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Multifractal nature of extensive air showers

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Cherenkov images are multi-fractal in nature. We show that multi-fractal behaviour of Cherenkov images arises due to multiplicative nature of pair production and bremsstrahlung processes in the longitudinal shower development passage.

1. Shower development as a multifractal process

A ultra relativistic γ-ray enters atmosphere from the top and interacts with air molecule to produce electron-photon cascade . The radiation length(x) for pair production and bremsstrahlung process is equal in UHE/VHE region. Particle -photon cascade in the atmosphere is sustained alternately by electron (e^-) -positron (e^+) pair production and bremsstrahlung process till the average energy per particle reaches critical value E_c below which energy loss process is mainly dominated by ionization process.

Shower development can be visualized a process in which a γ-ray of energy E_0 (= 1 Tev (say)) after traveling distance x (on average) produces electron-positron pair each having energy $\frac{E_0}{2}$. Since energy is getting divided into two equal parts , we can attribute a resolution of energy E =2^{-1}. In the next radiation length both electron and positron lose half of their energy (on average) and each radiates one photon. Thus in this radiation length there are two photons and two particles (e^{-1} and e^{+1}) each having energy $\frac{E_0}{4}$ which can be attributed to energy resolution E= 2^{-2}. At this stage fraction of photons (p_1 =$\frac{1}{2}$) is same as fraction of particles (p_2= $\frac{1}{2}$). As the shower develops into next radiation length both electron and positron lose half of their energy and produce one photon each. At the same time two photons produced in the previous radiation length interact with air molecule to produce electron (e^-) -positron (e^+) pair. Thus in third radiation length there are two photons and six particles each having energy $\frac{E_0}{8}$. This stage can be attributed to the energy resolution E=2^{-3}. It is important to note that, at this stage the process of equal division in energy between each photon and each particle continues but the process of unequal measure between photons and particles begins. In this radiation length the fraction of photons is p_1=$\frac{1}{3}$ and fraction of particles (electron +positrons) is p_2=$\frac{2}{3}$. The fourth radiation length corresponds to energy resolution E=2^{-4} as total of 16 photons and particles are produced each having energy $\frac{E_0}{16}$. However, there are 10 charged particles (5 positrons + 5 electrons) and 6 photons, a case of unequal energy division and unequal fraction.

At a distance of nx , the total number of particles and photons is 2^n , each having average energy $\frac{E_0}{2^n}$ and on an average shower consists of fraction of $\frac{2}{3}$ particles and $\frac{1}{3}$ photons even at nth stage. This corresponds to energy resolution of E=2^{-n}. At nth stage each particle or photon can be labelled sequentially with i=0,1,2,..... The probability or fraction of particles and photons can be written as p_i=$p_1^k p_2^{n-k}$. The partition function for finite energy can be written as $\Gamma(q,\tau,E) = \sum_{i=1}^{n} \frac{N_k p_i^q}{E_i^\tau}$ where N_k is the number of particles and photons. For the simplicity of calculations, we assume that incoming energy E is equal to unit energy. This will not make any difference to actual results but integrate it with other classical examples of multifractal behaviour, e.g. curdling of cantor set [9]. On the average shower development process has a recursive structure similar to cantor set [9] because both processes are inherently binomial multiplicative in nature. For τ=$\tau(q)$,we have $\Gamma(q,\tau(q),E) = 1$. In the limit of E $\rightarrow$ 0, the most dominant contribution to this partition function will survive when τ=$\tau(q)$, where $\tau(q)$ is the solution of the equation $(p_1^q E^{-\tau(q)} + p_2^q E^{-\tau(q)})^n$ =1. Above equation can be easily solved to get $\tau(q)$.

2. Case of equal energy and unequal fractions

In each radiation length the measure of photons and particles fluctuates but on the average the shower consists of $\frac{2}{3}$ positrons and electrons and $\frac{1}{3}$ photons. In the present case E= $\frac{1}{2}$, p_1= $\frac{1}{3}$, p_2=$\frac{2}{3}$. Using above equation and Stirling approximation, we have D_q= $\frac{1}{q-1}\frac{ln(p_1^q+p_2^q)}{lnE}$.

3. Case of unequal energy and unequal fractions

It is possible that as the shower develops deep into atmosphere, division of energy may not remain equal. With the possibility of $E_1 \neq E_2 \neq \frac{1}{2}$ and/or $p_1 \neq \frac{1}{3}$ and $p_2 \neq \frac{2}{3}$ shower development process can still be described as a multifractal process because either the energy or the probability or both need to be different for a multifractal process. The partition function for this case can be written as $(p_1^q E_1^{-\tau(q)} + p_2^q E_2^{-\tau(q)})^n$ =1. where $E_1 + E_2$=E and $p_1 + p_2$ =p. Above equation can be numerically solved for $\tau(q)$ if values of E_1,E_2,p_1 and p_2 are known and fixed for each radiation length.

