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On the 'ankle' of cosmic ray spectrum: Effects of the quasi-perpendicular shocks

A. Meli[a], P. L. Biermann[b,c] and J. K. Becker[a]

(a) Institut für Physik, Universität Dortmund, Germany
(b) Max Planck Institut für Radioastronomie, Bonn Germany
(c) Department of Physics and Astronomy, University of Bonn, Germany
Presenter: J. K. Becker (julia@physik.uni-dortmund.de), ger-becker-J-abs2-og14-poster

The acceleration rate and the maximum energy that cosmic rays acquire in the quasi-perpendicular and perpendicular non-relativistic shocks is evaluated. We examine numerically via Monte Carlo simulations, the effect of the diffusion coefficients and the obliquity of the magnetic field to the shock front, on the energy gain and the acceleration rate. We find and justify previous analytical work (Jokipii 1987) that in highly oblique shocks the smaller the perpendicular diffusion gets compared to the parallel diffusion coefficient values, the greater the energy gain of the cosmic rays to be obtained and under specific conditions the acceleration rate of the particles allows for critical energies to be reached. Specifically, the cosmic ray spectrum in high energies between 10^{15}eV and about 10^{18}eV can be explained. We estimate the upper limit of energy that cosmic rays could gain in plausible astrophysical regimes as they have interpreted by the scenario of cosmic rays which are injected by three different kinds of astrophysical sources, such as: Supernovae which explode into the interstellar medium; Red Supergiants, and Wolf-Rayet stars, where the two latter explode into their pre-supernovae winds (Biermann, 1993).

1. Introduction

Much work has been done over the years concerning studies of the kinetic theory of charged cosmic rays (e.g. [1, 2, 3, 4, 5]), propagating in (turbulent) electromagnetic fields. Furthermore the mechanism of particle shock acceleration is believed to account for the origin of cosmic rays in the whole cosmic ray spectrum range detected. In this paper we aim to test numerically a theoretical approach concerning the particles' diffusion behavior in *non-relativistic* parallel and highly oblique sub-luminal shocks, by applying Monte Carlo simulations. Specifically in the non-relativistic limit, Jokipii [2] theoretically showed that the direction of the average magnetic field at the shock seems to have a large effect on the acceleration mechanism when the parallel diffusion coefficient, $\kappa_{||}$ is much larger than the perpendicular diffusion coefficient, $\kappa_{\perp}$, and as he has particularly shown, the energy gain of the cosmic rays increases as $\kappa_{\perp}$ decreases. Briefly, he showed that if the perpendicular diffusion is much smaller then the parallel one, cosmic rays can gain considerable energy in quasi-perpendicular shocks compared to quasi-parallel ones. In his investigation he estimates that some limits may arise as we will confirm. The relation between the diffusion coefficients and the mean free path of the particle would be given by

$$\frac{\kappa_{||}}{\kappa_{\perp}} = 1 + (\lambda_{||}/r_g)^2 \; , \tag{1}$$

where $\lambda_{||}$ and r_g is the mean free path of the particle and the particle gyroradius respectively. Also it can be shown that the time for acceleration from an initial momentum p_o to a momentum p_1 may be written as

$$\tau_{acc} = \frac{3}{V_1 - V_2} \int_{p_o}^{p_1} (\frac{\kappa_1}{V_1} + \frac{\kappa_2}{V_2}) \frac{dp}{p} \; , \tag{2}$$

where we could assume that $\kappa_1 = \kappa$ as a function of momentum p upstream, and $\kappa_2 = \kappa$ in the downstream region of the shock. What has been shown is that the following should hold

$$\kappa > r_g V_{sh}, \tag{3}$$

with r_g and V_{sh} the particle's gyroradius and the velocity of the shock respectively or else [2]

$$\frac{\lambda_{||}}{v} < \frac{r_g}{V_{sh}}, \tag{4}$$

with v the velocity of the particle. The two foregoing 'limitations' and their effects to the acceleration is what we would like to test in a Monte Carlo simulation by simulating the trajectories of the cosmic rays as they scatter along and across the magnetic field lines in a highly oblique non-relativistic shock.

