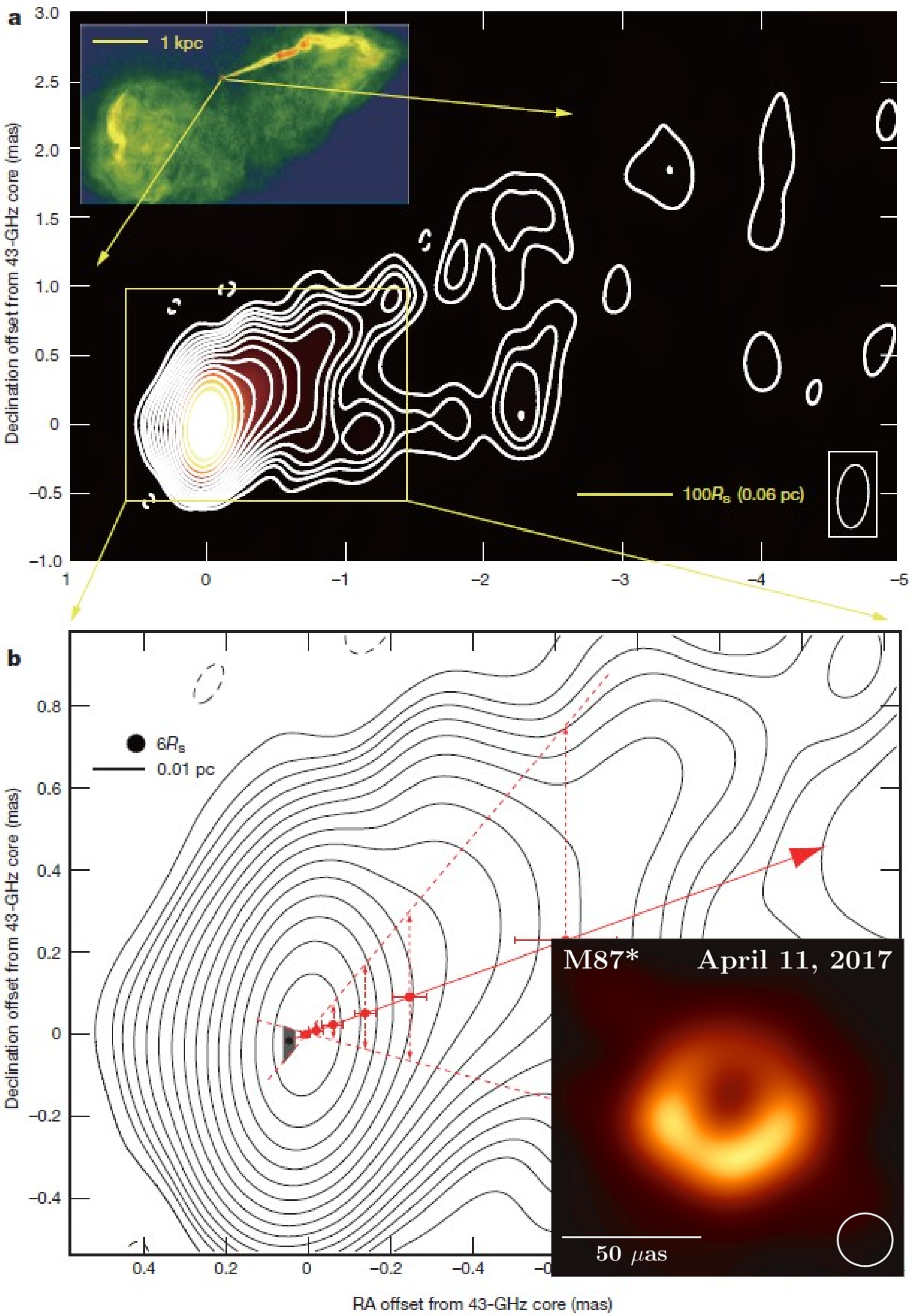
Active Galactic Nuclei Jet



Hilghlly collimated outflows from center of galaxy

- central engine

  supermassive black hole

  +

  accretion disk

Long term GRMHD simulations for the study of accretion flows onto BH and jet launching physics. Because The results for short duration includes memory of initial conditions, especially Initial toplogy of magnetic fileds.

We are depeloping a nre GRMHD code for robust and stable numerical calculations.

EHT(2019)
BH Shadow

M87 radio observation Hada +(2011)

Numerical scheme

- Finite volume method
- Time integration conservative form
- Numerical fluxes : approximate Riemann solver (HLL scheme)
  Gammie+2003, Anton+2006, Nagataki 2009, White+2016, Porth+2017
- Cell reconstruction (1^{st}, 2^{nd} or 3^{rd} accuracy : MUSCL)
- Time integration (2^{nd} or 3^{rd} order accuracy:
  TVD-Runge-Kutta)
- Flucx-CT: numerically to keep div B=0
- Primitive recovery: 1D or 2D NR method (Noble +2006)
  $[\rho, U, \tilde{u}^i, B^i]$ where $\tilde{u}^i = u^i + \alpha\gamma g^{ti}$

Basic Equations : ideal GRMHD Eqs.

