29th International Cosmic Ray Conference Pune (2005) **7**, 195–198

Multiple scattering of the fluorescence light from EAS

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One of the methods to study the highest energy cosmic rays is to observe the fluorescence (and Cherenkov) light emitted sideways by the extensive air showers (EAS) they produce in the atmosphere. To reconstruct the shower cascade curve, $N(X)$, from the observations of the light arriving from the directions towards the subsequent shower track elements, it may be necessary to take into account the multiple scattering that photons undergo on its way from the shower to the detector. This effect seems not to be a negligible one, particularly for distant showers. In contrast to some Monte-Carlo treatments, we present here some analytical and numerical results of our calculations of the Rayleigh scattering. We treat separately the consequent 'generations' of the scattered light. The angular smearing as well as the time delays are obtained for a point- and line light source. The results can be scaled to various distances when measured in the mean scattering length.

1. Introduction

One of the methods to determine the EAS primary energy is to register fluorescence light emitted along the shower track in the atmosphere (as in the Fly's Eye, HiRes and Auger experiments). The aim of this paper is to study the effect of this light being scattered in the atmosphere before reaching the detector . This leads to some broadening of the shower cascade curve (number of particles as a function of slant depth, $N(X)$), if not taken into account. Here we consider the Rayleigh scattering occurring in a uniform (constant density) atmosphere. Where possible we solve the problem analytically.

A distant shower can be treated as a point isotropic light source moving with light velocity c along the shower track. At the fluorescence detector camera, measuring the angular (and time -) distribution of the arriving light, it causes an elongated track of hit pixels (PMT's), each having a small angular field of view.

Usually, it is assumed that it is only the light emitted in the pixel's field of view which arrives, after some attenuation, at the pixel. However, the attenuation consists in scattering the photons away of the field of view of the considered pixel. Here we calculate what fraction of these photons comes back to the pixel or to its neighbours. To do this we first consider an instantaneous isotropic point source and calculate the angular and time distributions of photons arriving at a unit surface located perpendicularly at a given distance R. We then integrate the obtained distribution over position and time of the emitted light. We treat the scattered light as a sum of several generations: the first generation n_1 being the photons scattered exactly once, the second one n_2 - twice and so on.

2. Angular and time distribution of light from an instantaneous isotropic source

We first calculate $dn_1/d\theta$, being the angular distribution of photons scattered once, arriving at an angle θ to a unit perpendicular surface located at a distance R from the source, so that $\int_0^\pi \frac{dn_1}{d\theta} d\theta = n_1$, being the total number of photons crossing the unit surface (from both sides), independent of time. Figure.1 presents the geometry of the first scattering. All photons $dn_1(\theta)$ arriving to the surface ΔS at an angle $(\theta, \theta + d\theta)$ must have been scattered between the two surfaces of the two cones with opening angles θ and $\theta + d\theta$. So, it is not difficult to see that their number should be:

$$dn_1(\theta; R)\Delta S = \int_0^{\pi-\theta} d\beta \, \frac{1}{2} sin\beta \cdot e^{-\frac{x}{\lambda}} \cdot \frac{dx}{\lambda} \cdot f(\alpha)\Delta\Omega \cdot e^{-\frac{x'}{\lambda}} \tag{1}$$

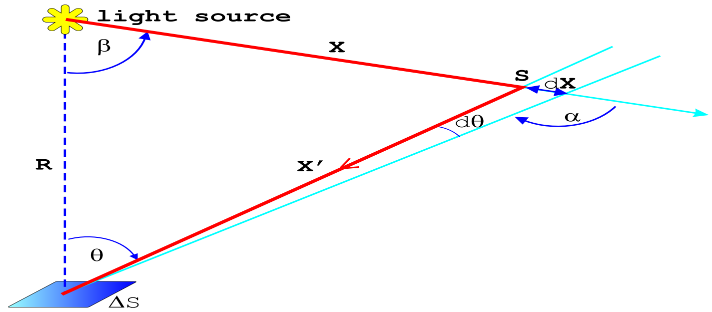

Figure 1. Geometry of the first scattering.

where λ is the mean scattering path length, $f(\alpha)\Delta\Omega$ is the fraction of photons scattered by angle α into the solid angle $\Delta\Omega = \Delta S\, cos\theta/x'^2$. This formula is for one emitted photon. Expressing x, x' and α as functions of β (for a fixed R and θ) and assuming Rayleigh scattering for $f(\alpha)$ we obtain

$$\frac{dn_1(\theta;R)}{d\theta} = \frac{3}{32\pi}\frac{k\,|cos\theta|}{R^2}\int_0^{\pi-\theta} exp\Big[-k\frac{sin\theta + sin\beta}{sin(\theta+\beta)}\Big]\cdot[1+cos^2(\theta+\beta)]d\beta \tag{2}$$

where $k \equiv R/\lambda$. Within a small angle θ_S around $\theta = 0$ the fraction of photons from the first generation Δn_1, as compared to the unscattered flux, can be found analytically:

$$\frac{\Delta n_1(\theta_S)}{n_0} \simeq \frac{9\pi}{16}k\,\theta_S(rad) \simeq 3.1\cdot 10^{-2}\,k\,\theta_S(in\,deg) \tag{3}$$

