日本物理学会（大阪市立大学）　2015年9月25日
シンポジウム「一般相対性理論100年の歩み」

一般相対性理論とその検証の理論

浅田秀樹
(弘前大　理工)

Clifford M. Will,
"The Confrontation between General Relativity and Experiment",
Living Rev. Relativity, 17 (2014), 4

http://www.livingreviews.org/lrr-2014-4

T. Clifton, P. G. Ferreira, A. Padilla, C. Skordis,
"Modified Gravity and Cosmology"
Physics Reports 513, 1 (2012)

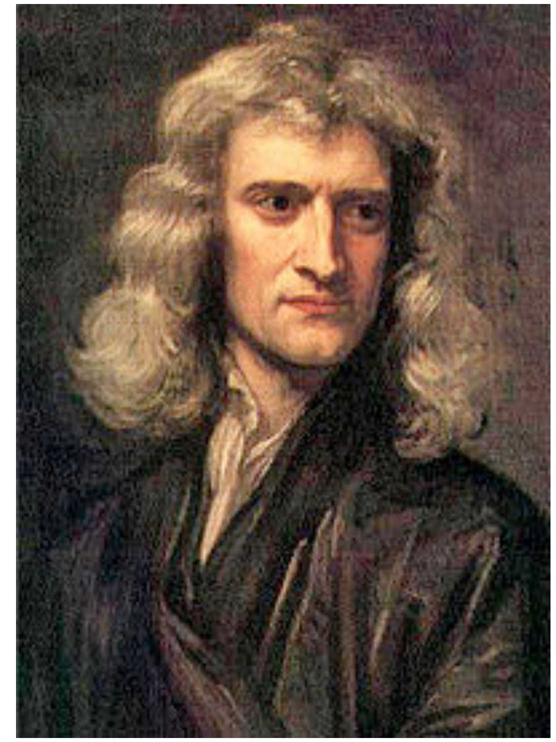
DOI:
10.1016/j.physrep.2012.01.001

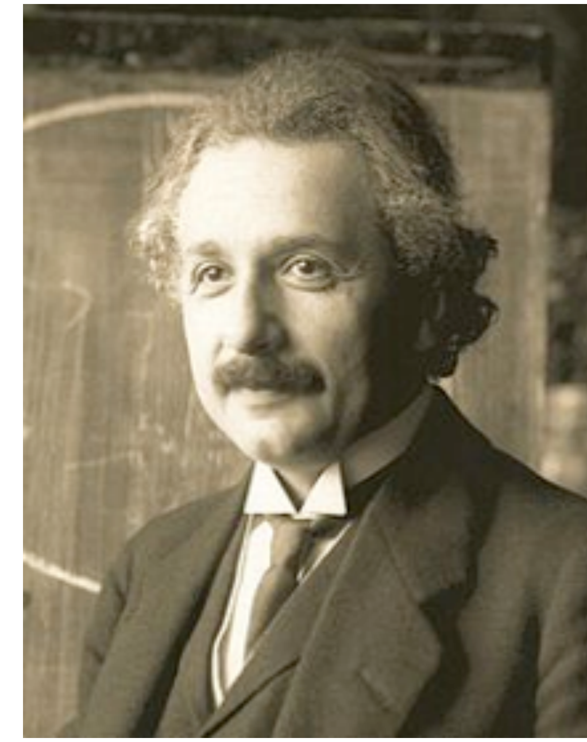
E. Berti, et al.
"Testing General Relativity with Present and Future Astrophysical Observations",

arXiv:1501.07274

1　イントロ

一般相対性理論(GR)の検証：





出典：wikipedia

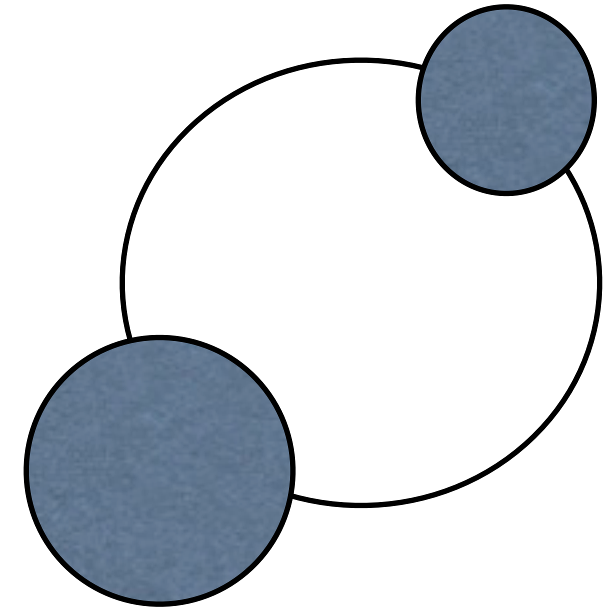
Einstein方程式

$$G_{\mu\nu}(+\Lambda g_{\mu\nu}) = \frac{8\pi G}{c^4} T_{\mu\nu}$$

が、自然界（物理実験と宇宙観測）と合うのか？

非線形の連立偏微分方程式なので

2体（連星系）でさえ厳密解は無理



（注）１体系（シュバルツシルト解やカー解）周りの性質や定式化（BH摂動法）は知られている

GRにおける近似法（摂動計算）が用いられる。

例）Post-Newton近似

Slow Motion $\dfrac{V}{c} \ll 1$ $(\sim 10^{-4})$

例：地球の公転

Weak Field $\dfrac{GM}{c^2 R} \ll 1$ $(\sim 10^{-5} - 10^{-8})$

例：太陽や惑星

太陽系はほとんど曲がっていないので、

(背景時空としての)Minkowski時空周りの摂動を考える

例えば、調和座標条件

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \qquad (\bar{h}^{\mu\nu} \equiv \sqrt{-g}\, g^{\mu\nu} - \eta^{\mu\nu})$$

を課すと、波動方程式の形

$$\Box \bar{h}_{\mu\nu} = \frac{16\pi G}{c^4} \tau^{\mu\nu} \rightarrow T^{\mu\nu} + O(\bar{h}^2)$$

(遅延グリーン関数を用いて解けるように見えるが)

$$g_{\mu\nu} = \eta_{\mu\nu} + g^{1PN}_{\mu\nu} + g^{2PN}_{\mu\nu} + g^{2.5PN}_{\mu\nu} + g^{3PN}_{\mu\nu} + g^{3.5PN}_{\mu\nu} + \cdots$$

$$1/c^2 \qquad 1/c^4 \qquad 1/c^5 \qquad 1/c^6 \qquad 1/c^7$$

２　PPN (Parameterized post-Newton)の定式化

Eddington (1922)　　　Will and Nordtvedt (1972)

