Toward precision measurements in Solar Neutrinos

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- LMA what is next
- Adiabatic perturbation theory and non-adiabatic corrections
- Earth matter effects: precise analytic description
- Neutrino oscillations in low density medium

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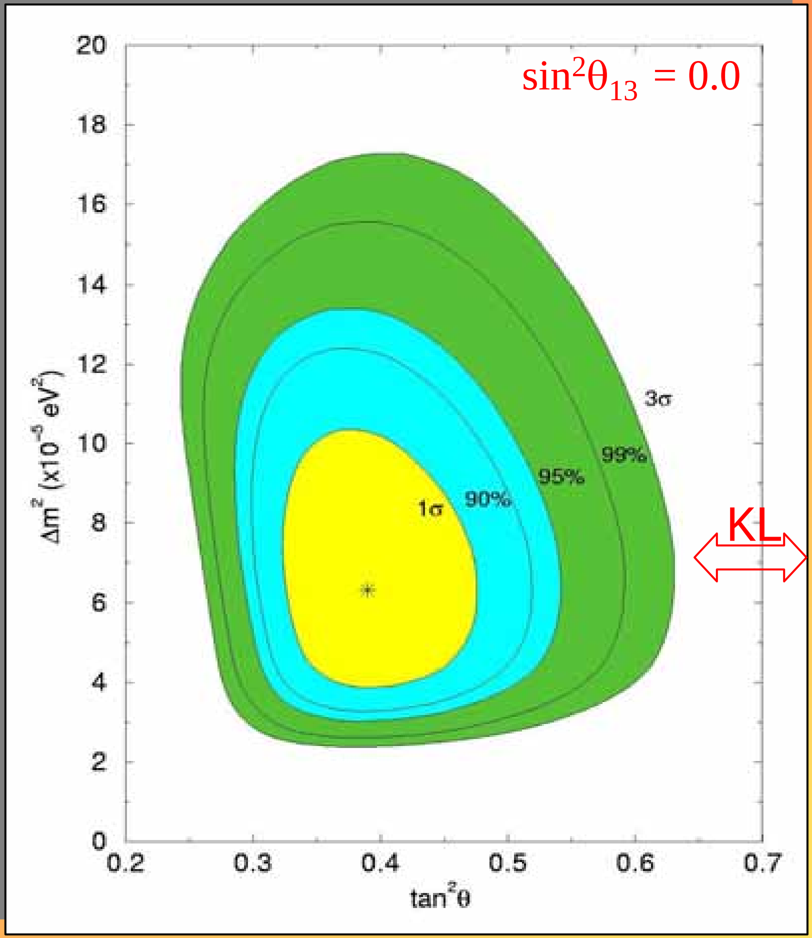
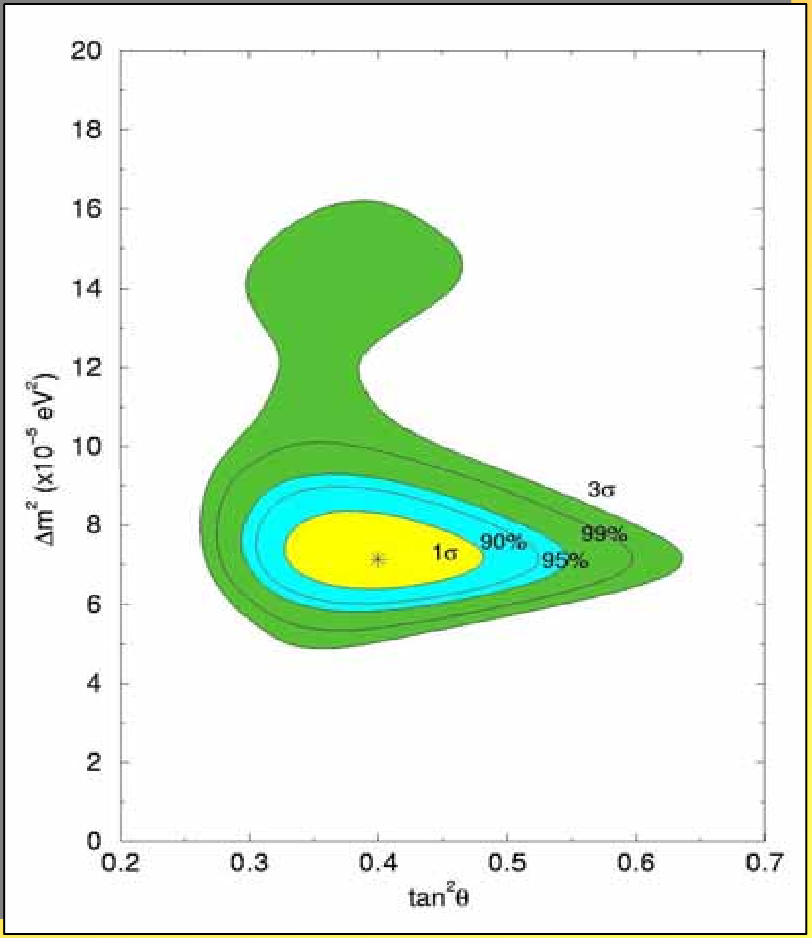
LMA MSW solution

P. de Holanda, A.S.

solar data



solar data + KamLAND



$\Delta m^2 = 6.3 \; 10^{-5} \; eV^2$
$\tan^2\theta = 0.39$

$\Delta m^2 = 7.1 \; 10^{-5} eV^2$
$\tan^2\theta = 0.40$

What is next?

Further tests of LMA,
Consistency checks,
searches for
the day-night asymmetry,
``upturn” of spectrum

Precise determination
of parameters, 1-2 mixing
searches for the effect of
1-3 mixing

Astrophysics
determination of
original neutrino
fluxes

Searches for sub-leading
effects, Bounds on physics
beyond LMA

- sterile neutrinos
- magnetic moment effects
- new interactions
- tests of fundamental symmetries, CPT...

