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Statistical Mechanics of Supernovae Revisited

F.C. Jones and R.E. Streitmatter

Exploration of the Universe Division, NASA/Goddard Space Flight Center, Greenbelt, MD 20771, USA

Presenter: F.C. Jones (frank.c.jones@gsfc.nasa.gov), usa-jones-FC-abs1-og13-oral

We have performed Monte Carlo calculations to study the spectrum arising from supernovae occuring at random times and positions. Each supernova injects an $E^{-2.2}$ power law spectrum and the diffusion coefficient varies as $E^{0.5}$.The resulting spectrum appears normal for energies below 10^{16} eV but above this energy the spectrum shows considerable variation from one realization to another and shows a lack of ultra high energy particles. It appears that obtaining ultra high energy particles requires a large number of supernovae at large distances from the observation point.

1. Introduction

The notion that the random nature of supernova explosions, which are believed to be the primary accelerator of cosmic rays in the Galaxy, could affect the observed spectrum and anisotropy of the radiation has been around for some time [1, 2, 3, 4, 5, 6]. Also see [7] and references therein.

We have employed a Monte Carlo technique to investigate the effect of the random nature of supernova explosions on the cosmic-ray spectrum as it appears from realization to realization.

2. The Model

We represent the Galaxy as a slab of half thickness,$L = 1.4$kpc between two free-escape planes with an infinite radial extent from the observation point. The cosmic-ray diffusion coefficient is taken to be 4×10^{28} cm^2/s at 1 GeV, increasing with energy as $E^{0.5}$. This yields a residence lifetime of 1.5×10^7 yrs. at 1 GeV, decreasing with energy as $E^{-0.5}$. These choices define the characteristic length and time scales for the problem and we allow cosmic rays to be injected with a spectrum proportional to $E^{-2.2}$ from 1 to 10^{13} GeV at random times in the past up to 10 characteristic times and at random radii, proportional to area, up to 10 characteristic scale lengths. In some realizations we allow the diffusion coefficient in the direction x, lying in the galactic plane, to be 4×10^{29} cm^2/s, 10 times the value of that in the direction perpendicular to the plane. This allows cosmic rays to arrive at the observation point from a distance about 3 times further away than otherwise.

3. Results

First of all we should point out that this model is highly idealized. We treat observations from a point about which the Galaxy is symmetric and extends to infinity, while in reality the scale we have chosen to simulate is close to the actual size of the Galaxy and we are observing cosmic rays at a point far from the center. Secondly, we know of no Galactic accelerator capable of producing a spectrum that extends to 10^{13} GeV, and even so the diffusion approximation should become invalid above 10^7 GeV since the diffusive lifetime above this energy is shorter than the free flight time to the edge of the Galaxy. This occurs in spite of the fact that the gyroradius of such a particle is only about 10pc in a 10^{-6} gauss magnetic field. Incidentally, we believe this to be a strong reason that a $E^{0.5}$ variation of the diffusion coefficient to be untenable. We intend to redo this study in the future with a more reasonable (in our opinion) dependence of the diffusion coefficient on energy.

