

量子重力理論の発展と ブラックホール

橋本幸士
(阪大理)

AdS_{d+1}

CFT_d

①

$1 + 1$

弦の世界膜

Wilsonループ

[Maldacena 1998]

②

$(d-1) + 1$

Dブレーン

クォークセクタ

[Karch, Katz 2002]

③

$(d-1) + 0$

柳-高柳 surface

エンタングルメント

[Ryu, Takayanagi 2006]

④

$d + 0$

Volume

量子計算複雑性

[Stanford, Susskind 2014]

⑤

$d + 0$ (境界有) Island

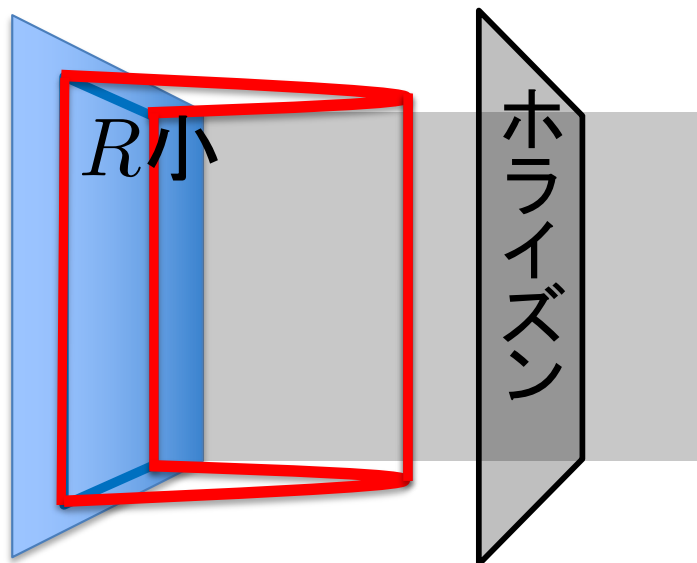
Page curve

[Penington 2019] [Almheiri, Engelhardt, Marolf, Maxfield 2019]

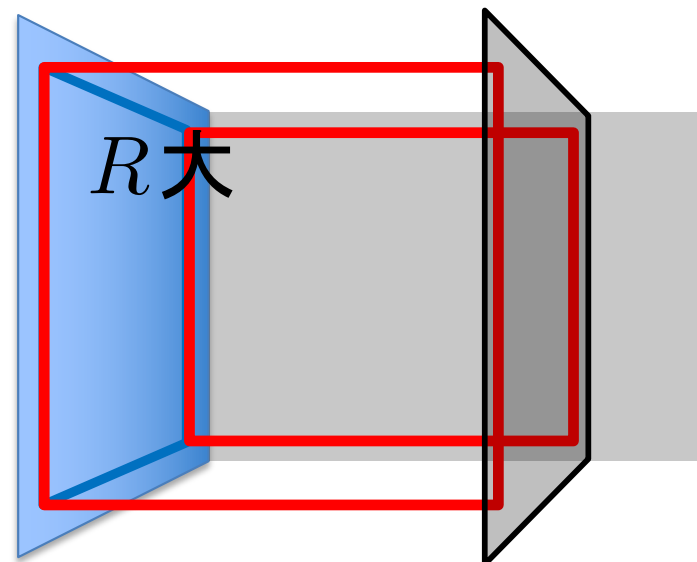
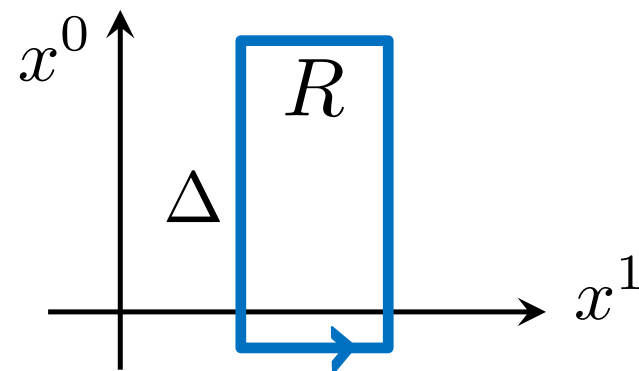
①

