Neutrino Mass and Cosmology

Masahiro Takada (IPMU)
Collaborators

Toshifumi Futamase (Tohoku U.)
Kiyotomo Ichiki (U. Tokyo)
Issha Kayo (IPMU)
Eiichiro Komatsu (UT Austin)

Shun Saito (U. Tokyo)

Tomo Takahashi (Saga)
Atsushi Taruya (U. Tokyo)
Neutrinos

• Known as one of fundamental elementary particles, involved in SM
• Only has weak interactions (no charge and very light), so very difficult to directly see
• Yet not know much about neutrinos, mass unknown yet

The Nobel Prize of Physics, 2002

Prof. Koshiba

Prof. Davis

No doubt neutrinos are very interesting particles to explore!
Cosmic Thermal History

Thermal equilibrium \( f(\varepsilon) = \left[\exp(\varepsilon / T) \pm 1\right]^{-1} \)

- Neutrinos didn’t annihilate to photons
  - \( \nu, \bar{\nu}, \gamma, e^-, e^+ \)
    \( T_\gamma = T_e = T_\nu \)
  - \( T \sim 0.5 \text{MeV}: \) electrons and positrons annihilate
    \( e^- + e^+ \rightarrow 2\gamma \)
    \( \nu, \bar{\nu} \rightarrow \gamma, \text{a few } p, e^- \) \( T_\gamma > T_\nu \)
  - \( T \sim 1 \text{eV}: \) matter-radiation equality
  - \( T \sim 0.24 \text{eV}(\sim 3000K): \) recombination, CMB
    \( \gamma, \quad e^- + p \rightarrow H \)

- Neutrinos decouple
  - \( T \sim \text{a few MeV}: \) neutrinos decouple
  - \( \gamma, \nu, \bar{\nu}, e^-, e^+ \)
    \( T_\gamma = T_e = T_\nu \)

- Today
  - \( n_{\nu}, 0 \sim 100 \text{ cm}^{-3} \)
Neutrinos mass!

- The experiments (Kamiokande, SK, SNO, KamLAND) imply the total mass, \( m_{\text{tot}} > 0.06 \text{ eV} \); but the mass scale yet unknown.
- Neutrinos became non-relativistic at redshift when \( T_{\nu, \text{dec}} \sim m_{\nu} \)

\[
1 + z_{nr} \approx 189 \left( \frac{m_{\nu}}{0.1 \text{eV}} \right)
\]

- If \( m_{\nu} > 0.6 \text{eV} \), the neutrino became non-relativistic before recombination, therefore larger effect on CMB, vice versa.

- The cosmological probes measure the total matter density: CDM + baryon + massive neutrinos.

\[
\Omega_{m0} = \Omega_{\text{cdm}0} + \Omega_{\text{baryon}0} + \Omega_{\nu0}
\]

\[
f_{\nu} \equiv \frac{\Omega_{\nu0}}{\Omega_{m0}} = \frac{m_{\nu, \text{tot}}}{94.1 \text{eV} \Omega_{m} h^2} > 0.005
\]
In particular the cosmological linearized perturbation theory is remarkably successful: gives very robust, secure model predictions in structure formation.
Effect of finite-mass neutrinos on CMB

\[ \Omega_{m0} = 0.26 \rightarrow \Omega_{m0} = 0.33 \]

\[ h = 0.72 \rightarrow h = 0.66 \]

The m\_nu effect on CMB absorbed by other paras
The $m_{\nu}$ effect on CMB degenerate with $h$ and $\Omega_m$ that are sensitive to the distance (Ichikawa+ 05)

- WMAP5: CMB alone $m_{\nu,\text{tot}} < 1.3\text{eV}$; **WMAP5 + SN + BAO (no galaxy $P(k)$)** $m_{\nu,\text{tot}} < 0.6\text{eV}$ (CMB + geometrical probes)
- Seems best-available constraint from this method; if $m_{\nu} < 0.6\text{eV}$, as neutrinos become non-rel. btw $z \sim 1100$ and today
• Given the precise CMB constraints, combining CMB and LSS allows to probe the evolution of structure formation over \(z=\left[0, 10^3\right]\), thereby tightening the neutrino mass constraints (Hu, Eisenstein & Tegmark 98)
\( \text{\textbf{\( \Lambda \)CDM: SF scenario}} \)

