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Cosmic rays transport in the fractal-like galactic medium

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We consider the problem of the cosmic ray spectrum formation assuming that cosmic rays are produced by galactic supernova remnants (SNR). Phenomenological anomalous diffusion (AD) model, in which the high energy cosmic ray propagation in the galactic medium is simulated as fractal walks, is used. In the framework of this approach we calculate the contribution of each nearby SNR and show that Loop I, Loop II are the main contributing sources of cosmic rays in the energy region $10 \div 10^6$ TeV.

1. Introduction

Numerous hypotheses on the propagation and acceleration mechanisms of galactic cosmic rays and their sources have been proposed to explain the steepening of the all-particle spectrum at $3{\cdot}10^{15}$ eV (the “knee”) and the flattening of the spectrum around 10^{19} eV (the “ankle”) (see, for example, the reviews [1–4]). However, in spite of considerable theoretical and experimental efforts, a self-consistent model, which can explain these features in the energy spectrum is not developed yet.

In our recent papers [5–8] we proposed an anomalous diffusion (AD) model for solution of the “knee” problem in primary cosmic-rays spectrum and explanation of different values of spectral exponent of protons and other nuclei at $E \sim 10^2 \div 10^5$ GeV/nucleon. The anomaly results from large free paths (“Lévy flights”) of particles between magnetic domains – traps of the returned type. These paths are distributed according to power law $p(r,R) \sim A(R,\alpha) r^{-\alpha-1}, r \to \infty, \alpha < 2$, being an intrinsic property of fractal structures. Here R is magnetic rigidity. It is also assumed that the particle can spend anomalously a long time in the trap. An anomalously long time means that the distribution of the particles staying in traps, $q(\tau,R)$, has a tail of power-law type $q(\tau,R) \sim B(R,\beta)\tau^{-\beta-1}, \tau \to \infty$ with $\beta < 1$ (“Lévy trapping time”).

Without energy losses and nuclear interactions, AD-propagator $G(\vec{r},t,R;R_0)$, describing such a process, obeys the equation [9]

$$\frac{\partial G}{\partial t} = -D(R,\alpha,\beta)\mathrm{D}_{0+}^{1-\beta}(-\Delta)^{\alpha/2} G(\vec{r},t,R;R_0) + \delta(\vec{r})\delta(t)\delta(R-R_0). \qquad (1)$$

Here, D is the AD coefficient, D_{0+}^{μ} denotes the Riemann-Liouville fractional derivative [10], $(-\Delta)^{\alpha/2}$ is the fractional Laplacian (called “Riss” operator) [10].

In the case of punctual impulse source of duration T with inverse power spectrum $S(\vec{r},t,R) = S_0 R^{-p}\delta(\vec{r})$ $\Theta(T-t)\Theta(t)$, $\Theta(t)$ being the Heviside function, cosmic ray concentration is

$$N(\vec{r},t,R) = \frac{S_0 R^{-p}}{D(R,\alpha,\beta)^{3/\alpha}} \int\limits_{\max[0,t-T]}^{t} \tau^{-3\beta/\alpha} \Psi_3^{(\alpha,\beta)}\Big(|\vec{r}|(D(R,\alpha,\beta)\tau^{\beta})^{-1/\alpha}\Big) d\tau, \qquad (2)$$

where the scaling function $\Psi_3^{(\alpha,\beta)}(r)$,

$$\Psi_3^{(\alpha,\beta)}(r) = \int\limits_0^{\infty} q_3^{(\alpha)}(r\tau^{\beta}) q_1^{(\beta,1)}(\tau)\tau^{3\beta/\alpha} d\tau,$$

is determined by three-dimensional spherically-symmetrical stable distribution $q_3^{(\alpha)}(r)$ $(\alpha \leq 2)$ and one-sided stable distribution $q_1^{(\beta,1)}(t)$ with characteristic exponent β [11, 12].

The AD coefficient is determined by the positive constants $A(R,\alpha)$, and $B(R,\beta)$ (in the asymptotic behaviour) for the "Lévy flight" (A) and the "Lévy waiting time" (B) distributions: $D(R,\alpha,\beta) \propto A(R,\alpha)/B(R,\beta)$. Taking into account that both the free path and the probability to stay in trap during the time interval τ for particle with charge Z and mass number A depend on particle magnetic rigidity R, we accept $D = (v/c)D_0(\alpha,\beta)R^{\delta}$. It has been shown [5–8] that in the framework of AD model it is possible to explain the locally observed basic feature of the cosmic rays in the wide energy range .