For larger angles the integral (2) has to be calculated numerically.
Having $\frac{dn_1}{d\theta}(\theta;R)$ one can calculate the angular distribution of the second generation $dn_2/d\theta$ arriving at the surface ΔS by a similar way as for $dn_1/d\theta$. The only difference is that point S in Figure.1 is now the site of the second scattering and the arriving photons have the distribution of angles $dn_1(\theta_1;x)/d\theta_1$ with the axial symmetry around the distance x. We obtain that

$$\frac{dn_2(\theta;R)}{d\theta} = 2|cos\theta|\int_0^{\pi-\theta} d\beta\, e^{-\frac{x'}{\lambda}}x^2\int_0^{\pi}\frac{d\theta_1}{|cos\theta_1|}\frac{dn_1(\theta_1;x)}{d\theta_1}\int_0^{\pi}(1+cos^2\alpha)d\varphi_1 \tag{4}$$

where φ_1 is the azimuth angle of the first generation photons at the second scattering point, and the scattering angle α can be found from the relation

$$cos\alpha = cos(\theta+\beta)cos\theta_1 - sin(\theta+\beta)sin\theta_1 cos\varphi_1 \tag{5}$$

The integral over φ_1 can be calculated analytically:

$$\int_0^{\pi}(1+cos^2\alpha)d\varphi_1 = \pi\big[1+cos^2(\theta+\beta)cos^2\theta_1 + \frac{1}{2}sin^2(\theta+\beta)sin^2\theta_1\big] \tag{6}$$

The remaining double integral has to be found by numerical calculations.
Any next $i-th$ generation can be found from the previous one $(i-1)th$ from formulae (4) - (6), where the index 2 is replaced by i and index 1 - by $i-1$. The resulting angular distributions of the first three generations, as a ratio of the $0-th$ generation (not scattered) $n_0 = \frac{1}{4\pi R^2}e^{-k}$, are presented in Figure.2, left.

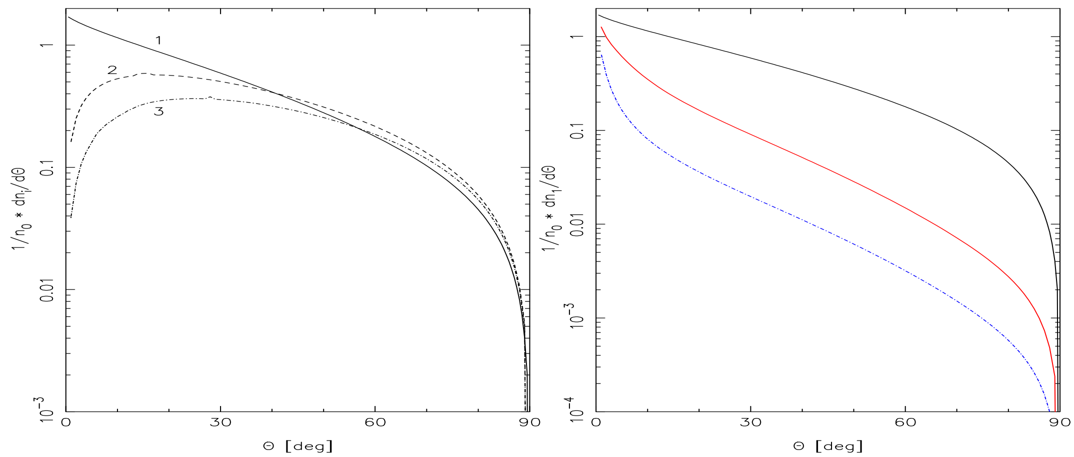

Figure 2. Left: Angular distribution of light $dn_i/d\theta$ of the three generations at $R = \lambda$, from a point isotropic source, as a fraction of the non-scattered light n_0. **Right:** First generation; upper curve: all times (the same as in the left figure), middle: $\tau \leq 1.05$, lower: $\tau \leq 1.01$

The field of view of a single pixel is usually small ($\sim 1.5°/2$ in the Auger experiment) so that one would expect that for $R = \lambda$ there will be about $3.1\% \cdot 1.5/2 = 2.3\%$ (formula (3)) of the first generation plus $\sim< 0.1 \cdot 2.3\% = 0.2\%$ of the second generation (Figure.3), with negligible contribution from the third one. Thus, roughly $\sim 2.5\%$ of the light, emitted in the field of view of a pixel and arriving to it without scattering, will be scattered away and back to its field of view (for $R = \lambda$). This number scales as $k^i = (R/\lambda)^i$ for the $i - th$ generation at $\theta_S \ll 1$.

