An approach to strong CP problem without axion

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TABLE OF CONTENTS
1. Introduction
2. Strong CP Problem
3. Summary

Introduction

Some accomplishments in $SU(1, 1)$ model

- **Spontaneous P-C-T Violation**
  - [K. Inoue and N. Y., PTP'08a]

- **Generations of Quarks and Leptons**
  - [K. Inoue, PTP'95]

- **Hierarchical Mass Structures**
  - [K. Inoue, PTP'95]
  - [K. Inoue and N. Y., PTP'08a]

- **SU(1, 1) Model**
  - Supersymmetric Vectorlike Model

- **μ–problem**
  - [K. Inoue and N. Y., PTP'08b]

- **Strong CP Problem**
  - [K. Inoue and N. Y., PTP'08b]

- **Dark Matter Candidate (without R-parity)**
  - [K. Inoue, PTP'95]
Introduction

General framework of SU(1,1) model

From Vector Model to Chiral Model

Supersymmetric Vectorlike Model
\( (\text{ = SU}(1,1)\text{ Model}) \)

SU(1,1) Horizontal Symmetry

P-C-T invariance

\[ \text{Supersymmetric Chiral Model} \quad (\text{ = the MSSM}) \]

SU(1,1) Horizontal Symmetry

P-C-T invariance

Spontaneous Generation of Generations

Quarks, Leptons and Higgses
( + their conjugates )
SU(1,1) infinite-dimentional rep.

Vectorlike Quarks:
\[ Q = \{ q_1, q_2, q_3, \ldots \} \]
\[ \bar{Q} = \{ \bar{q}_1, \bar{q}_2, \bar{q}_3, \ldots \} \]

Nonvanishing VEV finite-dim. rep.

Three Generations of Quarks and Leptons
One Generation of Higgses

Chiral Quarks:
\[ q_1, q_2, q_3 \]
Possible solutions to the strong CP problem and their conditions
\[ \theta \equiv \theta_{\text{QCD}} + \theta_{\text{QFD}} (< 10^{-10}), \quad L_\theta = \theta (g_c^2 / 64 \pi^2) \epsilon_{\mu \nu \lambda \sigma} G^a_{\mu \nu} G^a_{\lambda \sigma} \]

\section*{Strong CP Problem}

\begin{itemize}
    \item \( \theta < 10^{-10} \): Naturalness Problem
\end{itemize}

- \textbf{Peccei-Quinn Mechanism}
  \begin{itemize}
    \item \( \theta = \text{dynamical valuable: axion} \)
    \item \( \theta = 0 : \text{minimum of vacuum energy} \)
    \item \textit{Anomalous U(1)}
  \end{itemize}

- \textbf{Spontaneous CP Violation}
  \begin{itemize}
    \item \( \theta_{\text{QCD}} = 0 : \text{CP (or T) invariance} \)
    \item \( \theta_{\text{QFD}} < \theta \): upper bound
    \item \textit{CP (or T) invariance}
  \end{itemize}

- \textbf{Massless Quark}
  \begin{itemize}
    \item \( \theta = \text{unphysical} \)
    \item \textit{Quark Chiral Rotation}
    \item \textit{Massless Quark}
  \end{itemize}

References:
- [R.D. Peccei and H.R. Quinn, PRL, PRD’77]
- [S. Weinberg, PRL’78; F. Wilczek, PRL’78]
- [R.N. Mohapatra and G. Senjanovic, PLB’78]
- [A. Nelson, PLB’84; S. Barr, PRL’84]
- [G.’t Hooft, Plenum’80]
- [G.’t Hooft, PRL’76, PRD’76]
- [C.G. Callan, R.F. Dashen, D.J. Gross, PLB’76]
How to solve the strong CP problem in $SU(1, 1)$ model (outline)

**Assumption**
- Vector symmetry is exactly maintained in the vectorlike model.

**SU(1,1) Model (Supersymmetric Vectorlike Model)**
- P-C-T Invariance
- $SU(1,1)$ Horizontal Symmetry

**Spontaneous P-C-T Violation**
- $\theta_{QCD} = 0$

**Natural Hierarchy of Yukawa Coupling**
- $\theta_{QFD} = 0$

**A Solution to the Strong CP Problem**
Strong CP Problem

**Tree-level analysis**

Superpotential relating to \( \theta_{QFD} \) in minimal \( SU(1,1) \) model

\[
W_{\text{Yukawa}} = \bar{U}QH + \bar{D}QH' + U\bar{Q}\bar{H} + D\bar{Q}\bar{H}'
\]