2. Monte Carlo simulations - Results

A Monte Carlo code was constructed in order to simulate non-relativistic near parallel and oblique shocks with $0^o < \psi < 90^o$ where ψ is the angle between the shock normal and the magnetic field, seen in the shock frame. The velocities of the upstream plasma flow are kept between $0.01c$ and $0.05c$, which correspond to astrophysical environments with non-relativistic shocks such as Wolf-Rayet star winds, Red Supergiant winds, Super Novae that explode into their wind, etc. We mention that Super Novae shock speeds can reach up to $0.05c$. 10^4 particles of a weight equal to 1.0, are injected far upstream at a constant energy (e.g. Γ=5 - particles already relativistic (in the wind)). The cosmic rays are left to move towards the shock and along the way they collide with the scattering centers. Consequently as they keep scattering between the upstream and downstream regions of the shock (its width is much smaller than the particle's gyroradius) they gain a considerable amount of energy. A cross-field diffusion of the particles is present. That means that during the simulation, the particle is allowed to cross the assumed field lines and not only diffuse along them. We give a value of the particle's gyroradius -initially in units of $c \times 1sec$. While the particle is injected upstream, an initial θ and ϕ is given to it and both angles are randomized between each scattering in order to calculate the position of the particle in each step, following the relations: $x_o = r_g v_z \sin\psi, y_o = -r_g v_z \cos\psi$, where, v_z is given by, $v_z = \sin\theta_i \sin\phi_i - \sin\theta_{i-1} \sin\phi_{i-1}$ We keep the compression ratio r equal to 4. We note here that in the test-particle theory of diffusive shock acceleration, the spectral index σ depends on the compression ratio $r = V_1/V_2$ where V_1 and V_2 are the upstream and downstream plasma flow velocities, and $\sigma = 3r/(r-1)$ which for strong shocks in an ideal gas $(c_p/c_v = 5/3)$ gives $\sigma = 4$ ([6, 7, 8, 9]). A splitting technique is used similar to one used in Monte Carlo simulations of [10], [11], so that when an energy is reached such that only a few accelerated cosmic rays remain, each particle is replaced by a number of N particles of statistical weight $1/N$ so as to keep roughly the same number of cosmic rays being followed. For the diffusion along the pressumed magnetic field lines, a guiding center approximation is applied, where the particle trajectory is followed in two-dimensional, pitch angle and distance along **B** space. The mean free path is calculated in the respective fluid frames by the formula: $\lambda = \lambda_\circ p_{1,2}$, assuming a momentum dependence to this mean free path for scattering along and across the field and related to the effective diffusion coefficient. We note that for the description of the physical quantities necessary throughout the simulations we use the fluid rest frames, the normal shock frame and the de Hoffman-Teller frame. Furthermore as the theory for the oblique shocks postulates [12], from the conservation of the first adiabatic invariant we can find the new pitch angle in the downstream frame and similar transformations allow the particle scattering to be followed in this frame. Thus, the particle that crosses the shock from upstream is transmitted only if its pitch angle is less than the

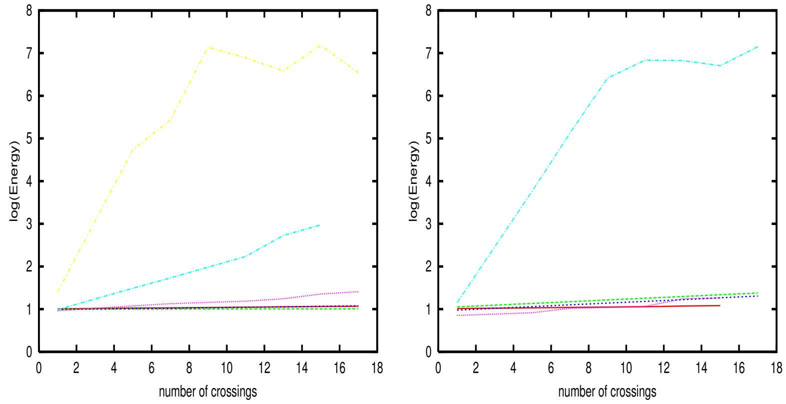

Figure 1. The log of the mean energy of the particles versus the number of shock crossings for $V_{sh} = 0.01c$ (left) and $V_{sh} = 0.05c$ (right). Starting from the bottom for 5^o, 25^o, 65^o, 80^o and 89^o magnetic field inclination, respectively. We see the difference in the energy gain of the particles for the almost perpendicular case compared to smaller shock inclinations. An effect that Jokipii (1987) pointed out as well. The y axis is in dimensionless units (gamma). As an indication 8 (which is $\log_{10}(10^8)$) corresponds to $\sim 10^{16}$eV for protons and $\sim 10^{17}$eV for iron.

critical pitch angle (cosine of the 'loss cone' angle): $\mu_c = \sqrt{\frac{\mathbf{B}_{HT,1}}{\mathbf{B}_{HT,2}}}$. Following the above relations of the conservation of the first adiabatic invariant, we can find the new pitch angle in the downstream frame and similar transformations allow the particle scattering to be followed in this frame. The simulation results are summarized in the following plot captions of figure 1 and 2.

