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On cosmic-ray self-generated turbulence of supernova remnant shocks

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It has been claimed that young Supernovae remnants (SNr) could be at the origin of cosmic-rays up to energies of the order of $10^{18} eV$ because of the strong amplification of the magnetic field by the stream of the accelerated particles in the precursor of the shock. Moreover the turbulence spectrum so generated would allow a Bohm regime for cosmic ray transport, which would provide the maximum efficiency of the acceleration. In this work, we have investigated these issues through a spectral analysis of the magnetic turbulence excited upstream the forward SNR shock. Both resonant and non-resonant regimes of the streaming instability are treated concurrently. The turbulence spectra in a mean magnetic field as well as the turbulence level at the shock are derived for a quasi-parallel shock configuration accounting for different non-linear transfer processes. We finally discuss the consequences in the downstream flow of the stationary upstream turbulence.

1. Introduction

The Fermi acceleration of cosmic rays in astrophysical shock fronts depends in a crucial way on their transport properties in the turbulent magnetic field on both sides of the shock. Often the turbulent field spectrum and intensity are arbitrarily prescribed, assuming that it has been built by the ambient medium independently of the shock acceleration process. However, the anisotropy of the cosmic ray distribution function triggers an instability upstream of the shock ([1], [2]). Recently, [3] have shown that the amplification of the turbulent magnetic energy could be quite significant and could produce a magnetic field intensity suitable for pushing the high energy cut-off of the proton distribution up to the "knee" of the cosmic ray spectrum ($E \sim 3 \times 10^{15}$ eV). This theory has then been developed further by [4] analysing the competition between turbulence generation and advection towards the shock front leading to the saturation of the spectrum. These authors have also carried out a preliminary examination of the role of the Kolmogorov cascade for the energy transfer among the excited waves. More recently [5] described a non-resonant regime of the streaming instability and has shown that its growth rate should be dominant in the high wavenumber range. In this work, we analyse the excitation of Alfvén waves as a function of the location in the upstream flow and of the wavenumber taking into account the two instability regimes (Section 2). In Section 3, we calculate the saturation mechanism of the instability considering the advection effect as a function of the wavenumber and the location in the upstream flow. We calculate the contribution due to two nonlinear effects: the transverse nonlinear transfer among turbulent Alfvén waves, and the non-linear backscattering of Alfvén waves off slow magneto-sonic waves. These two nonlinear processes are shown to be relevant and essential to the determination of the anisotropic turbulence spectra. We finally derive these spectra mandatory for calculating the transport coefficients of the cosmic rays. In section 4, we examine the astrophysical consequences of our work with some emphasizes on the phases of dominance of the two regimes of instability and on the properties the turbulence downstream the shock.

2. The two regimes of the streaming instability

The instability triggered by the super-Alfvénic flow of cosmic rays upstream of a shock has been analysed in two ways: one is related to the *resonant* interaction of the cosmic rays with the Alfvén waves (see [2]) and is essentially described by the kinetic theory. The other one has recently been proposed by [5], it emphasises the importance of non-resonant interactions in which the DC-electric current J_{cr} of cosmic-rays generates a Lorentz force responsible for the amplification of the MHD perturbations. In fact, this is the *return current* J_{pl} in the background plasma which generates the perturbations. The plasma remains locally neutral $\rho_{cr} + \rho_{pl} = 0$ with $\vec{J}_{cr} + \vec{J}_{pl} = \vec{0}$. Hereafter, B_∞ and $V_{a\infty}$ denote the mean magnetic field and the Alfvén velocity in the interstellar medium; i.e. far from the forward SN shock. We assumed that the magnetic field obliquity allows a de Hofmann-Teller transformation, such that the cosmic rays motion, at velocity $\vec{V}_s$ with respect to the upstream medium, is along the mean field.

