# Galactic Magnetic Field

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# Equipartition

- Energy-density equipartition between cosmic rays and magnetic fields

246 RAINER BECK

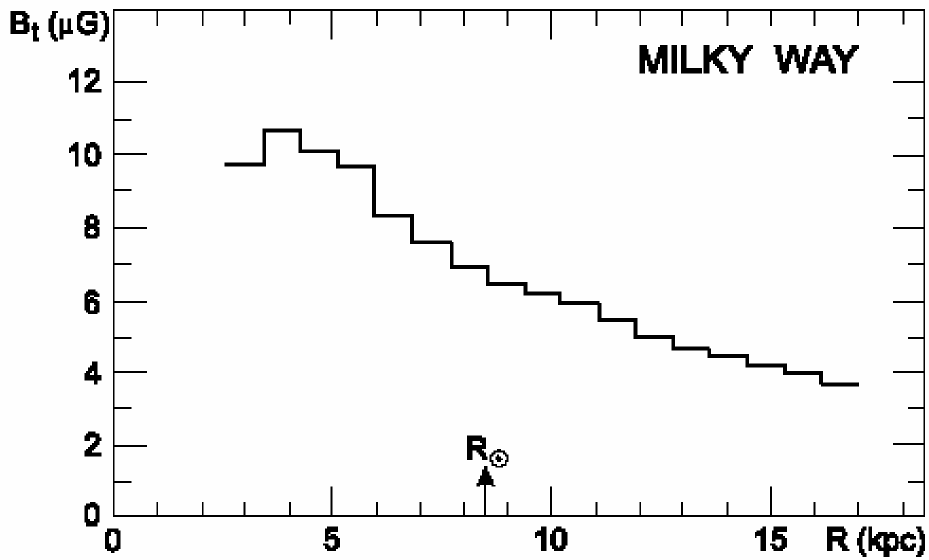



*Figure 1.* Strength of the total magnetic field in the Galaxy, averaged from the deconvolved surface brightness of the synchrotron emission at 408 MHz (Beuermann *et al.*, 1985), assuming energy equipartition between magnetic field and cosmic-ray energy densities (Berkhuijsen, personal communication). The accuracy is about 30%. The Sun is assumed to be located at $R = 8.5$ kpc.

Beuerman et al., A&Ap 153 (1985) 17

# Pulsars

- $<B_{reg}> = 1.4 \pm 0.2\ \mu G$ from measurement of dispersion measure and rotation measure