1 + 1 dim. 弦の世界膜 = Wilsonループ

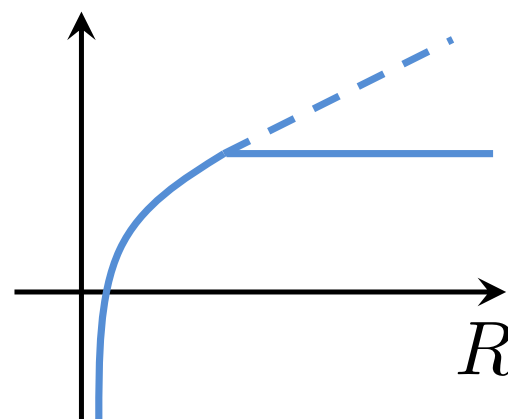
[Maldacena 1998]



$$W \equiv \text{tr} P \exp \left[i \int_{\square} A_{\mu}(x) dx^{\mu} \right]$$



$$E(R) \equiv -\Delta^{-1} \log[\langle W \rangle]$$

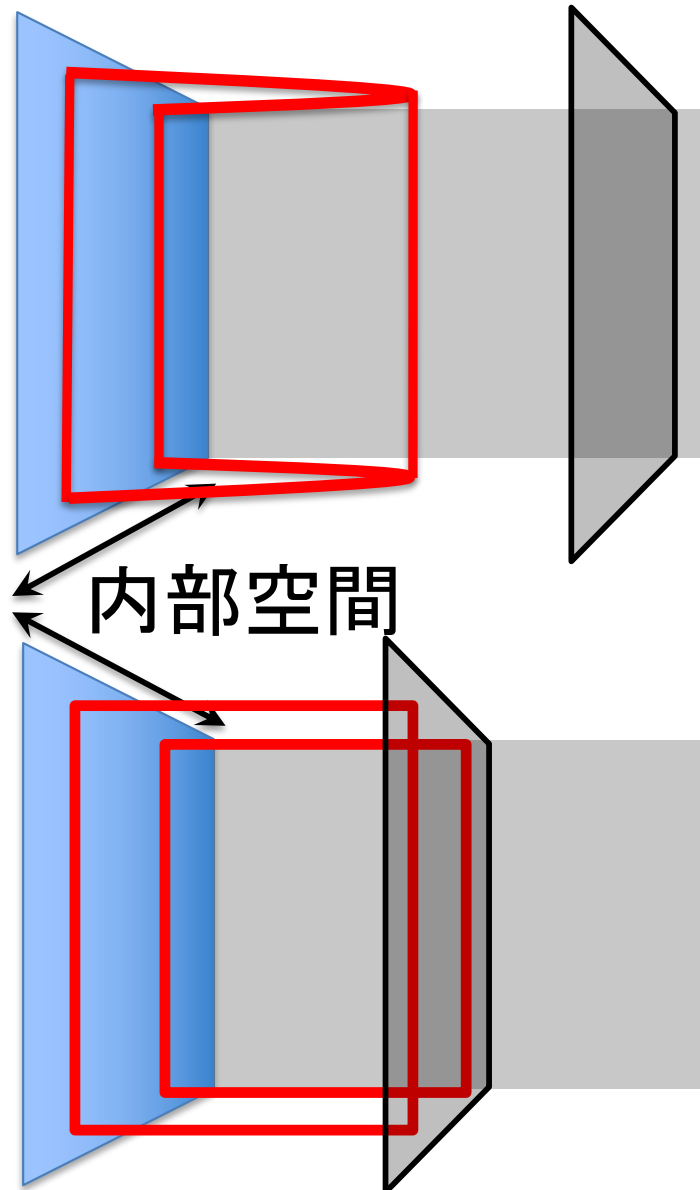


非閉じ込め
(デバイ遮蔽)

