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Test-Particle Diffusive Shock Acceleration in Relativistic Shocks

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We present results from a fully relativistic Monte Carlo simulation of diffusive shock acceleration (DSA) in unmodified (i.e., test-particle) shocks. The computer code uses a single algorithmic sequence to smoothly span the range from nonrelativistic speeds to fully relativistic shocks of arbitrary obliquity, providing a powerful consistency check. We show the dependence of the particle spectrum, which can differ strongly from the canonical $f(p) \propto p^{-4.23}$ result, on the obliquity and on the strength and "fineness" of scattering for a range of shock Lorentz factors. The Monte Carlo results are also shown to be consistent with a simple relation for the spectral index given by Keshet and Waxman[10] in parallel shocks when the diffusion is "fine" and isotropic.

1. Introduction

Most work on diffusive shock acceleration (DSA) in relativistic shocks has been restricted to the ultra-relativistic regime with magnetic fields assumed parallel to the shock normal. However, trans-relativistic shocks are certain to be important in some sources and, in general, relativistic shocks will have highly oblique magnetic fields. Here we consider test-particle diffusive shock acceleration in shocks of arbitrary obliquity[1] and with arbitrary Lorentz factors, γ_0. We consider only particle acceleration in plane, unmodified (i.e., test-particle) shocks where effects on the shock structure from superthermal particles are ignored. In contrast to nonrelativistic shocks, the details of particle scattering strongly influence the superthermal particle populations produced in trans-relativistic and ultra-relativistic shocks (see [1][2][12] for example). Unfortunately, these details are not known with any reliability so the results of all current models of DSA depend on the particular scattering assumptions made. We use a simple, parameterized model of particle diffusion (described in detail in [6]) which we believe adequately describes the main features of particle transport. Until a self-consistent theory of wave-particle interactions in relativistic shocks is produced, parameterization will be necessary.

2. Model and Results

The acceleration process is modeled with a Monte Carlo simulation where thermal particles are injected far upstream and followed as they diffuse through the shock (for full details, see [6] and references therein). Scattering is elastic and isotropic in the local plasma frame and the scattering mean free path λ in the local frame is proportional to the gyroradius r_g. We simulate small-angle scattering by allowing the tip of the particle's fluid-frame momentum vector $\mathbf{p}$ to undergo a random walk on the surface of a sphere. After a small time δt the momentum undergoes a small change in direction of magnitude $\delta\theta$ within a maximum angle $\delta\theta_{\max}$.

If the time in the local frame required to accumulate deflections of the order of 90° is identified with the collision time $t_c = \lambda/v_p$, [7] showed that $\delta\theta_{\max} = \sqrt{6\delta t/t_c}$, where δt is the time between pitch-angle scatterings. We take λ proportional to the gyroradius $r_g = pc/(eB)$ (e is the electronic charge and B is the local uniform magnetic field in Gaussian units), i.e., $\lambda = \eta_{\rm mfp}\, r_g$, where $\eta_{\rm mfp}$ is a measure of the "strength" of scattering. The strong scattering limit, $\eta_{\rm mfp} = 1$, is called Bohm diffusion and in this limit, cross-field diffusion is important. Setting $\delta t = \tau_g/N_g$, where N_g is the number of gyro-time segments δt, dividing a gyro-period $\tau_g = 2\pi r_g/v_p$, we have $\delta\theta_{\max} = \sqrt{12\pi/(\eta_{\rm mfp} N_g)}$. Large values of N_g result in "fine" scattering where it is assumed that magnetic field fluctuations exist on all scales so that $\delta\theta$ can be arbitrarily small. The DSA

[1] Oblique shocks are those where the angle between the upstream magnetic field and the shock normal, $\theta_{\rm B0}$, is greater than 0°. Parallel shocks are those with $\theta_{\rm B0} = 0^\circ$. Everywhere in this paper we use the subscript 0 (2) to indicate upstream (downstream) quantities.

distribution, $f(p)$, becomes independent of N_g when it is large enough such that $\delta\theta \ll 1/\gamma_0$, otherwise large-angle scattering (LAS) effects become important. Thus, the scattering properties of the medium are modeled with the two parameters $\eta_{\rm mfp}$ and N_g and, along with γ_0 and $\theta_{\rm B0}$, this yields, in the high Mach number limit, four independent parameters characterizing DSA.

