Einstein and Cosmology

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This review gives a historical account of how cosmology has developed since the 1917 paper of Albert Einstein. Today it is a frontier level science drawing on contemporary astronomy as well as contemporary physics, stretching both as far as extrapolations will permit. Thanks to numerous observations at different wavelengths, cosmologists today have their plates full. Extrapolations of laboratory tested physics are called for to understand all information within the framework of a standard model. The success and shortcomings of this approach are briefly discussed against the historical backdrop.

1. Historical Background

The year 1905 is a historical year for physics and for Albert Einstein. It was during this year that Einstein published three papers of a fundamental character[1-3], one on Brownian motion, another on the photoelectric effect and the third on special theory of relativity. The first gave a statistical description of molecular motion in fluids and enriched our understanding of the microscopic composition of fluids. The second was the initiator of the idea that light can also be described as made of particles subject to the discipline of quantum mechanics, while the third led to a revision of the very basic ideas of space-time measurements. It is but natural that we are this year celebrating the international year of physics as a centenary of the above developments.

Ten years after proposing his special theory of relativity, Einstein came out with a more comprehensive general theory of relativity which also provided a very unusual description of the phenomenon of gravity as a manifestation of curved spacetime around any presence of matter and energy. After coming out with the general theory in 1915, Albert Einstein[4] used it in an ambitious way to propose a model of the entire universe. This simple model assumed that the universe is homogeneous and isotropic and *also static*. Homogeneity means that the large scale view of the universe and its physical properties at any given epoch would be the same at all spatial locations. Isotropy demands that the universe look the same in all directions, when viewed from any spatial location. The requirement of a static universe was motivated by the then perception that there is no large scale systematic movement in the universe.

That was the general belief at the time. In fact the realization that there is a vast world of galaxies spread beyond the Milky Way had not yet seeped into the astronomical community. Although there were isolated measurements of nebular redshifts, these did not convey any impression that the universe as a whole is not static. However, to obtain such a static model Einstein had to modify his general relativistic field equations to include an additional *cosmological constant term* λ which corresponded to a long range force of repulsion.

The original equations were:

$$R_{ik} - 1/2g_{ik}R = - [8\pi G/c^4] \; T_{ik}. \tag{1}$$

Here the left hand side relates to the spacetime geometry of the universe and the right hand side describes the physical contents of the universe. These equations did not yield a static solution and so Einstein sought to modify them in the *simplest possible way*. This led him to the following equations:

$$R_{ik} - 1/2g_{ik}R + \lambda g_{ik} = - [8\pi G/c^4] \ T_{ik}. \tag{2}$$

In the 'Newtonian approximation' this additional term corresponds to an acceleration of λrc^2 between any two matter particles separated by a distance r. The constant is λ is called the cosmological constant since its value is very small, (today's estimate is $\sim 10^{-56}$ cm^{-2})and it does not affect the motion of matter isignificantly on any but the cosmological scale.

The Einstein universe, as the model came to be known described the universe by a spacetime metric given by

$$ds^2 = c^2 dt^2 - S^2 [dr^2/(1-r^2) - r^2(d\theta^2 + \sin^2\theta \ d\phi^2)], \tag{3}$$

where the spherical polar coordinates have their usual meaning on the surface of a hypersphere of radius S. The field equations (2) then give the density and radius of the universe in terms of the fundamental constants G, c and λ. To Einstein this was an eminently satisfactory outcome as it related physics of the universe to its spacetime geometry in a unique way. The gravity of the matter 'curled up' the space into a finite volume, showing the essence of the general relativistic relationship between gravity and space curvature. He felt that the uniqueness of the solution attached special significance to the model in terms of credibility.

He was in for disappointment on this count as within a few months de Sitter[5] found another solution to the same equations with the metric given by

$$ds^2 = c^2 dt^2 - e^{2Ht} [dr^2 - r^2(d\theta^2 + \sin^2\theta \ d\phi^2)], \tag{4}$$

where H = constant. The de Sitter universe was homogeneous and isotropic but *non-static*. It described an expanding but empty universe. One can say that whereas the Einstein universe had matter without motion, the de Sitter universe had motion without matter. In 1917 the astronomical data did not support the de Sitter model, which remained a mathematical curiosity.

