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A Novel Hybrid Scheme for Lya Line Transfer

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Collaborators

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Monte Carlo Schemes

Zheng & Miralda-Escude 2002, ApJ, 578, 33
Ahn et al, 2002, ApJ, 567, 922
Tasitsiomi, 2006, ApJ, 645, 792
Verhamme et al, 2006, A&A, 460, 397
Dijkstra et al. 2006, ApJ, 649, 14
Hansen & Oh, 2006, MNRAS, 367, 979
Semelin et al. 2007, A&A 474, 365
Laursen et al., 2009, ApJ, 696, 853
Pierleoni et al. 2009, MNRAS, 393, 872
Zheng et al. 2010, ApJ, 716, 574
Zheng et al. 2011, ApJ, 726, 38
Yajima et al. 2012, MNRAS, 424, 884
Abe, MU, et al. 2018, MNRAS, 476, 2664
(direct SPH version: see Poster by M. Abe)

Mesh Schemes

Tasitsiomi, 2006, ApJ, 648, 762

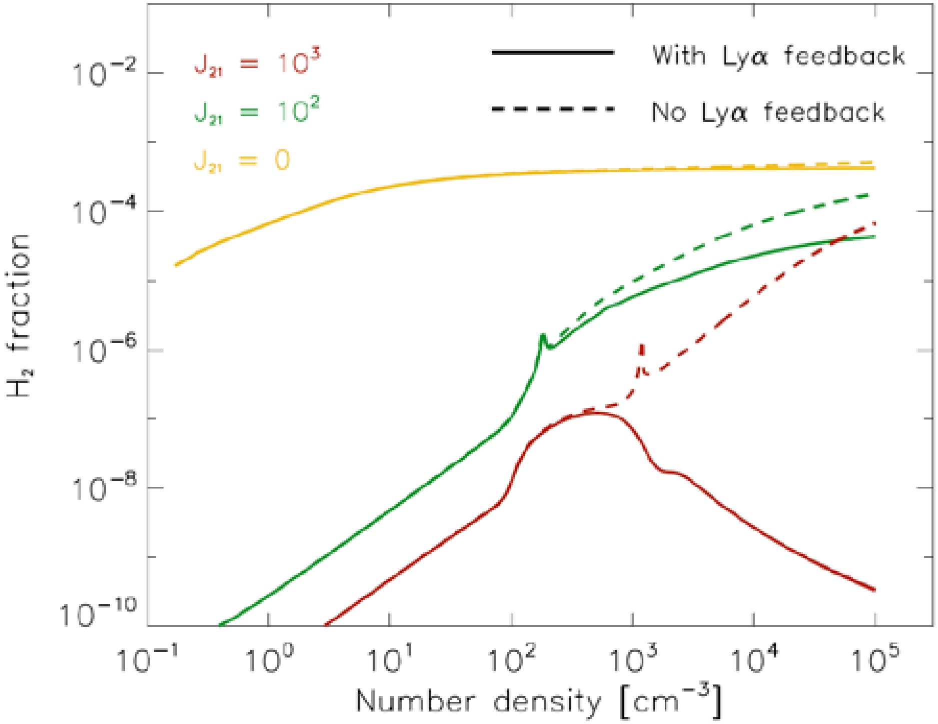
Toward RHD with Lya Transfer

- **Cooling radiation**
- **Line force (eg. Dijkistra & Loeb 2008; Smith et al 2017, Kimm et al. 2018)**
- **H^- Photodetachment**

H^- + hn ® H + e^-

for hn >0.76 eV

H^- Process in H_2 Formation
e^- + H ® H^- + hn
→ H^- + H ® H_2 + e^-



Johnson & Dijkstra 2017

One-zone model

Lya feedback

$J_{21} = J_{LW} / 10^{-21}$ erg $s^{-1} cm^{-2} Hz^{-1} str^{-1}$

Monte Carlo Schemes

- straightforward to implement
- subject to shot noises
- time consuming for all fluid elements to receive a sufficient number of photons