GM=c=1, a: dimensionless Kerr spin parameter
G: Gravitational constant, M:Black hole mass

Units L : $Rg=GM/c^2$ (=Rs/2), T : $Rg/c=GM/c^3$
e.g. ~9.0×10^{14}cm($M_{BH}/6 \times 10^9 M_{sun}$) ~3000s ($M_{BH}/6 \times 10^9 M_{sun}$)

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\rho u^\mu) = 0$$ Mass conservation Eq.

$$\partial_\mu(\sqrt{-g}T^\mu_\nu) = \sqrt{-g}T^\kappa_\lambda \Gamma^\lambda{}_{\nu\kappa}$$ Energy-momentum conservation Eq.

$$\partial_t(\sqrt{-g}B^i) + \partial_j(\sqrt{-g}(b^i u^j - b^j u^i)) = 0$$ Induction Eq.

$$p = (\gamma - 1)\rho\epsilon$$ EOS (γ: constant specific heat ratio)

Constraint equations.

$$\frac{1}{\sqrt{-g}}\partial_i(\sqrt{-g}B^i) = 0$$ No-monopoles constraint

$$u_\mu b^\mu = 0$$ Ideal MHD condition

$$u_\mu u^\mu = -1$$ Normalization of 4-velocity

Energy-momentum tensor

$$T^{\mu\nu} = (\rho h + b^2)u^\mu u^\nu + (p_{\rm g} + p_{\rm mag})g^{\mu\nu} - b^\mu b^\nu$$

$$p_{\rm mag} = b^\mu b_\mu/2 = b^2/2$$

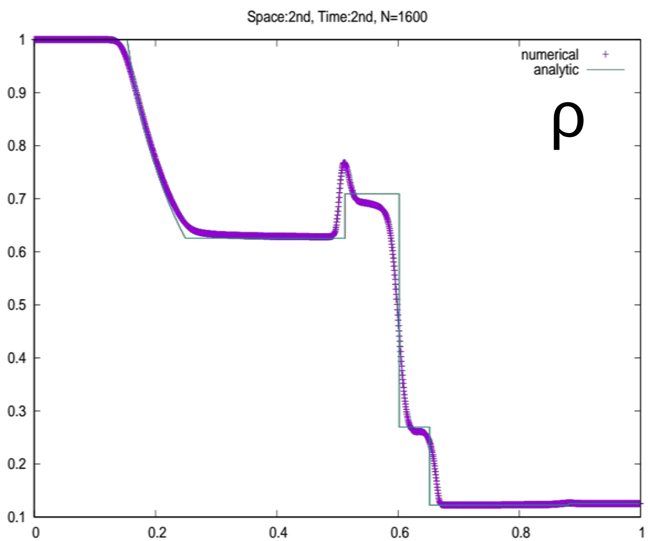
$$b^\mu \equiv \epsilon^{\mu\nu\kappa\lambda}u_\nu F_{\lambda\kappa}/2 \quad B^i = F^{*it}$$

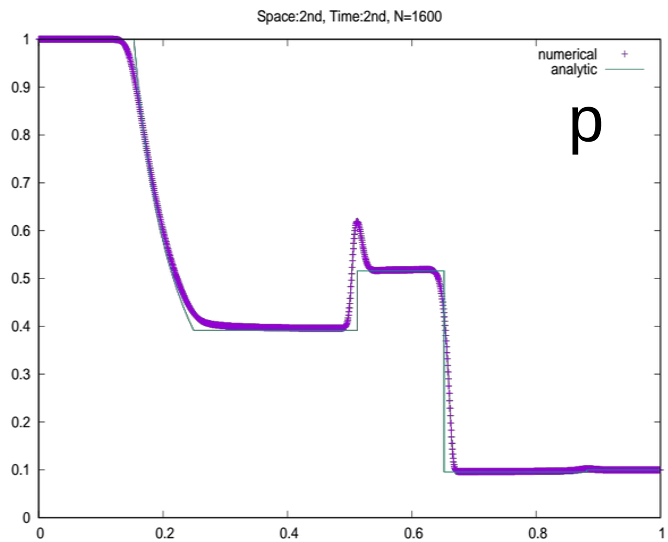
1D Shock tube test

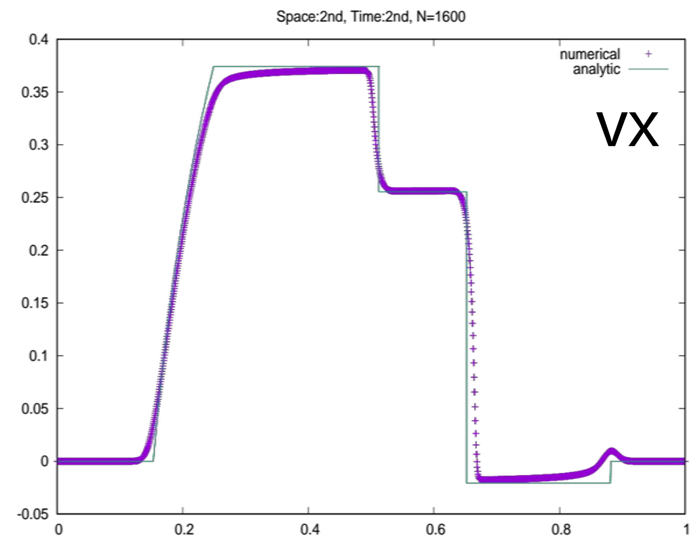
Problem	ρ	p_g	v^x	v^y	v^z	B^x	B^y	B^z
SHT01 $\gamma = 5/3$								
left state	10.0	13.3	0.0	0.0	0.0	0.0	0.0	0.0
left state	1.0	1×10^{-6}	0.0	0.0	0.0	0.0	0.0	0.0
SHT02 $(\Gamma = 2.0)$								
left state	1.0	1.0	0.0	0.0	0.0	0.5	1.0	0.0
right state	0.125	0.1	0.0	0.0	0.0	0.5	-1.0	0.0
SHT03 $(\Gamma = 5/3)$								
left state	1.0	30.0	0.0	0.0	0.0	5.0	6.0	6.0
right state	1.0	1.0	0.0	0.0	0.0	5.0	0.7	0.7
SHT04 $(\Gamma = 5/3)$								
left state	1.0	10^3	0.0	0.0	0.0	10.0	7.0	7.0
right state	1.0	0.1	0.0	0.0	0.0	10.0	0.7	0.7
SHT05 $(\Gamma = 5/3)$								
left state	1.0	0.1	0.999	0.0	0.0	10.0	7.0	7.0
right state	1.0	0.1	-0.999	0.0	0.0	10.0	7.0	7.0

