Search for Gravitational Waves from Inspiraling Compact Binaries : Data analysis

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Introduction

Detection method

Gravitational waves

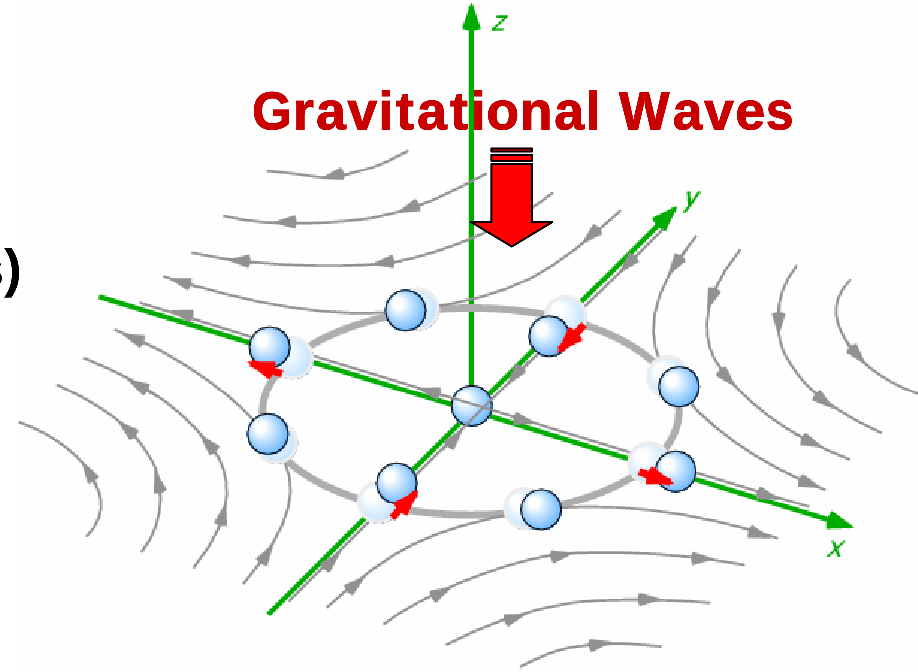
(Coalescences of compact binaries, Stellar core collapses)

➔ Change in distance between free masses

(Strain about 10^{-18})

Precise measurement
by a laser interferometer
(Mirrors behave as free masses)

Michelson interferometer



TAMA300

Fabry-Perot-Michelson interferometer with 300m arms

(with power recycling)

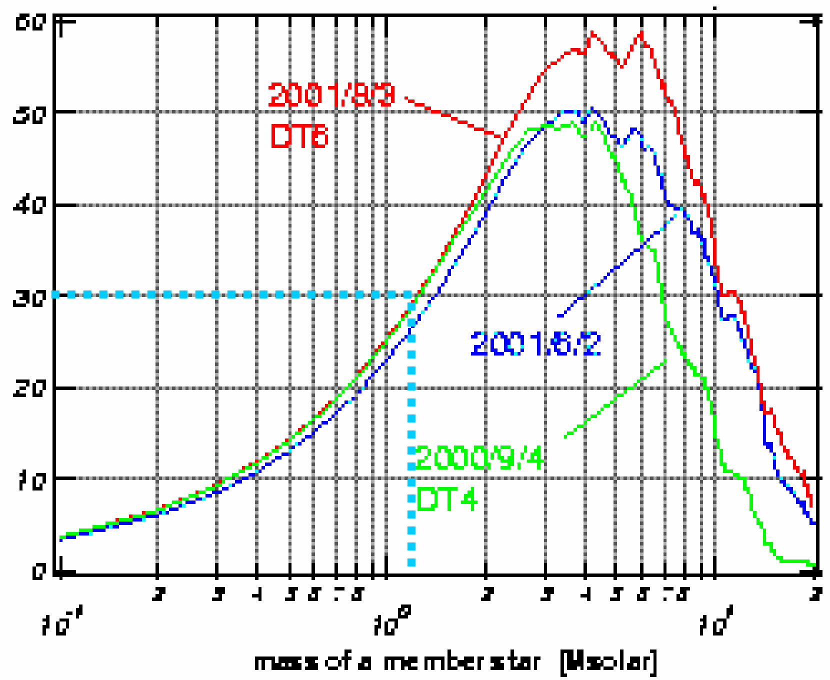
Detect gravitational wave from local group of our galaxy

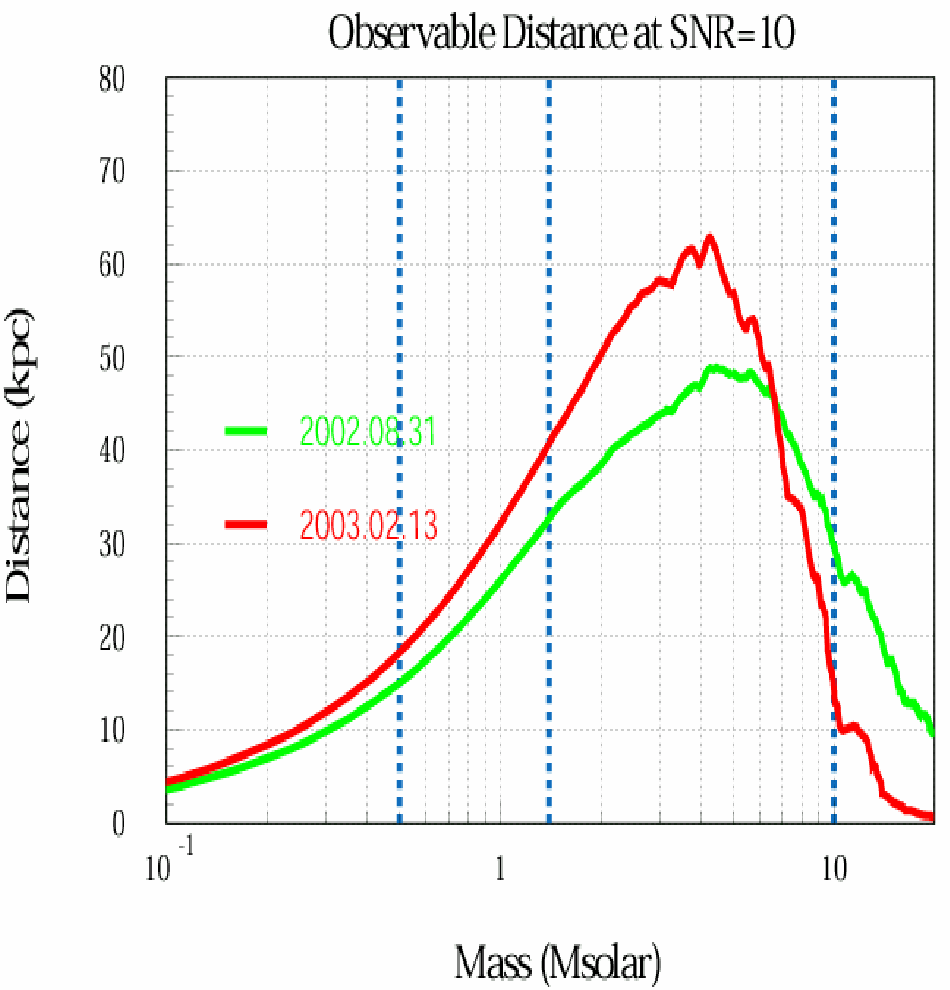
Research and development for a large-scale detector

