相対論的衝撃波の Particle-in-Cellシミュレーション

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宇宙線研小研究会「高エネルギー天体現象の多様性」

内容

1. 数値チェレンコフ不安定の抑制
   Ikeya & Matsumoto, PASJ, 2015

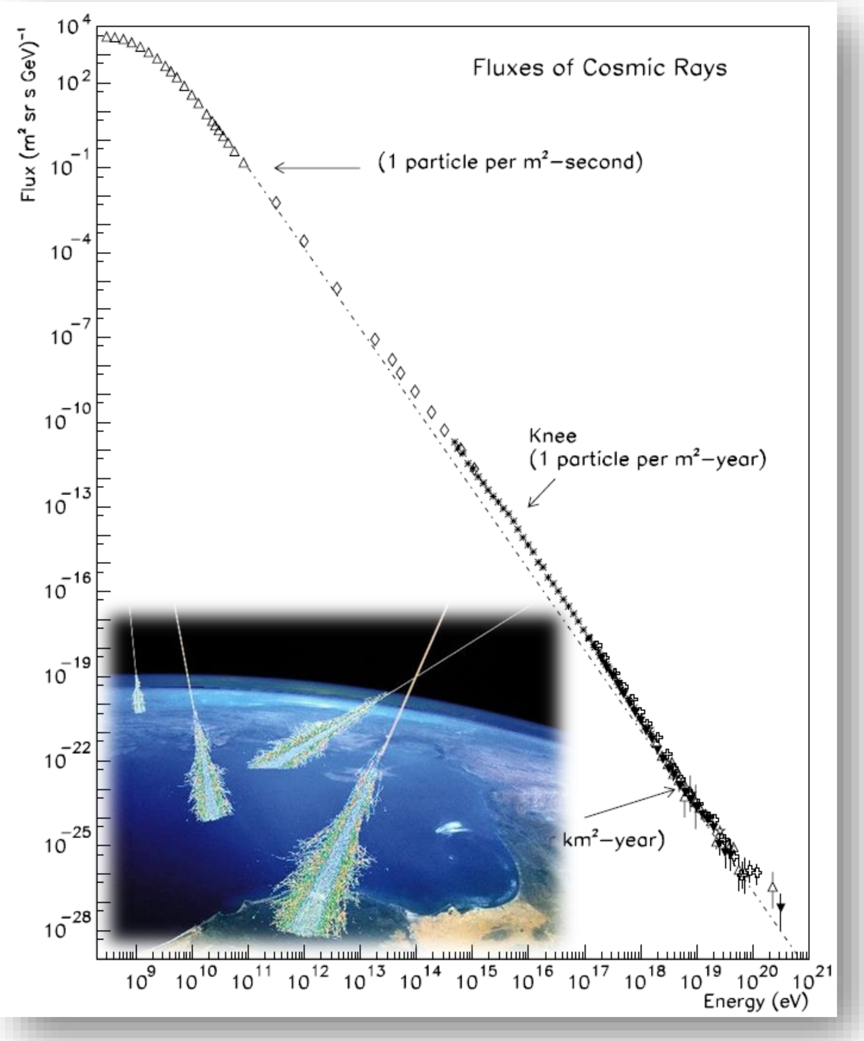
2. 相対論的衝撃波における大振幅先駆波励起と電子加速
   Iwamoto et al., ApJ, 2017; 2018

3. イオンワイベル不安定による磁場生成／飽和過程
   Takamoto, Matsumoto, Kato, ApJL, 2018

宇宙線の起源

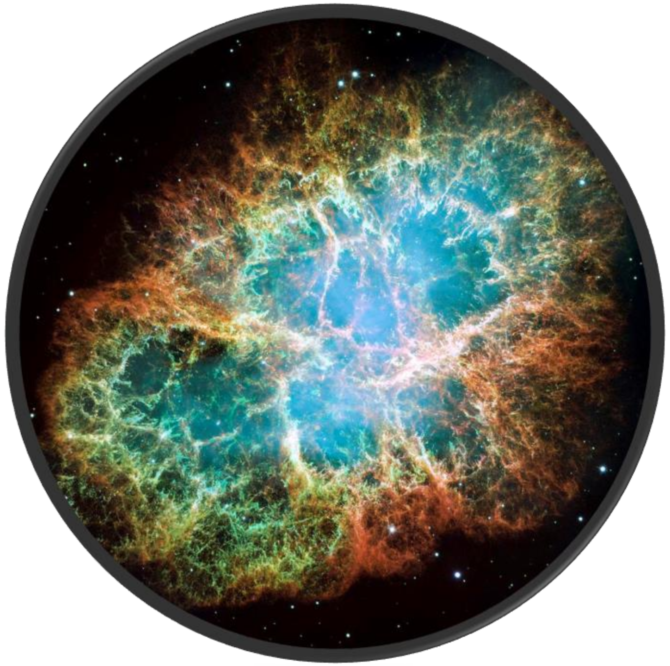
地球に届く宇宙線のフラックス

宇宙の爆発的現象に伴う**衝撃波**での
荷電粒子（陽子・電子）の加速メカニズムを探求



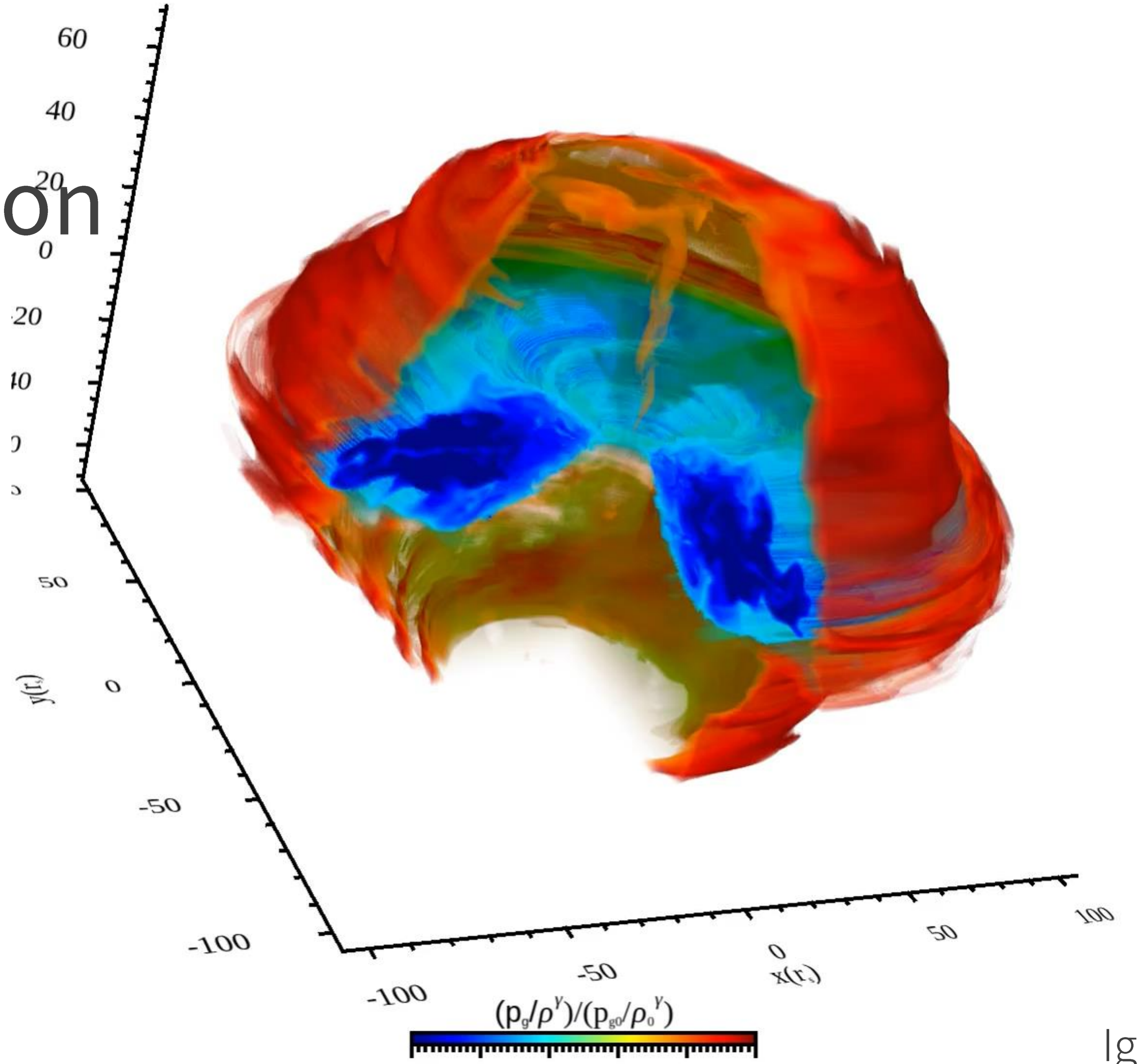
Swordy 01

超新星爆発
(銀河宇宙線)



宇宙ジェット
(超高エネルギー宇宙線)

CANS+R (MHD + radiation transfer)



Igarashi+, in prep.

Particle-in-Cell simulation

Vlasov eq. as particle motions

$$\frac{d\boldsymbol{x}_p}{dt} = \frac{\boldsymbol{u}_p}{\gamma_p}$$

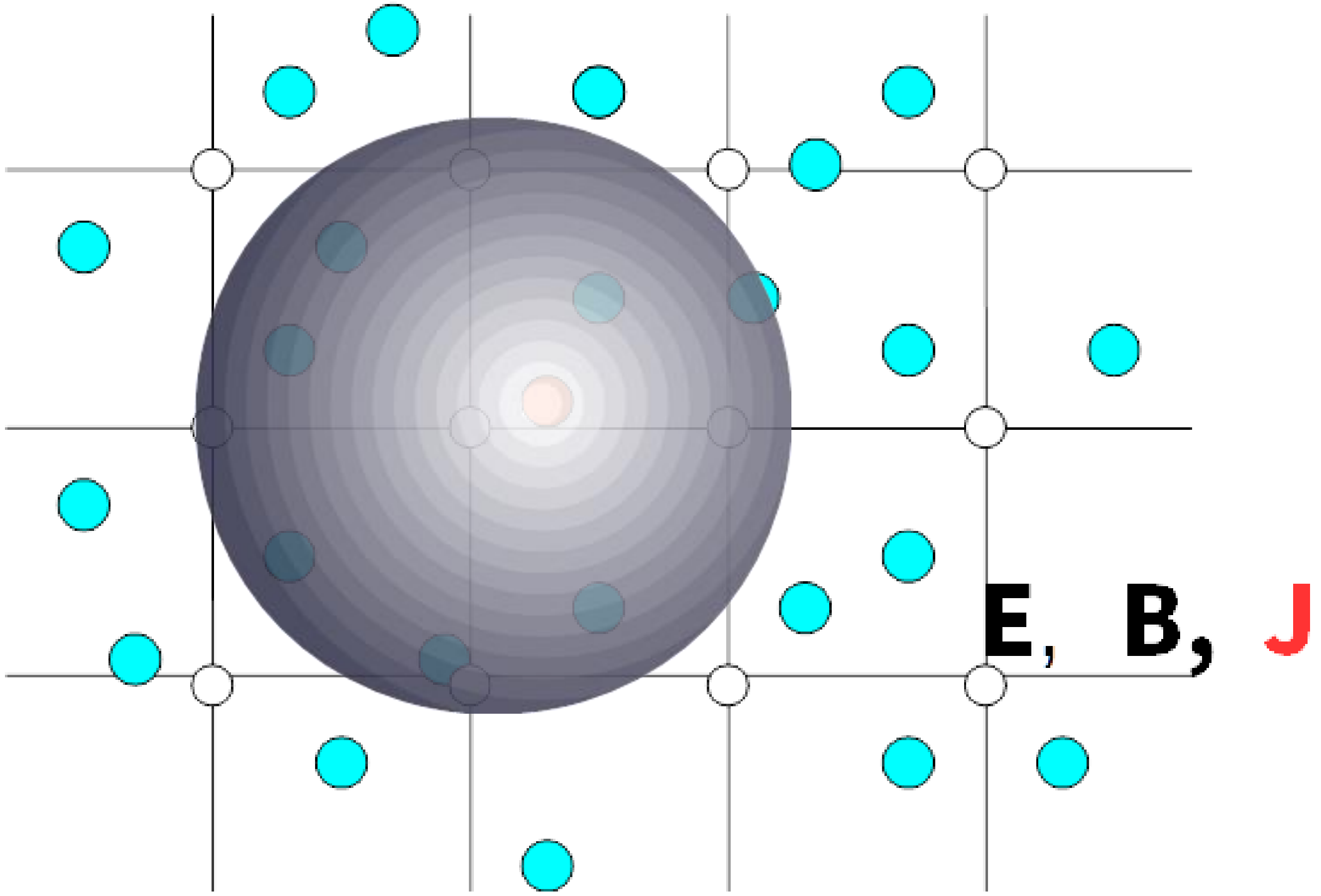
$$\frac{d\boldsymbol{u}_p}{dt} = \frac{q}{m}\left(\boldsymbol{E} + \frac{\boldsymbol{u}_p}{c\,\gamma_p} \times \boldsymbol{B}\right)$$

$$\boldsymbol{J} = \sum_p q_p \frac{\boldsymbol{u}_p}{\gamma_p}$$

Maxwell eqs. on grid points

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\,\nabla \times \boldsymbol{E}$$

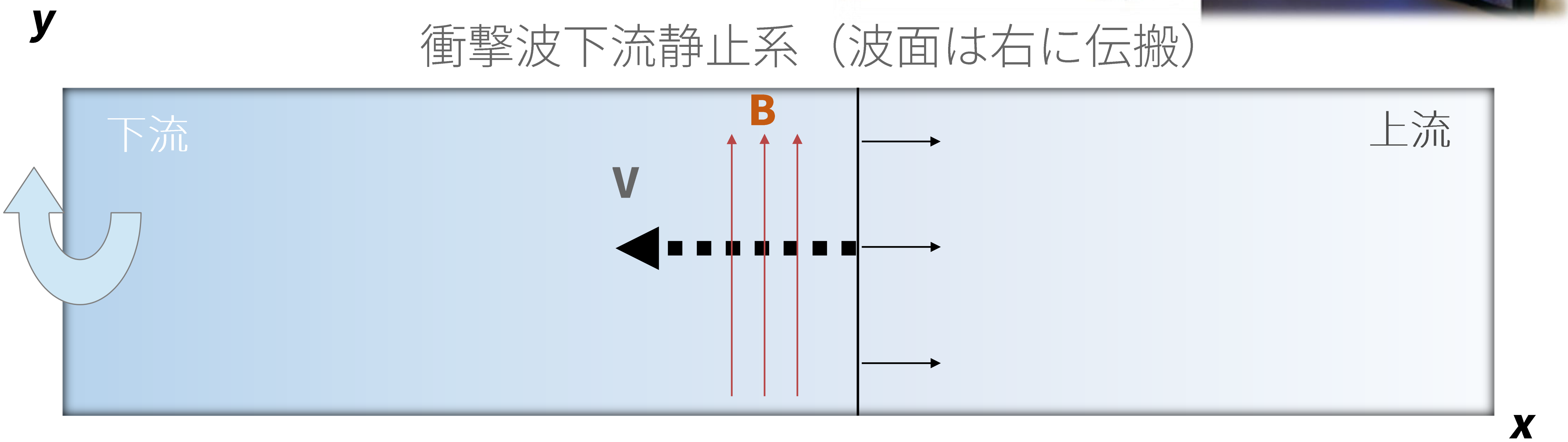
$$\frac{\partial \boldsymbol{E}}{\partial t} = c\,\nabla \times \boldsymbol{B} - 4\pi \boldsymbol{J}$$



スパコンで衝撃波を作る

- 自家製 2D/3D particle-in-cell コード
- SIMD最適化
- MPI+OpenMP ハイブリッド並列
- 10^5 cores までスケール

衝撃波下流静止系（波面は右に伝搬）



PICの世界動向 (TRISTAN-PIC 対 その他)

$\sigma = M_A^{-2}$

1

弱い衝撃波

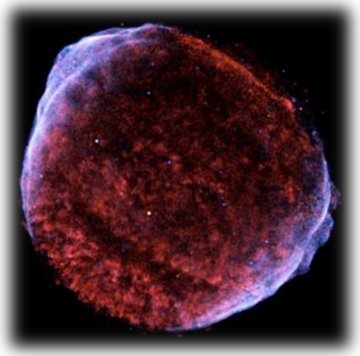
強い衝撃波

銀河団衝撃波

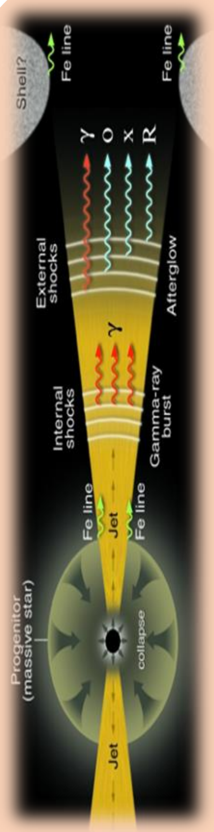
Weeren+ 2017

Matsukiyo+ 2011
Riquelme & Spitkovsky 2011
Guo+ 2014a,b
Matsukiyo & YM 2015
(太陽系衝撃波を入れると
もっとたくさん)

SNR衝撃波



Kato+ 2010; 2015
YM+ 2012;2013;2015;2017
Niemiec+ 2012; 2013
Park+ 2015
Wieland+ 2016
Bohdan+2017



Meszaros 2001

相対論的衝撃波

Kato 2007
Spitkovsky 2008
Hoshino 2008
Kechet+ 2009
Sironi+ 2009; 2011; 2013
Ikeya & YM 2015
Ardaneh+ 2016
Tomita & Ohira 2016
Iwamoto+ 2017;2018