- The density fluctuation field of total matter (mainly CDM) in the linear regime

\[
\delta_m(x,z) \equiv \frac{\rho_m(x,z) - \bar{\rho}_m(z)}{\bar{\rho}_m(z)} = D(z)\delta_m(x,z \approx 1000)
\]

- The 2nd-order diff. eqn. to govern the redshift evolution of density pert.: (FRW eqns + linearized Einstein eqns.) \( \delta G_{\mu\nu} = 8\pi G\delta T_{\mu\nu} \)

\[
\ddot{D} + 2H\dot{D} - 4\pi G\bar{\rho}_m D = 0
\]

Friction due to cosmic exp.  \hspace{1cm} Gravitational instability

where \( H^2(z) \equiv \left( \frac{\dot{a}}{a} \right)^2 = H_0^2\left[ \Omega_{m0}(1+z)^3 + \Omega_{de0}(1+z)^{3(1+w)} \right] \)

Matter  \hspace{1cm} Dark energy \hspace{1cm} \( (\Omega_{m0} + \Omega_{de0} = 1) \)

- Cosmic acceleration \( \Rightarrow \) the density growth is suppressed
The initial conditions on the perturbations are well constrained by the CMB.

• A variant in DE changes the growth of density perturbations.

• A test of gravity theory on cosmological scale:
  - CMB($z \sim 1000$)
  - Redshift survey ($0.5 < z < 1.3, z \sim 3$)
  - Weak Lensing ($0.2 < z < 1$)

• SCDM

• $\Lambda$CDM

Jenkins+99
Modeling nonlinear LSS formation - N-body simulations -

- The initial conditions of SF is now well constrained by CMB
- In a CDM model, gravity due to dark matter distribution plays a major role
- N-body simulation is the most powerful tool to study nonlinear clustering processes in structure formation
  - N-body particle = DM super particle; e.g. each N-body particle = 10^{11} M_{\text{sun}} = 10^{50} DM particles
  - Cold particle = no thermal velocity
- Simulations have been used in various cosmological studies
- A model with CDM plus neutrinos is still computationally challenging
\textbf{\( \nu + \Lambda \text{CDM} \) model}

- Neutrinos are very light compared to CDM/baryon
- The phase-space distribution of neutrinos, even after decoupling, obeys the relativistic FD dist. (specified by \( m_\nu \))
- The thermal velocity at redshift \( z \) relevant for LSS is larger than the gravity induced peculiar velocity

\[
\sigma_\nu(z) = \sqrt{\frac{p^2}{2m_\nu}} \approx 1800 \text{km/s} \left( \frac{m_\nu}{0.1 \text{eV}} \right)^{-1} (1 + z)
\]

- Even a massive cluster can’t much trap neutrinos

- \textit{The free-streaming scale}, the distance neutrino can travel with the thermal vel. during cosmic expansion

\[
\lambda_{fs}(z) \approx \sigma_\nu H^{-1} a^{-1} \Rightarrow k_{fs}(z) \approx \frac{0.037}{(1 + z)^{1/2}} \left( \frac{m_\nu}{0.1 \text{eV}} \right) \left( \frac{\Omega_m}{0.3} \right)^{1/2} h \text{ Mpc}^{-1}
\]

\( \lambda_{fs} \) is a 100Mpc scale, similar to BAO scales
**Suppression in growth of LSS**

- A mixed DM model: Structure formation is induced by the density fluctuations of total matter

\[
\delta_m = \frac{\rho_c \delta_c + \rho_b \delta_b + \rho_\nu \delta_\nu}{\rho_c + \rho_b + \rho_\nu} \equiv f_c \delta_c + f_b \delta_b + f_\nu \delta_\nu
\]

- The neutrinos slow down LSS on small scales
  - On large scales \(\lambda > \lambda_{fs}\), the neutrinos can grow together with CDM
    \[
    \delta_c = \delta_b = \delta_\nu
    \]
  - On small scales \(\lambda < \lambda_{fs}\), the neutrinos are smooth, \(\delta_\nu = 0\), therefore weaker gravitational force compared to a pure CDM case
    \[
    \ddot{\delta}_{cb} + 2H \dot{\delta}_{cb} - 4\pi G \bar{\rho}_m (1 - f_\nu) \delta_{cb} = 0, \quad \delta_\nu \approx 0
    \]

Total matter perturbations can grow! 

