Gravitational Waves

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1. Gravitational wave theory briefly
2. Sources of gravitational waves

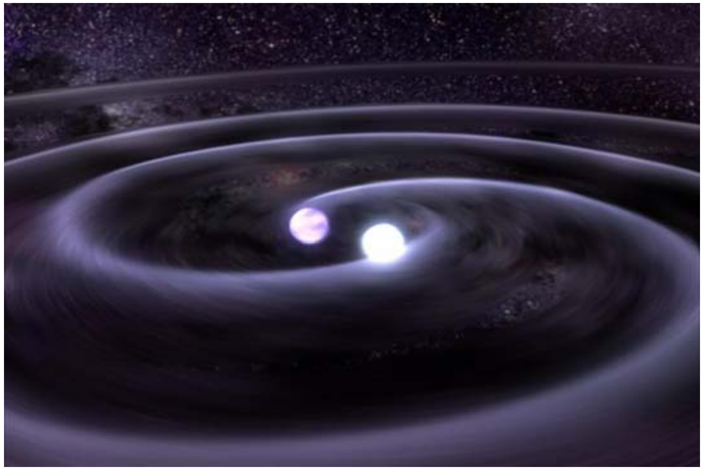
2A: High frequency ($f > 10$ Hz)

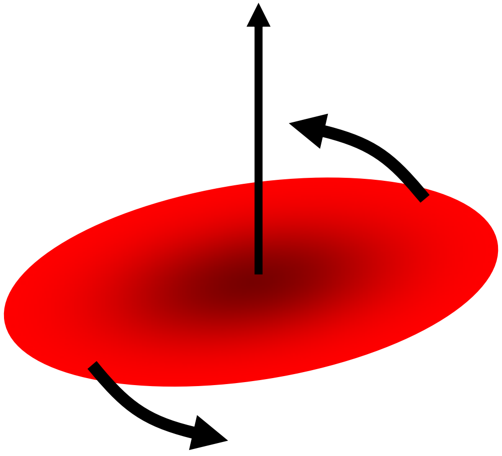
2B: Low frequency ($f < 10$ Hz)

(talk 2B only in the case I have time)

I Gravitational wave theory

- Dynamical (nonstationary), nonspherical massive objects emit gravitational waves (GWs);
- Specifically, GWs are emitted when tracefree-part of quadrupole moment of the system changes





Amplitude by Quadrupole formula

$$h_{ij} \approx \frac{G}{c^4} \frac{\ddot{I}_{ij}^{TF}}{D} \quad \begin{cases} I_{ij}^{TF} = \text{tracefree-part of quadrupole} \\ D = \text{distance to source} \end{cases}$$

Displacement $\Delta L \approx |h_{ij}| L$:

L=effective length of detector $\left\{ \begin{array}{c} L \sim \text{several 100 km} \\ \text{for Grand-based} \end{array} \right\}$

Order estimate

$$|\ddot{I}_{ij}^{TF}| \sim \frac{MR^2}{T^2}\varepsilon \sim Mv^2\varepsilon$$

$$\Rightarrow |h_{ij}| \approx 2\times 10^{-22}\left(\frac{100\ \mathrm{Mpc}}{D}\right)\left(\frac{M}{3M_\odot}\right)\left(\frac{v}{0.4c}\right)^2 \varepsilon$$

M : Mass, R : Characteristic radius,

v : Characteristic speed,

T : Characteristic dynamical time

ε: Nonsphericity (1 for binary, ~ 0.1 for SN)

This is amplitude at an instantaneous time.

Not the amplitude measured.

Effective amplitude ①

- In detection, amplitude is integrated by N, where N is number of cycle.
- N is approximately given by

$$N = f \times \max(\text{Obs. time } T_{obs}, \text{ emission time } \tau)$$

f is frequency of GWs.

- Emission time is $< \sim E/(dE/dt)_{\mathrm{GW}}$

$$\tau \sim \left(\frac{0.4c}{v}\right)^5 \frac{1}{f\varepsilon^2} \Rightarrow N \sim \left(\frac{0.4c}{v}\right)^5 \varepsilon^{-2}$$

where quadrupole formula

and virial relation $v^2 \sim GM/R$ is used.

Effective amplitude ②

$$h_{eff} = |h_{ij}|\sqrt{N}$$

$$\approx 2\times 10^{-22}\left(\frac{100\ \mathrm{Mpc}}{D}\right)\left(\frac{M}{3M_{\odot}}\right)\left(\frac{v}{0.4c}\right)^{-1/2}$$

for $T_{obs} > \tau$

$$\approx 2\times 10^{-22}\left(\frac{100\ \mathrm{Mpc}}{D}\right)\left(\frac{M}{3M_{\odot}}\right)\left(\frac{v}{0.4c}\right)^{2}\varepsilon\sqrt{fT_{obs}}$$

for $T_{obs} < \tau$

High velocity is not always important
for a source longterm integration is done

Maximum emission timescale

$$\tau \sim 2\times10^{6}\,\mathrm{sec}\left(\frac{f}{1\mathrm{Hz}}\right)^{-8/3}\left(\frac{M}{M_{\odot}}\right)^{-5/3}\varepsilon^{-2}$$

For $\tau < T_{obs} = 1\,\mathrm{yrs}$,

$$f_{\mathrm{crit}} \sim 0.3\mathrm{Hz}\left(\frac{M}{M_{\odot}}\right)^{-5/8}\varepsilon^{-3/4}$$

Low f (low v/c) source is weak emitter
→ emission timescale is longer.

Frequency

$$f \approx \frac{R}{\pi v}, \quad \pi f \approx \left(\frac{GM}{R^3}\right)^{1/2}$$

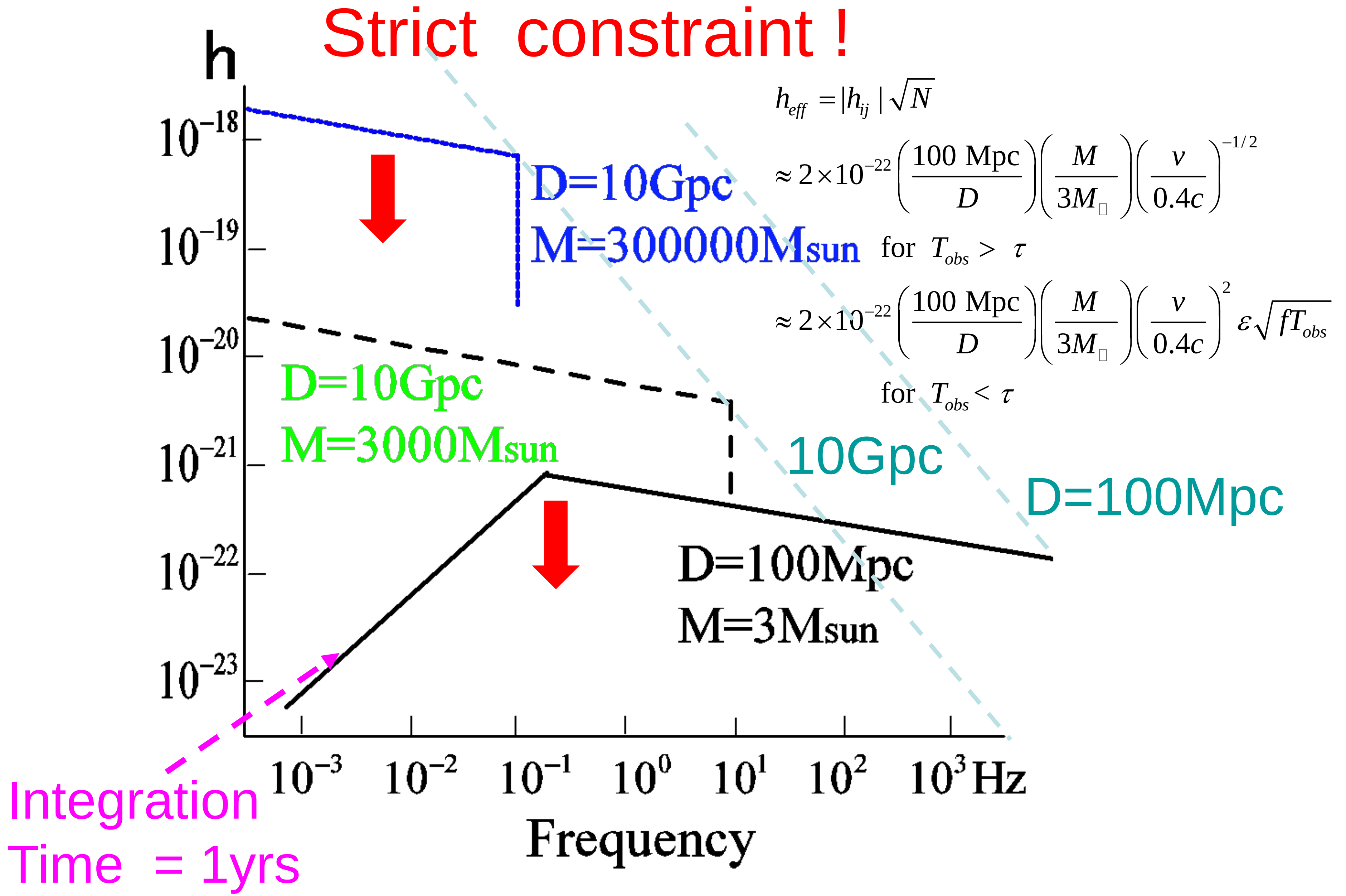
$$\Rightarrow \quad f \approx \frac{v^3}{\pi GM} \approx 4\text{kHz}\left(\frac{M}{M_\odot}\right)^{-1}\left(\frac{v}{0.4c}\right)^3$$