3. Summary and Conclusions

We examined numerically the effect of the obliquity of the magnetic field to the energy gain and the acceleration rate of accelerated particles in non-relativistic nearly perpendicular shocks using Monte Carlo simulations. Previous analytical work [2] is justified. The striking findings of this work are:
(1) We confirm that the spectrum is independent of which scattering coefficient is used.
(2) The individual steps to get to the spectrum are not in many small steps, but by a mixture of large and small steps in energy (not a gaussian distribution anymore, but more like a powerlaw distribution or a very asymmetric gaussian, with a long positive tail). Only less than 20% of the total number of cosmic rays used succeed to get to the highest energies achieved.
(3) The acceleration time scale is shorter, because the scattering is faster. In other words in highly oblique shocks the energy jumps come both very large and very small, without changing the spectrum, but shortening the time scale of acceleration.
(4) The cosmic rays reach very high energies (PeV) in a reasonable time limit frame using eq. (4) compared to standard Bohm limit ($r_g\, c/3$), see also figure (2). That means that highly oblique shocks help to get to 'ankle' (PeV) energies faster.
In conclusion the more oblique the shock is the greater the efficiency of the shock to accelerate the particles. Furthermore, this means that the diffusive shock acceleration mechanism is indeed efficient to accelerate particles up to high energies. It seems that it is quite plausible that a part of the spectrum just above the 'knee' and down close to the 'ankle' is likely due to the contribution of the cosmic rays accelerated by non-relativistic nearly perpendicular shocks such as the ones with particle diffusion behavior described in this work. The seed

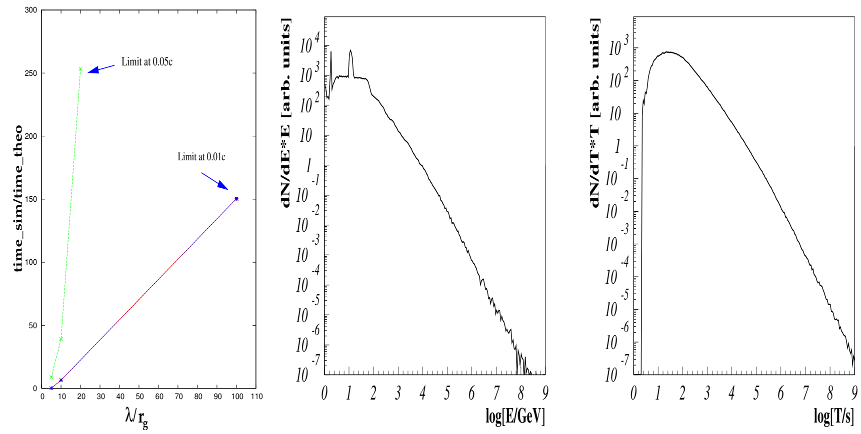

Figure 2. *Left*: The Jokipii Limit is calculated by: $\lambda_{||}/r_g < 1/V_{sh}$. For plasma flow velocity equal to 0.01c the maximum Jokipii limit is at $\lambda_{||}/r_g = 100$ and for plasma flow velocity equal to 0.05c the maximum Jokipii limit is at $\lambda_{||}/r_g = 20$. In y axis the time ratio is the ratio of the simulation time to the theoretical formula given by equation $\tau_{acc} = \frac{3}{V_1 - V_2}(\frac{\kappa_1}{V_1} + \frac{\kappa_2}{V_2})$. *Middle*: A typical spectrum for 85^o for $V_{sh} = 0.05c$ and $\lambda_{||}/r_g = 20$ (at maximum Jokipii limit). Note here that the spectrum is not changed while just two points are modified: (1) Getting to the maximum energy in a distribution of large and small jumps in momentum and, (2) The speed of the acceleration process (obviously affected by the limits) with which we reach the spectrum. *Right*: Calculation of the differential time spectrum for a single run at $V_{sh} = 0.05c$ at 85^o.

of particles to get accelerated in these shock fronts and in such energies, may come from the remnant winds of astrophysical sources with such magnetic field/plasma nearly perpendicular configurations [13].

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