Summary of Data taking

	period	actual data amount	take note
DT1	8/6-7/1999	~3 + ~7 hours	first whole system test
DT2	9/17-20/1999	31 hours	first physics run
DT3	4/20-23/2000	13 hours	
	8/14/2000	world best sensitivity $h=5\times10^{-21}\ [1/\sqrt{Hz}]$	
DT4	8/21-9/3/2000	167 hours	stable long run
DT5	3/1-3/8/2001	111 hours	
Test Run 1	6/4-6/6/2001	longest stretch of continuous lock 24:50	keeping running all day
DT6	8/1-9/20/2001	1038 hours duty cycle 86%	full dress run without recycling
DT7	8/31-9/2/2002	24hours duty cycle 76.7%	Recycling Simultaneous obs. with LIGO and GEO
DT8	2/14-4/14/2003	1168 hours duty cycle 81.1%	coincidence obs. with LIGO S2

History of TAMA300 Sensitivity
for inspiraling compact binaries





- TAMA300 observed during August 1 and September 20, 2001. (Data Taking 6)

  Total length of data amounted to 1039 hours.

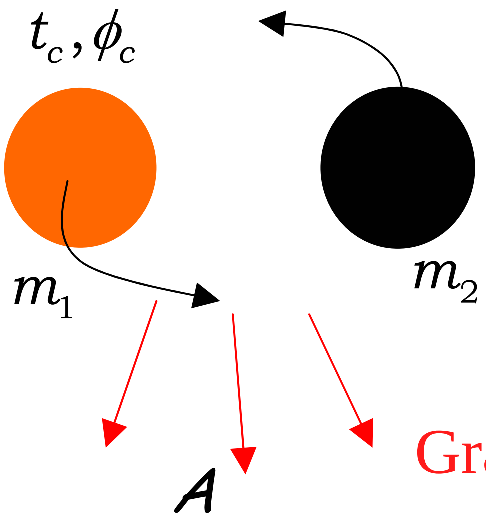
- TAMA300 also observed during February 14 and April 14, 2003. (Data Taking 8)

  Total length of data amounted to1163 hours.

We have tried a event search for inspiraling compact binaries using TAMA300 data.

Coalescing compact binaries

Inspiral phase of coalescing compact binaries are main target because expected event rate of NS-NS merger
:a few within 200Mpc / year , well known waveform etc.



Neutron stars
Black holes

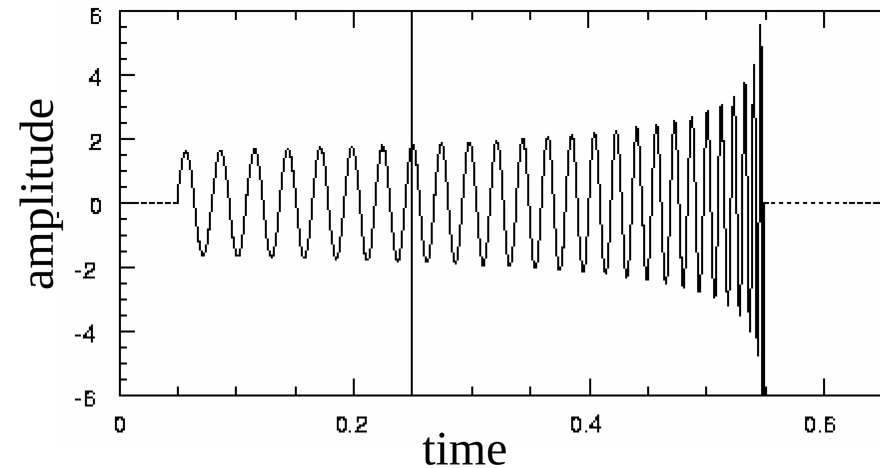
Gravitational Waves

$$\mathcal{M} = M\eta^{3/5}$$

M : total mass
η : reduced mass

Possibility of MACHO black hole

chirp signal



In this search, mass region: $1.0M_{solar} \le m_1, m_2 \le 2.0M_{solar}$ for DT6

$1.0M_{solar} \le m_1, m_2 \le 3.0M_{solar}$ for DT8

Matched filter

- Detector outputs: $s(t) = Ah(t) + n(t)$

  $h(t)$: known gravitational waveform (template)

  $n(t)$: noise.

2.5 Post-Newtonian approximation

- Outputs of matched filter:

$$\rho(t_c, m_1, m_2, \ldots) = 2\int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} df = (s \mid h)$$

- $S_n(f)$ noise power spectrum density

- Signal to noise ratio is $\mathrm{SNR} = \rho / \sqrt{2}$

- Best linear filter

波形の表式

$$h(t) = F_+(\overbrace{\theta, \varphi}^{\text{ソース方向}}, \overbrace{\psi}^{\text{偏極方向}})h_+(t) + F_\times(\theta, \varphi, \psi)h_\times(t)$$

$F_+, F_\times$:レーザー干渉計ビームパターン関数

チャープ信号は

Inclination angle

$$\begin{aligned} h_+ &= A'(t)(1+\cos^2 i)\cos(\Phi(t)), \\ h_\times &= 2A'(t)\cos i \sin(\Phi(t)), \end{aligned}$$

であるので，結局，重力波信号は

$$\begin{aligned} h(t) &= A[\cos(\Phi)\cos(\phi_c) - \sin(\Phi)\sin(\phi_c)], \\ &= A(t-t_c)\cos(\Phi(t-t_c)+\phi_c). \end{aligned}$$

t_c :合体時刻，ϕ_c :初期位相

m_1, m_2 :質量

基本パラメータ

（角度パラメータはすべて A, ϕ_c に含まれる）

Matched filteringは，以上のパラメータの中で最適なパラメータ，すなわち，ρ を最大とするパラメータを求めるプロセスである.

ϕ_c についての最大化

$$\begin{aligned}\rho &= (s|h)\\ &= (s|h_c)\cos(\phi_c) - (s|h_s)\sin(\phi_c)\\ &= \left[(s|h_c)^2 + (s|h_s)^2\right]^{1/2}\cos(\phi_c - \phi_a)\end{aligned}$$

よって

$$\max_{\phi_c} \rho = \sqrt{(s|h_c)^2 + (s|h_s)^2}$$

位相についての**max**は解析的に求められる.

合体時刻についての最大化

$$\begin{aligned}\tilde{h}(f) &= \int_{-\infty}^{\infty} dt e^{2\pi i f t} h(t - t_c) \\ &= \tilde{h}_{(t_c=0)}(f) \times e^{2\pi i f t_c}\end{aligned}$$

より

$$(s|h_{c/s}) = 2\int_{-\infty}^{\infty} df \frac{\tilde{s}(f)\tilde{h}_{c/s}(f)}{S_n(f)} e^{-2\pi i f t_c}$$

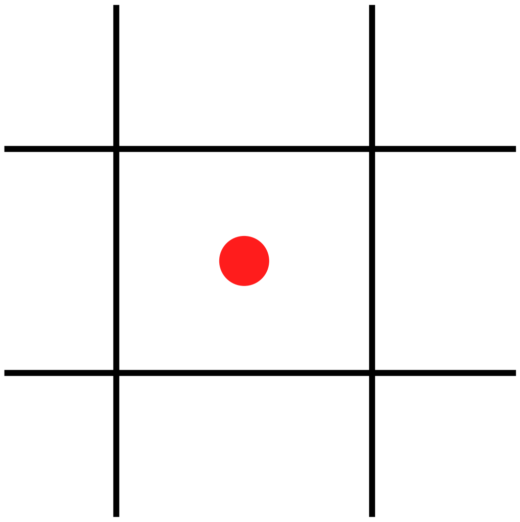
異なる合体時刻 t_c について ρ をもとめる計算は

逆 FFT でまとめて効率的に計算できる.