- Mass of sources of GWs: $M >\sim M_\odot$
- Velocity must be $< c$.

$\Rightarrow$ Frequency of GWs should be $< \sim$ 10kHz.

But no limit for low-frequency region.

Upper limit of GW amplitude



Nature of GW sources

- High mass
- Near periodic (longterm emitter)
- Small distance (frequent event)
- (high velocity is not very important)

→ Lower frequency is better region.

Discuss sources specifying frequency

2A Sources of ground detectors

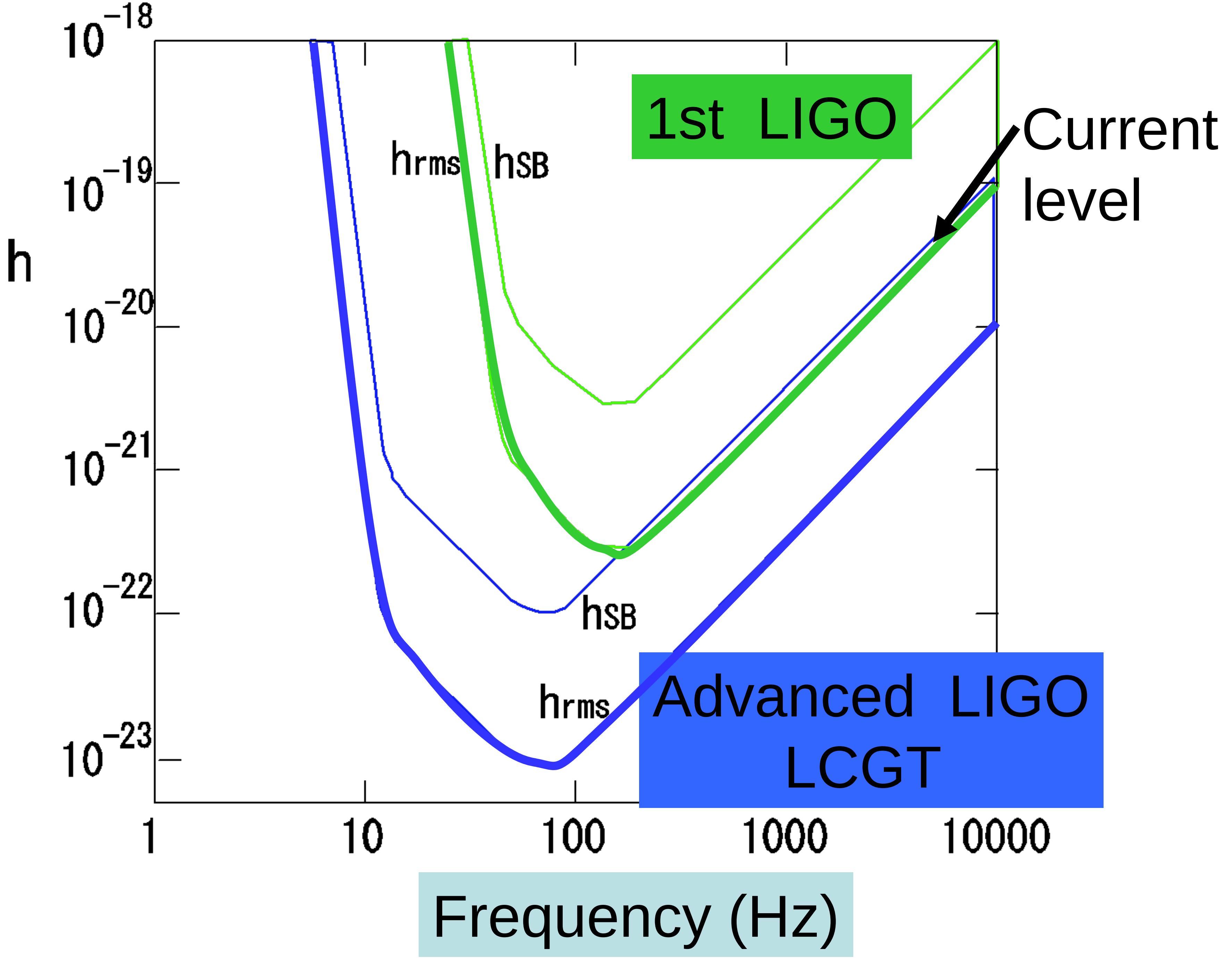
- Almost no sensitivity for $f < 10\ \mathrm{Hz}$ due to seismic noise
  → $10\ \mathrm{Hz} < f < 10\ \mathrm{kHz}$

$$f \approx 4\mathrm{kHz}\left(\frac{M}{M_\odot}\right)^{-1}\left(\frac{v}{0.4c}\right)^{3}$$

$$\Rightarrow \quad 0.4\left(\frac{v}{0.4c}\right)^{3} \le \frac{M}{M_\odot} \le 400\left(\frac{v}{0.4c}\right)^{3}$$

$$\Rightarrow \begin{cases} M = 2M_\odot : v/c \ge 0.07, & \text{mildly relativistic} \\ M = 20M_\odot : v/c \ge 0.15, & \text{relativistic} \end{cases}$$

$\Rightarrow$ Compact stars: Neutron star, black hole



1st LIGO
Current level
Advanced LIGO
LCGT
h
hrms
hSB
hSB
hrms
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
1
10
100
1000
10000
Frequency (Hz)

Sources (< ~200, 300M_{sun})

- Coalescence of neutron star (NS) and black hole (BH): NS/NS, BH/NS, BH/BH
- Supernova
- Oscillation of neutron stars