1PNの計量の係数の値は「理論」に依存する

仮定(陽)　ポテンシャルはPoisson方程式に従う

仮定(陰)　ポテンシャルはParityを破らない

１０個のパラメタ

Newton (GM)

$$
\begin{aligned}
g_{00} &= -1 + \underline{2GU} - 2\beta G^2U^2 - 2\xi G^2\Phi_W + (2\gamma + 2 + \alpha_3 + \beta_1 - 2\xi)G\Phi_1 \\
&\quad +2(1 + 3\gamma - 2\beta + \beta_2 + \xi)G^2\Phi_2 + 2(1 + \beta_3)G\Phi_3 - (\beta_1 - 2\xi)G\mathcal{A} \\
&\quad +2(3\gamma + 3\beta_4 - 2\xi)G\Phi_4 \\
g_{0i} &= -\frac{1}{2}(3 + 4\gamma + \alpha_1 - \alpha_2 + \beta_1 - 2\xi)GV_i - \frac{1}{2}(1 + \alpha_2 - \beta_1 + 2\xi)GW_i \\
g_{ij} &= (1 + 2\gamma GU)\delta_{ij}.
\end{aligned}
$$

$$
\begin{aligned}
U &= \int \frac{\rho'}{|\mathbf{x}-\mathbf{x}'|}d^3x', \\
U_{ij} &= \int \frac{\rho'(x-x')_i(x-x')_j}{|\mathbf{x}-\mathbf{x}'|^3}d^3x', \\
\Phi_W &= \int \frac{\rho'\rho''(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}\cdot\left(\frac{\mathbf{x}'-\mathbf{x}''}{|\mathbf{x}-\mathbf{x}''|} - \frac{\mathbf{x}-\mathbf{x}''}{|\mathbf{x}'-\mathbf{x}''|}\right)d^3x'd^3x'', \\
\mathcal{A} &= \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')]^2}{|\mathbf{x}-\mathbf{x}'|^3}d^3x', \\
\Phi_1 &= \int \frac{\rho' v'^2}{|\mathbf{x}-\mathbf{x}'|}d^3x', \\
\Phi_2 &= \int \frac{\rho' U'}{|\mathbf{x}-\mathbf{x}'|}d^3x', \\
\Phi_3 &= \int \frac{\rho'\Pi'}{|\mathbf{x}-\mathbf{x}'|}d^3x', \\
\Phi_4 &= \int \frac{p'}{|\mathbf{x}-\mathbf{x}'|}d^3x', \\
V_i &= \int \frac{\rho' v_i'}{|\mathbf{x}-\mathbf{x}'|}d^3x', \\
W_i &= \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')](x-x')_i}{|\mathbf{x}-\mathbf{x}'|^3}d^3x'.
\end{aligned}
$$

Table 2: The PPN Parameters and their significance (note that α_3 has been shown twice to indicate that it is a measure of two effects).

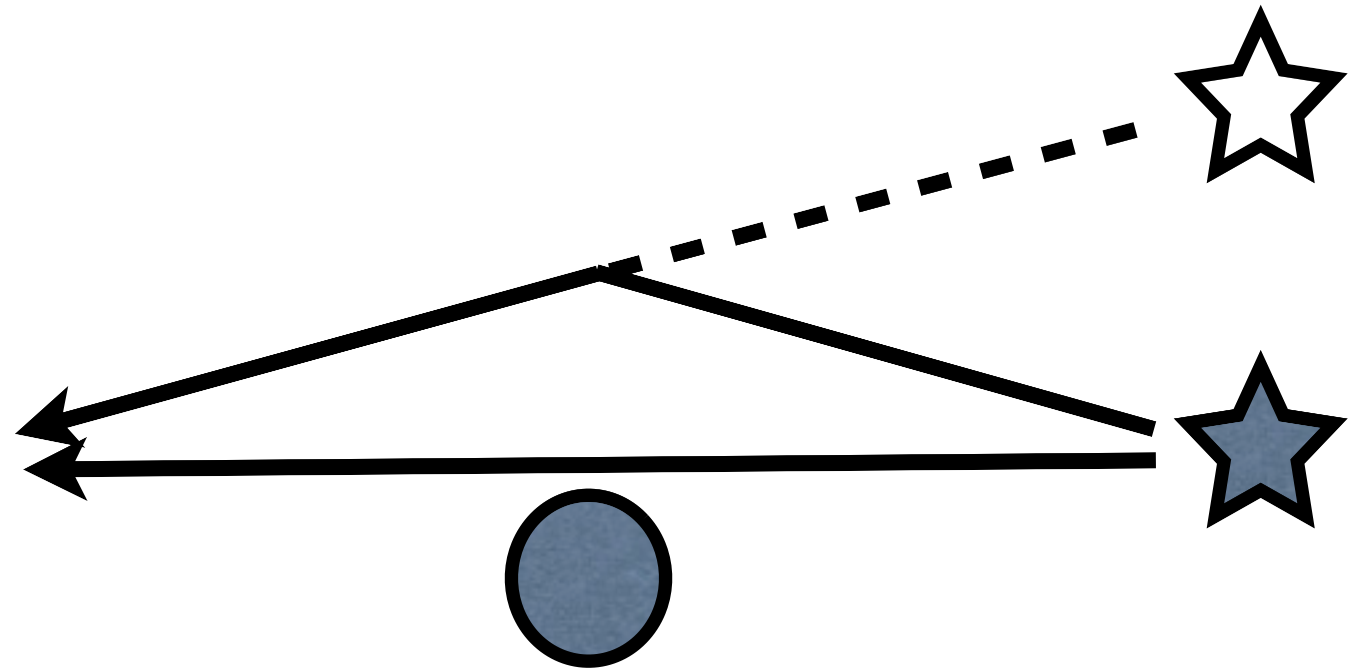
Parameter	What it measures relative to GR	Value in GR	Value in semiconservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
α_3	Violation of conservation of total momentum?	0	0	0
ζ_1		0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