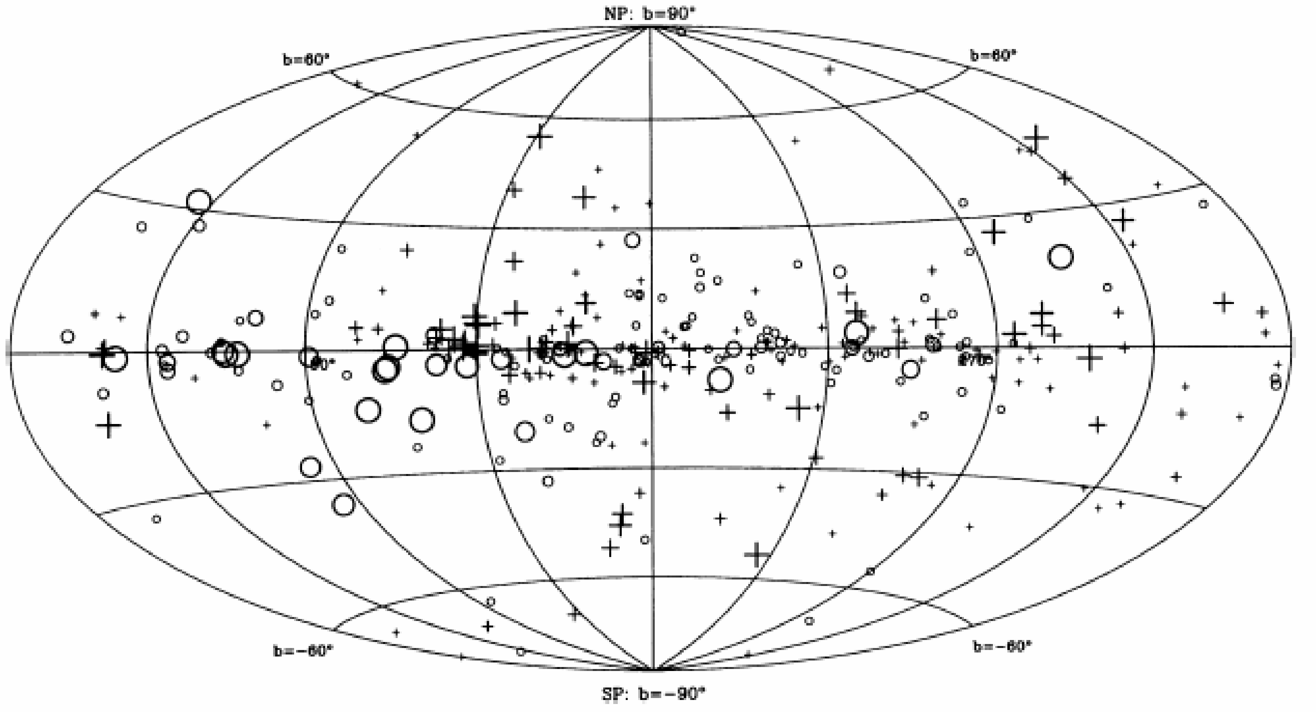



**Figure 8.** Distribution of $\langle B_{||} \rangle$ of all measured pulsar RMs in Galactic coordinates. Plus signs indicate that the average field is directed towards us, and circles indicate that the average field is directed away from us. The size of symbols is proportional to the field strength within limits of 0.8 and 2.5 μG.

Han et al., MNRAS 306 (1999) 405

# Plasma effects

- Maxwell eqn.

$$[\vec{E} = \mathbf{E}\exp(i\omega t - i\mathbf{k}\cdot\mathbf{x}), \quad \vec{B} = \cdots]$$

$$i\mathbf{k}\cdot\mathbf{E} = 4\pi\rho, \quad i\mathbf{k}\cdot\mathbf{B} = 0$$

$$i\mathbf{k}\times\mathbf{E} = i\frac{\omega}{c}\mathbf{B}, \quad i\mathbf{k}\times\mathbf{B} = \frac{4\pi}{c}\mathbf{j} - i\frac{\omega}{c}\mathbf{E}$$

- Electron motion

$$m\tfrac{\mathrm{d}}{\mathrm{d}t}\mathbf{v} = -e\mathbf{E}$$

$$\mathbf{v} = e\mathbf{E}/i\omega m$$

$$\mathbf{j} = -N_e e\mathbf{v} = \sigma\mathbf{E}, \quad \sigma = iN_e e/\omega m$$

- Charge conservation

$$i\omega\rho + i\mathbf{k}\cdot\mathbf{j} = 0$$

$$\therefore \rho = \omega^{-1}\mathbf{k}\cdot\mathbf{j} = \sigma\omega^{-1}\mathbf{k}\cdot\mathbf{E}$$

- Now Maxwell eqn.

$$\varepsilon \equiv 1 - 4\pi\sigma/i\omega$$

$$i\mathbf{k}\cdot\varepsilon\mathbf{E} = 0, \quad i\mathbf{k}\cdot\mathbf{B} = 0$$

$$i\mathbf{k}\times\mathbf{E} = i\frac{\omega}{c}\mathbf{B}, \quad i\mathbf{k}\times\mathbf{B} = -i\frac{\omega}{c}\varepsilon\mathbf{E}$$