In this connection it is important

to give precise description of the LMA conversion both in the Sun and in the Earth taking into account various corrections

to estimate accuracy of approximations we use

to find accurate analytic expressions for probabilities and observables as functions of oscillation parameters

Towards precision measurements in solar neutrinos

Accuracy of measurements
of the oscillation parameters

$\Delta(\tan^2\theta) < \tan^2\theta$

$\Delta(\Delta m^2) < \Delta m^2$

Possible effects of 1-3 mixing should be taken into account

$\Delta m^2 = 6.5 \; 10^{-5}$ eV ^{2}
$\tan^2\theta = 0.39$

LMA = Adiabatic solution

- $\nu_f = U_m \nu_m$ $\quad$ $U_m = U(\theta_m)$

  $\nu_f = (\nu_e, \nu_a)^T$ - flavor states
  $\nu_m = (\nu_{1m}, \nu_{2m})^T$ - eigenstates in matter

$$U(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- Mixing angle in matter

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{(\cos 2\theta - \varepsilon(x))^2 + \sin^2 2\theta}}$$

$$\varepsilon(x) = \frac{2EV}{\Delta m^2} \qquad V(x) = \sqrt{2}\, G_F n(x)$$

- Survival $\nu_e \to \nu_e$ probability:

$$P_{ee} = 0.5\,(1 + \cos 2\theta_m^0 \cos 2\theta)$$

$\theta_m^0 = \theta_m(x_0)$ - mixing angle in matter in the production point

Adiabatic approximation

- Evolution equation for the matter eigenstates:

$$i\, d\nu_m/dx = H_m(x)\, \nu_m$$

$$\nu_m = (\nu_{1m}, \nu_{2m})^T$$

$$H_m(x) = \begin{pmatrix} -\Delta_m(x)/2 & -i\, d\theta_m(x)/dx \\ i\, d\theta_m(x)/dx & \Delta_m(x)/2 \end{pmatrix}$$

$$\Delta_m(x) = \frac{\Delta m^2}{2E}\sqrt{(\cos 2\theta - \varepsilon(x))^2 + \sin^2 2\theta}$$

- Adiabatic approximation: if

$$\gamma = \frac{d\theta_m(x)/dx}{\Delta_m(x)} \ll 1$$

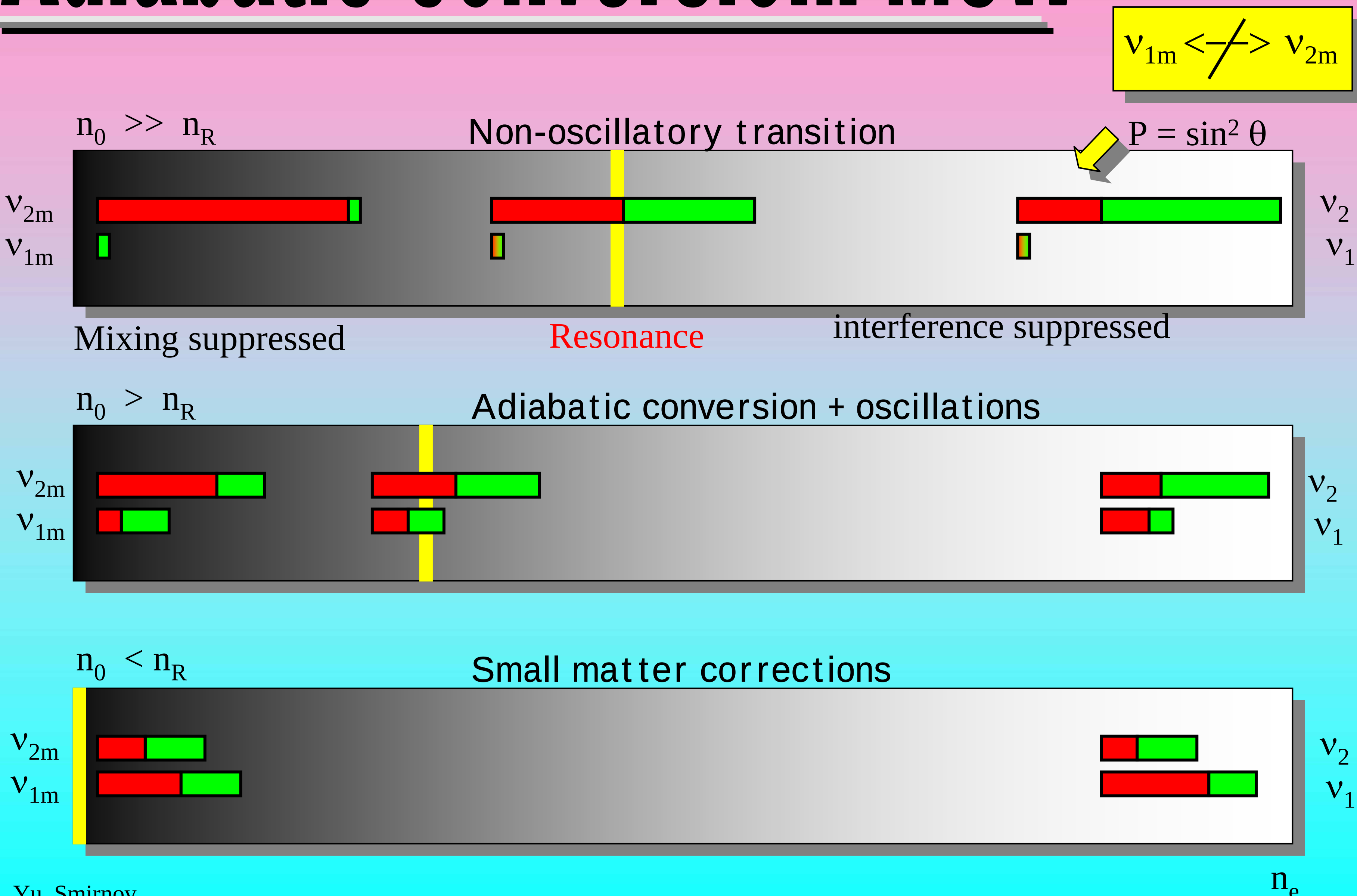
The off-diagonal terms can be neglected -> equation splits

