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Similarity of the extensive air showers — an application to their reconstruction from a fluorescence light detector

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Using CORSIKA simulations of the highest energy extensive air showers we show that all showers are similar when described by the shower age parameter: the angular and energy spectra of electrons at a given level in the atmosphere depend only on the shower age at this level. Moreover, electrons with a given energy have the same angular distributions at any level (age) of the shower. We have calculated these distributions and found analytical functions describing them quite well. The description of large showers in terms of age, instead of depth in the atmosphere, is very useful in interpreting data from experiments observing fluorescence light with admixture of Cherenkov.

1. Energy spectra of electrons

The energy and angular distributions of charged particles in pure electromagnetic cascades were studied in some detail by Hillas [1]. It was not clear how these distributions would behave in hadronic extensive air showers, being a superposition of many such cascades initiated on various depths by photons with various energies. It has turned out that the age parameter $s = \frac{3X}{X+2X_{max}}$ (proposed by Hillas for EAS), where X_{max} is the depth of the shower maximum, describes very well the shower as a whole. It has been shown by us [2] and independently by Nerling et al [3] that for large showers the shape of the energy spectrum of electrons at a given level X in the atmosphere depends only on the shower development stage at this level, i.e. on its age s. The shape of the spectrum at a given age s is the same independently of the nature of the primary particle (proton or iron) and its primary energy. For big showers the spectrum practically does not fluctuate from shower to shower; what does fluctuate (for a fixed primary energy) is the total number of particles $N(s)$ (see [5]). Thus, in the sense of the energy spectra of electrons, all large showers are similar. As it is believed that the fluorescence yield of a particle is proportional to its energy loss for ionisation (what in turn depends on the particle energy), the total fluorescence flux would depend on the energy spectrum of particles at the observed level s.

2. Angular distribution of electrons

As the electron angles with respect to the shower axis depend mainly on the Coulomb scattering process, where the scattering angles are inversely proportional to the particle energy, the angular distribution of electrons at a given level also depends only on the shower age at this level. We have shown in [4] that the two dimensional energy and angle distribution of electrons, $F(E, \theta; s)$, (normalised to 1) at a given level depends on the shower age s at this level only. Again, it does not depend on the primary particle mass or on its energy. What we show here is something more: the whole dependence of the distribution $F(E, \theta; s)$ on the shower age s is contained in the energy distribution of electrons $f(E; s)$, so that

$$F(E, \theta; s) = f(E; s) \cdot g(\theta; E) \tag{1}$$

where $g(\theta; E)$ is the angular distribution of electrons of a given energy and $\int_0^\pi g(\theta; E) \cdot 2\pi \sin\theta d\theta = 1$. This

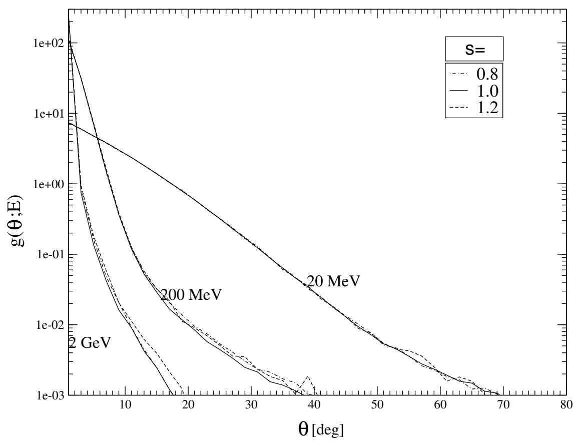

Figure 1. The angular distribution of shower electrons $g(\theta; E)$ for three values of electron energy ($\Delta \log E = 0.1$). For each E there are three distributions for $s = 0.8$, 1.0 and 1.2.

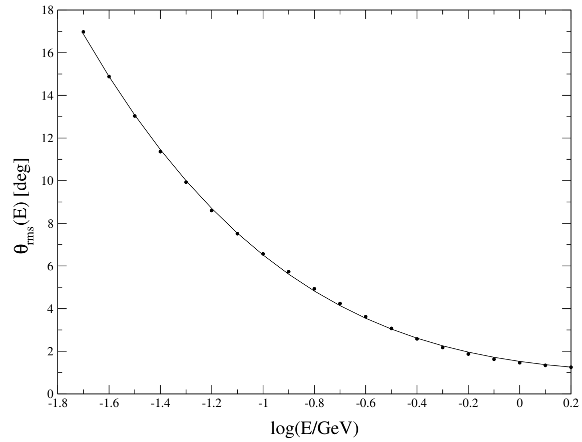

Figure 2. Root mean square of the angle between shower axis and the velocity of electron $\theta_{rms}(E)$. Points - simulations; line - polynomial fit (see Table).

means that electrons of a given energy E have the angular distribution independent of the shower age (see Figure 1). It is quite easy to understand. In a large shower any electron with a given energy E arises from a particle (electron or photon) with a much larger energy. The angles of the parents are much smaller that those of their daughters. As, on average, the angles add in quadrature, the angular distribution of the parent particles is irrelevant to that of the daughter with a much smaller energy E, the latter depending on the latest Coulomb scatterings, bremsstrahlung and/or pair production.