In the fluorescence experiments the arrival time window is limited, so that it is of interest to determine the angular distribution of light within a fixed range of time. We can get easily the $d^2n_1/d\theta dt$, as the integrand in (1), being a function of β, can be represented as a function of the the arrival time t ($t = 0$ is the emission time). It is convenient to express time in units of R/c, introducing $\tau = ct/R$. Then we obtain

$$\frac{d^2n_1}{d\theta d\tau} = \frac{3}{8\pi}\frac{k}{R^2}e^{-k\tau}\frac{(\tau - cos\theta)^4 + sin^4\theta}{\left[(\tau - cos\theta)^2 + sin^2\theta\right]^3}\, sin\theta|cos\theta| \tag{7}$$

As the delay time of the measured photons (with respect to R/c) is usually set as small ($\tau - 1 \ll 1$) we can calculate the angular distribution of the first generation by integrating (7) over $1 \leq \tau < \tau_0$, for $\tau_0 - 1 \ll 1$. This can be done analytically (for the lack of space we do not give here the formula) and the results are presented in Figure.2, right. It can be seen that the time-limited light flux is considerably smaller than the total one, particularly for large angles.

3. Light from a moving source

To determine the effect of the multiple scattering of the EAS fluorescence light we have to consider a moving, isotropic point source. Let $J(\theta, \varphi; R, t)d\Omega\, dt$ be the number of photons arriving in the solid angle $d\Omega(\theta, \varphi)$ in the time interval $(t, t + dt)$ at the unit surface at distance R from the point determined by $\theta = 0$ on the shower

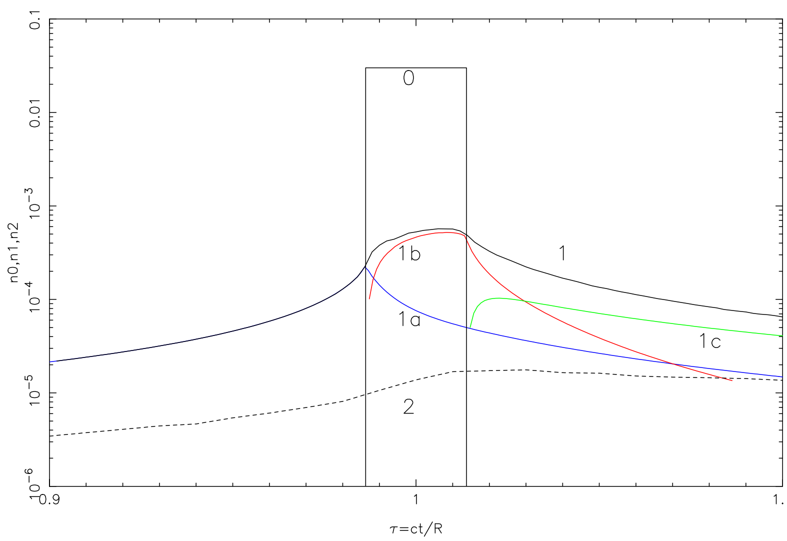

Figure 3. Time distribution of light observed by a pixel with radius $0.75°$ at angle $\delta = \pi/2$ to the line of sight, from a moving source (shower) at $R = \lambda$. $\tau = 0$ corresponds to the time when the source is in the middle of the pixel.

axis. It is not difficult to see that

$$J(\theta,\varphi,t;R) = \int_A^B j(\theta^{'},t';R')\cdot Cdl \quad where \quad j(\theta^{'},t';R') = \frac{1}{2\pi\, sin\theta' |cos\theta'|}\frac{d^2n}{d\theta' dt'} \tag{8}$$

A and B are the limiting points of the shower track and Cdl is the number of photons produced along dl. If the angle between the shower axis and the direction $\theta = 0$ is δ then $R' = R\sqrt{1 - \frac{2l}{R}cos\delta + \frac{d^2}{R^2}}$, where l is the distance of the element dl from the point on the shower with $\theta = 0$. We also have that $t' = t - l/c$ and $cos\theta' = cos\gamma\, cos\theta - sin\gamma\, sin\theta\, cos\varphi$, where γ is determined from: $l/R = sin\gamma/sin(\gamma + \delta)$.
To calculate how many scattered photons arrive at a particular pixel of the detector camera, one has to integrate $J(\theta,\varphi,t;R)$ over its field of view. Changing A and/or B we can calculate from which part of the shower the photons arrive. Figure.3 shows an example of the time distribution of the light arriving from a unit distance at a pixel, with angular diameter $1.5°$. The upper horizontal line shows the non-scattered light, the other three correspond to the first generation of photons produced: above the pixel field of view (1a), inside it (1b) and below it (1c), correspondingly; (1) is the sum of the three. We see that the contribution of the neighbouring pixels is rather small. The line denoted by (2) shows the total contribution from the second generation.

4. Conclusions

It is possible to treat the multiple scattering of the fluorescence light analytically, with some numerical help, at least for a homogeneous atmosphere. The Rayleigh scattering, causing some of the light come back to the direct flux, can be important only when the distance to the shower $R > \lambda$. Its contribution within an opening angle θ_S is less (if time limited) than $3.1\%\frac{R}{\lambda}\theta_S$ (deg).

Acknowledgements: Work supported by University of Lodz grant No 696