- Solution (S-matrix):

$$S^{ad}(x_0 \to x) = \begin{pmatrix} e^{i\Phi(x)/2} & 0 \\ 0 & e^{-i\Phi(x)/2} \end{pmatrix}$$

$$\Phi(x) = \int_{x_0}^{x} dy\, \Delta_m(y)$$

Adiabatic conversion. MSW



A. Yu. Smirnov

Physics of conversion

Averaged survival probability at the surface of the Sun

$$P = \sin^2\theta + \cos 2\theta \cos^2\theta_m^0$$

non-oscillatory part — oscillations

θ_m^0 is the mixing angle in matter in the production point

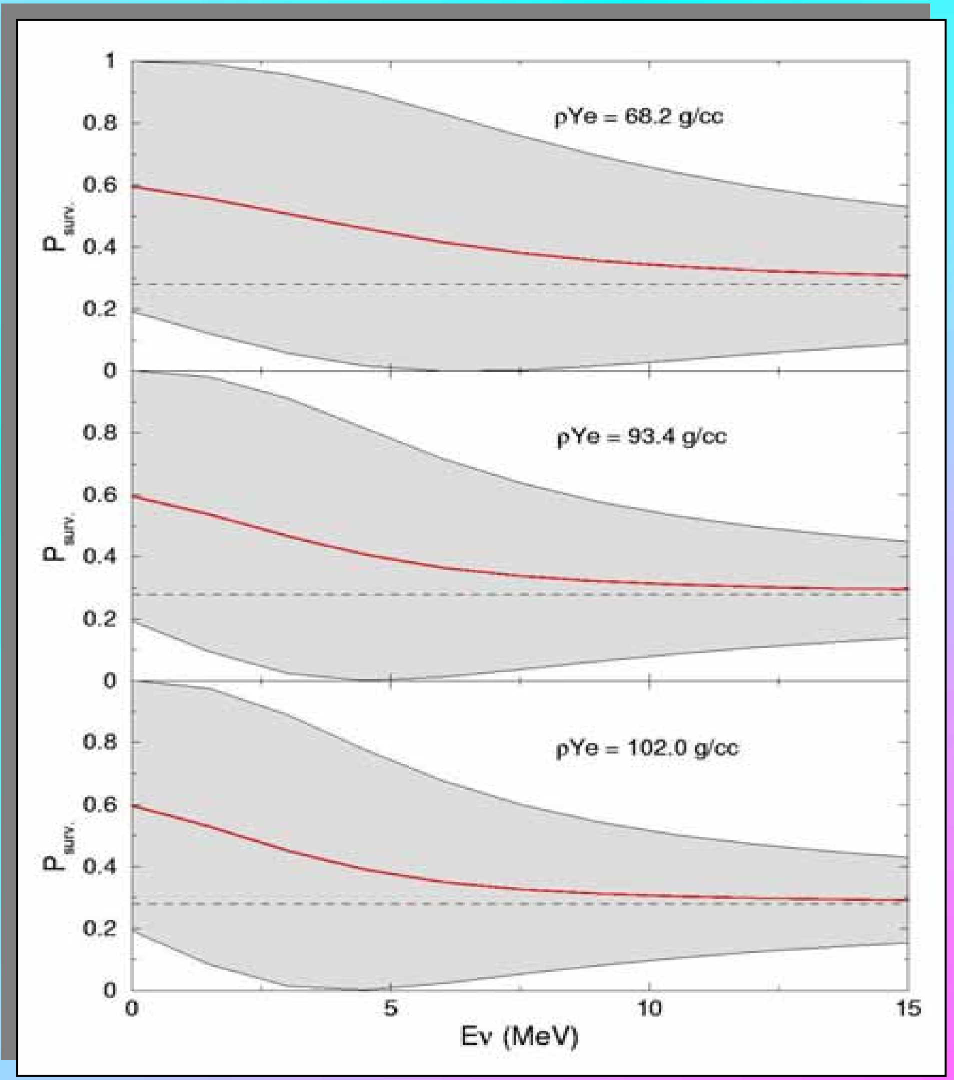
The depth of oscillations:

$$A = \sin 2\theta \sin 2\theta_m^0$$

At the detector:

$$P_{det} = P + \Delta P_{reg}$$

Earth regeneration



Non-adiabatic corrections

- Search for the solution in the form

$$S(x_0 \to x) = C(x)\, S^{ad}(x_0 \to x)$$

$$C(x) = \begin{pmatrix} 1 & c(x) \\ -c(x)^* & 1 \end{pmatrix}$$

Explicitly:

$$S(x_0 \to x) = \begin{pmatrix} e^{i\Phi(x)/2} & c(x)\, e^{-i\Phi(x)/2} \\ -c(x)^* e^{i\Phi(x)/2} & e^{-i\Phi(x)/2} \end{pmatrix}$$

c(x) gives the amplitude of transition between the eigenstates

- Inserting $S(x_0 \to x)$ in the evolution equation → Differential equation for c(x) → Solve the equation

→

$$c(x) = -\int_{x_0}^{x} dx' \frac{d\theta_m(x')}{dx'} \exp\left(i \int_{x'}^{x} dx''\, \Delta_m(x'') \right)$$

Applications to the Sun and the Earth

phase $\Phi(x' \to x)$

Non-adiabatic corrections inside the Sun

- With non-adiabatic corrections: $P_{ee} = 0.5\,[1 + (1 - 2P_c)\cos 2\theta_m^0 \cos 2\theta]$

$P_c = |c(x_0 \to x_f)|^2$ - jump probability (probability of $\nu_{1m} \leftrightarrow \nu_{2m}$)