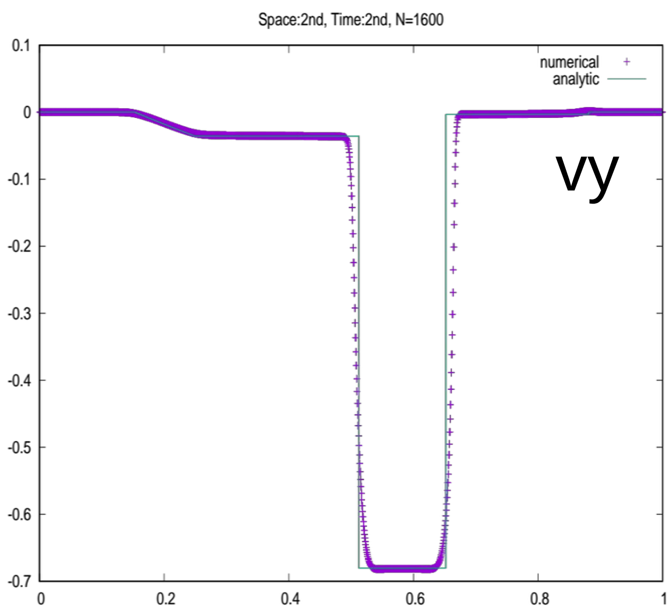
Analytic solution Giacomazzo & Rezzolla (2006)
Marti & Muller (2015)