Suppresses growth of total matter perturbations
Suppression of linear $P(k)$

$\Omega_m = 0.26$, $f_\nu = 0.08$ ($m_\nu = 1.0\text{eV}$)

$z = 0$

Note: linear theory
The suppression is stronger at lower redshifts and at larger \( k \).

\[
\frac{\Delta P(k)}{P_{\nu=0}(k)} \Bigg|_{z \sim 0, k >> k_{\text{fs}}} \sim -8 f_{\nu}
\]

\( k_{\text{fs}}(z) \approx 0.35(1 + z)^{-1/2} \left( \frac{m_{\nu}}{\text{leV}} \right) \left( \frac{\Omega_m}{0.26} \right)^{1/2} h\text{Mpc}^{-1} \)
Suppression of linear $P(k)$ (contd.)

- A more realistic $f_{\nu}\sim 0.01$ ($m_{\nu}\sim 0.1\text{eV}$): the neutrinos became non-relativistic after $z\sim 10^3$
- The power spectrum amplitude is suppressed by $\sim 8\%$
We observe visible to explores invisibles

Dark Energy  Dark Matter

Neutrinos
Caution: “light” is biased tracers of mass

Different types of galaxies (and clusters) trace the total matter (mostly DM) distribution in different ways.
Large-scale structure probes

$k_{fs} \sim 0.03 \, h/\text{Mpc}$

for $m_\nu \sim 0.1 \, \text{eV}$
Sensitivity window of each probe

\[ \frac{P_{f_\nu}(k)}{P_{f_\nu=0}(k)} - 1 \]

- **CMB**
- **Galaxy Survey**
- **Ly-alpha**
- **Weak Lensing**

\[ \Omega_m = 0.26, f_\nu = 0.008 \ (m_\nu = 0.1 \text{eV}) \]

\[ z = 1000 \]

\[ k \ (h \ \text{Mpc}^{-1}) \]
Cosmological constraints on $M_\nu$

- **CMB alone**
  - Pros: precise modeling available, linear scale
  - Cons: smaller effect if $M_{\nu}<0.6\text{eV}$

- **Galaxy survey**
  - Pros: relatively easier to model in the weakly NL regime, a unique way to probe the scale-dependent suppression
  - Cons: galaxy bias uncertainty degenerate with $M_{\nu}$

- **Weak lensing (CMB lensing, cosmic shear)**
  - Pros: directly probe mass clustering
  - Cons: degenerate with $z_s$, sensitive to NL clustering

- **Ly-alpha forest**
  - Pros: probe smallest, linear scales, higher statistical precision
  - Cons: not straightforward to model
Cosmological constraints on $M_\nu$ (contd.)

Note: Lab. $m_{e,\nu}<a$ few eV

- **CMB alone**
  - Ichikawa+(05) $m_{v,\text{tot}}<2\text{eV} \ (95\% \text{CL})$ for a flat model; WMAP5(Komatsu +08), 1.5 eV (note: 0.6 eV if BAO+SN added, CMB lensing)

- **Galaxy clustering**
  - 2dF: Elgaroy+(02) $m_{v,\text{tot}}<2\text{eV} \ (k_{\text{max}}=0.1h/\text{Mpc})$ with the prior on $\Omega_m$
  - SDSS: Tegmark et al (06) $m_{v,\text{tot}}<0.9\text{eV} \ (k_{\text{max}}=0.2h/\text{Mpc})$ when combined with WMAP

- **Weak lensing**
  - Ichiki, MT, Takahashi (09) CFHTWL ~34 deg$^2$+WMAP5, $m_{v,\text{tot}}<1\text{eV}$

- **Ly-alpha forest** (+ galaxy survey)
  - SDSS: Seljak, Slosar & McDonald (06); $m_{v,\text{tot}}<0.17\text{eV}$
• The linear theory ceases to be accurate even on these large length scales (~50Mpc: \( \delta \sim O(0.1) \))

• The empirical model is employed: nuisance parameters Q and b introduced

\[
P_g(k) = b^2 P_m^L(k) \frac{1 + Q_{nl} k^2}{1 + 1.4 k}
\]
$f_{\nu} = \frac{\Omega_{\nu}^{0.8}}{\Omega_{m}}$

$M_{\nu,\text{tot}} < 0.94 \text{eV (95\% C.L.)}$
The linear theory assuming $\delta_m \ll 1$ is not sufficient.