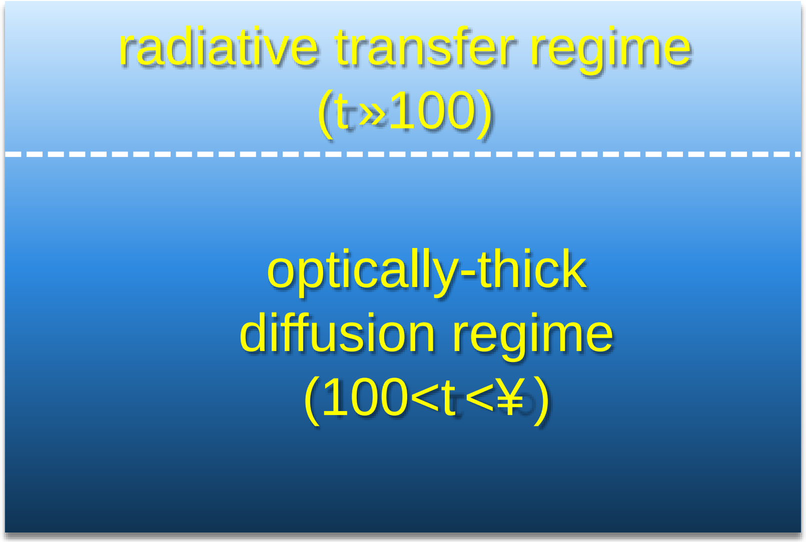
Mesh Schemes

- easy to couple with hydrodynamics
- transfer calculations in arbitrary optical-depth are time consuming

My talk
A novel mesh scheme coupling radiative diffusion with transfer

Hybrid Scheme

RDT: Radiative Diffusion and Transfer Scheme



radiative transfer regime
(t»100)

optically-thick
diffusion regime
(100<t<¥)

Ø Context:
Consider domains containing optical thick and thin regimes.

Ø Goal:
Speed up computation by coupling diffusion and transfer equations.
Maintain accuracy of a full transfer solution.

Ø Method:
Solve the diffusion equation in optically-thick regimes.
Solve the transfer equation in optically-thin regimes.
Use diffusion solution as boundary data for the transfer equation.

Diffusion equation of resonant line scattering

$$\boxed{\frac{1}{3f(x)^2}\frac{\partial^2 J(x)}{\partial \tau^2} = J(x) - \frac{1}{f(x)}\int R(x;x')J(x')dx'}$$

$x = \frac{\nu - \nu_0}{\Delta\nu_D}$ $f(x)$: line profile $R(x;x')$: redistribution function

For Lorentz profile $f(x) \simeq a/(\pi x^2)$ $(\tau \gg 1)$

Diffusion equation (Poisson-type)

$$\boxed{\frac{\partial^2 J}{\partial \tau^2}+\frac{\partial^2 J}{\partial \sigma^2} = -\frac{\sqrt{6}}{4\pi}\delta(\tau_s)\delta(\sigma_s)}$$

$$\sigma(x) \equiv \left(\frac{2}{3}\right)^{1/2}\int_0^x \frac{1}{f(x)}dx \simeq \left(\frac{2}{3}\right)^{1/2}\frac{\pi}{a}\frac{x^3}{3}$$