0

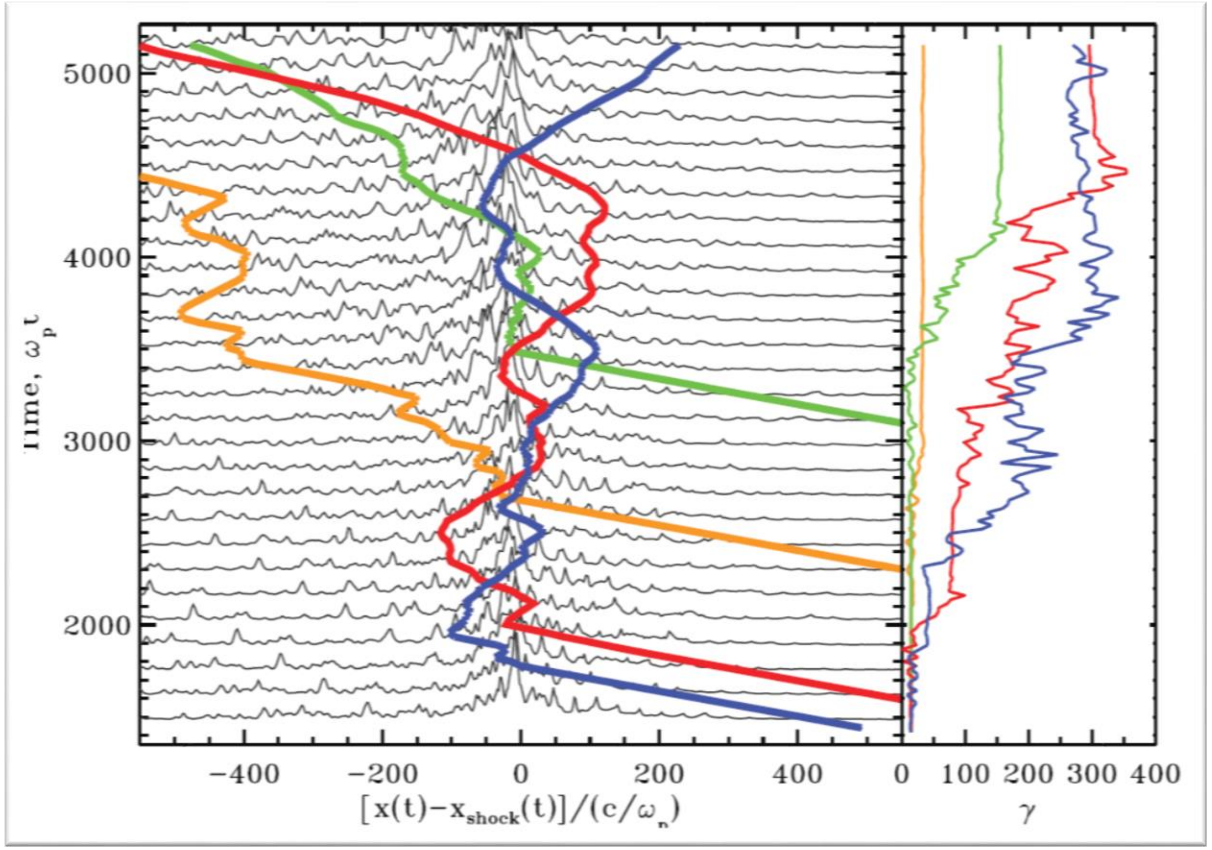
非相対論的 ← - - - - - → 相対論的

1

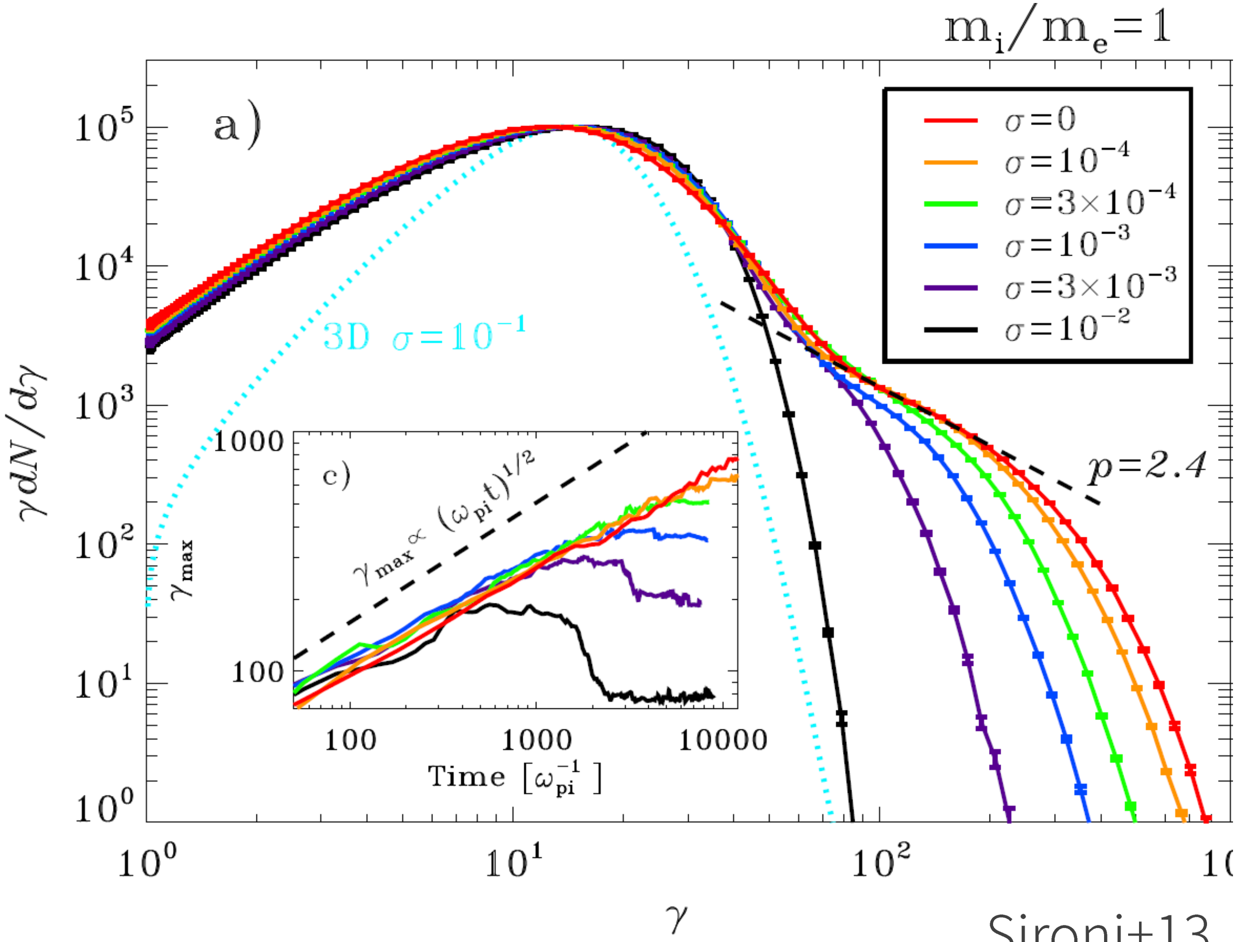
V_{shock}/c

相対論的衝撃波のPICシミュレーションと粒子加速

Weibel-mediated relativistic shock &
Fermi acceleration



Kato07; Spitkovsky08



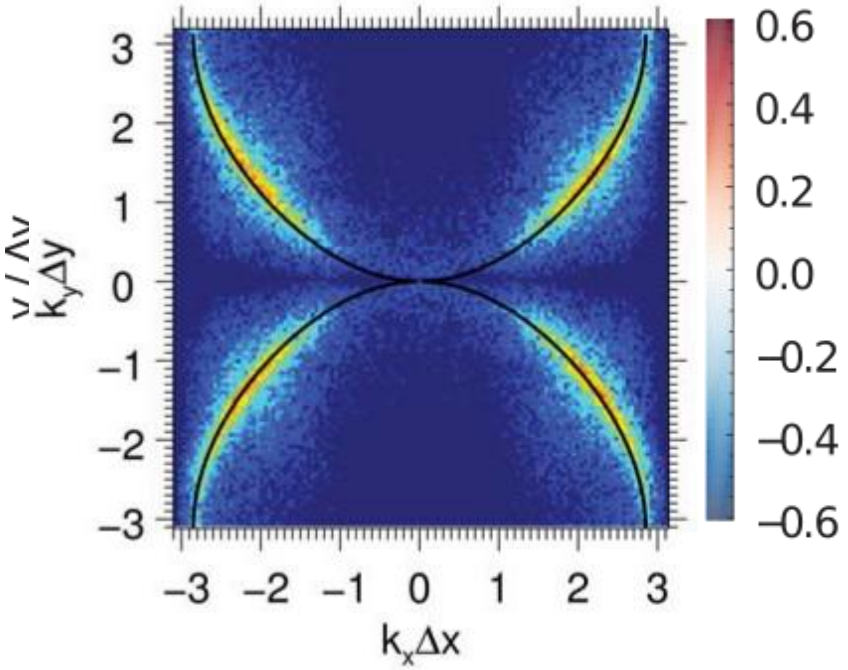
Sironi+13

数値チェレンコフ不安定の抑制

数値チェレンコフ不安定 (Godfrey74 JCP)

- ✓ 周期境界系
- ✓ X方向に相対論的流れ

（$\gamma = 100$）



離散化した空間での光モードの分散関係

$$\sin^2\left(\frac{\omega\Delta t}{2}\right) = \frac{c^2\Delta x^2}{\Delta t^2}\sin^2\left(\frac{k_x\Delta x}{2}\right) + \frac{c^2\Delta y^2}{\Delta t^2}\sin^2\left(\frac{k_y\Delta y}{2}\right)$$

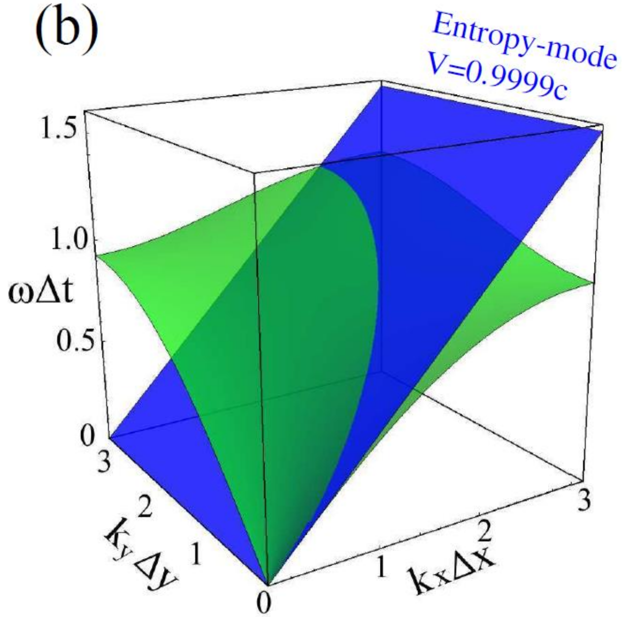
相対論的速度のビームとの共鳴

$$\omega = kV \ (V \sim c)$$

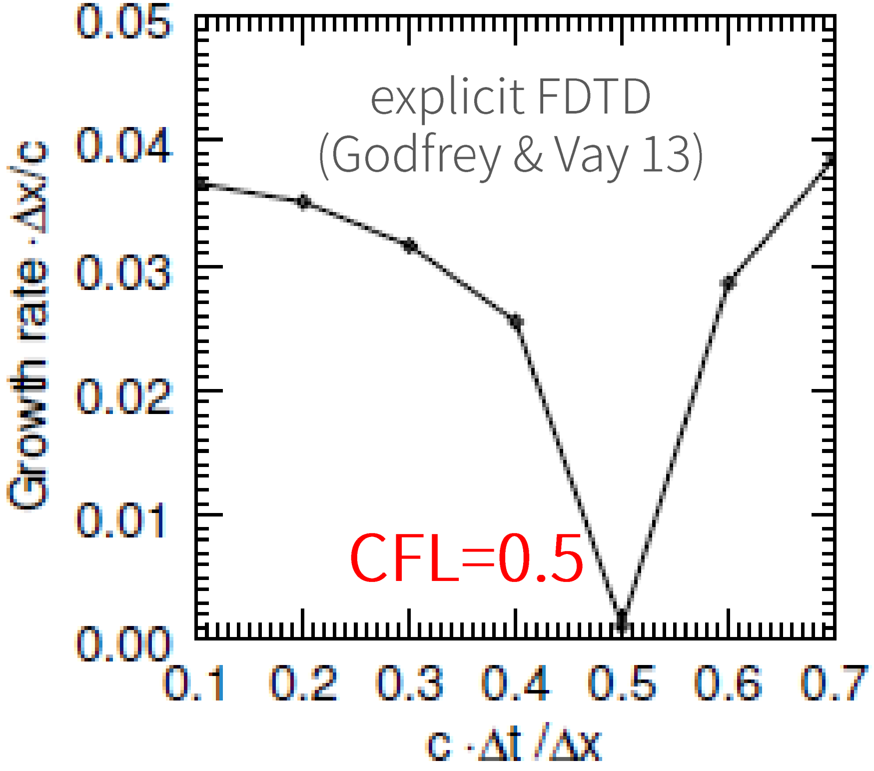
なので離散化された世界では物理的に正しい（エネルギー保存はされる）

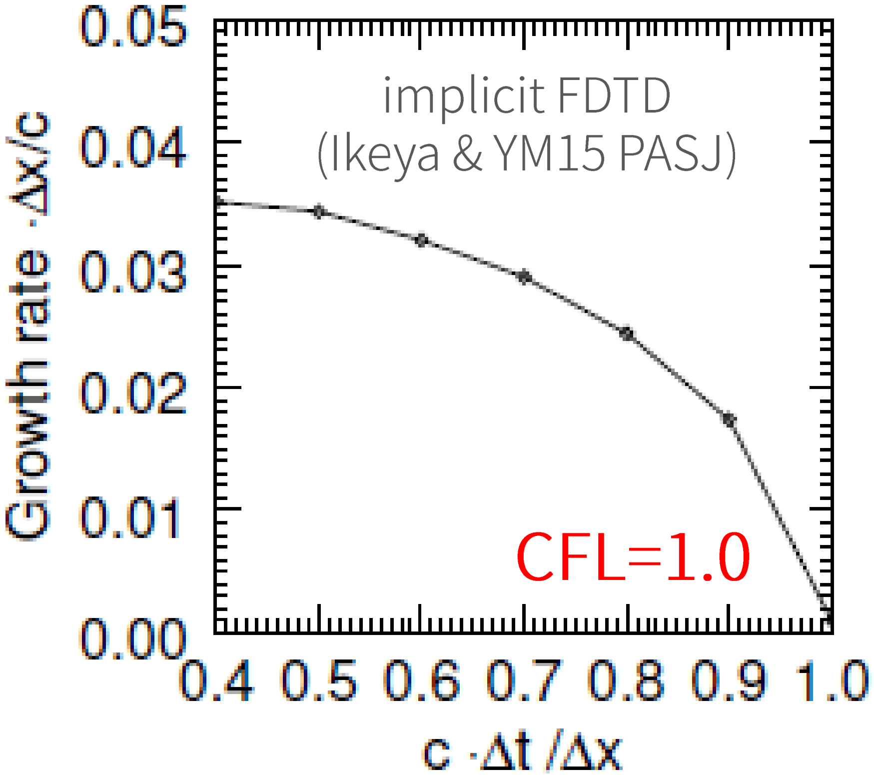
<u>対処方法</u>

- □ スペクトル法で解く (Kato07; Vay+ 13)
  - ✓ フーリエ空間でマクスウェル方程式を解く
  - ✓ 波数空間でNCIを除去
  - ✓ 大規模並列計算で効率が出ない
- □ FDTD (差分法)
  - ✓ J, E, B に対してフィルタリング (Greenwood+ 04; Sironi & Spitkovsky 09)
  - ✓ 物理的な電磁波も同時に落としてしまう



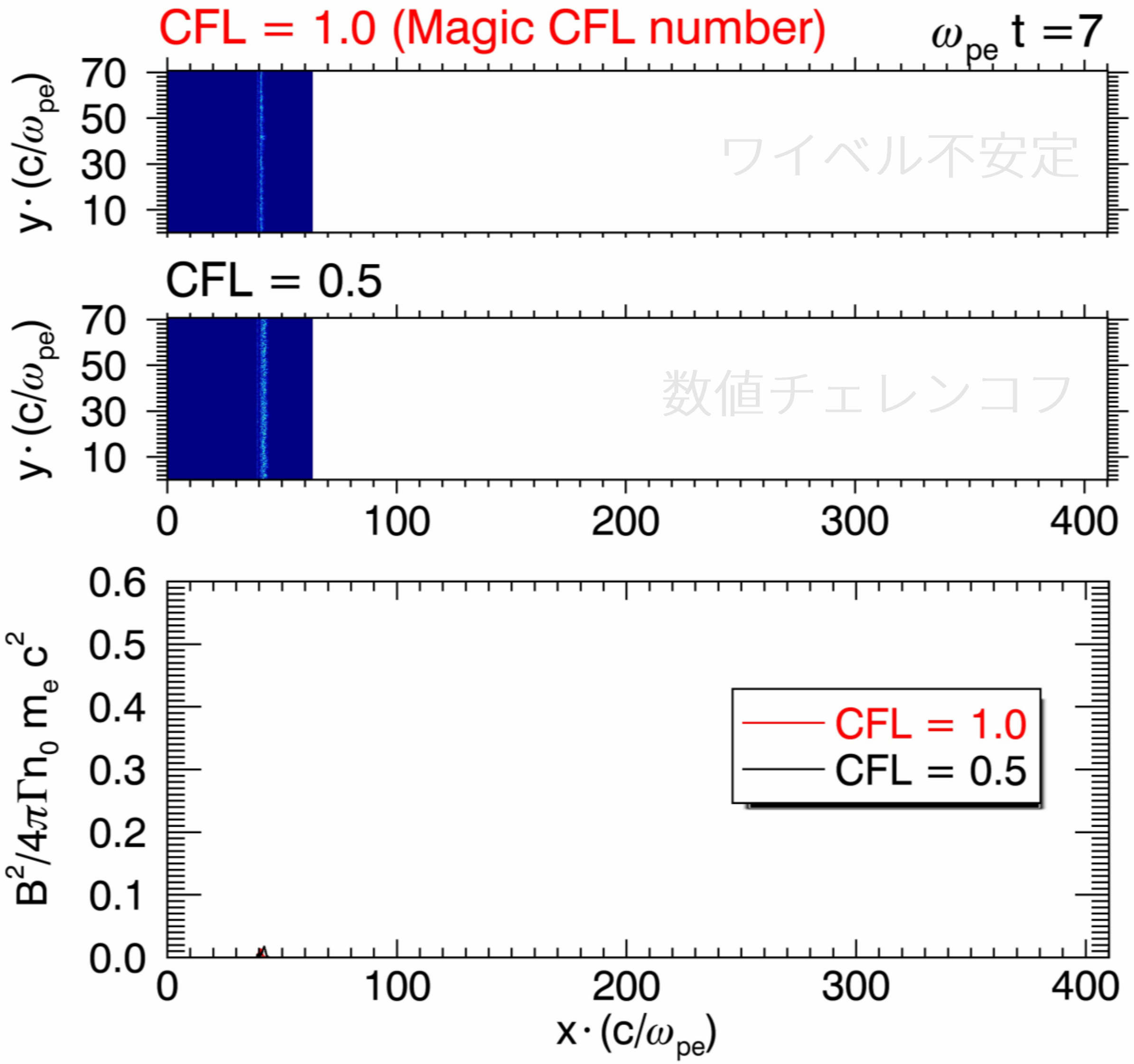
マジカルCFL数の発見 (Vay+11 JCP)





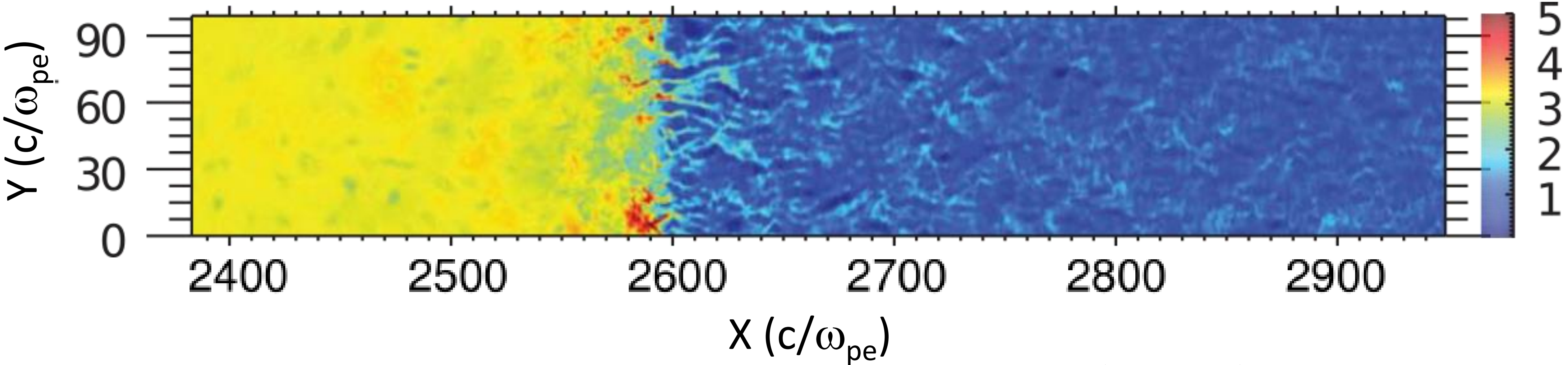
- NCIの成長率が特定のCFL数 ($c\Delta t/\Delta x$)で急激に減少することが発見 (Vay+ 11)
- PICアルゴリズムに強く依存
  - ✓ Esirkepovの電流計算法 (Esirkepov 00)
  - ✓ 電磁場に対して陽解法 (Godfrey & Vay 13) & 陰解法 (Ikeya & YM 15)
  - ✓ 粒子に対する電磁場の補間方法にも依存

