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A method to calculate the Galactic cosmic ray density with high spatial resolution, new developments

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The recent discovery of direct evidences for the acceleration of high energetic particles at the shell supernova remnant RXJ1713.7-3946 underlined the need to calculate the cosmic ray (CR) distribution in the Galaxy on a spatial grid fine enough to resolve the changes in the CR density due to these kind of objects. This is a progress report on the further development of a method to solve the time dependent CR propagation equation with high spatial resolution for arbitrary source distributions.

1. Introduction

The recent discovery of direct evidences for the acceleration of high energetic particles at the shell supernova remnant RXJ1713.7-3946 [1] underlined the need to calculate the cosmic ray (CR) distribution in the Galaxy on a spatial grid fine enough to resolve the changes in the CR density due to these kind of objects, what results in a huge numerical grid.

As described in [2, 3], we developed a method to solve the time-dependent CR propagation equation:

$$\frac{\partial N}{\partial t} - S = k\left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r} + \frac{1}{r^2}\frac{\partial^2 N}{\partial \varphi^2} + \frac{\partial^2 N}{\partial z^2}\right) - \Omega N \tag{1}$$

for the CR density N with high spatial and temporal resolution avoiding the need of a huge numerical grid. Eq. (1) takes into account diffusion and catastrophic losses, which is suitable for CR nuclei with $Z > 2$. $k = k(p,z)$ is the spatial diffusion coefficient, $\Omega = n_{\rm gas} v_{\rm CR} \sigma_{\rm p,CR}$ describes the rate of catastrophic losses and $S = S(r,\varphi,z,t,p)$ represents the sources. These quantities also depend on particle momentum p which is treated as a a free parameter. For our calculations, the Galactic disc is assumed to be filled with gas, with a distribution depending on z only. The disk also contains the sources. The Galactic disc is embedded in the Galactic halo, a region where particles may still be confined in the system. The diffusive volume is treated as a cylinder with radius R and height $2H$. Free escape boundary conditions are imposed at the boundaries. Cylindrical coordinates are used, which is suggested by the geometry of the Galaxy.

Under the condition that neither the diffusion coefficient k nor the loss term Ω depend on galactocentric radius r or azimuth φ, we apply a series ansatz for the CR density N:

$$N = \frac{1}{\pi}\sum_n \sum_\alpha \left(A_{nm}\cos(n\varphi) + B_{nm}\sin(n\varphi)\right)\frac{\jmath_n(\alpha_{nm} r)}{\left(\jmath_n'(\alpha_{nm} R)\right)^2} \tag{2}$$

α_{nm} is the mth the solutions of $\jmath_n(\alpha R) = 0$, and $\jmath_n$ the Bessel function of order n which transforms the three-dimensional spatial problem into a system of one-dimensional equations for the coefficients of this expansion, which then can be solved efficiently with standard numerical methods. The resolution in r, φ only depends on the number of coefficients in Eq. (2) and is limited only by the available CPU time.

2. New Developments

During the last year, the method was improved significantly. The calculation time has been reduced drastically by changing the numerical code from a DuFort Frankel to a Crank Nicholson scheme [4], which allowed to increase the time step by a factor of 100 and also proved to be numerically stable for a much larger parameter space.

On the technical side, we optimized the code for computer clusters by implementing the MPI framework [6]. Alternatively the already implemented lock-file mechanism can be chosen, which is suitable for inhomogeneous computer grids.

3. Extending the method

Eq. (1) describes the propagation of CR nuclei heavier than helium. Considering also electrons, protons and light nuclei, continuous losses have to be taken into account. In this new model, we further include effects of the Galactic wind and reacceleration. So we have instead of Eq. (1), using cylindrical coordinates

$$\begin{aligned}\frac{\partial N}{\partial t} &= S + k\left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r} + \frac{1}{r^2}\frac{\partial^2 N}{\partial \varphi^2} + \frac{\partial}{\partial z}\left(k\frac{\partial N}{\partial z}\right)\right) \\ &\quad -\frac{\partial}{\partial z}\left(V_{\mathrm{wind}}\, N\right) - \frac{\partial}{\partial p}\left(\kappa\frac{\partial N}{\partial p}\right) - \frac{\partial}{\partial p}\left(\dot{P}\, N\right) - \Omega N \end{aligned} \tag{3}$$

where V_{wind} is the speed of the Galactic wind, $\dot{P}$ describes continuous losses. κ is the diffusion coefficient for diffusion in momentum space. Again, we can assume that V_{wind}, $\dot{P}$, κ are independent of the galactocentric radius r and azimuth φ so we apply ansatz Eq. (2) to Eq. (3), which leaves us with equations for the expansion coefficients

$$\begin{aligned}\frac{\partial A_{nm}}{\partial t} &= S^A_{nm} - k\alpha^2_{nm}A_{nm} + \frac{\partial}{\partial z}\left(k\frac{\partial A_{nm}}{\partial z}\right) - \frac{\partial}{\partial z}\left(V_{\mathrm{wind}}\, A_{nm}\right) \\ &\quad -\frac{\partial}{\partial p}\left(\kappa\frac{\partial A_{nm}}{\partial p}\right) - \frac{\partial}{\partial p}\left(\dot{P}\, A_{nm}\right) - \Omega A_{nm} \end{aligned} \tag{4}$$

where $S^{(A)}_{nm}$ is given by

$$S^{(A)}_{nm} = \int_0^{2\pi}\int_0^R S(r,\varphi,z,p,t)\cos(n\varphi)\, J_n(\alpha_{nm} r)\, dr d\varphi \tag{5}$$

and similar equations for B_{nm}. Note that A_{nm}, B_{nm} now depend on z, t and p where the latter is no more a free parameter. Eq. (4) are solved with a ADI scheme [5], which is in the process of testing.

4. Summary

We presented a method that solves CR-propagation type PDEs for cylindrical geometries. The code is capable of dealing with all three spatial dimensions and time, where the solution may be obtained with high resolution (easy adjustable) in space and time. The change to the Crank Nicholson scheme drastically reduced the computation time. The method allows distributed computation and is usable on PC-style hardware or clusters (using MPI). We are currently expanding this method to also be usable for CR electrons and protons.

5. Acknowledgements

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