物理学会シンポジウム
重力波宇宙論の展望
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原始ブラックホールと重力波

Primordial Black Holes and Gravitational Waves

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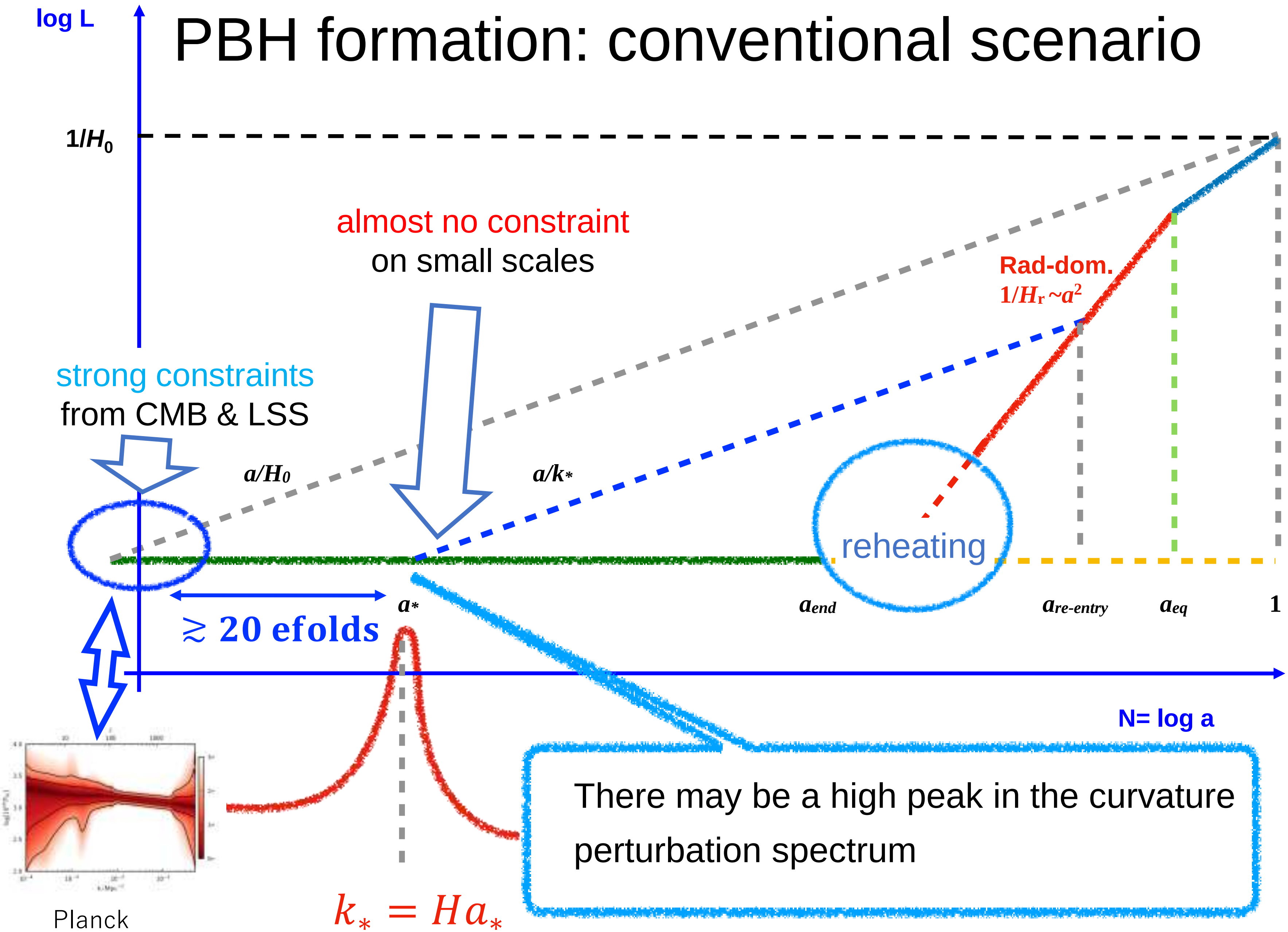
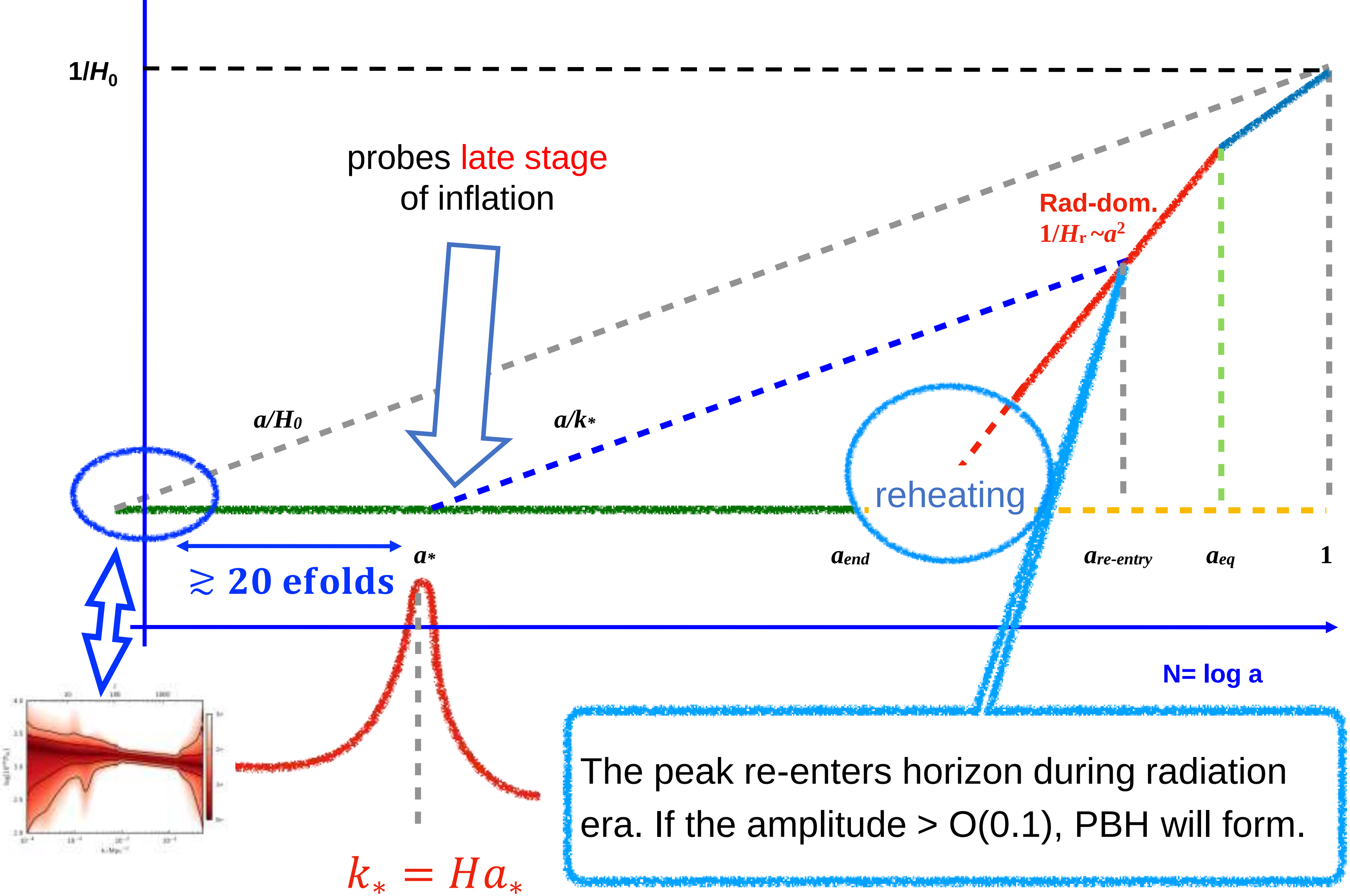
Primordial Black Holes