Harrington-Neufeld Solution for a Static Slab

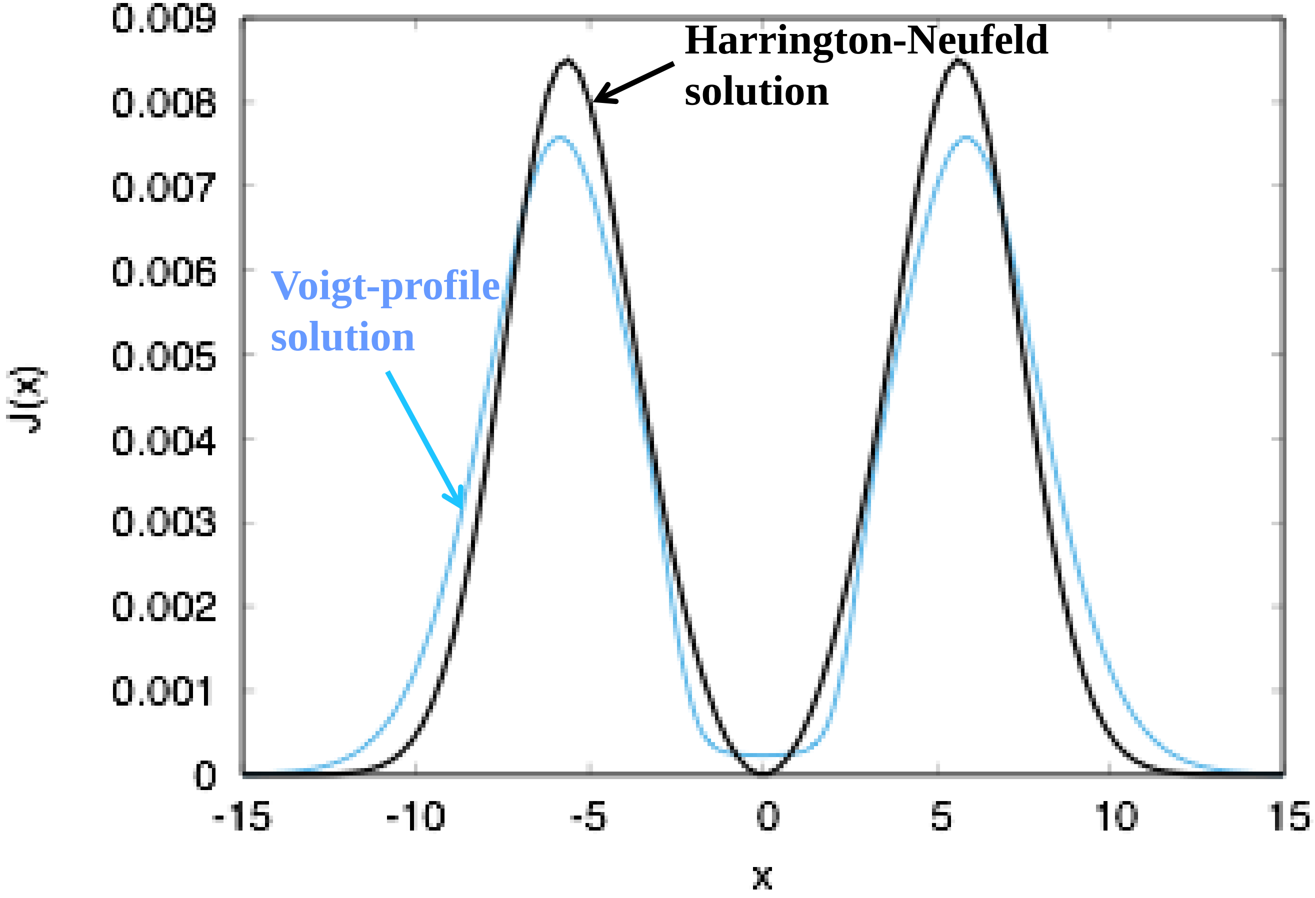
$$J(x) = \frac{\sqrt{6}}{24 a\tau_L}\frac{x^2}{\cosh\left[\sqrt{\pi^4/54}\left(\left|x^3\right|/a\tau_L\right)\right]}$$