Primary sources

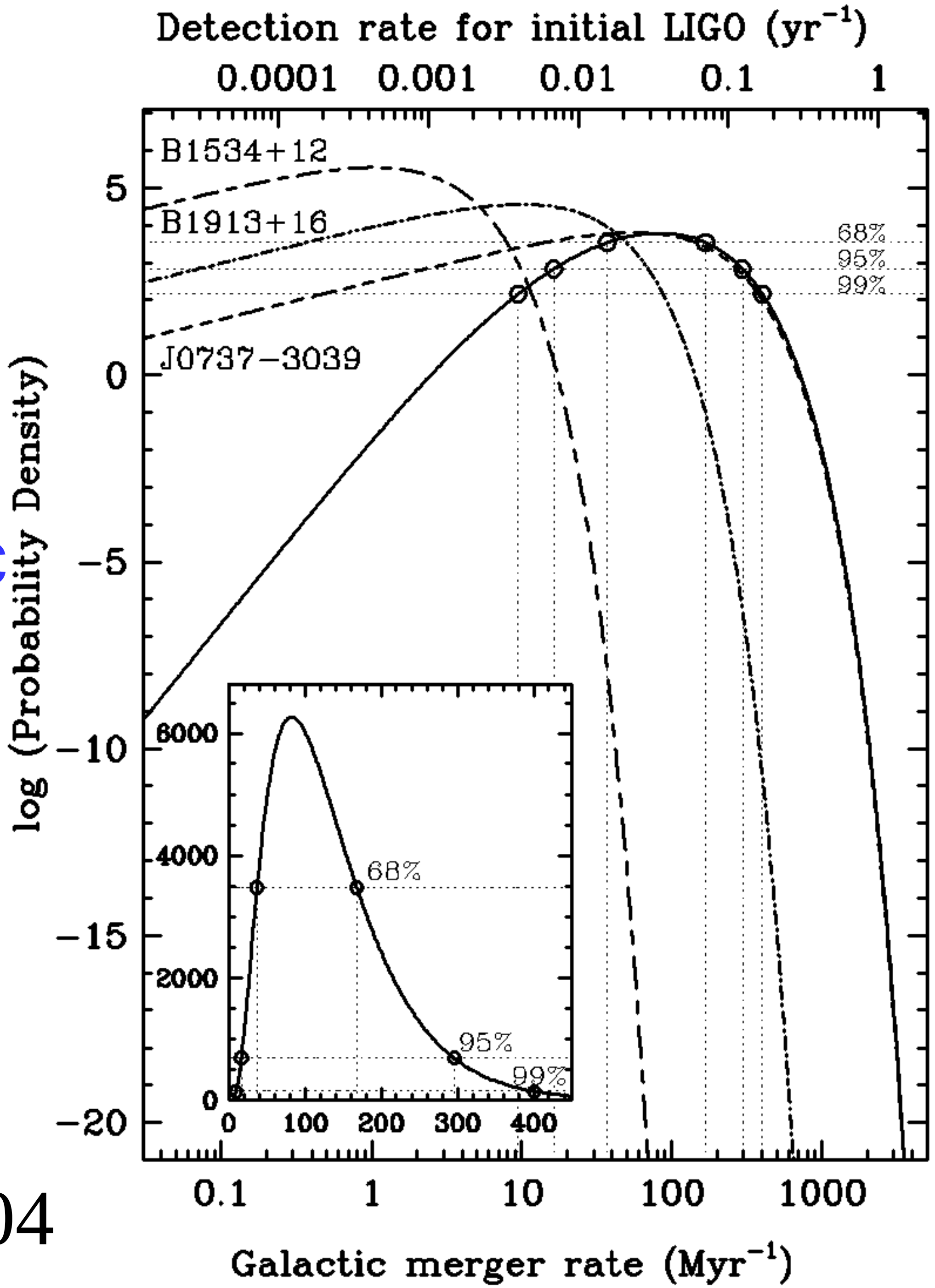
Merger rate of NS-NS

1 per ~10^4 yrs in our Galaxy

⇒1 per yrs in ~ 50-100 Mpc (<< 4 Gpc)