- **k**, **E**, **B** are RHS

$$kE = \omega B/c = \omega(\omega\varepsilon/kc)/c$$

$$\therefore c^2k^2 = \varepsilon\omega^2$$

$$\varepsilon = 1 - \left(\frac{\omega_p}{\omega}\right)^2, \quad \omega_p \equiv \frac{4\pi N_e e^2}{m}$$

$$k = c^{-1}\sqrt{\omega^2 - \omega_p{}^2}$$

# Dispersion in plasma

- Phase velocity

$$\upsilon_{ph} \equiv \omega / k = c / n > c$$

$$n = \sqrt{\varepsilon} = \sqrt{1 - \omega_p{}^2 / \omega^2}$$

- Group velocity

$$\upsilon_g \equiv \partial\omega / \partial k = c\sqrt{1 - \frac{\omega_p{}^2}{\omega^2}}$$

- Arrival time

$$t_a = \int \upsilon_g{}^{-1} \mathrm{d}\ell$$

$$\upsilon_g{}^{-1} = \left( c\sqrt{1 - \frac{\omega_p{}^2}{\omega^2}} \right)^{-1} \approx \frac{1}{c}\left( 1 + \frac{1}{2}\frac{\omega_p{}^2}{\omega^2} \right)$$

$$\text{for} \quad \omega >> \omega_p$$

- Arrival time dispersion

$$\therefore \frac{dt_a}{d\omega} = -\frac{4\pi e^2}{cm\omega^3} \int N_e \mathrm{d}\ell$$

# Dispersion measure

- Delay time in the arrival of signals as a function of frequency

$$t_a = 4.15 \times 10^9 \sec \frac{1}{\nu^2} \int N_e \mathrm{d}\ell$$

- Dispersion measure [sec Hz$^{-2}$]

$$\mathrm{DM} = \int N_e d\ell = 2.410 \times 10^{-16} (t_1 - t_2) / \left( {}^{1}\!/\!_{\nu_1{}^2} - {}^{1}\!/\!_{\nu_2{}^2} \right)$$

# Faraday rotation

- Polarization field

$$\mathbf{P} = (\mathbf{D} - \mathbf{E})/4\pi$$
$$= (\varepsilon - 1)\mathbf{E}/4\pi$$
$$= N_e e\mathbf{r}$$

- Particle in mag. field

$$e\mathbf{E} \pm \frac{eB\omega\mathbf{r}}{c} = -m\omega^2\mathbf{r}$$

$$\therefore \mathbf{r} = -\frac{e}{m}\left(\frac{1}{\omega^2 \pm eB\omega/mc}\right)$$

$$\therefore \varepsilon = 1 - \frac{4\pi N_e e^2}{m\omega(\omega \pm \omega_c)}, \quad \omega_c = \frac{eB}{mc}$$

- Phase rotation

$$\omega_p \equiv 4\pi N_e e^2 / m \text{ (plasma freq.)}$$

$$\varepsilon_{R,L} = 1 - \frac{\omega_p^{\ 2}}{\omega(\omega \pm \omega_c)^2}$$

$$\varphi_{R,L} = \int k_{R,L}\,\mathrm{d}\ell$$

$$k_{R,L} = \frac{\omega}{c}\sqrt{\varepsilon_{R,L}} \approx \frac{\omega}{c}\left(1 - \frac{\omega_p^{\ 2}}{2\omega^2}(1 \mp \frac{\omega_c}{\omega})\right)$$

for $\omega >> \omega_c, \omega >> \omega_p$

- Rotation of linear polarization

$$\Delta\varphi = \tfrac{1}{2}\int (k_R - k_L)\mathrm{d}\ell = \tfrac{1}{2}\int \frac{\omega_p^{\ 2}\omega_c}{c\omega^2}\mathrm{d}\ell$$

$$= \frac{2\pi e^3}{m^2c^2\omega^2}\int N_e B_{//}\mathrm{d}\ell \quad \propto \omega^{-2}$$