マジカルCFL数を用いた陽電子・電子相対論衝撃波計算 (上流ローレンツ因子=100)



Ikeya & YM 15 PASJ

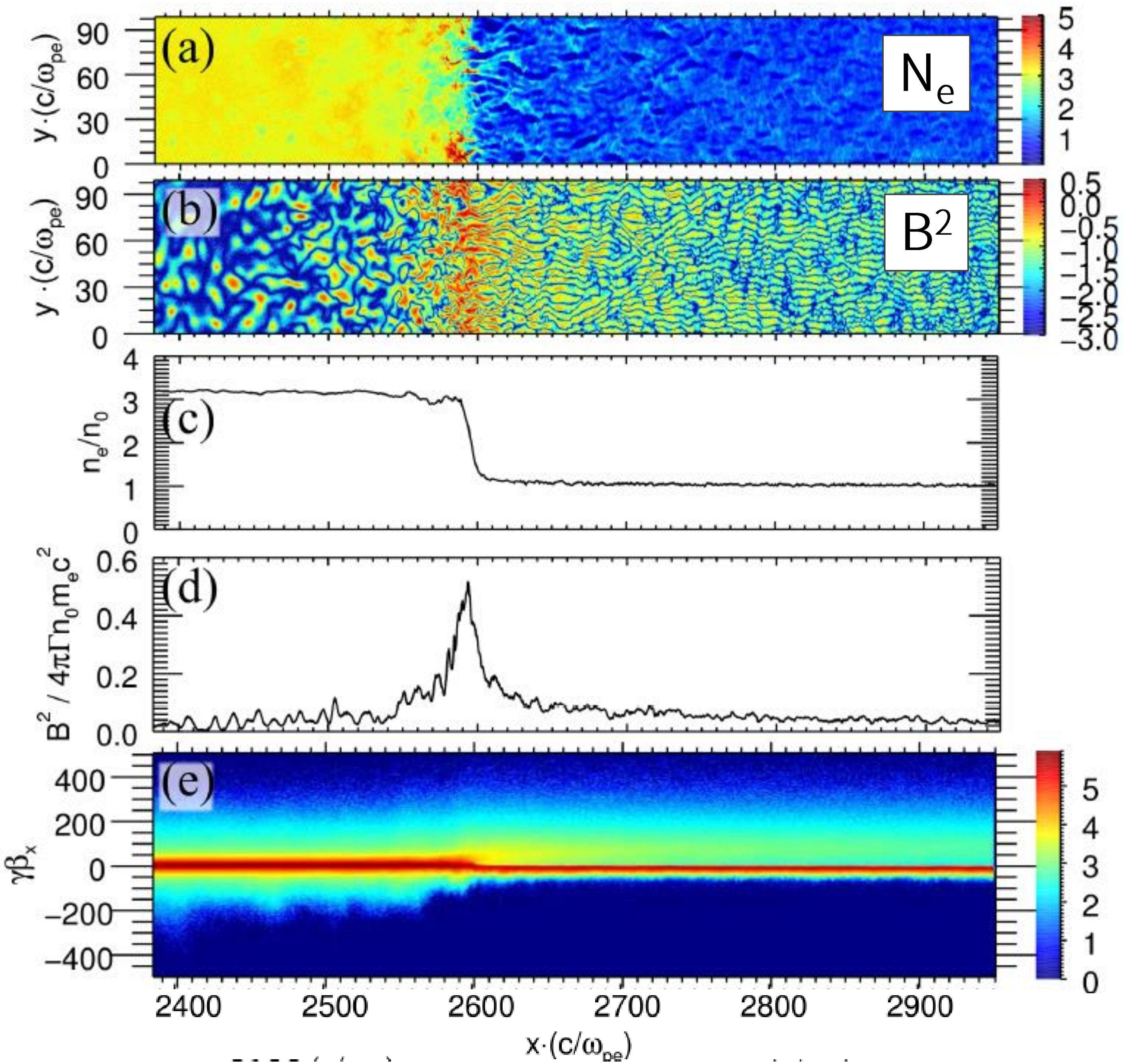
Long-term 2D PIC simulations



- ☐ Un-magnetized Weibel-mediated shocks in pair (e^+ & e^-) plasmas
- ☐ Magic CFL number method vs. previous results (smoothing current density, *cf.* Sironi+13)
- ☐ Long-term simulations $\omega_{pe}t \sim 6000$ to observe particle accelerations
- ☐ $\Gamma = 15$, N_0=8, 16, 25, 50 /cell in the upstream region
- ☐ $N_x \times N_y = 80{,}000 \times 1{,}400$
- ☐ 648 cores & 3.5TB memory on Cray XC30 at CfCA, NAOJ

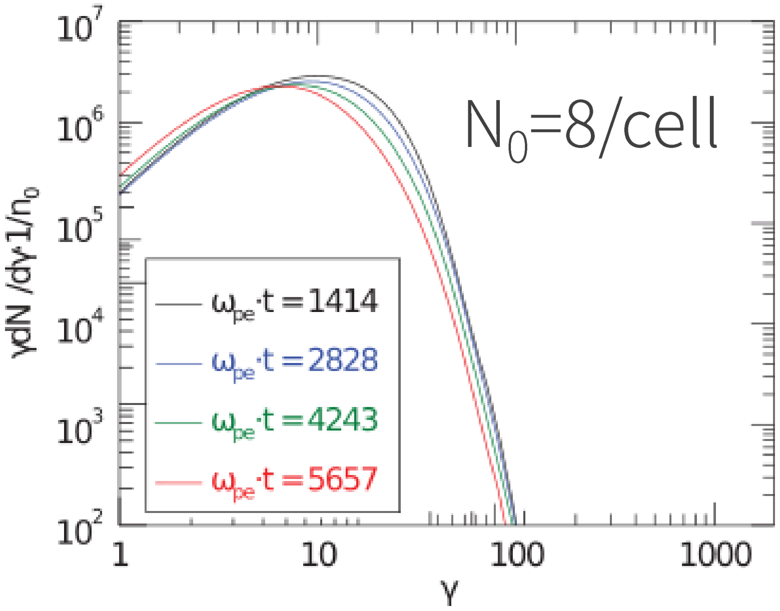
2D shock structure (N_0=50 /cell)

- Basically followed Sironi+ 13
- Weibel filaments in far upstream regions
- ~10 times stronger magnetic field turbulence
- Present method can maintain magnetic fields in larger amplitudes

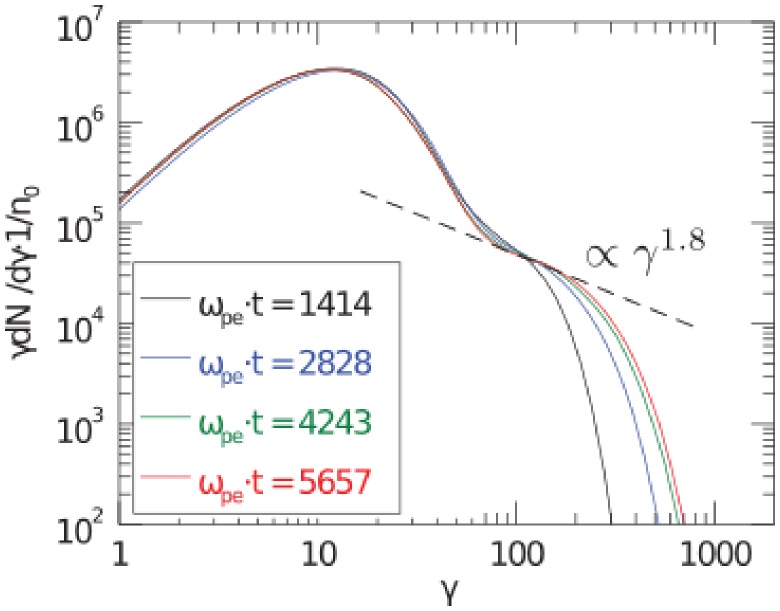


Time evolutions of downstream energy spectrum

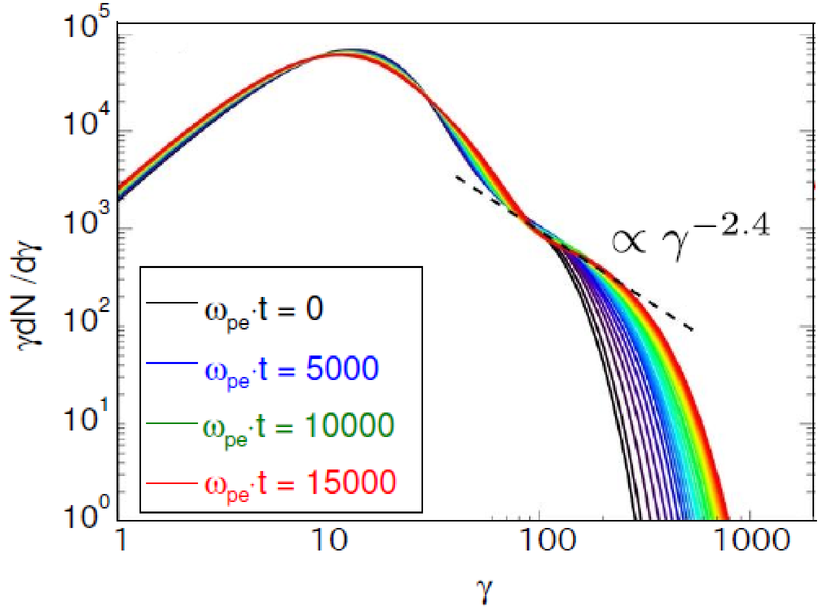
CFL=0.5 (Non-magic)



N$_0$=50/cell, CFL=1.0



Sironi+13

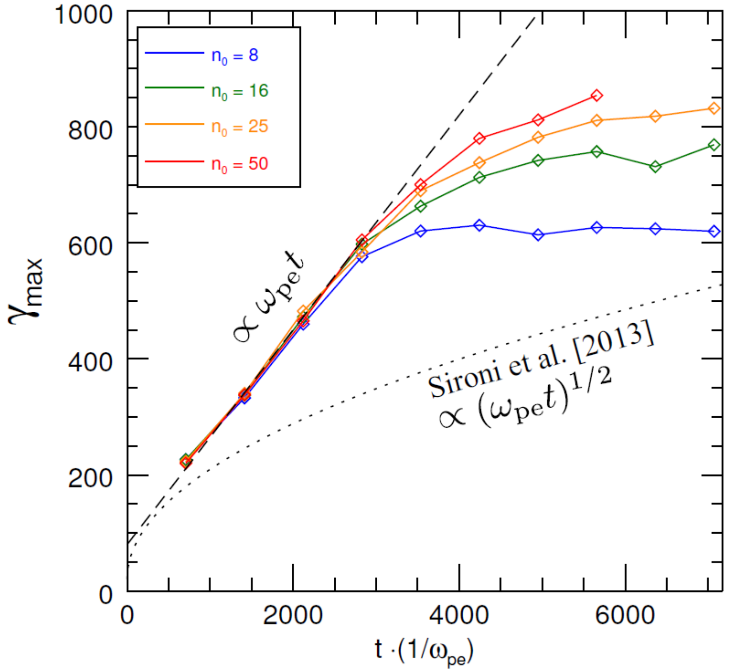


☐No efficient acceleration with NCI

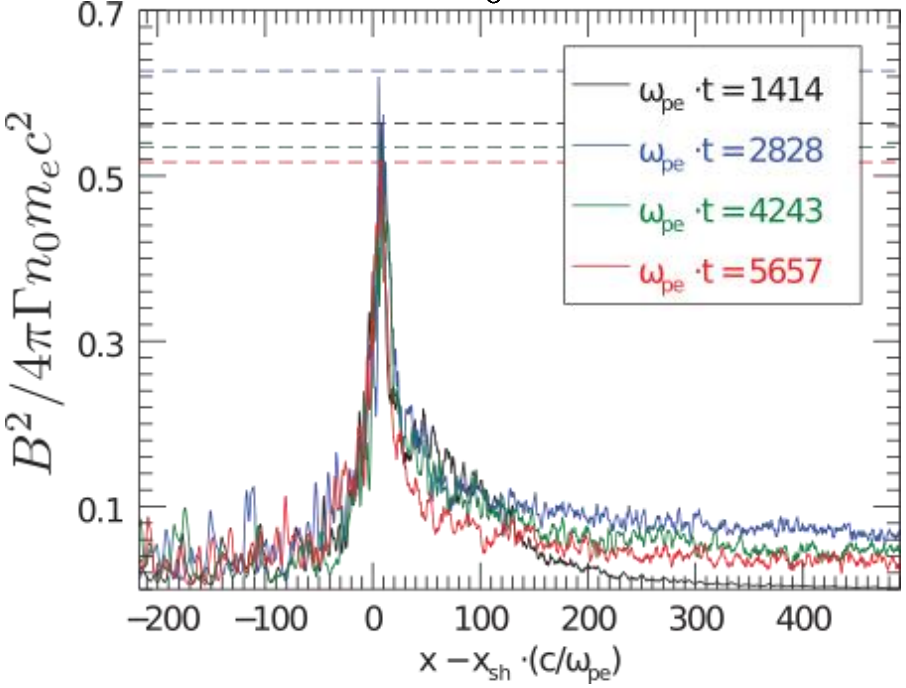
☐Efficient accelerations were obtained by using Magic CFL number method

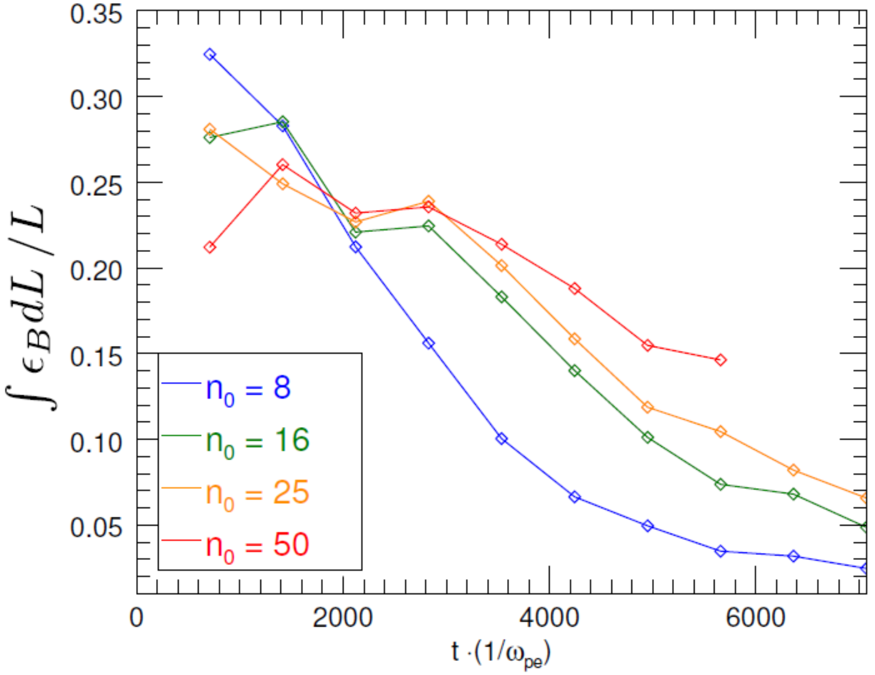
☐Spectral index -1.8 vs. -2.4

γ_{max} time evolutions



N_0=50





- ☐ Rapid acceleration until $\omega_{pe}t$ ~ 3000 due to strong magnetic turbulence around the shock front ($\gamma_{max} \propto t$)
- ☐ Followed by slow acceleration stage due to decay of the Weibel magnetic turbulence ($\gamma_{max} \propto t^{\frac{1}{2}}$ by Sironi+ 13)
- ☐ Results seemed to be converged with n_0=50