Not rare in the whole universe

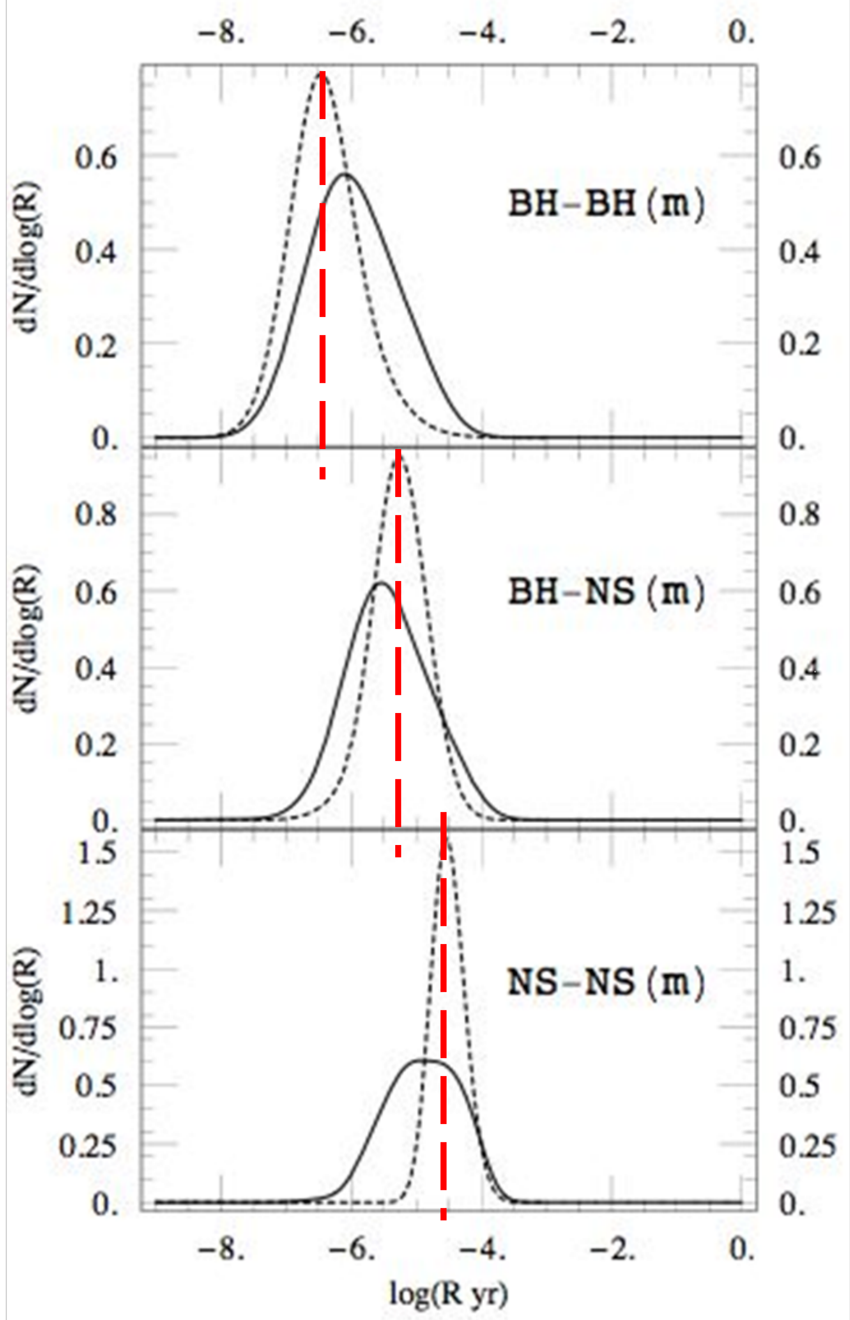
V. Kalogera et al. 04



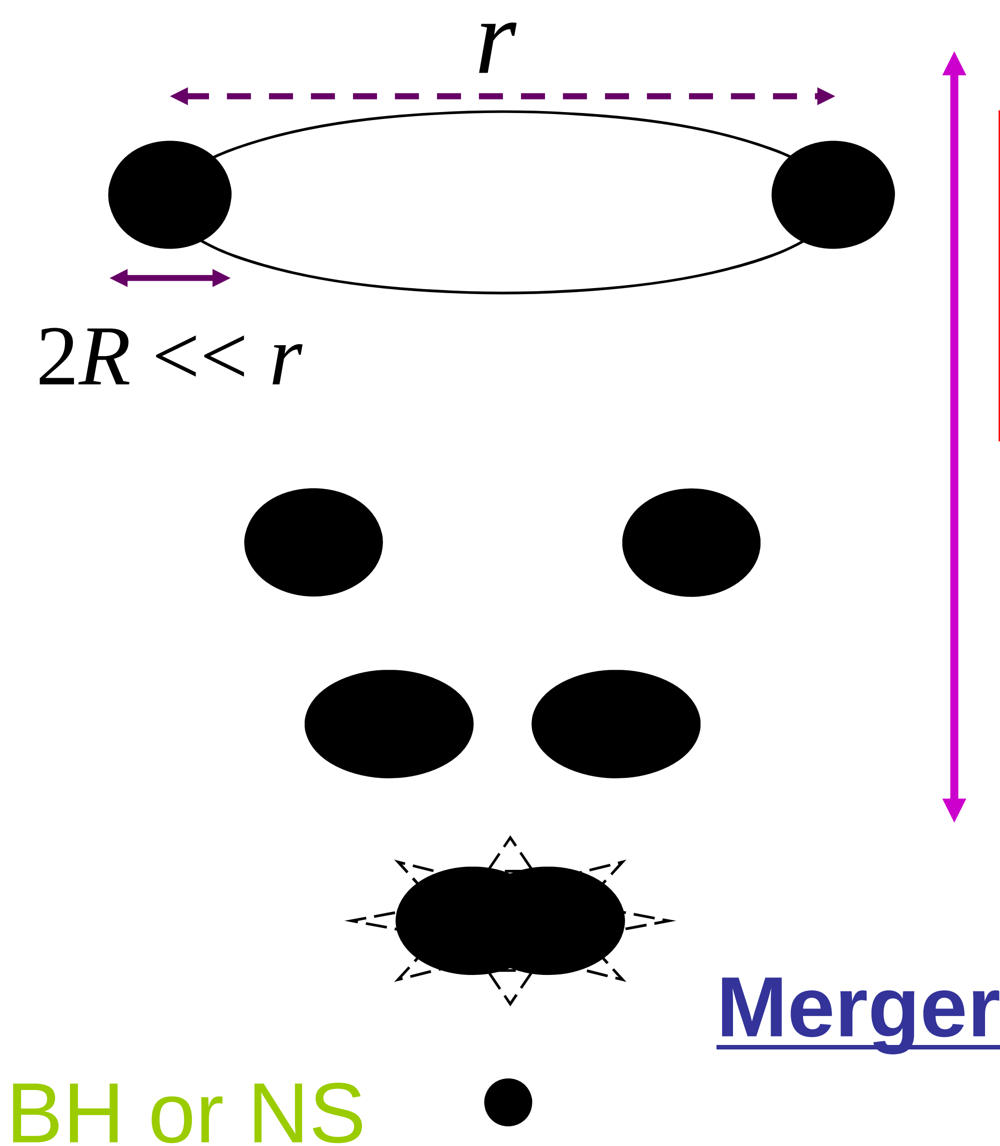
Merger rate predicted by population synthesis