- Inside the Sun: smooth density profile - integration can be done

$$c(x_0 \to x_f) = -i\,\gamma(x)\,\exp[\,i\Phi(x \to x_f)]\Big|_{x_0}^{x_f}$$

- Since at the surface $V = 0$, the lower limit of integration is relevant

$$P_c = |\gamma(x_0)|^2 = \left(\frac{l_m(x_0)}{4\pi h(x_0)}\right)^2 K(x_0)^2$$

$$K = \frac{\Delta m^2\, V \sin 2\theta}{2E\,\Delta_m^2} \sim 1$$

$l_m = 4\pi/\Delta_m \qquad h = V\,(dV/dx)^{-1}$

- Numerically: $P_c = (10^{-9} - 10^{-7})\,(E/10\text{ MeV})^2$

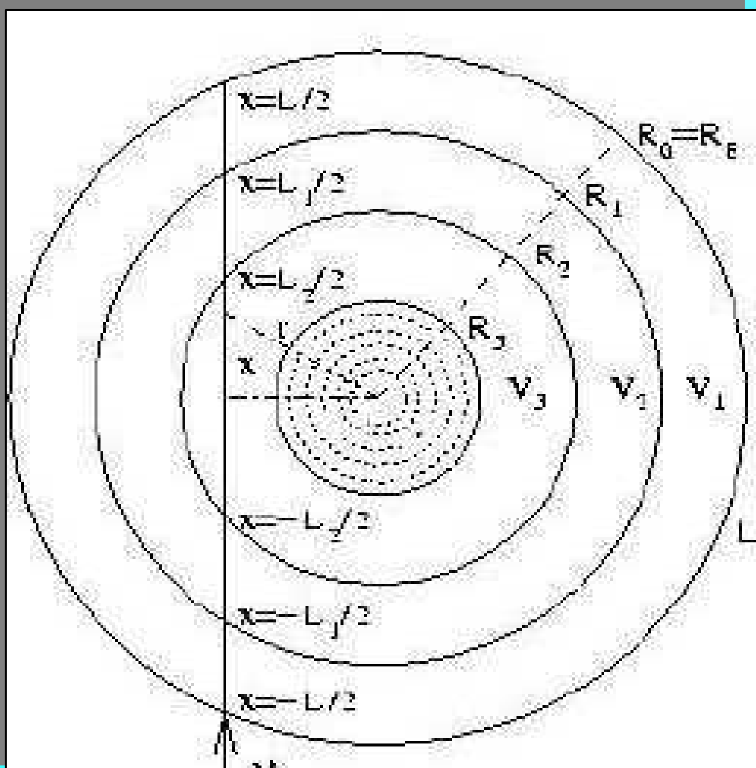
Negligible, still much large than the double exponential formula: 10^{-400}

Oscillations inside the Earth

1). Incoherent fluxes of ν_1 and ν_2 arrive at the surface of the Earth

2). In matter the mass states oscillate

3). the mass-to-flavor transitions, e.g. ν_2 --> ν_e are relevant



Regeneration factor:

$$P_{2e} = \sin^2\theta + f_{reg}$$

$$P_{ee} = 0.5[\,1 + \cos 2\theta_m^0 \cos 2\theta\,] - \cos 2\theta_m^0 f_{reg}$$

4). The oscillations proceed in the weak matter regime:

$$\epsilon(x) = \frac{2EV(x)}{\Delta m^2} \ll 1$$

Regeneration factor

$\nu_2 \dashrightarrow \nu_e$

$$P(\nu_2 \dashrightarrow \nu_e) = |\langle\nu_e|\, U(\theta_{mR})\, S(x_0 \to x_f)\, U^+(\theta_{mR})\, U(\theta)\, |\nu_2\rangle\, |^2$$

θ_{mR} - mixing angle at the surface of the Earth

$$f_{reg} = P_{2e} - \sin^2\theta$$

Regeneration factor in the first approximation in ε :

$$f_{reg} = \varepsilon\,(R)\, \sin^2 2\theta\, \sin^2 [\Phi^m(x_0 \to x_f)/2] + \sin 2\theta\, \mathrm{Re}\{c(x_0 \to x_f)\}$$

$$\varepsilon\,(R) = \frac{2EV(R)}{\Delta m^2}$$

- If adiabaticity is conserved the regeneration depends on the potential V(R) at the surface and total adiabatic phase
- Non-adiabatic conversion appears as the interference term and therefore - linearly
- Calculate $c(x_0 \to x_f)$ in two steps

1) estimation in a given layer
2) taking into account borders of the shells

Corrections inside the layer of the Earth

- $$c(x_0 \to x_f) = -i\,\gamma(x)\,\exp[\,i\Phi(x \to x_f)]\Big|_{x_0}^{x_f}$$

- $$\Delta f_{reg} / f_{reg} \sim \frac{l_m}{2\pi h_E \sin 2\theta} \sim 0.01 - 0.02$$

$h_E \sim R_E$ - typical scale of the density change is of the order of radius of the Earth

$l_m = 4\pi / \Delta_m$

We neglect these corrections

Effect of n-shells

- Consider the trajectory which crosses n shells (2n-1 layers)

  Neglect the adiabaticity violation inside shells -->
  contribution to $c(x_0 \to x_f)$ comes from the borders between layers

- $$\frac{d\theta_m}{dx} = \frac{\Delta m^2}{4E}\,\frac{\sin 2\theta}{\Delta_m^{\;2}}\,\frac{dV(x)}{dx}$$ ➡ Jumps of density (potential) between layers lead to δ - functions:

$$\frac{d\theta_m}{dx} = \frac{E \sin 2\theta}{\Delta m^2} \Sigma_{j=1\ldots n-1} \Delta V_j \,[\, \delta(x + L_j/2) - \delta(x - L_j/2)\,]$$

- Inserting in formula for c(x):

$$f_{reg} = \frac{2E \sin^2 2\theta}{\Delta m^2} \sin\Phi_0/2 \;\; \Sigma_{j=0\ldots n-1} \Delta V_j \sin \Phi_j/2$$

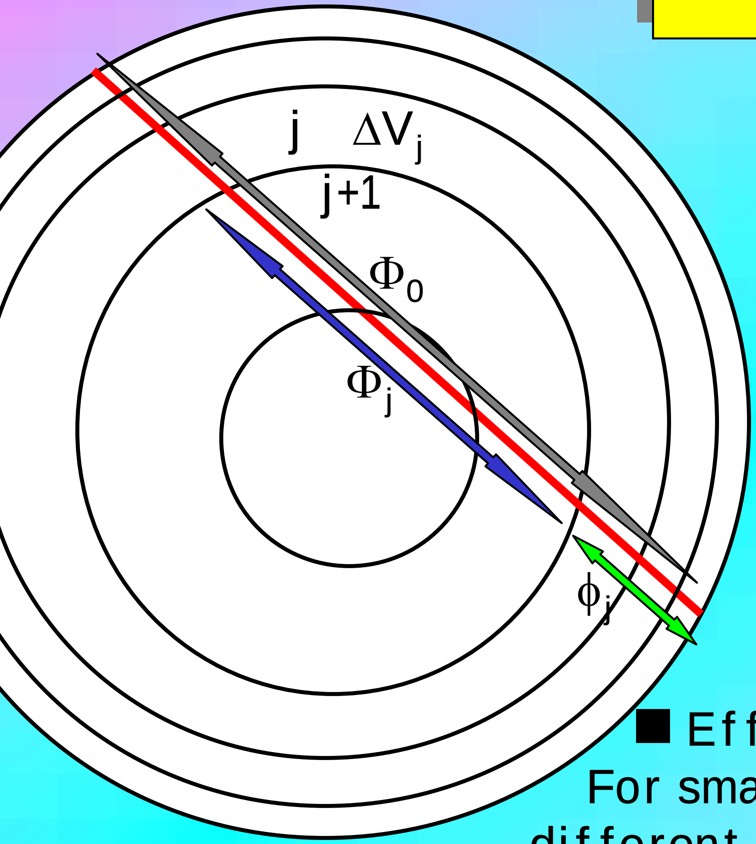
Φ_0 - adiabatic phase along the whole trajectory