(Yukawa couplings)  (“Mirror couplings”)

\[
W_{\text{Quark, Higgs}} = (Q\bar{Q} + \bar{U}U + \bar{D}D)\Psi^F + H\bar{H}\Psi + H'\bar{H}'\Psi'
\]

(Quark couplings)  (Higgs couplings)

Tree level quark mass term \((i,j,k,\ell = 0, 1, 2, 3, \ldots; m,n = 0, 1, 2.\))

\[
\bar{Q}_U = \{\bar{u}_{\beta+m}|\bar{u}_{\beta+g+i}|\bar{q}_{-\alpha-j}\}, \quad Q_U = \{\bar{q}_{\alpha+n}|\bar{q}_{\alpha+g+k}|\bar{u}_{-\beta-i}\},
\]

\[
\mathcal{M}_U = \begin{pmatrix}
\frac{y^u_{mn}(\epsilon_x, \epsilon, \langle h \rangle)}{y^u_{i+g,n}(\epsilon_x, \epsilon, \langle h \rangle)} & \frac{y^u_{m,k+g}(\epsilon_x, \epsilon, \langle h \rangle)}{x^u_{i,\ell+g}(\langle \psi^F_{-g} \rangle)} & 0 \\
0 & \frac{y^u_{i+g,k+g}(\epsilon_x, \epsilon, \langle h \rangle)}{x^u_{j+g,k}(\langle \psi^F_{-g} \rangle)} & 0
\end{pmatrix}.
\]
**Strong CP Problem**

**Naive evaluation**

Quark mass determinants

\[ \theta_{\text{naive}} = \arg \det[M_U M_D] = -g \phi_{PQ} + [\phi_{PQ}-\text{independent term}] \]

- **$SU(1,1)$ invariance requires**
  \[ c = -(g - 1) - [SU(1,1) \text{ weight-dependent term}]. \]

- **$SU(1,1)$ breaking ($g \neq 0$) makes $U(1)_{PQ}$ anomalous symmetry.**
- Nambu-Goldstone boson becomes an axion.
- The axion solves strong CP problem.

However,

- **$U(1)_{PQ}$ symmetry should not suffer from any anomaly in vectorlike model.**
**Strong CP Problem**

### Alternative evaluation

**Principle**
- \( U(1)_{PQ} \) symmetry is exactly maintained in vectorlike model.

**Possible effects for \( \theta \)**
- Quark mass matrix

\[
\mathcal{M}_U = \begin{pmatrix}
  y_{mn}^u (\epsilon_x, \epsilon, \langle h \rangle) & y_{m,k+q}^u (\epsilon_x, \epsilon, \langle h \rangle) & 0 \\
  y_{i+q,n}^u (\epsilon_x, \epsilon, \langle h \rangle) & y_{i+q,k+q}^u (\epsilon_x, \epsilon, \langle h \rangle) & x_{i,l+q}^u (\langle \psi_F \rangle) \\
  0 & x_{j+q,k}^u (\langle \psi_F \rangle) & 0
\end{pmatrix}.
\]

(1,2)-, (2,1)- and (2,2)-entries give undetermined effects for \( \theta : f(\epsilon_x, \epsilon, \langle h \rangle) \).

These infinite matrix components make \( U(1)_{PQ} \) symmetry exact.
Quark mass matrix determinant ($\theta = \arg \det [M_U M_D]$)

$$\theta_{\text{tree}} = (A + B + C + \cdots) \varphi_{PQ} + [\varphi_{PQ}\text{-independent term}].$$

Exact vector symmetry "$U(1)_{PQ}$" requires $(A + B + C + \cdots) = 0$.

Tree level analysis for single higgs sector

$$\theta_{\text{tree}} = c(- \arg[\epsilon_x \epsilon^3] - \arg[\epsilon_y \epsilon'^3] + 4 \arg[\langle \psi_0^{F'} \rangle] + \arg[x'u'd']) + \arg[f^\beta\alpha(\epsilon_x \epsilon^3)] + \arg[f^{\gamma\alpha}(\epsilon_y \epsilon'^3)] + \arg\det[Y_{u}^{mn}] + \arg\det[Y_{d}^{mn}].$$

Conditions for satisfying $\theta_{\text{tree}} = 0$ (mod. $\pi$):

$$\arg[\epsilon_x \epsilon^3] = \arg[\epsilon_y \epsilon'^3] = 0 \ (\text{mod. } \pi), \quad \arg[\langle \psi_0^{F'} \rangle] = 0 \ (\text{mod. } \pi/4).$$
Strong CP Problem

