Spatial Profile of the Emission from Pulsar Wind Nebulae with steady-state 1D Modeling

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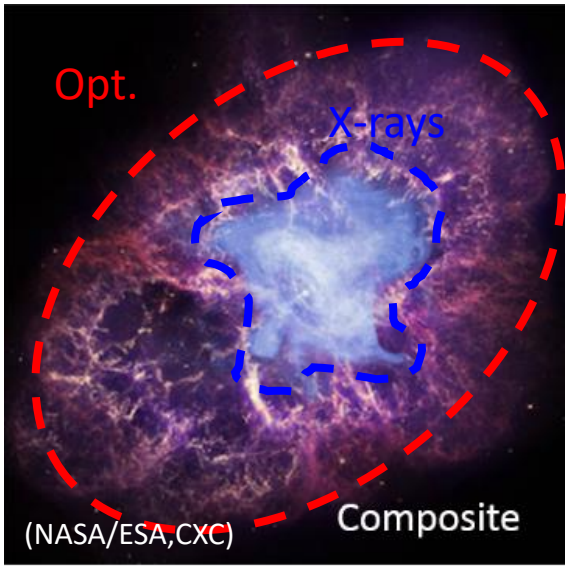
Abstract

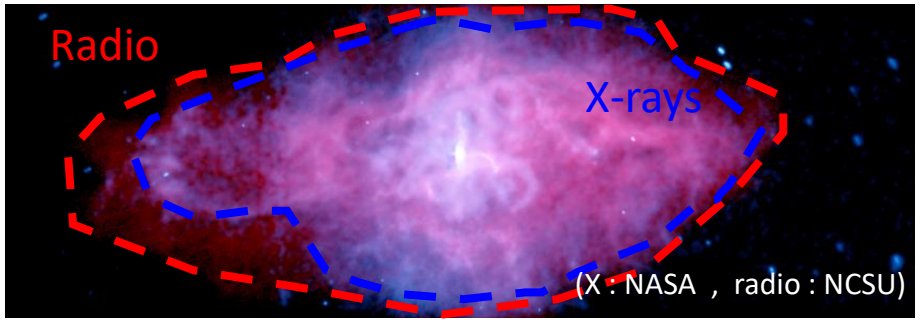
- The pulsar releases its rotational energy as highly relativistic wind
- This produces a pc-scale nebula, called as Pulsar Wind Nebula (PWN)
- The “standard model” of PWN based on Crab found general acceptance
- BUT the PWN which exhibits different feature from Crab is found
- We calculate both the entire spectrum and the spatial structure of emission spontaneously and examine the “standard model”

Crab Nebula



?

3C 58



The size of Crab nebula shrink with increasing frequency.
This fact is interpreted by synchrotron cooling.
BUT 3C 58 does not appear this feature…

Introduction

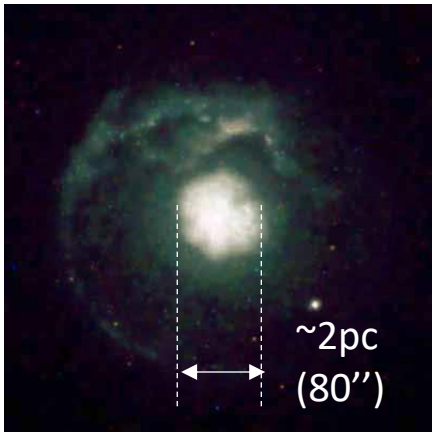
Introduction –Pulsar Wind Nebulae-

- Extended objects around the isolated pulsar
  - Center-filled morphology
  - Physical scale ~ a few pc
- Very broad non-thermal spectrum from radio to TeV-γ rays
  - Synchrotron and Inverse Compton radiation from shocked pulsar wind
- e.g. Crab Nebula (as known as “M1”)

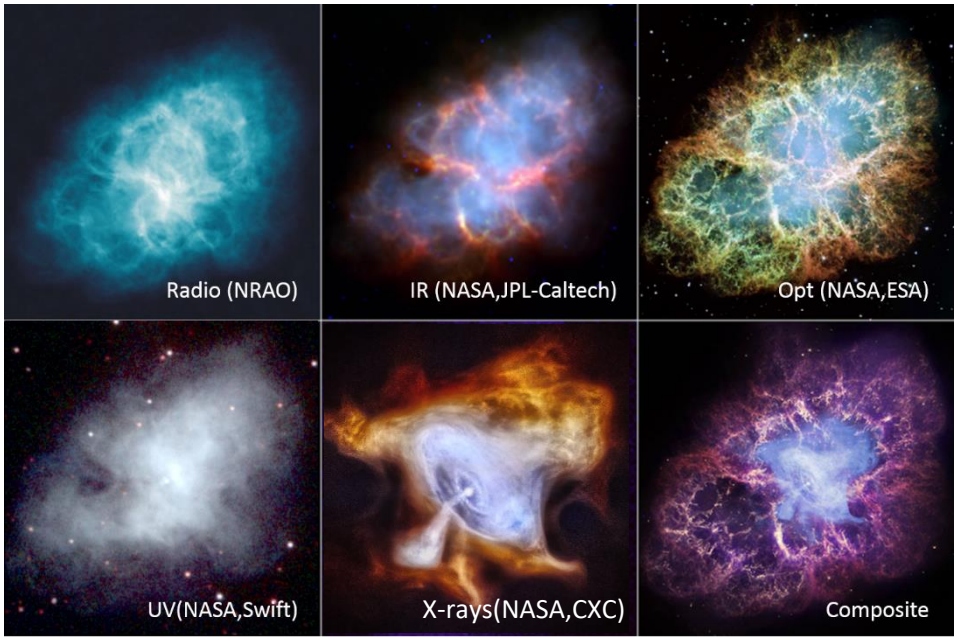
3C 58 (Blue :X , Red : Radio)

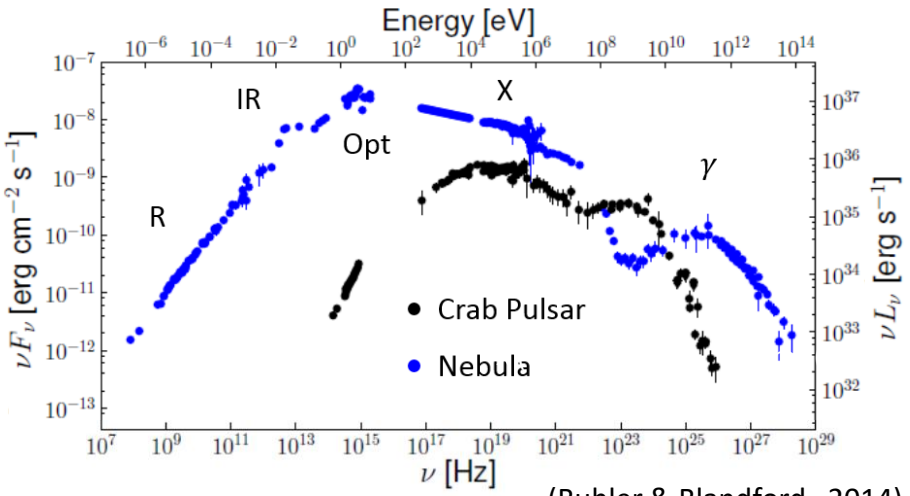
(X : NASA , radio : NCSU)

G21.5-0.9



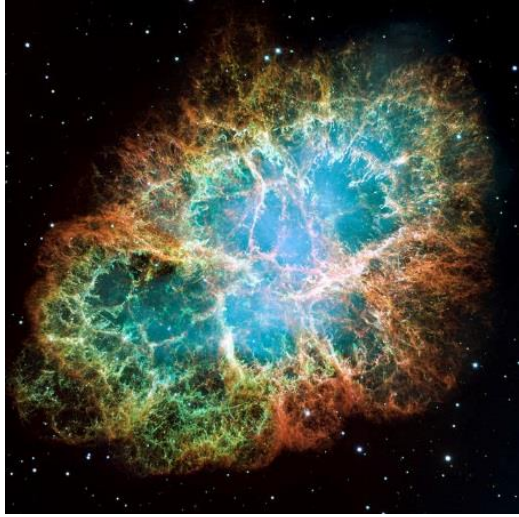
(NASA,CXC)



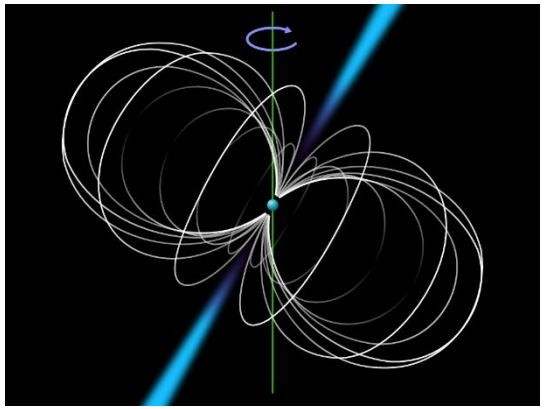


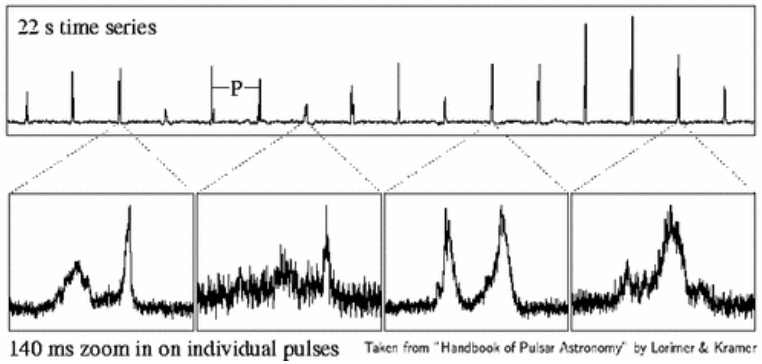
(Buhler & Blandford , 2014)