PBH formation: conventional scenario



PBH formation: conventional scenario

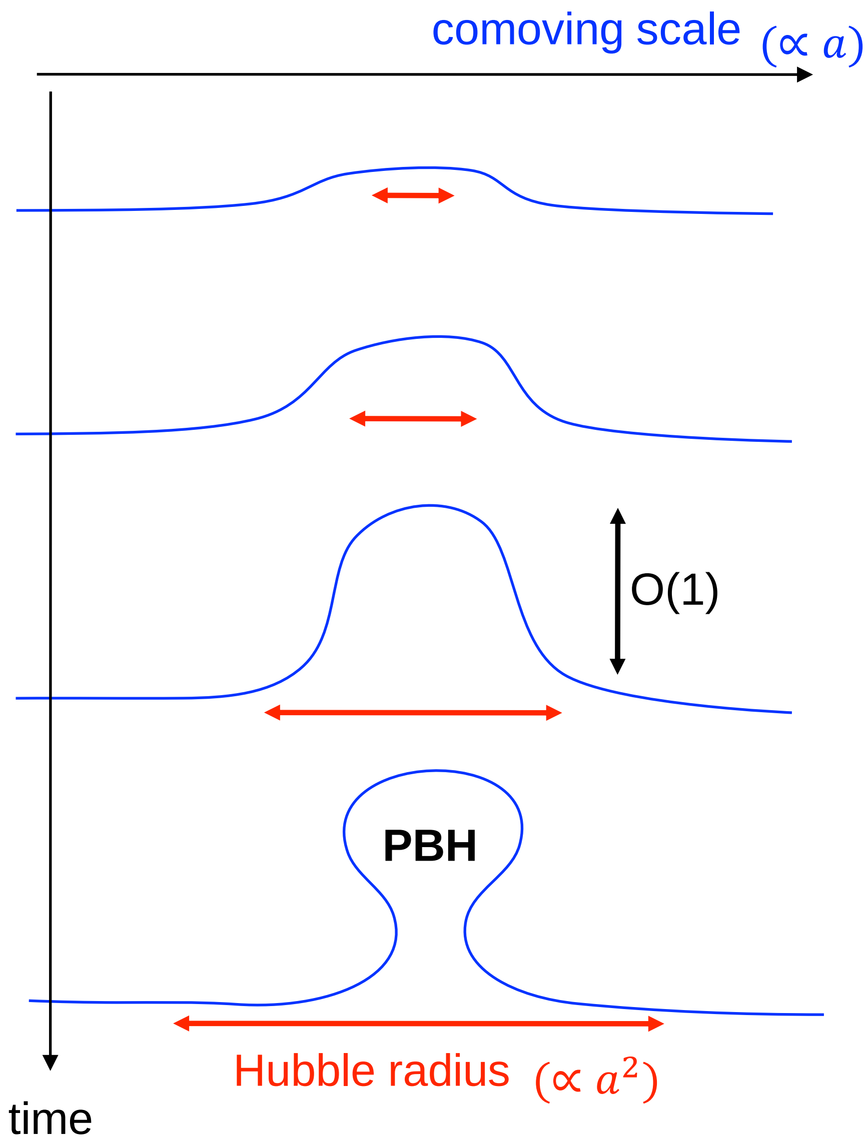


PBH in a nutshell

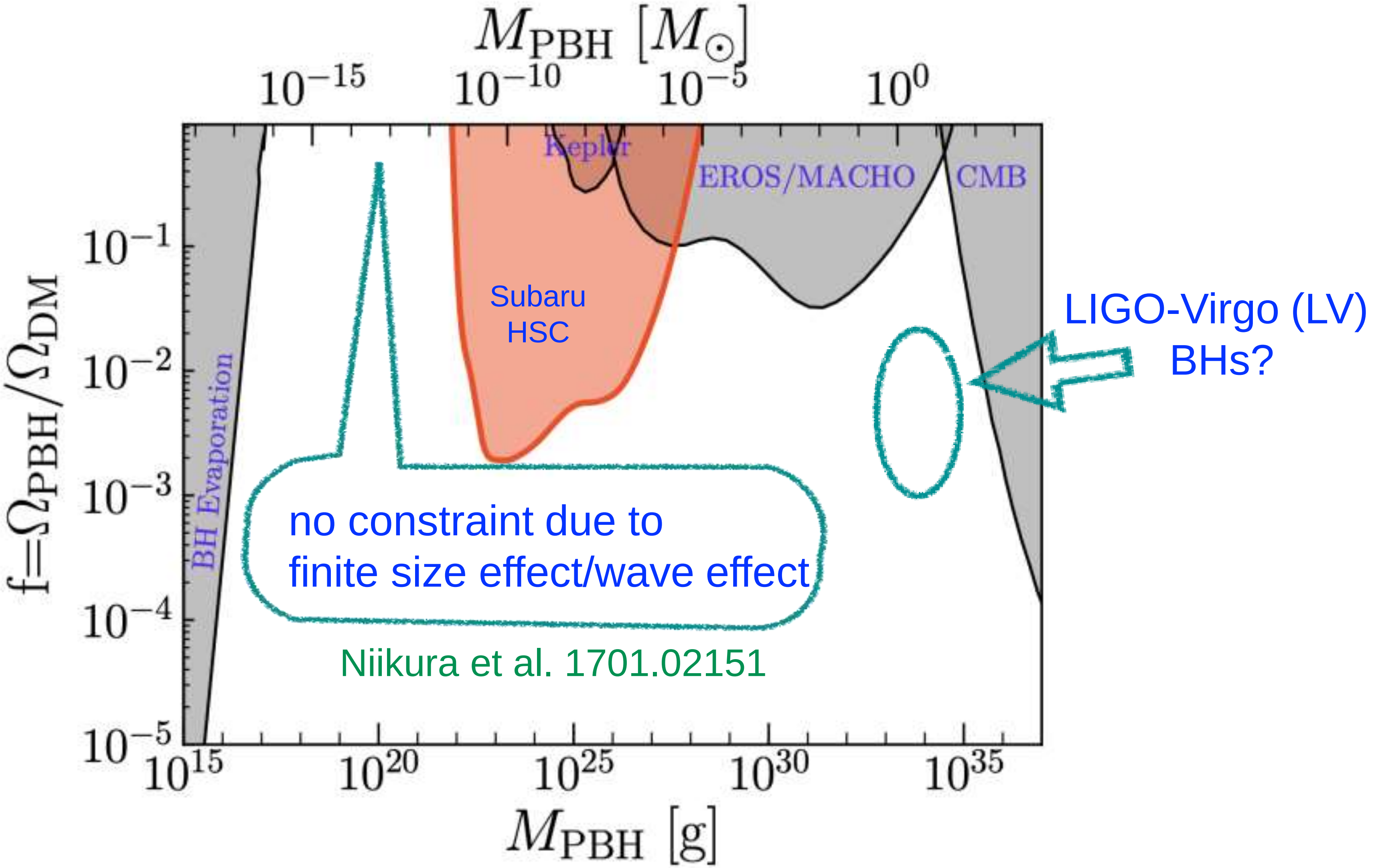
- Primordial Black Holes (PBHs) are those formed in the very early universe, conventionally when the universe was radiation-dominated.
- Presumably they originate from a large positive curvature perturbation produced during inflation (which hence should be a rare event).
- For a BH to form during radiation dominance, the perturbation must be O(1) on the Hubble horizon scale.

$$\boldsymbol{M_{\mathrm{PBH}} \sim M_{\mathrm{horizon}}}$$

$$\sim \left(\frac{100\mathrm{MeV}}{T}\right)^2 M_\odot \sim \left(\frac{\ell}{1\mathrm{pc}}\right)^2 M_\odot$$



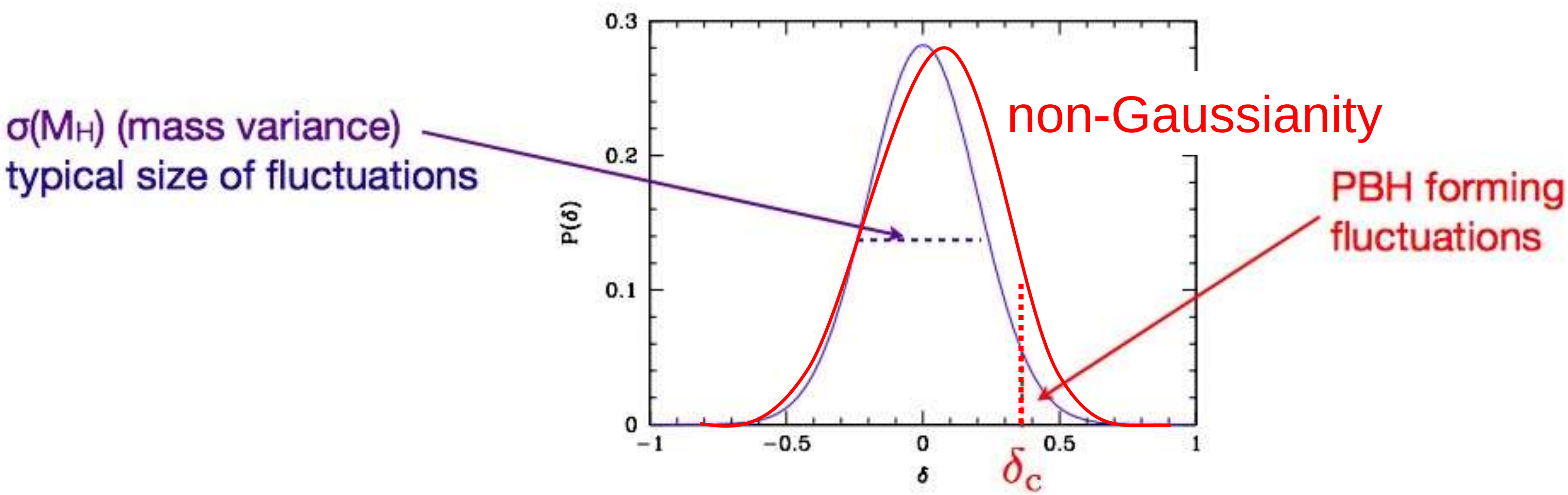
observational constraints



big window at $M_{\rm PBH} \approx 10^{17} - 10^{22}$g ⟷ $T_{\text{re-entry}} \sim 10^4$ -10^8 GeV

fraction β that turns into PBHs

for Gaussian probability distribution



- When $\sigma_M \ll \delta_c$, β can be approximated by exponential:

$$\beta \approx \sqrt{\frac{2}{\pi}}\frac{\sigma_M}{\delta_c}\exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right) \qquad \delta_c \equiv \left(\frac{\delta\rho_c}{\rho}\right)_{\rm crit} \sim 0.4$$

