# Depth Development of Extensive Air Showers from Muon Time Distributions 

L. Cazón ${ }^{a, b}$, R.A. Vázquez ${ }^{a}$ and E. Zas ${ }^{a}$<br>(a) Departamento de Física de Partículas, Universidade de Santiago de Compostela, 15782 Santiago, SPAIN<br>(b) Institut für Kernphysik, Forschungszentrum Karlsruhe, POB 3640, D-76021 Karlsruhe, Germany<br>Presenter: E. Zas (zas@fpaxp1.usc.es), ger-cazon-L-abs1-he14-oral

We develop an algorithm that relates the depth development of ultra high energy extensive air showers to the time delay of muons. The time distributions sampled at different positions at ground level by a large air shower array are converted into distributions of production distances using an approximate relation between production distance, transverse distance and time delay. The method is naturally restricted to inclined showers where muons dominate the signal at ground level but could be extended to vertical showers provided that the muon signal can be separated from that of electrons and photons. The method is tested using simulated showers by comparing the production distance distributions obtained using the method with the actual distances in the simulated showers. It could be applied in the search for neutrinos to increase the acceptance to highly penetrating particles, as well as for unraveling the relative compositions of protons and heavy nuclei. We also illustrate that the obtained depth distributions have minimum width when both the arrival direction and the core position are well reconstructed.

## 1. Relation between depth development and muon time distributions

When an Ultra High Energy Cosmic Ray (UHECR) particle enters the atmosphere it interacts producing an extensive air shower that propagates through it and reaches ground level. These showers can be detected by arrays of particle detectors that sample at ground level an enormous shower front which can exceed $10^{12}$ particles. In these arrays the relative times of the detected signals allow the reconstruction of the incoming particle arrival direction. The time distribution of the arriving signal has been known for long to be dependent on the depth distribution of the shower particles $[1,2,3,4]$ which is different for different primary particles. It has been recently shown [6] that the the arrival time distributions of muons in extensive air showers can be accounted for by the different path lengths traveled by the muons from their production point in a simplified model.

In [6] it was shown that most of the time delay of the muons is due to geometrical effects. Additional sources of delay such as multiple scattering and the kinematical delay, which is the delay due to the subluminal velocity of muons, are less important. The kinematical delay is dominant only in the region close to the shower axis, and in principle, could be taken approximately into account (see [7] for details).

The dominance of the geometrical delay implies that there is a one to one correspondece between the arrival time and the production distance of the muons, for a given perpendicular distance to the core [6]:

$$
\begin{equation*}
z=\frac{1}{2}\left(\frac{r^{2}}{c t_{g}}-c t_{g}\right)+\Delta \tag{1}
\end{equation*}
$$

where $z$ is the production distance, measured along the shower axis, $r$ is the perpendicular distance to the core, $t_{g}$ is the arrival time respect to the arrival of the shower plane and $\Delta=r \tan \theta \cos \zeta$, where $\theta$ is the arrival zenith angle and $\zeta$ is the azimuthal angle on the perpendicular plane. Then, one can convert the time histogram with $N_{i}$ entries corresponding to the $N_{i}$ muons detected by detector $i$ into a $z$ histogram, using this $t_{g} \rightarrow z$ correspondence.

| $\theta(\mathrm{deg})$ | $\phi(\operatorname{deg})$ | B | cut $\mathrm{r}(\mathrm{m})$ | $\Gamma$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | - | no | 900 | 1.23 |
| 30 | - | no | 1000 | 1.16 |
| 60 | - | no | 1500 | 1.07 |
| 70 | - | no | 2000 | 1.03 |
| 80 | - | no | 2900 | 1.00 |
| 80 | 0 | yes | 2900 | 0.97 |
| 80 | 90 | yes | 2900 | 0.89 |
| 86 | 0 | yes | 4000 | 0.86 |
| 86 | 90 | yes | 4000 | 0.85 |


| $\theta(\mathrm{deg})$ | $\phi(\mathrm{deg})$ | B | cut $\mathrm{r}(\mathrm{m})$ | $\Gamma$ |
| :---: | :---: | :---: | :---: | :---: |
| neutrino-like injected at $500 \mathrm{~g} / \mathrm{cm}^{2}$ vertical depth. |  |  |  |  |
| 80 | 0 | yes | 1400 | 1.02 |
| neutrino-like injected at $750 \mathrm{~g} / \mathrm{cm}^{2}$ |  | vertical depth. |  |  |
| 80 | 0 | yes | 900 | 1.14 |

Table 1. Table of deviation of the reconstruction respect to the real distribution. $\Gamma=<\frac{z_{\text {rec }}}{z_{\text {true }}}>$. The azimuth angle $\phi$ is measured counterclockwise respect to the local magnetic north. For high zenith angles the results obtained with and without magnetic field are compared.