Rate per galaxy
~ 0.01--0.1 NS-NS
→NOT SMALL
because of high M

Kalogera et al.
Astroph-0612144



Last ~ 10 minutes of coalescing binary



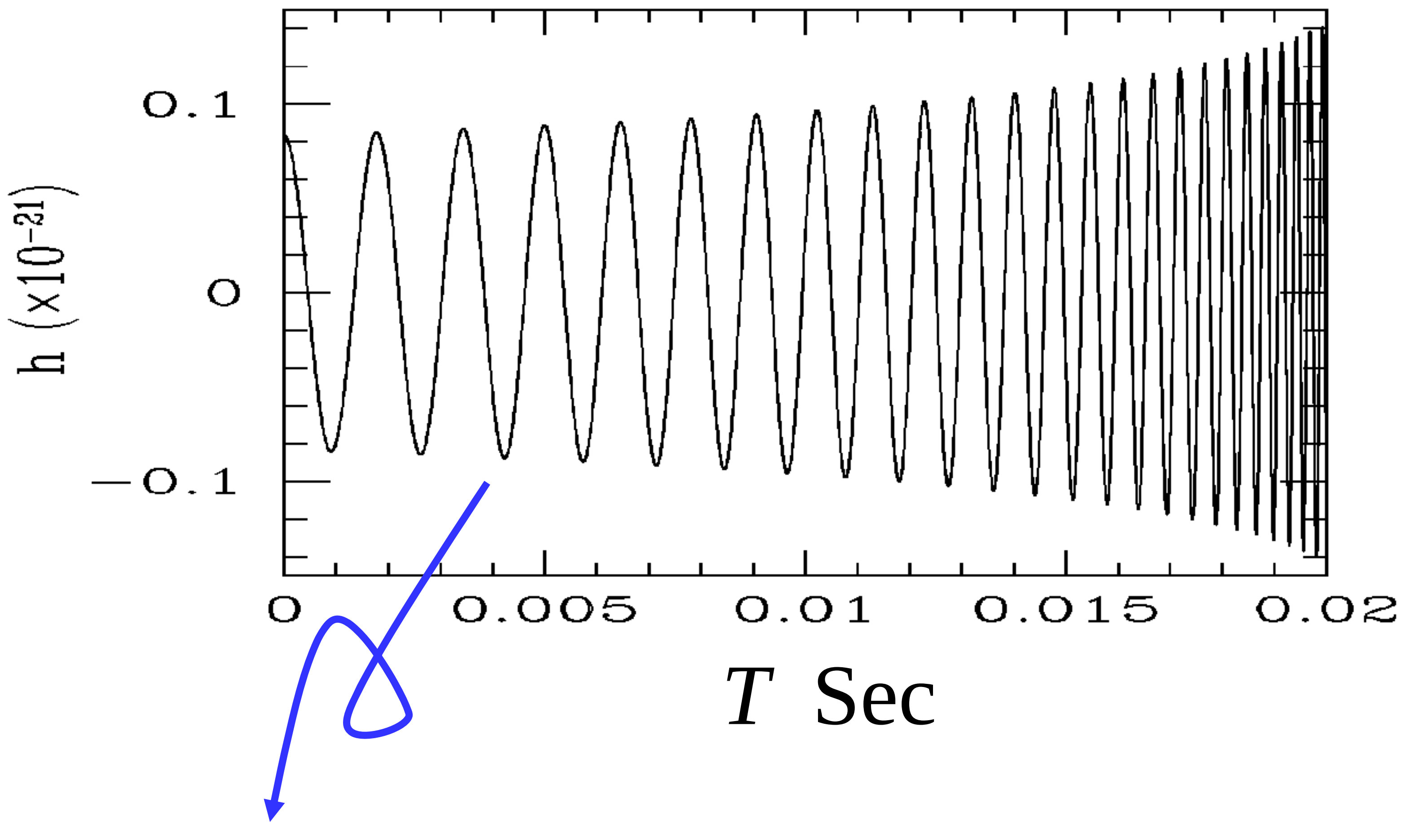
Adiabatic Inspiral

$r >> R$,

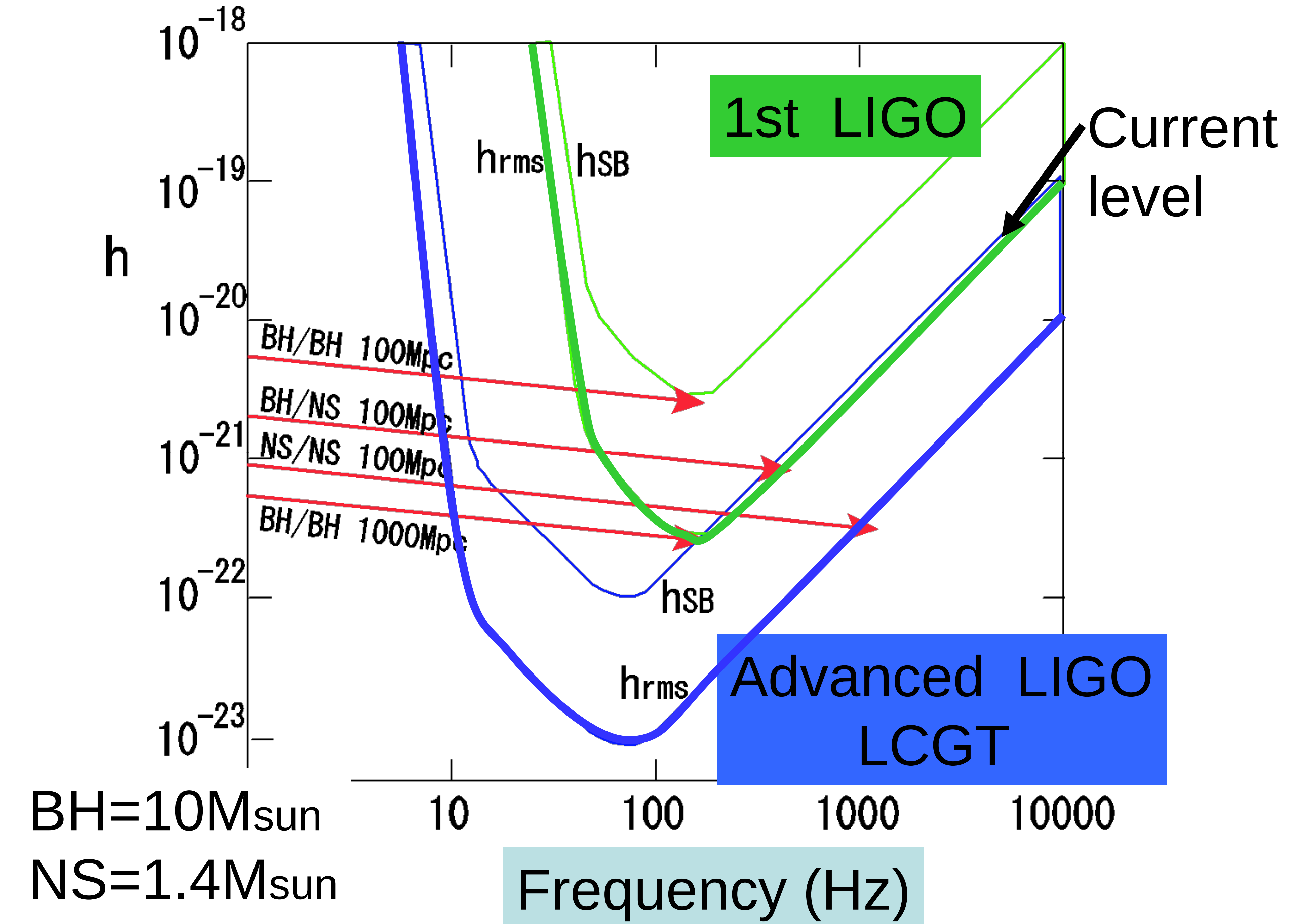
$T_{\text{emission}} >> P$

Orbital separation gradually decreases due to GW emission

Inspiral waveform



Determined only by
mass & spin of NSs, BHs

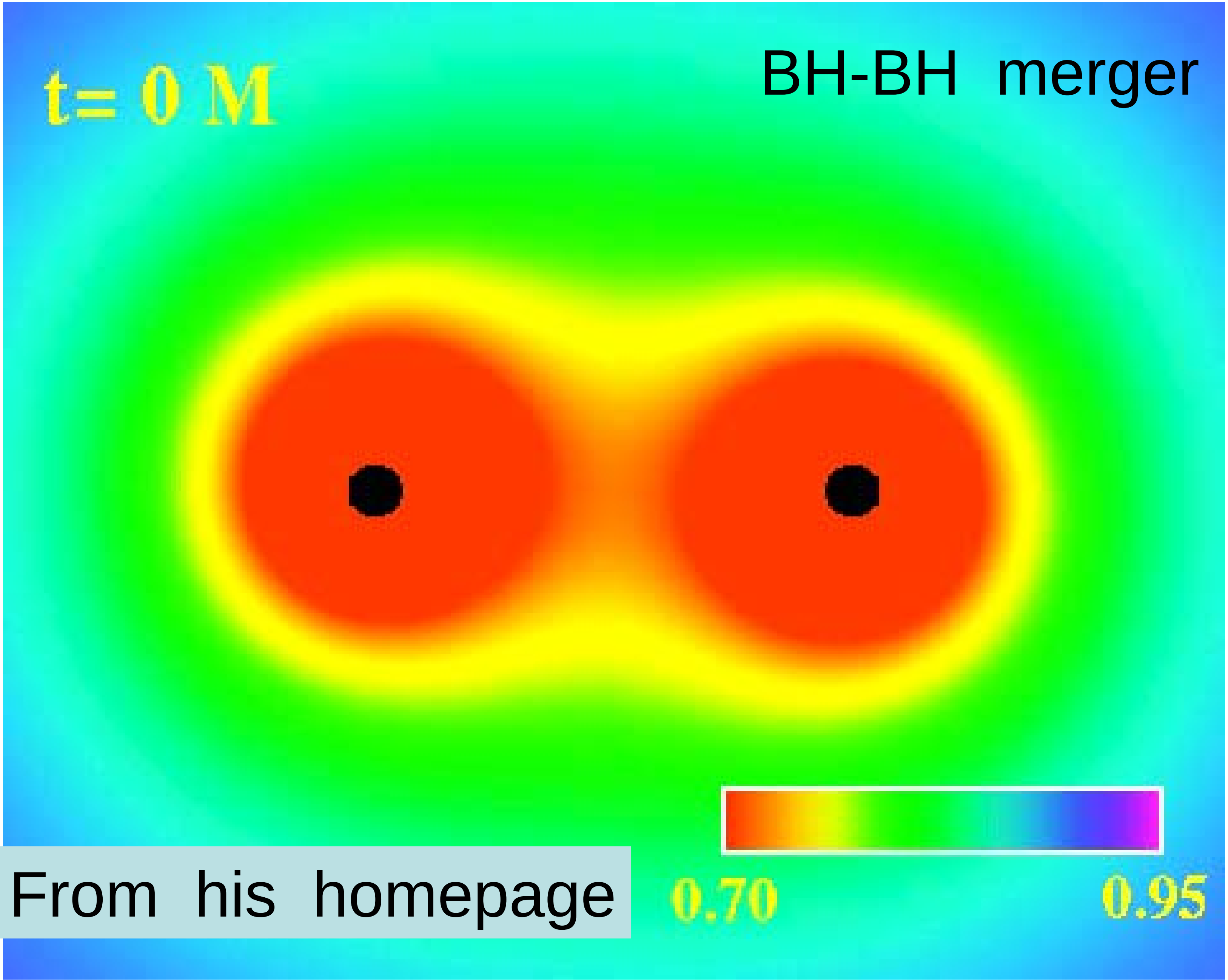


1st LIGO
Current level
h
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
hrms
hSB
BH/BH 100Mpc
BH/NS 100Mpc
NS/NS 100Mpc
BH/BH 1000Mpc
hSB
hrms
Advanced LIGO
LCGT
10
100
1000
10000
Frequency (Hz)
BH=10Msun
NS=1.4Msun