Dijkstra-Haiman-Spaans Solution for a Static Sphere

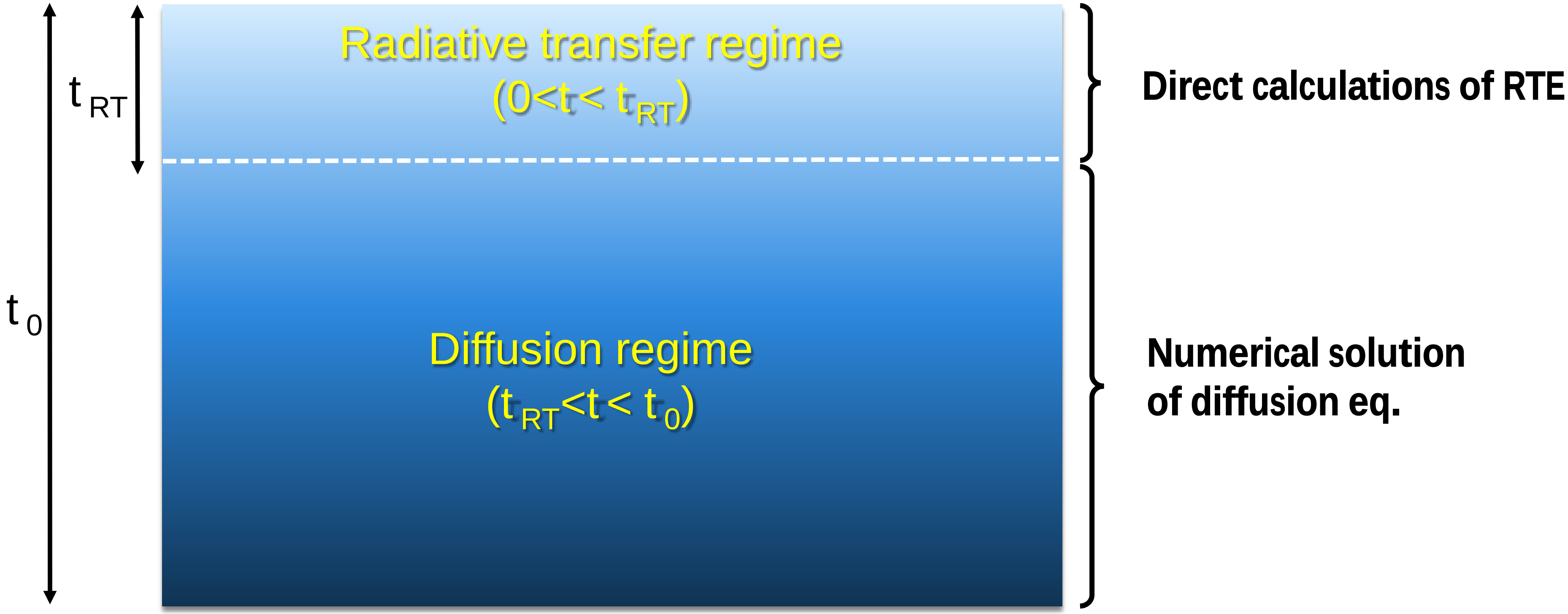
$$J(x) = \frac{\pi}{\sqrt{24 a\tau_L}}\times\frac{x^2}{1+\cosh\left[\sqrt{2\pi^4/27}\left(\left|x^3\right|/a\tau_L\right)\right]}$$