The time resolution of the detector, $\delta t$, sets an intrinsic limitation which translates directly into an uncertainty in the production distance $z, \delta z$. We get from Eq. $1 \frac{\delta z}{z} \simeq-\frac{c \delta t}{c t}\left(1-\frac{\Delta}{z}\right) \simeq-\frac{\delta t}{t}$. Depending on the distance to the core this may affect the accuracy of the $z$ reconstruction. As we look at the arrival time of muons closer to the shower axis, the time delay becomes smaller and the relative error in the $z$ distribution reconstruction diverges. To have $\frac{\delta z}{z}$ smaller than a given value, $e_{z}$, we obtain $r>r_{c}=\sqrt{\frac{2 z_{u} c \delta t}{e_{z}}}$. provided that we assume that muons are produced at $z<z_{u}$. Notice that this $r$ cut avoids the regions near the shower axis where the kinematical delay dominates over the geometrical delay, and also the region where the muonic component signal could be shadowed by the electromagnetic component.
We have checked our reconstruction method by applying it to the time distributions of showers simulated with the Aires Monte Carlo package. We have simulated sets of 500 showers at different zenith angles to test the method. Muons arriving at ground are arranged in a time histogram of 25 ns bins. We then apply Eq. 1 to all the muons to calculate $z$, where the geometric corrections and the kinematic corrections as explained in [7] are included. Finally an $r$ cut is applied.
We have verified that the reconstructed histogram is not very sensitive to small changes in $r_{c}$ but clearly these cuts in $r$ can have large impact on the statistics. In table 1 we show the average ratio between the reconstructed and the true z values, $\Gamma=<\frac{z_{\text {rec }}}{z_{\text {true }}}>$. As can be seen, the method works best for moderately inclined showers between $60^{\circ}$ and $80^{\circ}$. At very low zenith angles, there is an overestimation of the production distance, which could be due to an slight overestimation of the energy loss. On the other hand, at very high zenith angles the magnetic field effects starts to be important and the time-geometry relation underestimates the production distance. Nevertheless, the precision obtained is quite good.

In Fig. 1 we have compared the results of the reconstruction procedure applied to protons and to deeply injected protons arriving with $80^{\circ}$ zenith angle to illustrate how the method can be used to identify deeply interacting inclined showers at high zenith, which are natural neutrino candidates. A systematic study of the reconstruction procedure and the ability to identify neutrinos under realistic experimental conditions is left for future work.

## 2. Correlation with angular and core position uncertainties

The method has another intrinsic limitation because to convert the arrival time histogram into a histogram of production distances the incoming direction and the position of shower axis must be known, so that the appropriate values of $r$ can be introduced. We explore here the stability of the reconstruction to shifts in the core positions and angular directions. We assume an array of particle detectors and calculate the number of muons that crosses each of them. We have choosen detectors of $10 m^{2} \times 1.2 m$ (area $\times$ height) in a
hexagonal grid, separated 1500 m , corresponding to the Auger surface detector. This limits the statistics of the reconstructed distribution, $d N / d z$, in a realistic way. The right pannel of Fig. 1 shows an example of the simulated and reconstructed distributions for a $10^{19} \mathrm{eV}$ shower with $\theta=60^{\circ}$.


Figure 1. Left and central figures: Histograms of production distribution (Light fill: original distribution. Unfilled Thick Line: reconstructed), for 500 proton showers at $10^{19} \mathrm{eV}$ energy and $0^{\circ}$ zenith angle (left), $80^{\circ}$ zenith angle for normal protons and for protons injected at $500 \mathrm{r} / \mathrm{cm}^{2}$ of vertical depth to simulate a neutrino interaction (marked as $\nu$-like) (center). The geomagnetic field is included. Right figure: Distribution for a reconstructed $10^{19} \mathrm{eV}$ proton shower of $\theta=60^{\circ}$ (histograms) compared with the original distribution when we have finite sampling produced by a finite number of detectors.

We first recalculate the depth distributions assuming that the arrival direction has been misreconstructed by $(\Delta \theta, \Delta \phi)$ with respect to the actual arrival direction chosen for the simulation. For each angular shift both the mean and RMS width of the $z$-distributions were calculated. It is worth remarking that the mean value of $z$ is rather stable to shifts in azimuthal direction $\Delta \phi$, whereas there is a slight rise of the reconstructed $z$ when increasing the zenith angle $\theta$. There is an asymmetry in the behavior of $<z>$ versus changes in $\theta$ since the shift $\Delta \theta$ introduces an asymmetry between early and late regions. A completely analogous method was followed to study the core position and $z$-reconstruction correlations. The reconstructed impact points were


Figure 2. Effects of reconstruction direction shifts in the width of $z$-distribution for a $10^{19} \mathrm{eV}$ proton shower of $\theta=60^{\circ}$. Left RMS width of $\log _{10}(z / m)$ as a function of angular shifts $(\Delta \theta, \Delta \phi)$ with an $r$ cut $r>1500 \mathrm{~m}$. Right RMS of $\log _{10}(z / m)$ as a function of a core shift $(\Delta x, \Delta y)$ with an relaxed $r$ cut $r>500 \mathrm{~m}$.
shifted by $(\Delta x, \Delta y)$ in the ground plane with respect to the core of the simulated shower. We found that the mean value of $z$ is stable to shifts in core position.

The RMS width of the distribution displays a local minimum when the correct angle or impact point are used (see Fig 2). The plots display some discontinuities of statistical nature because the total number of muons in the detector is small and as the core position position is shifted, individual detectors are rejected or accepted because of the $r$ cut. The detectors close to the cut are those that have most muons and including or not including them affects the $z$-distribution. These discontinuities are also present in some circumstances for angular shifts because the relative position of the detectors also change but clearly the changes of angular reconstruction modify the distances to shower axis at a second order level. These discontinuities can become smoother by increasing statistics, for instance relaxing the $r$-cut.
This suggests that this method could be used either to reconstruct angle (shower impact points) independently or to check that the angle (core position) reconstruction obtained through conventional methods is consistent with the arrival time of the muons at large distances from shower core, on an individual shower basis.

## 3. Summary

We have developed a method that has the potential of reconstructing the production altitude for the muons in cosmic ray showers based on the time distribution of the muon signals in the detectors of an extensive air shower array. The method works best in the $60^{\circ}-80^{\circ}$ zenith angle range and it is fairly stable with respect to misreconstruction of the shower core and the incoming direction, for the mean of the distribution. The RMS width of the production distance distribution displays a minimum when the correct impact point and arrival direction are used in the reconstruction procedure. The reconstruction of depth development in inclined showers can also have important implications in improving the potential of air shower arrays to detect neutrinos.

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