Will, LRR

３　１PN（太陽系）でのテスト

（１）光の曲がり

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = 0$$



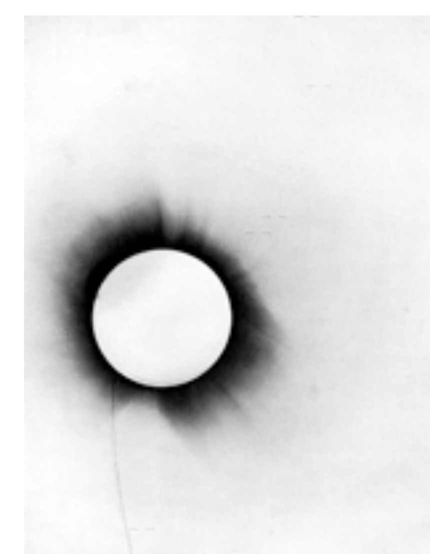
軌道方程式

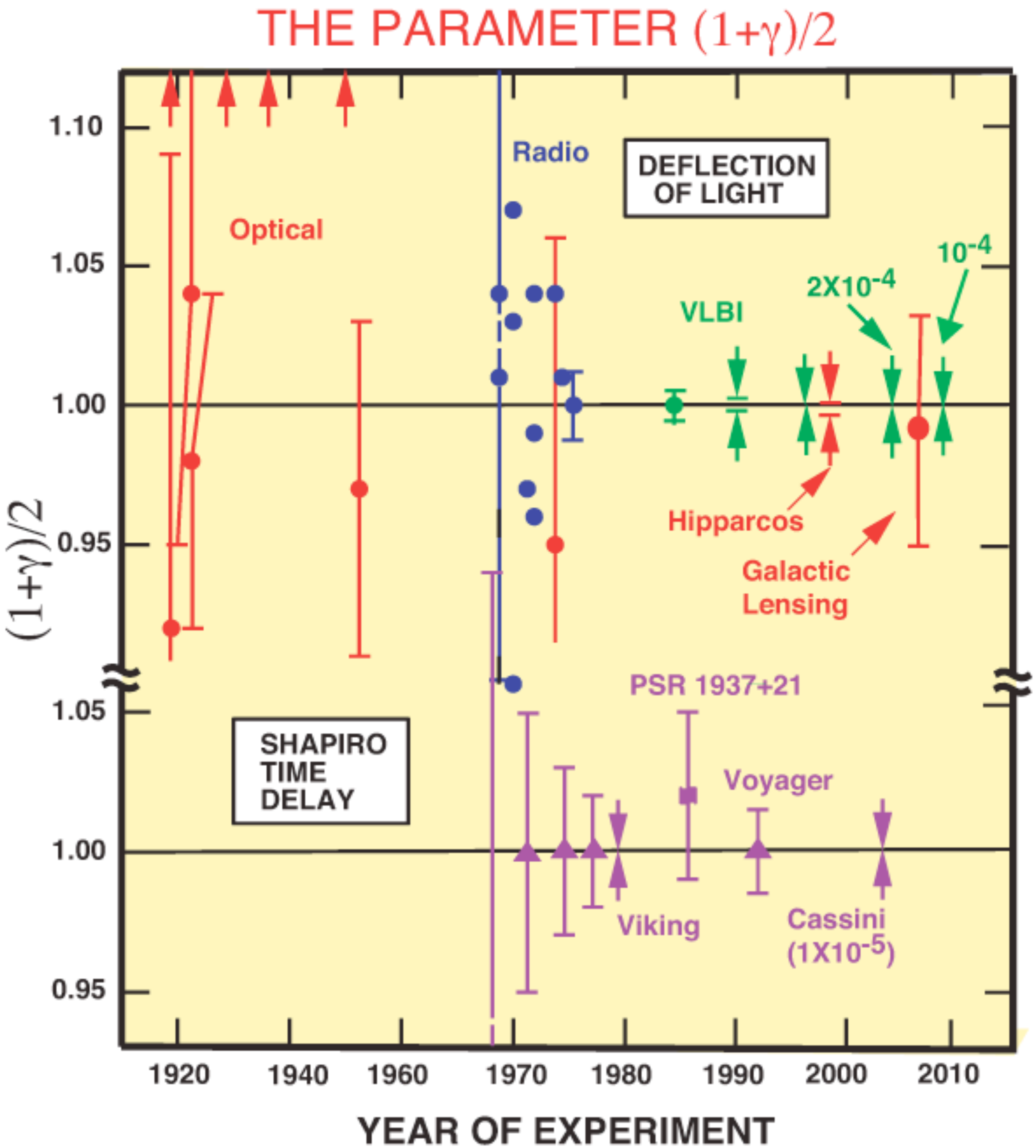
$$\frac{dr}{d\theta} = F(r, g_{\mu\nu})$$

GR $$\frac{4GM_\odot}{c^2 b} = 1.75"$$

PPN $$\frac{2(1+\gamma)GM_\odot}{c^2 b}$$

Eddington (1919)




THE PARAMETER (1+γ)/2
(1+γ)/2
1.10
1.05
1.00
0.95
Optical
Radio
DEFLECTION OF LIGHT
VLBI
$2X10^{-4}$
10^{-4}
Hipparcos
Galactic Lensing
SHAPIRO TIME DELAY
PSR 1937+21
Voyager
Viking
Cassini ($1X10^{-5}$)
1920
1940
1960
1970
1980
1990
2000
2010
YEAR OF EXPERIMENT

Will, LRR

（２）水星の近日点移動

Bertrandの定理 (古典力学)

中心力で閉軌道になるポテンシャルは

2種類のみ（1/r、 r^2）

GR (or PPN)は、(GM/r c^2)^2　型なので

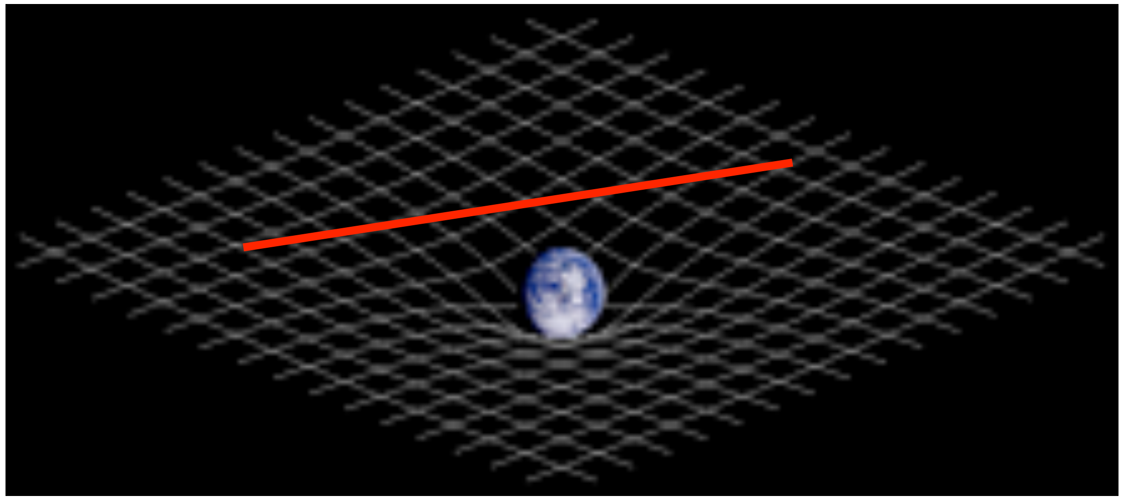
近日点が移動する

$$\dot{\omega} = 42.''98 \left(\frac{1}{3}(2 + 2\gamma - \beta) + 3 \times 10^{-4} \frac{J_2}{10^{-7}} \right)$$