Radiative correction

\[ \tilde{Q}_u = \{ \bar{u}_\beta | \bar{q}_{-\alpha} \}, \quad Q_u = \{ q_\alpha | u_{-\beta} \}, \]

\[ \mathcal{M}_u = \left( \begin{array}{c|c} X^{ij} (\epsilon's, \langle h \rangle's) & \Psi_{i,\ell+g}^u (\langle \psi_{-g}^F \rangle, \langle hh' \rangle) \\ \hline \Psi_{j+g,k}^{\prime \prime} (\langle \psi_{-g}^F \rangle, \langle hh' \rangle) & \mathcal{M} (\epsilon's, \langle h \rangle's) \end{array} \right). \]

These effects for \( \theta \) from \( X \) and \( \tilde{M} \)

\[ \theta \ni \varphi_{PQ} A, \quad B, \]

\[ A = [a_0 + a_1 (m_{\text{SUSY}}/M)^2 + \cdots], \quad B = (m_{\text{SUSY}}/M)^2 [b_1 + b_2 (m_{\text{SUSY}}/M)^2 + \cdots]. \]

- \( A = 0 \) because of maintaining vector symmetry \( U(1)_{PQ} \)
- \( B = O(m_{\text{SUSY}}^2/M^2) \approx O(10^{-26}). \)

\[ \Rightarrow \text{} O(1) \text{ effects are canceled for } \theta. \]
Consideration for this cancellation

- Exact vector symmetry $U(1)_{PQ}$ exists because of vectorlike Model.
  
  $\Rightarrow$ The origin of this cancellation is the vectorlike feature of the model.
  
  $\Rightarrow$ The vertex corrections of $X$ and $\tilde{M}$ with $\langle \Psi \rangle$'s will be also canceled.

$SU(1, 1)$ Model

- All radiative corrections for $\theta$ will be canceled when $\theta = 0$ is naturally realized at the tree level.
- We expect that $\theta$-term do not receive radiative corrections when $\theta_{\text{tree}} = 0$

Vacuum structure (hypothesis)

- In a model with some discrete symmetry, the directions of the vacua from the origin will be determined by the tree level analysis.
Summary

$SU(1, 1)$ model is a supersymmetric vectorlike model

- This model has $SU(1, 1)$, P, C and T symmetries.
- Yukawa couplings are systematic structures.
- Vector symmetry is not anomalous in the vectorlike model.

$\Rightarrow$ Simple conditions give $\theta_{\text{tree}} = 0$ at tree-level.

- Vacuum phase structure is determined by tree level analysis when some discrete symmetry exist.

$\Rightarrow$ $\theta = 0$ at quantum-level when $\theta_{\text{tree}} = 0$ at tree-level.
Q.1 Why three chiral generations of Quarks and Leptons exist?
Q.2 Why mass hierarchies of Quarks and Leptons exist?

<table>
<thead>
<tr>
<th>Generation</th>
<th>Mass (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>100</td>
</tr>
<tr>
<td>Second</td>
<td>10</td>
</tr>
<tr>
<td>Third</td>
<td>1</td>
</tr>
</tbody>
</table>

Natural coupling constant in Standard Model

[W. M. Yao et al., (PDG) '06]
SU(1,1) Symmetry

SU(1,1) is the simplest noncompact nonabelian group.

\[ g_{\mu\nu} = \text{diag}(+1, -1) \] is invariant under the SU(1,1) transformation: \( U\eta U^\dagger = \eta. \)

(cf. SU(2) case: \( \eta_{\mu\nu} = \delta_{\mu\nu} = \text{diag}(+1, +1): UU^\dagger = \delta). \)

<table>
<thead>
<tr>
<th>symmetry</th>
<th>SU(1,1)</th>
<th>SU(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>transformation</td>
<td>( U = e^{i \sum_{i=1,2,3} \theta_i H_i} )</td>
<td>( U = e^{i \sum_{i=1,2,3} \theta_i H_i} )</td>
</tr>
<tr>
<td>generators ( H_i (i = 1, 2, 3) )</td>
<td>( [H_1, H_2] = -iH_3 ) ( [H_2, H_3] = +iH_1 ) ( [H_3, H_1] = +iH_2 )</td>
<td>( [H_1, H_2] = +iH_3 ) ( [H_2, H_3] = +iH_1 ) ( [H_3, H_1] = +iH_2 )</td>
</tr>
<tr>
<td>unitary representation</td>
<td>infinite-dimensional</td>
<td>finite-dimensional</td>
</tr>
<tr>
<td>nonunitary representation</td>
<td>finite-dimensional</td>
<td>infinite-dimensional</td>
</tr>
</tbody>
</table>

**SU(1, 1) model**

- Infinite-dim. rep. \( \Leftrightarrow \) the MSSM superfields
- Finite-dim. rep. \( \Leftrightarrow \) spontaneously breakdown of SU(1, 1)