質量パラメータについての最大化　(cf. **Owen, PRD53, 6749('96)**)

検出器１台の場合，massパラメータについて最適パラメータを探すプロセスが主な計算である.

質量パラメータ空間に格子をはり，格子点上の質量について調べ，全部調べたことにする.



格子点上から信号のパラメータがずれていると，信号の振幅　ρ は小さくなる

ρ が３%小さくなるとすると

約１０%の本物重力波イベントを失う

⇒ ３%以下になるように格子間隔を決める

典型的な質量パラメータの組み合わせ数

DT6 : 700　$(1.0M_{solar} \le m_1, m_2 \le 2.0M_{solar})$

DT8 : 600　$(1.0M_{solar} \le m_1, m_2 \le 3.0M_{solar})$

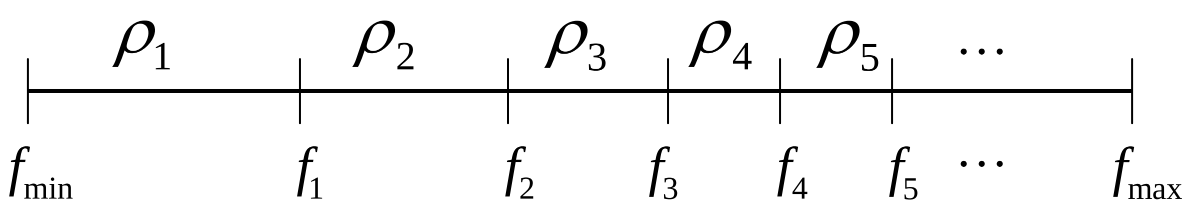
The real data contained large amount of non-stationary and non-Gaussian noise.

In order to remove the influence of such noise, we also introduce χ^2 (B.Allen et al,PRL,**62**,1489(1999))

Divide each template into n mutually independent bins in frequency domain.

Test whether the contribution to ρ from each bins agree with that expected from chirp signal

$$\rho \equiv (s \mid h) \left(= 2\int \frac{\tilde{s}(f)\tilde{h}^*(f)}{S_n(f)} df \right)$$

