

# Energy loss of muons and taus through inelastic scattering on nuclei

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A hybrid model [1] was used to describe the energy loss of very high-energy taus and muons in matter due to inelastic scattering on nuclei. The model involves soft and semihard photonuclear interactions as well as the deep inelastic scattering including the weak neutral current processes. For the lepton scattering off nuclei all important nuclear effects, the shadowing, anti-shadowing, EMC and the nucleon binding, were taken into account. Approximating formulas for the muon and tau energy loss portion by inelastic scattering off nuclei in water are given for wide energy range.

## 1. Introduction

The muon inelastic scattering off nuclei contributes noticeably to the energy loss of cosmic rays muons. The influence of this interaction on the shape of ultra-high energy muon spectra at the great depth of a rock/water is still unknown in detail. The tau-lepton energy loss is of interest in view of ability of the atmospheric or extraterrestrial muon neutrinos to transform to the tau neutrinos which may in turn produce taus in  $\nu N$ -interactions.

In Ref. [1], the hybrid (two- and three-component) model was proposed to describe high-energy interactions of charged leptons with nuclei. Calculations of differential cross sections for lepton-nucleon inelastic scattering at the HERA energies were checked making a comparison with H1 and ZEUS measurements of electron and positron scattering on protons. With this model muon and tau energy loss spectra due to lepton-nuclear inelastic scattering were computed as well as the energy loss rate for leptons passing through standard rock. Now we apply the two-component version to study the energy loss in the scattering on nuclei of very-high energy muons and taus in their passage through water and rock. The difference in the scattering of opposite charged leptons that might originate from the weak neutral current processes is considered.

## 2. Charged lepton inelastic scattering off nuclei

The hybrid two-component (2C) model for inelastic interactions of high-energy muons and taus with nuclei involves photonuclear interactions at low and moderate momentum transfer squared as well as the deep inelastic scattering (DIS) processes at high  $Q^2$ . For virtuality  $0 < Q^2 < 5 \text{ GeV}^2$  the Regge based parametrization [2, 3] for the electromagnetic structure function  $F_2^\gamma$  was applied and the cross section of lepton-nucleon scattering at  $Q^2 \leq 5 \text{ GeV}^2$  was computed with the formula

$$\frac{d^2\sigma}{dQ^2 dy} = \frac{4\pi\alpha^2}{yQ^4} \left[ 1 - y - \frac{Q^2}{4E^2} + \frac{y^2}{2(1+R)} \left( 1 - \frac{2m_l^2}{Q^2} \right) \left( 1 + \frac{Q^2}{E^2 y^2} \right) \right] F_2^\gamma(x, Q^2), \quad (1)$$

where the ratio  $R = \sigma_L/\sigma_T$  is taken into account according to Ref. [4];  $x = Q^2/(2MEy)$ . In the DIS range the cross section of nonpolarized lepton scattering off nonpolarized nucleon can be written in the form [5]

$$\frac{d^2\sigma}{dQ^2 dy} = \frac{4\pi\alpha^2}{yQ^4} \left[ \left( 1 - y - \frac{Q^2}{4E^2} + \frac{y^2}{2} - \frac{y^2 m_l^2}{Q^2} \right) F_2^{NC} \pm \left( \frac{y^2}{2} - y \right) x F_3^{NC} \right], \quad (2)$$

where we put  $R = Q^2/(Ey)^2 = 4M^2x^2/Q^2$ , that is equivalent to the Callan-Gross relation,  $F_2 = 2xF_1$ ; signs “ $\pm$ ” stand for  $\ell^\pm$  ( $\ell = \mu, \tau$ ). In Eq. (2) used notations are:

$$F_2^{NC} = F_2^\gamma - g_V^\ell \eta_{\gamma Z} F_2^{\gamma Z} + (g_V^{\ell^2} + g_A^{\ell^2}) \eta_{\gamma Z}^2 F_2^Z, \quad F_3^{NC} = -g_A^\ell \eta_{\gamma Z} F_3^{\gamma Z} + 2g_V^\ell g_A^\ell \eta_{\gamma Z}^2 F_3^Z; \quad (3)$$

$$\eta_{\gamma Z} = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \frac{Q^2}{M_Z^2 + Q^2}, \quad g_V^\ell = -\frac{1}{2} + 2\sin^2\theta_W, \quad g_A^\ell = -\frac{1}{2}. \quad (4)$$