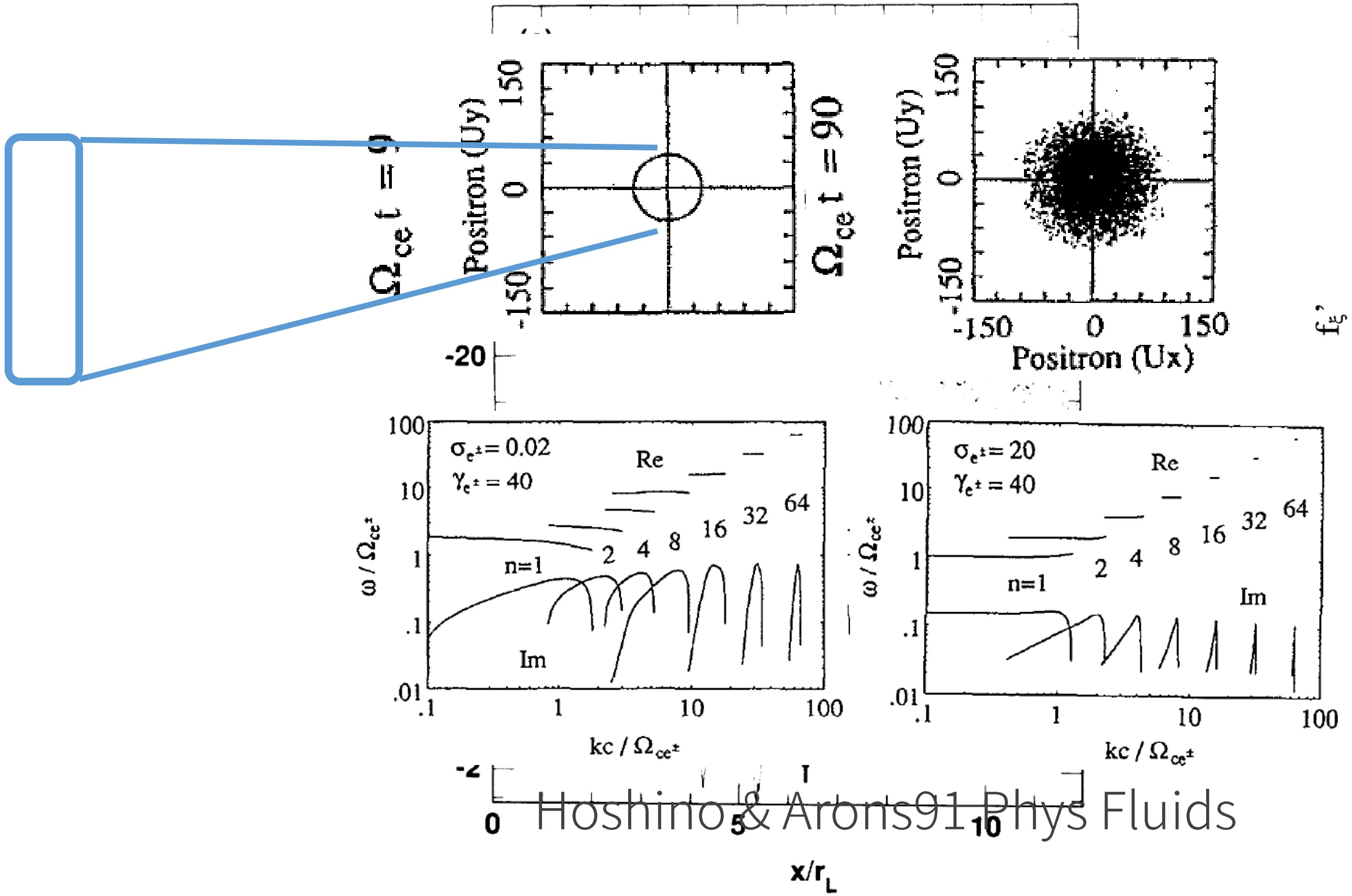
相対論的衝撃波からの大振幅先駆波励起と電子加速

1990s~ 1D PICシミュレーション

最初のPIC計算

synchrotron maser instability

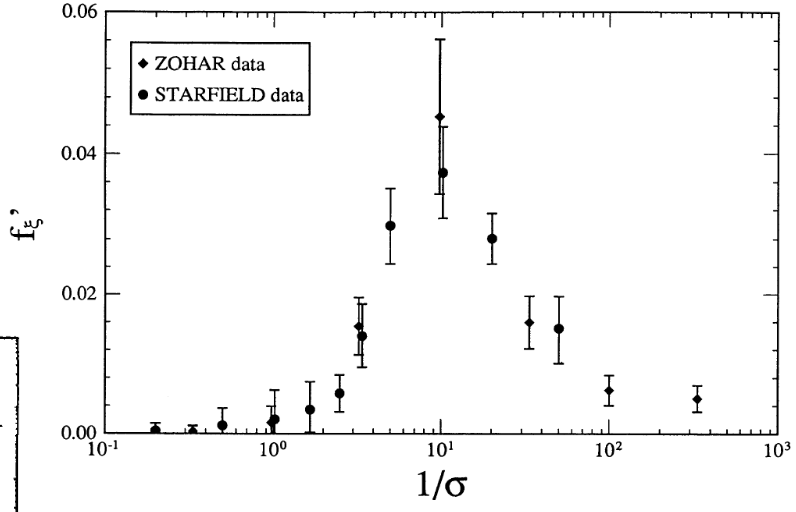
precursor wave



Hoshino & Arons91 Phys Fluids

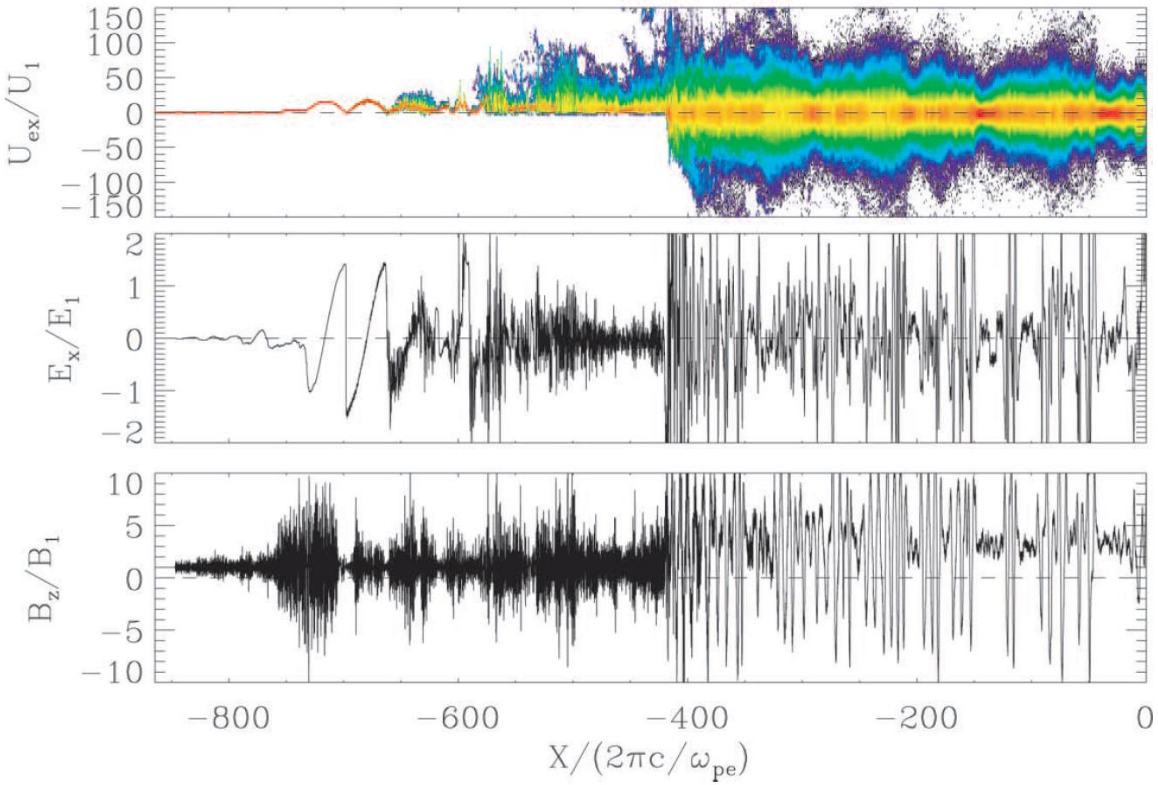
Langdon, Arons, Max88 PRL

emission efficiencies for shock magnetizations



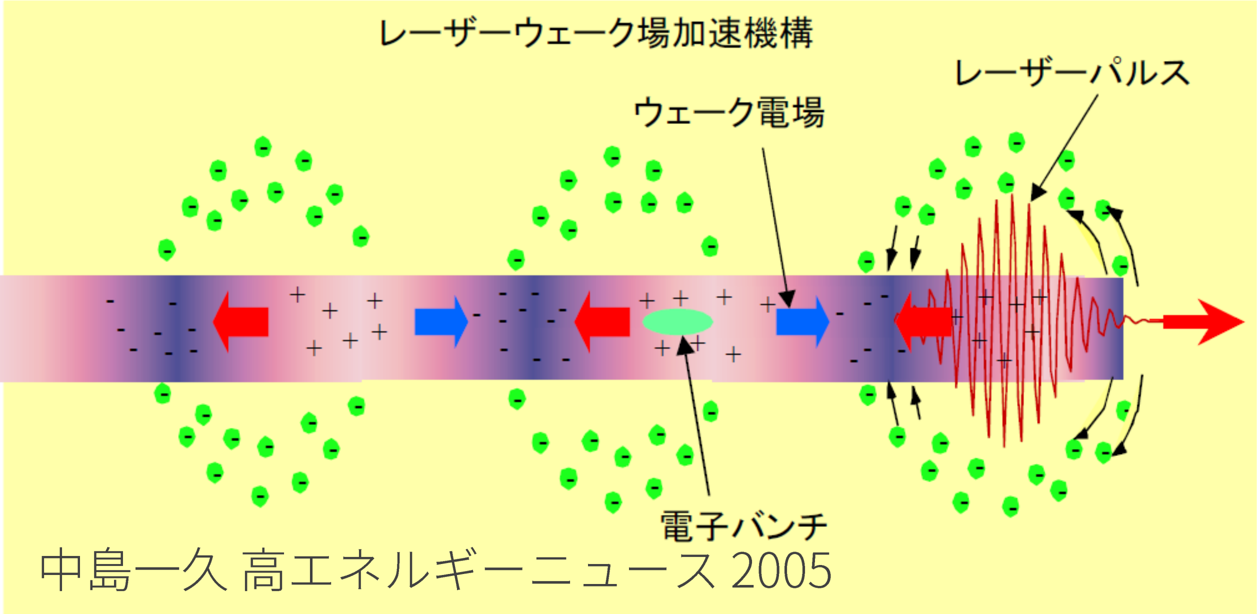
Gallant+91 ApJ;
Hoshino+92 ApJ

先駆波励起と航跡場加速
(1D PIC, Lyubarsky06; Hoshino08)



Hoshino08

- Tajima & Dawson 79 PRL



中島一久 高エネルギーニュース 2005

- レーザー（光モードの電磁波）のポンデロモーティブ力（$-\frac{e^2}{4m\omega^2}\nabla E^2(x)$）によって電子が掃きだされ、ウェークに強い静電場（ラングミュア波）が励起
- ラングミュア波とのランダウ共鳴

Iwamoto+17;18 ApJ

□2次元PICシミュレーションでワイベル不安定との競合を調べた

□上流バルクΓ＝40

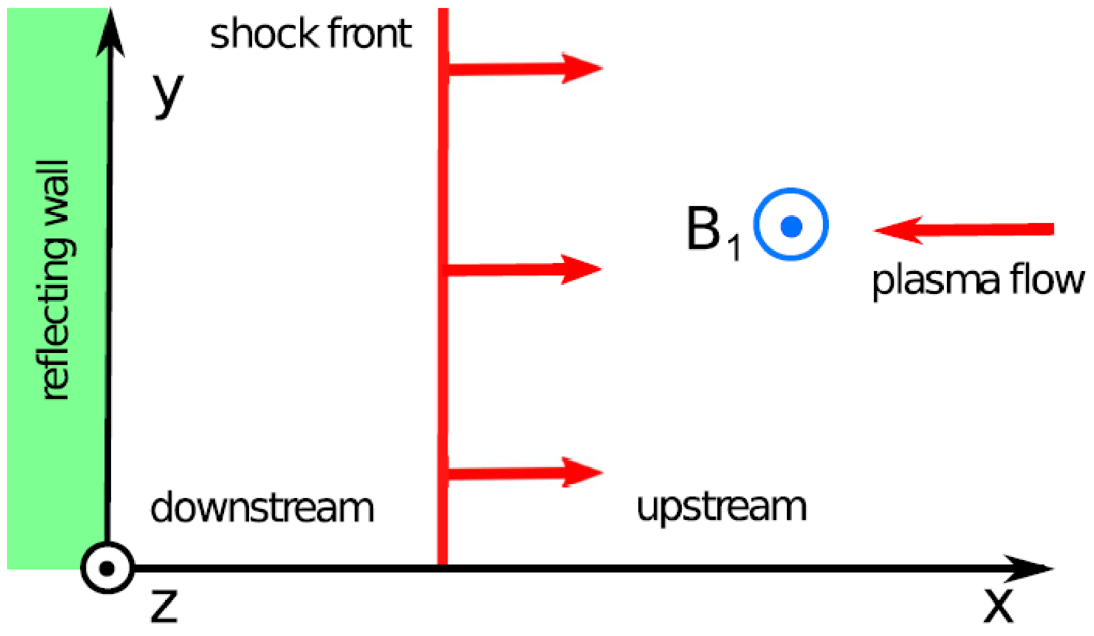
□陽電子ー電子系（M/m=1）

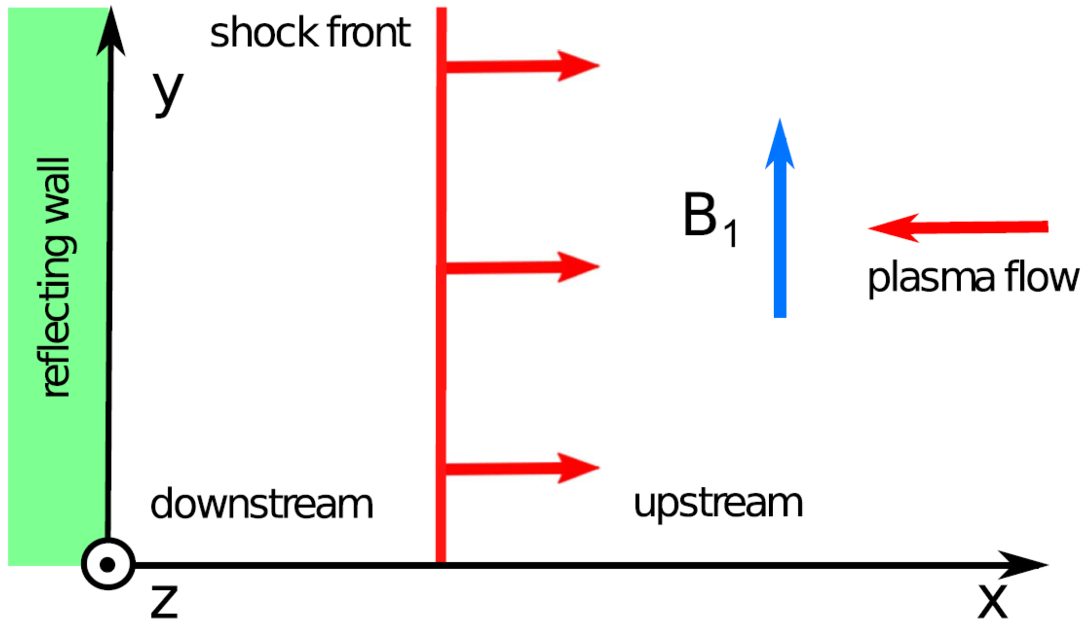
□イオンはないので航跡場加速は起きない。Precursor waveの放射効率のみを議論

□マジカルCFL法

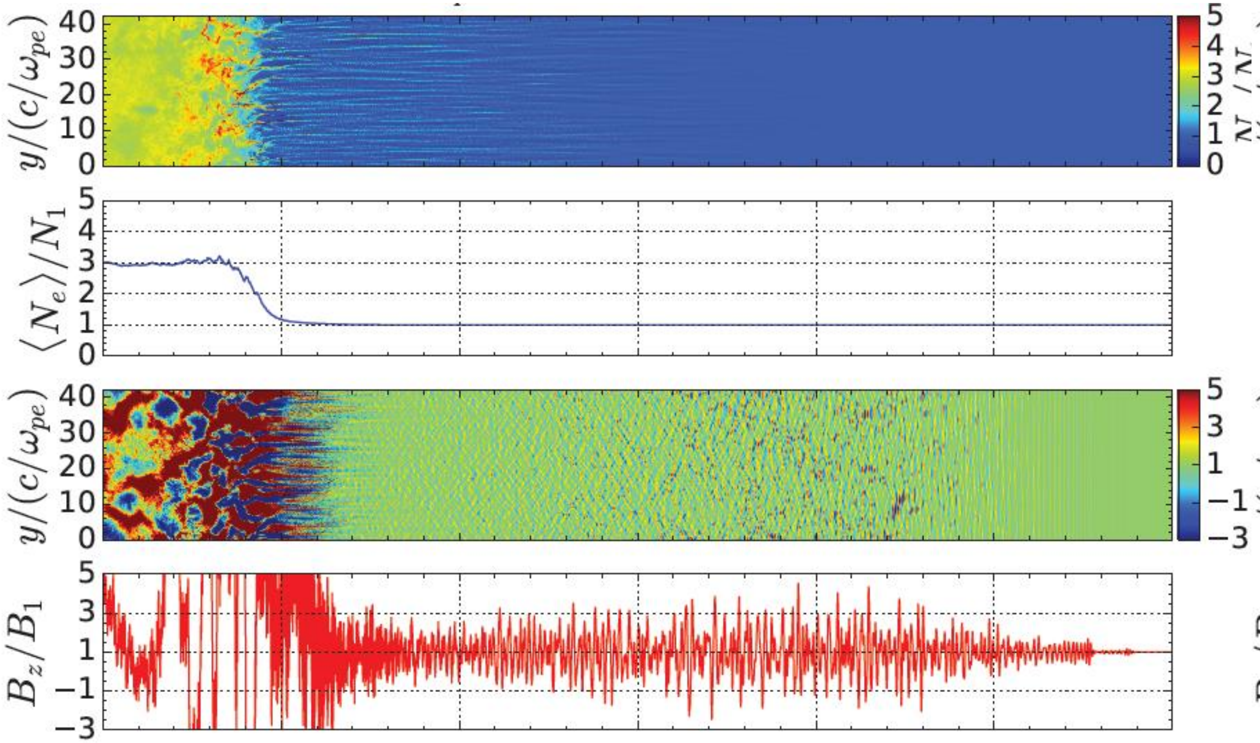
□N=64 /cell/species

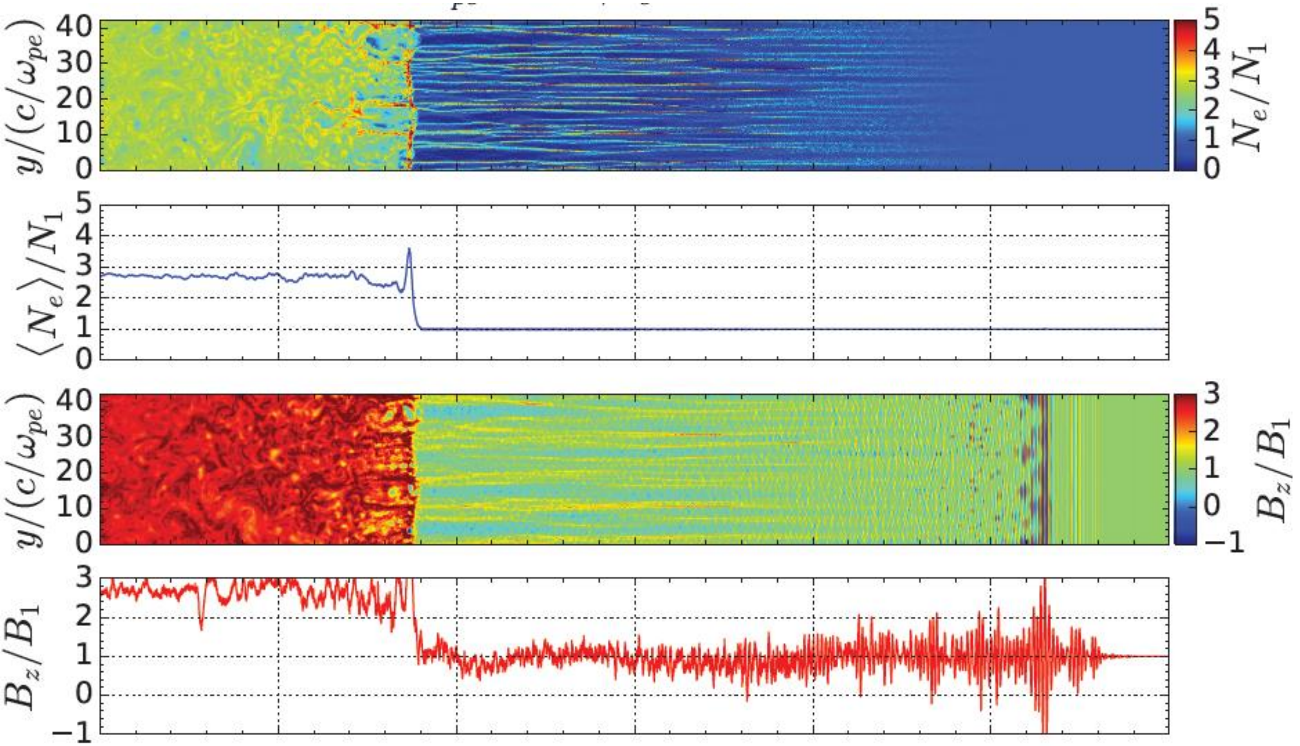
□$dx/(c/\omega_{pe}) = 1/40$ (typically ~ 1/10)





