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Approximation of measured longitudinal development of EAS at ultra high-energies with errors taken into account

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The problem of description of characteristic of UHE cascades with account physical fluctuations and accuracy of measuring process is discussed. To estimate energy and kind of the primary particle the longitudinal development of an individual extensive air shower should be accurately described: both the specific form of longitudinal development and errors of measurements should be taken into account. It is suggested to use function form of $A\exp(-(x-c)^2/(a(x-c)+2b^2))$ which is a generalization of the normal law as a distribution function of measuring errors. The maximum likelihood method has been applied to find out the parameters of an approximation. The values of parameters for the event observed by Fly's Eye in October 1991 have been found.

1. Introduction

Characteristics of longitudinal development of an individual EAS (for example number of particles at the maximum, depth of the maximum, and also more complex characteristics such as the area under a cascade curve) can be used for estimation of the energy and kind of a primary particle [1]. The values of these characteristics are subject by fluctuations of two kinds: 1. Physical fluctuations, defined by fluctuations of interactions and cascade development, 2. Fluctuations of process of measurement (errors) including hardware and software. The revealing and separation of influence of these essentially various fluctuations are special and as rule very difficult problem.

Since the fluctuations change the characteristics at random value, the analysis of values of these characteristics the approximation procedures of results of measurement is used. The procedure of approximation is, that, having set some form of function $\varphi(x,\alpha,\beta\ldots)$, the values of parameters α, β, …, most probably agreed to the measured results (x_i, y_i) are calculated. For example, to describe the longitudinal development of giant ($3.2\pm0.9\times10^{20}$ eV) EAS detected by Fly's Eye the well known formula Gaisser-Hillas as the functional form is used.

To the account for the second kind fluctuations using of general maximum likelihood method requires know the theory of probability distribution functions of the errors. Difficulties of researching of the fluctuations and their influence on results of measurement force to make various assumptions concerning the form of this distribution function, that result to various methods and final results. So, assumption, that all measuring errors are distributed normally, i.e. $f(y,a,b,\ldots)=A\exp(-(y-c)^2/2\sigma^2)$ and with identical dispersions $\sigma_i=\sigma$ $i=1\div n$ at all measured values, results to the well known least squares method:

$$\min\sum_i^n(\varphi(x_i,\alpha,\beta,\ldots)-y_i)^2 \quad (1).$$

If dispersions are various, this assumption results to least squares method with weight coefficients:

$$\min\sum_i^n(\varphi(x_i,\alpha,\beta,\ldots)-y_i)^2/\sigma_i^2$$

Generally, when the distribution functions of errors of measurement in various points differ not only dispersions, but are asymmetrical functions, we used the form

$$f(y,a,b,c)=A\exp(-(y-c)^2/(a(y-c)+2b^2)), \quad (2)$$

the same way resulting to

$$\min \sum_i^n (\varphi(x_i,\alpha,\beta,...)-y_i)^2 /(a_i(\varphi(x_i,\alpha,\beta,...)-y_i)+2b_i^2) \tag{3}$$

Thus account of fluctuations of the first kind (physical) is carried out by function $\varphi(x,\alpha,\beta\ldots)$, and the account of fluctuations of the second kind (measuring) is carried out by functions $f(y, a_i, b_i, c_i)$, which determine the local metrics.
The work [2] is devoted to the description of process of measurement of the above mentioned giant EAS from physical side as basic. The results of longitudinal development of the cascade are represented in form of 12 obviously axially-asymmetric rectangular crosses and their approximation under the Gaisser-Hillas formula (see left part of a Figure 1).