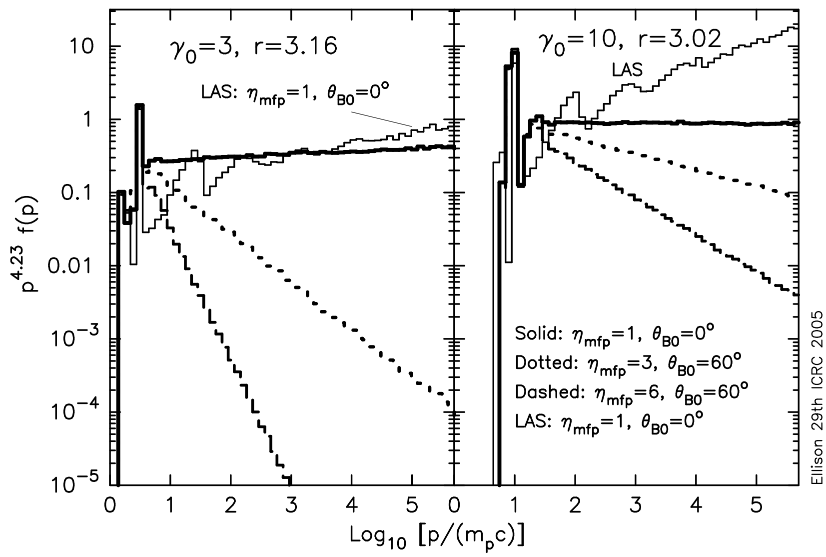

Figure 1. Test-particle spectra for shock Lorentz factors $\gamma_0 = 3$ (left panel) and $\gamma_0 = 10$ (right panel) (note that $p^{4.23}f(p)$ is plotted). For $\gamma_0 = 3$, the shock compression ratio, as determined in [4], is $r = 3.16$. For $\gamma_0 = 10$, $r = 3.02$. The listing of line types applies to both panels. The light-weight solid curves labeled LAS show the effects of large-angle scattering, all other curves have $\delta\theta \ll 1/\gamma_0$. All spectra are calculated at the shock in the shock frame.

In Fig. 1 we show examples where these four parameters are varied. We plot $p^{4.23}f(p)$ for a trans-relativistic $\gamma_0 = 3$ and a more fully relativistic $\gamma_0 = 10$. In all cases except the two marked LAS, N_g is large enough so that $\delta\theta \ll 1/\gamma_0$. In the two LAS cases, the effects of large-angle scattering are clearly seen.

In the limits $\delta\theta \ll 1/\gamma_0$ and $\gamma_0 \gg 1$, DSA produces a power law $f(p) \propto p^{-4.23}$, independent of $\theta_{\rm B0}$ or $\eta_{\rm mfp}$ (see [2]). The heavy solid curve in the right panel of Fig. 1 shows that for $\theta_{\rm B0} = 0°$ and $\eta_{\rm mfp} = 1$, the canonical power law is obtained. However, even with $\delta\theta \ll 1/\gamma_0$, $f(p)$ can be much steeper than this for larger values of $\theta_{\rm B0}$ and $\eta_{\rm mfp}$. We showed in [6] that effects of $\theta_{\rm B0}$ and $\eta_{\rm mfp}$ diminish with increasing γ_0 but persist noticeably to at least $\gamma_0 = 30$.

The curves labeled LAS in Fig. 1 both have $\theta_{\rm B0} = 0°$ and $\eta_{\rm mfp} = 1$, but here N_g is small enough so particles make large deflections in individual scattering events. Large-angle scattering will be important whenever $\eta_{\rm mfp}N_g$ is small enough so that $\delta\theta_{\rm max} \not\ll 1/\gamma_0$ and, in principle, could occur even in the shocks expected when a γ-ray burst fireball expands ultra-relativistically into the interstellar environment.

Keshet and Waxman[10] have presented the following analytic expression for the power-law spectral index $\sigma_{\rm KW}$

$$\sigma_{\rm KW} = \left(3\beta_0 - 2\beta_0\beta_2^2 + \beta_2^3\right) / \left(\beta_0 - \beta_2\right) \ , \quad (1)$$

where β_0 (β_2) is the speed of the upstream (downstream) fluid normalized to the speed of light. This expression, which assumes that the shock is parallel and that $\delta\theta \ll 1/\gamma_0$, depends only on the shock Lorentz factor and