# Linear polarization

- Stokes parameters

$$E_x = a_1 \cos(2\pi\nu t - \mathbf{k}\cdot\mathbf{r} + \phi_1)$$

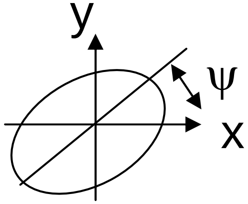
$$E_y = a_2 \cos(2\pi\nu t - \mathbf{k}\cdot\mathbf{r} + \phi_2)$$

$$I = a_1^{\;2} + a_2^{\;2}$$

$$Q = a_1^{\;2} - a_2^{\;2} = I\cos 2\chi \cos 2\psi$$

$$U = 2a_1a_2\cos\phi = I\cos 2\chi \sin 2\psi$$

$$V = 2a_1a_2\sin\phi = I\sin 2\chi$$




- Linear polarization and position angle

$$L = \sqrt{\overline{Q}^2 + \overline{U}^2}$$

$$\tan 2\psi = U / Q$$

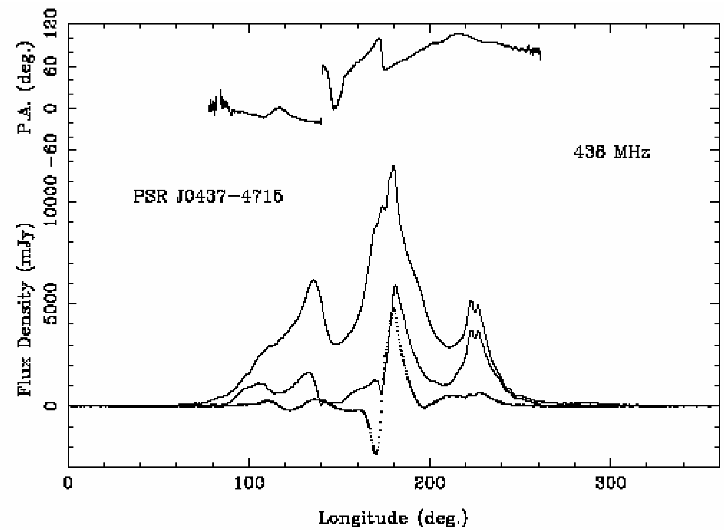



FIG. 1.—Mean pulse profile and polarization properties for PSR J0437−4715 at 438 MHz. In the lower part, the upper solid line is the total intensity (Stokes *I*), the lower solid line is the linearly polarized component (*L*) and the dotted line is the circularly polarized component (Stokes *V*). The dots are spaced at the sampling interval, *P*/1024. Profile baselines were defined by setting to zero the mean level between pulse phases 0.45 and 0.55 relative to the main peak. Polarized intensities are shown only where the total intensity exceeds the baseline level by five times the baseline rms deviation. Position angles above the ionosphere at 438 MHz are shown in the upper part of the Figure. Error bars are $\pm 2\,\sigma$ and position angles are plotted when the $2\,\sigma$ error is less than 10°. The zero of longitude is arbitrary.

# Rotation measure

- Faraday rotation of electric vector of linear polarization through a region in which the magnetic field is uniform

$$\phi / \lambda^2 = 8.12 \times 10^3 \int_0^1 N_e B_{//} \, \mathrm{d}\ell$$

- Rotation measure [rad/m$^2$]

$$\mathrm{RM} = (\phi_1 - \phi_2)/(\lambda_1{}^2 - \lambda_2{}^2)$$

# Line-of-sight magnetic field

$$\langle B_{//} \rangle = \int N_e B_{//} dl \Big/ \int N_e dl = \mathrm{RM}/(0.812\mathrm{DM})$$

(B in μG)