ΔV_j - jump of the potential between j-th and j+1 shells

Φ_j phase acquired within borders of with jumps ΔV_j and $-\Delta V_j$

Effect of small structures

$$f_{reg} = \frac{2E \sin^2 2\theta}{\Delta m^2} \sin\Phi_0/2 \ \Sigma_{j=0 \ldots n-1} \Delta V_j \sin\Phi_j/2$$



Defining $\phi_j = 0.5(\Phi_0 - \Phi_j)$

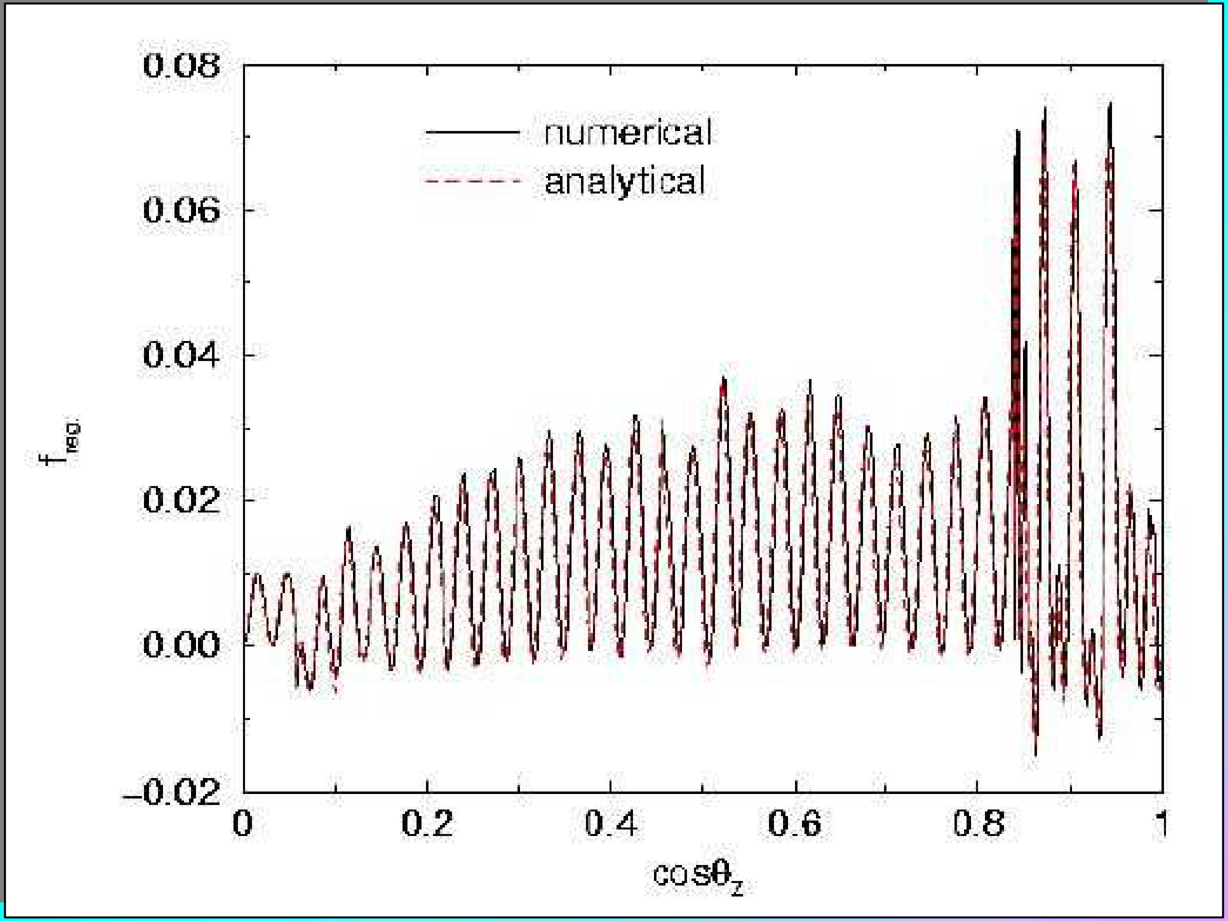
$$f_{reg} = \frac{2E \sin^2 2\theta}{\Delta m^2} \times$$

$$\Sigma_{j=0 \ldots n-1} \Delta V_j [\sin^2\Phi_0/2 \cos\phi_j \ - \ 0.5 \sin\Phi_0 \sin\phi_j]$$

- If ϕ_j is large - averaging effect. This happens for remote structures, e.g. core

- Effect of shells at small depth (~ 10 km) is important. For small $\cos\theta_Z$ - interference of contributions from different shells - oscillatory behaviour of f_{reg} For large $\cos\theta_Z$ - the distance is small and they can be accounted as one layer.