（3）Shapiroの時間の遅れ

平坦な時空に比べて、光が到着するまでに
余計に時間がかかる

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = 0$$

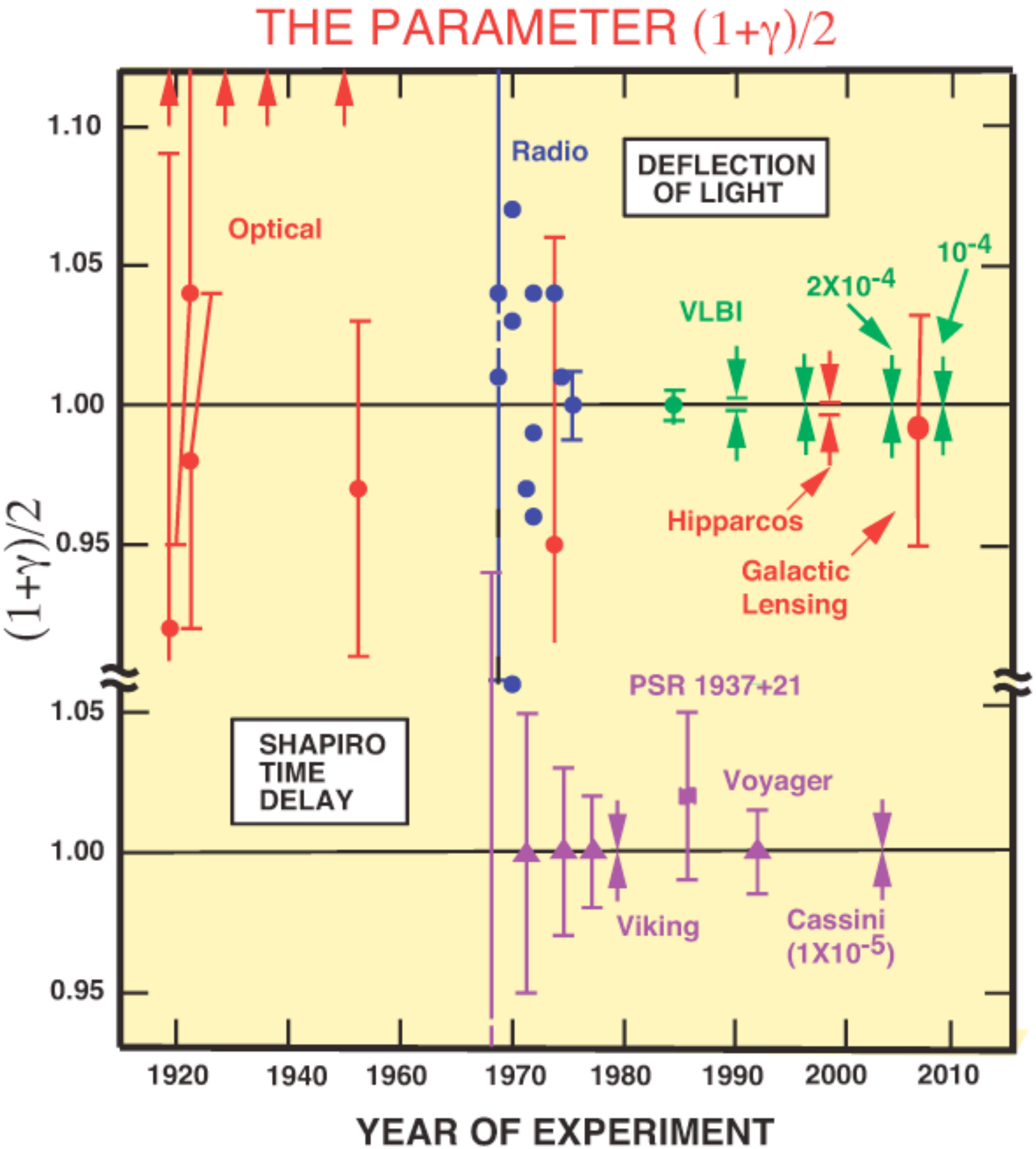


図（wikipedia）に加筆

dt = ・・・

$$\delta t = \int_{emit}^{obs} T(r, g_{\mu\nu})dr - \frac{D_{Euclid}}{c}$$

$$\delta t \propto 2(1+\gamma)M$$


THE PARAMETER (1+γ)/2
(1+γ)/2
1.10
1.05
1.00
0.95
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10-4
Hipparcos
Galactic Lensing
PSR 1937+21
SHAPIRO TIME DELAY
Voyager
Viking
Cassini (1X10-5)
1920
1940
1960
1970
1980
1990
2000
2010
YEAR OF EXPERIMENT

Will, LRR

（4）ジャイロの歳差

$$g_{0i} = -\frac{1}{2}(4\gamma + 4 + \alpha_1)V_i$$

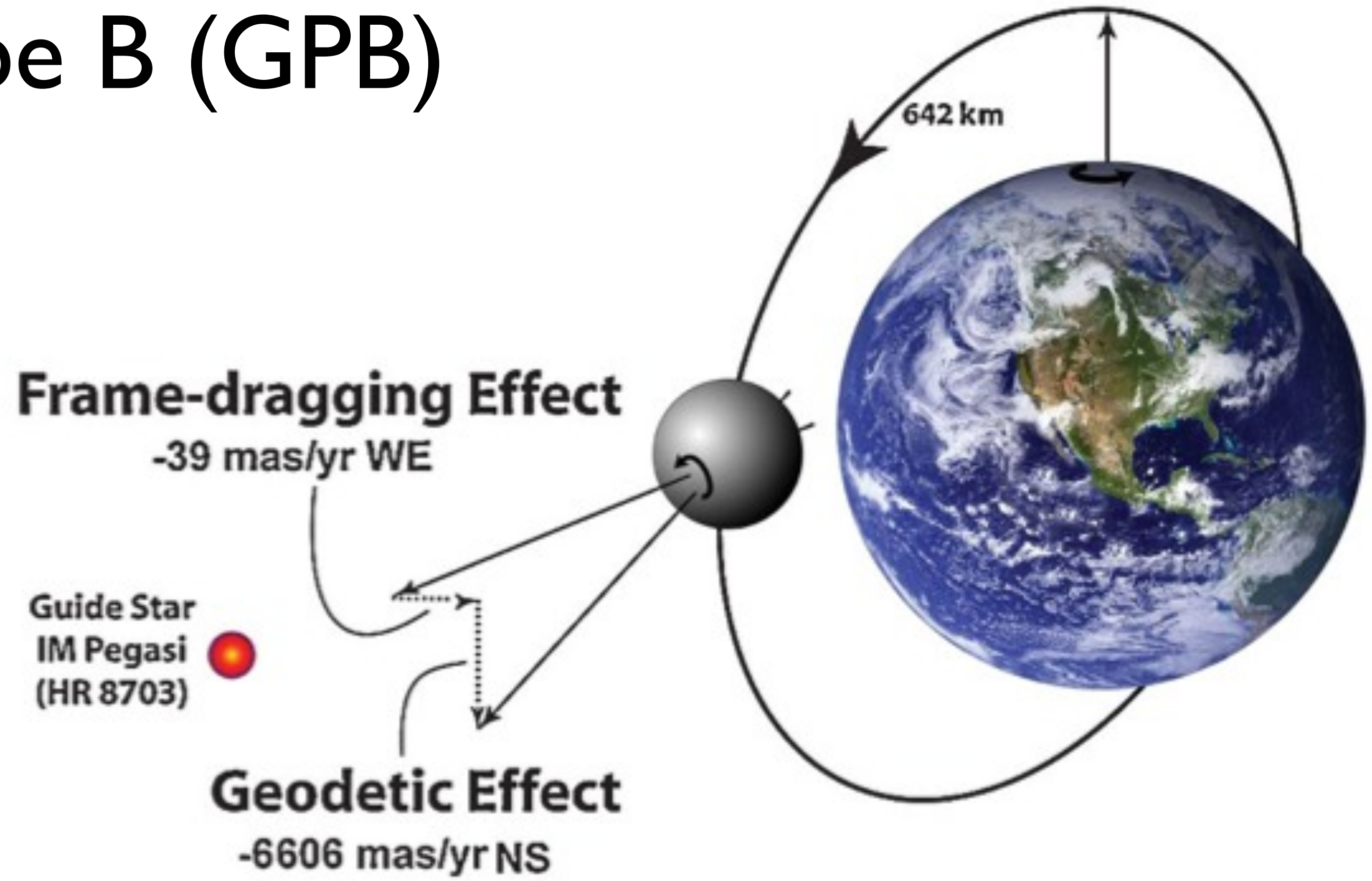
Gravitomagnetism $\quad \vec{B}_g = \nabla \times (g_{0i}\vec{e}^{\,i})$