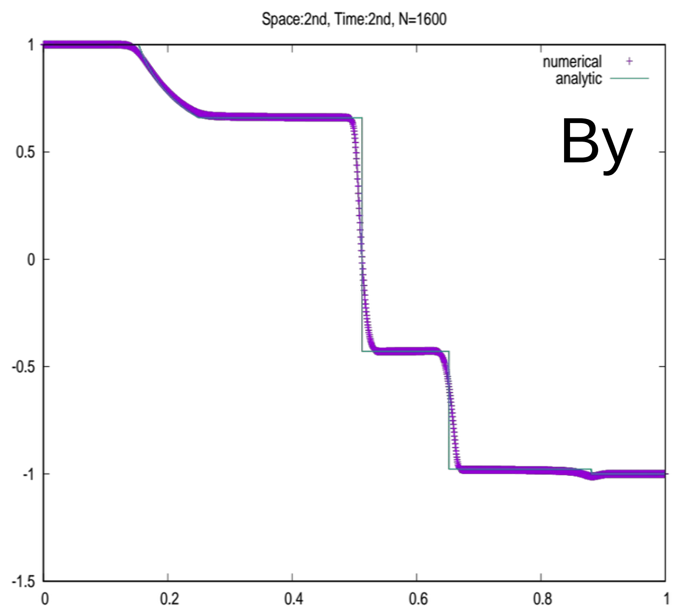
1D Shock tube test (cont.) SHT02



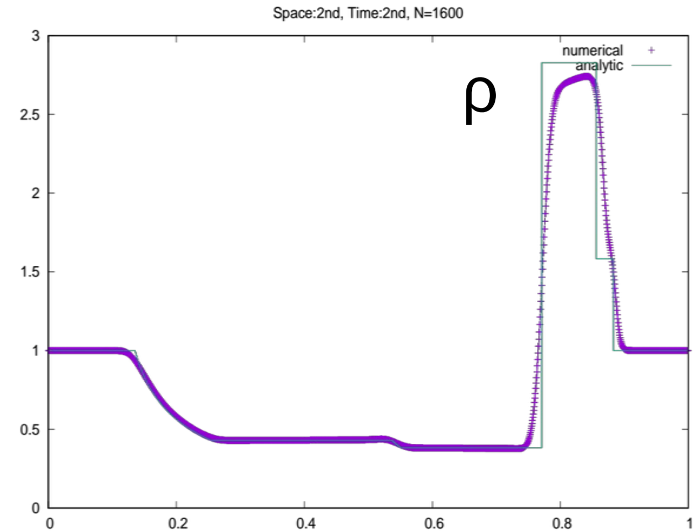


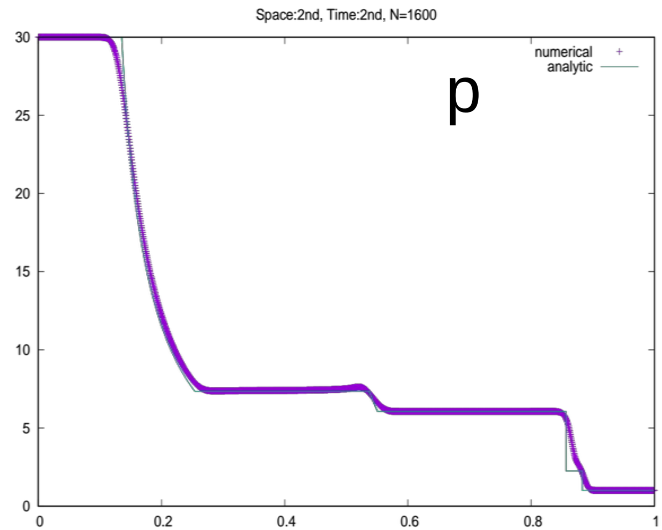


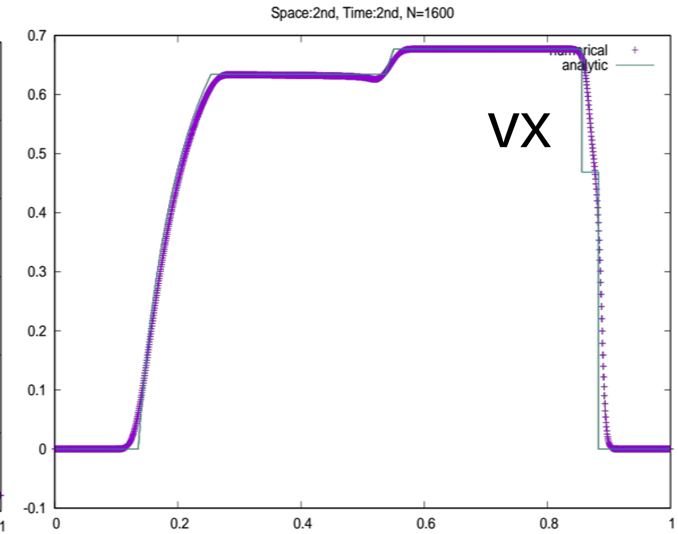


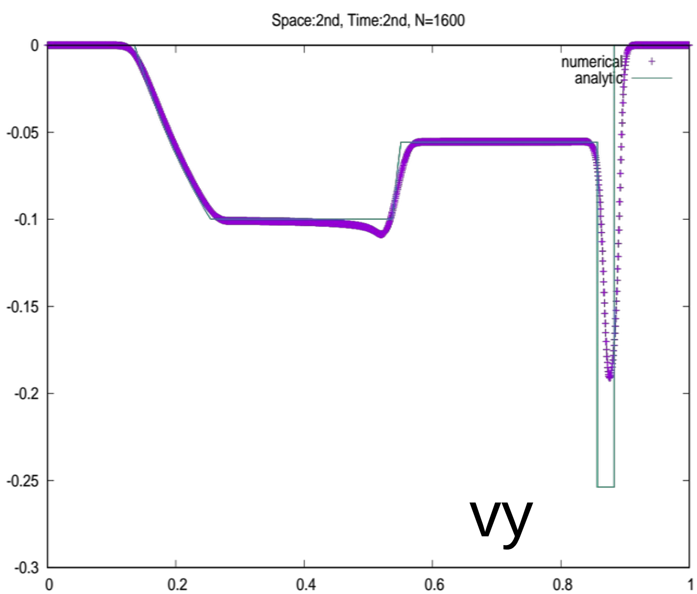


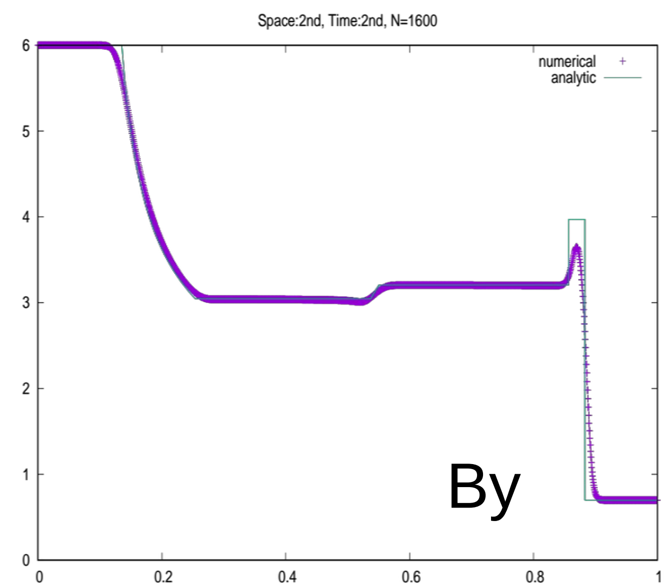