Carr, ApJ 201, 1 (1975), …

- Recent studies indicates enhanced production: $\delta_c \sim 0.2$

$\underbrace{\text{}}_{\text{using peak theory}}$

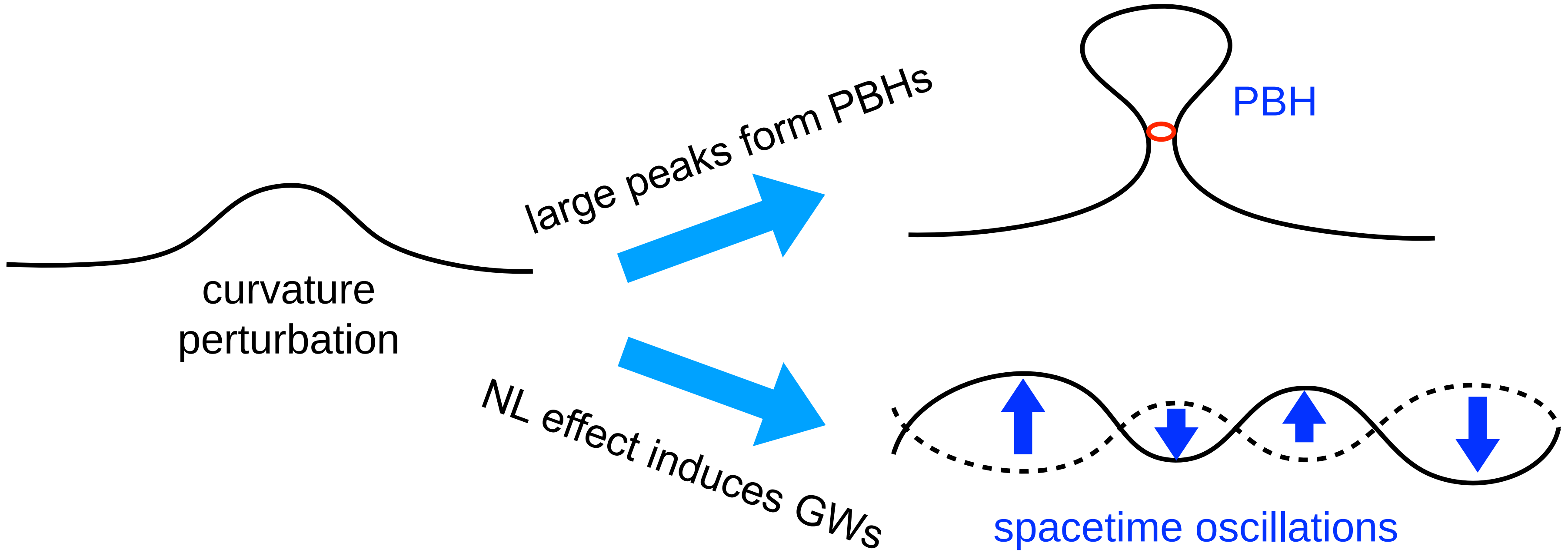
Yoo, Harada, Garriga & Kohri, 1805.03946

- Non-Gaussianity may significantly affect β

induced GWs

(iGWs)

GWs can capture PBHs!



PBHs = CDM with M_{PBH} ~10^{21}g
generates iGWs with f~10^{-3} Hz

Background GWs
at LISA band

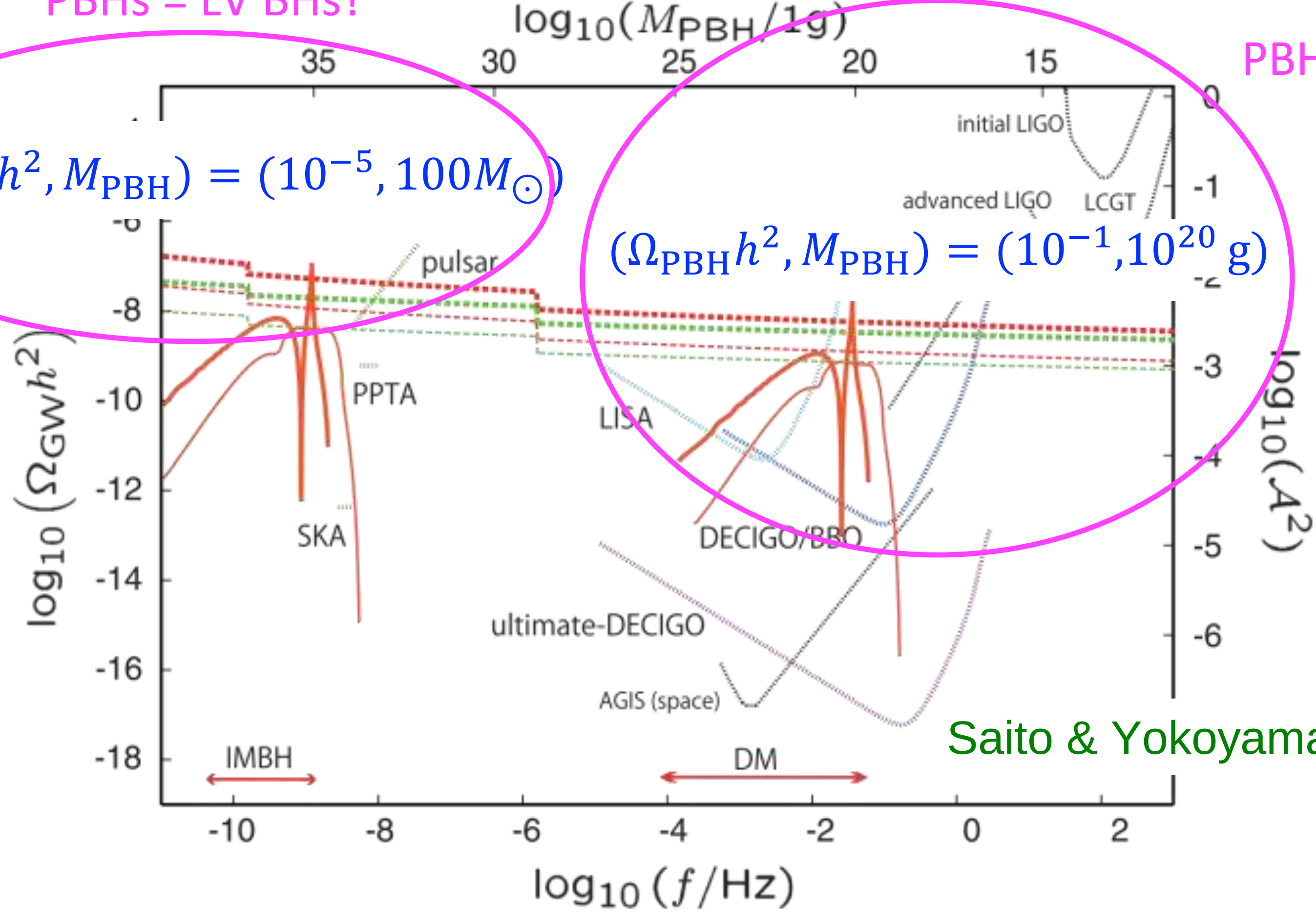
PBHs =LV BHs with M_{PBH} ~$10M_\odot$
generates iGWs with f~10^{-8} Hz

Background GWs
at PTA band

GWs can test PBH scenario!

PBHs = LV BHs?

PBHs = CDM?



$(\Omega_{\rm PBH}h^2, M_{\rm PBH}) = (10^{-5}, 100M_\odot)$

$(\Omega_{\rm PBH}h^2, M_{\rm PBH}) = (10^{-1}, 10^{20}\,{\rm g})$

Saito & Yokoyama 0812.4339

- PBHs =LV BHs scenario is already constrained by NANOGrav(PTA)

