# On the Estimation of Solar Energetic Particle Injection Timing from Onset Times near Earth 

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A common technique for estimating the start time of solar energetic particle (SEP) injection consists of a linear fit to the observed onset time versus the inverse of particle velocity. This is based on a concept that the first arriving particles move directly along the magnetic field with no scattering. We examine the accuracy of this technique by performing numerical simulations of the transport of solar protons of different energies from the Sun to the Earth, by means of a finite difference method to numerically solve the Boltzmann equation. We then analyze the results using the inverse velocity fit. We find that in most cases, the onset times align close to a straight line as a function of inverse velocity. Despite this, the estimated injection time can be in error by several minutes. Also, the estimated path length can deviate greatly from the actual path length along the interplanetary magnetic field. The major difference between the estimated and actual path lengths implies that the first arriving particles cannot be viewed as moving directly along the interplanetary magnetic field.

## 1. Introduction

An important issue when studying solar events is the exact time when SEPs are first released from the Sun or its vicinity, $t_{0}$. When inferring this, one has to take into account the many different processes acting on the SEPs from their release until the time of detection, $t_{\text {onset }}$, at spacecraft or Earth-based instruments. These include the finite duration of injection at the Sun, the streaming along the interplanetary (IP) magnetic field, the pitch-angle scattering due to resonant interactions with magnetic field irregularities, and effects due to the solar wind speed, such as convection and adiabatic deceleration. These various effects can be taken into account to precisely determine the start time of injection at the Sun [1].
A popular approximation is to consider that the first observed SEPs move approximately parallel to the mean magnetic field. By doing this one neglects the effects of IP scattering at onset. Then, combining measurements at different energies, both the injection time and the path length travelled by the SEPs (typically interpreted as distance along the magnetic field from the Sun to the Earth) are estimated from a fit of the detection onset times and inverse velocities to a straight line. This "onset time versus $1 / \beta$ " method has already become a common practice [2], reinforced by the generally good alignment of experimental data along a straight line in this plot.
However, the basic hypothesis of negligible scattering and motion at zero pitch angle is hard to reconcile with the well-established theories of particle transport. Considerable delays in the detected onset can arise both from IP scattering and a finite duration of the particle injection [3]. The onset time can also be affected by other physical processes such as solar wind convection, and also by the technical difficulties in measuring the onset above the pre-event particle background. In this paper we investigate the validity and systematic error of the approximation that the first arriving SEPs have undergone no scattering. We employ state-of-the-art numerical simulations of particle transport, and analyze the resulting onset time versus inverse velocity. We then compare the estimated start time of injection at the Sun and path length with those actually used in the simulation to estimate the systematic error in the estimated values.

## 2. Numerical experiments

We describe the propagation of protons ensuing from a solar event by numerically solving a Fokker-Planck equation of pitch-angle transport that includes the effects of IP scattering, adiabatic deceleration and solar wind convection [4]. We assume transport along the mean magnetic field, as expected when there is good magnetic connection between the source and the observer. We consider an initial injection of SEPs into the IP medium from the proximity of the Sun, with zero pitch angle. Starting from this, we construct the time profiles for the intensity at the position of Earth's orbit, for seven values of the rigidity $P$, corresponding to proton kinetic energies of $E=2,6,20,60,200,600$, and 2000 MeV . It has been argued that the following results apply equally well to electrons of the same velocity range ( $E \approx 1 \mathrm{keV}$ to 1 MeV ) [5].
We consider three different cases of rate of SEP injection: an impulsive injection (a delta function in time) at $t=t_{0}$, a triangular profile with the same full width at half maximum (FWHM) for every value of $P$ (we take FWHM $=12 \mathrm{~min}$ ), and a triangular profile with different FWHM for each value of $P$. As deconvolution techniques have shown that the injection duration tends to decrease with increasing rigidity [6], for this third case we use values of the FWHM as $75,50,30,20,12,8$, and 4.8 min ., for the seven energy values listed above. Note that in all three cases, the injection is taken to start simultaneously for every rigidity.
For typical conditions of IP turbulence, the diffusive component of particle transport is characterized by some value of the radial mean free path $\lambda_{r}$ on the order of 0.1 to 1 AU . We take into account the variability in $\lambda_{r}$ by running our simulations with different assumptions for its value: a low constant value $\lambda_{r}=0.2 \mathrm{AU}$, a high constant value $\lambda_{r}=1.0 \mathrm{AU}$, or a value depending on rigidity, $\lambda_{r} \propto P^{\alpha}$. We explore both positive and negative values of the exponent, $\alpha=-1 / 3,1 / 3$ and 1 (in addition to the assumptions of constant $\lambda_{r}$, that we can express as cases with $\alpha=0$ ). In the rigidity-dependent cases, we fix the value of $\lambda_{r}$ for our lowest energy, $\lambda_{0}$, in such a way that the values for the other energies will roughly remain inside the range $0.2-1.0 \mathrm{AU}$.