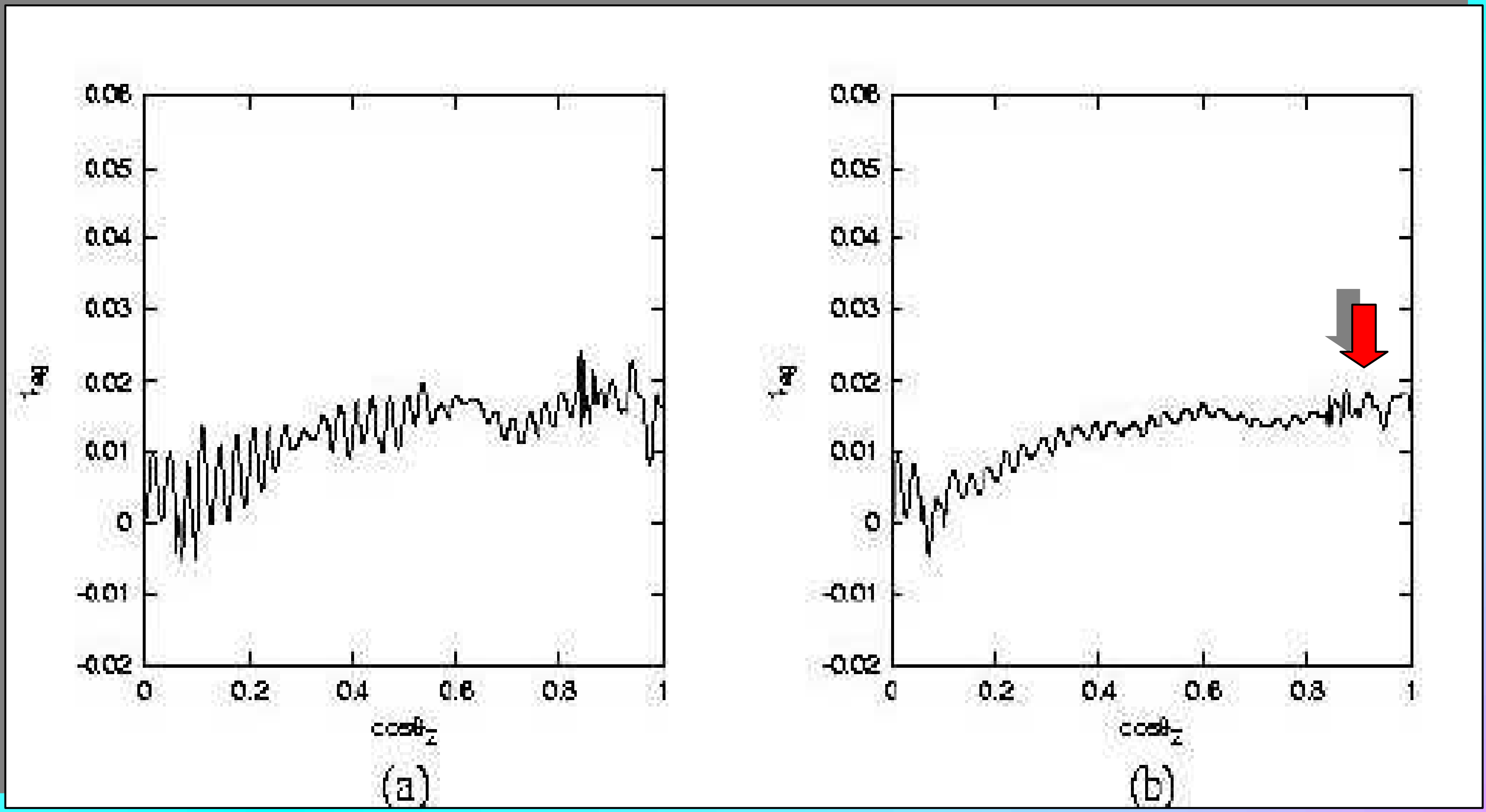
Analytic vs. numerical results



Regeneration factor as function of the zenith angle

E = 10 MeV, $\Delta m^2 = 6\ 10^{-5}$ eV^2, $\tan^2\theta = 0.4$

Averaging regeneration factor



Regeneration factor averaged over the energy intervals
E = (9.5 - 10.5) MeV (a), and E = (8 - 10) MeV (b).

No enhancement for core crossing trajectories in spite of larger densities

Neutrino oscillations in low density medium

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hep-ph/0404 060

$V(x) \ll \Delta m^2 / 2E$

Potential $\ll$ kinetic energy

For LMA oscillation parameters applications to

Solar neutrinos
Supernova neutrinos

Small parameter:

$$\varepsilon(x) = \frac{2\,E\,V(x)}{\Delta m^2} \ll 1$$

$\varepsilon(x) \sim (1 - 3)\,10^{-2}$

perturbation theory in $\varepsilon(x)$

ε- perturbation theory

Weak matter effect:
mixing in matter ~ mixing in vacuum,
$\theta_m \sim \theta$

→ theory in terms of mass eigenstates
$\nu_{mass} = (\nu_1, \nu_2)^T$

- Mass states mix in matter:

$$\nu_{mass} = U' \nu_m$$

$\nu_m = (\nu_{1m}, \nu_{2m})^T$
eigenstates in matter

$$U' = \begin{pmatrix} \cos\theta' & \sin\theta' \\ -\sin\theta' & \cos\theta' \end{pmatrix}$$

- $\theta' = \theta'(V)$ - mixing angle of mass states in matter

$$\sin 2\theta' = \frac{\varepsilon(x)\sin 2\theta}{\sqrt{(\cos 2\theta - \varepsilon(x))^2 + \sin^2 2\theta}} = \varepsilon(x)\sin 2\theta_m$$

small

- Evolution equation:

$$i d\nu_{mass}/dx = H(x)\,\nu_{mass}$$

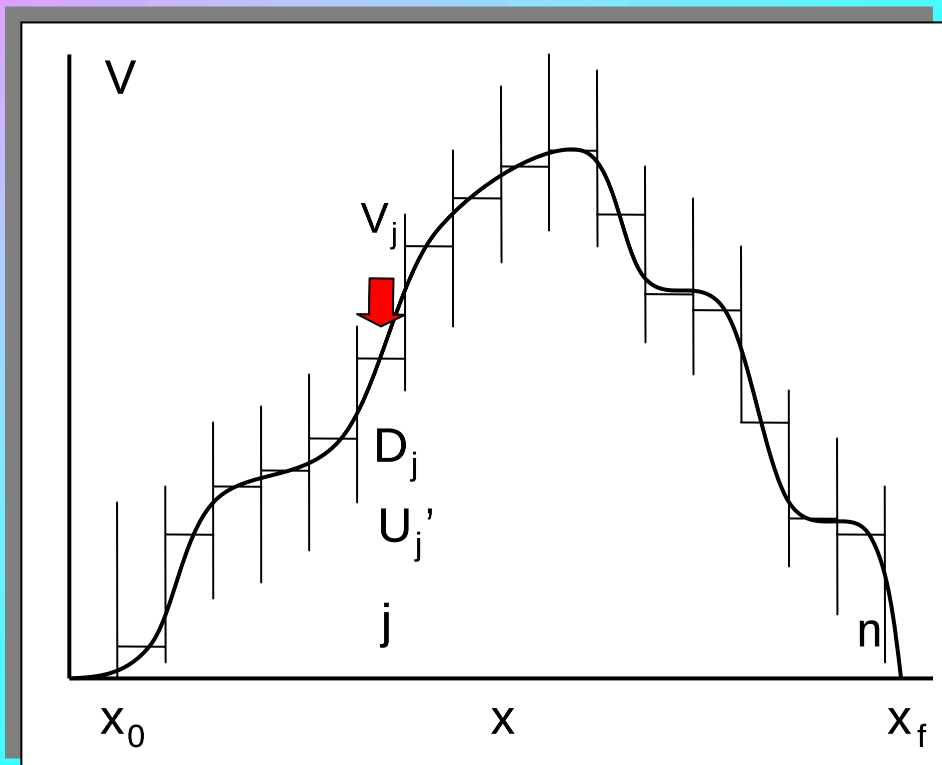
$$H(x) = U'(x)\begin{pmatrix} 1 & 0 \\ 0 & \Delta_m(x) \end{pmatrix} U'(x)^+$$

$$\Delta_m(x) = \frac{\Delta m^2}{2E}\sqrt{(\cos 2\theta - \varepsilon(x))^2 + \sin^2 2\theta}$$

S-matrix

S-matrix in the basis of mass eigenstates $\nu_{mass} = (\nu_1, \nu_2)^T$

$$S(x_0 \to x_f) = (U_n' D_n U_n'^{+}) \;\ldots\; (U_j' D_j U_j'^{+}) \ldots (U_1' D_1 U_1'^{+})$$



Following procedure of the numerical computations . . .