②

$(d-1) + 1$ dim. D-brane = クォークセクタ

[Karch, Katz 2002]

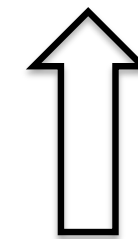


温度低

対称性: $U(1)_v$

温度高

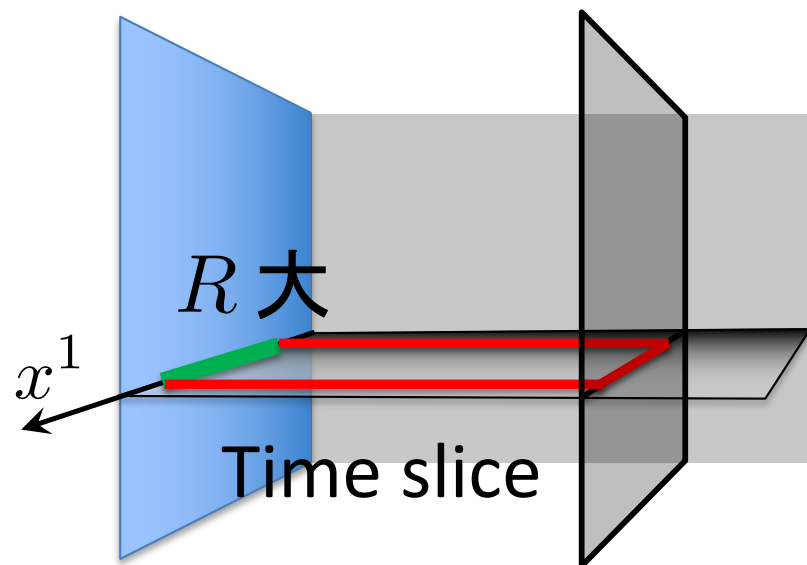
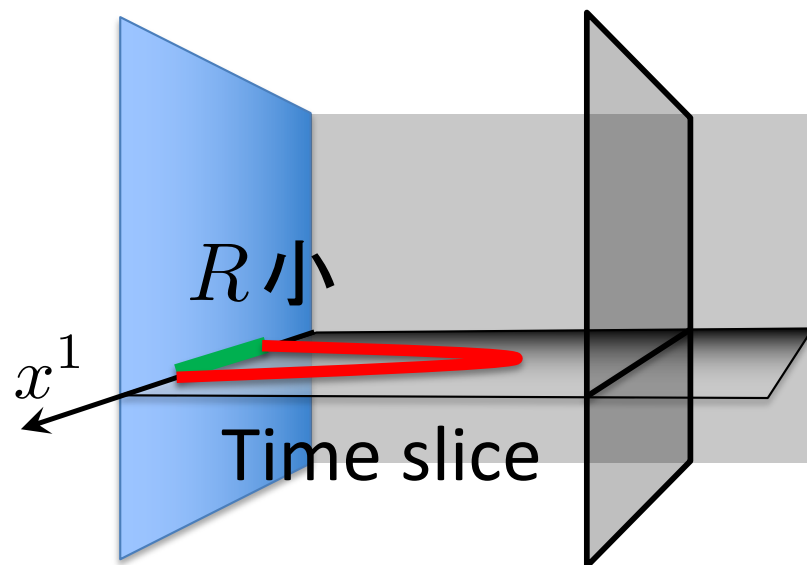
対称性: $U(1)_L \times U(1)_R$