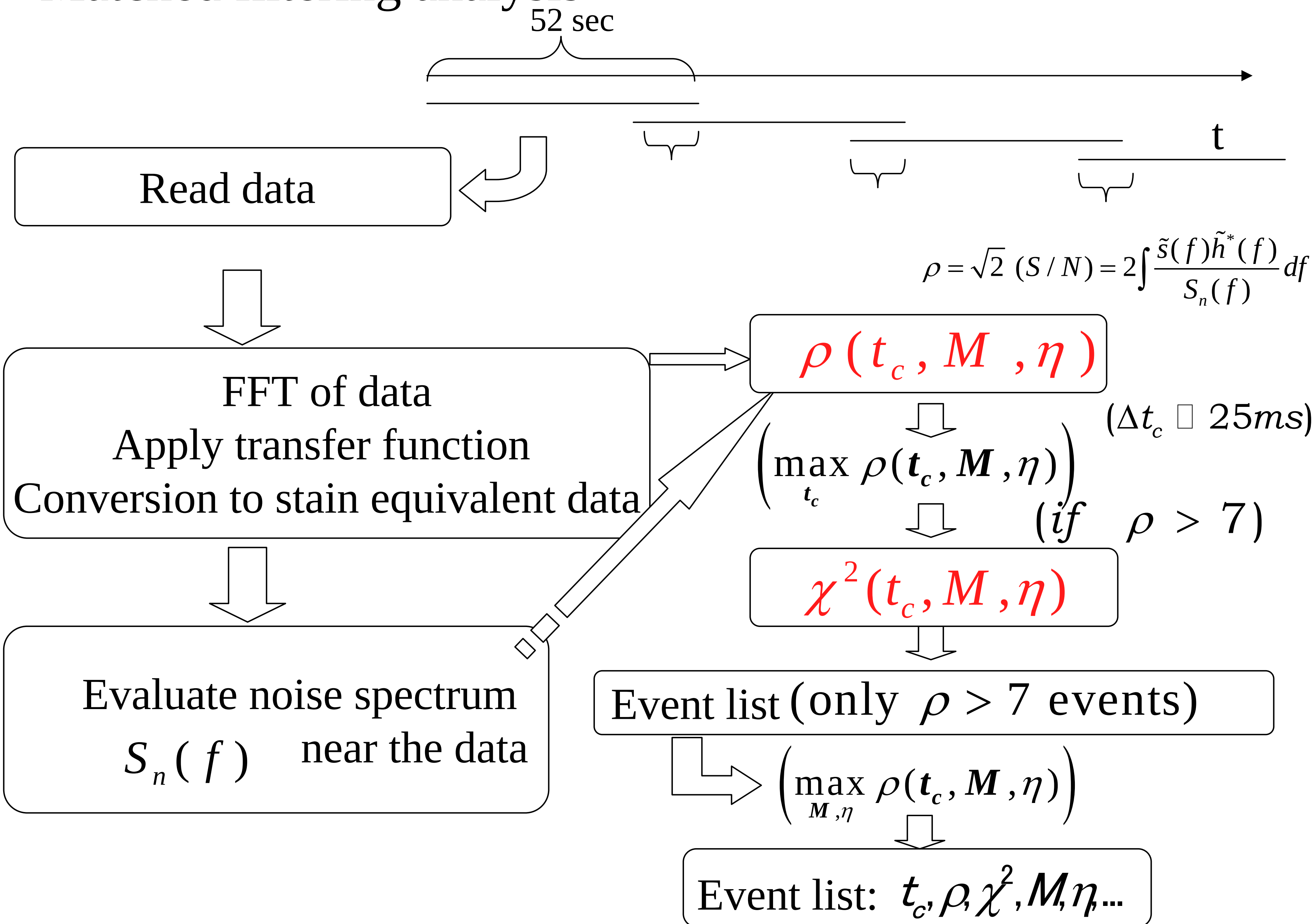


$$\chi^2 \equiv n\sum(\rho_i - \bar{\rho}_i)^2$$

$$, \bar{\rho}_i = \langle \rho_i \rangle$$

$$\tilde{\chi}^2 = \chi^2 / (2n-2)$$

Matched filtering analysis



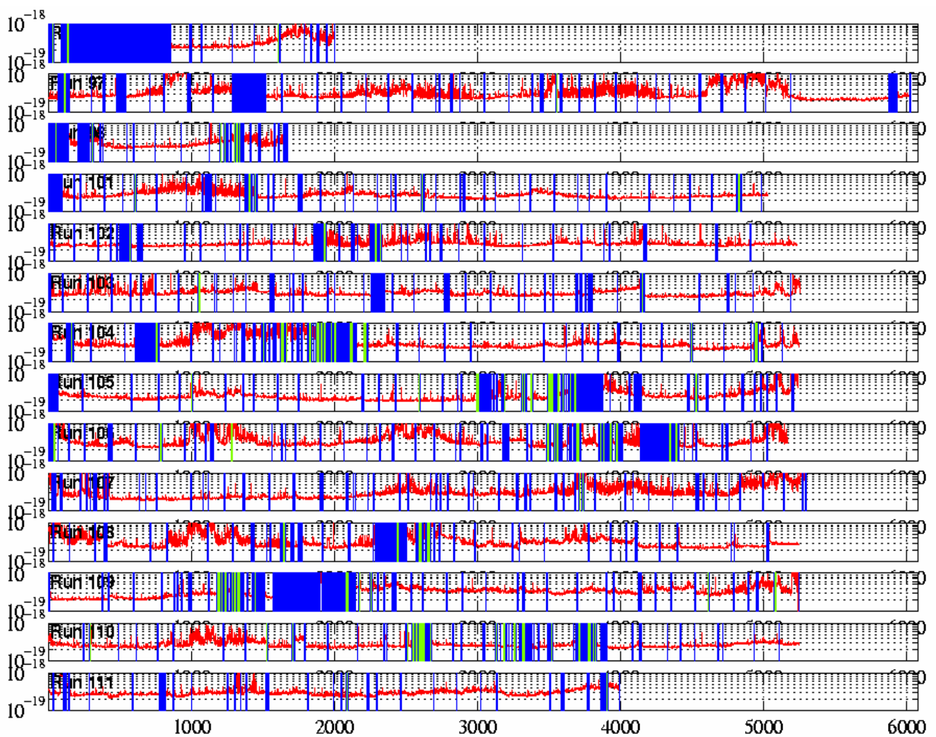
Variation of Noise power (1 minute average)

Before the matched filter analysis, we evaluate the fluctuation of noise power.

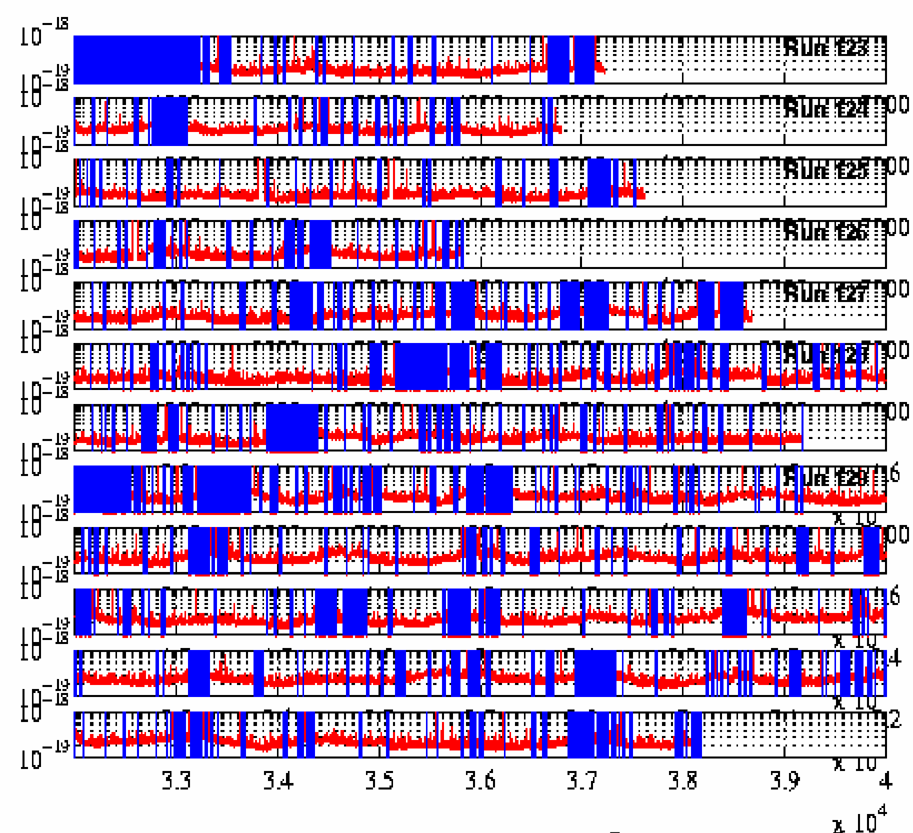
$$\left(4 \int_{f_{min}}^{f_{max}} df \, \frac{f^{-7/3}}{S_n(f)} \right)^{-1/2}$$