1D Shock tube test (cont.) SHT03



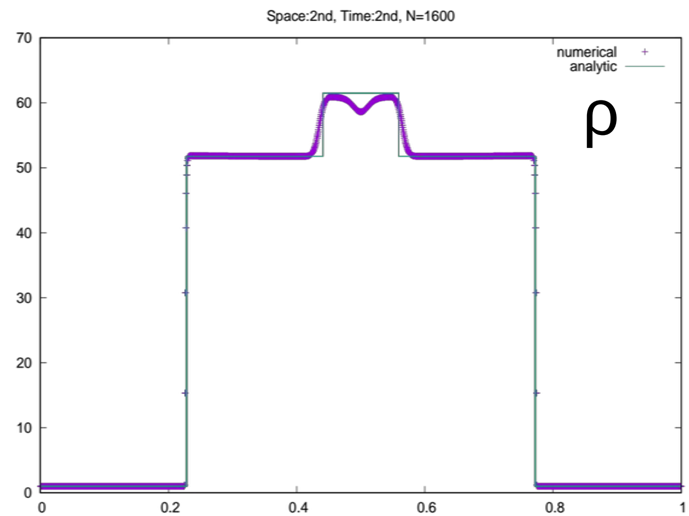


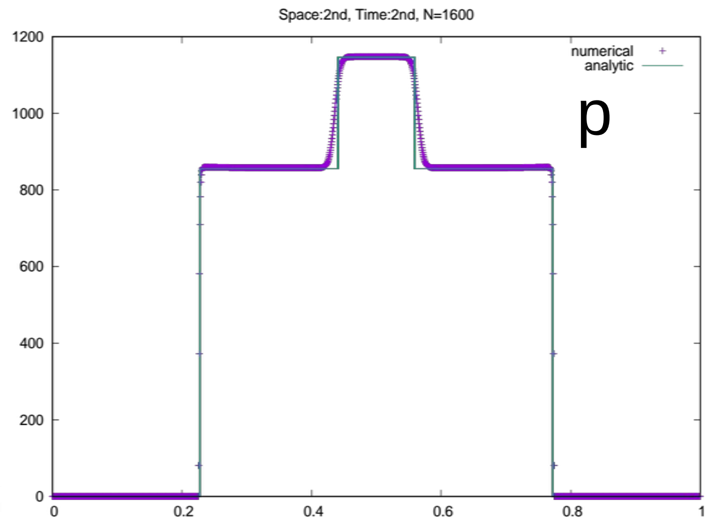


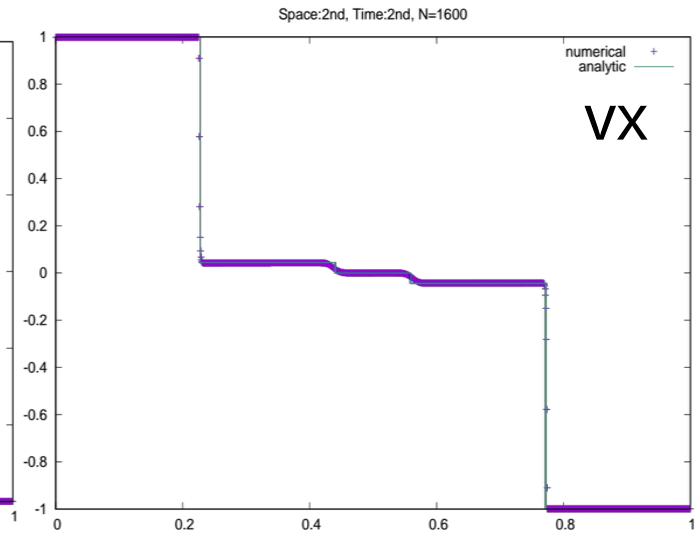


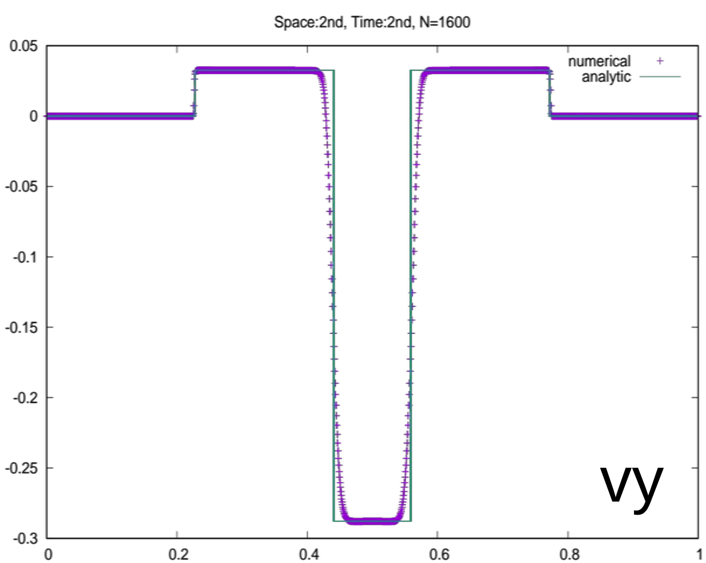


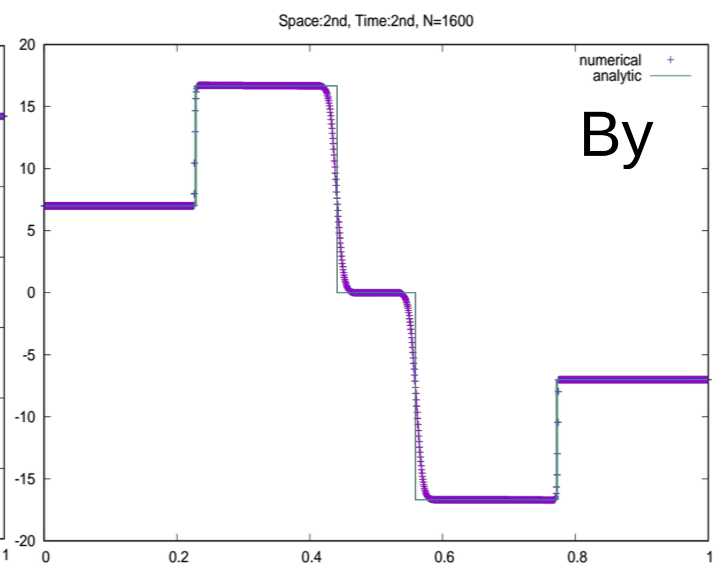