カイラル
対称性の
自発的破れ

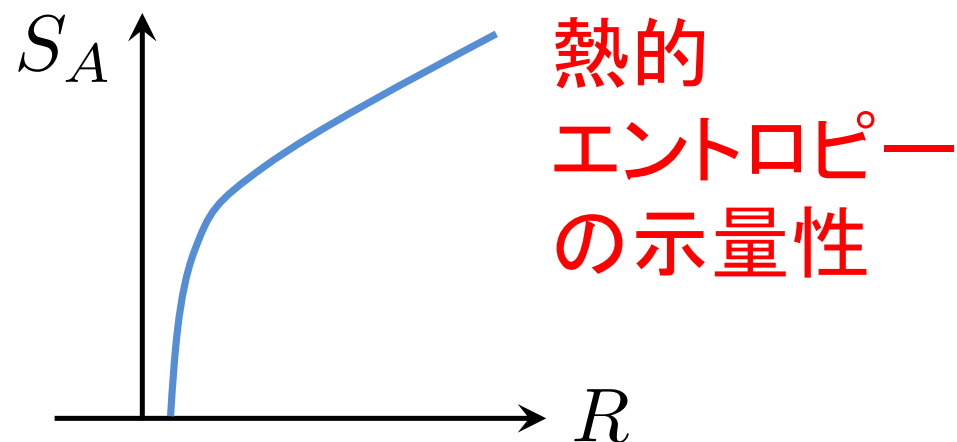
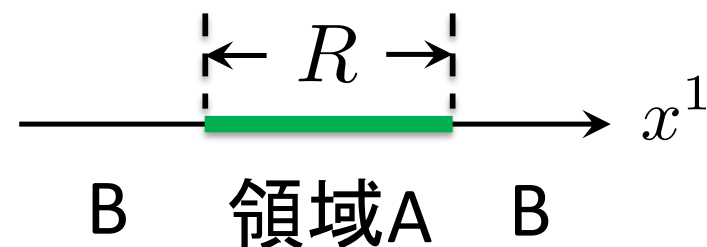
$$S_{\text{add}} \equiv \int d^4x i[\bar{\psi}_L \not{D} \psi_L + \bar{\psi}_R \not{D} \psi_R]$$

③ (d-1) + 0 dim. 柳-高柳 surface = エンタングルメント エントロピー
[Ryu, Takayanagi 2006]