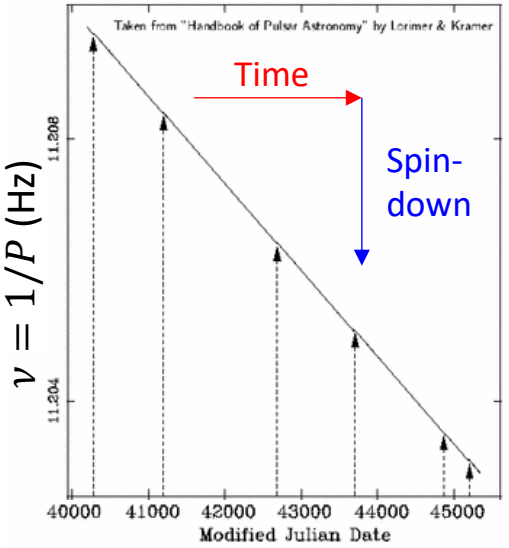
Introduction –Pulsar Wind Nebulae-



- Rotation-Powered Pulsar
  - Periodic radiation : $P \sim 1s$
  - Magnetic braking by strong $B \sim 10^{12}$G
- Pulsar releases its rotational energy
  - Time derivative of P is well observed : $\dot{P} \sim 10^{-(12-13)} ss^{-1}$
  - $L_{sd} = \left|I\Omega\dot{\Omega}\right| = 5 \times 10^{38}\mathrm{erg/s}\left(\frac{P}{33ms}\right)^{-3}\left(\frac{\dot{P}}{4.21\times10^{-13}}\right)$ (Crab pulsar)
- Pulsar Wind Nebula(PWN) is driven by pulsar
  - Pulse luminosity $\sim 1\% \times L_{sd} \ll L_{sd}$
  - (Nebula luminosity)+(Expanding power) $\sim L_{sd}$
  - Most of L_{sd} is injected to PWN

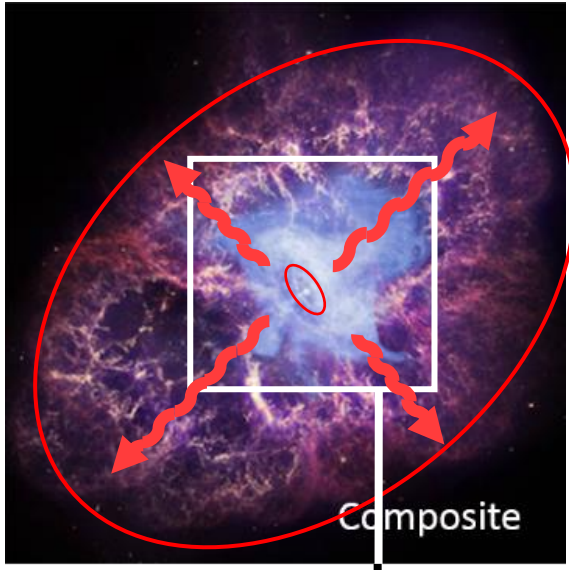






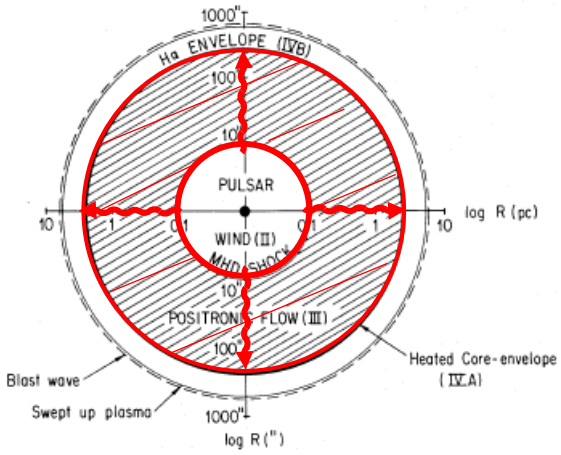
Introduction –Pulsar Wind Nebulae-

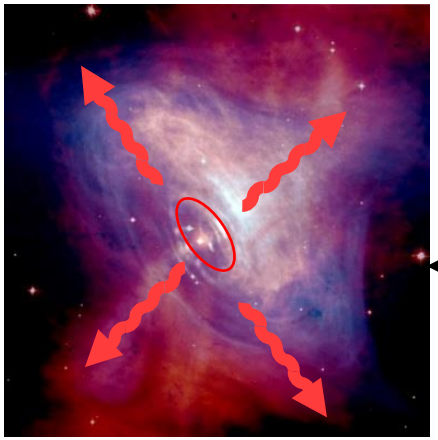
Blue : X-rays
Red : Opt.



(NASA/ESA)

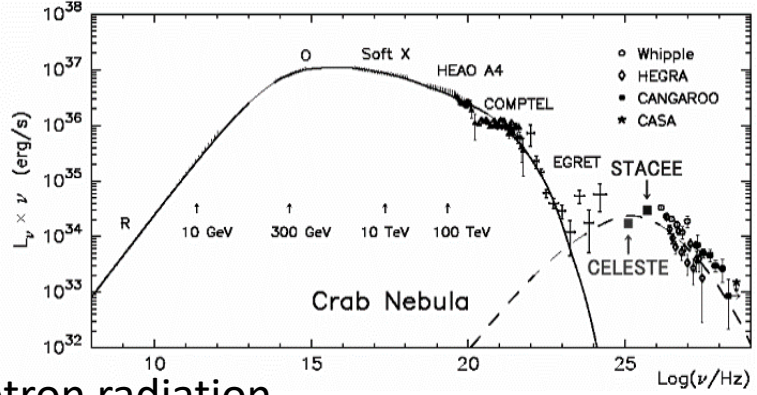
- Pulsar Wind
  - Outflow which brings out pulsar rotational energy
  - Highly relativistic wind (bulk $\Gamma \sim 10^6$) consists of $e^{\pm}$
  - The strong shock is formed due to the interaction with interstellar matter
- Kennel & Coroniti (1984); Standard model for PWN (hereafter KC model)
  - 1D and steady state MHD model
  - The non-thermal $e^{\pm}$ are produced by shock acceleration
  - Due to the effect of radiative cooling, the emission region becomes narrow with increasing frequency





(NASA/CXC/SAO)

Introduction -1D steady model-

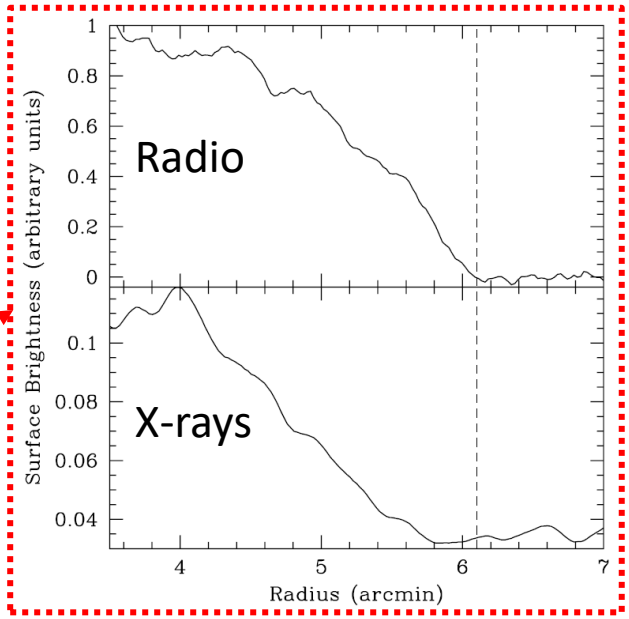


- Previous studies of 1D steady model for PWN
  - Kennel & Coroniti (1984) ; MHD model + synchrotron radiation
  - Atoyan & Aharonian (1996)
    - Reproducing the whole spectrum of Crab nebula by considering the Inverse Compton
- Raised problems of 1D steady model
  - KC model is ONLY confirmed by Crab nebula
    → Can KC model explain a spectrum of general PWN?
  - There are some X-ray PWNe that extend the same as radio
    → Different behavior from Crab nebula…

Entire spectrum
＋
Spatial structure of emission

Can we reproduce these simultaneously?

↑3C58 (X(blue) : NASA , radio(red) : NCSU)



Model

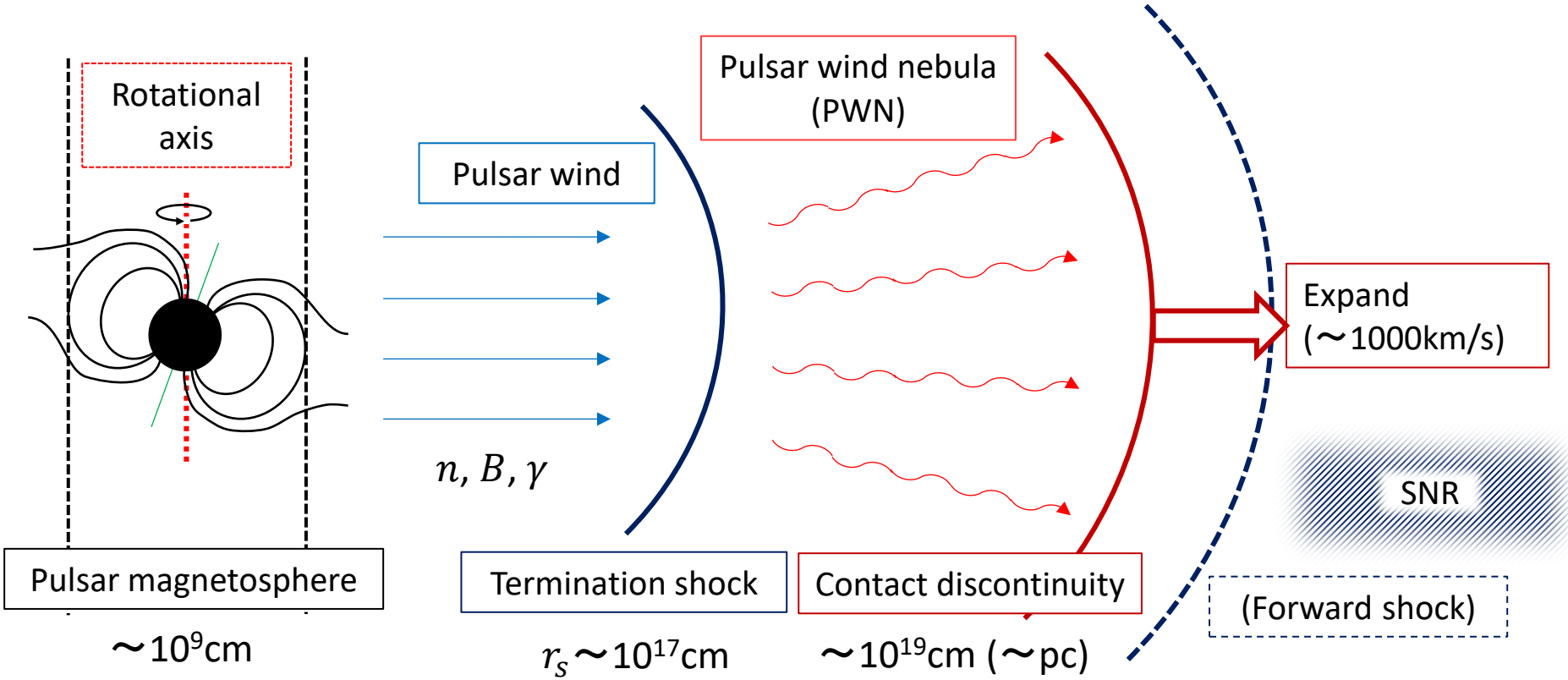
Model –Overview-

- Schematic view of KC model
  - Pulsar wind forms the shock structure
  - Downstream flow is free stream
  - Assuming that the energy distribution at r_s
  - Propagating in PWN with radiative cooling