Comparison of Diffusion Solutions

$$\tau_0 = 10^4,\ T = 10\ \mathrm{K}$$



Test Calculations for Ly a Transfer



t_0

t_{RT}

Radiative transfer regime

$(0<t< t_{RT})$

Diffusion regime

$(t_{RT}<t< t_0)$

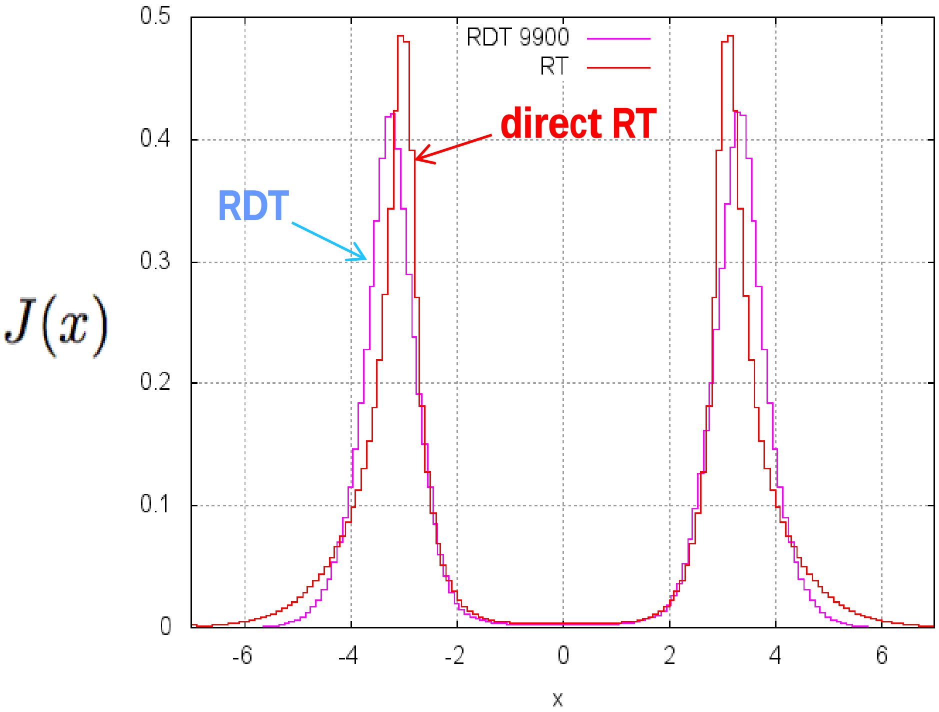
Direct calculations of RTE

Numerical solution of diffusion eq.

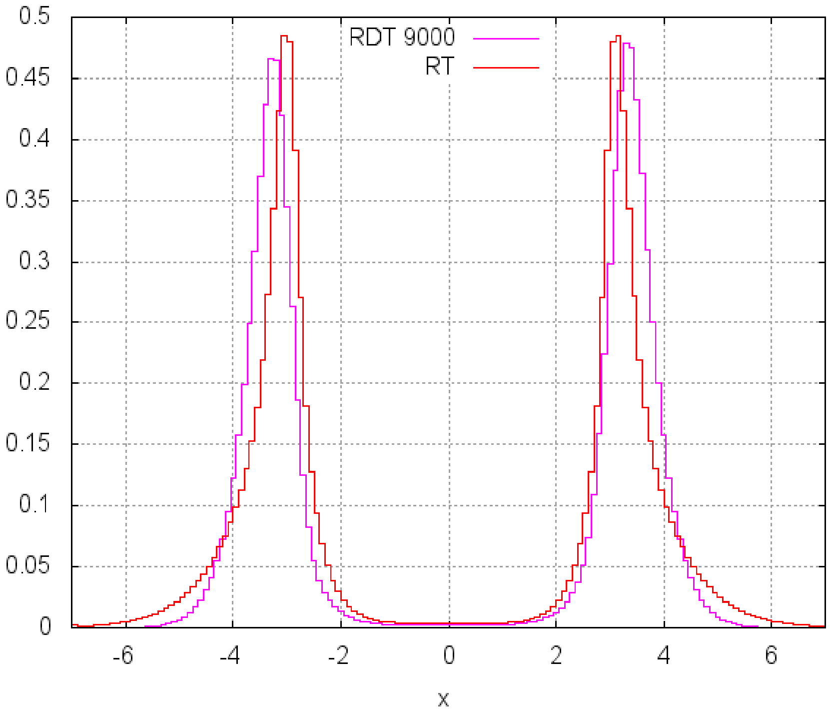
RDT vs RT

$t_0 = 10^4$, T= 10^4 K

t_{RT}=100=t_0/100



t_{RT}=1000 =t_0/10



Ø **Resultant mean intensity is insensitive to t_{RT}.**

Ø **RDT with t_{RT}=t_0/100 give mean intensity with an accuracy of a few - 10 %.**

Computational Time

$t_0 = 10^4$, D$t = 1$ (10^4meshes) $\qquad$ $t_{diff} = t_0 - t_{RT}$

Method	t_{RT}	t_{diff}	Iteration #	Computational time by one core	Acceleration
RT(Direct)	10000	0	262,920	44.4days	1
RDT	1000	9000	12,664	15hrs	70
RDT	100	9900	556	8min	8000

Summary

- We’ve developed a novel hybrid scheme, RDT (Radiative Diffusion and Transfer), with the diffusion solution based on the Voigt profile and the exact redistribution function for non-coherent resonant line scattering in arbitrary optical-depth media on meshes.

- RDT calculations with $t_{RT}= t_0/100$ give the mean intensity with an accuracy of a few - 10 %. The accuracy of radiation force is enhanced with increasing t_0.

- RDT scheme can reduce the computational cost dramatically and allow us to properly calculate the formation of Pop III objects or LAEs incorporating Lya feedback.