The knowledge about the angular distributions of particles in the shower is necessary to predict the contribution of the direct Cherenkov light to the fluorescence flux. We have fitted the angular distributions $g(\theta; E)$ by some analytical functions of $x = \frac{\theta}{\theta_{rms}(E)}$, with 4 parameters, as follows

$$g(\theta; E) = \begin{cases} a_1 \cdot e^{-(c_1 \cdot x + c_2 \cdot x^2)} & \text{for } x < 2.7 \\ a_2 \cdot x^{-\alpha} & \text{for } x > 2.7 \end{cases} \tag{2}$$

The dependence of parameters $\theta_{rms}, a_1, c_1, c_2$ and α on $y = \log(E/GeV)$ can be approximated by the following polynomials: $parameter = b_0 + b_1 y + b_2 y^2 + \ldots$ We have found the polynomials of $\log E$ fitting best the points and our proposition of its values is presented in the table below:

parameter	b_0	b_1	b_2	b_3	b_4
θ_{rms}	1.5012	-1.9647	1.4731	-1.5771	
a_1	2.1801	1.39171	0.32708		
c_1	4.2924	1.44073	-0.64280	-0.39655	
c_2	-0.38098	-0.42941	0.16231	0.12449	
α	3.3903	-0.29816	-0.48017	-0.81585	-0.06869

The root mean square angle, θ_{rms}, as a function of energy is shown in Figure 2. The flattening of the θ_{rms} for $E > 1$ GeV is an artifact, as our smallest angular bin is $\theta \leq 1°$.

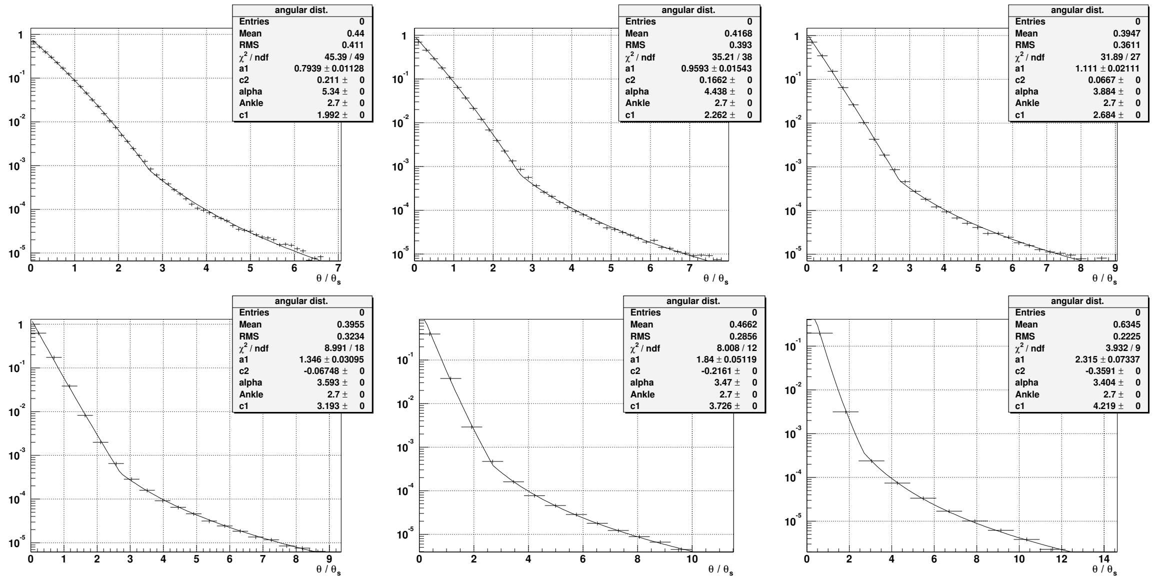

Figure 3. Angular distributions of shower electrons $g(\theta; E) \times \theta^2_{rms}(E)$ for several values of their energy versus θ/θ_s: upper-left — $\log E = -(1.6 \div 1.5)$ (22 MeV); each next graph corresponds to an increase of $\log E$ by 0.3 ($\sim$ factor of 2 in energy), so that the bottom-right is for $\log E = (-0.1 \div 0)$ (891 MeV). The histograms correspond to simulations, the lines are the analytical curves with the parameters described by the polynomials.