1D Shock tube test (cont.) SHT05





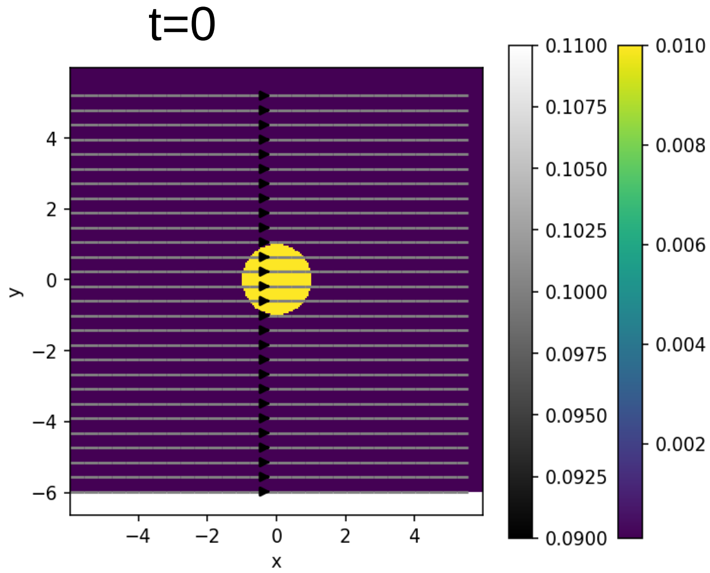


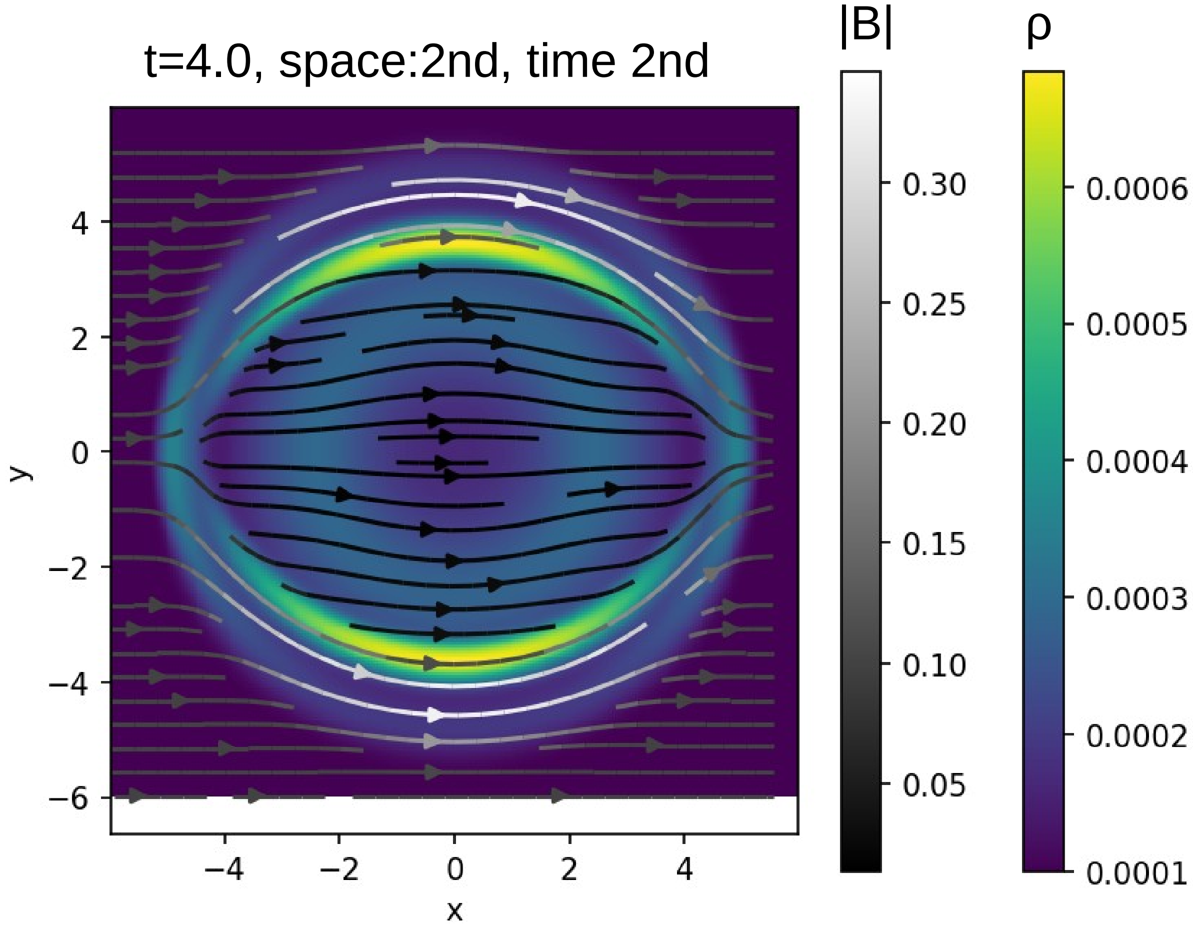




2D Cyrindrical Explosion test (SR test)

Domain: 6x6 (xy plane)
Grid # : [250x250]
t_end=4.0
γ=4/3
Bx=0.1
|r|<1
    ρ =0.01, p=1
|r|>1
    ρ=1.e-4, p=5.e-4