$$S_A \equiv -\text{Tr} \rho_A \log \rho_A$$

$$\rho_A \equiv \text{Tr}_B \rho_{\text{total}}$$



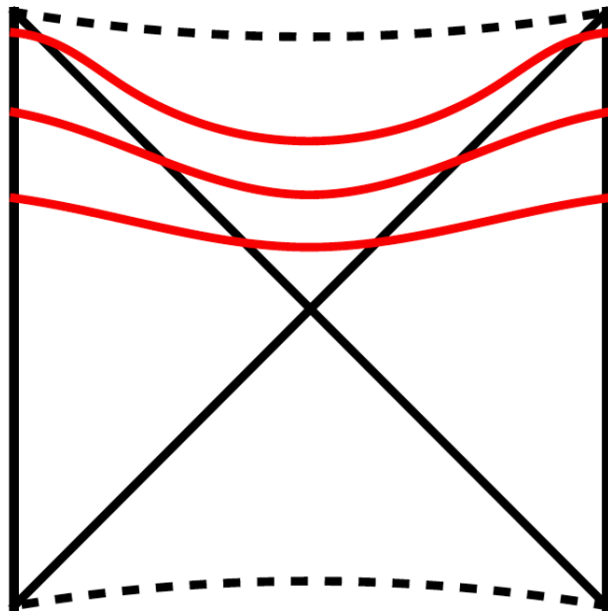
4

d + 0 dim. Volume

=

量子計算複雑性

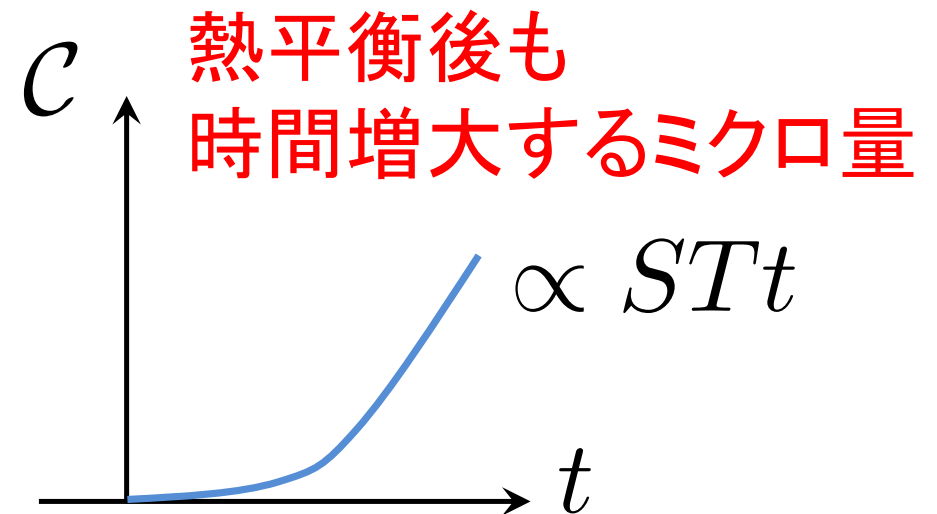
[Stanford, Susskind 2014]



Eternal AdS Schwarzschild
black hole

$\mathcal{C}$: $|\text{TFD}(t)\rangle$ を $|\text{TFD}(t=0)\rangle$
から構築するのに必要な
量子ゲートの最小数

$$|\text{TFD}\rangle \equiv \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_L \otimes |n\rangle_R$$



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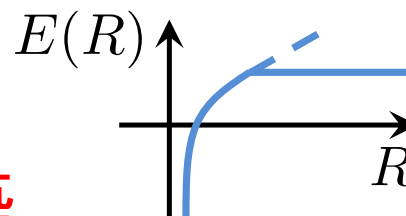
Wilsonループ

[Maldacena 1998]

ブラックホール時空

[KH 2020]

バルク再構築



③

$(d-1) + 0$

柳-高柳 surface

エンタングルメント

[Ryu, Takayanagi 2006]

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Volume

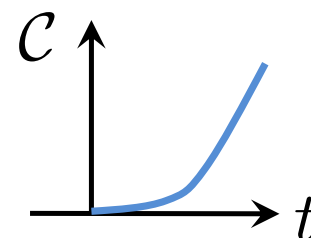
量子計算複雑性

[Stanford, Susskind 2014]

ブラックホール時空
(horizonの内側も)

[Watanabe, KH to appear]

バルク再構築



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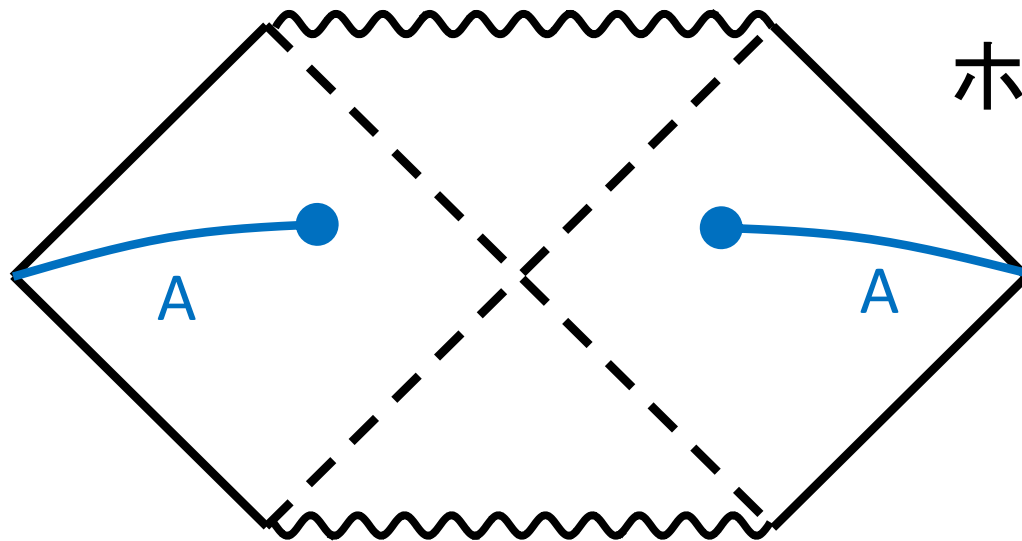
[Penington 2019] [Almheiri, Engelhardt, Marolf, Maxfield 2019]