Cai, Pi, Wang & Yang 1907.06372

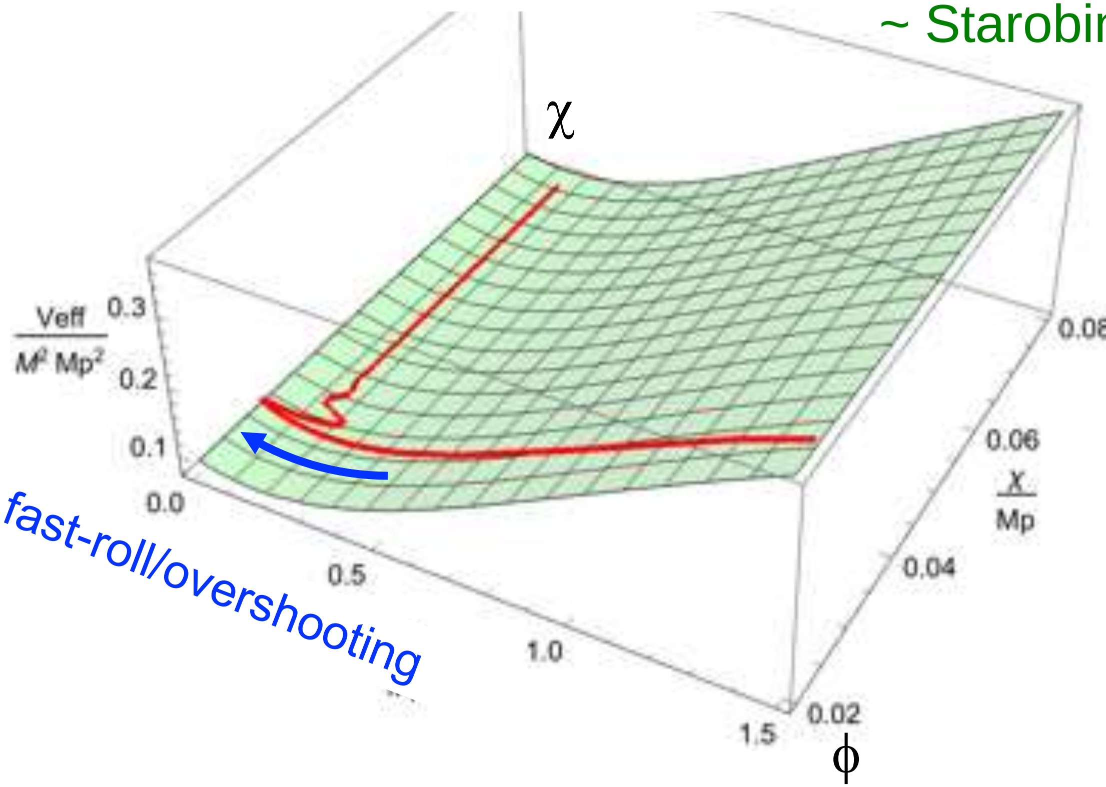
Inflation models

(1) Two-field inflation model

Pi, Zhang, Huang & MS, 1712.09896

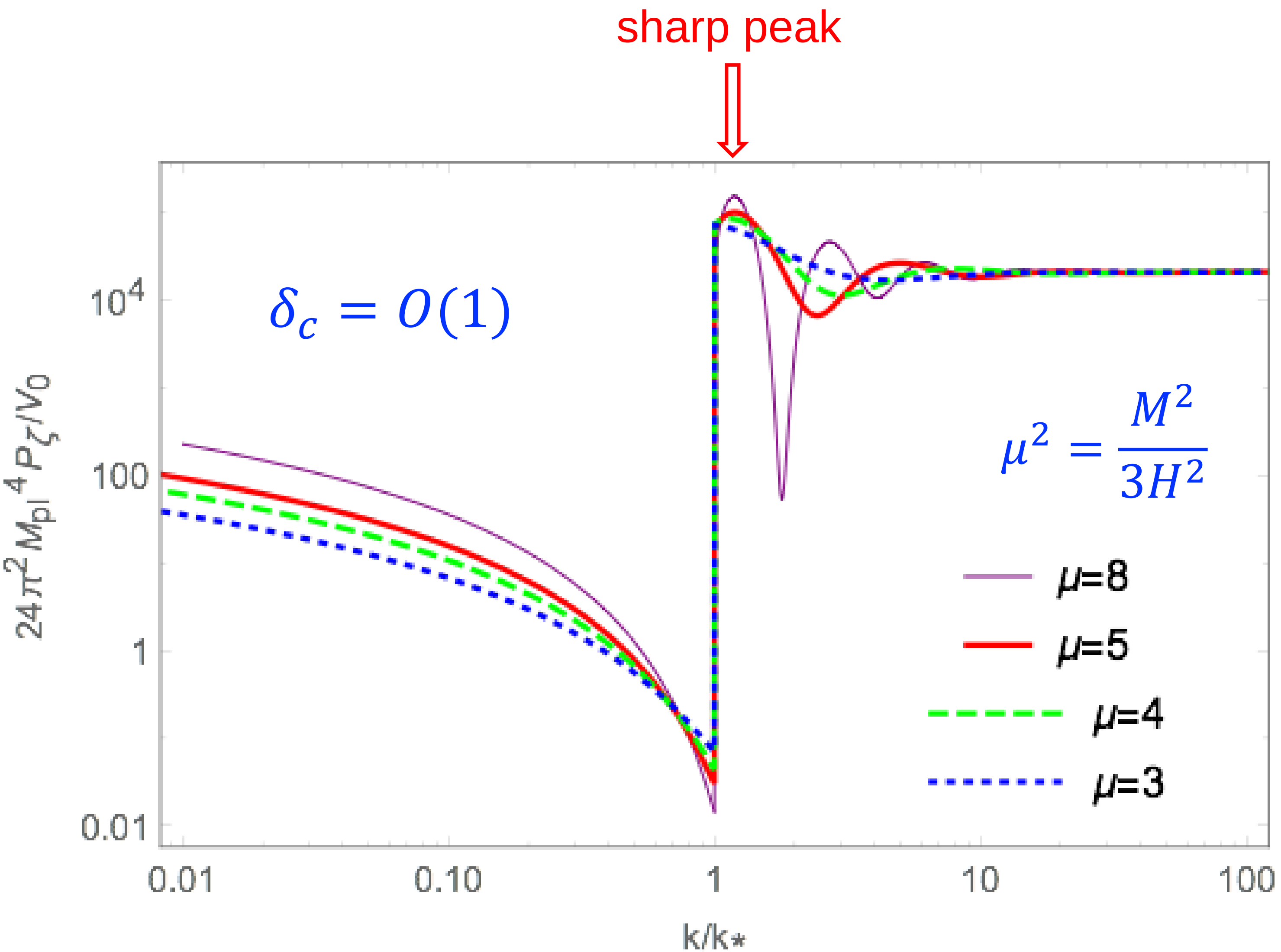
$$S_J = \int d^4x\sqrt{-g}\left\{\frac{M_{\rm Pl}^2}{2}\left(R+\frac{R^2}{6M^2}\right)-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi)-\frac{1}{2}\xi R\chi^2\right\}.$$

~ Starobinsky (scalaron) + curvaton



many 2-stage models can account for PBH formation

- Scalaron ϕ becomes massive at the end of the 1st stage.
- Field χ plays the role of inflaton at the 2nd stage.



scalaron + x model can produce a sharp peak in the curvature perturbation spectrum at small scale

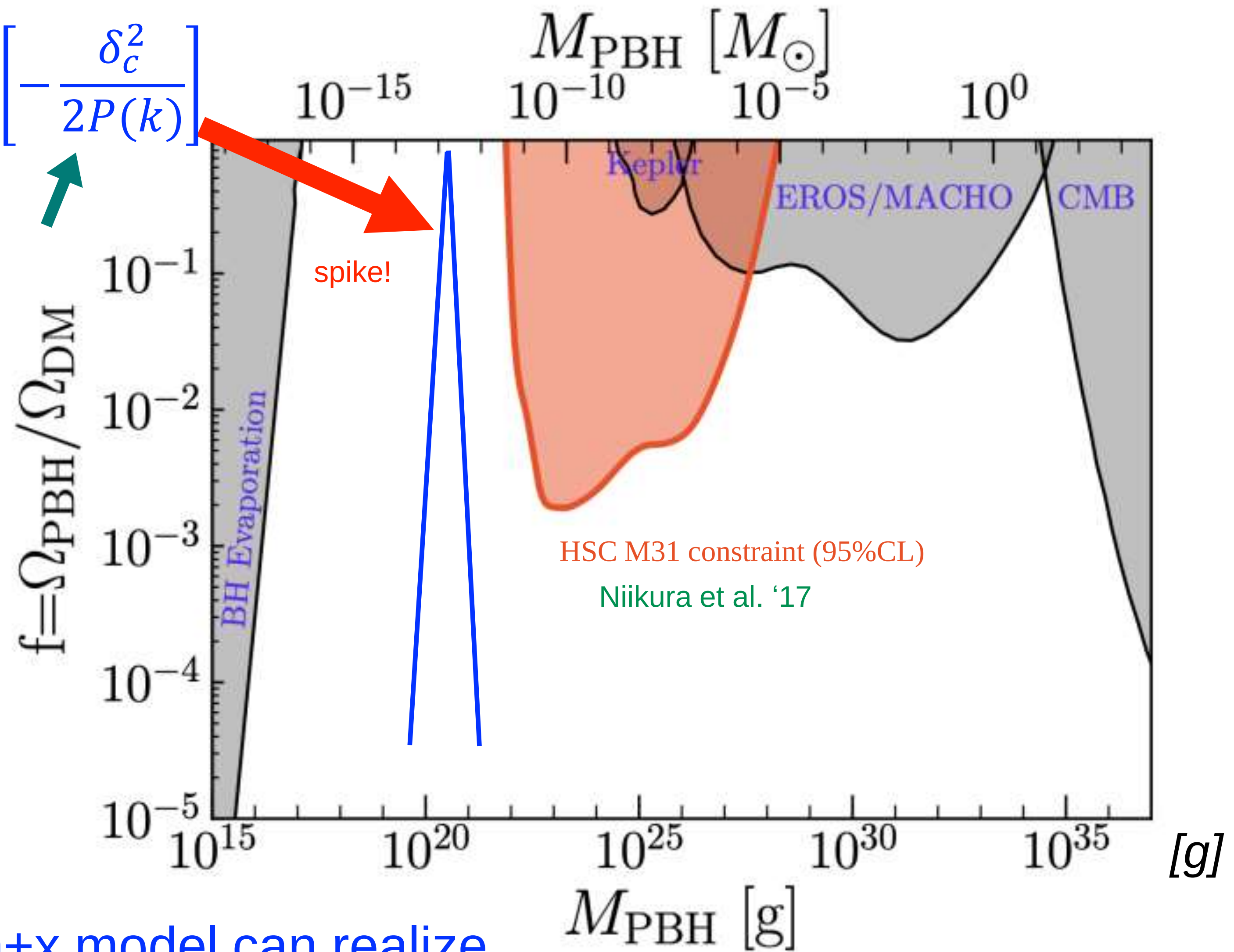
(local) non-Gaussianity is usually small in this kind of models

PBH mass function

$$\delta_c = O(1)$$

$$f(M_{\rm PBH}) \propto \exp\left[-\frac{\delta_c^2}{2P(k)}\right]$$

sharp peak



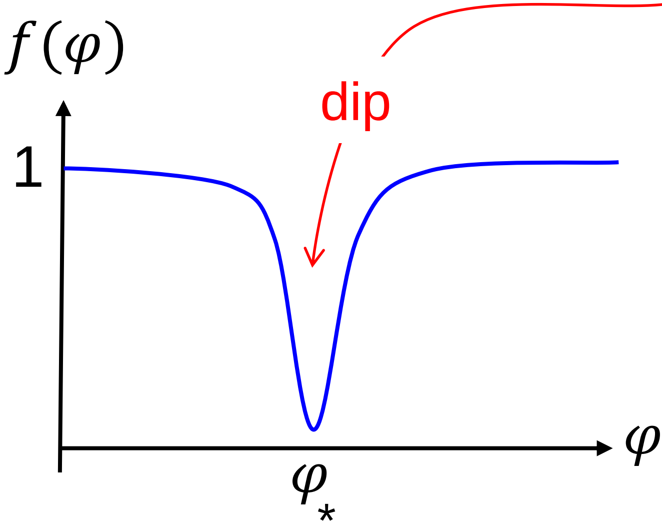
scalaron+x model can realize

PBH=CDM scenario with a monochromatic PBH mass!