TABLE 1

ROTATION AND DISPERSION MEASURES

| PSR | $l$ (deg.) | $b$ (deg.) | Frequency Range (MHz) | DC ($10^{16}$ Hz) | DM ($cm^{-3}$ pc) | RM (rad $m^{-2}$) | $\langle B_{l,e} \rangle$* (microgauss) |
|---|---|---|---|---|---|---|---|
| 0329+54.... | 145 | − 1 | 280–485 | 11.110 ±0.002 | 26.776±0.005 | −63.7±0.4 | −2.93 ±0.02 |
| 0525+21.... | 184 | − 7 | 281–421 | 21.07 ±0.05† | 50.8 ±0.1† | −39.6±0.2 | −0.960±0.006 |
| 0531+21.... | 185 | − 6 | 365–414 | 23.5705‡ | 56.805‡ | −42.3±0.5 | −0.92 ±0.02 |
| 0809+74.... | 140 | +32 | 365–421 | 2.42 ±0.03 | 5.84 ±0.06 | −11.7±1.3 | −2.5 ±0.3 |
| 0818−13.... | 236 | +13 | 365–421 | 16.97 ±0.04 | 40.9 ±0.1 | − 2.8±1.7 | −0.08 ±0.05 |
| 0834+06.... | 220 | +26 | 365–414 | 5.35 ±0.02 | 12.90 ±0.04 | +24.5±2.5 | +2.3 ±0.3 |
| 0950+08.... | 229 | +44 | 280–421 | 1.230 ±0.003 | 2.965±0.007 | + 1.8±0.5 | +0.7 ±0.3 |
| 1133+16.... | 242 | +69 | 280–421 | 2.006 ±0.003 | 4.834±0.007 | + 3.9±0.2 | +0.99 ±0.06 |
| 1237+25.... | 252 | +87 | 365–414 | 3.840 ±0.004 | 9.254±0.008 | − 0.6±0.4 | −0.07 ±0.05 |
| 1508+55.... | 91 | +52 | 281–421 | 8.133 ±0.005 | 19.60 ±0.02 | + 0.8±0.7 | +0.05 ±0.04 |
| 1604−00.... | 11 | +36 | 365–410 | 4.45 ±0.02 | 10.72 ±0.05 | ... | ... |
| 1642−03.... | 14 | +26 | 365–421 | 14.816 ±0.004 | 35.71 ±0.01 | +16.5±2.5 | +0.58 ±0.09 |
| 1706−16.... | 6 | +14 | 365–421 | 10.37 ±0.03 | 24.99 ±0.08 | ... | ... |
| 1818−04.... | 26 | + 5 | 365–421 | 35.06 ±0.04 | 84.48 ±0.08 | +70.5±7.5 | +1.0 ±0.1 |
| 1911−04.... | 31 | − 7 | 365–414 | 37.10 ±0.02 | 89.41 ±0.04 | ... | ... |
| 1929+10.... | 47 | − 4 | 365–410 | 1.318 ±0.001 | 3.176±0.003 | − 8.6±1.8 | −3.3 ±0.7 |
| 1933+16.... | 52 | − 2 | 365–421 | 65.78 ±0.02 | 158.53 ±0.05 | − 1.9±0.4 | −0.015±0.003 |
| 2016+28.... | 68 | − 4 | 365–421 | 5.88 ±0.01 | 14.16 ±0.03 | −34.6±1.4 | −3.0 ±0.2 |
| 2021+51.... | 88 | + 8 | 365–414 | 9.369 ±0.002 | 22.580±0.004 | − 6.5±0.9 | −0.36 ±0.05 |
| 2045−16.... | 31 | −33 | 281–410 | 4.775 ±0.004 | 11.51 ±0.01 | −10.8±0.4 | −1.15 ±0.04 |
| 2111+46.... | 89 | − 1 | 365–414 | 58.6 ±0.2 | 141.4 ±0.4 | −223.7±2.2 | −1.95 ±0.03 |
| 2217+47.... | 98 | − 8 | 365–414 | 18.06 ±0.02 | 43.52 ±0.05 | −35.3±1.8 | −1.00 ±0.05 |
| 2303+30.... | 98 | −27 | 365–410 | 20.70 ±0.05 | 49.9 ±0.2 | ... | ... |

* A positive field component is directed toward the observer.