The density perturbation is still small $O(\delta_m) \sim 0.1$ on relevant length scales.

The perturbation theory offers a yet another method for structure formation in the weakly nonlinear regime (Makino, Suto, Sasaki 92; Jain & Bertschinger 94).

$$\delta_m = \delta_m^{(1)} + \delta_m^{(2)} + \delta_m^{(3)} + \cdots$$

Mass conservation eq.
Euler eq.
Poisson eq.

$$P_m(k) = \left\langle \left( \delta_m^{(1)} + \delta_m^{(2)} + \delta_m^{(3)} + \cdots \right)^2 \right\rangle = P_m^{(11)} + P_m^{(22)} + P_m^{(13)} + \cdots$$
Modeling NL $P(k)$ for a MDM model

(Saito, MT, Taruya PRL 08)

• The first attempt to analytically model $P(k)$ in the weakly NL regime, based on cosmological perturbation theory (PT)

• Have to work with multi-component fluid system
  – NL clustering on small scales is mainly driven by CDM + baryon
  – Neutrinos with light masses remain to stay in the linear regime (can’t be much trapped by halos)

\[
\delta_{\text{cdm+baryon}} \equiv \delta_{\text{cb}} = \delta_{\text{cb}}^{(1)} + \delta_{\text{cb}}^{(2)} + \delta_{\text{cb}}^{(3)} + \cdots \quad \Rightarrow \text{Apply PT}
\]

\[
\delta_{\nu} \approx \delta_{\nu}^{(1)} \quad \Rightarrow \text{Linear theory (Solve Boltzmann eqns)}
\]

• NL $P(k)$ for a MDM model up to the 1-loop correct.

\[
P_{m}(k) = \left\langle \left( \frac{\delta \rho_m}{\bar{\rho}_m} \right)^2 \right\rangle = \left\langle \left\{ f_{cb} \left( \delta_{cb}^{(1)} + \delta_{cb}^{(2)} + \delta_{cb}^{(3)} \right) + f_{\nu} \delta_{\nu}^{(1)} \right\}^2 \right\rangle
\]
WFMOS achieves a few % accuracy in measuring $P(k)$ at each $k$ bins over $k=[0.03,1]$.

The suppression effect on $P(k)$ due to neutrinos is enhanced in the weakly nonlinear regime.

The PT model explicitly tells the valid $k$-range of linear theory. PT can be applied to larger $k_{\text{max}}$.

Saito, MT, Taruya, PRL, 2008
A MDM Simulation
(Brandbyge, Hannestad08)

256^3 CDM particles + 512^3 neutrino particles

CDM

M_nu=0.6eV

M_nu=0.3eV

z_i=4

512Mpc/h(Δ=10Mpc/h)

z=0
Nice agreement with the PT results (private communication)

Brandbyge, Hannestad08

\(\frac{\Delta P_m}{P_m} [\%] \)

\(k \ [h \text{Mpc}^{-1}]\)
Galaxy bias

In weakly nonlinear regime, straightforward to include a galaxy biasing effect in a perturbation theory manner, if galaxy bias is a local type

\[ \delta_g (x) = f[\delta_m(x)] \]

\[ = b_1 \delta_m(x) + \frac{1}{2} b_2 [\delta_m(x)]^2 + \frac{1}{3!} b_3 [\delta_m(x)]^3 + \cdots \]

\[ = b_1 \left[ \delta_m^{(1)} + \delta_m^{(2)} + \delta_m^{(3)} + \cdots \right] + \frac{1}{2} b_2 \left[ \delta_m^{(1)} + \delta_m^{(2)} + \delta_m^{(3)} + \cdots \right]^2 + \cdots \]

\[ P_g(k) = b_1^2 \left[ P_m(k) + b_2 P_{b2,m}(k) + b_2^2 P_{b22} \right] + N \]