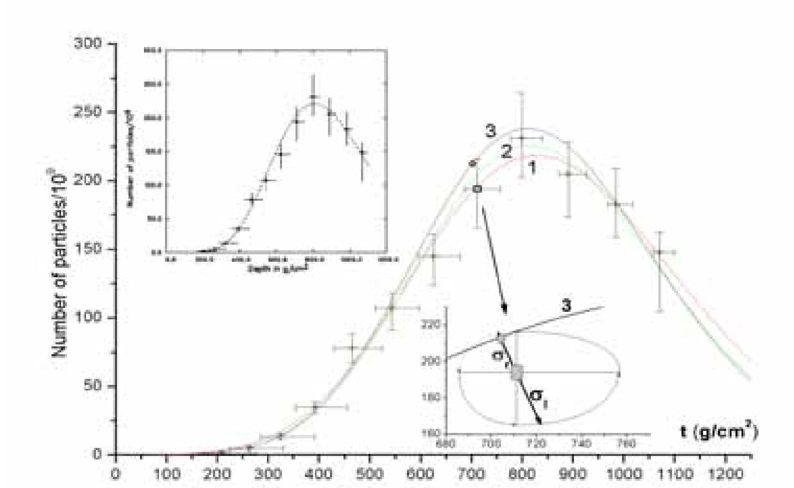

Figure 1. Variants of approximation of measuring results longitudinal profile 51 Joule EAS detected by Fly's Eye. Enlarged fragment area of measure at 711g/cm^2 shows creation of local metric for variant № 3.

The submitted approximation (see dotted line) supposes ambiguous interpretation. In [2] the exact definition of the characteristics of the distribution function of errors is not given (that quite naturally for so complex process of measurement). Obviously however, that on approximation asymmetry of the errors should be taken into account.

2. Method of the account asymmetry of measuring errors.

Asymmetry of measuring errors can be presented by various ways. It represent to us by most natural two ways:

1. indication mode and also W_l - left and W_r - right parts of width of distribution at the level e^{-1} [3]. (For normal distribution at this level $W_l = W_r = \sigma/\sqrt{2}$.);
2. indication mean value and also σ_l - left and σ_r - right parts of standard error [4]. (For normal distribution $\sigma_l^2 = \sigma_r^2 = \frac{1}{2}\sigma^2$,

and generally

$$\sigma^2 = \int_{-\infty}^{+\infty}(x-<X>)^2 f(x)dx = \int_{-\infty}^{<X>}(x-<X>)^2 f(x)dx + \int_{<X>}^{+\infty}(x-<X>)^2 f(x)dx = \sigma_l^2 + \sigma_r^2)$$

For the both causes one can easy calculate [4] the distribution function of the form (2).
So, the cross defines local metric in direction of coordinate axes.
For the complete task of the local metrics on a plane (i.e. for the description of errors not only in a direction of coordinate axes) we set that the crosses-pieces in each quadrant define the quarter of the ellipse (see central fragment of a Figure 1) with semiaxises equaled these crosses-pieces (turn with compressing/decompressing, in particular for axially-symmetric crosses thus it will turn out to two dimensional normal distribution). The ellipse parts define ends of cross turning on any direction. Thus, distribution function form (2) can be calculated for any point of the plane.
The local contiguous of an approximation was estimated through distance from the given site of the cross's node up to the approximation (piece of a perpendicular dropped from the site (x_i, y_i- node of the cross with number i) to approximation $(\tilde{x}_i, \tilde{y}_i)$). For usual least squares method (1) the contiguous point is taken in direction ordinate). So, each given measurement (x_i, y_i) corresponds one point of approximation $(\tilde{x}_i, \tilde{y}_i) = (\tilde{x}_i, \varphi(\tilde{x}_i, \alpha, \beta ...))$ (see squares and rings in Figure 1) and for the point distribution function form (2) can be calculated (see Figure 2).

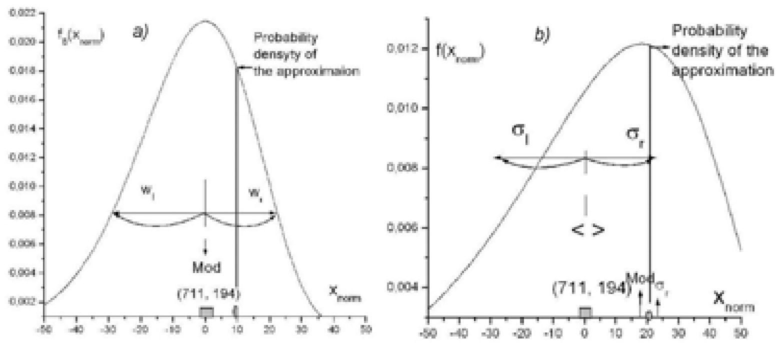

Figure 2. Probability density distribution of errors at t ≈711 g/cm^2 defined by two way:
a) Value of the mode and parts of the width. b) Value of the mean and parts of the variance.