Primary goal of LIGO/LCGT

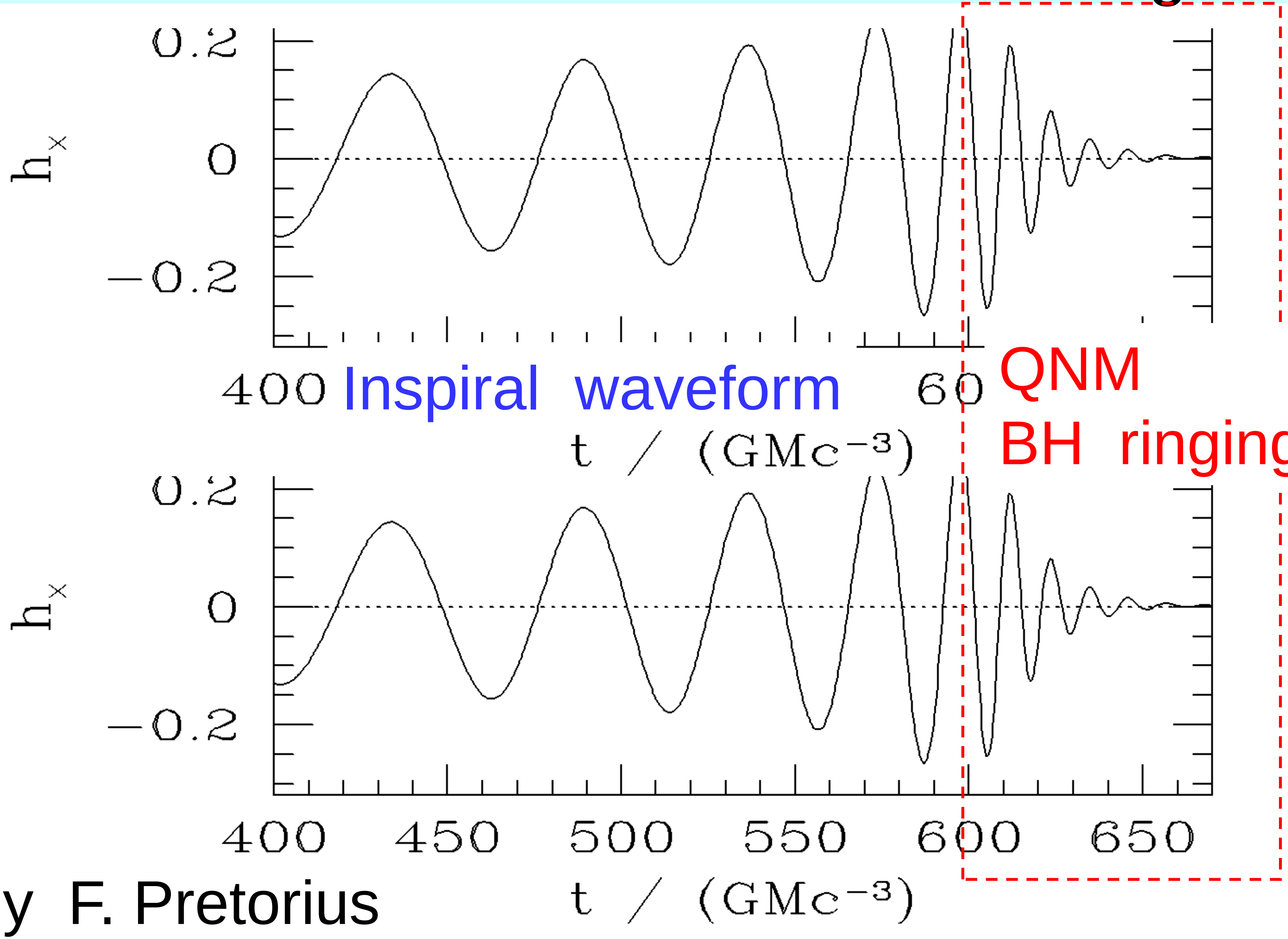
- Detection of GWs
- Detection of inspiral waveforms → Distribution of mass & spin of NS/BH
- Detection of quasi-normal mode of black hole → Prove BH exists and observe highly curved spacetime directly
- Determining the central engine of short gamma-ray bursts

Simulation by Pretorius: Lapse



From his homepage

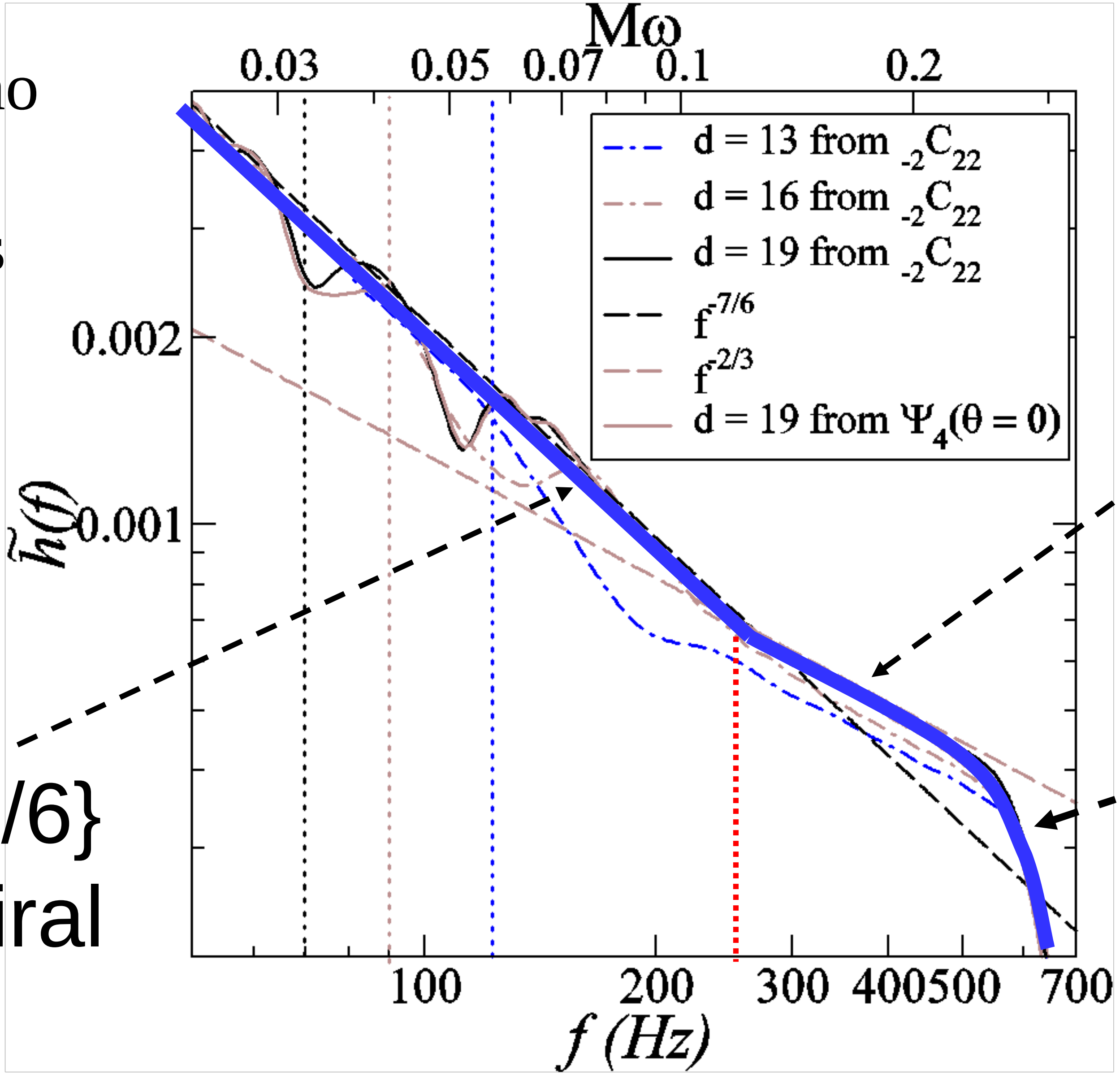
Gravitational waves from BBH merger



By F. Pretorius

Fourier spectrum (15+15Msun)