$\Longleftrightarrow$ Electromagnetism $\quad \vec{B} = \nabla \times \vec{A}$

$$\frac{d\mathbf{S}}{d\tau} = \mathbf{\Omega}_{\rm LT} \times \mathbf{S}, \qquad \mathbf{\Omega}_{\rm LT} = -\frac{1}{2}\left(1 + \gamma + \frac{1}{4}\alpha_1\right)\frac{\mathbf{J} - 3\mathbf{n}(\mathbf{n}\cdot\mathbf{J})}{r^3}.$$

(Einstein-) Lense-Thirring effect (1917)

Gravity Probe B (GPB)



Everitt et al. PRL (2011)

Frame-dragging detection with 20% precision

Geodetic effect with 0.3% precision

"The full technical and data analysis details of GPB are expected to be published as a special issue of Classical and Quantum Gravity in 2015" in Will, LRR

GPB result はそれ以前の結果とも合う

LAGEOS 衛星 (Ciufollini et al. Nature 2004)

20 - 30 ％レベル

LARES(2012打ち上げ) が１%レベルを目標に運用中

(Ciufollini et al. 2013)

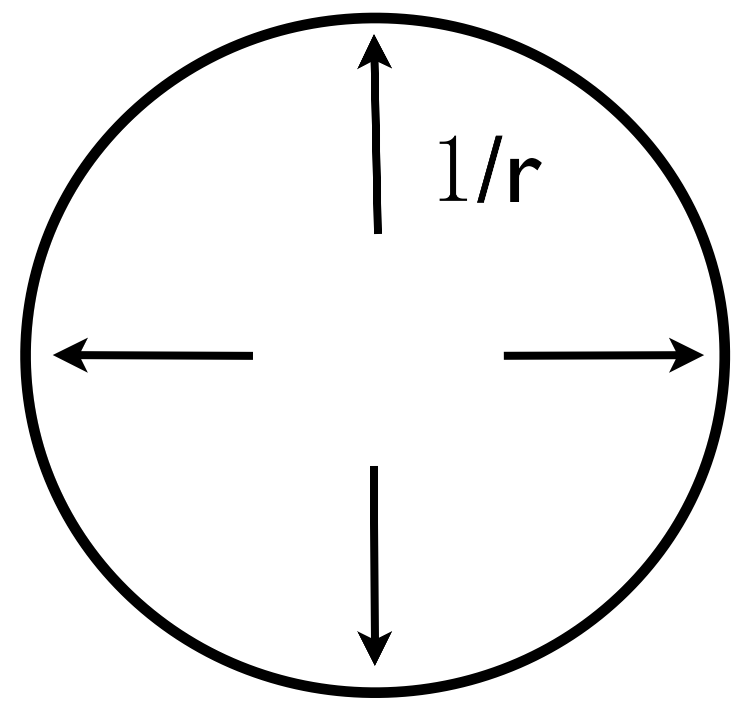