In 1922-24, Alexander Friedmann[6], however, showed that one can obtain homogeneous and isotopic solutions without the cosmological term, but they describe models of an *expanding universe*. In 1927, Abbé Lemaitre[7] also obtained similar solutions, but these, along with the Friedmann models were considered as mathematical curiosities.

Meanwhile, on the observational side, the early (pre-1920) perception of a universe mostly confined to the Milky Way Galaxy with the Sun at its centre, eventually gave way to the present extra-galactic universe in which our location has no special significance. Indeed this 1905 quotation of Agnes Clerke in her popular book on astronomy expresses the current dogma of those times:

The question whether nebulae are external galaxies hardly any longer needs discussion. It has been answered by the progress of research. No competent thinker, with the whole of the available evidence before him, can now, it is safe to say, maintain any single nebula to be a star system of co-ordinate rank with the Milky Way. A practical certainty has been attained that the entire contents, stellar and nebula, of the sphere belong to one mighty aggregation, and stand in ordered mutual relations within the limits of one all embracing scheme.

This perception represented the majority view which was still current in 1920 when the famous Shepley-Curtis debate took place. Shapley spoke in support of this view while Curtis represented the slowly emerging view that many of the faint nebulae were external galaxy far away from the Milky Way.

During the 1920s Edwin Hubble gradually established this picture in which spiral and elliptical galaxies are found all over the universe. The erroneous observations of Van Maanen contradicting this picture and arguing that all spiral nebulae were galactic, had been influential in the delay in accepting this revised picture. These were eventually set aside. In 1929, Hubble established what is today known as the

Hubble Law[8] which is generally interpreted as coming from an expanding universe. In this Hubble spectroscopically determined the Doppler radial velocities of galaxies and found these to vary in proportion to their distances. The constant of proportionality is called the Hubble constant and today it is denoted by H. Thus one may write Hubble's law in terms of redshifts as:

$$z = (H/c) . D \qquad (5)$$

where D is the distance of the extragalactic object with redshift z. The Friedmann-Lemaitre models now no longer were mathematical curiosities but were seen as the correct models to explain Hubble's law. They were all describable with the line element

$$ds^2 = c^2 dt^2 - S^2 [dr^2 / (1 - kr^2) - r^2 (d\theta^2 + \sin^2 \theta \; d\phi^2)], \qquad (6)$$

where the parameter k takes values 1, 0 or -1. The Einstein universe had $k = 1$ whereas the de Sitter universe had $k = 0$. The coordinates r, θ, ϕ are constant for a typical galaxy and may be called its comoving coordinates. The motion of the galaxy is manifest through the scale factor $S(t)$. The redshift is interpreted in terms of this model as coming from a time-dependent increasing scale-factor $S(t)$: if the light signal from the source left at time t_1 and it reached the observer at time t_0 then we have

$$1 + z = S(t_0) / S(t_1). \qquad (7)$$

The scale-factor $S(t)$ and the curvature parameter k were to be determined from Einstein's field equations. Einstein also decided that his cosmological constant was no longer needed and gave it up. Incidentally the much-publicised remark by Einstein that the cosmological constant was the 'greatest blunder' of his life has no direct authentication in Einstein-literature. It has been ascribed to George Gamow who claimed that this is what Einstein said to him[10].

The stage was thus set to launch cosmology as a discipline wherein the theoretical predictions based on relativistic models could be tested by observations of the extragalactic universe.

2. Early Cosmology

During the 1930s, cosmologists led by Eddington[10] and Lemaitre[11] discussed the theoretical models of the expanding universe and all these led to the concept of a 'beginning' when the universe was dense and very violent. Lemaitre called the state that of a *primeval atom*. Later, Fred Hoyle, an opponent of this idea referred to the state as of 'big bang', a name that caught on when the model became more popular.

The crucial effect in Hubble's law was the redshift found in the spectra of galaxies and its progressive increase with the galactic distances. The linear law discovered by Hubble was believed to be an approximation of the exact functional relationship between redshift and distance according to any of the various Friedmann-Lemaitre models. Attempts were made by succeeding astronomers to carry out deper surveys to test the validity of this extrapolation. This will be discussed later.