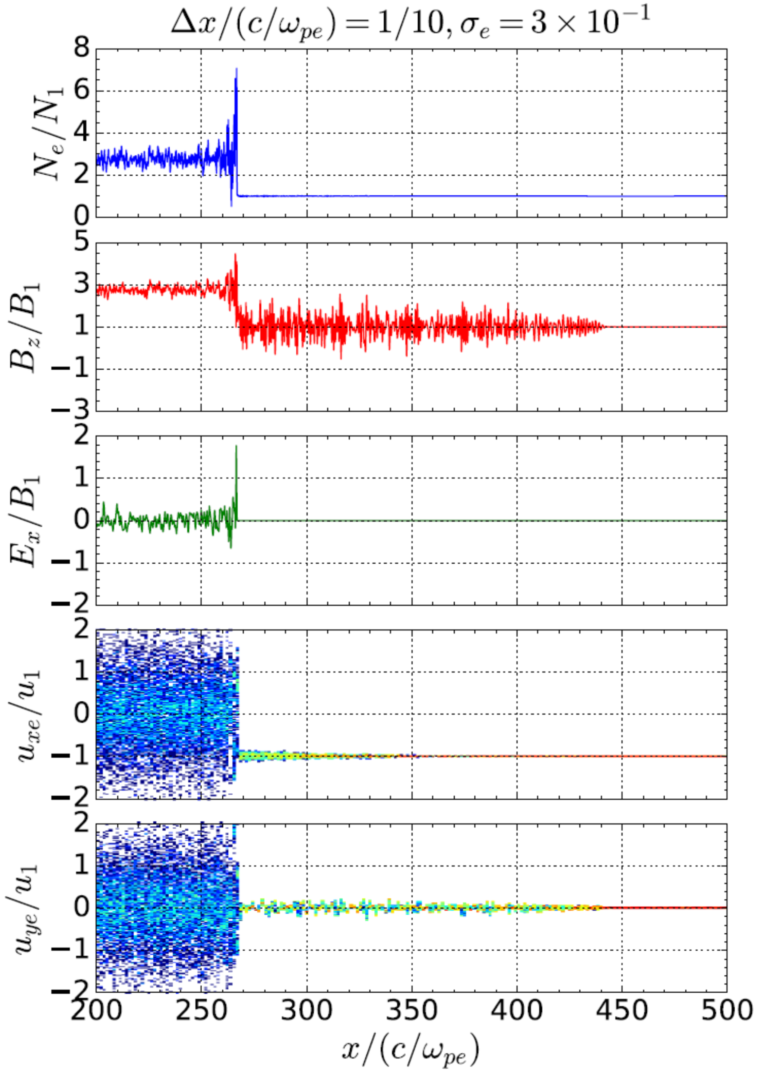
Persistence of precursor waves (out-of-plane B_0)


$\sigma_e = 3 \times 10^{-3}$

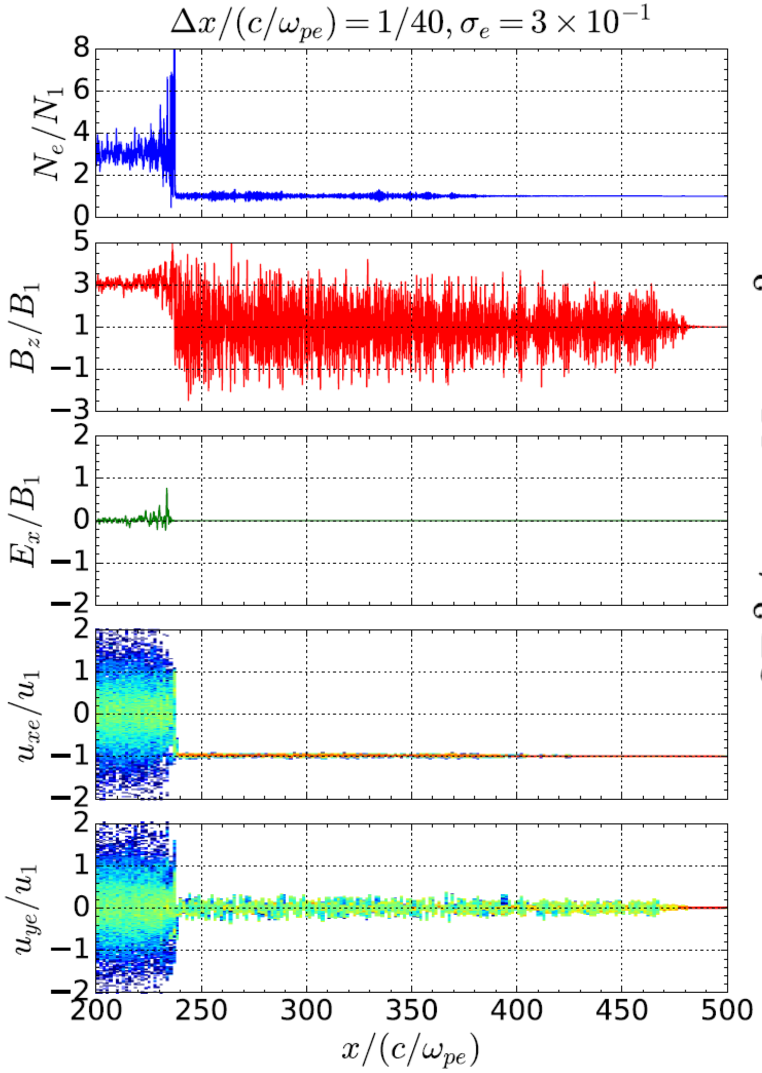

$\sigma_e = 3 \times 10^{-1}$

- 多次元計算では放射されないとされていたが（Sironi+11;13）低σでも大振幅の電磁波放射（ワイベル不安定と共存が可能）！
- 多次元性による効果：電磁波の自己集束による上流密度揺らぎ

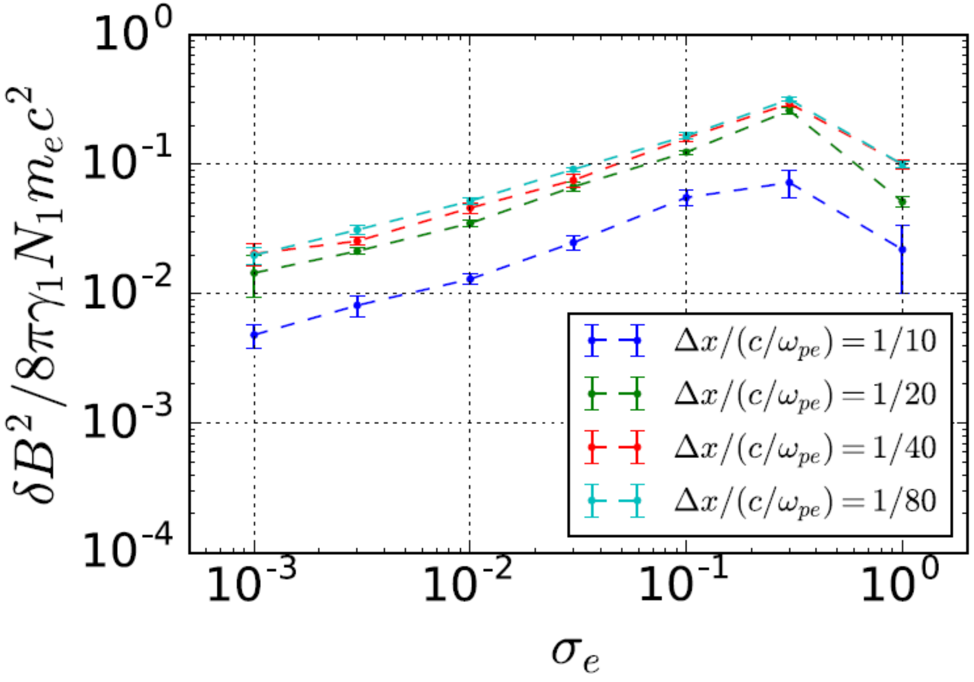
解像度依存性



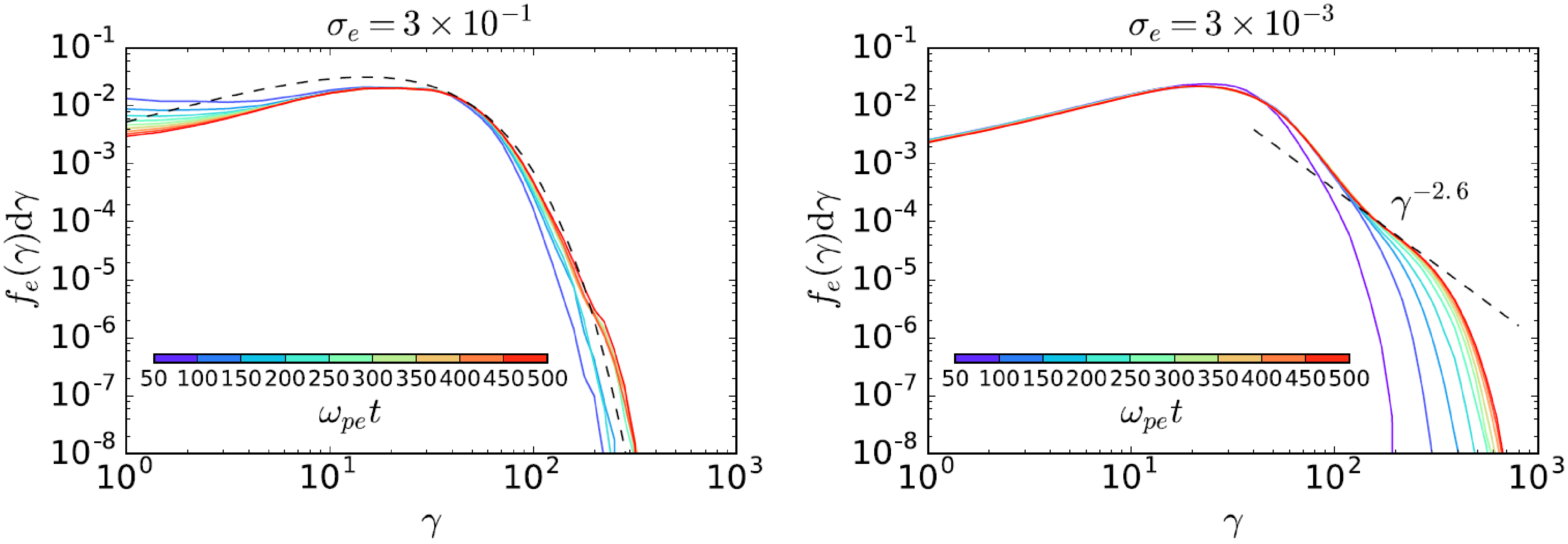
$\Delta x/(c/\omega_{pe}) = 1/10, \sigma_e = 3 \times 10^{-1}$



$\Delta x/(c/\omega_{pe}) = 1/40, \sigma_e = 3 \times 10^{-1}$

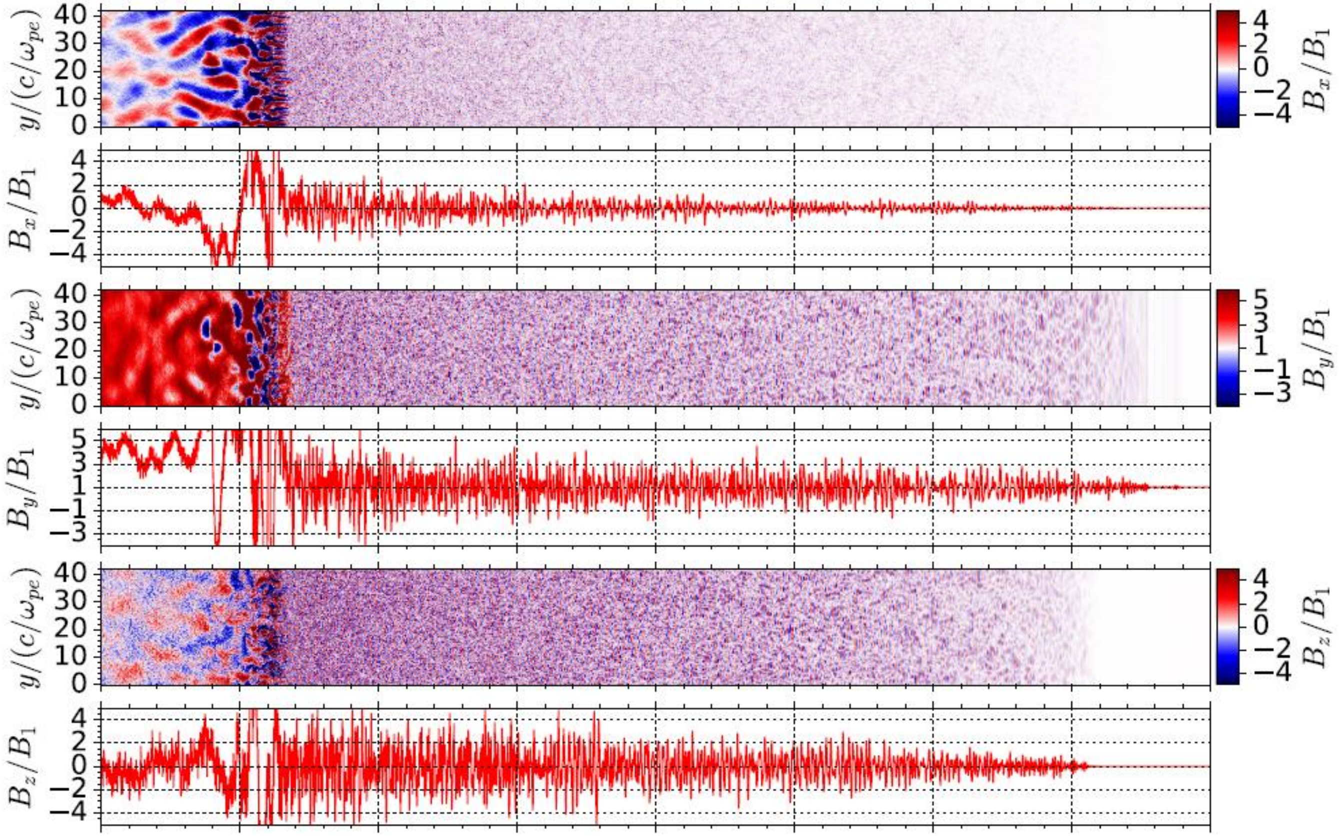


下流のエネルギースペクトル



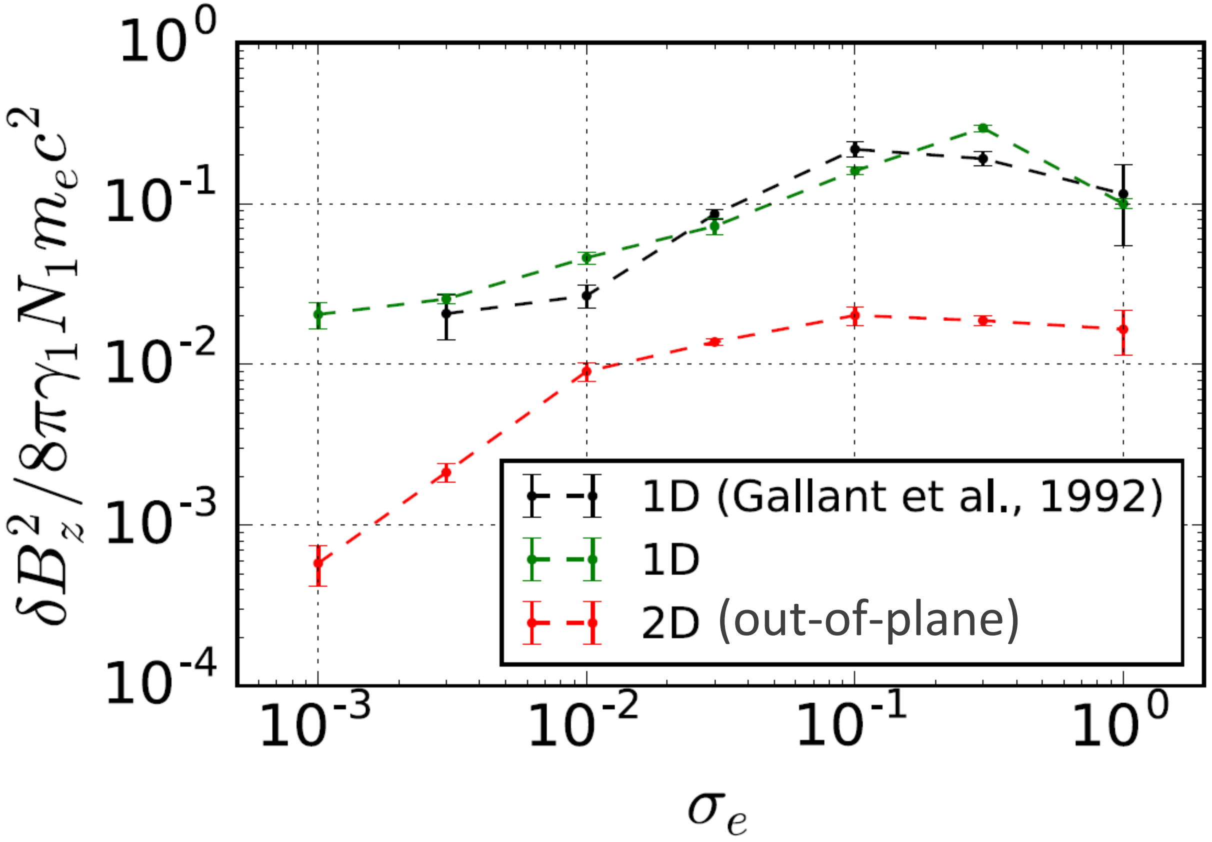
- □陽電子・電子系なので、航跡場加速は起きない
- □低σ_eでの非熱的粒子加速はWeibel乱流によるフェルミ加速（Sironi+ 13)
- □Sironi+13より速い時間スケールでの加速（マジカルCFL法の恩恵）

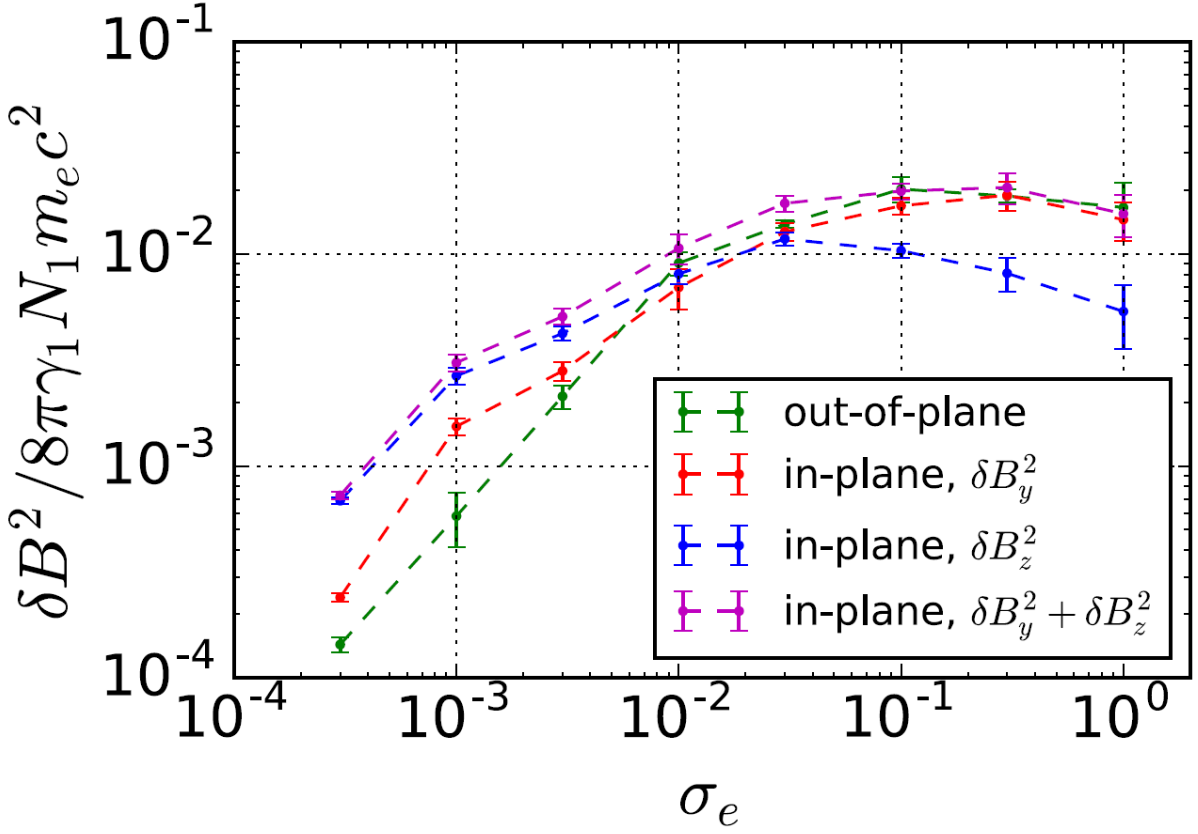
Precursor wave emission enhanced by the Weibel instability (in-plane B_0)