5

$d + 0 \text{ dim. island} \Rightarrow \text{Page curve}$

[Penington 2019]

[Almheiri, Engelhardt, Marolf, Maxfield 2019]

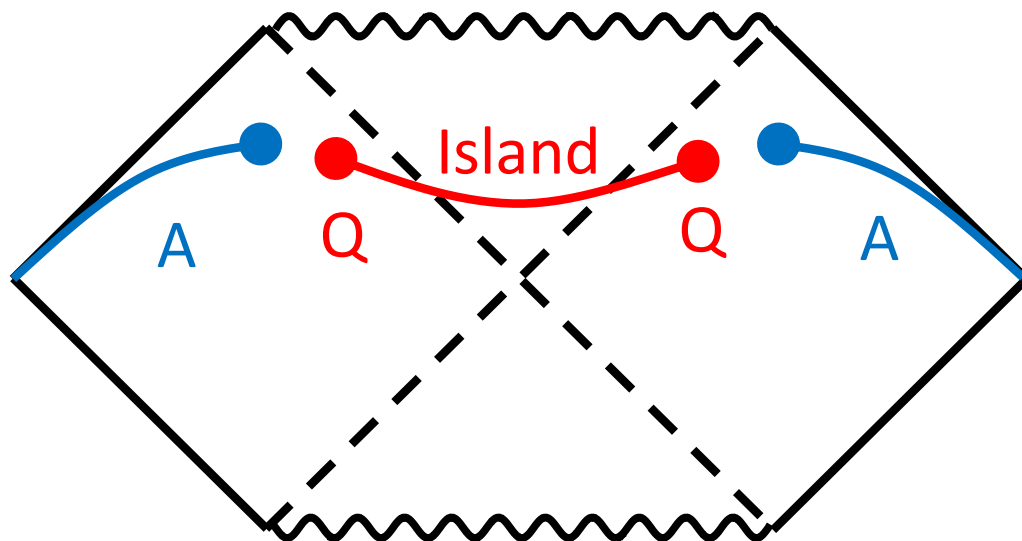


ホーキング輻射のエントロピー
(= 領域 A のEE)

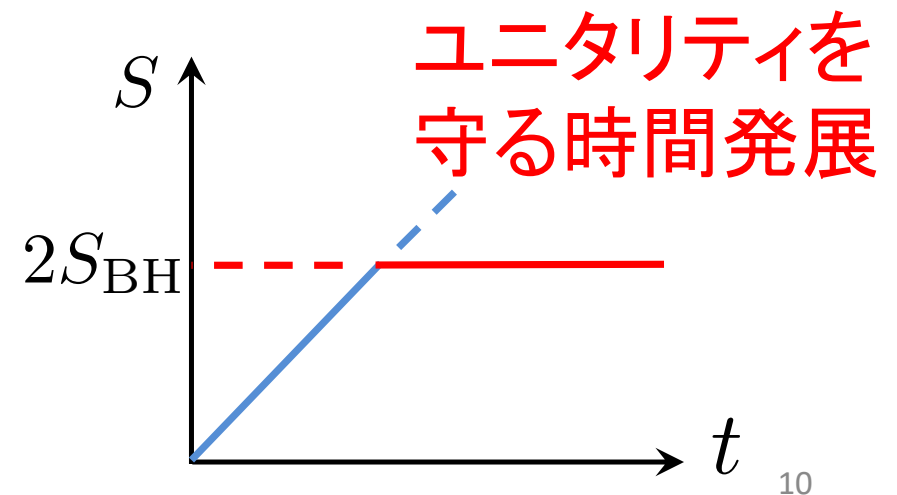
$$S = \text{Min.} \left[\text{Ext.} \left[\frac{\text{Area}(\mathbf{Q})}{4G_N} + S_{\text{matter}} \right] \right]$$