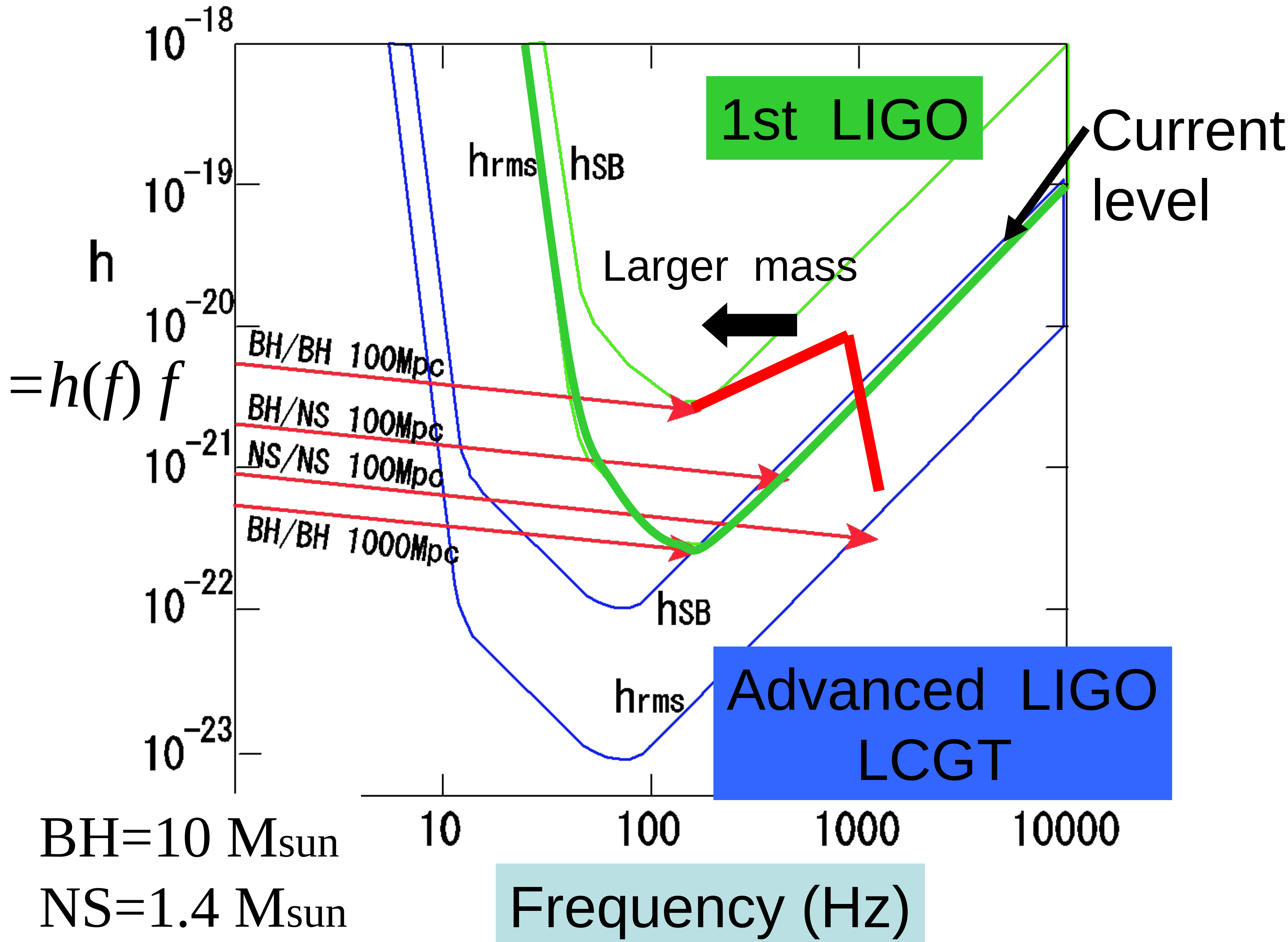
Buonanno
Cook
Pretorius
(2006)



f^{-2/3}
merger

f^{-7/6}
inspiral

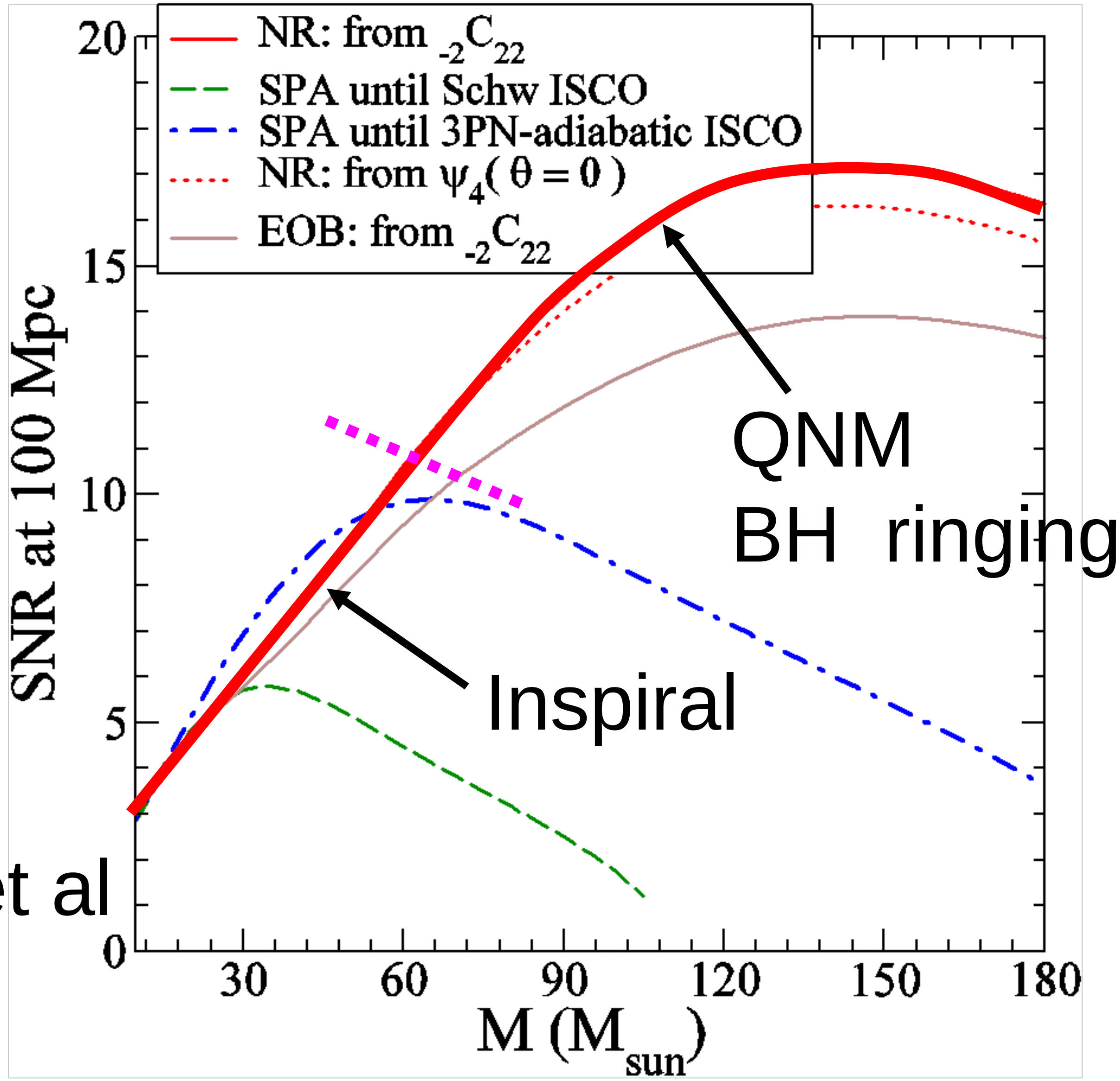
e^{−αω}
damp



1st LIGO
Current level
Larger mass
h
$=h(f)\,f$
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
hrms
hSB
BH/BH 100Mpc
BH/NS 100Mpc
NS/NS 100Mpc
BH/BH 1000Mpc
hSB
hrms
Advanced LIGO
LCGT
10
100
1000
10000
BH=10 Msun
NS=1.4 Msun
Frequency (Hz)