- いわゆるSMIで励起される電磁場（X-mode）は背景磁場方向に変動成分（δB_y）
- δB_z成分も遅れて強い励起（電磁場O-mode）
- 衝撃波面近傍の強いワイベル磁場がδB_z成分を生成するため、粒子のジャイロ運動は紙面垂直方向に傾くため、$(\delta B_y, \delta B_z)$を励起
- 上流領域（$B\sim B_y$）でO-modeにモード変換

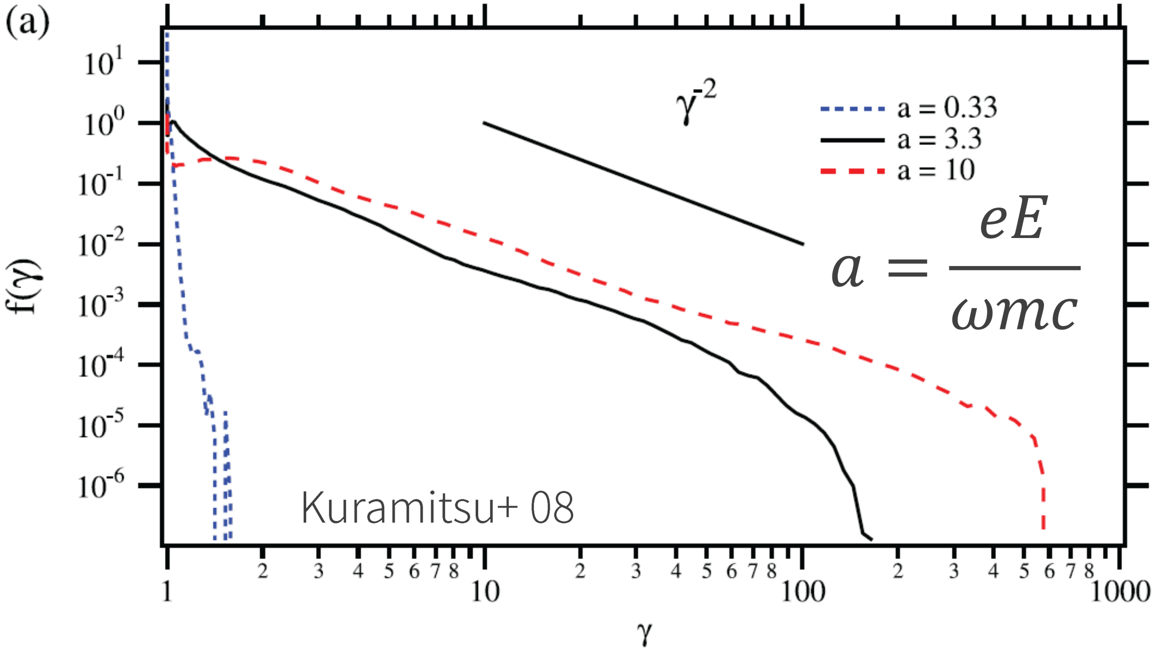
先駆波の振幅のσ依存性

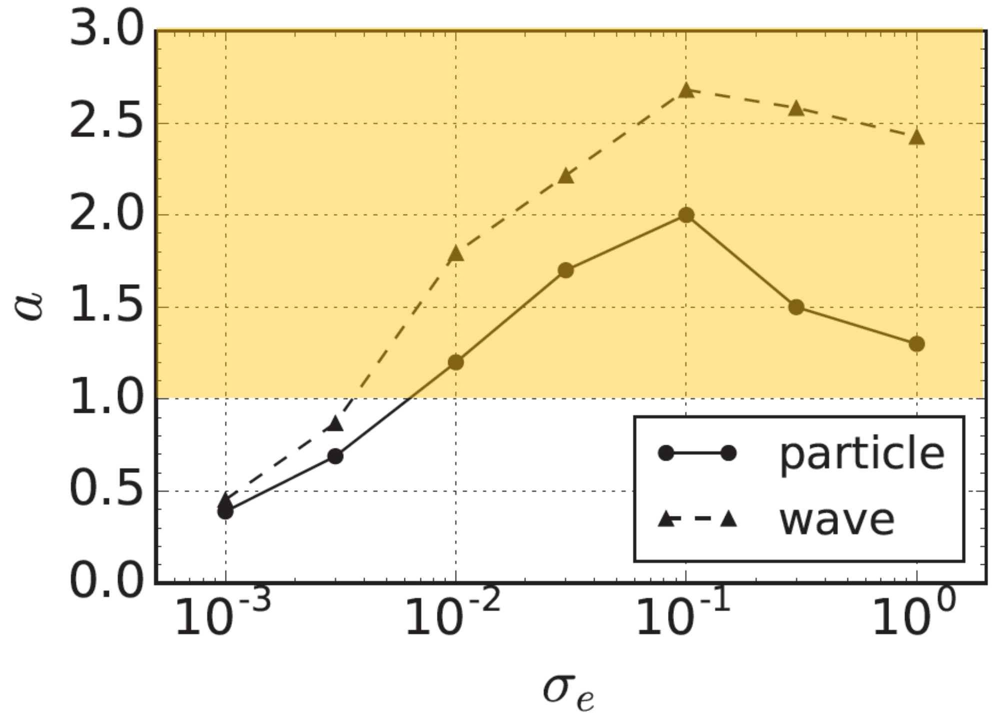


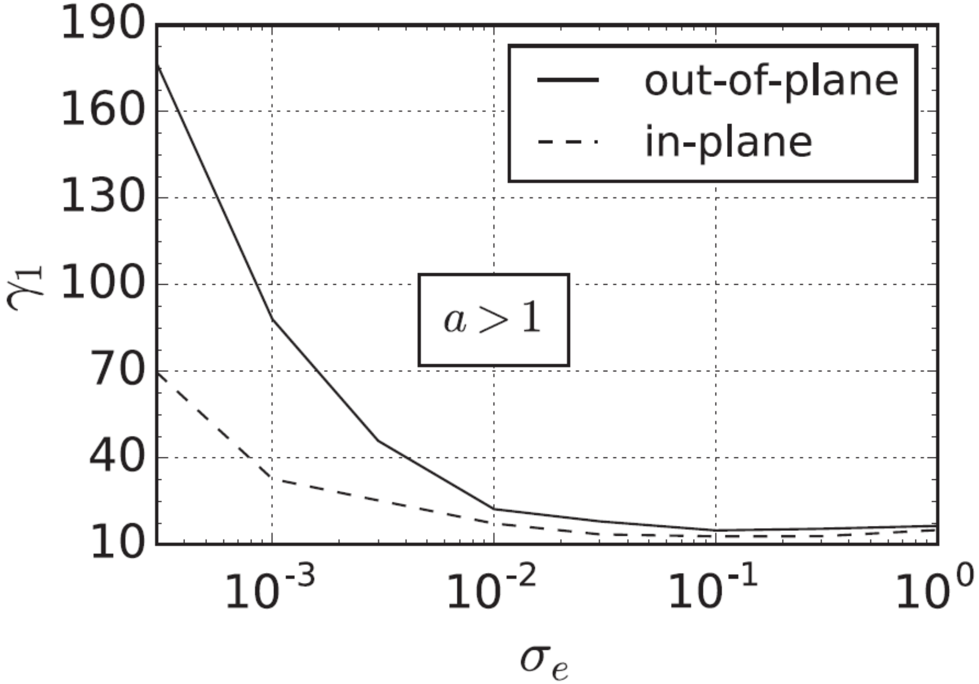


3D計算結果を待ちましょう！

航跡場加速への期待







- □Strength parameter a >1で効率的な航跡場加速（Kuramitsu+ 08）
- □幅広いσでa>1の大振幅電磁波励起が実現

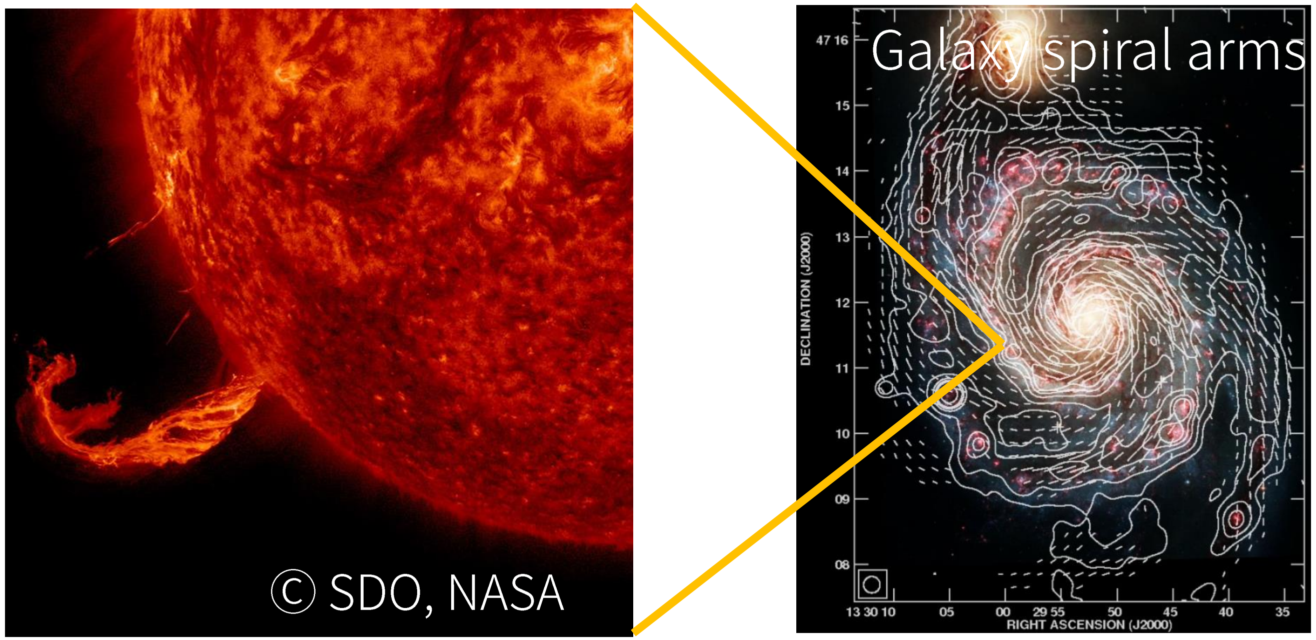
イオンワイベル不安定による磁場生成と飽和過程

Origin of the seed magnetic field (non-MHD process)

$$\frac{\partial B}{\partial t} = -c\nabla \times E = \nabla \times (v \times B)$$

MHD dynamo
for global magnetic fields

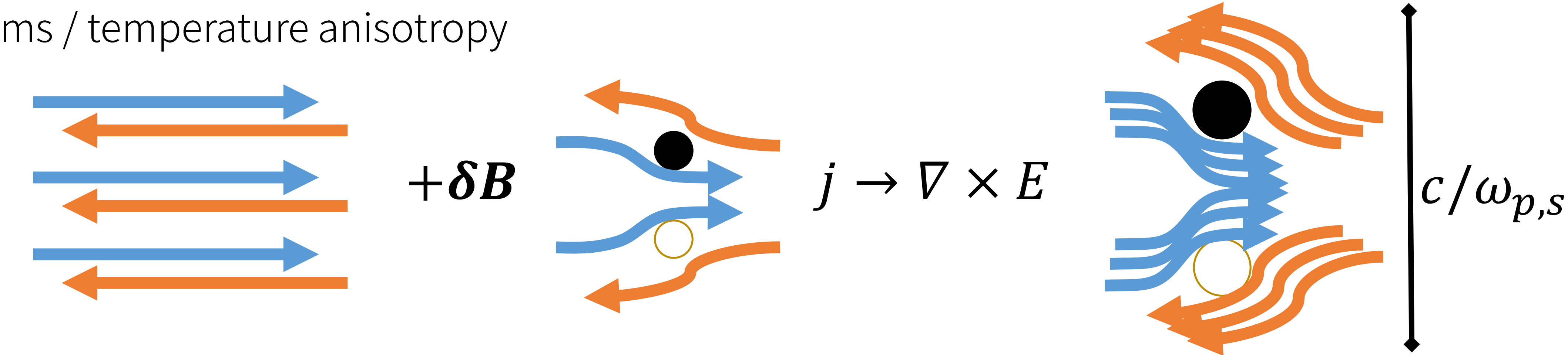
Seed magnetic field?


© SDO, NASA

Fletcher+11

Weibel instability (Weibel, PRL, 1959; Fried, Phys. Fluid, 1959)

Beams / temperature anisotropy



$+\boldsymbol{\delta B}$

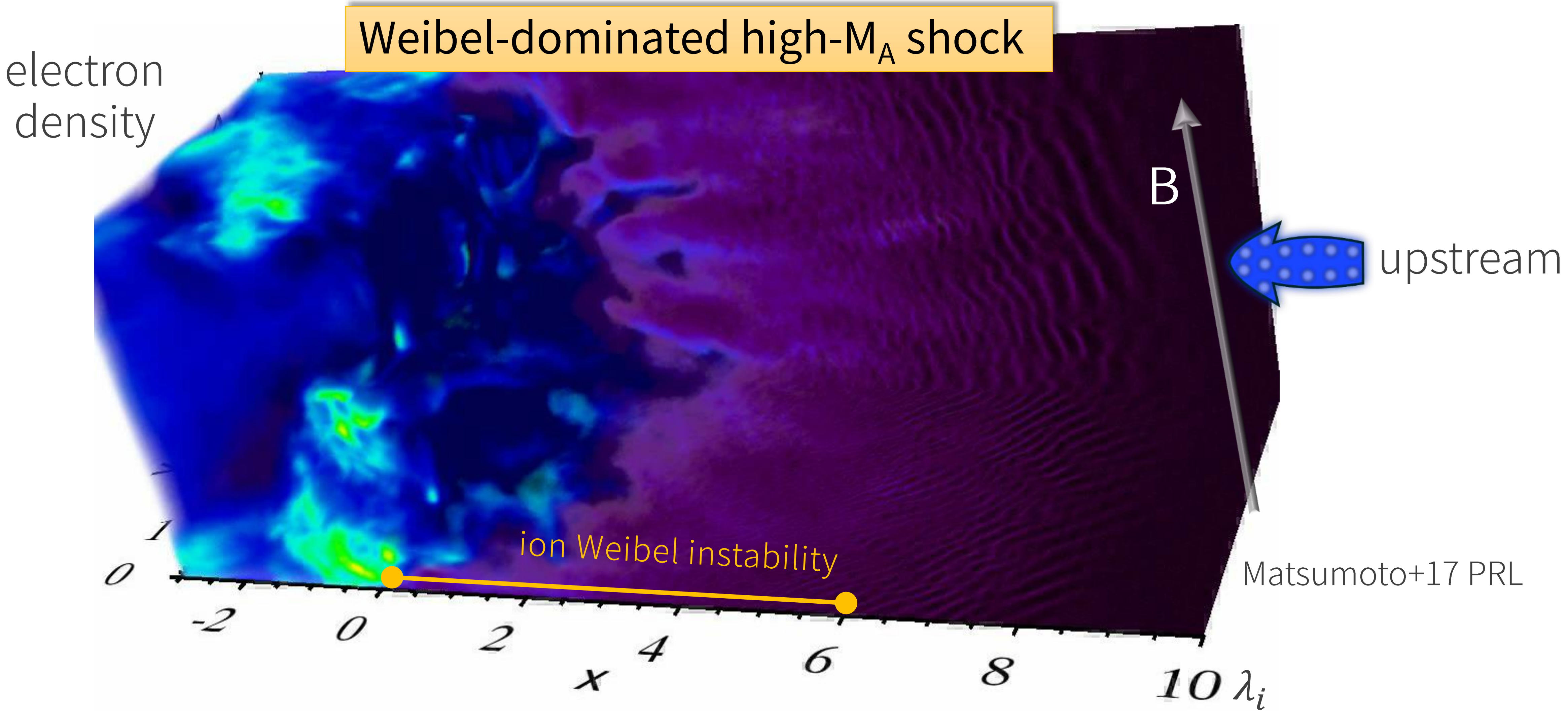
$j \rightarrow \nabla \times E$

$c/\omega_{p,s}$

Weibel instability in high-energy astrophysics

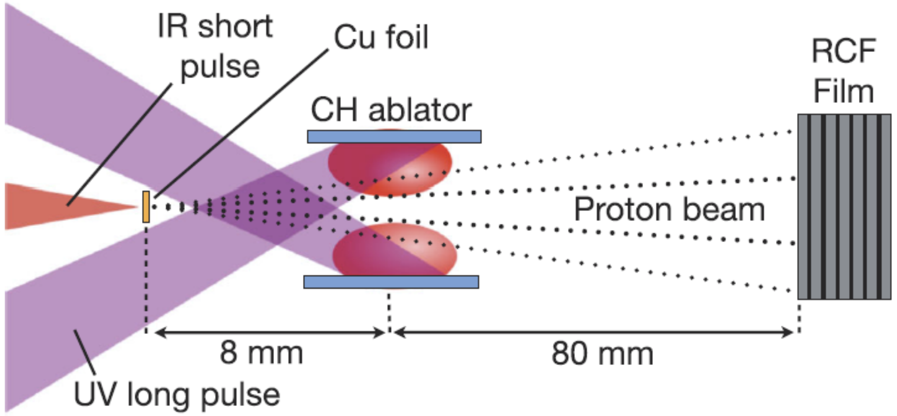
- Weibel-mediated shocks　(Kato07; Kato & Takabe08)
- Particle accelerations at shocks (Spitkovsky08; Sironi+11,13; Matsumoto+15,17)