Hubble's own priorities on the observational side, were elsewhere[12]. He wanted to fix the value of the mathematical parameter k of the model by observing galaxies and counting them to larger and larger distances. He made several unsuccessful attempts before realizing that the ability of the 100-inch Hooker telescope fell short of making a significant test of the relativistic models. The 5-metre telescope at the Palomar Mountain was proposed by him for this very reason as this bigger telescope was expected to settle this cosmological problem. By the time the telescope was completed and began to function (late 1940s) Hubble had realized that his observational programme was not a realistic one and the telescope in fact came to be used for other important works.

The reason Hubble's programme was unworkable was that in order to detect the effects of spacetime curvature through galaxy counts, one needed to look very far, out to redshifts of the order unity, and this requirement was hard to satisfy for two reasons. (1) Observational techniques were not yet sophisticated enough to detect galaxies of such large redshifts. (2) The number of galaxies to be counted was enormously large if one were to use the counts to be sensitive enough to draw cosmological conclusions. There was a third difficulty with the number count programme, to which I shall return in section 9.

3. The Advent of Radio Astronomy

Astronomy became more versatile after World War II, after radio astronomy came into existence as a viable tool of observations. In their enthusiasm about the new technique, radio astronomers felt that they could undertake Hubble's abandoned programme by applying it to the counts of radio sources. In the 1950s radio astronomers in Cambridge, England and in Sydney as well as Parkes, Australia, began their attempts to solve this problem by counting radio sources out to very faint limits. Radio astronomy apparently got round the two difficulties mentioned above. Radio galaxies could be observed, it was felt, to greater distances than optical galaxies and there were far fewer of them to count.

The basic test of counting of radio sources went thus. If one accepts that radio sources are of uniform luminosity and are homogeneously distributed in the universe, then in the static Euclidean model, it can be easily shown that the number (N) - flux density (P) relation satisfies the relation

$$\log N = -1.5 \log P + \text{constant}. \tag{8}$$

The relation for a typical expanding Friedmann-Lemaitre model shows a relation starting with (8) at high flux end and getting flatter at low fluxes. If, however, one put in an ad-hoc assumption that the number density of radio sources per unit comoving coordinate volume was higher than at present, then one could get slopes steeper than -1.5.

While the Australians felt that within the existing error-bars, their surveys did not show any evidence inconsistent with the Euclidean model, the Cambridge group under the leadership of Martin Ryle made several claims to have found a steep slope. While the early Cambridge data were later discounted as being of dubious accuracy, the data in the early 1960s (the 3C and 4C surveys) did show a slope of -1.8 at high flux density, which subsequently flattened at low flux densities. The steepness was claimed by Ryle to have confirmed the big bang models. However, it later became clear that these radio surveys might tell us more about (1) local inhomogeneity and (2) the physical properties of the sources rather than about large scale geometry of the universe[13].

4. The Steady State Theory

In 1948, there emerged a rival to the classic big bang theory. Authored by Hermann Bondi, and Thomas Gold[14] and independently by Fred Hoyle[15], this theory was based on a model of the universe with the de Sitter metric, but which had a constant non-zero density of matter. Such a model can be obtained from Einstein's gravitational equations (without the cosmological term), provided on the right hand side one introduces a negative energy field, called originally the *C*-field. Hoyle and later Maurice Price (private communication) worked on the *C*-field concept and a theory based on a scalar field derivable from an action principle emerged in 1960. This idea was developed further by Hoyle and Narlikar[16,17]. Although the concept of a negative energy scalar field was considered by physicists to be unrealistic in the 1960s, today, four decades later it is appreciated that the currently popular phantom fields are no different from the *C*-field.

Since, as the name implies, the steady state theory described an unchanging universe (on a large enough scale), the observational predictions of the theory were unambiguous and this was cited as a strength of the theory. Ryle's main attack was directed against this theory with the assertion that the radio source counts disproved this theory. This claim was refuted by Hoyle and Narlikar with the demonstration that in a more realistic structure of the universe inhomogeneities on the scale of 50-100 Mpc (megaparsec: 1 parsec is approximately 3 light years) would give rise to steep slopes of the $\log N - \log P$ curve for radio sources.