For each particle energy, we define the time of detected onset, $t_{\text {onset }}$, from the simulated time profiles at Earth orbit, as the moment when these profiles surpass a certain threshold value, which we take to be a constant fraction of the maximum intensity at each energy value. We explore the cases in which the onset times are determined by assuming a low threshold, corresponding to a constant fraction of $0.01 \%$ of the peak, a medium threshold, at $2 \%$ of the peak, or a high threshold, at $60 \%$ of the peak.
The estimation technique examined in this paper assumes that the first detection of protons at a given energy occurs for those protons arriving at their maximum velocity $v$ along the field line, which would be consistent with the supposition that particles with roughly zero pitch angle would suffer negligible scattering. If this holds, then $t_{\text {onset }}$ as a function of the particle velocity follows the simple relation

$$
\begin{equation*}
t_{\mathrm{onset}}=t_{\mathrm{inj}}+\frac{s}{v} \tag{1}
\end{equation*}
$$

where $t_{\mathrm{inj}}$ is the time when the injection starts and $s$ is the path length travelled by the particles. These last two parameters can be tuned to fit the data points by equation (1), a straight line in the $t_{\text {onset }}$ vs. $1 / v$ plot.
The results of the inverse velocity fits for all the cases studied in this paper are plotted in Figure 1. As can be seen, in most of the cases the fits yield an injection time that is correct to within a few minutes and a high value of the path length. All the simulations are well fit by straight lines in $t_{\text {onset }}$ versus $1 / v$. However, some of these apparently good fits yield path lengths over 2 times too long or injection time errors of over 10 minutes. Thus the goodness of fit alone does not validate the results or assumptions of the inverse velocity fits. Even in some of the best fits, the parameters differ considerably from the correct values, especially the path length. This means that for typical values of the solar wind speed, the IP turbulence effects on the transport of particles of different energies can produce onset times near Earth that align extremely well in the inverse velocity plot, but whose fit leads to a value of the path length $20-50 \%$ longer than the actual length of the field line.


Figure 1. Times of injection vs. path lengths estimated from the inverse velocity fits in the different cases studied. Left panels: results of the low threshold assumption ( $0.01 \%$ of the peak). Center panels: results of the medium threshold assumption ( $2 \%$ of the peak). Right panels: results of the high threshold assumption ( $60 \%$ of the peak). Top panels: impulsive injection. Middle panels: extended injection with constant absolute width ( 12 min ). Bottom panels: extended injection with width depending on particle rigidity (see text). Different symbols denote different mean free path assumptions: crosses: $\lambda_{0}=1.0 \mathrm{AU}, \alpha=-1 / 3$; triangles: $\lambda_{0}=1.0 \mathrm{AU}, \alpha=0$; squares: $\lambda_{0}=0.2 \mathrm{AU}, \alpha=0$; stars: $\lambda_{0}=0.2 \mathrm{AU}, \alpha=1 / 3$; diamonds: $\lambda_{0}=0.2 \mathrm{AU}, \alpha=1$. The intersection of dashed lines indicates the actual values of the start time of injection and path length used in performing all the simulations. From [5].

## 3. Discussion and Conclusions

The results of the pitch-angle transport simulations are not compatible with the assumption of no scattering for the first detected particles. The onset time for each energy is always influenced by IP scattering, the duration of injection, and solar wind convection, each of which may have a different relative importance for different energies. By combining the onset times at different energies in the inverse-velocity plot, a simple fit leads to an estimation of the start time of the injection, $t_{\mathrm{inj}}$, and the path length, $s$. Often a good linear fit is obtained fortuitously, even when the fit parameters deviate substantially from the true values. Thus the goodness of the fit should not be taken as an indication that the estimated values or underlying assumptions are correct.

Our results are comparable to previous investigations on the validity of the inverse velocity method [7]. While these authors found some cases with a major error in the timing estimation (even of hours), this was never the case when they considered only proton energies greater than 1 MeV . We do not find deviations in the timing estimation larger than several minutes in our results for protons of 2 MeV to 2 GeV .

In contrast, the path lengths obtained from the linear fits are frequently very different from the actual path length along the local IP magnetic field line, and always larger. Thus, the first arriving particles cannot be viewed as moving directly along the IP magnetic field, with $v_{z}=v$ and zero pitch angle. Empirically, a better assumption would be that the first arriving particles travel with an energy-independent value of the pitch angle cosine, $\mu=v_{z} / v$, that is less than unity. Indeed, this is a feature of the coherent pulse concept of focused transport theory [8] for a scattering mean free path that depends only weakly on energy. For a path length $z$, the "onset" particles empirically arrive roughly at time

$$
\begin{equation*}
t_{\mathrm{onset}}=t_{\mathrm{inj}}+\frac{z}{v_{z}}=t_{\mathrm{inj}}+\frac{s}{v} \tag{2}
\end{equation*}
$$

where $s=z / \mu$ for an unknown value of $\mu$ that is apparently roughly constant with energy. The coherent pulse concept can help explain why the inverse velocity fits provide estimates of $s$ that do not represent the actual path length, but rather the path length magnified by some factor $(1 / \mu)$, and estimates of the start time of injection that are accurate to the order of several minutes.

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