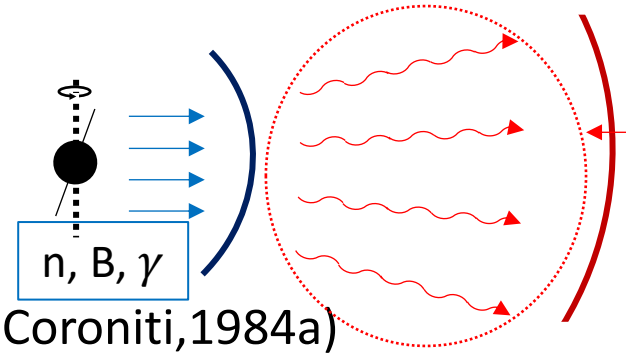
⇒Jump condition at termination shock

⇒Sub-sonic steady flow

⇒Evolution of energy distribution



Model –Flow Solution-



- MHD model of PWN (Kennel & Coroniti,1984a)
  - Up-stream : Pulsar wind (unshocked)
    - Assumptions : Cold Plasma, toroidal field & Highly relativistic wind
  - σ parameter
    - Assuming that the downstream is adiabatic flow, downstream is only characterized by (up-stream) σ parameter.

$$\sigma = \frac{B_1^2/4\pi}{n_1 u_1 \gamma_1 mc^2} = \frac{(\text{Poynting flux})}{(\text{particle energy flux})}$$

$$\frac{d}{dr}\left[nur^2\right] = 0$$

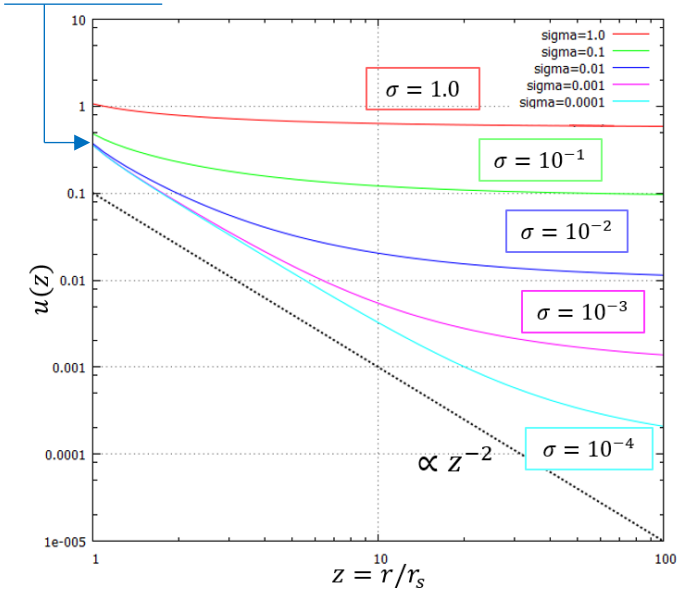
$$\frac{d}{dr}\left[\frac{ruB}{\gamma}\right] = 0$$

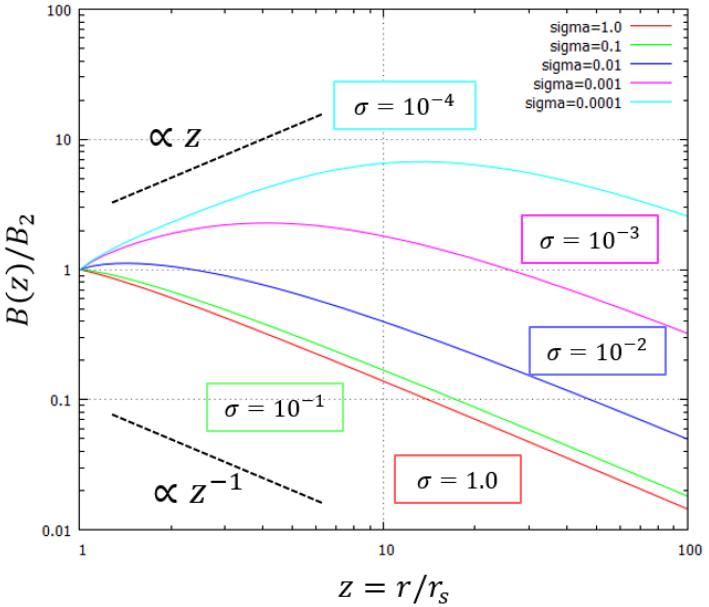
$$nur^2 \frac{d}{dr}\left[\gamma\mu + \frac{B^2}{4\pi n\gamma}\right] = 0$$

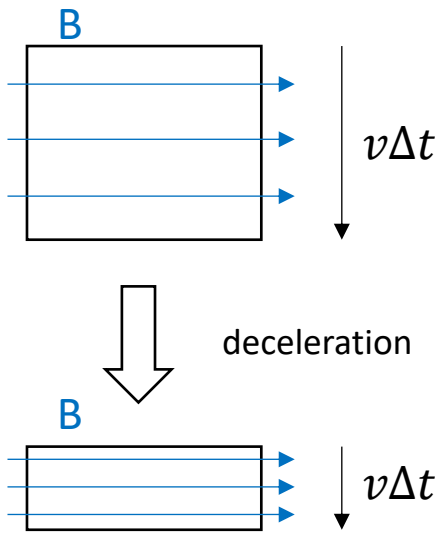
$$\frac{d}{dr}\left(ur^2 ne\right) + p\frac{d}{dr}\left(ur^2\right) = 0$$

$$\mu = mc^2 + \frac{\Gamma_c}{\Gamma_c - 1}\left(\frac{P}{n}\right)$$

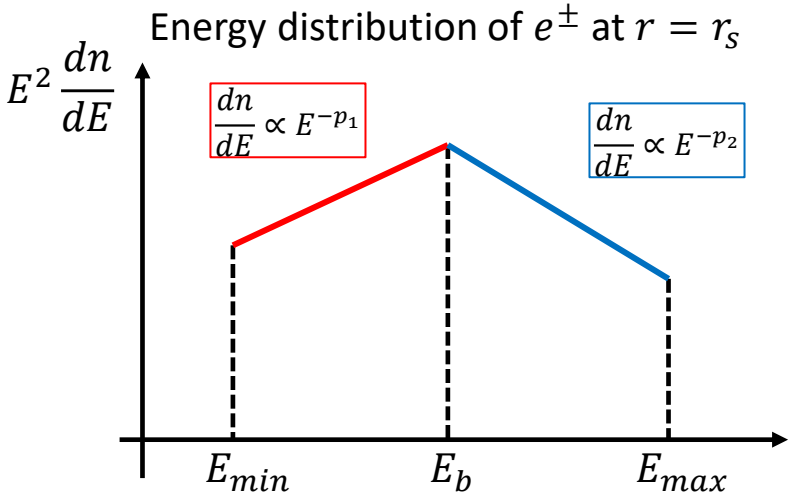
$v \sim \frac{c}{3}$



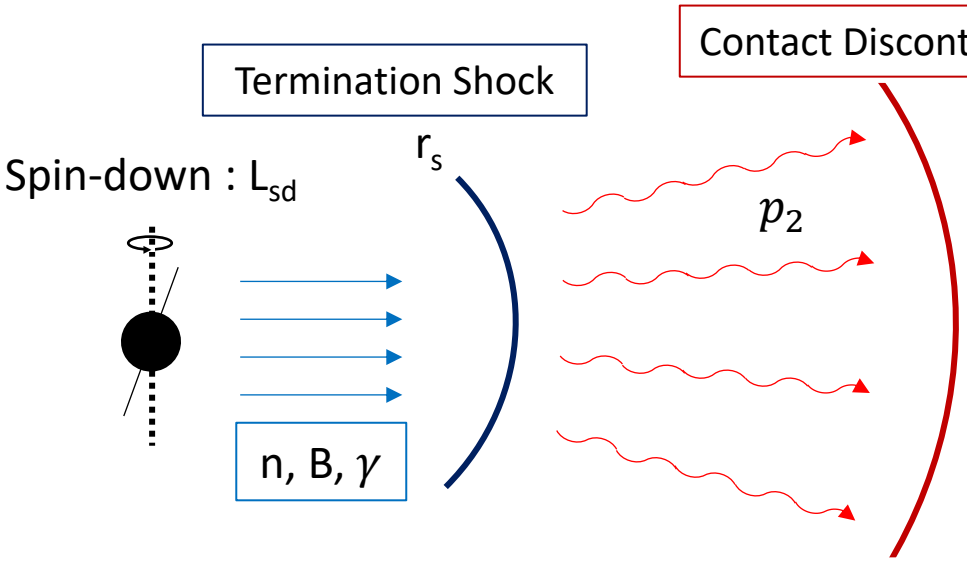




Model –Injection & Parameters-



- Boundary condition
  - Presuming broken power-law form
- Assumption/Observable quantities
  - $E_{\min} = 10mc^2$ is fixed
  - $E_{\max}$ is determined by "size limited" : $E_{\max} = eB_{up}r_s$
  - p_1 is corresponding to radio index, which is well observed. (e.g. Crab $p_1 \sim 1.6$)
  - All the spin-down power L_{sd} is converted to non-thermal $e^{\pm}$ and flow kinetic energy