Thus approximation can be constructed, calculating (3) as well as in one-dimensional case, where a_i, b_i, c_i are defined by the metrics received as a result of the above described turn with compressing/decompressing of given crosses.

3. Results of approximation.

The calculations were carried out for Gaisser-Hillas approximating the functional form
$\varphi(t) = N_m \exp((t_m - t_1)/\lambda \;\; \ln((t - t_0)/(t_m - t_0)) - (t - t_m)/\lambda)$ with 4-in free parameters: a=t_m, b=N_m, g=t_0, d=l. The calculation 1 was carried out in usual least squares method (1), i.e. without the account of errors of measurements and contiguous point was taken in direction ordinate. The calculations 2 and 3 were carried out in (3), i.e. the errors of measurements were taken into account by distribution (2). At calculation 2 the asymmetry was accounted by way a), i.e. the nodes of crosses were considered as modes of the distributions form (2) and cross-pieces were considered as parts of width of the distributions at the level e^{-1} (see Figure 2 a). At calculation 3 the asymmetry was accounted by way b), i.e. the nodes of crosses were considered as mean values of the distributions form (2) and cross-pieces were considered as parts of standard errors of the distributions (see Figure 2 b). The results of approximation are presented by the Table 1. and Figure 1.

Table 1. The values of the parameters for the variants of approximation.

№	a	b	c	d	min
1	827.08	218.76	-650.61	44.161	(517.4)
2	815.89	225.95	-639.59	39.037	4.11
3	810.88	238.35	-660.79	36.968	1.79

From Figure 1 it is visible, that the greatest divergence of variants is observed in the area of the cascade maximum. The approximations with account of errors are moved together (in comparison with variant 1 designed without the account of errors) in the direction of measurements with smaller values of local errors. The account asymmetry of errors is shown that the approximation aspires to occupy a position on the side of smaller part of the error (both for width and variance, since from this side density of probability is more (see Figure 2)). And in case of variant 3 the exactest approximation (i.e. with the greatest value of probability density) turns out moved of the mean value to the mode, because greatest of probability density corresponds to mode, instead of mean value. The shift do not seen from approximation of [2] (see dashed in left side of Figure 1).

4. Discussion

Difference of the fitting procedures (weight coefficients, account asymmetry) influences results. The specific least-square procedure, which is used in [2] to determine the shower parameters from recorded data is not known for us in details. It is rather possible, that crosses-pieces there are any borders of probable errors with the indication on them asymmetry, i.e. crosses are the characteristics only of device as such including the hardware and software, but the position of cross-nodes corresponds to results of the measures. The EAS is the single-valued phenomenon and consequently the indication of the device, as the single sample of asymmetric distribution, most probably sets in area of a mode, but not in area mean value. In such case most adequate is the description of asymmetry by a way a) and result of approximation is the variant 2 which difference from approximation [2] is not significant (see Figure 1).

5. Conclusion

The account of an asymmetry's errors of measurements requires the careful attention and formal description in detail. Method of account of asymmetry for least-squares fitting procedure is presented above.

If at specifying errors of measurement of the giant ($3.2\pm0.9 \times 10^{20}$ eV) EAS detected by Fly's Eye, the nodes of the crosses define mean values and the cross-pieces are components of variance – the estimation of energy is useful to check. Only on last stage of the approximation it is possible a few correction (about 10%) to the upper side.

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References

[1] L.G.Dedenko, G.T.Zatsepin. Yad.Fiz. 68, p.444-467, №3, 2005 (in russian).
[2] D.J. Bird, S.C. Corbato, H.Y. Dai, et al. Astropysical Jour., 441,(1995) 144.
[3] A.A. Kirillov, I.A. Kirillov. Astropart. Phys. 19. (2003) 101.
[4] L.G.Dedenko, A.A. Kirillov, I.A. Kirillov et al, Izvestia RAN, phys., 69 (2005), p. 359-362.
[5] A.A. Kirillov, I.A. Kirillov. Proc. 28th. Int. Cosmic Ray Conf., Tsucuba. vol. 2. 2003, p. 535.