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Numerical Studies of Diffusive Shock Acceleration at Spherical Shocks

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We have developed a cosmic-ray (CR) shock code in one dimensional spherical geometry in which the particle distribution and the gas flow can be followed numerically in a frame comoving with a spherical shock. We find the spatial grid spacing required for numerical convergence is less stringent in this code, compared to the typical Eulerian code. This is because the shock stays in the same grid zone in the comoving frame. In addition, the adaptive mesh refinement and the coarse-grained momentum volume method have been implement into the new code in order to achieve the highest possible computational efficiency. Using this code with the realistic Bohm type diffusion model we have calculated the CR acceleration and the nonlinear feedback at supernova remnant shocks during Sedov-Taylor stage. Similarly to plane-parallel shocks with $M_s \approx 10 - 15$, about 50 % of the kinetic energy that has passed through the shock is converted to CR energy.

1. Introduction

Collisionless shocks form ubiquitously in tenuous cosmic plasmas via collective, electromagnetic viscosities. The formation process of such shocks inevitably produces suprathermal particles, which can be further accelerated to very high energies through the interactions with resonantly scattering Alfvén waves in the converging flows across a shock [1, 2].

In the kinetic approach to study numerically the CR acceleration at shocks, the diffusion-convection equation for the particle momentum distribution, $f(p)$, is solved with suitably modified gasdynamic equations. This numerical task is challenging, because the full CR shock transition includes a very wide range of length scales associated with the particle diffusion lengths, $\kappa(p)/u_s$, from CR injection scales near the shock to outer diffusion scales for the highest energy particles. To follow the acceleration of highly relativistic CRs from suprathermal energies, we have developed the CRASH (Cosmic-Ray Amr SHock) code in one dimensional (1D) plane-parallel geometry by combining a powerful Adaptive Mesh Refinement (AMR) technique and a shock tracking technique [3]. Time-dependent nonlinear simulations of DSA found that $10^{-4} - 10^{-3}$ of incoming thermal particles can be injected into the CR population via thermal leakage at quasi-parallel shocks, and that up to 50-60 % of the shock kinetic energy can be converted into CRs at strong shocks with $M_s > 10$ [4, 5]. The presence of a preexisting CR population is equivalent to having efficient thermal leakage injection at the shock.

It is believed that the CR pressure is important in the evolution of the interstellar medium (ISM) of our galaxy and most of galactic CR protons with energies up to $\sim 10^{14}$ eV are accelerated by supernova remnant (SNR) shocks [6]. Simulations of DSA in spherical SNRs also indicate that CRs can absorb up to 50% of the initial blast energies [7, 8].

In this paper we describe a new CRASH code in 1D spherical geometry. We solve the flow equations in a frame comoving with the spherical shock, so the shock and refined region around it stay at the same grid locations. We will present the numerical simulation results for a typical SNR expanding into the uniform hot ISM.

2. Comoving Spherical Grid

In order to ensure that the shock remains near the middle of the computational domain at all levels of refined grids, for a spherically expanding shock, we must define a comoving frame which expands with the instantaneous shock speed. Following the conventional cosmological approach [9], the *comoving* radial coordinate, $x = r/a$, is adopted, where a is the expansion factor and $a = 1$ at the start of simulations. The expansion rate, $\dot{a} = (u_s - v_s)/x_s$, is found from the condition that the shock speed is zero at the comoving frame. Here u_s and v_s are the shock radial velocities in the Eulerian frame and in the comoving frame, respectively. Then the *comoving* density and pressures are defined as $\tilde{\rho} = \rho a^3$, $\tilde{P}_g = P_g a^3$, and $\tilde{P}_c = P_c a^3$. The gasdynamic equation with CR pressure terms in the spherical comoving frame can be written as follows:

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{1}{a}\frac{\partial}{\partial x}(\tilde{\rho} v) = -\frac{2}{ax}\tilde{\rho} v \tag{1}$$

$$\frac{\partial(\tilde{\rho} v)}{\partial t} + \frac{1}{a}\frac{\partial}{\partial x}(\tilde{\rho} v^2 + \tilde{P}_g + \tilde{P}_c) = -\frac{2}{ax}\tilde{\rho} v^2 - \frac{\dot{a}}{a}\tilde{\rho} v - \ddot{a} x \tilde{\rho}, \tag{2}$$

$$\frac{\partial(\tilde{\rho}\tilde{e}_g)}{\partial t} + \frac{1}{a}\frac{\partial}{\partial x}(\tilde{\rho}\tilde{e}_g v + \tilde{P}_g v + \tilde{P}_c v) = -\frac{v}{a}\frac{\partial \tilde{P}_c}{\partial x} - \frac{2}{ax}(\tilde{\rho}\tilde{e}_g v + \tilde{P}_g v) - 2\frac{\dot{a}}{a}\tilde{\rho}\tilde{e}_g - \ddot{a} x \tilde{\rho} v - \tilde{L}(x,t). \tag{3}$$