Injection power to PWN

$$L_{sd} = 4\pi r_s^2 cn\gamma^2 mc^2(1+\sigma)$$

Observable

n, B, γ → L_{sd}, σ, γ → L_{sd}, σ, E_b

Jump condition

4 free parameters σ, E_b, r_s, p_2

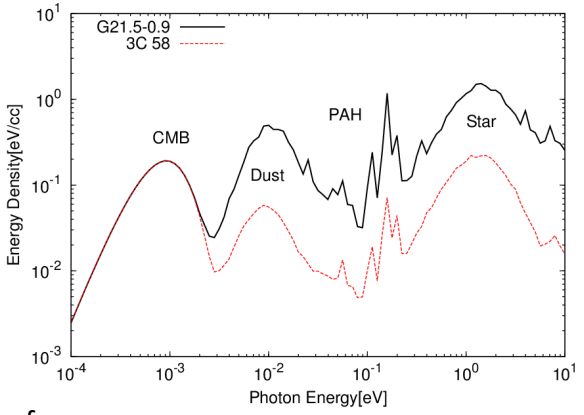
Model –Energy distribution & Radiation-

- Evolution of non-thermal $e^{\pm}$
  - n(E,r) : the energy spectrum of $e^{\pm}$ at radius r
  - $\frac{\partial n}{\partial t}^{0} + u\frac{\partial n}{\partial r} = \frac{\partial}{\partial E}\left(Q_{\rm syn}E^2 n\right) + \frac{\partial}{\partial E}\left(Q_{\rm IC}E^2 n\right) + \frac{\partial}{\partial E}\left(\frac{c}{3r^2}\frac{d}{dr}\left(ur^2\right)En\right) - \frac{c}{r^2}\frac{d}{dr}\left(ur^2\right)n$

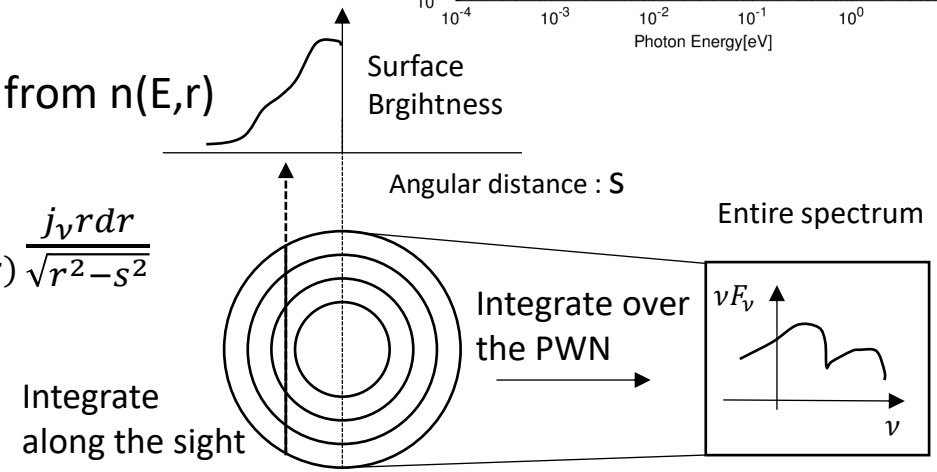
Flow solution given by MHD model

Convective derivative | Synchrotron cooling | IC cooling | Adiabatic cooling | Volume expansion

- Calculation of photon spectrum
  - Synchrotron radiation
    - The magnetic field B(r) given by flow solution
  - Inverse Compton scattering
    - The model of seed photon is taken from GALPROP
    - Fully taking into account the Klein-Nishina effect



- Observable quantities
  - The spectral emissivity j_ν is obtained from n(E,r)
  - Flux : $F_\nu = \frac{1}{4\pi D^2}\int_{r_s}^{r_{\rm N}} j_\nu 4\pi r^2 dr$
  - Surface brightness : $B_\nu(s) = \int_{\min(r_s,s)}^{r_{\rm N}} \frac{j_\nu r dr}{\sqrt{r^2-s^2}}$



Test Calculation

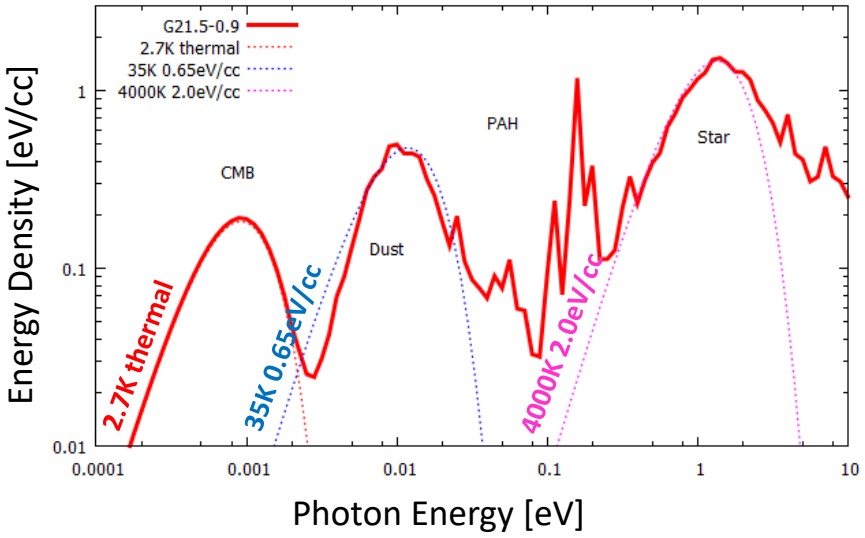
Test Calculation -configurations-

$$t_{\rm adv} \equiv \int_{r_s}^{r_{\rm N}} \frac{dr}{cu(r)}$$

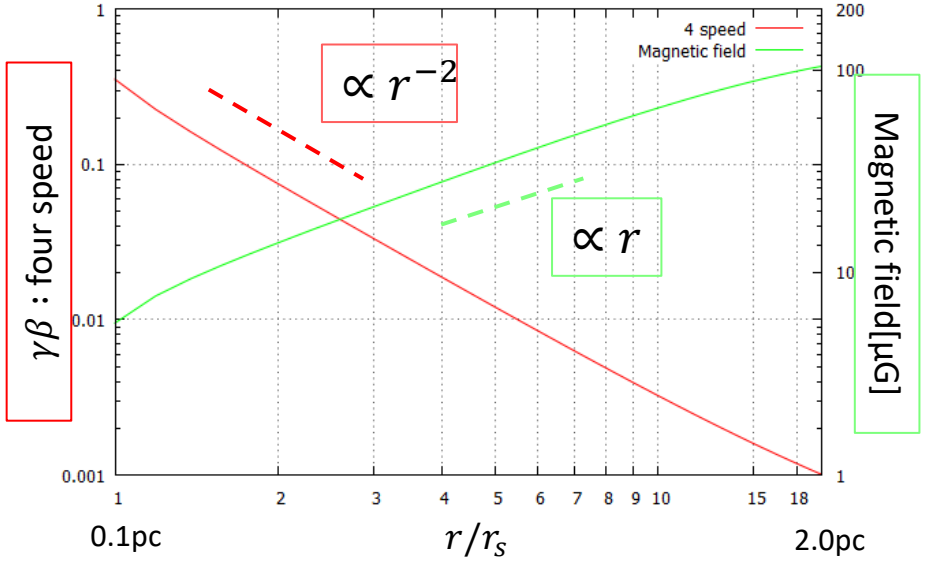
- Parameters

Free Parameter	Adopted Parameter	Obtained parameter
$\sigma = 1.0 \times 10^{-4}$	$L_{sd} = 10^{38}$[erg/s]	$E_{\rm max} = 3.4 \times 10^{8} mc^2$
$r_s = 0.1$[pc]	$E_{\rm min} = 10mc^2$	$B_1 = 1.9$[μG]
$E_b = 10^5 mc^2$	$p_1 = 1.1$	$n_1 = 4.5 \times 10^{-12}$[/cc]
$p_2 = 2.5$	$D = 2$[kpc]	$t_{\rm adv} = 2411$[yr]
	$r_N = 2$[pc]	

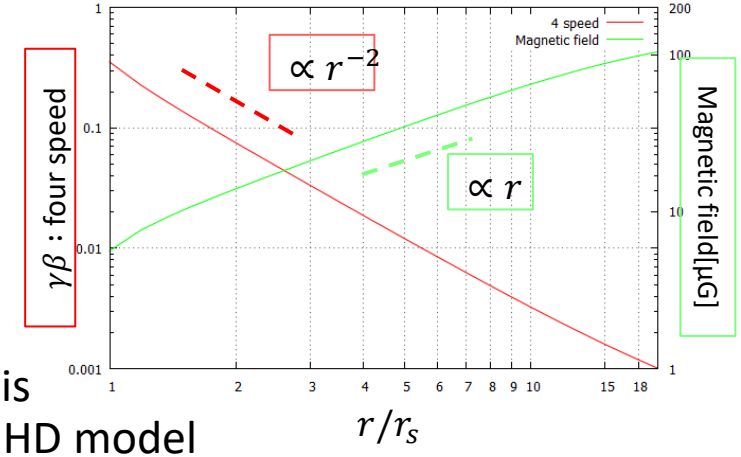
- Interstellar Radiation field



- Flow structure



Test Calculation -energy spectrum-



- Energy spectrum of non-thermal $e^{\pm}$
  - At $r = r_s$, absolute value of distribution function is determined so as to keep the consistency with MHD model
  - So the synchrotron life-time $\propto E^{-1}$, maximum energy of spectrum becomes lower with increasing radius r
  - Effect of inverse Compton cooling is negligible
  - Adiabatic cooling appears as a simultaneous energy-loss