(2) Non-minimal curvaton model

Pi & MS, 2112.12680

$$\mathscr{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi) - \frac{1}{2}\underline{f(\varphi)}^2(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 \quad (m_\chi^2 \ll H^2)$$

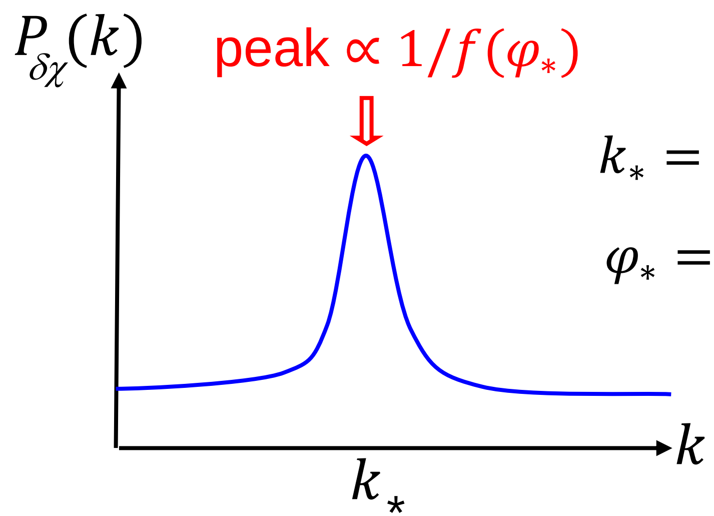


- assume f <<1 when $\varphi \sim \varphi_*$
- vacuum fluctuation: $f(\varphi)\delta x = \dfrac{H}{2\pi}$

$\Rightarrow$ $\delta x = \dfrac{H}{2\pi f}$

- $\delta\chi$ is enhanced at $\varphi = \varphi_*$

$\Rightarrow$ leads to PBH formation



$k_* = Ha(t_*)$

$\varphi_* = \varphi(t_*)$

Fully non-Gaussian curvature perturbation

$$\boxed{e^{4\zeta}-\left[\frac{4r}{3+r}\left(1+\frac{\delta\chi}{\chi}\right)^2\right]e^{\zeta}+\left[\frac{3r-3}{3+r}\right]=0}$$

MS, Valiviita & Wands, astro-ph/0607627

ζ = curvature perturbation on uniform density slices

$r = \rho_\chi/\rho_{\rm tot}$ at epoch of curvaton decay

- PBH formation criterion is fully non-Gaussian:
  criterion $\zeta > \zeta_{cr}$~0.5 gives a highly nonlinear expression in $\delta\chi/\chi$

$$\mathrm{Prob}_\Delta(\Delta)d\Delta = \mathrm{Prob}_\zeta[\zeta]d\zeta = \mathrm{Prob}_{\delta\chi}[\delta\chi]d(\delta\chi)$$

$\Delta(\mathrm{R})$: $\delta\rho/\rho$ smoothed over comoving scale R

$$\boxed{\Delta(\mathrm{R}) \approx \frac{4k_*^2R^2}{9}\zeta(R)}$$

for spectrum peaked at $k=k_*$

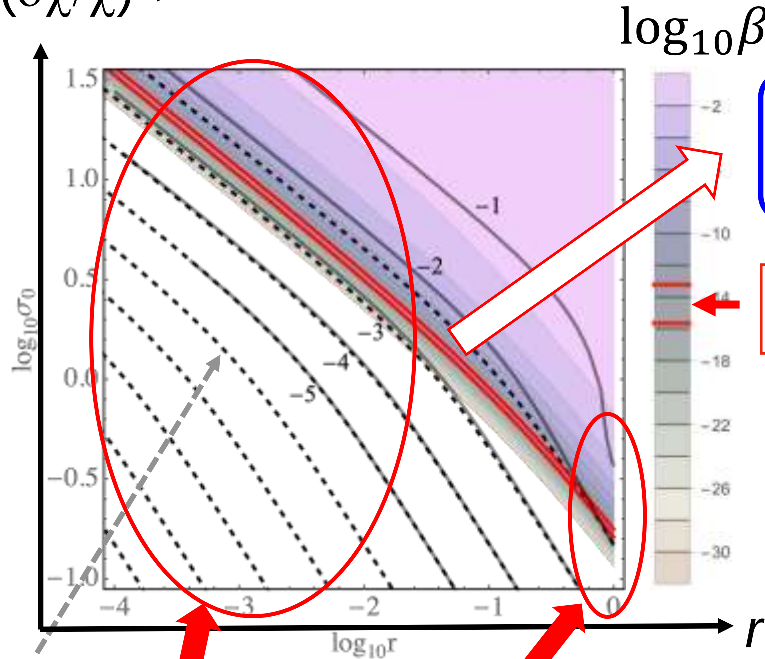
- Yet, power spectrum is well approximated by that of $\delta\chi^2$ for r <<1, while by that of $\log(1+\delta\chi/\chi)$ for r ~1.

$$P_{\delta\chi}(k_*) \nearrow \quad \zeta = \frac{\delta\chi}{\chi}+\frac{3}{4r}\left(\frac{\delta\chi}{\chi}\right)^2 \quad \text{for r<<1} \searrow$$

$$P_{\delta\chi}(k_*) \searrow \quad \zeta = \frac{1}{3}\ln\left(1+\frac{\delta\chi}{\chi}\right)^2 \quad \text{for r~1} \nearrow \quad P_\zeta(k_*)\left(\approx P_{\mathcal{R}}(k_*)\right)$$

$<(\zeta-<\zeta>)^2>$ & β_{tot} contours and Ω_{GW}

$<(\delta\chi/\chi)^2>$



$$P_\zeta(k_*) \gtrsim 3\times 10^{-3}$$

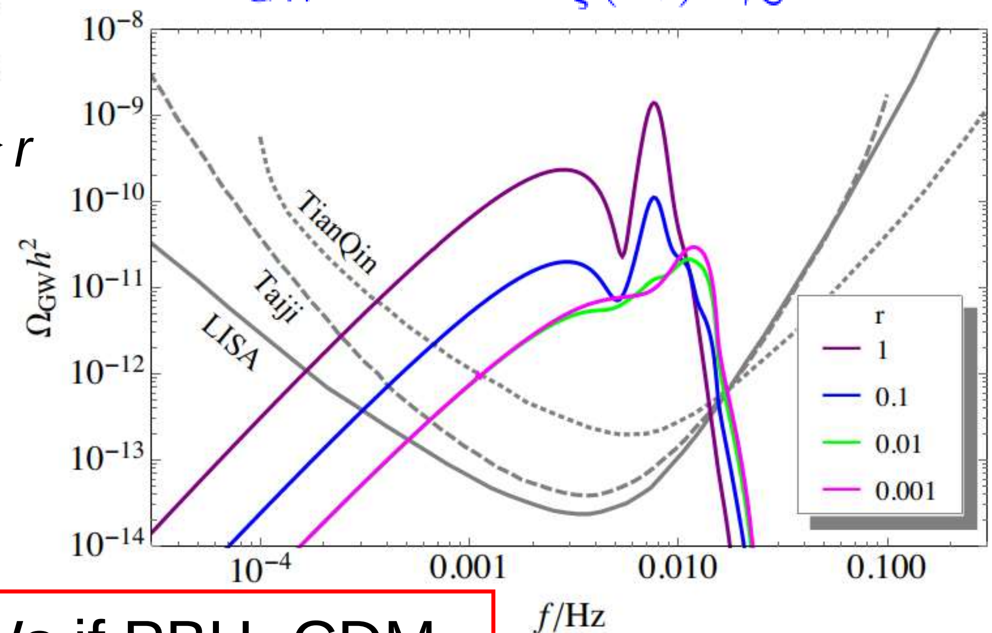
PBH as DM

quadratic approx.

$$\zeta = \frac{1}{3}\ln\left(1+\frac{\delta\chi}{\chi}\right)^2$$

$$\zeta = \frac{\delta\chi}{\chi} + \frac{3}{4r}\left(\frac{\delta\chi}{\chi}\right)^2$$

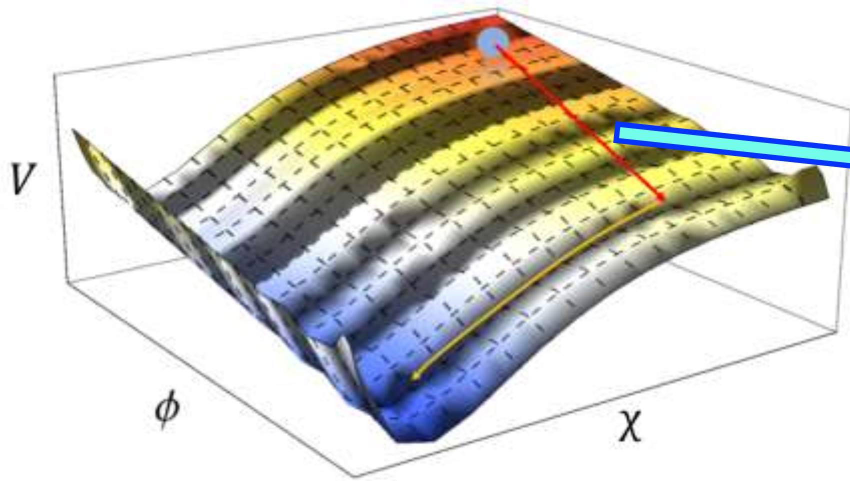
$$\Omega_{GW} \sim 10^{-6}P_\zeta(k_*)^2 \gtrsim 10^{-11}$$