† Manchester (1971*b*).

‡ Richards *et al.* (1970).

Manchester, ApJ 172 (1972) 43

# Structure of the Regular field

- Pitch angle ~ -8°
  Cf. Spiral arms: -18°(stars) -13°(gas)
- Field reversals

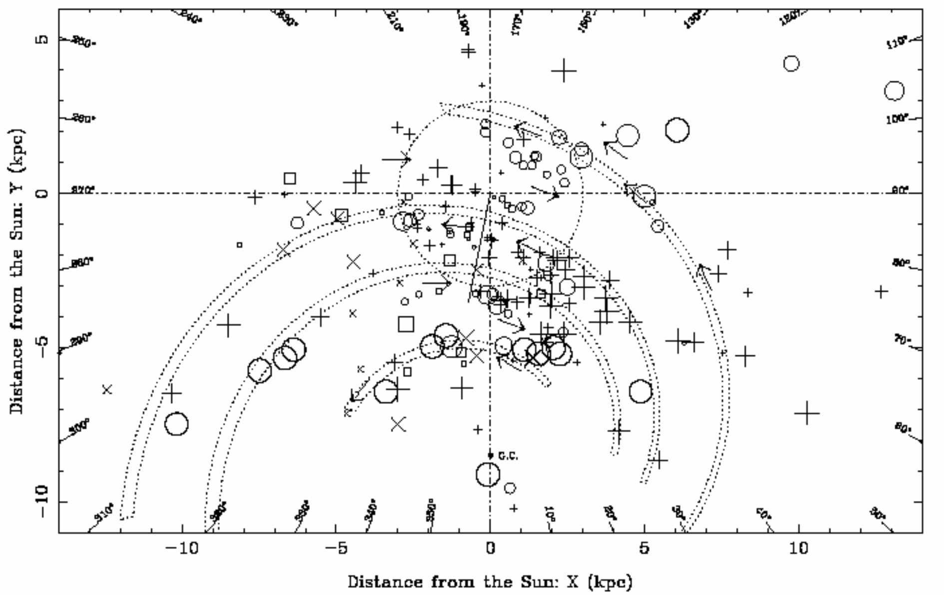



*Figure 2.* The distribution of the *RM*s of pulsars within 8° of the Galactic plane. Positive *RM*s are shown as crosses, negative *RM*s as circles. The most recent *RM* data are indicated by × and open squares. The symbol sizes are proportional to the square root of $|RM|$, with the limits of 5 and 250 rad $m^{-2}$. The directions of the bisymmetric field model are given as arrows. The approximate location of four spiral arms is indicated as dotted lines. The dotted circle has a radius of 3 kpc (from Han *et al.*, 1999a).

Han et al., MNRAS 306 (1999) 405

# Turbulent fields

- Sync. emission along the local arm (regular field vanishes) $\Rightarrow$ there is turbulent field!
- Starlight and synchrotron polarization data $\Rightarrow$ $B_{turb} \sim 5\ \mu G$, $L_{turb} \sim 55$ or 10-100 pc
- Depolarization by turbulent fields at cm-radio $\Rightarrow$ $L_{turb} \sim 25 pc\ f^{1/3}$ (f: filling factor of turbulent cells)
- Faraday dispersion by turbulent fields at dm-radio $\Rightarrow$ $L_{turb} \sim 7 pc\ f^{-1}$
- $L_{turb} \sim 20$pc and f~0.4?

# Summary

- Regular field: 1.8±0.3 μG
- Local total field: ~5 μG
- Stronger towards the Galactic center
- At least 3 (perhaps 5) field reversals in our Galaxy, separated by spiral arms.

References:

J.L. Han, Ap&SS 278 (2001) 181-184

R. Beck, Sp.Sci.rev. 99 (2001) 243-260

Text books