160eV/cc

$$\left(\frac{dE}{dt}\right)_{\rm syn} = \frac{4}{3}\sigma_T c\gamma^2 U_B$$

2eV/cc

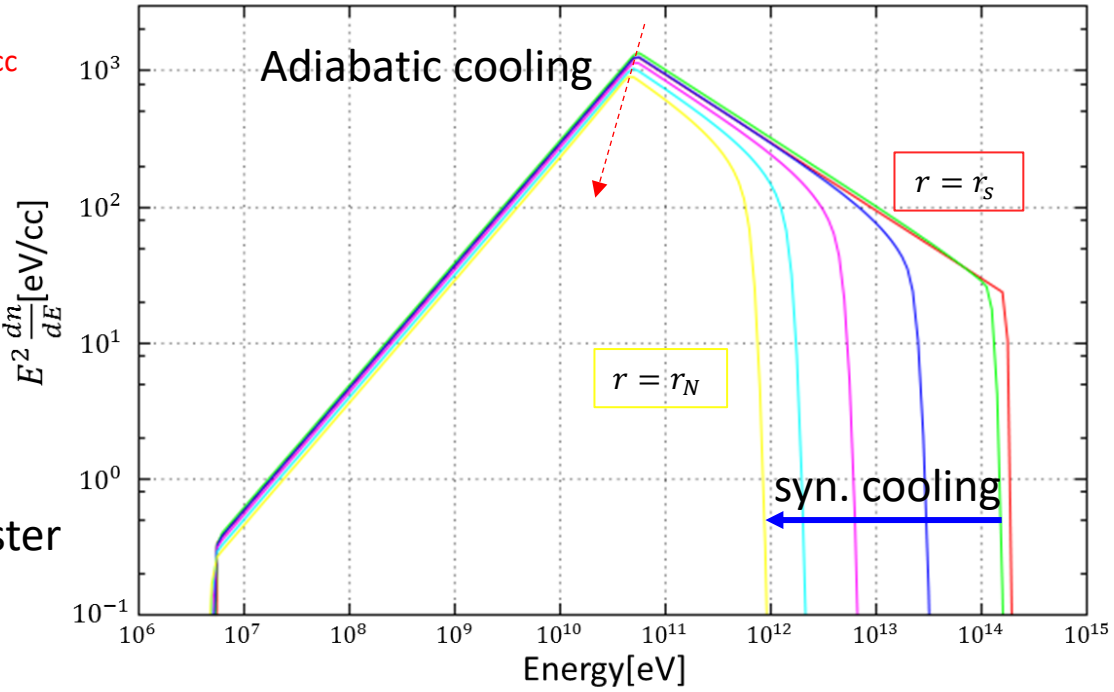
$$\left(\frac{dE}{dt}\right)_{\rm IC} \sim \frac{4}{3}\sigma_T c\gamma^2 U_{\rm ph}$$

$U_B \gg U_{\rm ph}$ is established

⇒ Synchrotron cooling dominated

$$t_{\rm cool} \sim \frac{E}{\left(\frac{dE}{dt}\right)_{\rm syn}} \propto E^{-1}$$

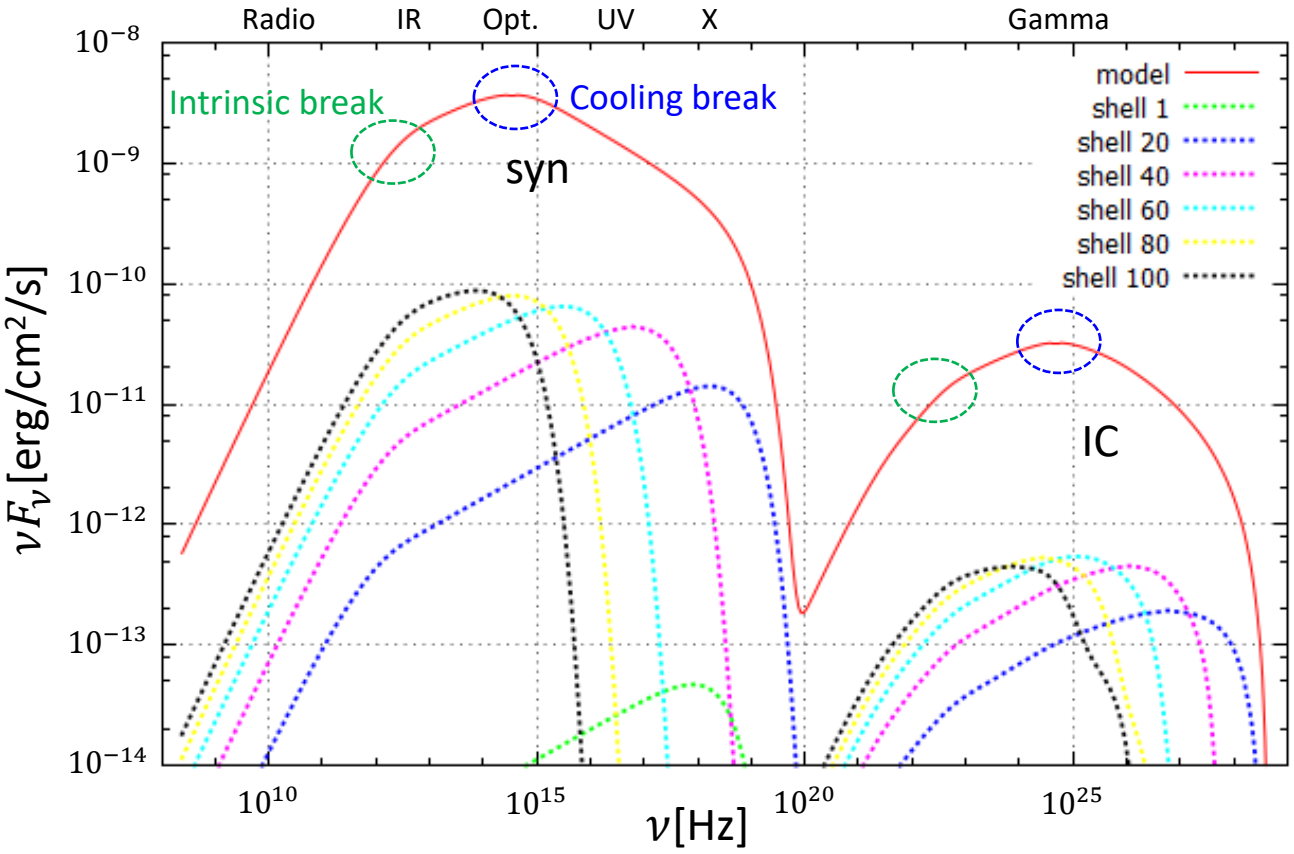
⇒ Higher energy electrons exhaust faster



Test Calculation -Spectrum-

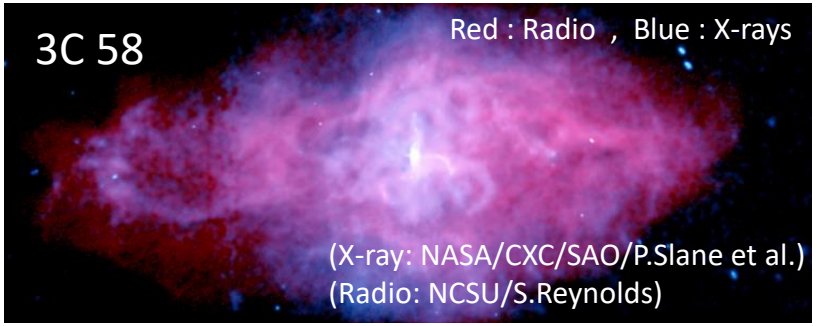
- Photon spectrum of entire nebula
  - Two breaks appear : intrinsic break & cooling break
  - Synchrotron spectrum is harder than IC

    → Due to the increasing magnetic field strength with radius r (In contrast, $U_{\rm ph}$ is uniform)

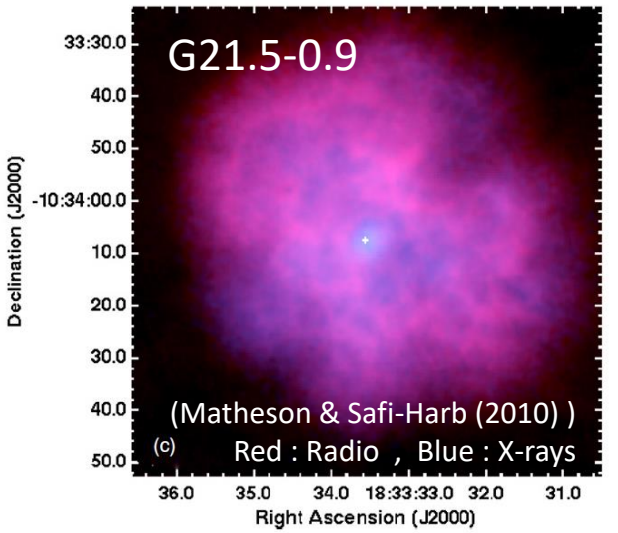


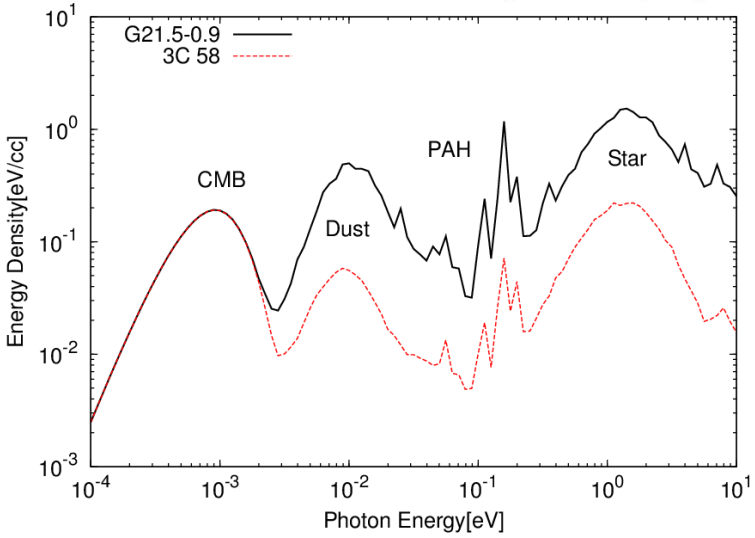
Application

Target



- Selection criterion
  - Enough flux measurement at various frequencies
  - X rays PWN extend same as radio
- G21.5-0.9
  - Angular size : 40'' in radius
  - Distance : D=4.8kpc (Tian & Leahy 2008)
    → Nebula radius : $r_{\mathrm{N}} = 0.9\mathrm{pc}$
  - Central pulsar : PSR J1833-1034
  - P=61.9ms , $\dot{P} = 2.02 \times 10^{-13} ss^{-1}$
    → $L_{sd} = 3.3 \times 10^{37}$[erg/s]
- 3C 58
  - Angular size : 5' × 9'
  - Distance : D=2.0kpc (Kothes et al.,2013)
    → Nebula radius : $r_{\mathrm{N}} = 2.0\mathrm{pc}$
  - Central pulsar : PSR J0205+6449
  - P=65.7ms ,$\dot{P} = 1.94 \times 10^{-13} ss^{-1}$
    → $L_{sd} = 2.7 \times 10^{37}$[erg/s]