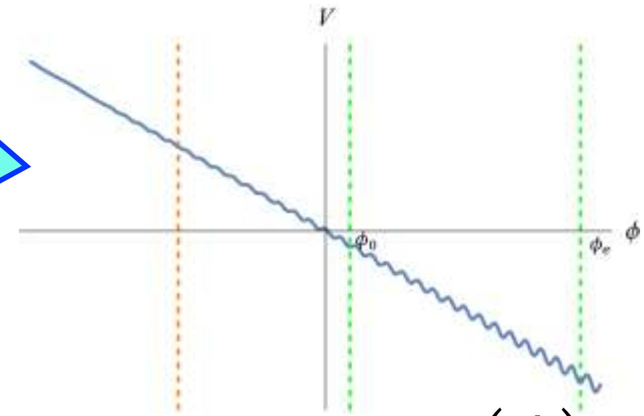


LISA will detect iGWs if PBH=CDM

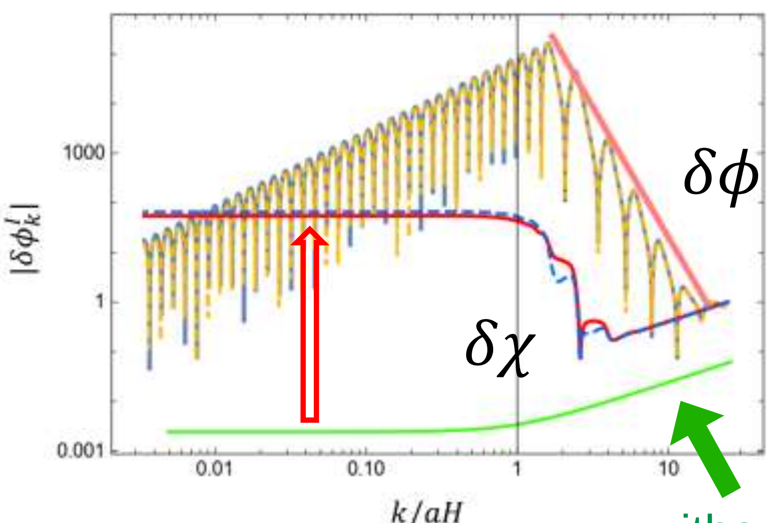
(3) Resonant Amplification Model

Z. Zhou, J. Jiang. Y-f. Cai, MS & S. Pi, 2010.03537





$$\boldsymbol{V(\phi) \sim \Lambda(\phi)} \cos\left(\frac{\phi}{f_a}\right)$$



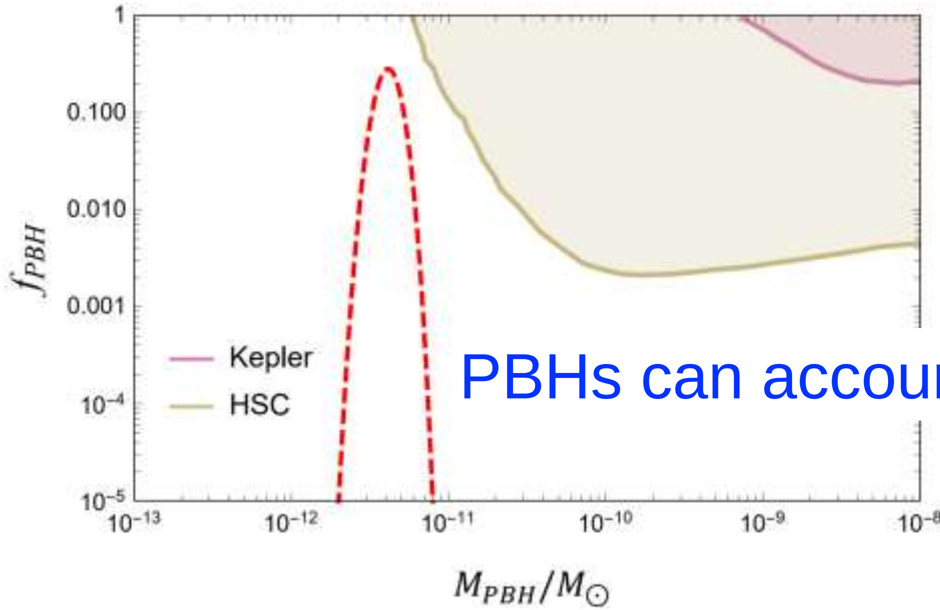
without amplification

$\delta\phi$: amplified by oscillating potential

$\delta\chi$: amplified through coupling to $\delta\phi$

This leads to enhanced curvature perturbation

Curvature pertn, PBH mass fcn, Induced GWs



$$P_{\mathcal{R}}(k) = \frac{H^2}{8\pi^2 M_p^2 \epsilon_{\chi\chi}} \mathcal{A}^2(k)$$

$\Delta = O(1)$

$$\mathcal{A}^2(k) = 1 + \mathcal{A}^2(k_*) \exp\left(-\frac{\ln^2(k/k_*)}{2\Delta^2}\right)$$

amplification factor

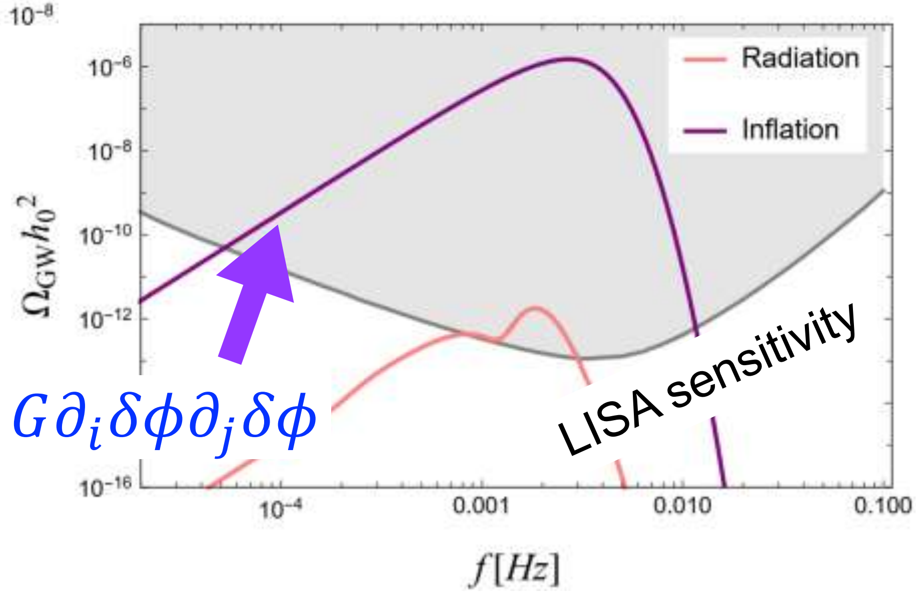
$\sim 10^6$

PBHs can account for CDM

GW probes $\delta\phi$

PBH probes $\delta\chi$

$\Box h_{ij} \sim G\partial_i\delta\phi\partial_j\delta\phi$

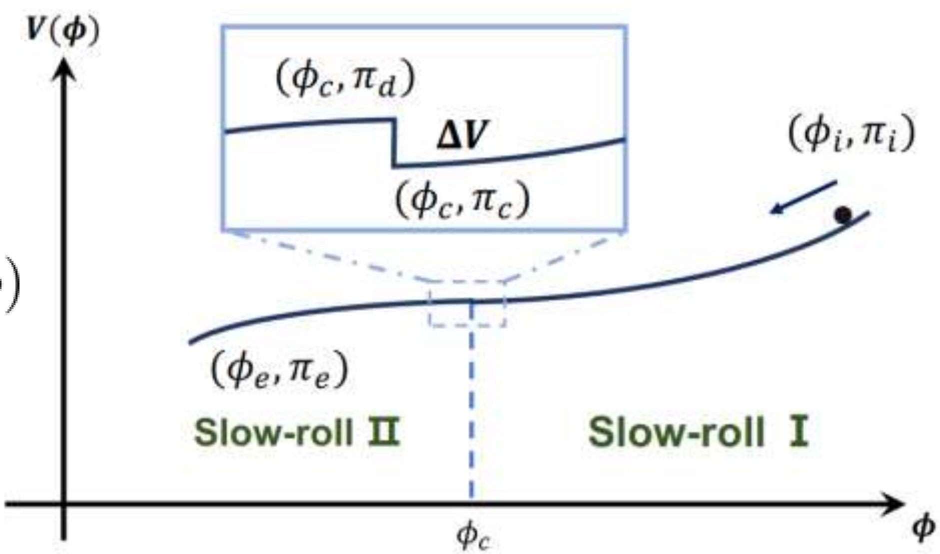


(4) A new mechanism : potential with an upward step

Cai, Ma, MS, Wang & Zhou, 2112.13836

- eqn of motion

$$\begin{aligned} \frac{d\phi}{H dt} &= \pi \\ \frac{d}{dt}(H\pi) &= -3H^2\pi - V'(\phi) \\ 3M_p^2 H^2 &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ &= \frac{1}{2}H^2\pi^2 + V(\phi) \end{aligned}$$