$\mathbf{Q}$: 量子極曲面

[Engelhardt, Wall 2015]



[Iizuka, Matsuo, KH 2020]



5

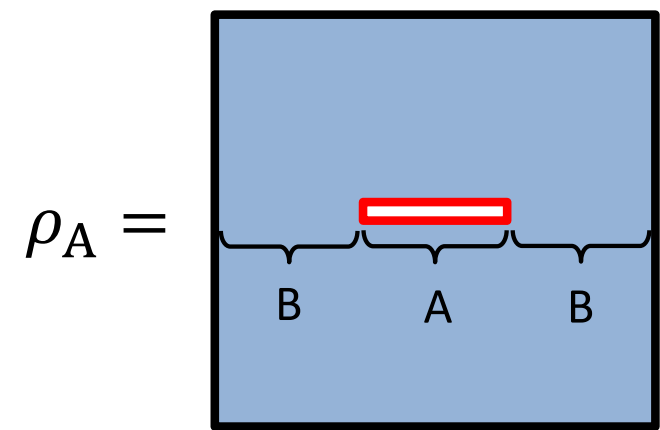
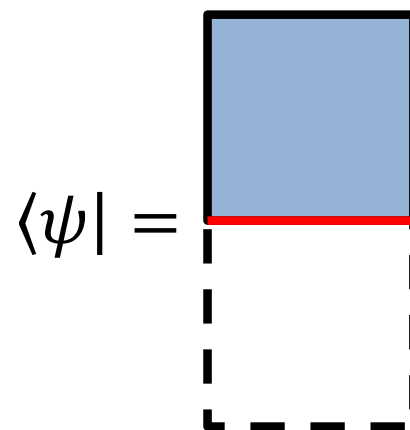
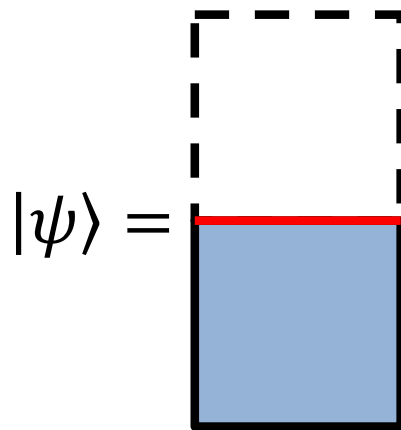
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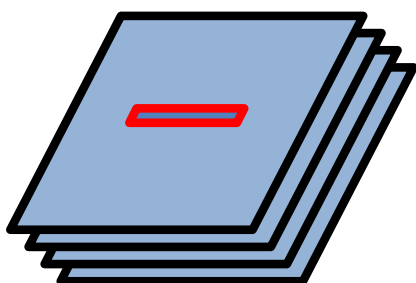
[Penington 2019]

[Almheiri, Engelhardt, Marolf, Maxfield 2019]

EE $S = -\lim_{n \rightarrow 1} \frac{\partial}{\partial n} \text{tr} \rho_A^n$ を計算するレプリカ法