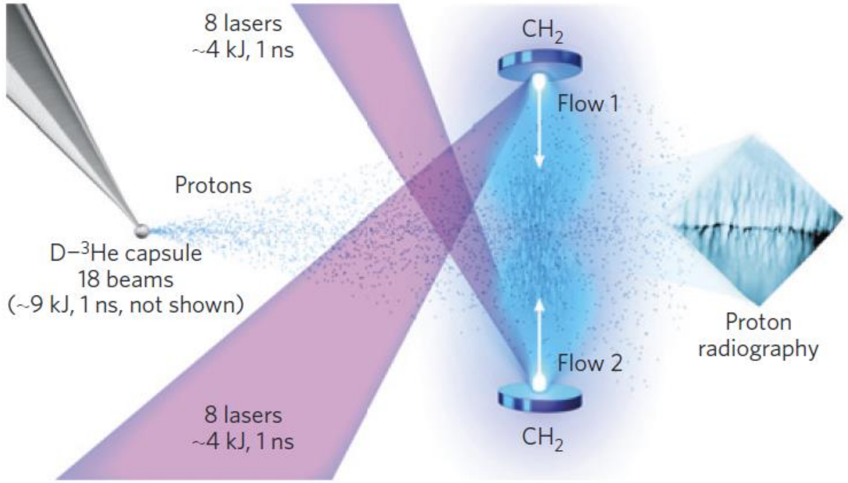
Weibel-dominated high-M_A shock



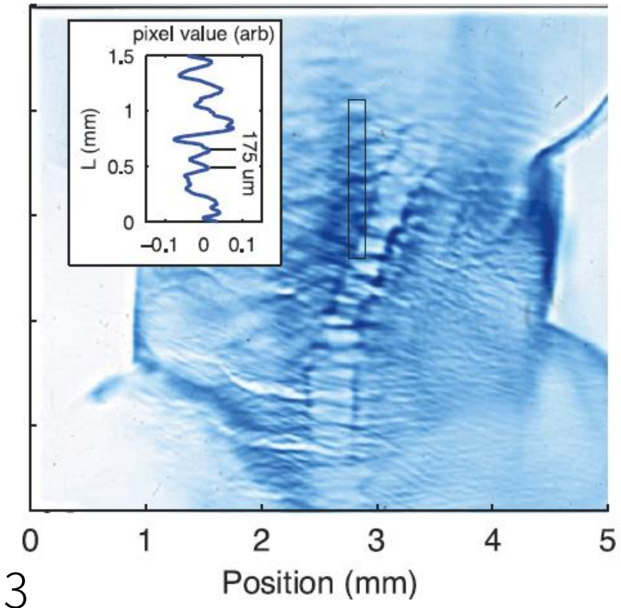
Matsumoto+17 PRL

Weibel instability in laboratory experiments

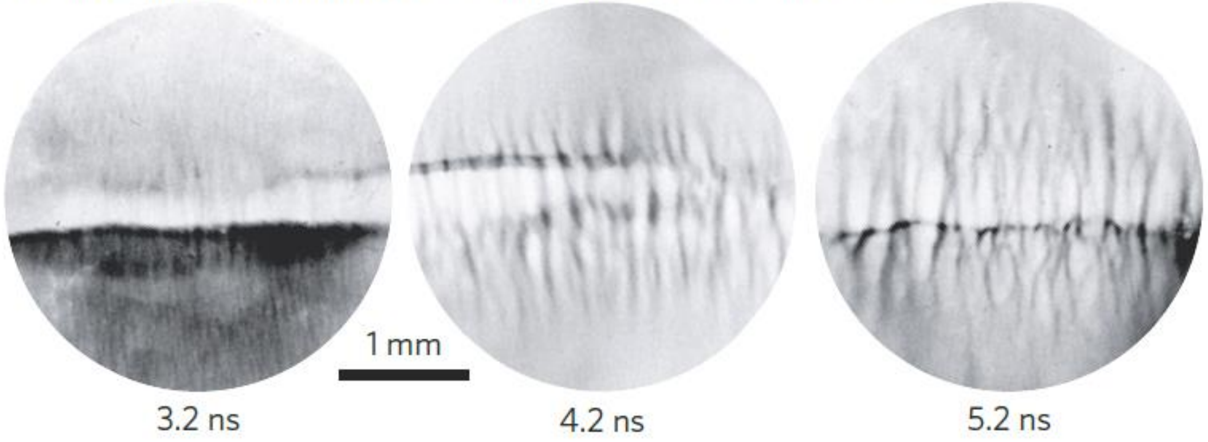




t = 5.8 ns



Experimental proton radiographs from 14.7 MeV (D–^{3}He) protons



Fox+ PRL 2013

Huntington+, Nat Phys 2015

Saturation mechanisms of the ion Weibel instability ⇔ maximum magnetic energy

- Linear growth : $\omega_{pi}^{-1}, c/\omega_{pi}$
- Inverse cascade
  - coalescence of current filaments with same polarity
  - $I = j\pi R(t)^2, j = const.$
  - $R(t) \propto \mathrm{t}^2(\mathrm{M/m})^{-\frac{1}{2}}$ (Ruyer+15;16)
- Upper limit of the current
  - Alfven current limit (Alfven38)
  - Bio-Savart law

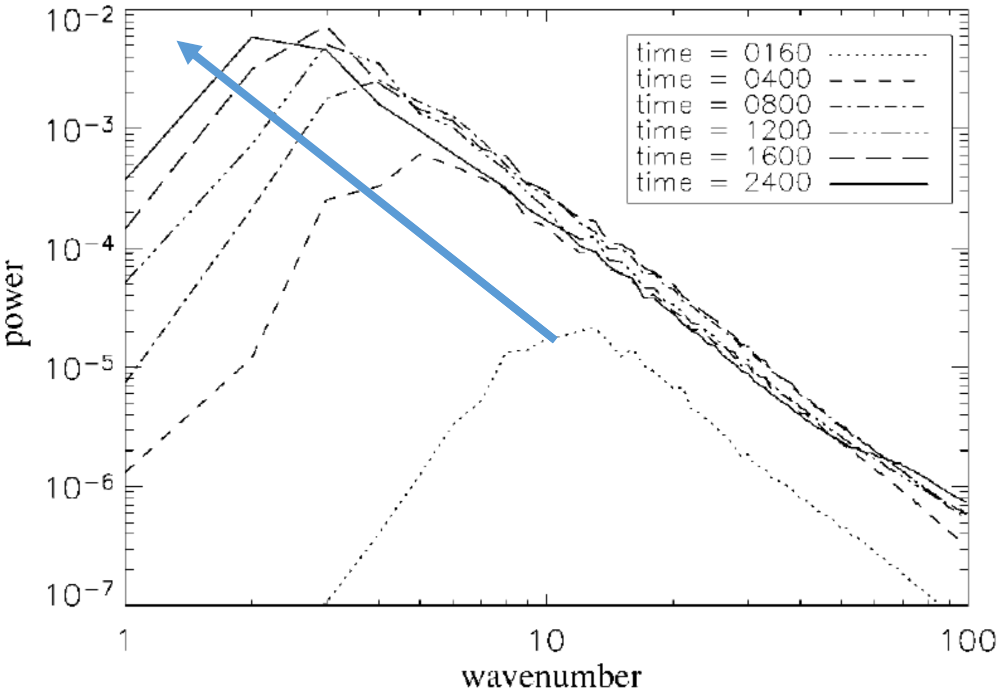
$$B = \frac{2Ir}{cR^2}\ (\mathrm{r} < R)$$

$$= \frac{2I}{cr}\ (\mathrm{r} > R)$$

  - When $r_{ge} \sim R(t)$ in self-generated magnetic field

$$I_A = \frac{mc^2\gamma c}{q}$$

  - Constant I_A with growing R results in monotonic decay of |B| after saturation



Frederiksen+04 ApJL

Alfven current limit in the Weibel instability

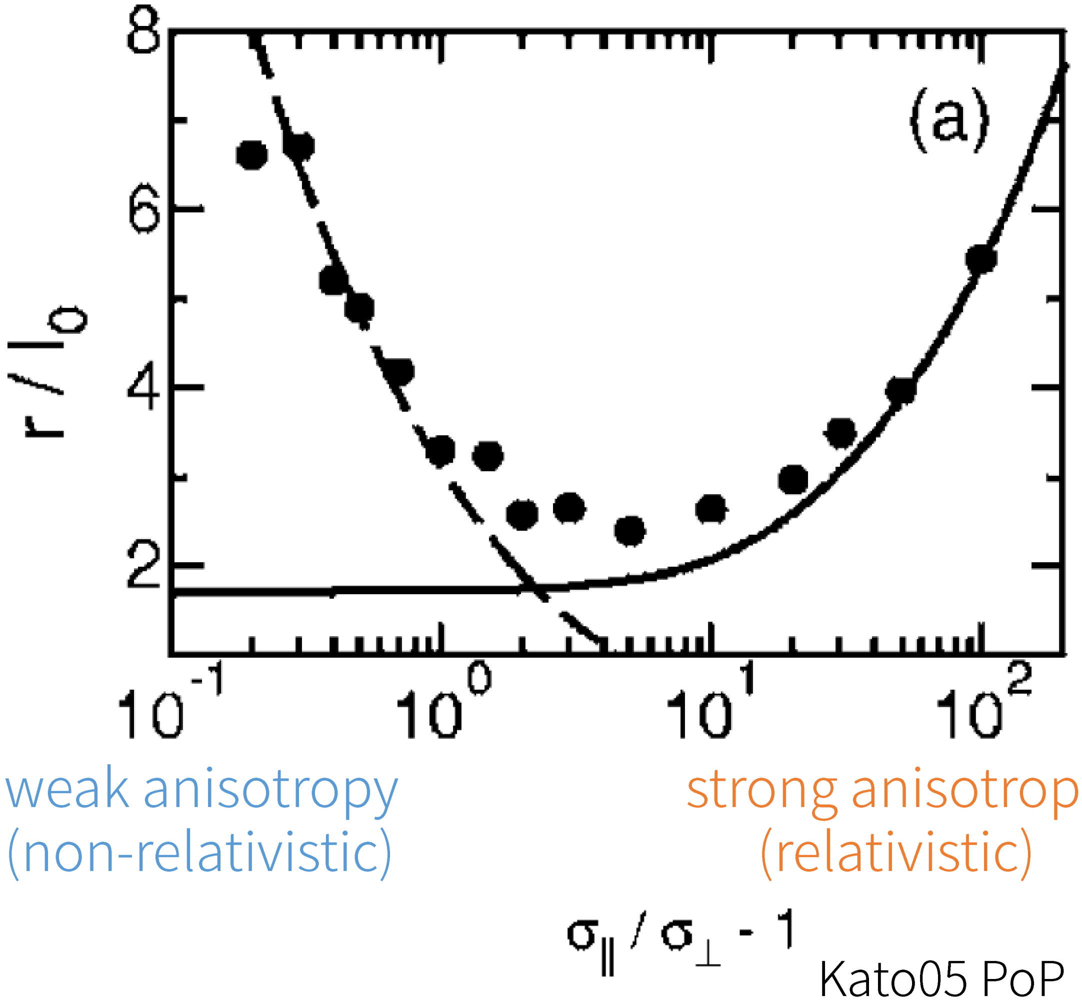
- 2D electron-positron plasmas (Kato05 PoP)
- In the strong anisotropy, $I_A = \pi R^2 qn\beta c$ gives the filament size at saturation as

$$\boldsymbol{r_P \sim 2\sqrt{\gamma}\frac{c}{\omega_{p,s}}}$$

- Magnetic field energy at saturation:

$$\varepsilon_B = \frac{B_{max}^2}{8\pi\gamma\rho c^2} \text{ with } B_{max} = \frac{2I_A}{cr_p}$$

$$\Rightarrow \boldsymbol{\varepsilon_B \sim \frac{\beta^2}{2} \sim \frac{1}{2}}$$



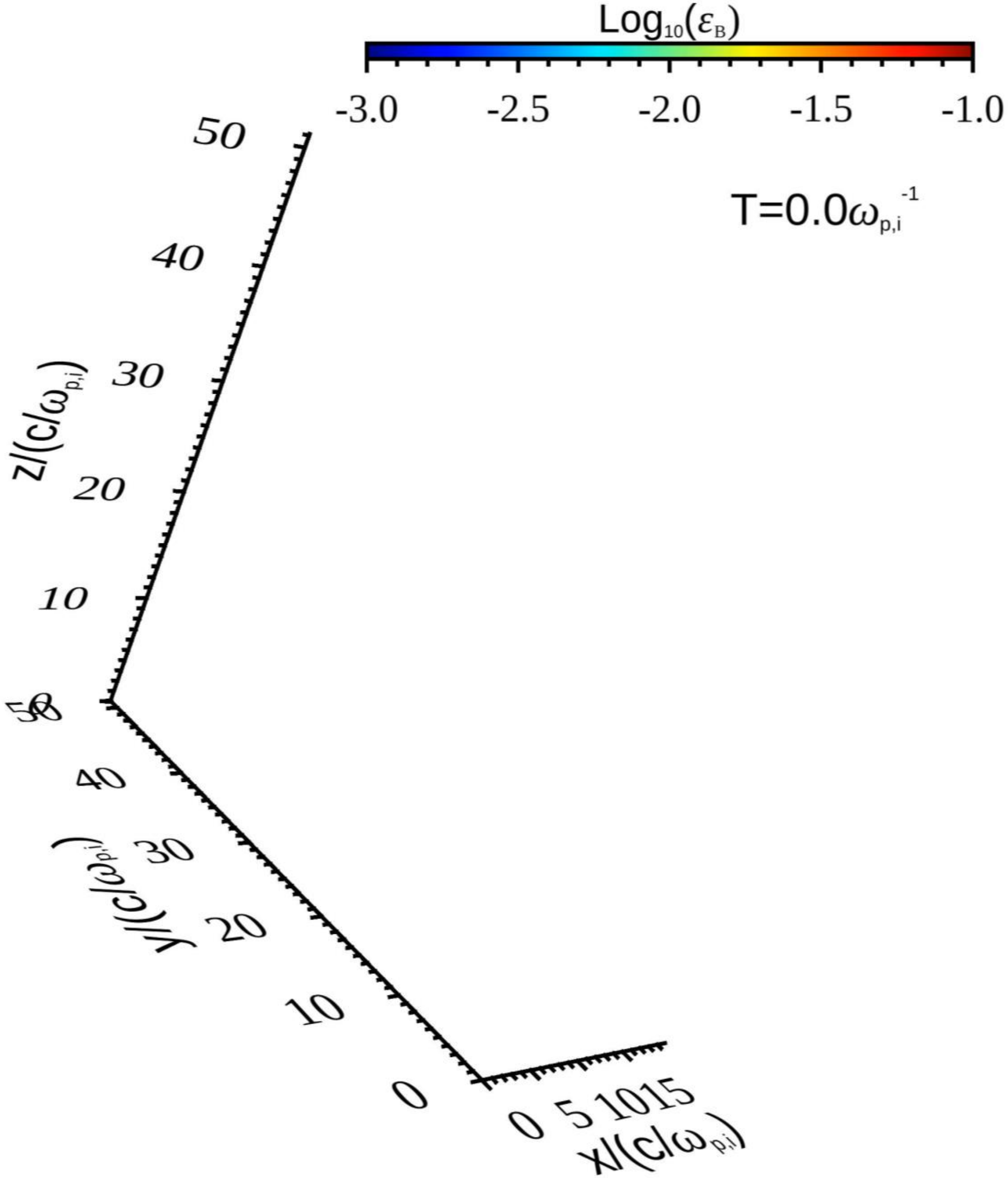
weak anisotropy
(non-relativistic)

strong anisotropy
(relativistic)

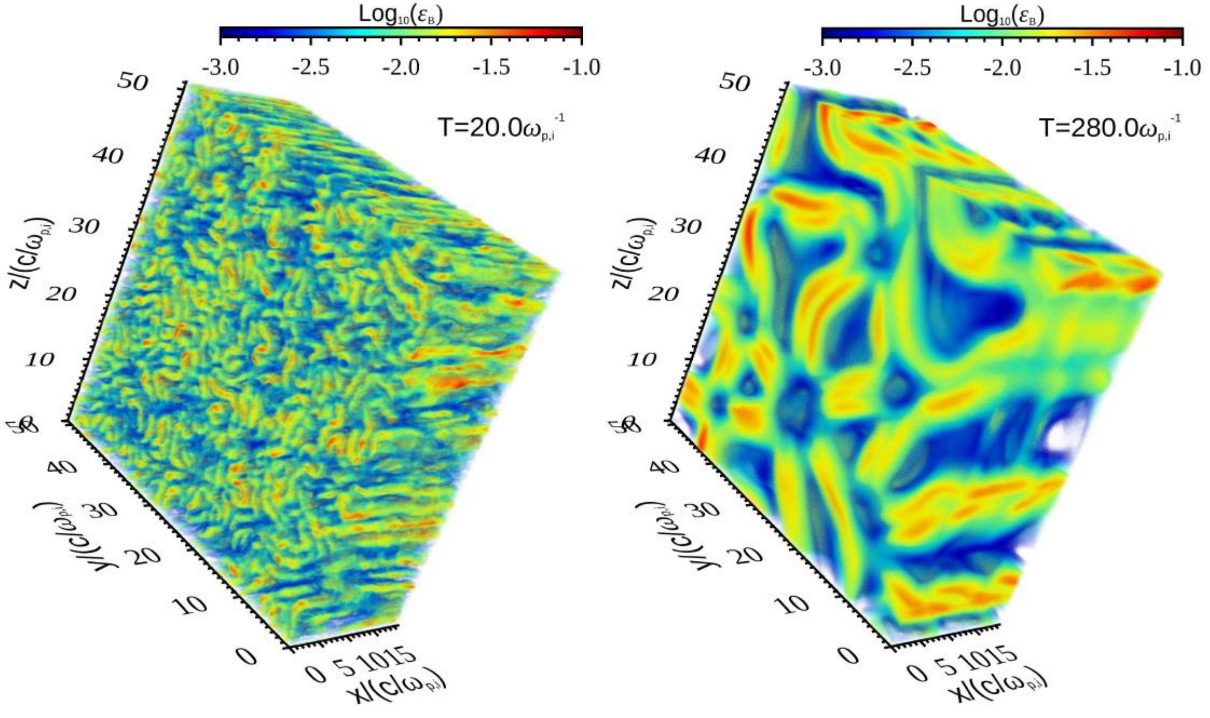
Kato05 PoP

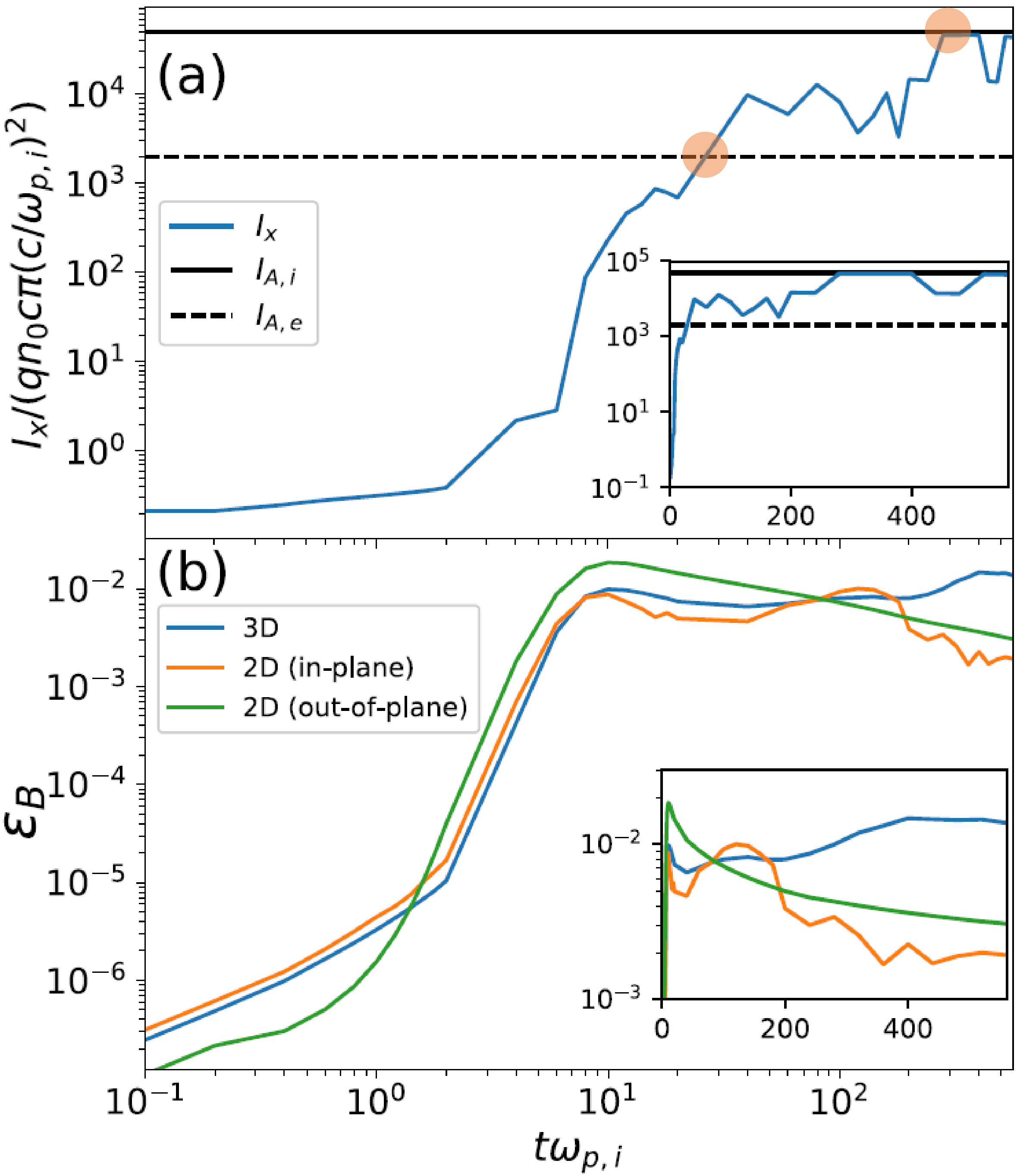
Saturation of the Ion Weibel instability