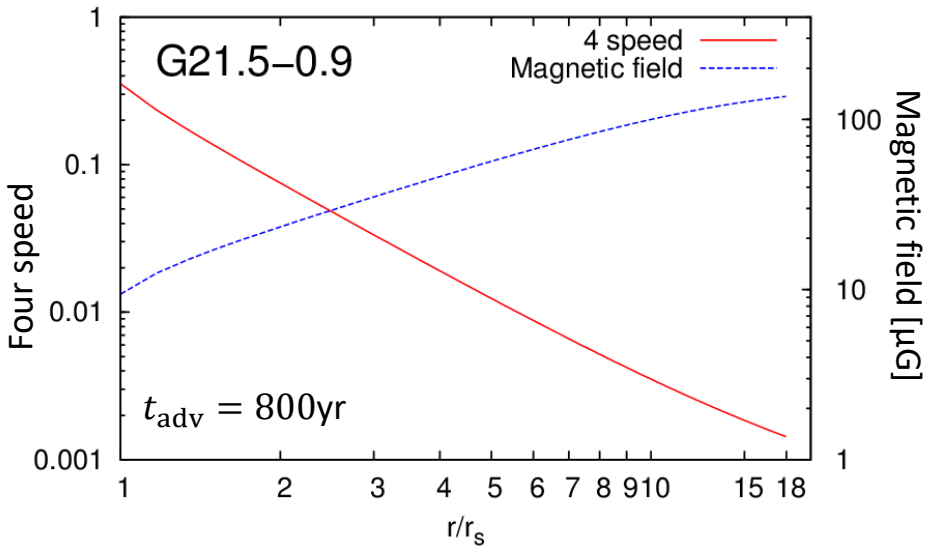


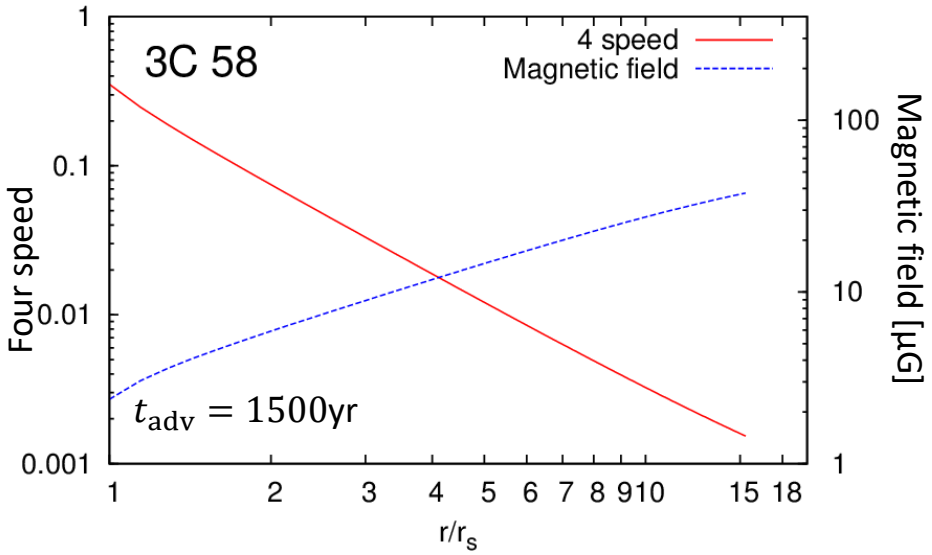


Result -Entire Spectrum-

	G21.5-0.9	3C 58
σ	2.0×10^{-4}	1.0×10^{-4}
r_s	0.05 pc	0.13 pc
E_{b}	$5.0 \times 10^4 mc^2$	$6.0 \times 10^4 mc^2$
p_2	2.3	3.0

- Fitting
  - Due to the enrichment of observational data, the model parameters can be obtained by fitting the entire spectrum only
- Note
  - In GeV range, there is large systematic error caused by subtracting the contribution of the central pulsar
    → We regarded the Fermi detection of 3C 58 as upper limit
  - There are NO natural parameter sets that can reproduce the X-ray index
    → We referred to only the absolute value of flux at X-ray datas





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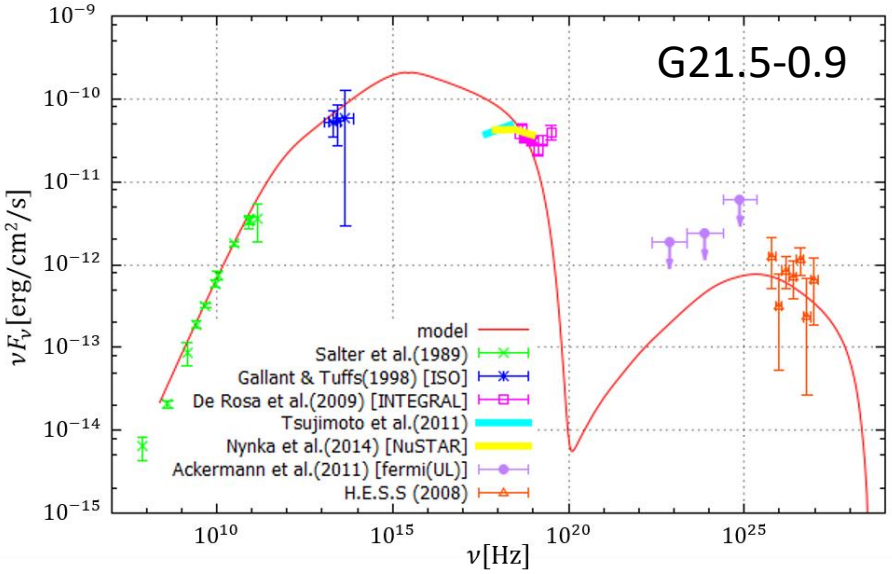
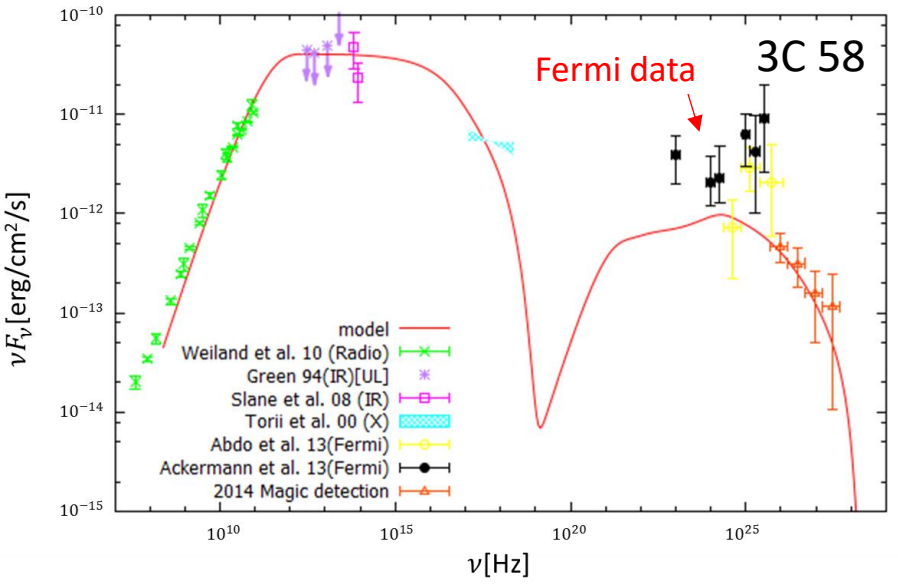
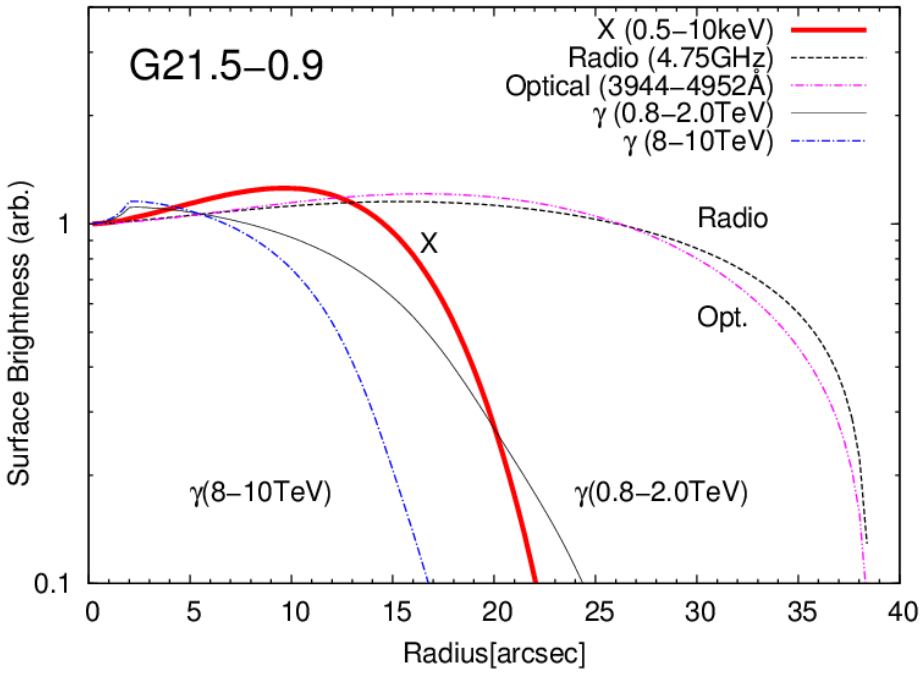