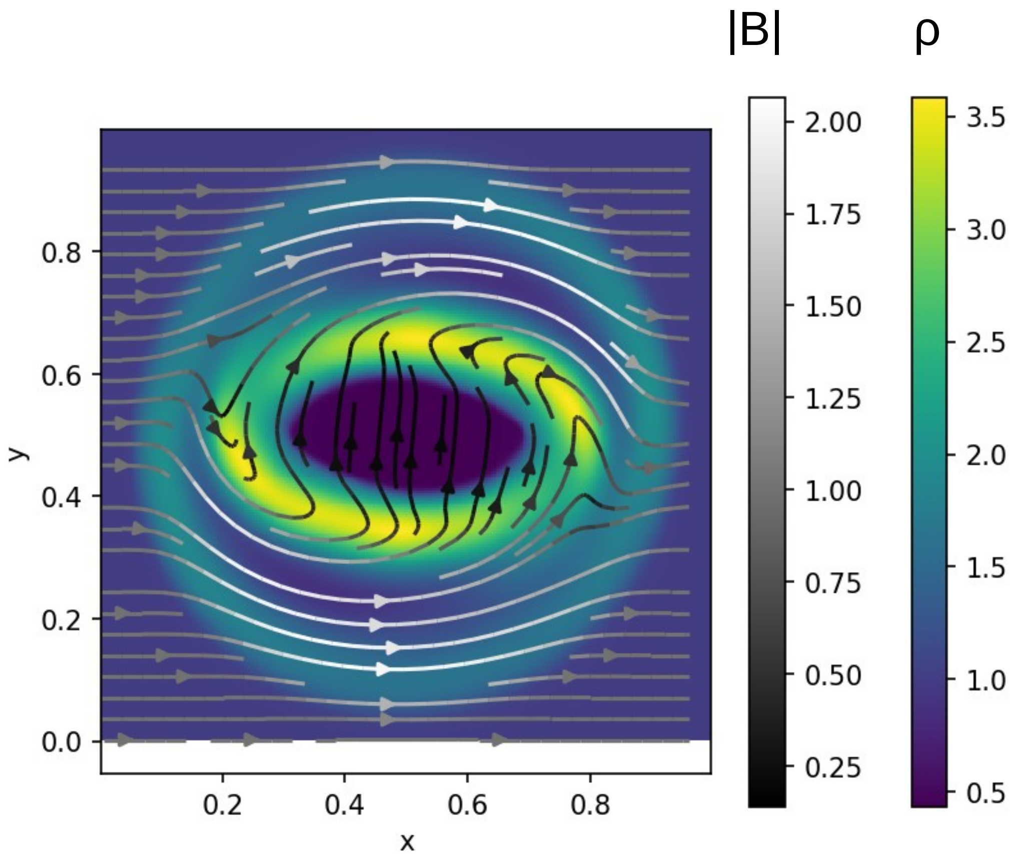


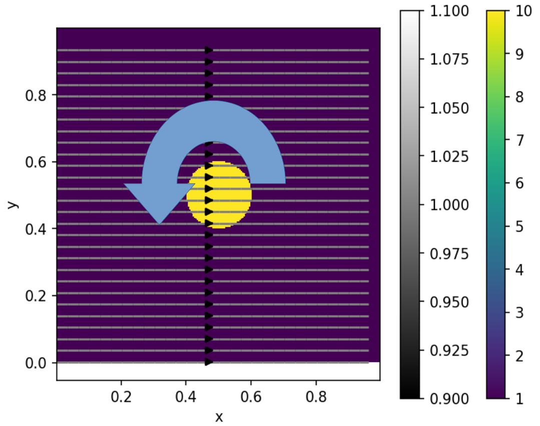


2D Magnetic rotator (SR test)

Domain: 1x1 (xy plane)
Grid # : [250x250]
t_end=0.4
γ=5/3
Bx=0.1, By=Bz=0

Circle r=0.5
inside
ρ =10, p=1
Rigid rotation ω=0.995
outside
ρ=1, p=1



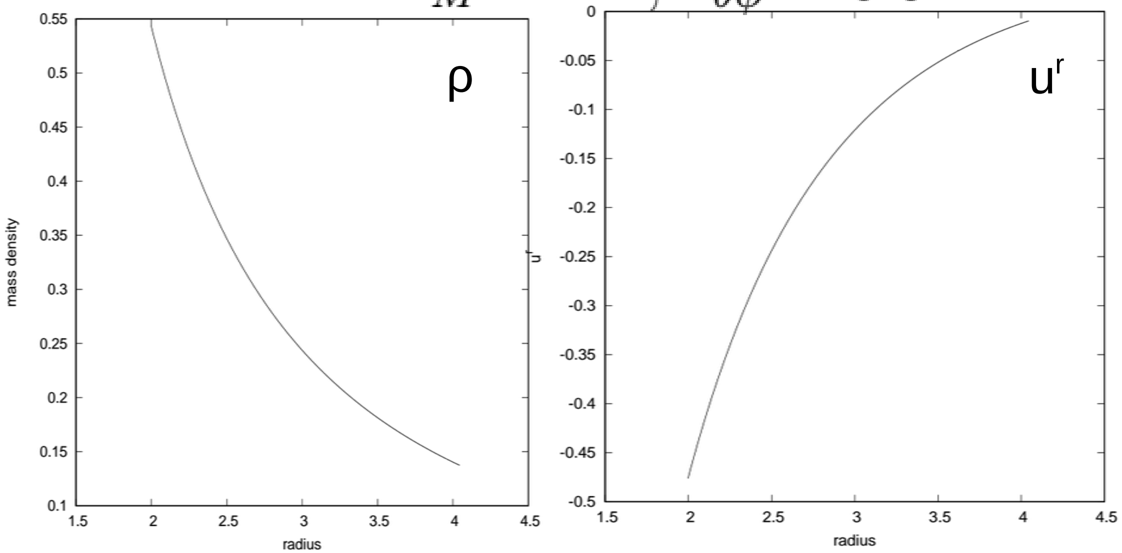


Gammie's Flow problem(1D GR test)

Gammie's Flow problem (Gammie 1999)

- special case of Takahashi+1990
- 1D accretion flow onto rotating BH
  solution is restricted @ equator (θ=π/2)
  and $r_H < r < r_{MSO}$
- assumptions
  1.steady state
  2.axis-symmetric
  3.ideal MHD,
  4.cold flow (very low pressure),
  5. $u^\theta = B^\theta = 0$

a=0.5 $F_M = -1, F_{\theta\phi} = 0.5$



This system has 5 conservative quantities.

$$F_\mathrm{M} = 2\pi r^2 \rho(r) u^r(r)$$

$$F_\mathrm{L} = 2\pi r^2 \left(\rho(r) u^r(r) u_\phi(r) - \frac{g^{rr} g^{\theta\theta}}{4\pi} F_{r\theta}(r) F_{\theta\phi} \right)$$

$$F_\mathrm{E} = -2\pi r^2 \left(\rho(r) u^r(r) u_t(r) + \frac{g^{rr} g^{\theta\theta}}{4\pi} F_{r\theta}(r) F_{t\theta} \right)$$

$$F_{\theta\phi}(r) = \mathrm{const}$$

$$F_{t\theta}(r) = \mathrm{const}$$

\+ physical condition@critical point (fast sonic point)