The injection energy loss term, $L(r,t)$, accounts for the energy of the suprathermal particles injected to the CR component at the subshock. The deceleration rate is calculated numerically by $\ddot{a} = (\dot{a}^n - \dot{a}^{n-1})/\Delta t^n$. The diffusion-convection equation for the function $\tilde{g} = p^4 \tilde{f}$, where $\tilde{f}(p,r,t)$ is the *comoving* CR distribution function, is given by

$$\frac{\partial \tilde{g}}{\partial t} + \frac{v}{a}\frac{\partial \tilde{g}}{\partial x} = [\frac{1}{3ax^2}\frac{\partial}{\partial x}(x^2 v) + \frac{\dot{a}}{a}](\frac{\partial \tilde{g}}{\partial y} - 4\tilde{g}) + 3\frac{\dot{a}}{a}\tilde{g} + \frac{1}{a^2 x^2}\frac{\partial}{\partial x}[x^2 \kappa(x,y)\frac{\partial \tilde{g}}{\partial x}], \tag{4}$$

where $y = \ln(p)$ and $\kappa(x,p)$ is the diffusion coefficient.

3. Numerical Models and Results

We considered a supernova explosion with $E_o = 10^{51}$ ergs and $M_{sn} = 10 M_{sun}$ in a uniform medium with $n_{\rm H} = 3\times10^{-3}{\rm cm}^{-3}$. The physical quantities are normalized, both in the numerical code and in the plots below, by the following constants: $\rho_o = 7.0\times10^{-27}{\rm g\ cm}^{-3}$, $t_o = 6.1\times10^3$ yr, $r_o = 28.5$ pc, $u_o = 4.6\times10^3{\rm km\ s}^{-1}$, $P_o = 1.5\times10^{-9}{\rm erg\ cm}^{-3}$, and $\kappa_o = 4.0\times10^{28}{\rm cm\ s}^{-1}$. We assume a Bohm type diffusion coefficient, $\kappa = (3\times10^{22}{\rm cm}^2\ {\rm s}^{-1})p/B_\mu$, where $B_\mu = 5$ is the ISM magnetic field strength in units of microgauss and p is the particle momentum in units of mc. The pressure of the background gas is set to be $P_{g,0} = 10^{-12}{\rm erg\ cm}^{-3}$ ($T_O \approx 10^6$K), and the Mach number of the initial shock is 13. In the code units $\hat{P}_{g,0} = 1.67\times10^{-4}$ and $\hat{\kappa}(p) = 1.5\times10^{-7}p$. It is assumed that there exits a pre-exiting CR population $f(p) \propto p^{-4.5}$, corresponding to an upstream CR pressure, $P_{c,0} = 0.5 P_{g,0}$.

The simulation is initialized at $(t/t_o) = 1$ by the Sedov-Taylor similarity solutions which are characterized by the shock position and speed, $(r_s/r_o) = \xi_s (t/t_o)^{2/5}$ and $(u_s/u_o) = (2/5)\xi_s (t/t_o)^{-3/5}$ with $\xi_s = 1.15167$. The spatial grid resolution in the code unit is $\Delta\hat{r}_0 = 6.0\times10^{-4}$ at the base grid and $\Delta\hat{r}_8 = 2.3\times10^{-6}$ at the *8th* refined grid, which is the finest refined grid for this simulation. When the simulation is repeated with 10 levels of refined grids, P_c increases less than 0.5 %, indicating true convergence in the simulation with 8 levels. This grid spacing is much larger than the diffusion length for $p_{min} \approx 10^{-2}$, $\hat{l}_{diff} = 2.5\times10^{-9}$, which is contrary to what we found in previous simulations in Eulerian grid [3]. The faster convergence at lower resolution seems