The non-resonant streaming instability:
Following the analysis of the Alfvén wave equation in [5] one can obtain for $V_{A\infty}/V_s \ll 1$, an unstable branch of the dispersion relation when $\omega^2 = -V_{A\infty}^2(k_\| k_c - k_\|^2) < 0$. The wavenumber k_c reads as $k_c \equiv |\rho_{pl} B_\infty|/\rho_0 V_{A\infty}^2 = |\chi_p - \chi_e| eB_\infty/(m_p V_{A\infty})\ V_s/V_{A\infty}$ where $\rho_0 = m_p\, n_0$, $\chi_p n_0$ and $\chi_e n_0$ are the density fraction in proton and electron cosmic-ray respectively. Once the instability grows, the magnetic field and the Alfvén speed in k_c have to include the perturbed magnetic field component and are written as barred quantities. For instance, we have $\bar{V}_a = V_{a\infty}/(1-\eta)^{1/2}$. The quantity $\eta \equiv \delta B^2/(\delta B^2 + B_\infty^2)$ determines the strength of turbulence; $\eta \to 1$ corresponds to $\delta B/B_\infty \to +\infty$. The unstable mode is a purely growing mode of right or left circular polarization depending on the main composition of the CR-fluid and the orientation of the magnetic field. For the likely case of a proton dominated CR-fluid, the mode is *left-handed* with respect to $\vec{B}_\infty$; in other words, it rotates in the same sense as the protons. This instability does apply downto a scale $\delta_0 = c/\omega_{pi}$ beyond which the MHD approximation is no longer valid; i.e. for $k_c\delta_0 = |\chi_p - \chi_e| V_s/\bar{V}_A$. As stated by [5] at short waves $1/r_* \ll k_\| \ll k_c$, the non-resonant growth rate is $G_{n-res}(k_\|) \simeq \bar{V}_A (k_c k_\|)^{1/2}$. The Larmor radius r_* corresponding to the minimum CR distribution momentum is an increasing quantity with the distance to the shock. Note that this non-resonant instability is not operative for a wavenumber $k_\|$ at distances $x \ll \ell_D(r_L = 1/k_\|)$ since the corresponding $r_*(x) \ll 1/k_\|$. The exact spatial dependence of the growth rate will be specified further on. For long waves, $k \ll 1/r_*$ the CR-response dominates, i.e. the non-resonant instability is inactive, the resonant instability takes over.

The resonant streaming instability:
Both electrons and ions, moving forward or backward, can resonate either with forward modes or backward modes. The small instability growth rate is the same, within an angular factor of order unity, and is given by $G_{res}(k_\|, x) = G_0(k_\|, x)\phi(x/\ell_D(k_\|))$, where $\ell_D(k_\|)$ should be understood as $\ell_D(r_L = 1/k_\|)$; the exact spatial dependence of ϕ is given by the convection-diffusion equation; in the case of uniform diffusion coefficient D, $\phi = \exp(-x/\ell_D)$. The growth rate G_0 (from [6]) is

$$G_0(k_\|, x) = \frac{\pi}{4}\frac{\alpha_0(\varepsilon)}{\delta_0}\frac{n_*}{n_0}\left(\frac{\cos\theta}{|\cos\theta|}V_s - \frac{4+\varepsilon}{3}V_A\right)(k_\| r_*(x))^{1+\varepsilon}\ ,$$

the coefficient $\alpha_0(\varepsilon) = \frac{1}{2}(1+\varepsilon)(4+\varepsilon)\int |\mu|^{1+\varepsilon}(1-\mu^2)d\mu$ according to [4]. Only modes propagating forward are destabilized when V_s is sufficiently larger than the Alfvén velocity. Note that the backward waves are damped at the same rate than the forward waves are amplified. The growth rate $G_0(x)$ depends on x due to the amplification of the magnetic field by the non-resonant instability. To derive the previous relation, we had to assume a CR power-law distribution $f(p) \propto p^{-4-\varepsilon}$ between p_{max} and p_{min}. The parameter ε is either positive or negative depending on the back-reaction effects of the CR and turbulence generation on the shock structure.