4　2.5PN（連星パルサー）を用いたテスト

GRでは、ポテンシャルの一部が波動方程式に従う

重力波

（電磁波と同様に）Poynting flux

$$\sim \oint \dot{h}\dot{h}r^2 d\Omega$$



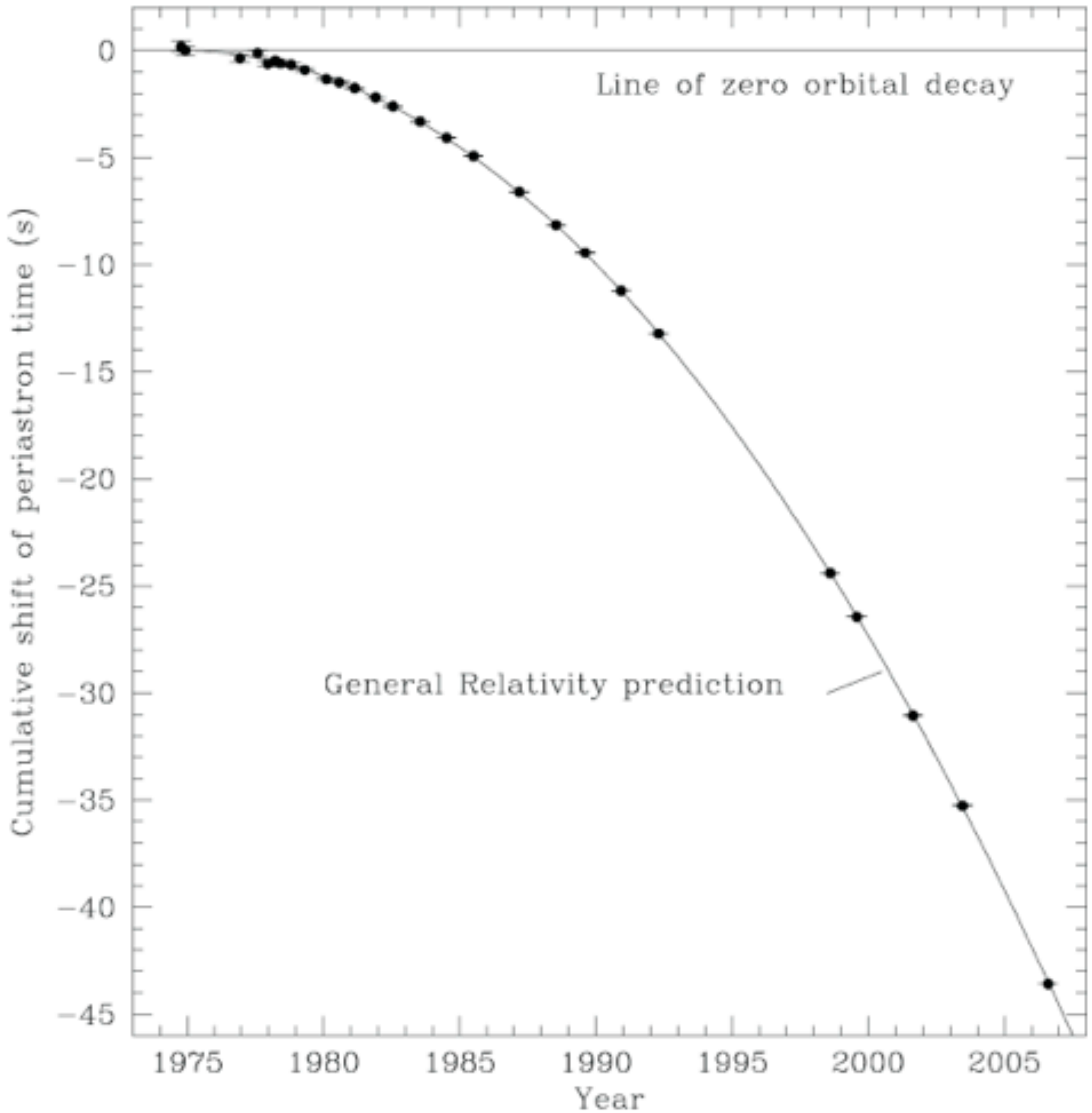
重力波のLuminosity

$$\frac{d}{dt}E_{Binary} = -L_{GW}$$

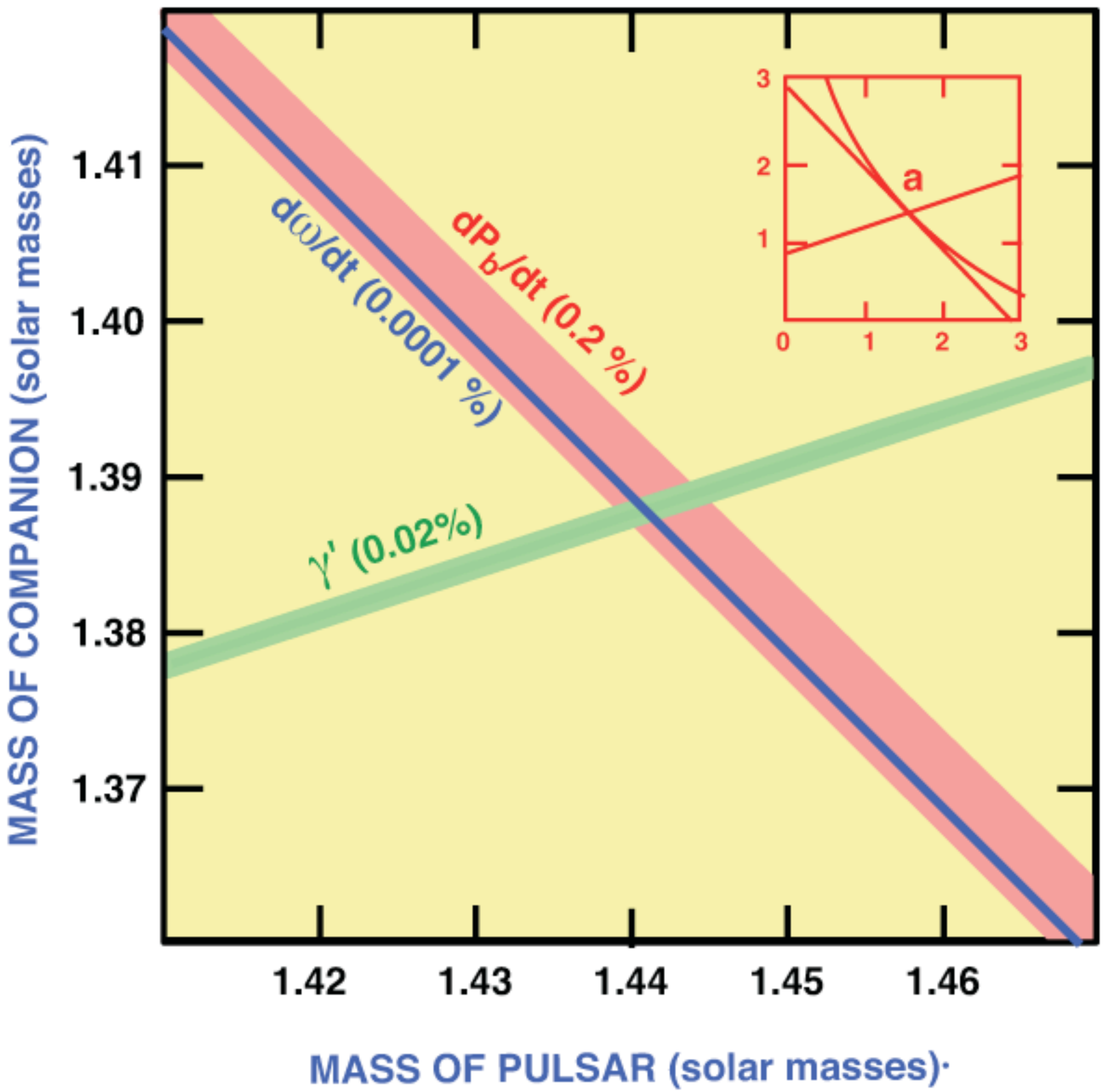
連星の公転振動数の時間変化率

$$\dot{f}_{\mathrm{b}} = \frac{192\pi}{5} f_{\mathrm{b}}^2 (2\pi \mathcal{M} f_{\mathrm{b}})^{5/3} F(e),$$

$$F(e) = (1-e^2)^{-7/2} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$



Will, LRR



MASS OF COMPANION (solar masses)
1.41
1.40
1.39
1.38
1.37
$d\omega/dt$ (0.0001 %)
dP_b/dt (0.2 %)
γ' (0.02%)
a
1.42
1.43
1.44
1.45
1.46
MASS OF PULSAR (solar masses)

Will, LRR

THE ASTROPHYSICAL JOURNAL, 345:434–450, 1989 October 1
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FURTHER EXPERIMENTAL TESTS OF RELATIVISTIC GRAVITY USING THE BINARY PULSAR PSR 1913+16