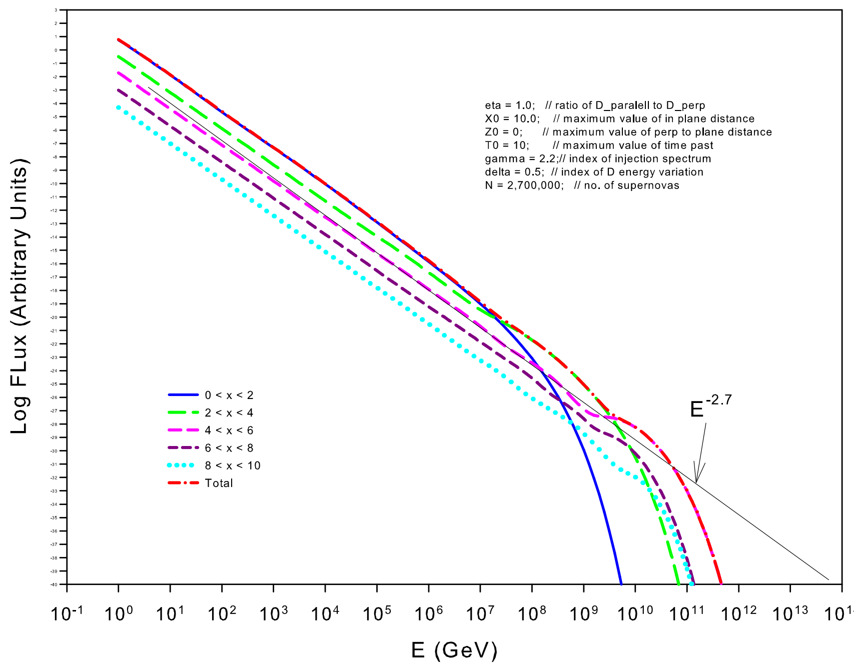

Figure 1. A realization displayed by space sections, $D_{\parallel} = D_{\perp}$.

We, nevertheless, believed that this model was the most favorable to producing the smooth spectrum yielded by analytical studies so that any deviations from this smoothness would be significant.

We can see in Figure 1 that the spectrum above $10^{16} \sim 10^{17}$ eV is rather bumpy and we will see in Figures 3 and 4 that it can vary in this energy region from one realization to another. Note: The straight line in these plots represents the $E^{-2.7}$ slope predicted by the simple analytic theory. We note further that at low ($< 10^{15}$ eV) the CR flux follows the analytic solution quite well and is dominated by the closer supernovae, particles coming from more distant ones tend to leak out of the Galaxy before they can reach the observer, and the particles spread in space more, increasing the depletion at the observing point. However, at higher energies ($> 10^{16}$ eV) the flux becomes richer with particles from far away. This is because the time scale for high energy particles grows short and the probability of seeing them from the, relatively few, nearby events is small compared to the more numerous events at large distance. There is also some randomness in the various cutoffs depending on the time of the most recent supernova in the particular distance range as is discussed in the next paragraph.

The times of the supernova injections of cosmic rays ran from $t = 10$ leakage times down to 1.4×10^{-5} leakage times, zero being not allowed since the diffusion solution is singular there and can not be calculated. The lower limit is due to the grid spacing of the random number generator. This lower limit being non-zero causes the cutoff at the high end of the spectrum. The flux from injections integrated from $t = t_1$ to ∞ is given by

$$\frac{\exp\left(-\frac{Dt_1}{L^2}\right)}{(D/L^2)} = \exp(-t_1 * E^{0.5})/E^{0.5}$$

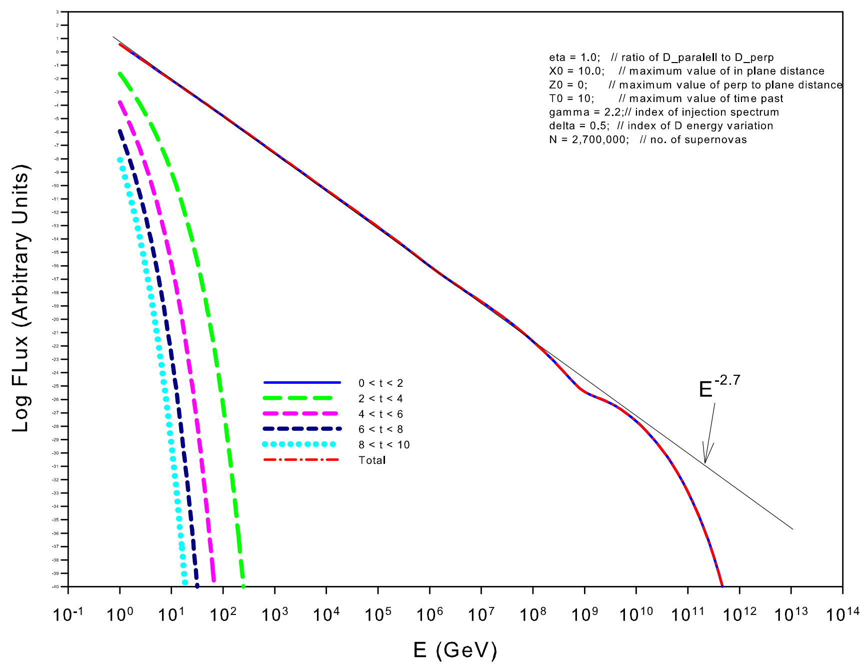

Figure 2. Another realization displayed by time sections, $D_{\|} = D_{\perp}$.