3. Stationary spectra and saturation mechanisms

Upstream turbulence:
It is convenient to describe the profiles of the wave spectra with the help of a dimensionless variable y defined by $dx = \ell_D(r_L = 1/k_\parallel, x)dy$, then the function $\phi = e^{-y}$. The quantity $r_*(x)$ is defined by $y = 1$ for $r_L = r_*$, Since $\ell_D(r_L = 1/k_\parallel, x) = (1/3)(c/V_s)k_\parallel^{-1}(k_\parallel/k_{min})^{\beta-1}\eta^{-1}(x)$ hence $y = \left(k_\parallel r_*\right)^{2-\beta_{NR}}$. The non-resonant dominated regime is to be taken in the interval $1 < y < y_{max} = (k_{min}r_*)^{2-\beta_R}$ (far from the shock front) while the resonant regime is to be taken in the interval $0 < y < 1$ (close to the shock front). The parameters β_{NR} and β_R stem for the non-resonant and resonant spectral indices respectively. The spectrum evolution equation normalised the timescale to the advection time $\tau_a(k_\parallel) = \ell_D(k_\parallel)/V_s$ can be constructed with the dimentionless variable y. The perpendicular k spectrum is fixed by the non-linear transfer produced in Alfvénic turbulence (see [7], [8]) while the parallel spectrum is fixed by the dominant streaming instability regime in the shock precursor. For instance, using the Goldreich-Sridhar scaling for strong turbulence the solutions have the form $S_{3D}(k_\parallel, k_\perp) \propto k_\parallel^{-\beta}\ k_\perp^{-q}$ with $3\alpha + 2\beta = 7$ and $\alpha = q - 1$. Hence for $\beta = 1$ we get $\alpha = 5/3$ corresponding to a Kolmogorov-type spectrum.
At high y or at large distances from the shock front, the non-resonant instability prevails and if one compares the instability growth to the advection, the evolution equation leads to a hard spectrum ($\beta_{NR} = 0$) and a very high turbulence level proportional to $r_{Lmax}/r_{Lmin} = p_{max}/p_{min}$, which can be larger than 10^6. In fact, the non-resonant growth should saturate much earlier, either through a quenching at $k_c r_* = 1$ or through non-linear transfer, leading to a 1D spectrum $S(k_\parallel) \propto k_\parallel^{-1}$, i.e. $\beta_{NR} = 1$. The saturation level is $k_\parallel S(k_\parallel, y) \simeq (\xi_{CR}/\Phi)V_s^3/(y\ V_{a\infty}^2\ c)$ where $\xi_{CR} = P_{CR}/\rho_0 v_s^2$ is the ratio of CR pressure to the kinetic energy at the shock front (see [5]) and $\Phi \simeq ln(p_{max}/p_{min}) \simeq 10$. Closer to the shock, the resonant streaming instability takes over, the spectrum is built from the solution at y = 1. Balancing the advection and the growth rate leads to saturation level $k_\parallel S(k_\parallel) \simeq (\xi_{CR}/\Phi)(V_s/V_{a\infty})$ and a 1D spectrum $S(k_\parallel) \propto k_\parallel^{-1}$, i.e. $\beta_R = 1$. However, in the resonant regime, for the CR to be scattered back and forth the shock, backward Alfvén modes have to be generated efficiently upstream the shock. The Alfvèn turbulence transfers the energy only in the perpendicular direction to the magnetic field and one have to invoke another non-linear transfer mechanism to transfer energy from the forward Alfvén waves destabilised by the CR towards the backward Alfvén waves. This transfer is possible in magnetic field dominated plasma with $\beta_p = P_{th}/P_B \leq 1$ through the interplay of sonic waves (see [1]). This is the only way to produce similar spectrum for both forward and backward Alfvén waves.