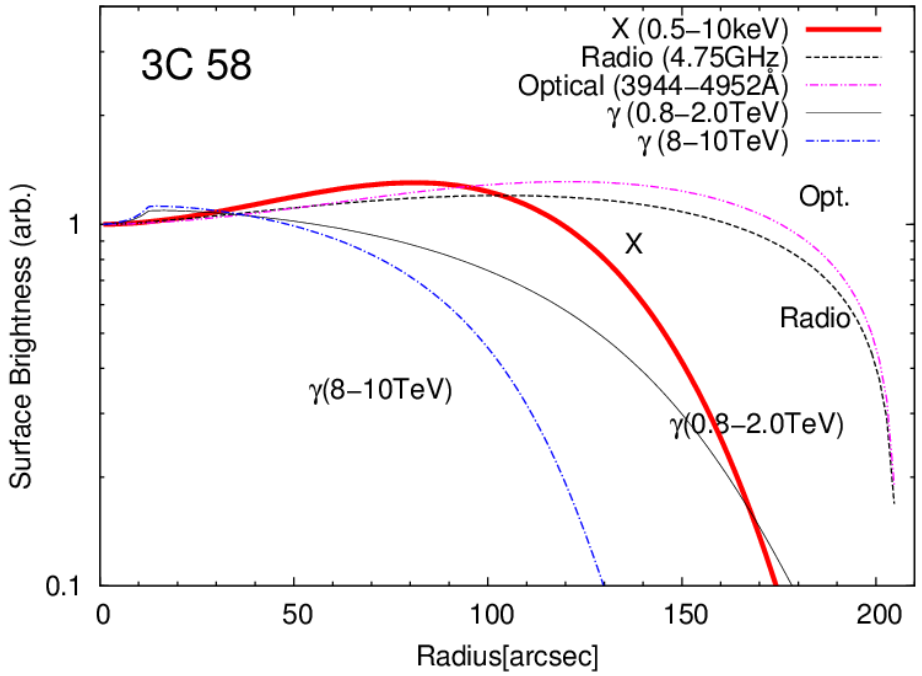


Result -Surface Brightness-

	G21.5-0.9	3C 58
σ	2.0×10^{-4}	1.0×10^{-4}
r_s	0.05 pc	0.13 pc
$E_{\rm b}$	$5.0 \times 10^4 mc^2$	$6.0 \times 10^4 mc^2$
p_2	2.3	3.0

- Surface brightness
  - Radial profile of the surface brightness for various frequencies
  - The Cut-off feature appears due to the two different reasons
    - At the edge of nebula -> “Confinement”
    - Inner nebula -> “Synchrotron Cooling”
  - The X-rays nebulae are smaller than radio on both G21.5-0.9 and 3C 58



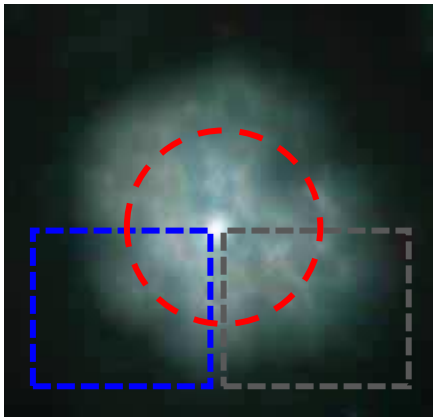


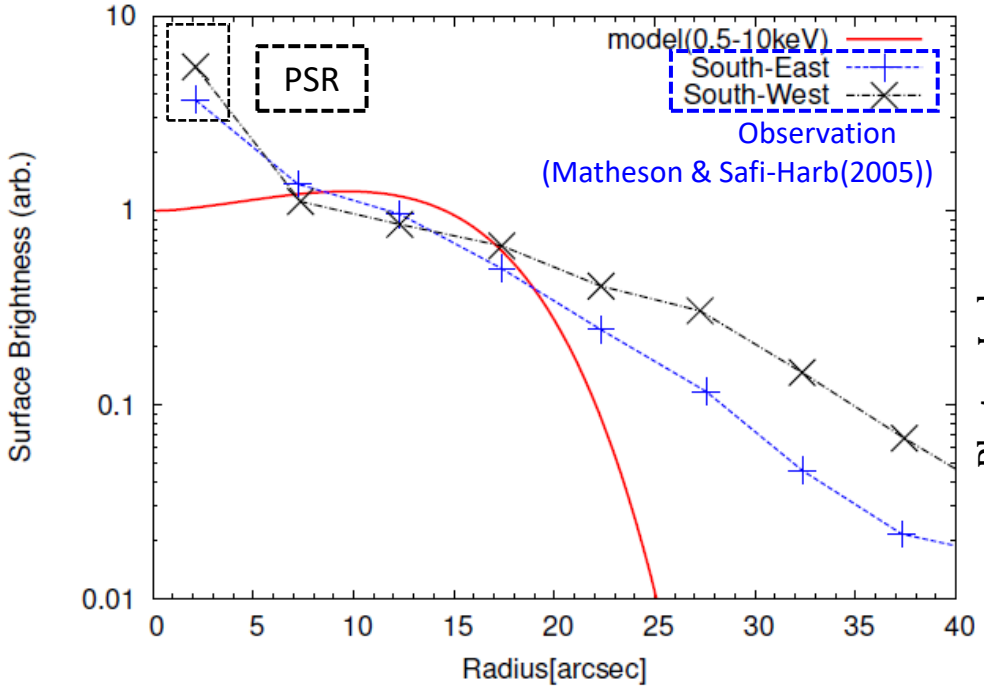
Result -Surface Brightness-

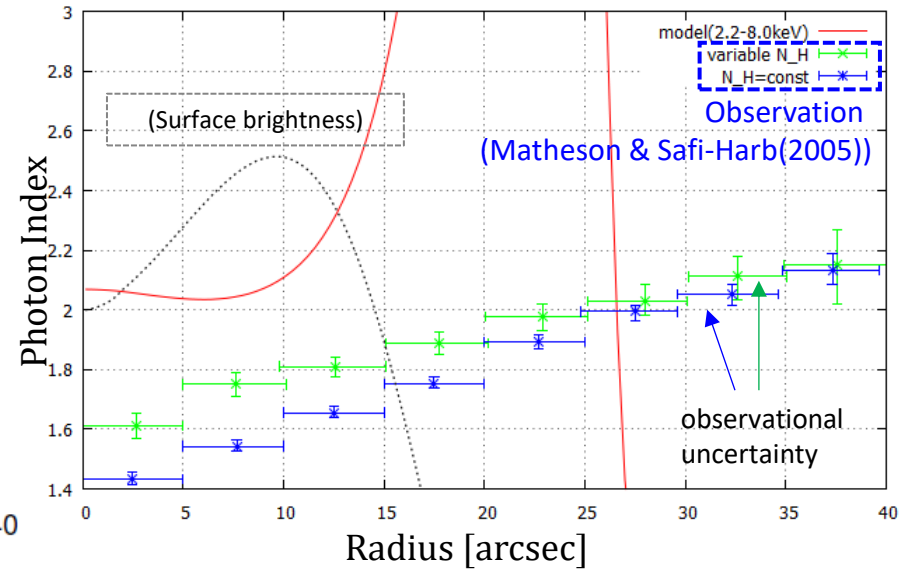
	G21.5-0.9	3C 58
σ	2.0×10^{-4}	1.0×10^{-4}
r_s	0.05 pc	0.13 pc
$E_{\rm b}$	$5.0 \times 10^4 mc^2$	$6.0 \times 10^4 mc^2$
p_2	2.3	3.0

- Comparison with observation
  - Surface brightness
    - Calculated emission region is smaller than observation
  - Photon index
    - The sudden steepening appears

The model can reproduces the entire nebula spectrum, BUT the spatial structure of emission is incompatible