Here

\[ P_{b2,\delta}(k) = 2 \int \frac{d^3q}{(2\pi)^3} P_m^L(q)P_m^L(|k - q|) \mathcal{F}_\delta^{(2)}(q, k - q), \]

\[ P_{b22}(k) = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} P_m^L(q)[P_m^L(|k - q|) - P_m^L(q)]. \]
• Nonlinear bias parameter $b_2$ introduces a scale-dependent modification on $P(k)$
• Even so, the galaxy $P(k)$ amplitude is suppressed by neutrino effect
There seems a space of bias parameters to reproduce the SDSS power spectrum and the simulated halo power spectrum. More physically reliable model, compared to Q_nl model

\[
P_g(k) = b^2 p_m^L(k) \frac{1 + Q_{nl}^2 k^2}{1 + 1.4k}
\]
Applying to SDSS DR4

- WMAP5+SDSS (PT model)  
  \[ M_{\nu,\text{tot}} < 1.01 \text{eV} \text{(95\%C.L.)} \]
- WMAP5+SDSS (Q-model)  
  \[ M_{\nu,\text{tot}} < 0.84 \text{eV} \text{(95\%C.L.)} \]
- Can be further improved by adding SN constraints
- Quantify a bias in parameter estimation for Q-model

Saito et al. in prep.

\[
L / L_{\max}
\]

\[ M_{\nu,\text{tot}} [\text{eV}] \]
Improving PT model

Include higher-order loop corrections

Error bars = $\sim 100 \text{ Gpc}^3$ (all-sky)
$\sim 5 \times \text{(Error)}$ for SUMIRE(5Gpc$^3$)

$z=0$

RPT (analytic model)

Linear

Crocce & Scoccimarro 08
Also Suto & Sasaki 91
Jeong & Komatsu 06
Taruya & Hiramatsu 08
Matsubara 08
Constraining neutrino mass with cosmological WL

Ichiki, MT, Takahashi 09

- Apply the NL model of $P(k)$ to CFHT weak lensing data ($\sim 60\,\text{deg}^2$)
- WL directory probe total matter (free of galaxy bias)
- Even though the data is from a small sky coverage ($60\,\text{deg}^2$), the constraint on $M_{\nu}$ is powerful: $M_{\nu,\text{tot}} < 0.54$ (WMAP5+SN +BAO)
BOSS: sampling the cosmic density field w/ galaxies

IPMU/U Tokyo is a full participating institute of SDSS-III

SDSS-I and SDSS-II

BOSS

M. White

The SDSS’s 2.5m telescope at Apache Point Observatory

Horizon simulation: A slice 500 $h^{-1}$Mpc across and 10 $h$ Mpc thick at $z=0.5$

http://www.sdss3.org

David Schlegel, Davis Fest, 18 Jan 2008
Planck+BOSS: $M_{\nu,\text{tot}} < 0.176\text{eV (95\%C.L.)}$

Ignoring neutrino mass in the parameter estimation may cause a bias in DE equation of state: not negligible

Japanese team is now trying to start the neutrino working group for BOSS
WL+galaxy P(k)+Planck: $M_{\nu} \sim 0.1\text{eV}(95\%\text{C.L.})$ achievable
Cosmological probes are, albeit indirect, a powerful method for constraining neutrino masses (total mass of three flavors). CMB + large-scale structure is particularly powerful, including galaxy clustering, weak lensing, Ly-alpha. Need to model structure formation up to the nonlinear regime for a mixed dark matter model, using perturbation theory methods and hybrid simulations. Future cosmological surveys look very promising, with the accuracy of 0.1eV (95% C.L.) achievable with SUMIRE and potential contributions to dark energy experiments. There is a lot of room to improve the neutrino mass constraints through bispectrum, redshift distortion, combining WL and galaxy P(k). CMB lensing has the potential to achieve 0.1eV (95%) with CMBPol.