Structure functions  $F_2^Z, F_3^Z$  represent the weak neutral current (NC) contribution,  $F_2^{\gamma Z}, F_3^{\gamma Z}$  are taking into account the electromagnetic and weak current interference. Nucleon structure functions,  $F_2^\gamma, F_2^{\gamma Z}, F_2^Z, xF_3^{\gamma Z}, xF_3^Z$ , are defined in the quark-parton picture:

$$\left[ F_2^\gamma, F_2^{\gamma Z}, F_2^Z \right] = x \sum_q \left[ e_q^2, 2e_q g_V^q, g_V^{q^2} + g_A^{q^2} \right] (q + \bar{q}), \quad \left[ F_3^{\gamma Z}, F_3^Z \right] = \sum_q \left[ 2e_q g_A^q, 2g_V^q g_A^q \right] (q - \bar{q}), \quad (5)$$

where  $g_V^q = \pm\frac{1}{2} - 2e_q \sin^2\theta_W$ ,  $g_A^q = \pm\frac{1}{2}$ . Sign + (–) corresponds to  $u, c, t$  ( $d, s, b$ )-quarks. For the range of  $Q^2 > 5 \text{ GeV}^2$  electroweak nucleon SFs are computed with the CTEQ6 [6] and MRST [7] sets of parton distributions. Linear fits for the nucleon SFs are used in  $5 < Q^2 < 6 \text{ GeV}^2$  range. For the scattering on nuclei effects of the nucleon shadowing, anti-shadowing as well as EMC effect are taken into account according to Ref. [8, 9] (see also [10] and [11] for details).

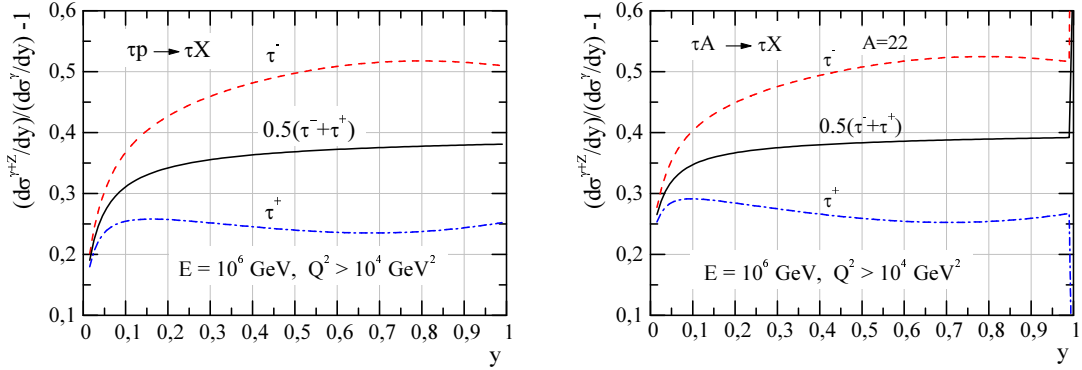
### 3. Results

The energy loss spectra for lepton passing a substance with nuclear weight  $A$  can be derived from the differential cross-section:

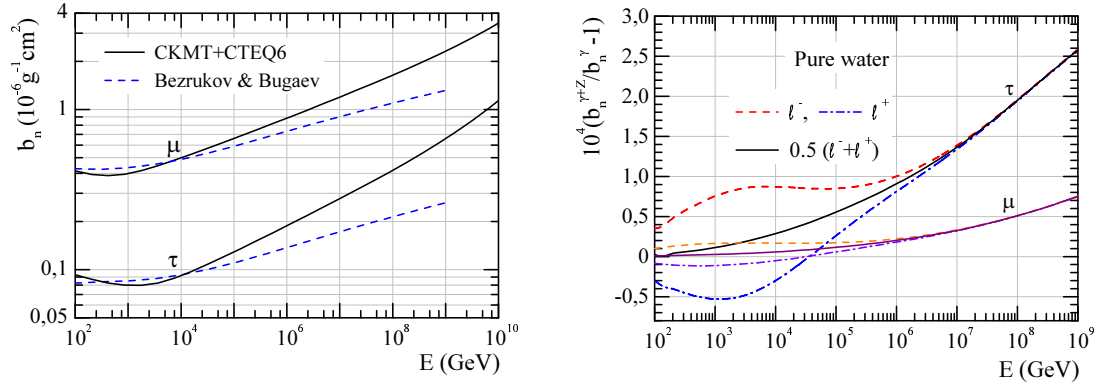
$$N_0 y \frac{d\sigma^{\ell A}}{dy} = N_0 y \int_{Q_{min}^2}^{Q_{max}^2} dQ^2 \frac{d^2\sigma^{\ell A}}{dQ^2 dy}, \quad y = \frac{E - E'}{E} = \frac{\nu}{E}, \quad (6)$$

where  $N_0 = N_A/A$ . The energy loss rate due to lepton-nucleus interactions is defined as

$$b_n^{(\ell)}(E) \equiv -\frac{1}{E} \frac{dE}{dh} = N_0 \int_{y_{min}}^{y_{max}} y \frac{d\sigma^{\ell A}}{dy} dy. \quad (7)$$



**Figure 1.** NC contribution to cross sections for  $\tau$  scattering off protons (left panel) and standard rock nuclei ( $A=22$ ) at  $E = 1 \text{ PeV}$  and  $Q^2 > 10^4 \text{ GeV}^2$ .