$f_{min} = 100\mathrm{Hz}, \; f_{max} = 2500\mathrm{Hz}$

DT6:8/1-9/20/2001



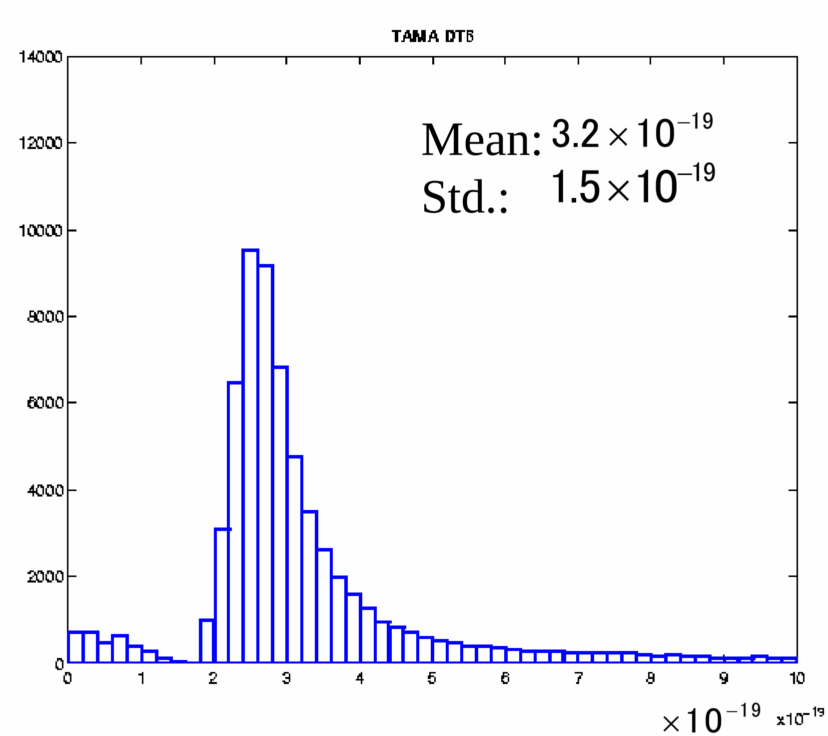
DT8:2/14-4/14/2003



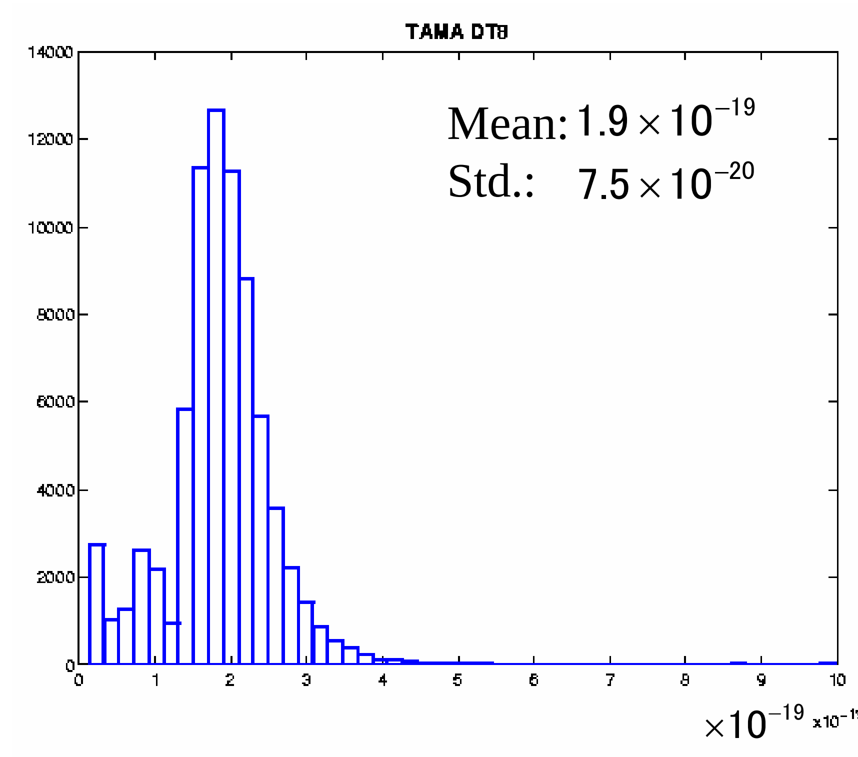
[1.09minutes]

Variation of Noise power (histogram) (2)

DT6



DT8



The fluctuation of noise power in DT8 is small.

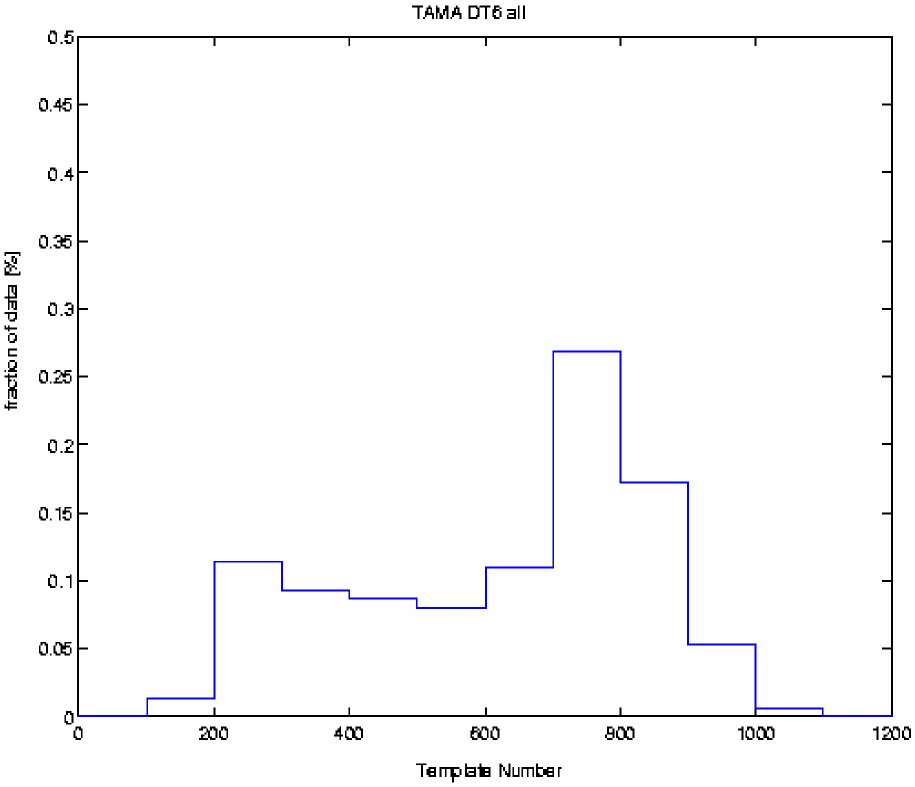
We can say that the detector operation in DT8 are more stable than that in DT6.

Distribution of template number

• Templates were placed so that maximum loss of SNR becomes less than 3%

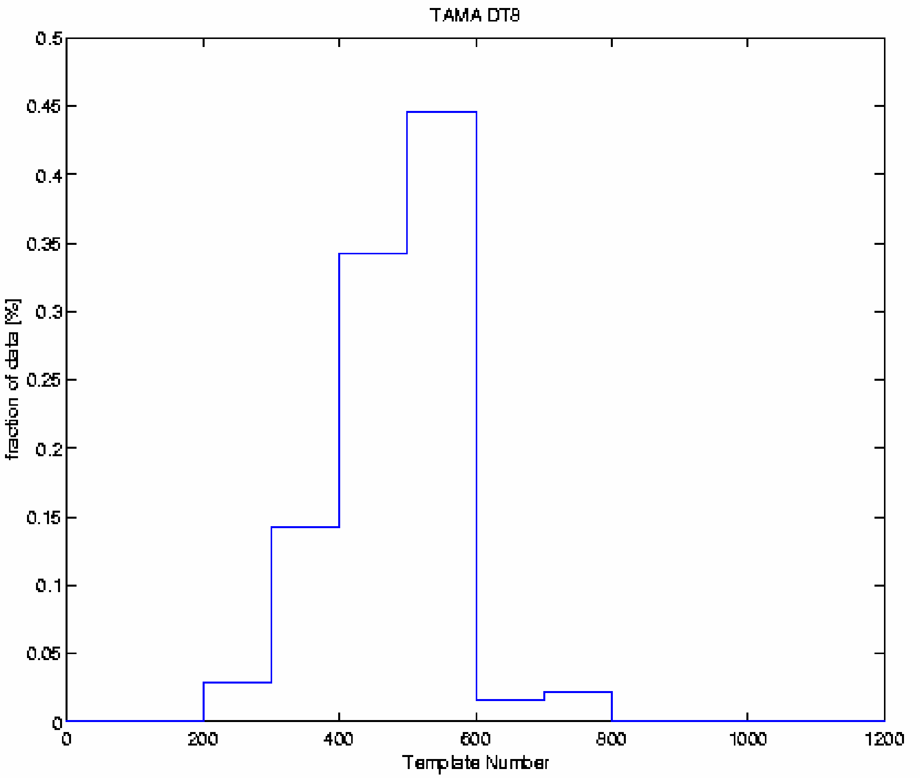
• The variation of number of templates is due to the variation of shape of noise power spectrum.