4. Case of loss of energy

As the shower develops deep into atmosphere most realistic situation is that there will be energy loss in all radiation lengths. So a generalized scenario can be described in which $E_1 \neq E_2 \neq \frac{1}{2}$ and $p_1 \neq \frac{1}{3}$ and $p_2 \neq \frac{2}{3}$ with the condition $E_1 + E_2 <$ E in each radiation length. Each shower is unique because shower development process is random and there are no unique values of E_1, E_2, p_1 and p_2.

5. Simulation studies

Any simulation / experimental arrangement to measure EAS will consist of set of detectors which can measure only a sample or fraction of EAS products. This sample may consist of distribution of charged particles / photons. The set of detectors which measure this distribution represents a geometric support and multifractal measures can be related to this geometric support. This can be done by calculating multifractal moments of this distribution and obtain generalized dimensions from scaling properties of multifractal moments. The definitions of multifractal properties given in previous section are not defined with respect to any support and hence can not be applied directly to simulated / experimental data.

Simulated cherenkov images were generated for γ-rays and protons using CORSIKA code for TACTIC configuration. γ-rays of energy 50 TeV and protons of energy 100 TeV are considered for multifractal studies. Each simulated image is divided into M= 2^ν where ν=2,4,6,8, is the scale. The multifractal moments are G_q= $\sum_{i=1}^{M}(\frac{k_j}{N})^q$ where k_j is the number of photoelectrons in the kth cell and N is the total number of photoelectrons in whole image. G_q shows a power law behaviour with M, i.e. $G_q = M^{\tau(q)}$ where $\tau(q)$ is related to generalized multifractal dimension by D_q=$\frac{\tau(q)}{q-1}$ We have calculated average values of D_q for 1000 images each for γ-rays and protons. We have repeated above studies for 30 TeV γ-rays and 60 Tev protons. No energy dependence of D_q on q was observed.