The four independent parameters a_1, c_1, c_2 and α (a_2 is determined by the matching condition of the two functions at $x = 2.7$) have been found for the consequent energy bins. Figure 3. represents the angular distribution of electrons, $g(\theta; E)$, averaged over $\Delta \log E = 0.1$, for several energy bins. Our aim is to describe the electron angular distributions well not only for those angles where there are most particles (per unit solid angle), i.e. for small angles, but also for larger angles ($x > 2$), so in our best fitting procedure we assumed "the error" in the fraction of particles in each angular bin ($1°$) as 10% (see the values of χ^2/ndf as a measure of goodness of the fits). The energy range we are dealing with is determined by the application concerning the Cherenkov contribution in the fluorescence air shower experiments. The energy threshold for the Cherenkov light to be emitted in the air by an electron is about 21 MeV at see level (increasing as $X^{-\frac{1}{2}}$), thus our lower limit in energy. The upper limit is determined by the angles of interest ($\gtrsim 1°$) reached at about 1 GeV (Figure 2.).

3. Prediction of the number of Cherenkov photons

A distant shower is seen in fluorescence light as a line. Usually, most of the light is fluorescence, with some admixture of the Cherenkov which, however, can not be neglected. The fluorescence light emitted by any small path element of a single electron is isotropic, so that the light emitted by all electrons in a shower track element will also be isotropic, independently of the angular distributions of electrons at this point. The Cherenkov (Ch) light is emitted at very small angles with respect to the particle direction ($< 1°$ in the air). The main Ch problem is that this light is scattered by the atmosphere aside, so that it adds to the fluorescence light observed at large angles to the shower axis. As the scattering is mainly by the Rayleigh process (at least high in the atmosphere), i.e. by large angles, and the particles propagate at rather small angles with respect to shower axis (see Figure 2.), the scattered Ch light has practically the same angular distribution as if all particles travelled exactly along the shower axis. There are, however, a fraction of showers with axes inclined by angles less than 30° to the telescope line of sight, and then the Ch light just produced at the observed shower path element (direct Ch light), may even exceed the fluorescence flux. This directly produced Ch flux has the angular distribution (down to angles less than 3°) practically the same as that of the electrons. To reconstruct a shower one has to be able to predict the amount of the Ch contribution for any track element of a shower. Thus, the number of photons Δn_i emitted towards the camera $i - th$ pixel, seeing the shower track element ΔX_i at depth X_i, collecting light from the (small) solid angle $\Delta\Omega$ at an angle θ_i to the shower axis, consists of the two components: the fluorescence and the Cherenkov light:

$$\begin{array}{l} \Delta n_i = \Delta n_{i,fl} + \Delta n_{i,Ch} \\ \text{with} \quad \Delta n_{i,fl} = k \cdot N(X_i) \cdot \left\langle \frac{dE}{dX} \right\rangle \cdot \Delta X_i \Delta\Omega(\theta_i)/4\pi \end{array} \tag{3}$$

where k is the proportionality constant between the number of fluorescence photons emitted per unit path (in $g \cdot cm^{-2}$) and the energy loss rate for ionisation [6]. The second term in (3) consists again of two components: the direct and the scattered Ch light. The number of the direct Ch photons produced in the field of view equals:

$$\Delta n_{i,Ch-d} = N(X_i) \int_{E_{th}(h(X_i))}^{E_0} Y_{ch}(E) f[E; s(X_i)] g(\theta_i; E) dE \cdot \frac{\Delta X_i}{\cos\theta_i} \Delta\Omega(\theta_i) \tag{4}$$

where $Y_{ch}(E)$ is the number of Ch photons emitted per unit path by an electron with energy E.

The scattered Ch light has been discussed by us in detail in [2], and equals:

$$\Delta n_{i,Ch-sc} = Y_0 \frac{\rho(X_i)}{\lambda_a(X_i)} f_s(\theta_i) \int_0^{X_i} T(X', X_i) N(X') Fr[X'(s,h)] dX' \cdot \Delta\Omega \Delta l_i^{av} \tag{5}$$

where: $\rho(X_i)$ - air density at X_i, $\lambda_a(X_i)$ - light attenuation length, $f_s(\theta)$ - angular distribution of the scattered light, $T(X', X)$ - fraction of photons produced at X' arriving to X_i, $Fr[X'(s,h)]$ - effective fraction of electrons emitting Ch light at given level and Δl_i^{av} - average path length (in meters) of Ch photons in shower length element $\Delta l_i(\Delta X_i)$ (being slightly larger than Δl_i). How to determine $N(X)$ - see [5].

4. Conclusions

The similarity of showers allows an accurate prediction of both - fluorescence and Cherenkov light emitted from any level of a shower, once $N(s)$ is known (see [5]).

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