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AND

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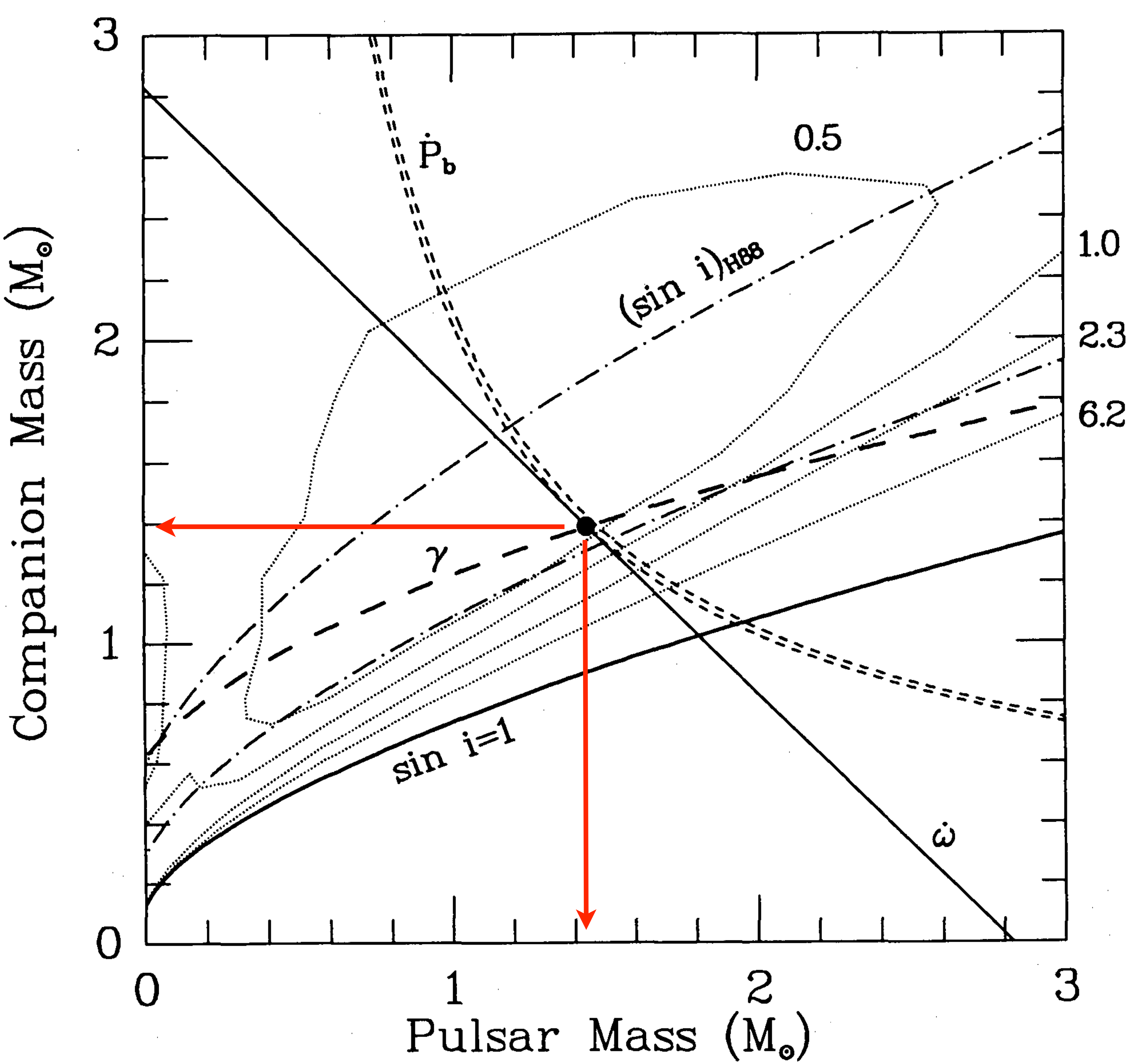
Received 1989 February 13; accepted 1989 March 24

ABSTRACT

Pulse time-of-arrival observations of the binary pulsar PSR 1913+16 now extend over approximately 14 years. The data are consistent with a straightforward model allowing for the motion of the Earth, special and general relativistic effects within the solar system, dispersive propagation in the interstellar medium, relativistic motion of the pulsar in its orbit, and deterministic spin-down behavior of the pulsar itself. The results show that at the present level of precision, the PSR 1913+16 system can be modeled dynamically as a pair of orbiting point masses. A total of five Keplerian and five "post-Keplerian" orbital parameters can now be determined, most of them with remarkably high precision. The masses of the pulsar and its companion are determined (within general relativity) to be $m_1 = 1.442 \pm 0.003$ and $m_2 = 1.386 \pm 0.003$ times the mass of the Sun, respectively, and the orbit is found to be decaying at a rate equal to 1.01 ± 0.01 times the general relativistic prediction for gravitational radiation damping.

Our results represent the first experimental tests of gravitation theory not restricted to the weak-field, slow-motion limit in which nonlinearities and radiation effects are negligible. The excellent agreement of observation with theory shows conclusively that gravitational radiation exists, at the level predicted by general relativity. We also use the results to calculate improved upper limits on the rate of change of the Newtonian gravitational constant, and the fractional energy density (relative to closure density) of a cosmic background of ultra–low-frequency gravitational radiation. These limits are, respectively, $\dot{G}/G = (1.2 \pm 1.3) \times 10^{-11}$ yr^{-1}, and $\Omega_g < 0.04$ at frequencies 10^{-9} to 10^{-12} Hz.

Subject headings: gravitation — pulsars — radiation mechanisms — relativity — stars: binaries



Taylor and Weisberg (1989)を基に加筆

ボーナス

GRからの数式(1PNレベル)を用いて、

パルサー（中性子星）の質量が直接求まった！

（天文学の経験則を用いずに）

（注）ニュートン重力（ケプラーの第3法則）

では、「二つの質量の和」なので、

各質量を個別に決めるのは不可能

５　PPNの発展

Chern–Simons項を加える

Witten(1989) Ashtekhar et al (1989) Jackiw&Pi (2003)

$$S_{\mathrm{CS}} = \frac{1}{16\pi G}\int d^4x \frac{1}{4} f R^{\star} R$$

$$R^{\star}R = \tfrac{1}{2} R_{\alpha\beta\gamma\delta}\epsilon^{\alpha\beta\mu\nu}R^{\gamma\delta}{}_{\mu\nu}$$

Parityを破る

Alexander&Yunes (2007)

新しいPPNパラメタの導入

LAGEOS+GPBで制限される

地上実験（干渉計）で、
この新しいPPNパラメタを制限できないか？

（大河原広樹　2013年修士論文）

PRL **109,** 231101 (2012)　　PHYSICAL REVIEW LETTERS　　week ending 7 DECEMBER 2012

Possible Daily and Seasonal Variations in Quantum Interference Induced by Chern-Simons Gravity

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(Received 28 May 2012; published 4 December 2012)

Possible effects of Chern-Simons (CS) gravity on a quantum interferometer turn out to be dependent on the latitude and direction of the interferometer on Earth in orbital motion around the Sun. Daily and seasonal variations in phase shifts are predicted with an estimate of the size of the effects, wherefore neutron interferometry with ~ 5 m arm length and $\sim 10^{-4}$ phase measurement accuracy would place a bound on a CS parameter comparable to the Gravity Probe B satellite.