For the minimum time used here the exponent becomes one at $E = 5.1 \times 10^9$ GeV producing the cutoff seen in Figure 1

In Figure 2 we show a realization with the results shown in time sections. It is clear that the higher time bands ($T > 2$) contribute a negligible amount since the previously discussed cutoff for $t = 2$ is 4 GeV.

Finally, in Figure 3 we show four realizations of the total flux with $D_{\|} = D_{\perp}$ and in Figure 4 four more realizations with $D_{\|} = 10 \times D_{\perp}$.

4. Discussion

We see that in the present study the high end of the spectrum is strongly dependant on the most distant sources. The average spectrum derived from the solution of the steady state, uniformly distributed sources, diffusion equation with leakage proportional to $E^{0.5}$ produces a spectrum shown by the straight line drawn in all of the figures. Our plots represent individual realizations of the possible supernova history and would not be expected to follow the average curve; however, one would naively expect that there would be as many excursions above the average as below it. It appears that in reality the excursions above the average at these high energies are extremely large bursts from recent, nearby supernova events which are extremely rare, and, in this simulation limited to times no more recent than $1.4 \times 10^{-5} * t_{\mathrm{escape}} \approx 200$ years. Their rarity is due primarily to the short timescale of such high energy particles in the diffusion approximation. In fact, their short duration makes them resemble gamma-ray bursts in their elusiveness. They are, therefore, unlikely to be seen and we did not

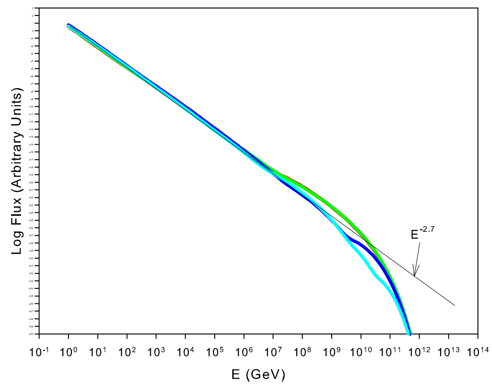

Figure 3. Different realizations of the total flux, $D_{\|} = D_{\perp}$.

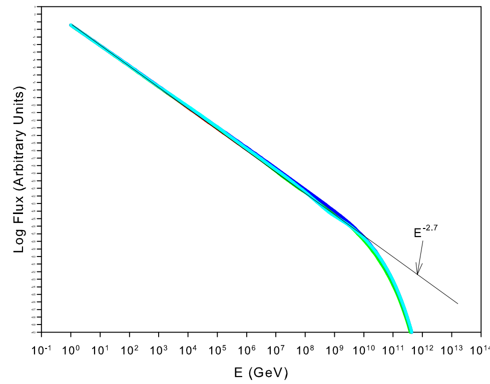

Figure 4. Different realizations of the total flux, $D_{\|} = 10 \times D_{\perp}$. Although we have plotted four realizations they are difficult to see, the results being essentially identical.

see any, at least not enough to bring the average flux up to the analytically predicted average. This is partly due, we believe, to the choice of a diffusion coefficient that increases with the square root of the energy and one increasing as $E^{0.3}$ would remove some of this effect. Furthermore, the high-energy cutoff just discussed would be at considerably higher energies ($\approx 1.5 \times 10^{16}$ GeV) than seen here. Such a propagation model could be more realistic in this respect and we shall investigate such models in the near future.

Another aspect of this result is the demonstration that the diffusion coefficient in the plane of the galaxy strongly affects the smoothness of the cosmic-ray spectrum. If it is the same as the perpendicular component it is difficult to see cosmic rays from sources that are very far away. If, on the other hand, it is considerably larger then contributions from far-away events can contribute significantly to the spectrum smoothing out some of the fluctuations that would be seen otherwise.

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