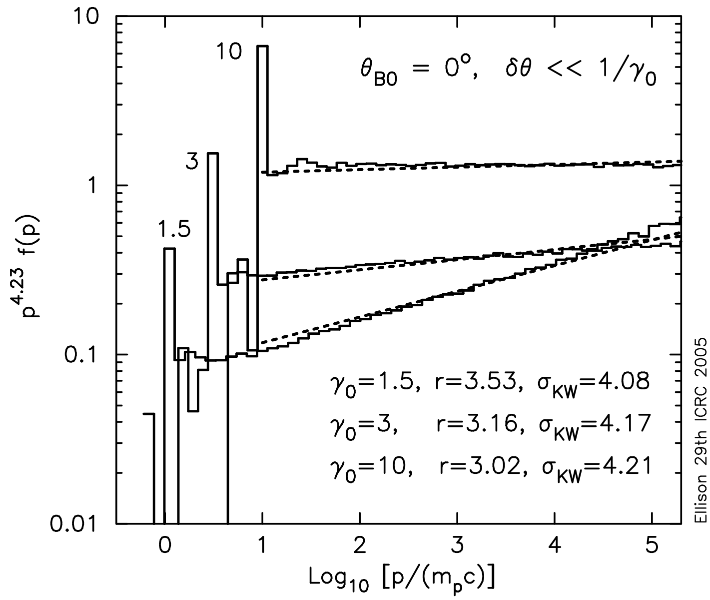

Figure 2. Comparison of Monte Carlo spectra (solid histograms) with the analytic result given in [10] (dotted lines). $p^{4.23}f(p)$ is plotted and the normalization is arbitrary.

the compression ratio. The compression ratio must be determined independently. In Fig. 2 we compare Monte Carlo spectra to eq. (1) for several γ_0. The Monte Carlo results use the solution to the shock jump conditions (including the equation of state) calculated in [4] to determine r and this value is used in eq. (1) (i.e., $\beta_2 = \beta_0/r$) and listed in Fig. 2. The match between the two models is quite good in this trans-relativistic range and will improve outside of this range.

3. Discussion and Conclusions

In nonrelativistic shocks, the test-particle power law from DSA depends only on the shock compression; it is independent of the details of diffusion, on the strength of scattering, and on the obliquity (i.e., independent of N_g, $\eta_{\rm mfp}$, and $\theta_{\rm B0}$). In relativistic shocks, particularly trans-relativistic ones, test-particle spectra can depend strongly on these parameters. We have shown examples with various N_g, $\eta_{\rm mfp}$, and $\theta_{\rm B0}$ and, in general, the power law steepens if $\eta_{\rm mfp} > 1$ and/or $\theta_{\rm B0} > 0°$. The parameter N_g determines the fineness of scattering and if N_g is small enough so that $\delta\theta_{\rm max} \not\ll 1/\gamma_0$, large departures from a power law can occur and the spectrum can become harder than $p^{-4.23}$ even for ultra-relativistic shocks. In addition, as have shown that the simple analytic result given in eq. (1) [10] matches our Monte Carlo results for parallel shocks and when $\delta\theta \ll 1/\gamma_0$.

The parameterization of diffusion used in the Monte Carlo technique is useful for investigating the behavior of DSA in relativistic shocks. It makes fewer approximations than analytic models and can cover a far larger dynamic range than any current particle-in-cell (PIC) simulation. Nevertheless, the actual diffusion depends on the self-consistent generation of magnetic turbulence by the accelerated particles and is inherently complex and has not been described adequately even for nonrelativistic shocks (see, for example, [11]). In fact, the correct form for the magnetic power spectrum in self-generated, relativistic turbulence will not be known until PIC simulations can perform this calculation. Unfortunately, the treatment of these problems using PIC codes is beyond current computing capabilities for three basic reasons: (i) To correctly describe cross-field diffusion and other physics of the viscous subshock, PIC simulations must be done fully in 3-D [8][9]. If 1- or 2-D simulations are used, cross-field diffusion, an essential element in DSA, is unrealistically suppressed; (ii) If electrons are to be understood, electrons and protons must be modeled simultaneously (except for pair plasma shocks), and the simulation must treat widely disparate inertial scales, greatly adding to the run time; and (iii) In order for nonlinear effects to become apparent or field generation to occur on large scales, simulations must be run long enough in a large enough box with enough particles for a significant population of extremely energetic particles to be produced. The combination of these three requirements means that, while PIC simulations can add to our knowledge of critical aspects of the injection problem and the start of magnetic field generation process (e.g., [13]), they will not be able to model the injection and nonlinear acceleration of electrons and ions to energies relevant to ultra-high energy cosmic rays or GRBs until computers more powerful than exist today become available.

While we have only discussed test-particle acceleration, nonlinear effects, where the accelerated particles modify the shock structure, may be important in relativistic shocks (see [5]), and in fact, most models of γ-ray bursts assume that relativistic shocks efficiently accelerate electrons. If this is the case, test-particle spectra may not be good approximations. This is certainly true for DSA in trans-relativistic shocks, where the harder intrinsic power law will naturally lead to a nonlinear feedback on the shock structure.

Nonlinear effects will also influence the distribution of energy between electrons and protons (e.g., [3]) and modify the shape of the distribution at low energies, i.e., near the sharp peaks at $\sim \gamma_0$ in Figs. 1 and 2 (i.e., [5]). These issues are critical for γ-ray burst models.

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