DT6



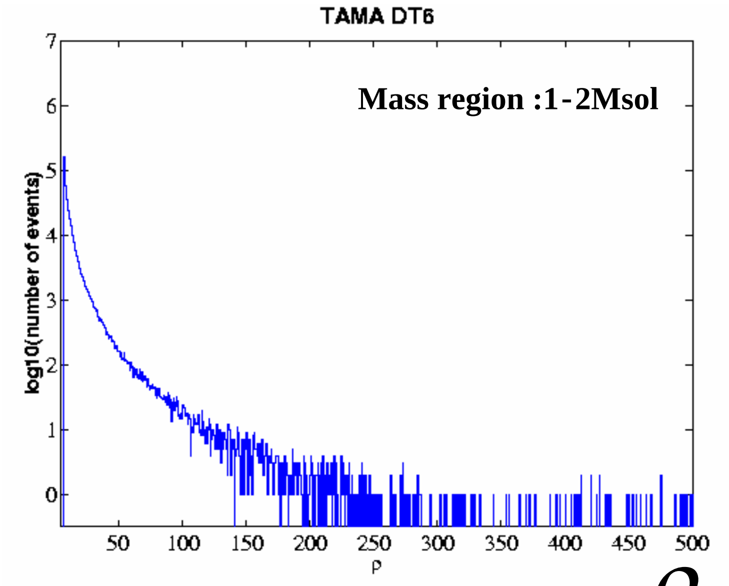
$$1.0M_{solar} \le m_1, m_2 \le 2.0M_{solar}$$

DT8

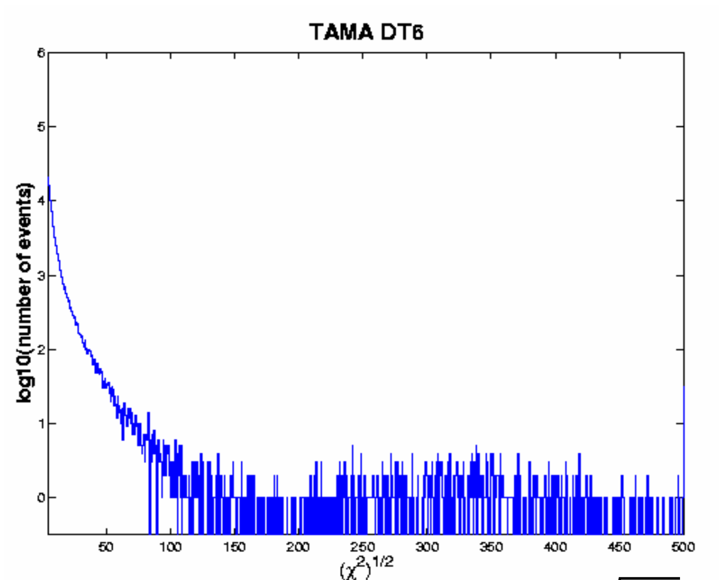


$$1.0M_{solar} \le m_1, m_2 \le 3.0M_{solar}$$

DT6

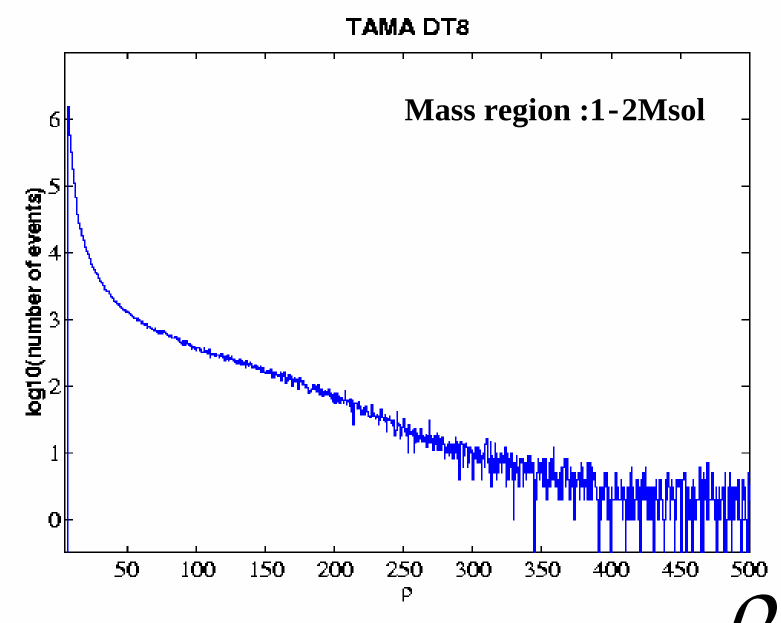


ρ

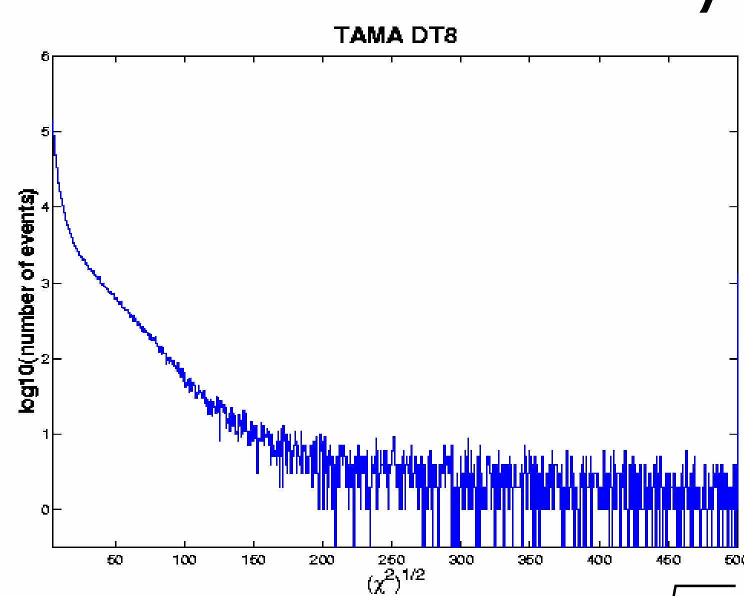


$\sqrt{\hat{\chi}^2}$

DT8



ρ



$\sqrt{\hat{\chi}^2}$

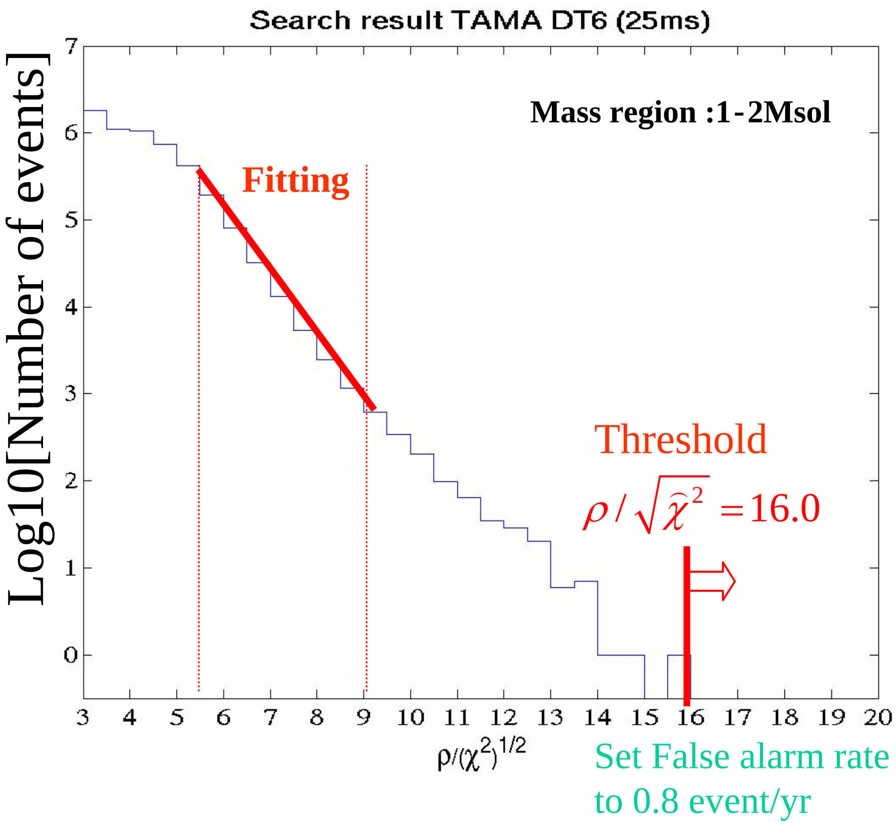
$\rho/\sqrt{\widehat{\chi}^2}$ statistics