The aim of the present paper is to calculate, in the framework of the anomalous diffusion approach, the contribution of nearby SNRs to cosmic ray flux and to evaluate the number of contributing sources.

2. Mechanism of cosmic rays spectrum formation

The physical arguments and the calculations indicate that the bulk of observed cosmic rays with energy $10^8 \div 10^{10}$ eV is formed by numerous distant ($r > 1$ kpc) sources. It means that the contribution of these sources to the observed flux may be evaluated in the framework of the steady-state approach. Using our results [13] we present the flux of the particles of type i from all distant sources in the form $J_G^i(r > 1 \text{ kpc}) = v_i C_{0i} E^{-p-\delta/\beta}$, where v_i is a particle velocity, C_{0i} is a constant evaluated via fitting of experimental data.

The contribution of the nearby ($r \leq 1$ kpc) relatively young ($t \leq 10^5$ yrs) sources defines the spectrum in the high energy region and, as it was shown in previous papers [6, 7], provides the "knee". We represent this component in the form

$$J_L^i = \frac{v_i}{4\pi}\sum_j N(\vec{r}_j, t_j, E), \tag{3}$$

where $(\vec{r}_j, t_j)$ are the coordinate and the age of the source (j), $N_i(\vec{r}_j, t_j, E)$ is the cosmic ray concentration (2). List of the supernova remnants, used in our calculations, is given in [14] and in Fig. 2.

The similar separation of the flux into two components with significantly different properties is frequently used in the studies of cosmic rays. However, the presence of the large free paths of the particles (the "Lévy flights") in our model leads us to the introduction of the third component. This third component is formed by the particles which pass a distance between an acceleration site of a source and solar system without scattering. The flux of non-scattered particles J_{NS}^i is determined by the injected flux ($\propto S_{0i}E^{-p}$) and the "Lévy flight" probability $p(>r)$. Taking into account that for the particle with energy E the probability $p(>r) \sim A(E,\alpha)r^{\alpha} \sim E^{\delta_L}$, we have $J_{NS}^i = C_{1i}^0 E^{-p+\delta_L}$. We assume that this component defines the spectrum in the ultrahigh energy region $E \geq 10^{18}$ eV and provides the flattening of the spectrum. In other words, in our model the "ankle" in primary cosmic ray spectrum is also due to the "Lévy flights" of the cosmic ray particles.

Thus, the differential flux J_i of the particles of the type i from all Galactic sources may be presented in the form $J_i(E) = J_G^i(E) + J_L^i(E) + J_{NS}^i(E)$.

3. Parameters of the model

The first free path distribution of the particle traveling through highly inhomogeneous medium of fractal type was investigated numerically. We obtained that first free path distribution in the medium with mass fractal dimension $0 < d_M < 2$ has power-law asymptotic $p(r) \sim r^{-\alpha-1}$. The index α dependence on fractal dimension of the medium under different assumptions on cross-section of particle interaction with elementary

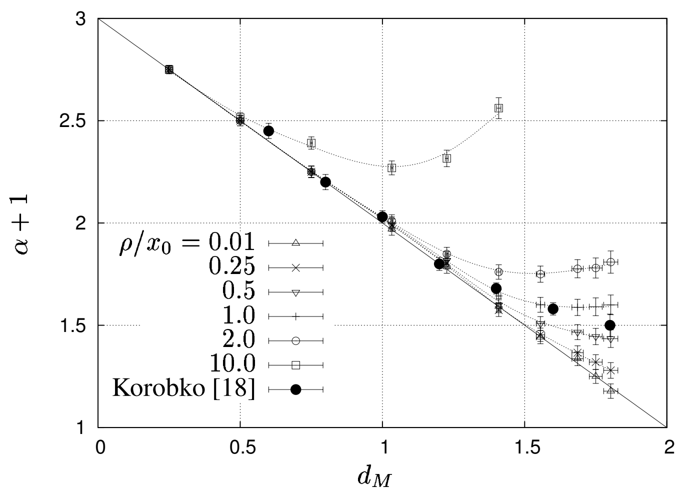

Figure 1. Dependence of α on fractal dimension of the medium under different assumptions on cross-section of particle interaction.

structures of the medium (parameter ρ/x_0) is shown in Fig. 1. In case of small cross-sections the relation $\alpha + 1 = 3 - d_M$, obtained in [15], is confirmed by our calculations. The violation of linear dependence of $\alpha(d_M)$ appears with increasing cross-section as a consequence of finiteness of medium and also overlapping inhomogeneities of the medium. Thus, assuming fractal dimension of the Galaxy as $d_M \sim 1.7$ (see e.g. [16]), we find $\alpha = 0.3$ (see [17] for details).