- energy conservation (=continuity of H) at the step

$$\pi_d^2 = \pi_c^2 - \frac{2\Delta V}{H^2} \approx \pi_c^2 - 6\frac{\Delta V}{V} ; \qquad \pi_c \approx -\frac{V'}{3H^2} = -\sqrt{2\epsilon_I}$$

$$\epsilon_I \equiv \frac{V'^2}{2V^2}(\phi_c+)$$

$$\Rightarrow \quad \pi_d = -\sqrt{\pi_c^2 - 6\frac{\Delta V}{V}}$$

(in $M_{\rm Planck} = 1$ units)

curvature perturbation on comoving slices

δN computation (for $\phi > \phi_c$)

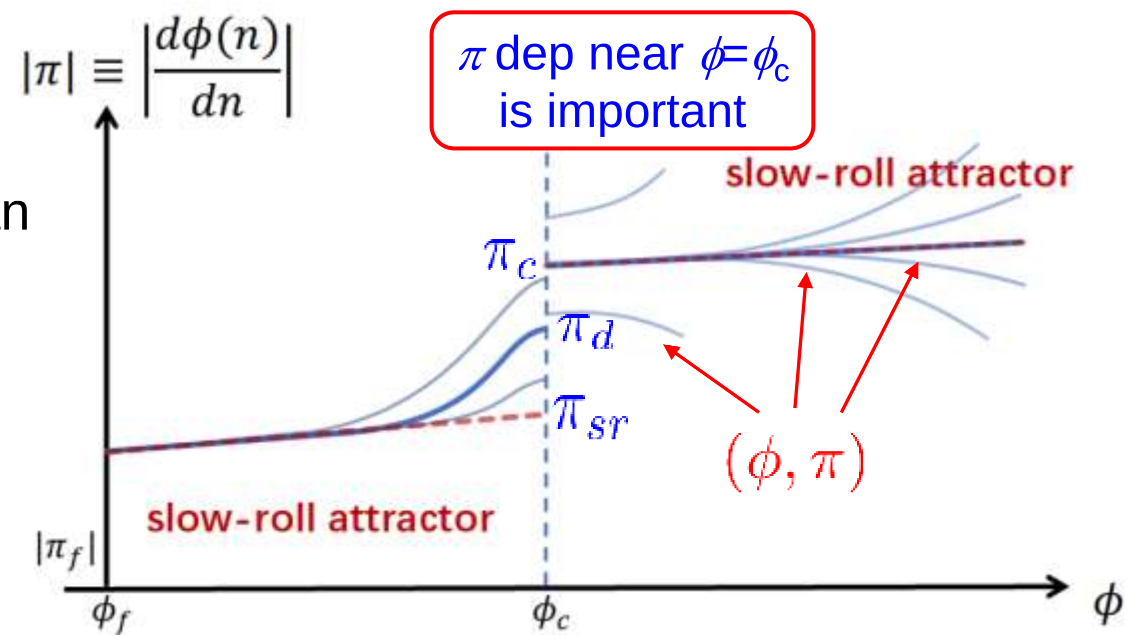
$$N(\phi,\pi) \simeq \frac{\phi-\phi_c}{\sqrt{2\epsilon_I}} + \frac{1}{\eta_{II}}\ln\left(-2\eta_{II}\pi_d(\phi,\pi) - 6\sqrt{2\epsilon_{II}}\right) + const. \qquad \eta \equiv \frac{V''}{V}$$

$$\Rightarrow \delta N = \mathcal{R}_c \simeq \frac{\delta\phi}{\sqrt{2\epsilon_I}} + \frac{2}{|h|}\left(1-\sqrt{1+\frac{2\eta_I}{g}\frac{\delta\phi}{\pi_d}}\right) \qquad h \equiv \frac{6\sqrt{2\epsilon_{II}}}{\pi_d} = -6\frac{\pi_{sr}}{\pi_d}$$

$$\simeq \frac{2}{|h|}\left(1-\sqrt{1-|h|\mathcal{R}_G}\right) \text{ for } g \ll 1 \qquad g \equiv \frac{\pi_d}{\pi_c} = \sqrt{1-6\frac{\Delta V/V}{\pi_c^2}}$$

$$\mathcal{R}_G \equiv -\frac{\eta_I}{g}\frac{\delta\phi}{3\pi_{sr}} \sim \text{Gaussian}$$

enhancement for $g \ll 1$



amplification of power spectrum

$$\boxed{\mathcal{R}_c \simeq \frac{2}{|h|}\left(1-\sqrt{1-|h|\mathcal{R}_G}\right)}$$

$$g = \frac{\pi_d}{\pi_c} \ll 1$$

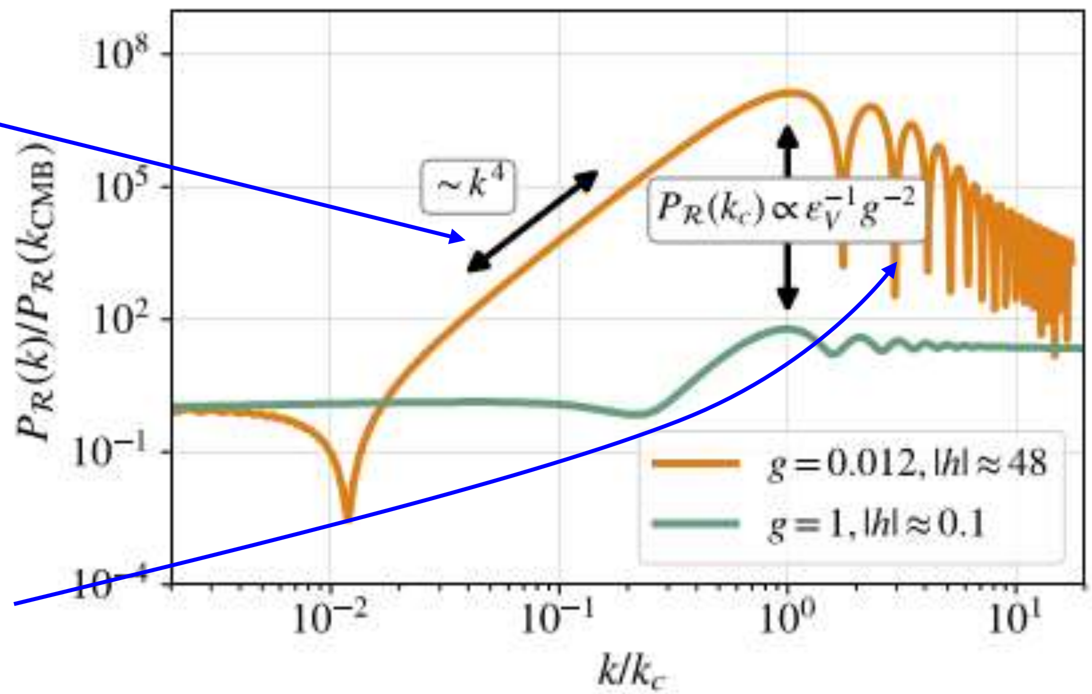
$$\mathcal{R}_G = -\frac{\eta_I}{3\pi_{sr}g}\delta\phi$$

even for a tiny step, $\Delta V << V$, $P_{\mathcal{R}}(k)$ is enhanced by $1/g^2$ if $g << 1$

⇨ may lead to copious PBH formation

superhorizon enhancement due to non-slowroll π

~ ultra slowroll inflation near $\phi = \phi_c$



negative frequency mode mixing inside horizon

non-perturbative non-Gaussianity at tail of distribution

- perturbative non-Gaussianity is small if $-h \equiv \dfrac{6\sqrt{2\varepsilon_V}}{|\pi_d|} \ll 1$

$$\mathcal{R} = \mathcal{R}_G + \frac{|h|}{4}\mathcal{R}_G^2 + \frac{|h|^2}{8}\mathcal{R}_G^3 + \cdots \quad \Longrightarrow \quad \mathcal{P}(k) \approx \mathcal{P}_G(k)$$

power spectrum is given by Gaussian part

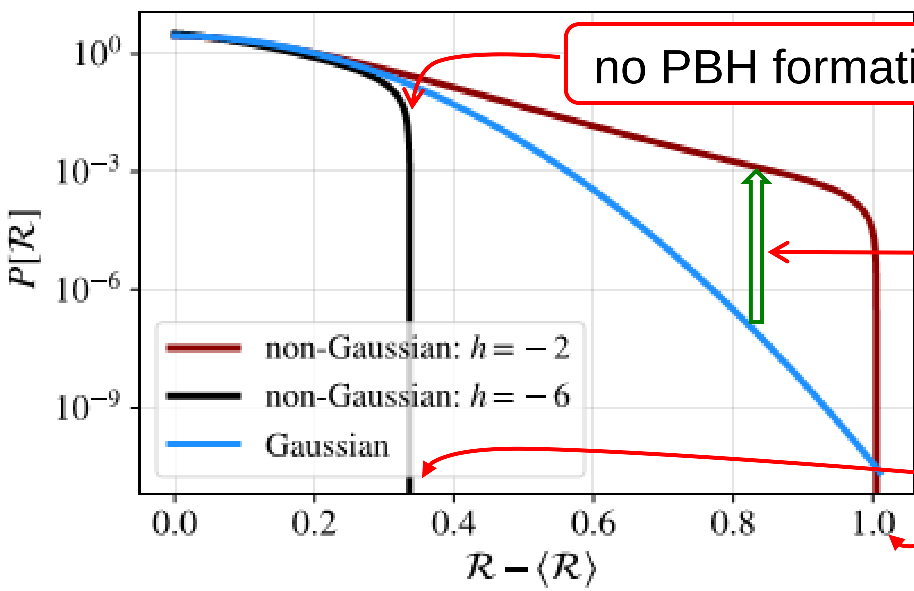
- tail of distribution is extremely non-Gaussian

$$P[\mathcal{R}] = \frac{2 - |h|\mathcal{R}}{\Omega} \exp\left[-\frac{\mathcal{R}^2(4 - |h|\mathcal{R})^2}{32\sigma_{\mathcal{R}}^2} \right]$$

$$\left| \frac{d\mathcal{R}_G}{d\mathcal{R}} \right|$$

$\sigma_{\mathcal{R}}^2 = \mathcal{P}_G(k)\Delta \ln k$ ······· Gaussian power spectrum