Downstream turbulence:
The turbulence properties downstream (level of turbulence, spectral index) can be constrained using the observation of the X-ray filaments in young SNr (see [9]). It is shown that the relativistic electrons with energies about a few tens of TeV producing the observed synchrotron radiation have a diffusion coefficient close to the Bohm value. However the previous analysis has been made assuming an isotropic turblence spectrum, which is not correct at least for two reasons: the turbulence is already anisotropic upstream and the magnetic field amplified upstream is compressed in the direction parallel to the shock front.
The way the turbulence acts on a relativistic particle can be characterized by competing the non-linear Alfvén transfer time $t_{nl}(k_\parallel) = (k_\parallel \bar{V}_A(k_\parallel))^{-1} \simeq (l_\parallel/\bar{V}_A)(l_\parallel k_\parallel)^{(\beta-3)/2}S_0^{-1/2}$, with the downstream return timescale $t_{ret} = \kappa\,(c/V_s)^2\,2/3t_s$, with $t_s = (2\pi)^{-\beta}\eta^{-1}(l_\parallel/c)(r_L/l_\parallel)^{2-\beta}$. The prefactor $\kappa < 1$ accounts for the shortening of the return timescale in compressed turbulence. The condition $t_{ret} < t_{nl}$ (using $\beta = 1$) means the particle does explore distances smaller than the relaxation length of the turbulence downstream and particles experience compressed rather than relaxed turbulence during their journey downstream. Using typical values of the magnetic field, mean density and shock velocity in our problem (see next section) and acknowledging

a saturation level of $M_a \xi_{CR}/\Phi$, we find the previous condition is fulfilled unless the shock velocity is lower than 10^{-2} c.

4. Astrophysical consequences

Both regimes of streaming instability occur in SNr but the dominance of one regime over the other depends strongly on the Alfvenic Mach number of the shock $\mathcal{M}_a = V_s/V_{A\infty}$. This number depends as much as on the evolution phase of the SN remnant than on the physical properties of the external interstellar medium (ISM). Let us consider the two first phases of SN evolution: the very early free expansion phase where the shock velocity reaches values as high as $V_s \simeq 0.1c$, the late free expansion phase (or early Sedov self-similar phase) where the shock velocity drops to $V_s \simeq 10^{-2}c$. In the latter phase the remnant may expand either in a hot rarefied ($\mathrm{T} \simeq 10^6$ K, $\mathrm{n} \simeq 10^{-3/-2}$ cm^{-3}) interior of a massive star wind bubble or in a warm partially ionised *typical ISM* ($\mathrm{T} \simeq 10^4$ K, $\mathrm{n} \simeq 10^{-1/0}$ cm^{-3}). The mean ISM magnetic field in both cases is conservatively taken as 3μGauss. In the very early phase of the SNr evolution, the medium is probably much denser with $\mathrm{n} \simeq 10 - 100$ cm^{-3}. The ratio $S_{NR}/S_R \simeq \mathcal{M}_{\dashv}\, V_s/c \simeq 500\; B_{3\mu G}^{-1}\sqrt{n}V_{s-1}^2$, where V_{s-1} is the shock velocity in units of 0.1c. The non-resonant regime appears to dominate for the very early free-expansion phase as already pointed out by [5], while the resonant regime dominates in the late free-expansion phase. The resonant instability is reinforced in low density and highly magnetised media.

5. Conclusions

Upstream of an astrophysical shock, the cosmic ray streaming triggers an instability occuring under either resonant condition and dominates the larger wave-lengths of the Alfvén spectrum off resonance and dominates at shorter wavelengths. The non-resonant instability prevails at large distance to the shock front and can saturates either through a balance of non-linear transfer cascade with the growth rate or by a quasi-linear quenching. The stationary 1D spectrum solution scales as $k_{\parallel}^{-1}$. The main saturation mechanism of the resonant instability stems from the fact that the shock front catches up with the growing waves over a diffusion length. This mechanism determines the spectrum, namely scaling also in $k_{\parallel}^{-1}$. The $k_{\perp}$ dependence of the spectrum is remodeled by the non-linear cascade of Alfvén waves, that essentially works transversally, the transfer time is short enough compared to the advection time. The resonant regime is found to always dominate in the late free-expansion phase once the shock velocity has enough decreased. The compression effects downstream have to be taken into account in the transport of the particles during a Fermi cycle.

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