Although the steady state theory survived Ryle's challenges, it appeared to receive a mortal blow in 1965 by the discovery of the cosmic microwave background. Also, it could not account for the rather large fraction (~25%) by mass of helium in the universe. To understand the implications of this result one needs to look back at the studies of the early universe in relativistic cosmology.

5. The Early Hot Universe

In the mid-1940s, George Gamow[18,19] started a new programme of studying the physics of the big bang universe close to the big bang epoch. For example, calculations showed that the universe in its early epochs was dominated by relativistically moving matter and radiation and that the temperature T of the universe, infinite at the big bang, dropped according to the law:

$$T = B/S \, . \quad B = \text{constant.} \tag{9}$$

Thus it fell to about ten thousand million degrees after one second. In the era 1-200 second, Gamow expected thermonuclear reactions to play a major role in bringing about a synthesis of the free neutrons and protons that were lying all over the universe. Were all the chemical elements we see today in the universe formed in this era?

This expectation of Gamow turned out to be incorrect. Only light nuclei, mainly helium could have formed this way. Also, one could adjust the density of matter in the universe over a wide band to produce the right cosmic abundance of helium. The heavier elements could, however, be formed in stars, as was shown later by the comprehensive work of Geoffrey and Margaret Burbidge, William Fowler and Fred Hoyle[20]. Today it looks as if the light nuclei were made in Gamow's early universe, as the stars do not seem to be able to produce them in the right abundance. It was because of this circumstance that the steady state universe which did not have a very hot era, failed in the production of helium.

Apart from this evidence, there was another prediction[21] made by Gamow's younger colleagues, Ralph Alpher and Robert Herman, namely that the radiation surviving from that early hot era should be seen today as a smooth Planckian background of temperature of around 5K. This prediction has been substantiated. In fact in 1941, McKeller[22] had deduced the existence of such a background of temperature 2.3K from spectroscopic observations of CN and other molecules in the Galaxy. This result was not widely known or appreciated at the time. In fact it was the serendipitous observation of an isotropic radiation background in 1965 by Arno Penzias and Robert Wilson[23] that drew physicists and cosmologists to the big bang model in a big way. Penzias and Wilson found the temperature to be 3.5K.

The post-1965 development of cosmology took a different turn. The finding of the cosmic microwave background radiation (CMBR)[24] was taken as vindication of the early hot universe and on the observational side efforts were made to observe the spectrum of the radiation as accurately as possible. In 1990, the COBE satellite gave a very accurate Planckian spectrum thus providing confirmation of the Alpher-Herman expectation of a relic black body spectrum. Another expectation, of finding small scale inhomogeneities in the background was also fulfilled two years later when COBE found[25] such fluctuations of temperatures $\Delta T / T$ of the order of a few parts in a million. On the theoretical side the emphasis shifted from general relativistic models to models of a very small scale universe with high

temperature corresponding to fast moving particles. Theorists also began to come to grips with the problem of formation of large scale structure ranging from galaxies to superclusters. We will consider these developments next.

6. Physics of the Early and Very Early Universe

The cosmic microwave background radiation (CMBR) prompted many physicists to look in depth at the physics of the post and pre nucleosynthesis era. For example, as the universe cools down, the chemical binding can become important and trap the free electrons into protons to make neutral hydrogen atoms. This eliminates the major scattering agency from the universe and radiation can subsequently travel freely. Calculations[26] show that this epoch was at redshift of around 1000 – 1100.

If instead we explore epochs *earlier* than the nucleosynthesis one, we would encounter larger temperature and more energetic activity. This has attracted particle physicists to the big bang models for here they have a possibility of testing their very high energy physics. The very early epochs when the universe was 10^{-38} second old had particles of energy so high that the might have been subject to the grand unification scheme which could therefore be tested. Energies required for such testing are, however, some 13 orders of magnitude higher than what can be produced by the most powerful accelerators on the Earth.