DOI: 10.1103/PhysRevLett.109.231101　　PACS numbers: 04.80.Cc, 04.25.Nx, 04.50.−h

量子干渉実験を想定して

Okawara, Yamada, HA (2012,2013)

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi = \left(\frac{1}{2m}\left(\vec{p} + mc\vec{h}_0\right)^2 + \frac{1}{2}mc^2 h_{00}\right)\psi}$$

Aharonov-Bohm効果と似ていて

$$\Delta = \frac{mc}{\hbar}\left(\int_{C_1}\vec{h}_0\cdot d\vec{r} - \int_{C_2}\vec{h}_0\cdot d\vec{r}\right)$$

$$= \frac{mc}{\hbar}\oint_C \vec{h}_0\cdot d\vec{r}$$

$$\boxed{= \frac{mc}{\hbar}\int_S (\vec{\nabla}\times\vec{h}_0)\cdot d\vec{S}}$$

光干渉計（サニャック計など）でも原理的に可能

（菊池大貴、大本直哉、2014年卒業論文）

PHYSICAL REVIEW D **90,** 064036 (2014)

Possible altitudinal, latitudinal, and directional dependence of the relativistic Sagnac effect in Chern-Simons modified gravity

Daiki Kikuchi, Naoya Omoto, Kei Yamada, and Hideki Asada

Faculty of Science and Technology, Hirosaki University, Hirosaki 036-8561, Japan
(Received 29 May 2014; revised manuscript received 28 August 2014; published 23 September 2014)

Toward a test of parity violation in a gravity theory, possible effects of Chern-Simons (CS) gravity on an interferometer have been recently discussed. Continuing work initiated in an earlier publication [H. Okawara, K. Yamada, and H. Asada, Phys. Rev. Lett. 109, 231101 (2012)], we study possible altitudinal and directional dependence of relativistic Sagnac effect in CS modified gravity. We compare the CS effects on Sagnac interferometers with the general relativistic Lense-Thirring (LT) effects. Numerical calculations show that the eastbound Sagnac interferometer might be preferred for testing CS separately, because LT effects on this interferometer cancel out. The size of the phase shift induced in the CS model might have an oscillatory dependence also on the altitude of the interferometer through the CS mass parameter m_{CS}. Therefore, the international space station site as well as a ground-based experiment is also discussed.

DOI: 10.1103/PhysRevD.90.064036 PACS numbers: 04.25.Nx, 04.50.-h, 04.80.Cc

Scalar-Tensor Theories

(1) (Jordan-) Brans-Dicke (1955,1961)

$$S = \int d^4x \sqrt{-g} \left(\frac{\phi R - \omega \frac{\partial_a \phi \partial^a \phi}{\phi}}{16\pi} + \mathcal{L}_{\mathrm{M}} \right)$$

$$\gamma(\omega) = \frac{1+\omega}{2+\omega}$$

Cassini衛星（Shapiro の時間遅れ）

$$\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$$

Bertotti et al (2003)

ω > 40000

(2013年打ち上げのGAIAが10^{-6}レベルを目標に観測中)

(2) Horndeski Theory (1973)

Ostrogradsky stability

Most general, stable Scalar-Tensor
with 2nd order E.O.M.

$$S = \sum_{i=2}^{5} \int d^4x \sqrt{-g} \mathcal{L}_i[g_{\mu\nu}, \phi] + S_m[g_{\mu\nu}, \chi_m] \,.$$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

(bi-scalar-tensorへの拡張: Ohashi+, ArXiv:1505.06029)

Hofmann, ArXiv:1506.04253

$$\gamma(r) = \frac{2\omega + 3 - e^{-m_\psi r}}{2\omega + 3 + e^{-m_\psi r}}$$

$$\begin{aligned}\beta(r) = 1 + \frac{1}{(2\omega + 3 + e^{-m_\psi r})^2} \Bigg\{ & \frac{\omega + \tau - 4\omega\sigma}{2\omega + 3} e^{-2m_\psi r} \\ & + a(r) \left[e^{-m_\psi r} \ln(m_\psi r) - (m_\psi r + e^{m_\psi r}) \,\mathrm{Ei}(-2m_\psi r) - \frac{1}{2} e^{-2m_\psi r} \right] \\ & + b(r) \left[e^{m_\psi r} \mathrm{Ei}(-3m_\psi r) - e^{-m_\psi r} \mathrm{Ei}(-m_\psi r) \right] \Bigg\}, \qquad (\end{aligned}$$

テスト粒子と天体との距離に依存する

（CS修正重力でも同様、Smith et al 2008）

5-B　重力波を用いた検証

連星からの重力波振動数の進化

仮定：GR、インスパイラル期（PN的）、

スピン無し、(準)円軌道

2.5PN　　+1PN　　+1.5PN

$$\dot{f} = \frac{96\pi}{5} f^2 (\pi \mathcal{M} f)^{5/3} \left[1 - \left(\frac{743}{336} + \frac{11}{4}\eta \right) (\pi m f)^{2/3} + 4\pi(\pi m f) + \left(\frac{34103}{18144} + \frac{13661}{2016}\eta + \frac{59}{18}\eta^2 \right) (\pi m f)^{4/3} + \mathcal{O}[(\pi m f)^{5/3}] \right],$$

+2PN

パラメタを入れる等

e.g. Yunes&Pretorius 2009, Mishra et al 2010

６　結論

修正重力模型の多様化により

70年代の「PPN定式化」が改良

例）パラメタが増える、距離依存(PPN関数)へ

今後の課題（ひとつの例）

修正重力の幾つかの模型

（およびダークエネルギー候補）

通常の物質とは異なり、

「補助場」が天体の外部にも存在してよい

漸近的平坦でない設定？

1）光の曲がり角の精密計算

例：Kottler時空（M, Λ）での論争

Islam, Lake, Rindler, Ishak, Sereno, Peacock, Arakida, Kasai, ...

2）重力波のエネルギー計算

例：BondiエネルギーやLandau-Lifshitz擬テンソルの正当性は（ヌルの）無限遠方の平坦性を要する

(初期宇宙の背景重力波のエネルギー密度評価の危うさ？)

ご清聴、ありがとうございます

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