The other parameters of the model (p, δ, β, D_0) were evaluated from experimental data. We found $p \approx 2.85$, $\delta \approx 0.27$, $D_0 \approx 3 \cdot 10^{-6}\mathrm{pc}^{0.3}\mathrm{yr}^{-0.8}$ (see [9]).

4. Results

Possible candidates of the cosmic ray sources, located within 1 kpc with ages less than $4 \cdot 10^5$ yrs, are presented in Fig. 2. We illustrate the contribution of each source to proton flux in Fig. 2, assuming that the output of protons from each supernova is $Q_p(> 1 \text{ GeV}) = 4 \cdot 10^{49}$erg/SN. We found, that only two SNRs (Loop-I,II) give significant contribution to proton flux in the energy region $(10 \div 10^6)$ TeV. Such a small number of sources contradicts to the available information on the CR anisotropy. Hence, it can be supposed, that SNR are not the sources of CR in the given energy range. Astrophysical aspects of this affirmation will be discussed elsewhere.

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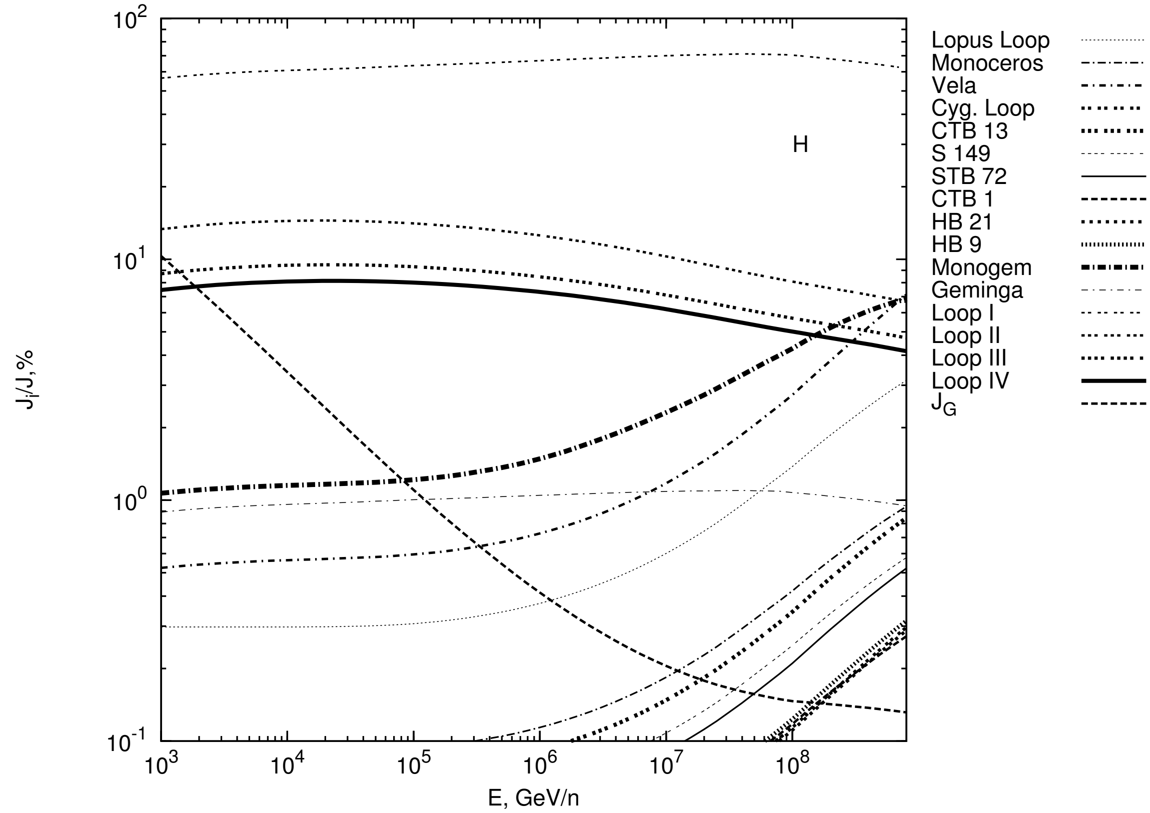

Figure 2. Relative contribution of each nearby SNRs to proton flux near the solar system

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