Such a combination of disciplines is called *astroparticle physics*. One of its most influential 'gifts' has been the notion of inflation[27]. This is the rapid exponential expansion of the universe lasting for a very short time, produced by the phase transition that took place when the grand unified interaction split into its component interactions (the strong and electroweak interactions). Inflation is believed to solve some of the outstanding problems of the standard big bang cosmology, such as the horizon problem, the flatness problem, the entropy problem, etc.

7. Dark Matter and Dark Energy

One of the conclusions of inflation is that the space part of the universe is flat. Theoretically it requires the matter density to be $\rho_c = 3H^2/8\pi G$. Here H is the Hubble constant and G is the gravitational constant. This value, sometimes known as the *closure density*, leads straightaway to a conflict with primordial nucleosynthesis which tells us that at this density there would be almost no deuterium produced. Even if we ignore inflation, and simply concentrate on the empirical value of matter density determined by observations, we still might run into a serious conflict between theory and observation: there is evidence for greater matter density than permitted by the above deuterium constraint.

For, while the visible matter in the form of galaxies and intergalactic medium leads to a value of density which is less than 4% of the closure density, there are strong indications that additional *dark* matter may be present too[13]. The adjective 'dark' indicates the fact that this matter is unseen but exerts gravitational attraction on visible matter. Such evidence is found in the motions of neutral hydrogen clouds around spiral galaxies and in the motions of galaxies in clusters. Even this excess matter would cause problem with deuterium.

To get round this difficulty, the big bang cosmologists have hypothesized that the bulk of dark matter is *non-baryonic*, that is it does not influence nucleosynthesis. Writing the ratio of the density of non-baryonic matter to the closure density as Ω_{nb} and the corresponding ratio for baryonic matter as Ω_b, we should get as per inflation $\Omega_{nb} + \Omega_b = 1$. Thus if the baryonic matter is 4%, the non-baryonic matter should be 96%.

However, even this idea runs into difficulty as there is no direct evidence for so much dark matter. A solution is provided, however, by resurrecting the cosmological constant that Einstein had abandoned in the 1930s. We can define its relative contribution to the dynamics of expansion through a parameter analogous to the density parameter:

$$\Omega_\Lambda = 3\lambda H^2 / c^2. \tag{10}$$

Thus we now get something like: $\Omega_b = 0.04$, $\Omega_{nb} = 0.23$, and $\Omega_\Lambda = 0.73$. This extra energy put in is called *dark energy.* The total of these values is meant to add up to unity, as expected by the inflationary hypothesis.

8. Structure Formation

These issues are important to the understanding of how large scale structure developed in the universe. To this end, the present attempts assume that small fluctuations were present in the very early universe and these grew because of inflation and subsequent gravitational clustering. Various algorithms exist for developing this scenario. One of the basic inputs is the way the total density is split up between baryonic matter, non-baryonic matter and dark energy. The non-baryonic dark matter can be hot (HDM) or cold (CDM) depending on whether it was moving relativistically or non-relativistically at the time it decoupled from ordinary (baryonic) matter.

A constraint to be satisfied by this scenario is to reproduce the observed disturbances found in the CMBR by these agents and also the observed extent of clustering of galaxies today. For, observations of small inhomogeneities of the CMBR rule out various combinations and also suggest what kind of dark matter (cold or hot or mixed) might be required. Currently the model favoured is called the ΛCDM-model to indicate that it has dark energy and cold dark matter.

9. Observational Tests

Like any physical theory cosmology also must rely on observational tests and constraints. There are several of these. There have been tets of cosmological models of the following kinds:

      (I)   Geometry of the universe

      (II)   Physics of the universe.

The first category includes the measurement of Hubble's constant, the redshift magnitude relation to high redshifts, the counting of radio sources and galaxies, the variation of angular size with redshift and the variation of surface brightness with redshift. The measurement of Hubble's constant has been a tricky exercise right from the early days dating back to Hubble's original work. The problem is to be sure that no systematic errors have crept in the distance measurement, as these have not yet been fully debugged. Which is why we still have serious observing programmes yielding values close to 70 km/s/Mpc as well as to 55 km/s/Mpc. At the time of writing this review, the majority opinion favours the higher value but 'rule of the majority' has not always been a successful criterion in cosmology.