In j-th layer:

- Mixing matrix of mass states:

$$U_j' = \begin{pmatrix} \cos\theta_j' & \sin\theta_j' \\ -\sin\theta_j' & \cos\theta_j' \end{pmatrix}$$

$$\theta_j' = \theta'(V_j)$$

- Evolution matrix of the eigenstates in matter:

$$D_j = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Phi^m_j} \end{pmatrix}$$

- Phase:

$$\Phi^m_j = \Delta x \;\; \Delta_m(V_j)$$

S-matrix

- Each block can be reduced to

$$(U_j' D_j U_j'^{+}) = D_j + G_j$$

$$G_j = 0.5 (e^{i \Phi^m_j} - 1) \sin 2\theta_j' \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + O(\varepsilon^2)$$

$\sim \Delta_m(V_j) \Delta x$

- $$S(x_0 \to x_f) = D_n \ldots D_j \ldots D_1 + \Sigma_j D_n \ldots D_{j+1} G_j D_{j-1} \ldots D_1 + O(G_j G_k) + \ldots$$

$D_j = O(1)$ $G_j = O(\varepsilon)$ expansion in power of G_j

- Limit n -> infty, $\Delta x \to 0$

$\Sigma_j \Delta x \to \int dx$

$$\Sigma_j \Phi^m_j = \Sigma_j \Delta x \Delta_m(V_j) \to \int dx \Delta_m(x)$$

$$\Phi^m(x_0 \to x_f) = \int_{x_0}^{x_f} dx \Delta_m(x)$$

S-matrix

S-matrix in the basis of mass eigenstates $\nu_{mass} = (\nu_1, \nu_2)^T$

$$S(x_0 \to x_f) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Phi^m(x_0 \to x_f)} \end{pmatrix}$$

$$+ 0.5\, i \sin 2\theta \int_{x_0}^{x_f} dx\, V(x) \begin{pmatrix} 0 & e^{i\Phi^m(x_0 \to x)} \\ e^{i\Phi^m(x \to x_f)} & 0 \end{pmatrix}$$

$$+ O(V^2)$$

The amplitude of the oscillation transition $\nu_a \to \nu_b$

$$A_{a \to b}(x_0 \to x_f) = \langle \nu_b | S(x_0 \to x_f) | \nu_a \rangle$$

Main result. Regeneration factor

Mass-to-flavor transition: $\nu_2 \to \nu_e$

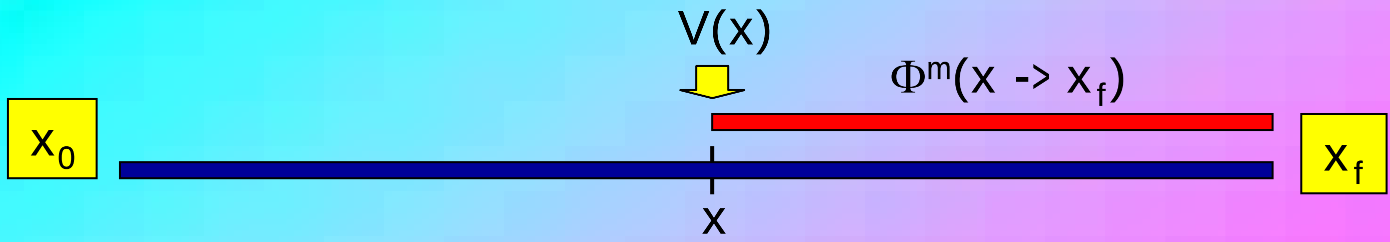
$$P_{2e} = \sin^2\theta + f_{reg}$$

Regeneration factor

$$f_{reg} = 0.5 \sin^2 2\theta \int_{x_0}^{x_f} dx\, V(x) \sin \Phi^m(x \to x_f)$$

$$f_{reg} = 0.5 \sin^2 2\theta \int_{x_0}^{x_f} dx\, V(x) \sin\left(\frac{\Delta m^2}{2E} \int_{x}^{x_f} dy \sqrt{\left(\cos 2\theta - \frac{2EV(y)}{\Delta m^2}\right)^2 - \sin^2 2\theta} \right)$$

Integration limits:



The phase is integrated from a given point to the final point

Sensitivity to structures of the density profile

For mass-to-flavor transition V(x) is integrated with $\sin \Phi^m(d)$

$d = x_f - x$ the distance from structure to the detector

larger d → larger $\Phi^m(d)$ → stronger averaging effects → weaker sensitivity to structure of density profile

Integration with the energy resolution function R(E, E'):

$$\overline{f_{reg}} = \int dE' \; R(E, E') \; f_{reg}(E')$$

The effect of averaging:

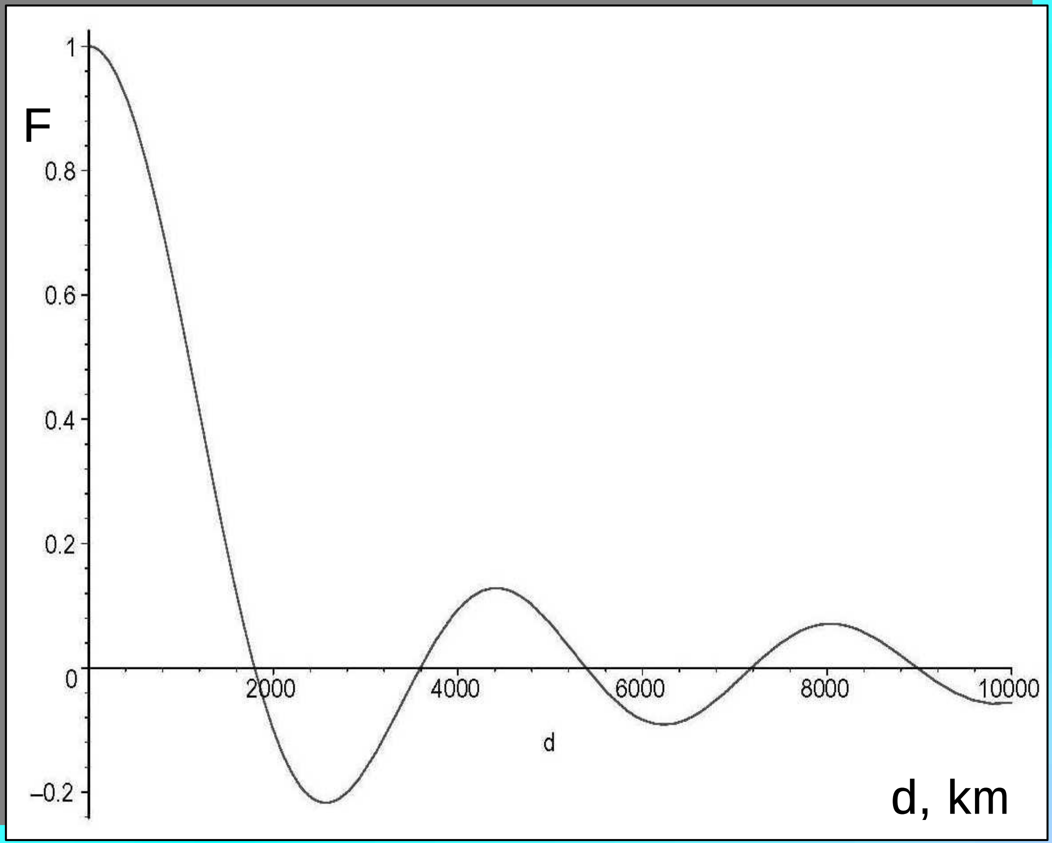
$$\overline{f_{reg}} = 0.5 \sin^2 2\theta \int_{x_0}^{x_f} dx \; V(x) \; F(x_f - x) \sin \Phi^m(x \to x_f)$$

averaging factor

For box-like R(E, E') with width ΔE:

$$F(d) = \frac{l_\nu \; E}{\pi \; d \; \Delta E} \sin\left(\frac{\pi \; d \; \Delta E}{l_\nu \; E}\right)$$

Sensitivity to structures of profile



The width of the first peak

$d < l_\nu E/\Delta E$

l_ν is the oscillation length

The sensitivity to remote structures is suppressed:

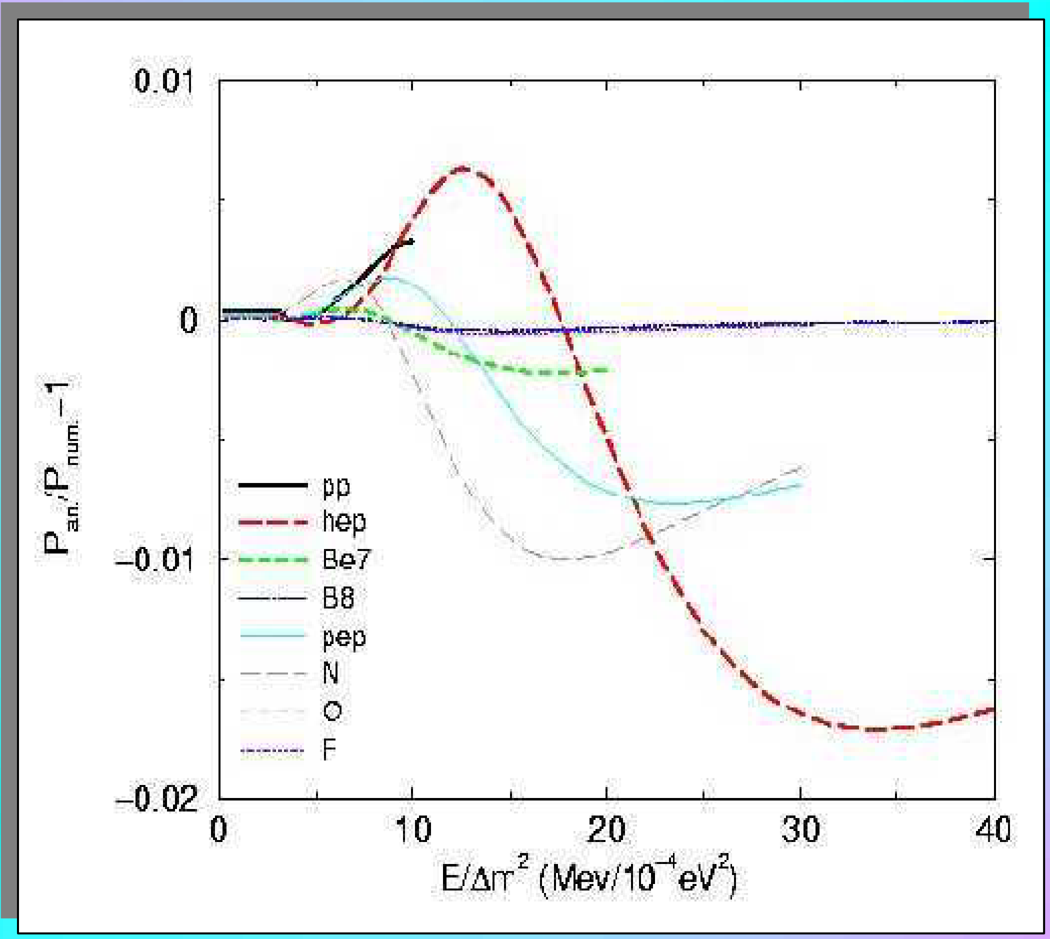
- Effect of the core of the Earth is suppressed
- Small structures at the surface can produce stronger effect
- The better the energy resolution, the deeper penetration

Summary

- For LMA solution one can use the adiabatic perturbation theory to describe the conversion both in the Sun and in the matter of the Earth
- Non-adiabatic corrections for propagation in the Sun are negligible

  Precise analytic analytic expression for the probability averaged over the production region in the Sun have been obtained
- Precise (1 -2 %) analytic formula for the Earth matter effects has been obtained which allows us to explain detailed features of regeneration effect
- Precise description of the oscillation effects in the low density medium is given using the ``epsilon- perturbation" theory
- The obtained formulas substantially simplify numerical computations allow to study the sensitivity of the oscillation effects to structure of the density profile

Averaging over production region



Precision of approximation