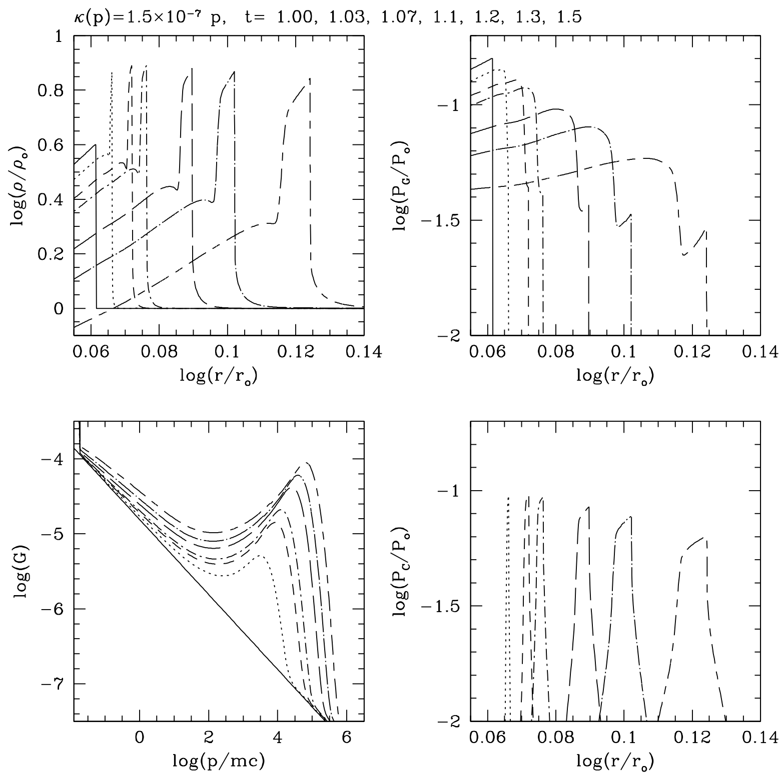

Figure 1. Evolution a typical SNR expanding into the uniform ISM. The model parameters are $E_o = 10^{51}$ ergs, $M_{sn} = 10M_{sun}$, $n_{\rm H} = 3 \times 10^{-3}{\rm cm}^{-3}$, and $B_\mu = 5$. It assumes a preexisting CR population of $f(p) \propto p^{-4.5}$, with $P_{c,0} = 0.5P_{g,0}$, but thermal leakage is not included. The lower left panel shows the integrated particle spectrum, $G(p) = 4\pi \int r^2 dr f(r,p)p^4$. The time $t = 1$ corresponds to 6100 years. See text for other normalization constants. The initial condition at $(t/t_o) = 1.0$ (solid line) is set by the Sedov-Taylor similarity solution.

to result from the fact that the shock stays in the same grid zone in the comoving frame. We use 230 uniformly spaced logarithmic momentum zones in the interval $\log(p/mc) = [\log p_{min}, \log p_{max}] = [-3.0, +6.0]$.

The CR modified shock structure and the CR momentum distribution inside the simulation box, $G(p) = 4\pi \int_{r_{min}}^{r_{max}} r^2 dr f(r,p)p^4$, are shown in Fig. 1 at $t/t_o = 1.0 - 1.5$. The density in the precursor, ρ_1, and in the postshock region, ρ_2, immediately before and after the subshock, respectively, are shown in the top panel of Fig. 2. The middle panel shows the Mach number of the subshock, while the bottom panel shows the CR pressure and gas pressure in units of the ram pressure of unmodified Sedov-Taylor similarity solution, *i.e.*, $\rho_0 u_{ST}^2 \propto t^{-1.2}$.

We note the following important observations from these figures:

- The CR protons are accelerated to the proton Knee energy (10^{14}eV, $p/mc \sim 10^5$) in several thousand years, as expected from the standard estimate [10].

- The ratios of both postshock P_c and P_g relative the shock ram pressure approach to time-asymptotic values quickly. The postshock P_c is about 50 % of the shock ram pressure, while the gas pressure takes only 20 %.

- Both the CR momentum distribution at the shock, $g(r_s,p) = f(r_s,p)p^4$, and the integrated distribution, $G(p)$, exhibit characteristic concave curvature, reflecting the nonlinear velocity structure in the precursor.

- A significant precursor develops due to nonlinear feedback from the CR pressure, so the subshock weakens to $M_s \approx 3$. The density compression factor through the precursor is $\rho_1/\rho_o \approx 2-3$, while the compression over the total transition is $\rho_2/\rho_o \approx 7-8$. However, the subshock does not completely disappear, as in the case of plane-parallel shocks.

These results are consistent with what we found for shocks with similar Mach numbers in plane-parallel geometry. Since the diffusion length of the highest momentum particle ($p/mc \approx 10^6$), $l_{diff}/r_o \sim 0.25$, is still smaller than the shock radius, $r_s/r_o \sim 1.3$, at $t/t_o = 1.5$, the diffusion in spherically expanding volume should have only minor effects. Although the shock expands and slows down, the CR acceleration efficiency (*i.e.* $P_{c,2}/[\rho_0 u_{ST}^2]$) is similar to that for plane- parallel shocks with similar shock parameters.

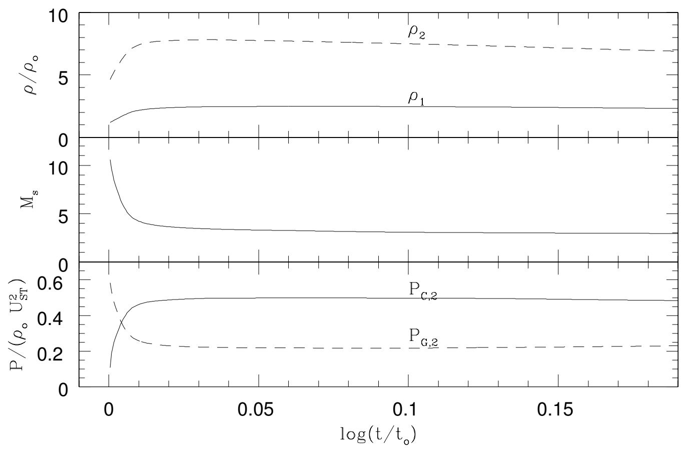

Figure 2. Preshock density, ρ_1, postshock density, ρ_2, the shock Mach number, M_s, the postshock CR and gas pressure in units of the ram pressure of Sedov-Taylor solution, $\rho_0 U_{ST}^2 \propto t^{-1.2}$.

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