密度行列 $\rho_A = \text{tr}_B |\psi\rangle\langle\psi|$



$\text{tr} \rho_A^n =$  $= \frac{Z_n}{Z_1^n}$ $Z_n: n\text{-レプリカ時空の分配関数}$

5

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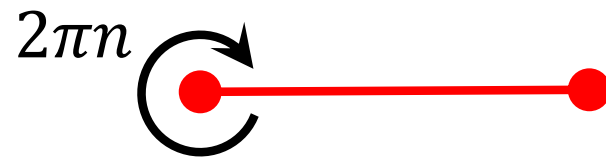
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曲がった時空中の物質場の1-loop有効作用

$$\log Z = \frac{c_0}{\epsilon^4} \int d^4x + \frac{c_1}{\epsilon^2} \int d^4x R + \dots$$



$$R \propto (n-1)\delta(x-x_1)$$

n -レプリカ時空のカットの
端で conical特異点

[Lewkowycz, Maldacena 2013]

$$S \propto \frac{\text{Area}}{\epsilon^2} + (\text{non-local}) \dots$$

5

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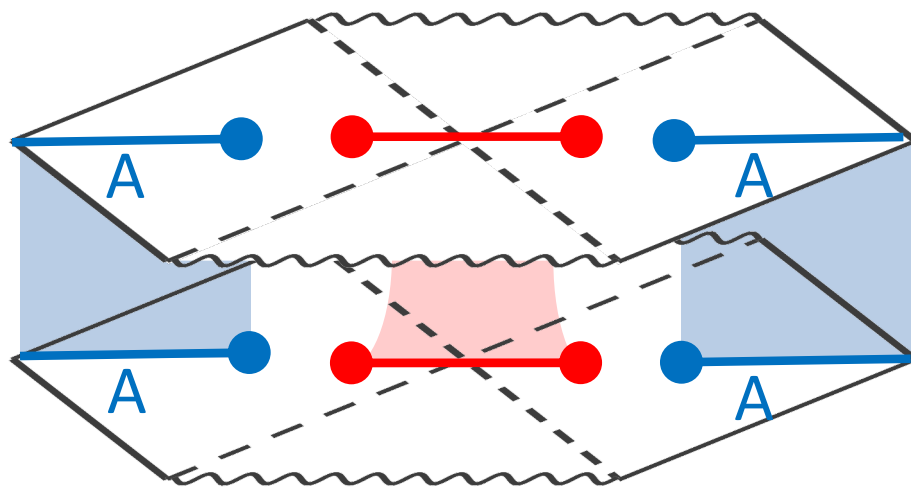
[Penington 2019]

[Almheiri, Engelhardt, Marolf, Maxfield 2019]

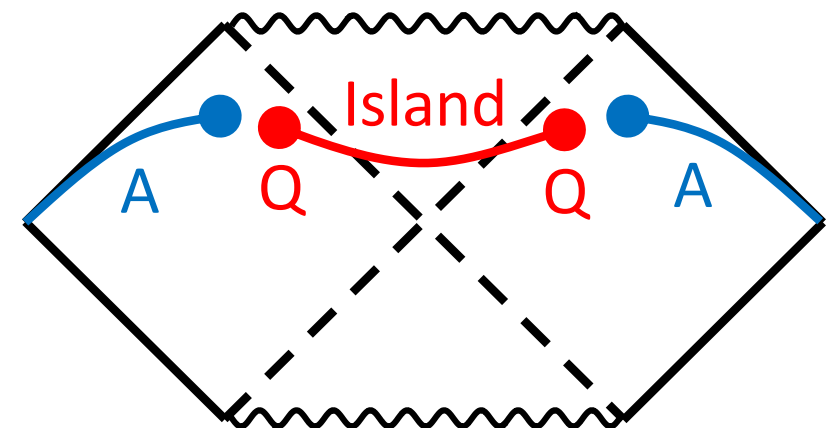
重力の下でレプリカ法を使うと、 n -レプリカ時空を繋ぐ
ワームホールがありうる

[Penington, Shenker, Stanford, Yang 2019]

[Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini 2019]



レプリカ
ワームホール



$$S = \text{Min.} \left[\text{Ext.} \left[\frac{\text{Area}(\text{Q})}{4G_N} + S_{\text{matter}}(A \cup \text{Island}) \right] \right]$$

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