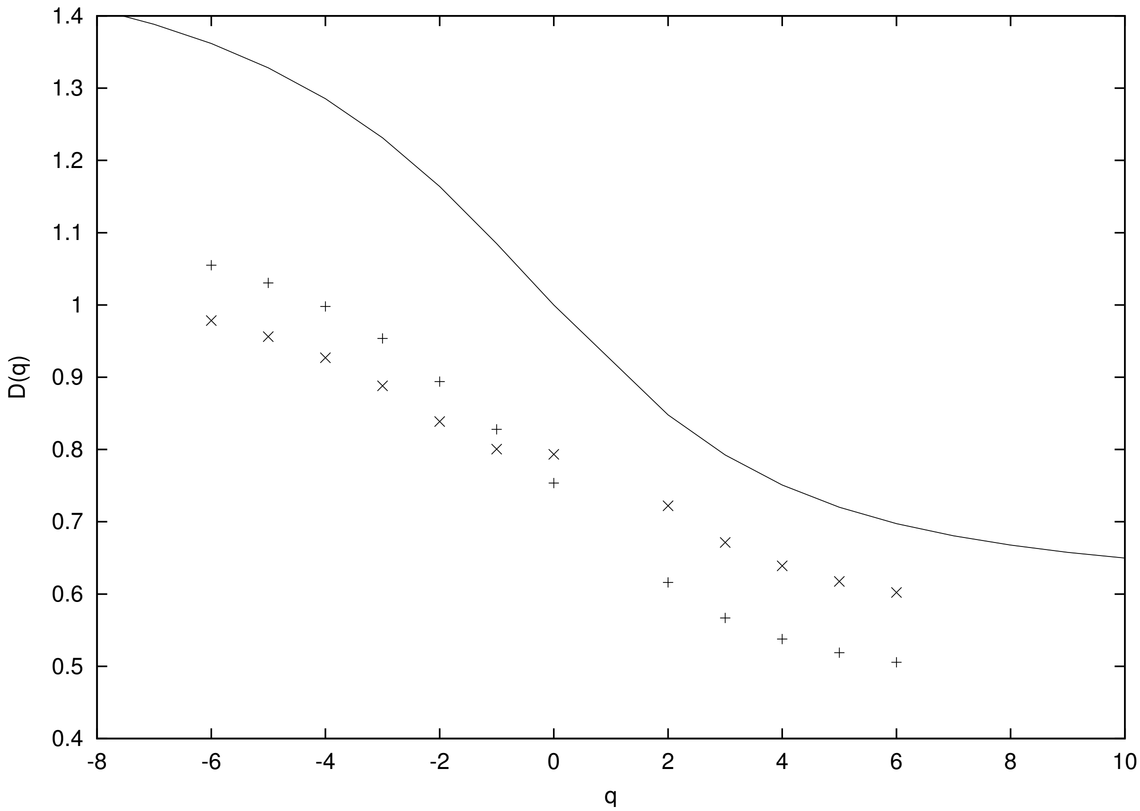

Figure 1. D_q v/s q behaviour

6. Results and Discussion:

As the shower develops deep into the atmosphere, values of E_1,E_2,p_1 and p_2 in various radiation lengths may not remain fixed but may fluctuate. It is not possible to solve equation (4) for all fluctuating values in each radiation length. Hence we have to take average values. Figure 1 depicts D_q dependence on q for various average values. Continuous curve has been obtained in figure 1 for the case E_1=E_2=$\frac{1}{2}$, p_1=$\frac{1}{3}$ and p_2=$\frac{2}{3}$. Simulated data corresponding to protons is represented by (+) sign and simulated data corresponding to γ-rays is represented by (x) sign. D_q values of each cherenkov image corresponding to γ-rays and protons were calculated for 1000 images and average was computed. In figure 1, these average values of D_q for both protons and $gamma$-rays have been shown. It is clear from figure that theoretically known average values of shower progression does not match with average values of simulated results.

Figure 2 shows average values of D_q versus q behaviour for five different cases. Curve 'a' is actually part (of continuous curve) of figure 1, shown here for the purpose of comparison. Average values of D_q corresponding to proton (+ sign shown as curve d) and γ-rays (x sign shown as curve e) have been replotted. The continuous corresponding to 'b' and 'c' have been obtained . Curve 'b' corresponds to E_1=$\frac{13}{35}$, E_2=$\frac{22}{50}$, p_1=$\frac{2}{5}$ and p_2=$\frac{3}{5}$. Curves 'b' correspond to 23 % loss of energy in proton initiated showers in all radiation lengths on the average and curve 'c' corresponds to average loss of 18 % energy in all radiation lengths by γ-ray initiated shower.

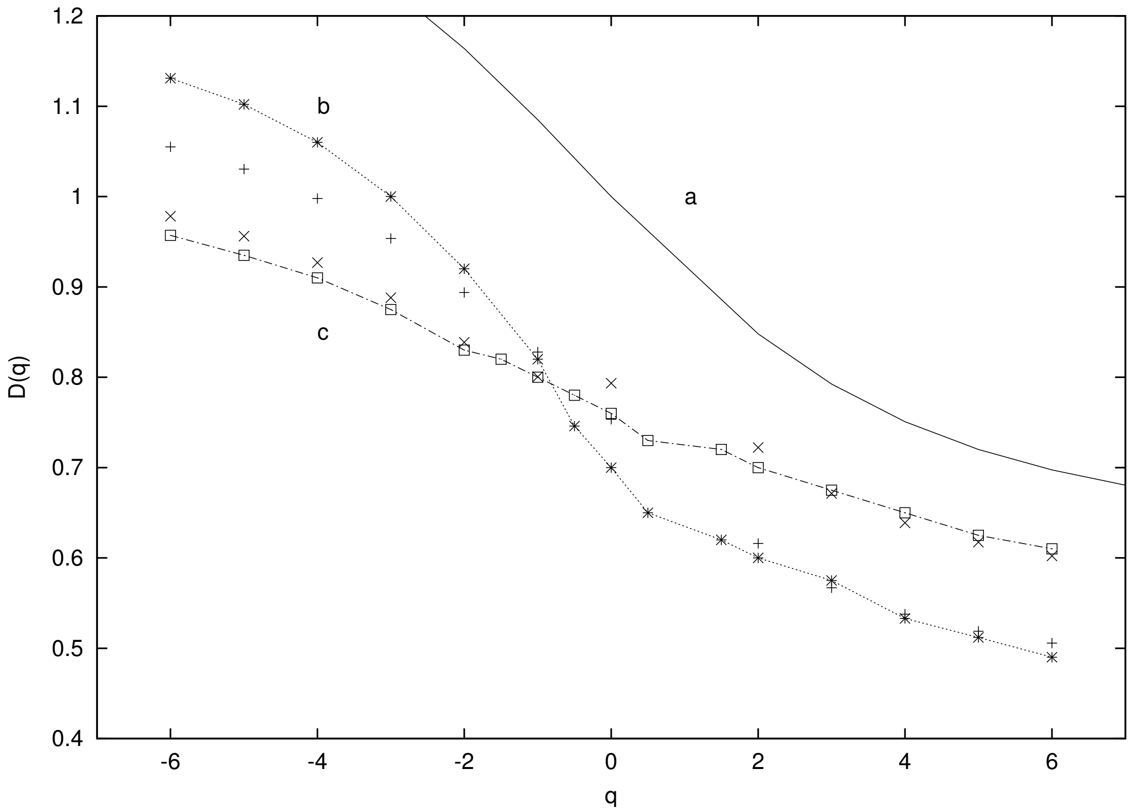

Figure 2. D_q v/s q behaviour

Curve 'a' corresponds to case of no energy loss. It is clear that theoretical model of shower development considered with average loss of energy in each radiation length is a good approximation of showers.

References

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