• We found that the value of $\hat{\chi}^2$ becomes larger, when the amplitude of signal becomes larger even if the events are real. In such situation, if we reject events simply by the value of $\hat{\chi}^2$, we may lose real events with large amplitude.

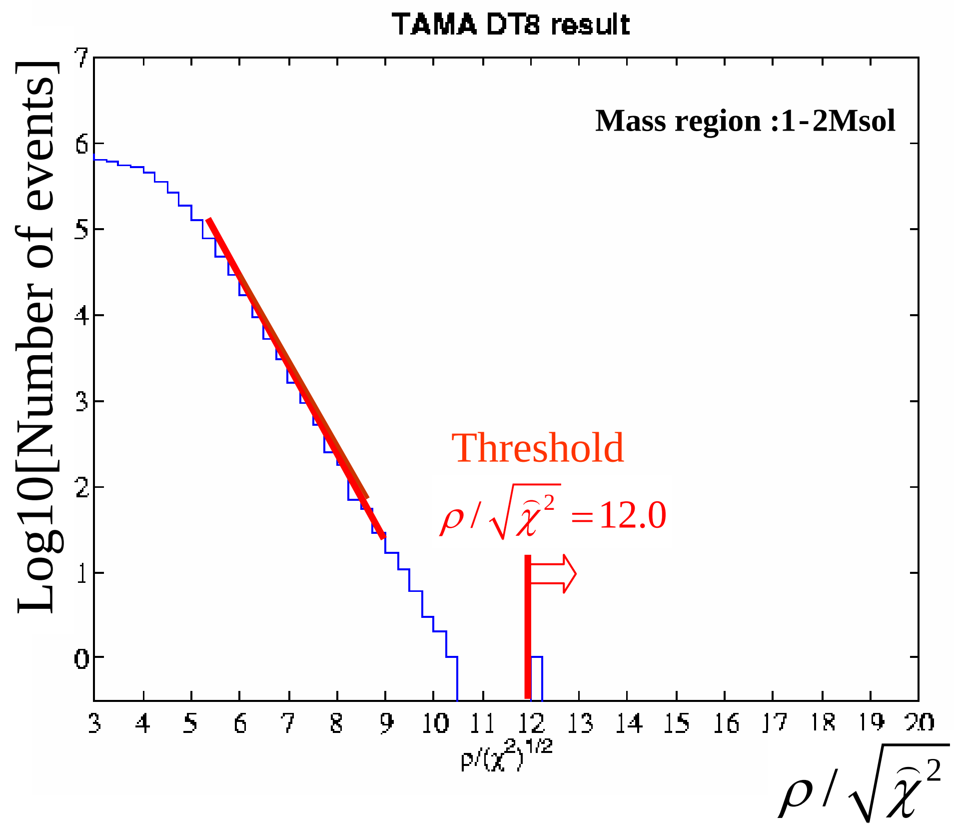
• We thus introduce a statistic $\rho/\sqrt{\widehat{\chi}^2}$, to distinguish between candidate events and noise events

Compare DT6 results to DT8 results

DT6



DT8



If GW events really happened, the value of $\rho/\sqrt{\hat{\chi}^2}$ would become much larger than tail of distribution.

In matched filtering analysis, we do not see events which exceed the tail of the distribution of events significantly.
Even in this case, we can estimate the upper limit to the event rate.