- M/m=25, relativistic cold beams (Γ=5) in a periodic box
- $(N_x,N_y,N_z)=(1000,2560,2560)$
- $(L_x, L_y, L_z) = (20, 51.2, 51.2)\ c/\omega_{pi}$
- 10 ptcls/cell/species
- 131,072 processor cores with 200TB memory on the K computer
- $\epsilon_B = B^2/8\pi\rho_0 c^2(\gamma-1)$ ⇨



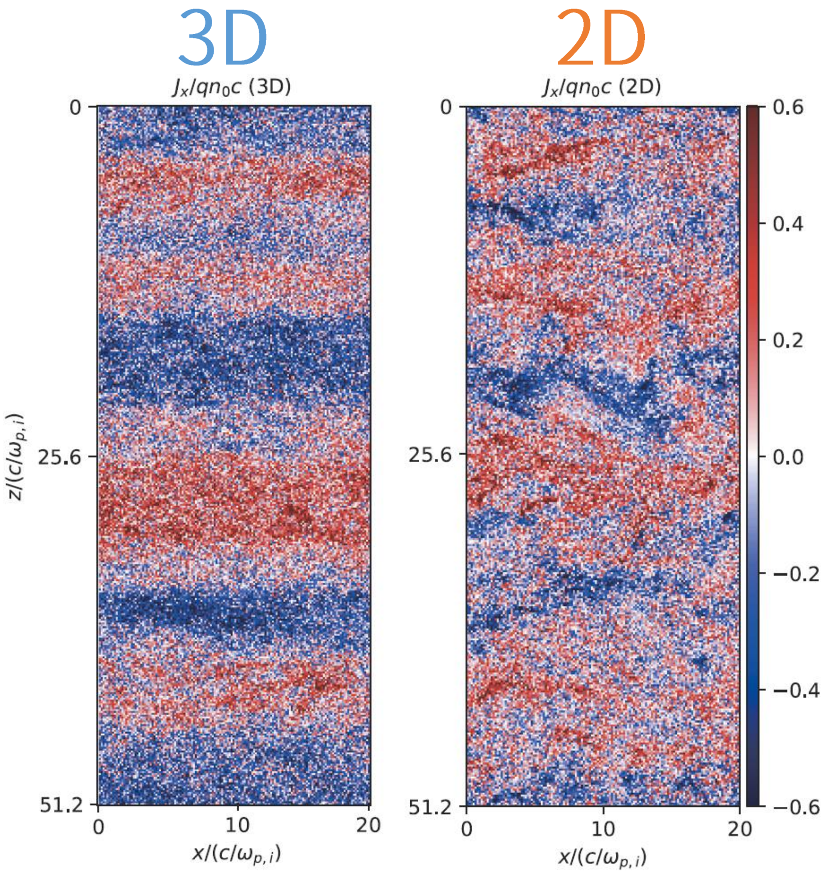
Time evolution

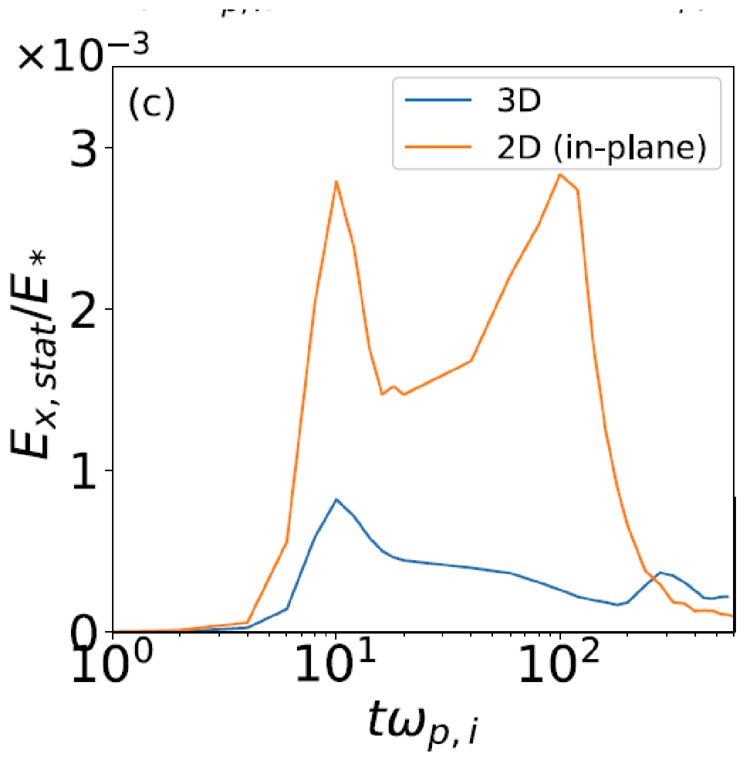




- First to present true saturation of the ion Weibel instability reaching Alfven current limit
- 3D ion Weibel instability could sustain strong magnetic field (ε_B~1.5%) after saturation (contrary to the rapid decay in 2D)

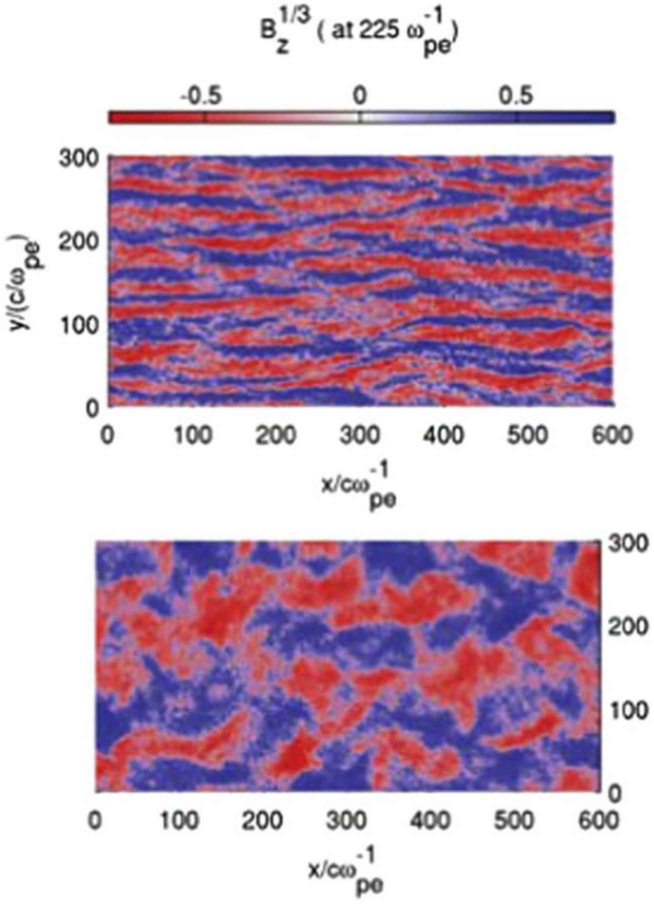
3D vs. 2D in-plane





Takamoto+18ApJL

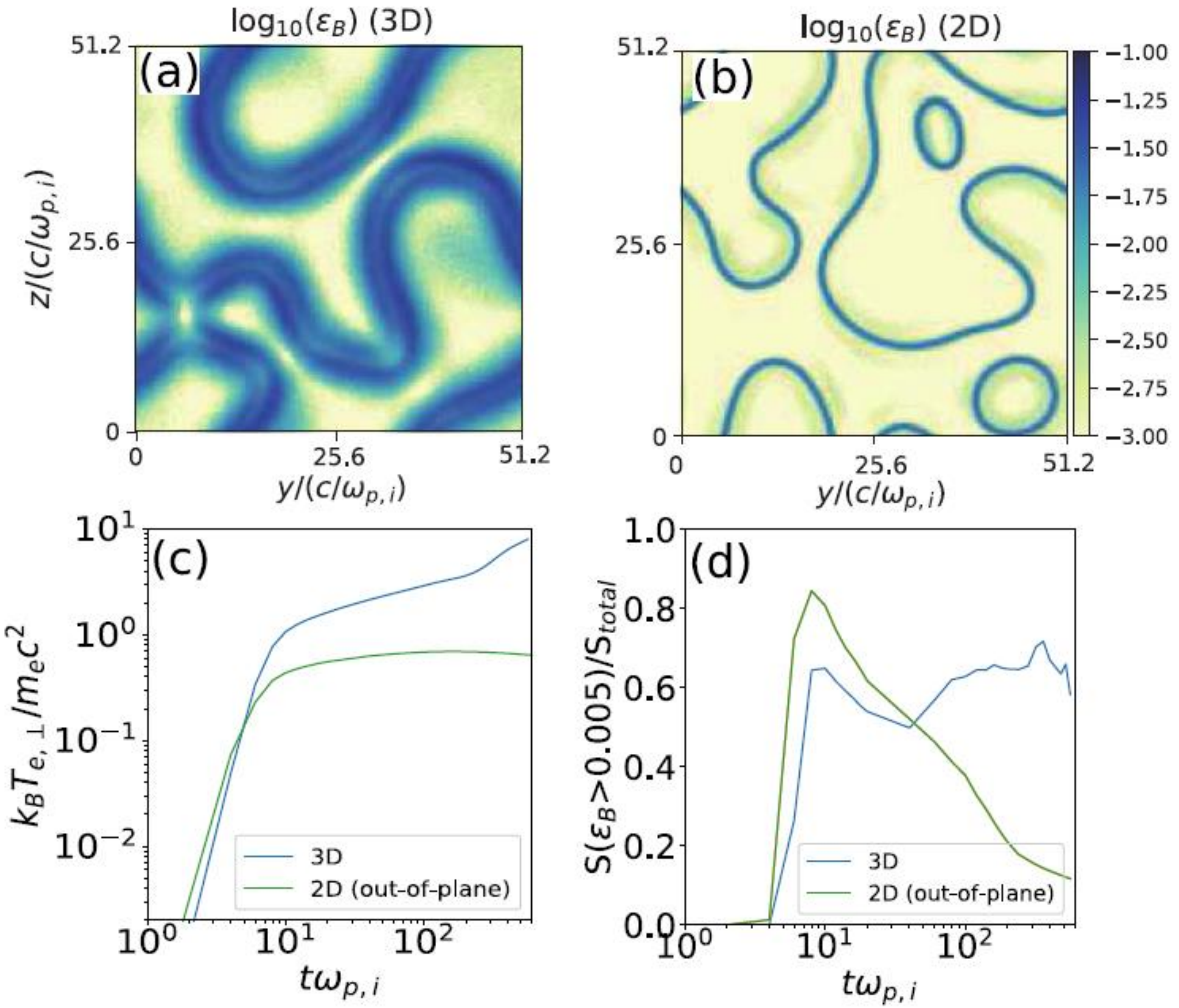
breakup of filaments in 2D in-plane case



Kumar+15ApJ

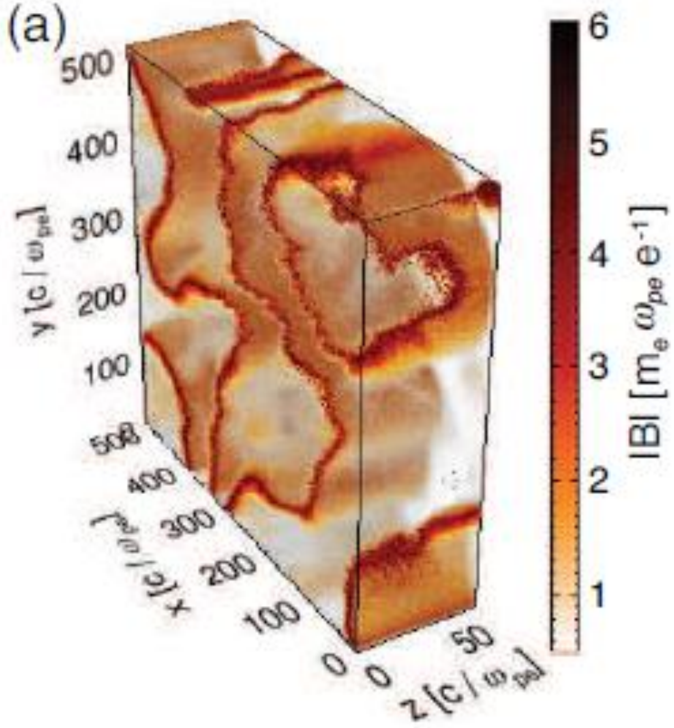
Dimensional limitation could cause two-stream electrostatic instabilities, resulting in breakup of current filaments in the 2D in-plane case (cf. Kumar+15)

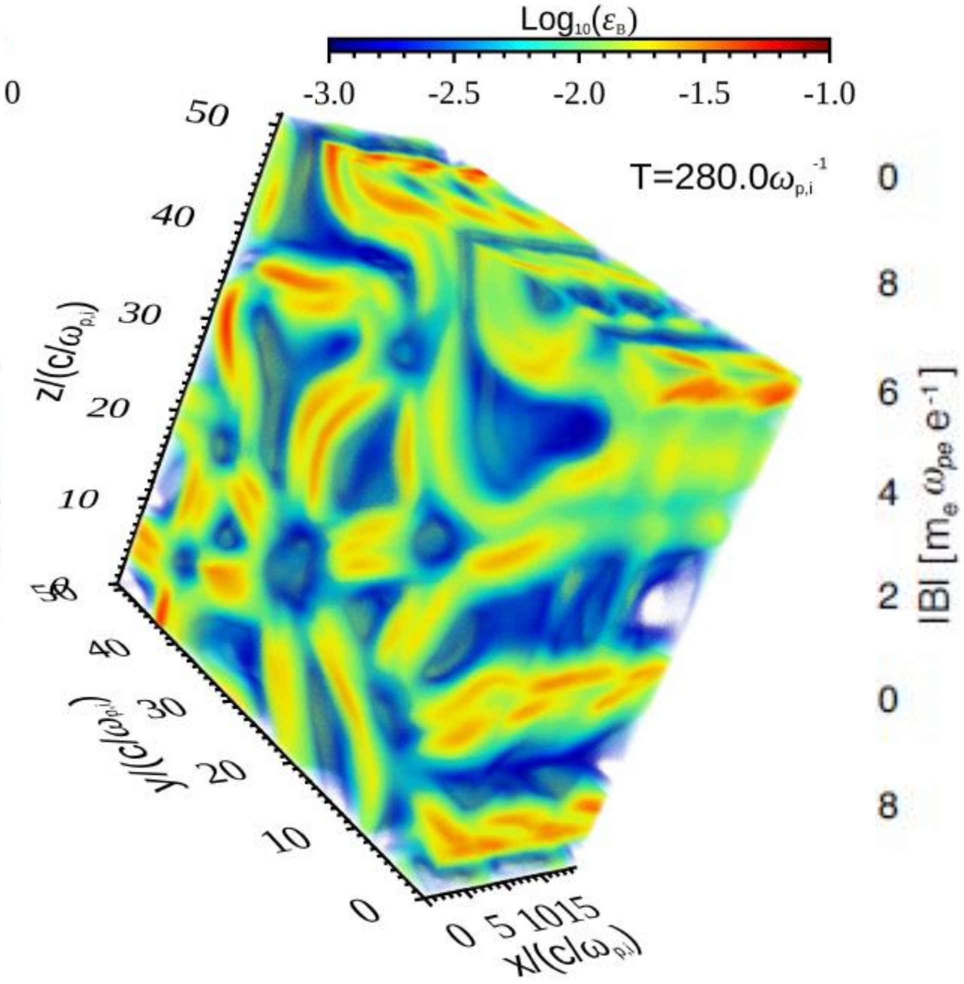
3D vs. 2D out-of-plane

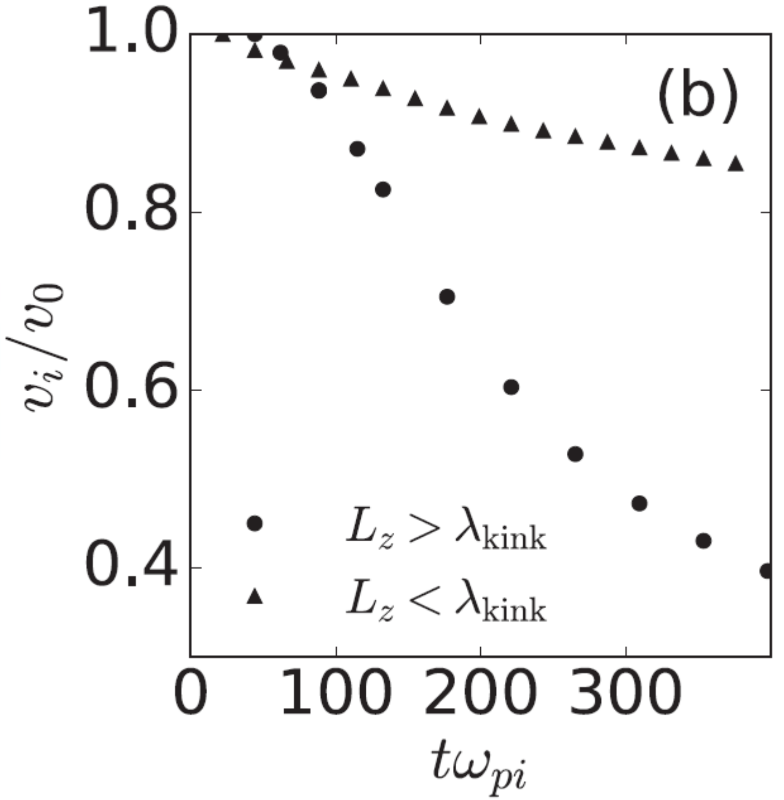


- Larger filing factor of large $\varepsilon_B$regions in 3D
- Continuous electron heating during coalescence of filaments un-magnetized electrons with relativistic temperature $(T_e \sim 10mc^2)$
- Relativistically hot electrons contributed ion-scale magnetic field generation

ion Weibel vs. Kink mode







Ruyer and Fiuza, PRL, 2018

まとめ

- Ikeya & Matsumoto15で開発したマジカルCFL法で、実用上数値チェレンコフ不安定問題は解決し、相対論的衝撃波の多次元構造が急速に明らかになりつつある
- Precursor waveの相対論的衝撃波面からの放射は幅広いσで普遍的に存在している（Iwamoto+17; 18）
- イオン・電子系での航跡場加速（Iwamoto+, in prep.）
- イオン・電子系での乱流リコネクション加速（$\sigma \ll 1$, cf. Matsumoto+ 15 Science）
- 超相対論的衝撃波（$\Gamma > M/m$）では、イオン加速も
- 3次元計算もしたい…けど「京」は来年度中に運用停止

Post-K supercomputer

- Next generation Japanese flagship supercomputer
- Present-K operation will terminate in FY2019
- Operation will start in 2021 (Approved to proceed to manufacturing phase from CPU design phase)
- No accelerator!
- CPU: Arm, 48cores, 512bitSIMD, 2.7TFlops
- Memory: HBM2 high band-width memory, but <1GB/core (32GiB/cpu)
- B/F=0.37 : high as GPUs
- Total Flops < 1Exa Flops, but x100 application speed up
- expected # of CPUs: 1EFlops/2.7TFlops ~ $3\text{x}10^5$ ($\sim 10^7$cores)