$$\partial_r F_E(r_\mathrm{crit}, u^r_\mathrm{crit}; F_L) = 0,$$

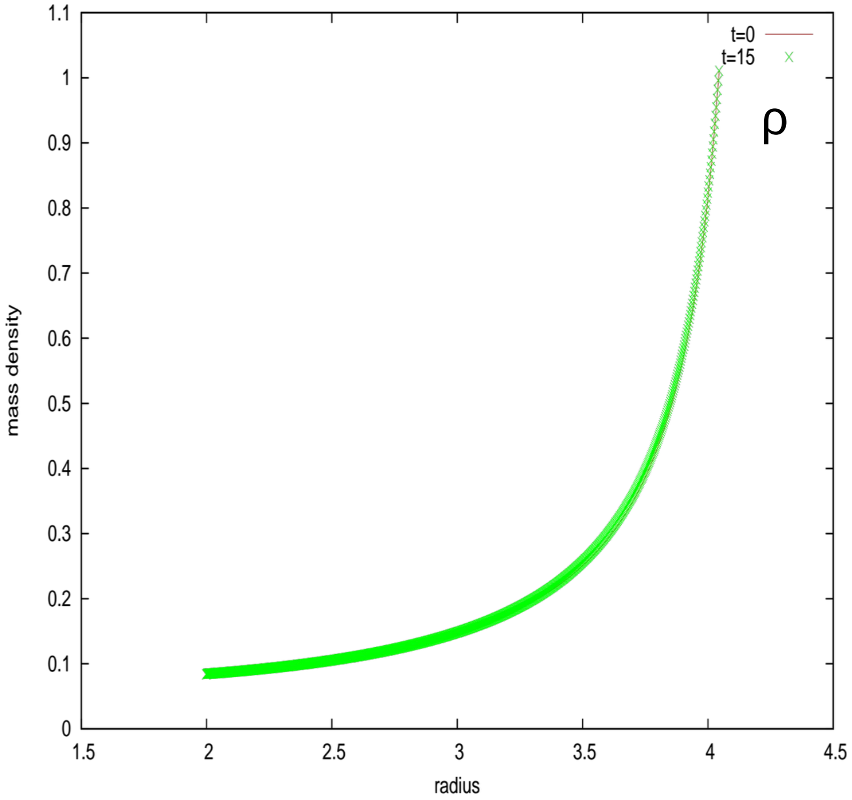
$$\partial_{u^r} F_E(r_\mathrm{crit}, u^r_\mathrm{crit}; F_L) = 0.$$

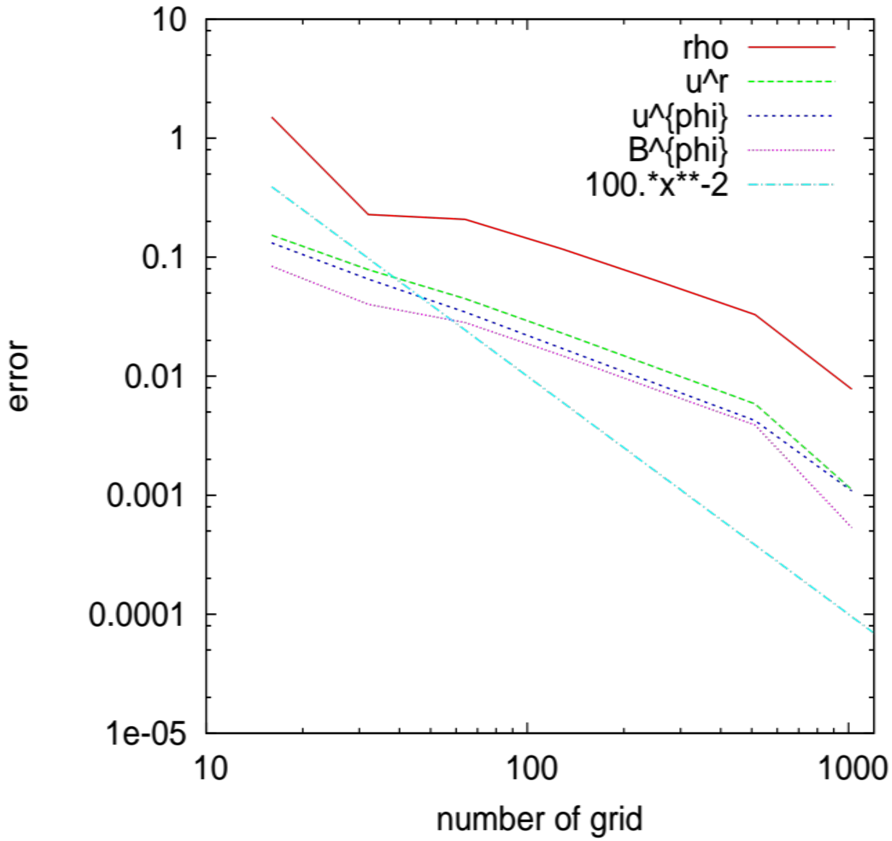
\+ boundary condition@mso

$$F_E(r_\mathrm{crit}, u^r_\mathrm{crit}; F_L) = F_E(r_\mathrm{mso}, u^r = 0; F_L),$$

→ two free parameters

Gammie's Flow problem(1D GR test), cont.





Start with steady state initial condition.
Run until t=15, then see the difference between initial condition and results.

Errors are evaluated by

$$\Sigma_{i=1}^{i=N} |a(\text{final}) - a(\text{initial})| / \Sigma_{i=1}^{i=N} |a(\text{initial})|$$

Numerical errors are still large compared with other group's results
Gammie+2004, Hawley+2004, Anton+2006...

Summary

Development of new GRMHD code is ongoing
The code passed SR tests
    (1D shock tube, 2D Cyrindrical explosion test, 2D magnetic rotator)
GR code test is ongoing
    1D Gammie flow problem (still some problems remain)
    2D Fishbone Moncrief torus (hydrostatic torus)
        + magnetic filed

We will include another technics
    HLLC flux (resolve 3 waves)
    PPM, MP5 reconstructions