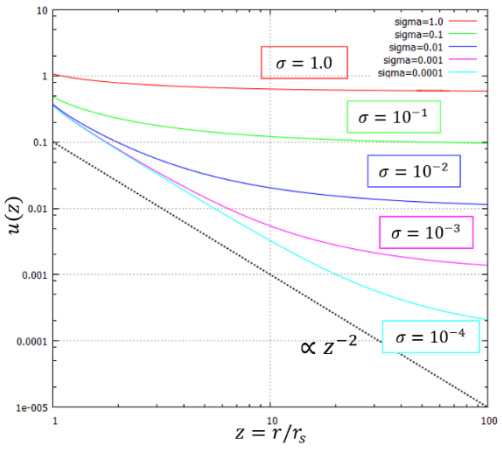






Discussion & Conclusion

Discussion –Parameter dependence-

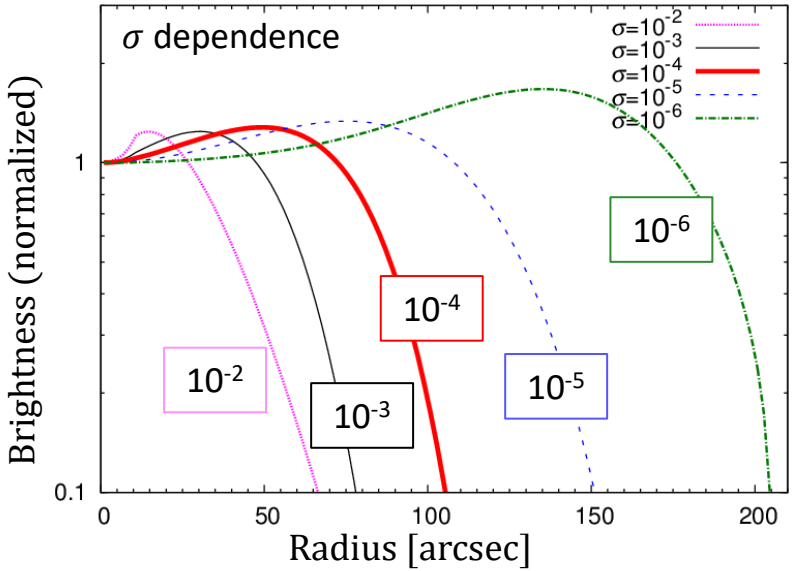


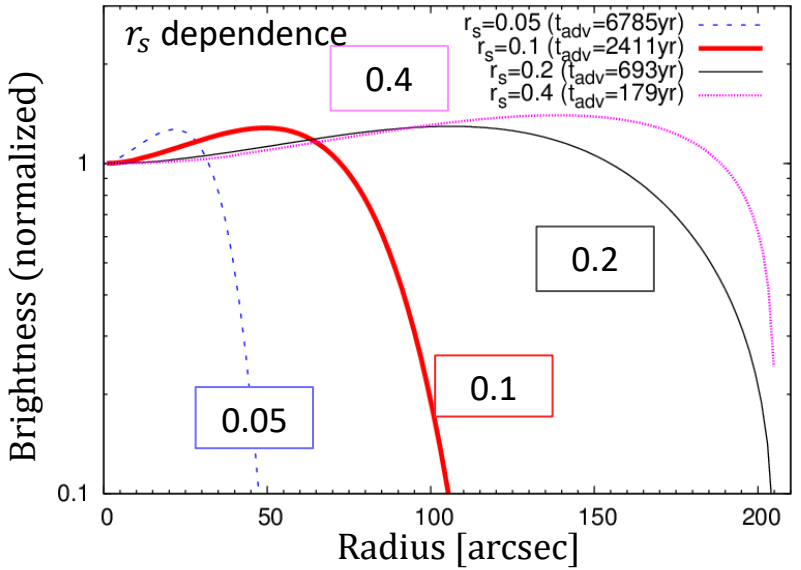
- Which parameter does change the emission profile?
  - σ : determines the flow structure
  - r_s : determines the characteristic scale of flow
  - Both σ and r_s are determines the strength of magnetic field
- The parameter dependence of surface brightness
  - σ →small : Magnetic field↘ ⇒ Cooling becomes inefficient
  - r_s →large : Scale of spatial variation ↗

$$B_1 = \sqrt{\frac{L_{sd}}{cr_s^2}\frac{\sigma}{1+\sigma}}$$

$$\left(\frac{dE}{dt}\right)_{syn} = \frac{4}{3}\sigma_T c\gamma^2 U_B$$

To reproduce the larger X-ray PWN, smaller σ and larger r_s is required





Discussion -Parameter dependence-

$$B_1 = \sqrt{\frac{L_{sd}}{cr_s^2}\frac{\sigma}{1+\sigma}}$$

$$\left(\frac{dE}{dt}\right)_{\rm syn} = \frac{4}{3}\sigma_T c\gamma^2 U_B$$

$$\left(\frac{dE}{dt}\right)_{\rm IC} = \frac{4}{3}\sigma_T c\gamma^2 U_{\rm ph}$$

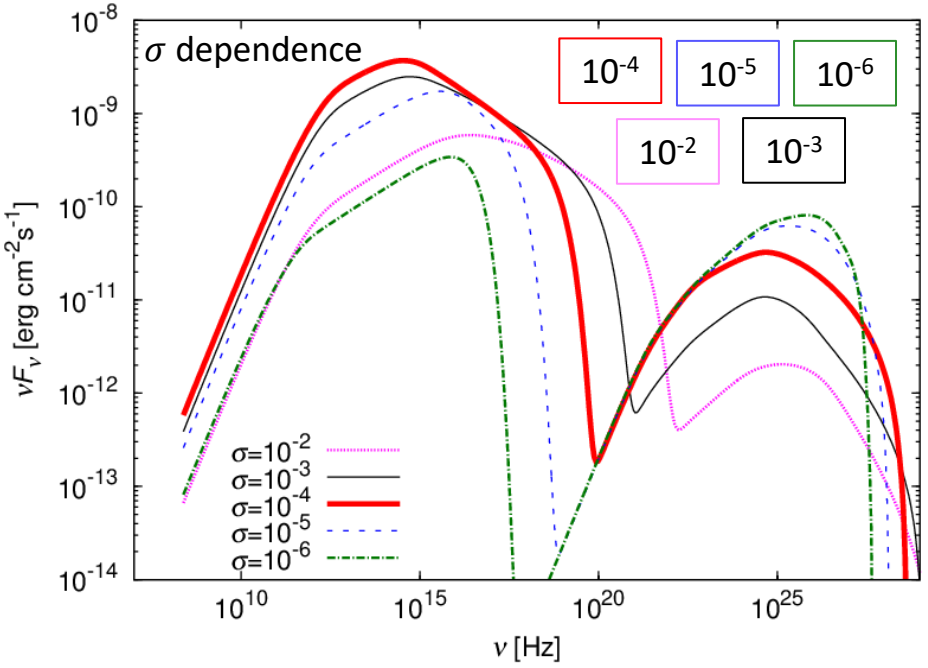
$$\frac{P_{\rm syn}}{P_{\rm IC}} = \frac{U_B}{U_{\rm ph}} \propto \sigma r_s^{-2}$$

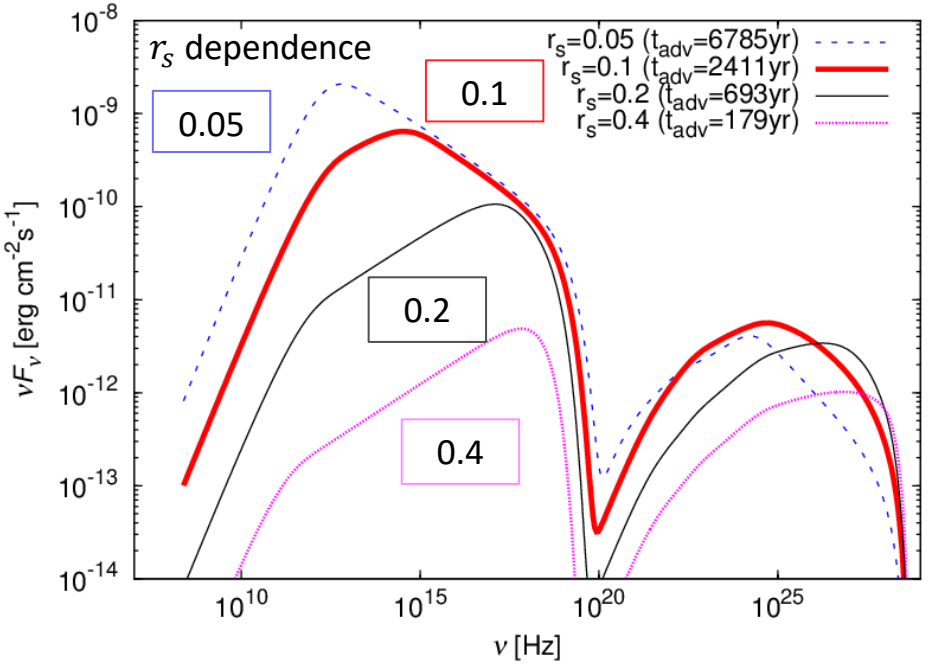
- Smaller σ and Larger r_s
  - These mean to take the lower magnetic field
  - In the case of G21.5-0.9,

$$\frac{P_{\rm syn}}{P_{\rm IC}} \gtrsim 10 \Rightarrow B \gtrsim 30[\mu{\rm G}] \quad \text{(c.f. Best-Fit } B \sim 120[\mu{\rm G}])$$

The magnetic field expected by fitting entire spectrum is too strong to reproduce the observed expanse of the X-ray PWN.

⇒ KC model CANNOT explain the spatial structure of PWNe!



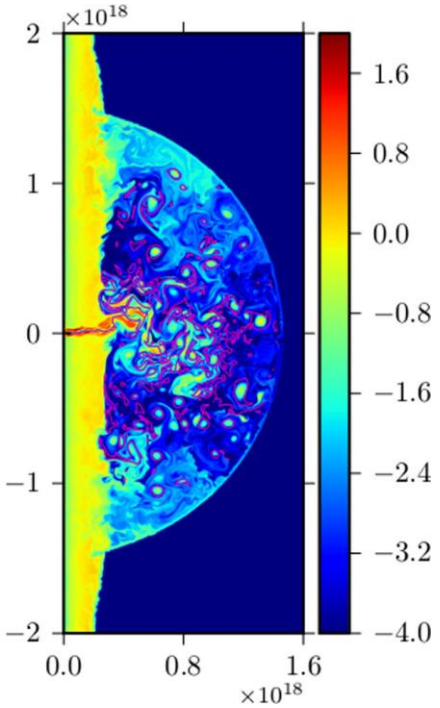


Summary

- We investigate the 1-D model of PWNe based on Kennel & Coroniti (1984), and apply this model to two objects, G21.5-0.9 and 3C58.
- We find that the KC model can reproduce the entire spectrum but the spatial structure (surface brightness and photon index) is incompatible.
- In this model, the magnetic field required by the entire spectrum is too strong to reproduce the spatial structure of emission.
- That is to say, the high-energy electrons that emits the X-rays suffer the excessive synchrotron cooling.

Future Work

- Observational study
  - The low energy component of electrons does not suffer synchrotron cooling.
    ⇒ Observing PWNe by radio, we can obtain the information about the spatial structure of the magnetic field and electron density. BUT these degenerates.
  - If we investigate the spatial structure by γ-rays, we can obtain the spatial distribution of electrons independently
    ⇒ CTA will achieve the spatially resolved observation of PWN
- Improvement of the model
  - Recent study by MHD simulation implies the existence of turbulence in PWN (e.g. Porth+14)
  - Approach
    - Diffusion: Spatial diffusion by disturbed magnetic field
      ⇒ This will spread the high-energy electrons further
    - Re-acceleration : Stochastic acceleration by turbulence
      ⇒ This will suppress the synchrotron cooling



(Porth et al. ,2014)