$\Omega \equiv \sqrt{2\pi\sigma_{\mathcal{R}}^2}[1 + \mathrm{Erf}(1/(|h|\sqrt{2\sigma_{\mathcal{R}}^2}))]$ ···· normalization



no PBH formation if $\mathcal{R}_{\rm cut} < \mathcal{R}_{\rm crit}$~0.5

exponential enhancement for $|h|$<O(1)

$\mathcal{R}_{\rm cut} \equiv \dfrac{2}{|h|}$ ··· ∃cutoff

Isocurvature

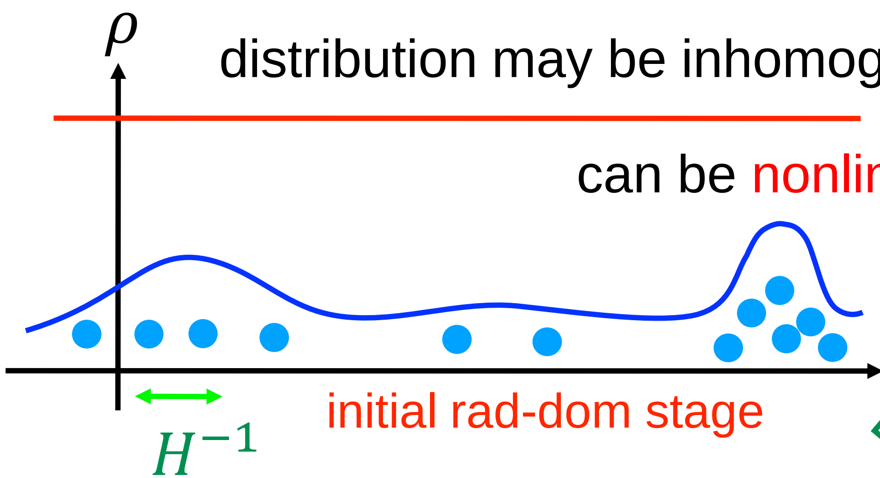
PBHs from Isocurvature Perturbation

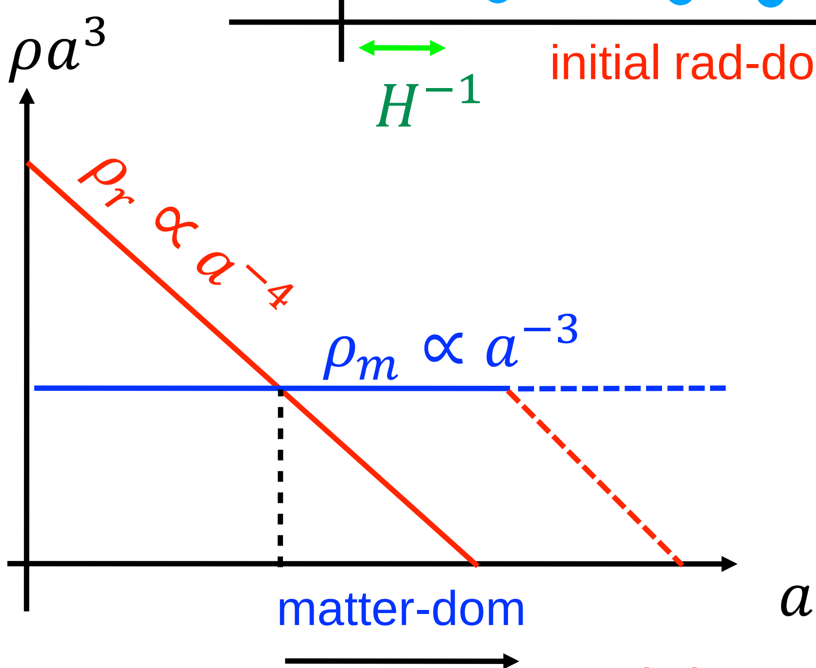
eg, E. Cotner, A. Kusenko, MS & V. Takhistov,1907.10613

non-grav formation of compact objects/Q-balls/etc

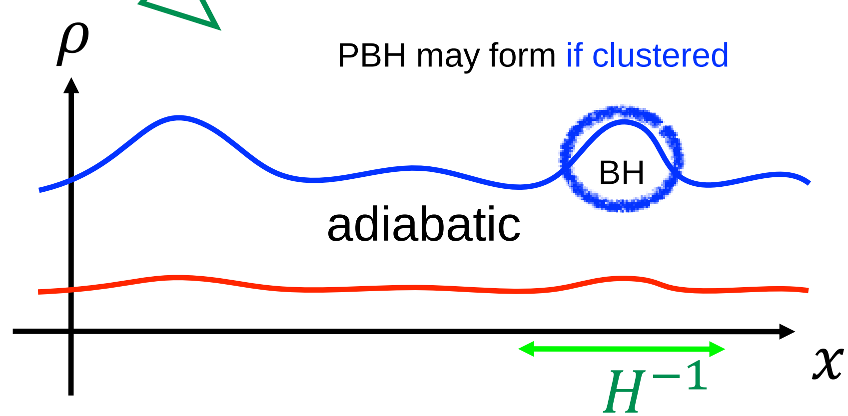
distribution may be inhomogeneous = isocurvature

can be nonlinear from the beginning





PBH may form if clustered



matter-dom stage

rad-dom again if objects decay to radiation

Constraints on CDM isocurvature on small scales

- Putting aside all possible nonlinear corrections, one can derive model-independent constraints on primordial CDM isocurvature perturbation

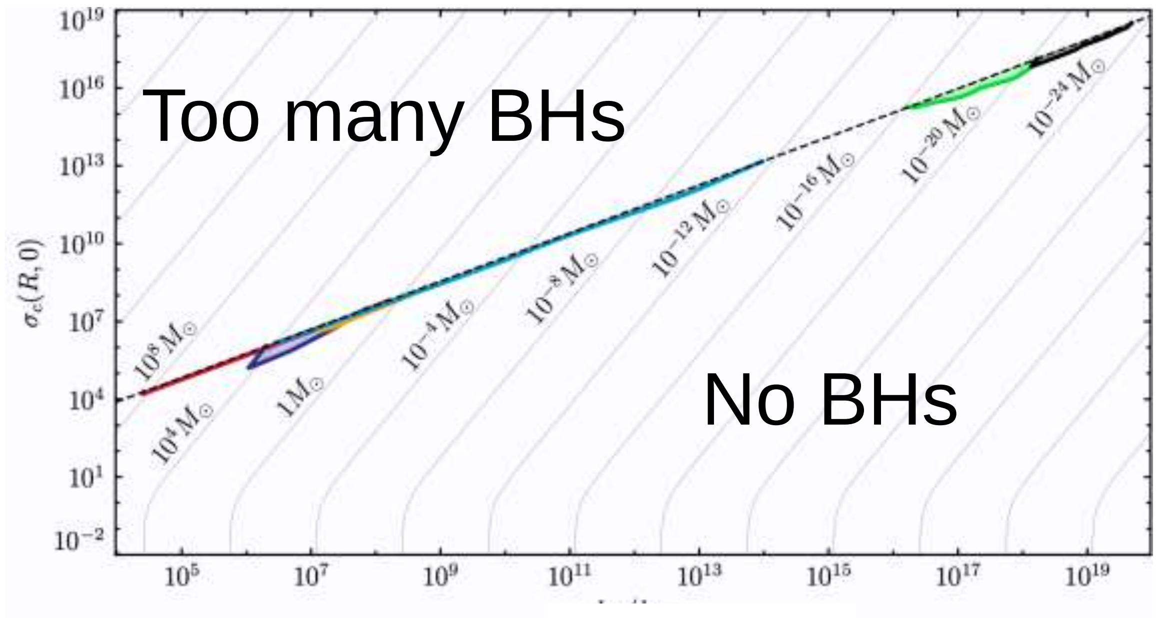
Passaglia & MS, 2109.12824

$\sigma_c(R)$

$$\sigma_c(R) = \sqrt{\langle S^2(R) \rangle}$$

$$S \equiv \delta_{CDM} - \frac{3}{4}\delta_\gamma$$

must be highly non-Gaussian as S>-1



$k_R/k_{\rm eq} \propto 1/R$

- S>>1 perturbations would collapse during radiation-dominance might lead to interesting secondary effects like induced GWs.

Domenech, Passaglia & Renaux-Petel, 2112.10163

take-home message:

- late stage of inflation can be probed by PBHs and the associated secondary/induced GWs
- there are various inflation models for PBH formation with specific testable predictions
- (nonlinear) isocurvature perturbations may play important roles in PBH cosmology
- PBHs may play central roles in GW cosmology

PBH-GW Cosmology!