The measurement of *z-m* relation had been attempted by Allan Sandage for quite a long time and during the period 1960-1990 the overall view was that the relation as applied to brightest galaxies in clusters treated as standard candles, favoured *decelerating* models. These models are naturally given by the Friedmann solutions *without* the cosmological constant. However, in the late 1990s, the use of Type Ia supernovae has led to a major reversal of perception and the current belief[28] is that the universe is *accelerating.*

The other tests like number counts or angular size variation have not been so clearcut in their verdict as they get mixed up with evolutionary parameters. Apart from the difficulties encountered by Hubble in the 1930s, any cosmological test using source populations of a certain type necessarily gets involved with the possibility that the source yardstick may be evolving with age.

Currently cosmologists are most attracted to measurements of the angular power spectrum of the microwave background inhomogeneities. These can be related to other dynamical features of the universe, given a cosmological model satisfying Einstein's equations with the cosmological constant. Using the details[29] from WMAP satellite one can get a range of models with $k = 0$. Among these models those with a positive cosmological constant are favoured. As mentioned before the favoured solution has $\Omega_b = 0.04$, $\Omega_{nb} = 0.23$, and $\Omega_\Lambda = 0.73$. We recall that the low value of baryonic density is required to understand the abundance of deuterium.

Many cosmologists feel that there is now a 'concordance' between various tests that suggest the above combination for the energy content of the universe together with the higher of the two values of the Hubble constant mentioned above. It is felt that this set of parameters describes accurately most of the observed features of the universe. With this optimistic view one may be tempted to think that the quest for the model of the universe that began with Einstein in 1917 is coming to an end.

10. Need for caution and alternatives

However, there needs to be some caution towards this optimism. The concordance has been achieved at the expense of bringing in a lot of speculative element into cosmology. Thus there is as yet no independent evidence for the non-baryonic dark matter, nor any for the dark energy. When one finds that these two make up more than 96% of matter in the universe leaving only about 4% to the astronomer for direct observation, one wonders whether the claims based on the unseen and the untested are really as firm as one wants in science. Then a lot revolves round the concept of inflation which is still not describable as a process based on a firm physical theory. Nor is the inflationary era observable by any telescopes today. The densities of matter one is talking about when inflation took place were some 10^{50} times the density of water. Recall how much investigation went into the equation of state for neutron stars where the matter density was a mere 10^{15} times the density of water. Yet one finds no such discussions of such matter amongst the cosmologists. Likewise, the inflationary time scales of the order of 10^{-38} second defy any operational physical meaning. These are some twenty five orders of magnitude smaller than the shortest measurable time scale known to physics, viz. those measured by the atomic clocks. So a physicist may wonder if the concordance cosmology is a rigorous physical exercise at all.

The concordance picture looks good today if one is happy with the number of epicycles that have gone into it. Non-baryonic dark matter and dark energy are two of them. They had to be introduced in order to ensure the survival of the model: they have no independent direct confirmation. These are examples of extrapolations of known physics to epochs that are astronomically unobservable. While indirect observations showing an overall consistency of these assumptions are necessary for the viability of the concordance model, they cannot be considered sufficient.

This is why there appears to be need for new ideas in cosmology especially alternative scenarios that are less speculative and follow very different tracks from the above standard scenario. Some attempts are in vogue at present, like the Quasi-Steady State Cosmology (QSSC)[30] or the Modified Newtonian Dynamics (MOND)[31], which are, however very much minority efforts. Perhaps by 2017, a hundred years after Einstein's paper on cosmology we may have a more realistic perception of how complex our universe is. I can do no better than end with a quotation from Fred Hoyle[32]:

... I think it is very unlikely that a creature evolving on this planet, the human being, is likely to possess a brain that is fully capable of understanding physics in its totality. I think this is inherently improbable in the first place, but even if it should be so, it is surely wildly improbable that this situation should just have been reached in the year 1970 ...

Fred Hoyle said this at the Vatican Conference held towards the end of the 1960-70 decade when cosmologists were making equally confident remarks about how well the universe was being understood. This was before inflation, dark matter, dark energy, etc. were even thought of. Are today's cosmologists sure that they have all pieces of the jigsaw puzzle that make up our universe?

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