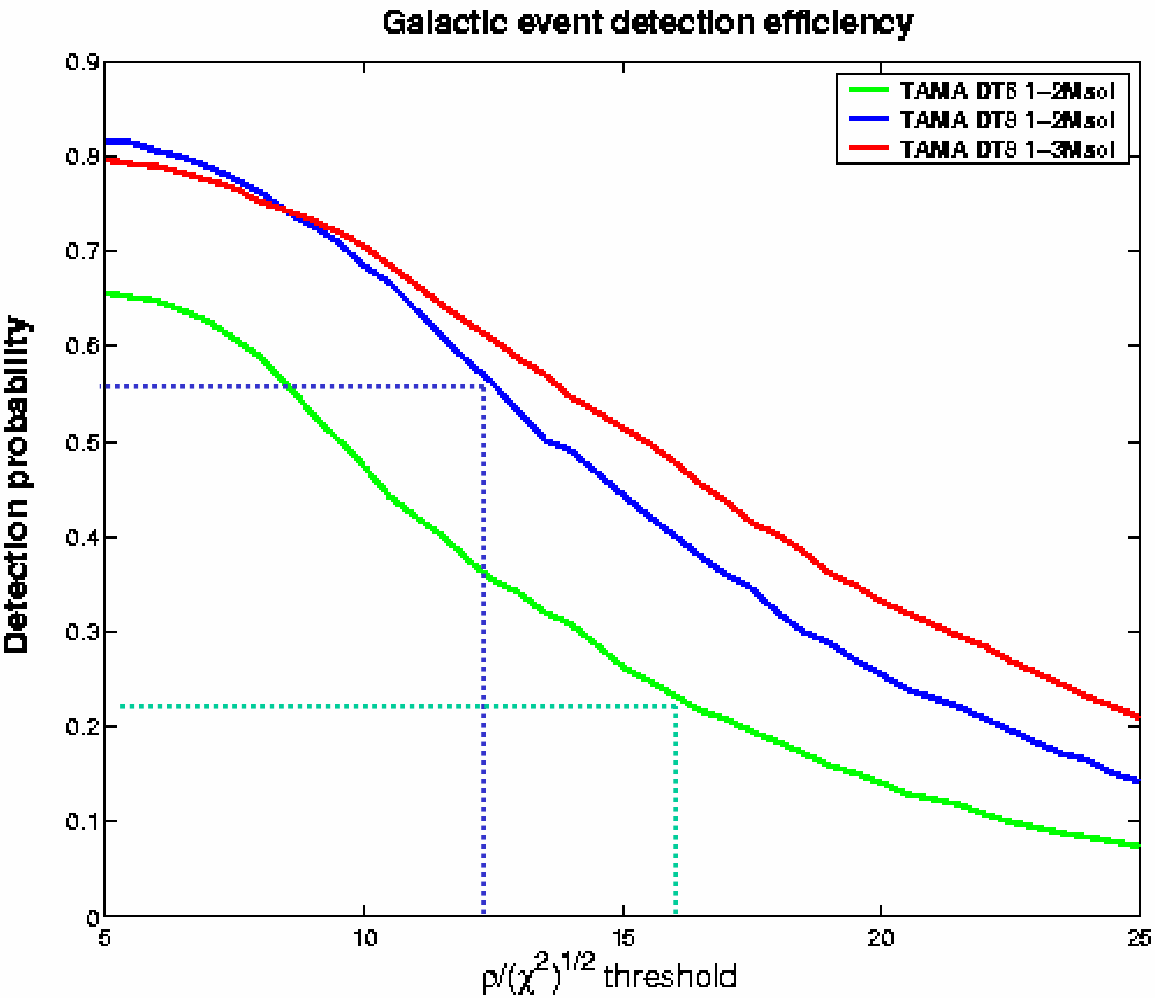
Upper limit to the Galactic event rate

$$\frac{N}{T\,\varepsilon}$$

- N : Upper limit to the average number of events over certain threshold
- T : Length of data [hours]
- ε : Detection efficiency

Galactic event detection efficiency

To estimate detection efficiency, we perform **Galactic event simulation**



	Threshold	efficiency
DT6	16	0.23
DT8	**12**	**0.58**

(False alarm rate = 0.8 / year)

search mass region:

$$1.0M_{solar} \le m_1, m_2 \le 2.0M_{solar}$$

Efficiency of DT8 becomes three times better than that of DT6

Search results
for inspiraling compact binaries

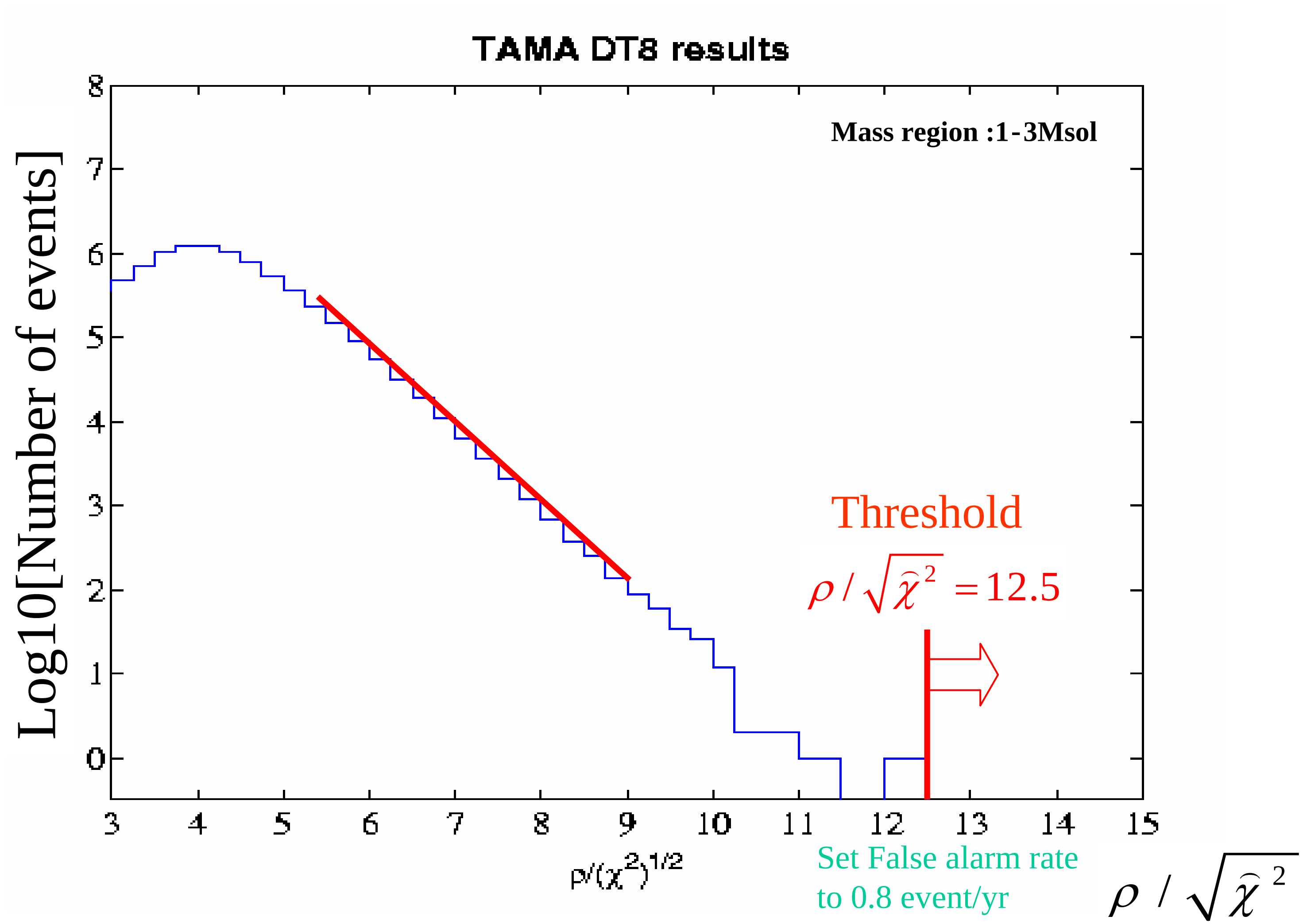
	threshold	number of evtnts(CL90)	obs. time	detection effici.	Galactic event rate (CL90)	
DT6	16	0obs,0.1bg ,2.3	1039	0.23	0.0095	event/h
					= 83	event/yr
DT8	**12**	**1obs,0.1bg,3.8**	**1163**	**0.58**	**0.0056**	**event/h**
					= 49	**event/yr**

(search mass region: $1.0M_{solar} \leq m_1, m_2 \leq 2.0M_{solar}$)

(False alarm rate = 0.8 / year)

We can obtain that upper limit of DT8 becomes about two times more stringent than that of DT6.

Uusing the **DT8 search results (mass region:1-3Msol)**, we estimate the upper limit to the galactic event rate.



Upper limit to the Galactic event rate

- threshold=12.5 (～S/N = 9)

(False alarm rate = 0.8 / year)

- detection efficiency from Galactic event simulation: $\varepsilon = 0.61$

- We also obtain upper limit to the average number of events over threshold by standard Poisson statistics analysis

➡ N = 2.3 (C.L. = 90%)

- Observation time T ＝ 1163 hours

$$\Rightarrow \frac{N}{T\varepsilon} = 0.0033 \quad event/\mathrm{hour}$$

= 29 event/yr (C.L. 90 %)

$$1.0 M_{solar} \le m_1, m_2 \le 3.0 M_{solar}$$

Summary

We performed a event search for inspiraling compact binaries using TAMA300 data.

DT6 (2001)

Range (SNR=10) : 33kpc

Mass range : 1-2Msol Upper limit : 0.0095 event/hour

(=83 event/yr)

DT8 (2003)

Range (SNR=10) : 42kpc

Mass range : 1-2Msol Upper limit : 0.0056 event/hour

(= 49 event/yr)

Mass range : 1-3Msol Upper limit : 0.0033 event/hour

(= 29 event/yr)