**Figure 2.** Energy loss due to inelastic scattering of charged leptons in water (left panel) and NC contribution (right) to the lepton energy loss due to  $\ell^\pm$ -nucleus inelastic scattering in water.

Figure 1 illustrates the charge-dependent NC contribution to scattering leptons and on protons (left panel) and on nuclei ( $A=22$ ) at  $E = 1$  PeV,  $Q^2 > 10^4$  GeV<sup>2</sup> as function of  $y$ . The effect of  $Z$  exchange is fairly seen just for very large  $Q^2$ . In Fig. 2 presented are the energy loss rate by interactions of muons and taus with nuclei in water (left panel). Dashed lines are results of calculations according to the vector-meson dominance model [12] for photonuclear interactions of muons and tau-leptons. The right panel of Fig. 2 shows the difference of energy loss ratio calculated for  $\ell^+$  and  $\ell^-$  particles. For whole energy range this effect is too little ( $10^{-4}$ ) to be of practical interest.

In the second column of the Table 1, the 2C model calculation of the muon and tau energy loss in standard rock and that are presented along with recent predictions [10, 13, 14]. One can see that present calculations of tau-lepton energy loss in standard rock are close to the results of other authors. As concerns to muons, this work result for  $b_n^{(\mu)}$  differs apparently from that of Ref. [14].

**Table 1.** The muon and tau energy loss in standard rock ( $A = 22$ )

$E$ , GeV	$b_n^{(\ell)}(E)$ , $10^{-6}$ cm <sup>2</sup> ·g <sup>-1</sup>			
	This work	Ref. [13]	Ref. [10]	Ref. [14]
Muon				
$10^5$	0.62	0.60	0.68	0.70
$10^6$	0.82	0.80	0.90	1.08
$10^8$	1.53	1.50	—	2.25
$10^9$	2.16	2.15	—	3.10
Tau				
$10^5$	0.13	0.12	—	0.14
$10^6$	0.19	0.18	—	0.21
$10^8$	0.41	0.40	—	0.50
$10^9$	0.65	0.60	—	0.72

The energy dependence of the inelastic scattering energy loss rate of muons and taus traveling through water

may be fitted for the range ( $10^2 - 10^9$ ) GeV with formula ( $\ell = \mu, \tau$ ):

$$b_n^{(\ell)}(E) = (c_0 + c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4) \cdot 10^{-6} \text{ cm}^2/\text{g}, \quad \eta = \lg(E/1 \text{ GeV}), \quad (8)$$

where coefficients are:

$$\begin{aligned} \mu : \quad & c_0 = 1.06416, \quad c_1 = -0.64629, \quad c_2 = 0.20394, \quad c_3 = -0.02465, \quad c_4 = 0.0013; \\ \tau : \quad & c_0 = 0.35697, \quad c_1 = -0.24437, \quad c_2 = 0.07403, \quad c_3 = -0.00940, \quad c_4 = 0.00051. \end{aligned}$$

## 4. Conclusions

Recent calculations [1], [10], [13], [14] of the energy loss in the tau-nuclear interactions are compatible at least for lepton energy up to  $10^9$  GeV. However there is the discrepancy between predictions for high-energy behavior of the muon energy loss,  $b_n^{(\mu)}(E)$ , in Refs. [13] and [1] on the one side, and that in Ref. [14] on the other side, likely due to diverse ways in considering of nuclear effects and high  $Q^2$  processes.

The neutral current (Z exchange) contribution to energy loss of muons and taus is found to be negligible both in water and standard rock on whole energy range (up to  $10^{12}$  GeV). Though the ratio of the cross section for inelastic scattering of  $\tau^-$  to that of  $\tau^+$  is a sizeable for  $Q^2 > 10^4$  GeV<sup>2</sup> at not too large energies, the effect for the energy loss ( $\Delta b_n \sim 10^{-4} \cdot b_n$ ) seems too small in the cosmic ray physics context.

## 5. Acknowledgements

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