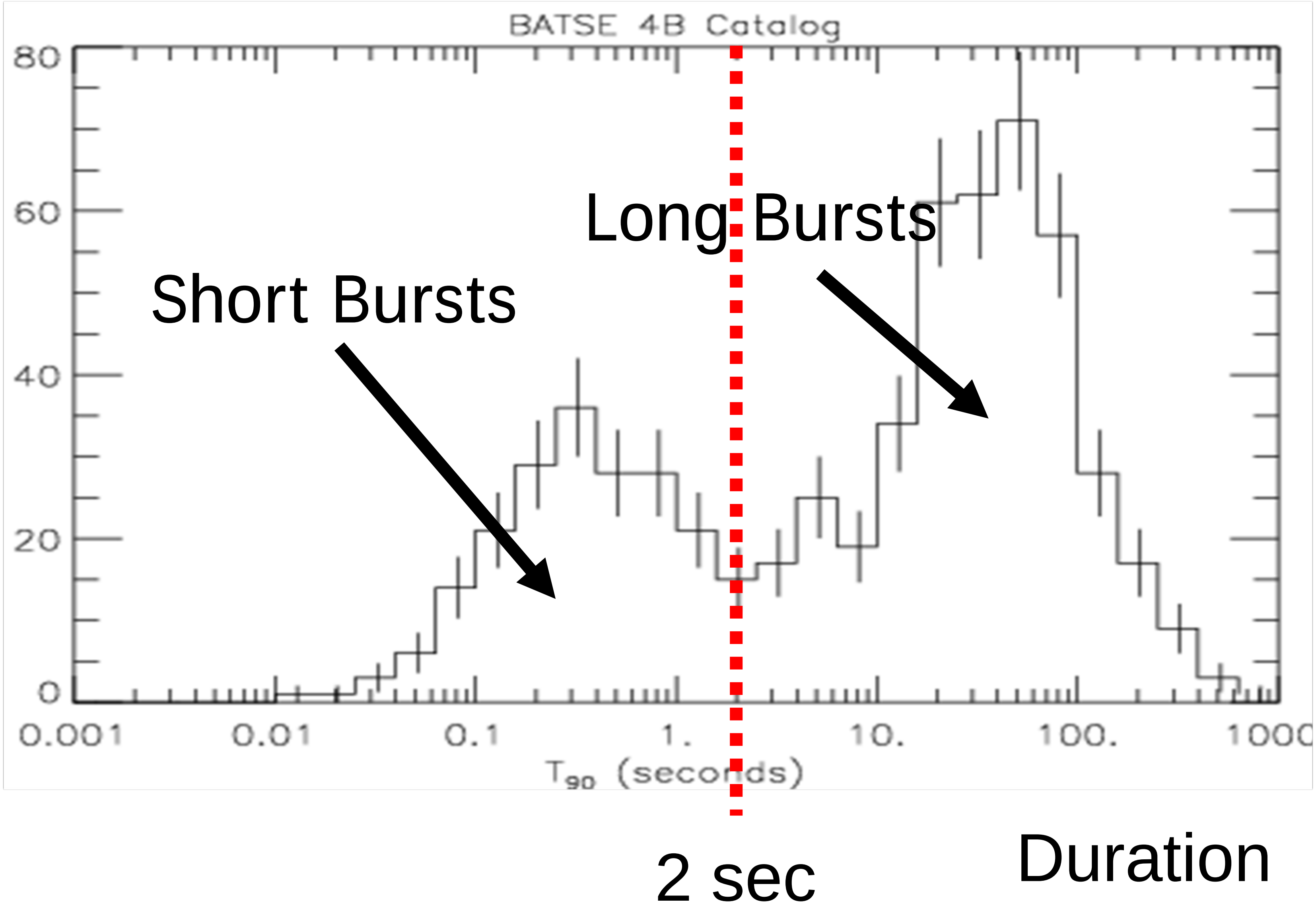
Signal-to-Noise ratio at 100Mpc

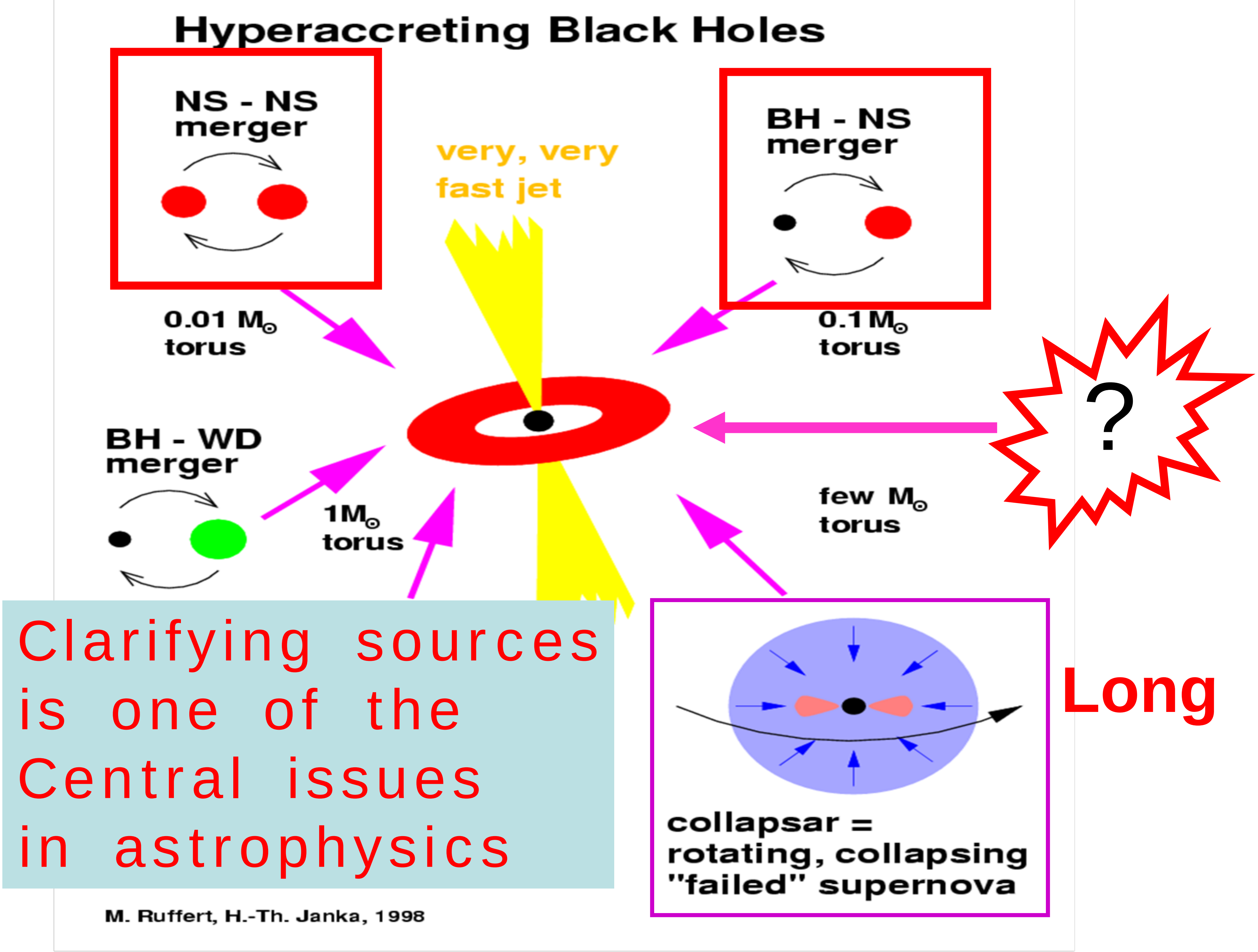
For LIGO



Buonanno et al
(2006)

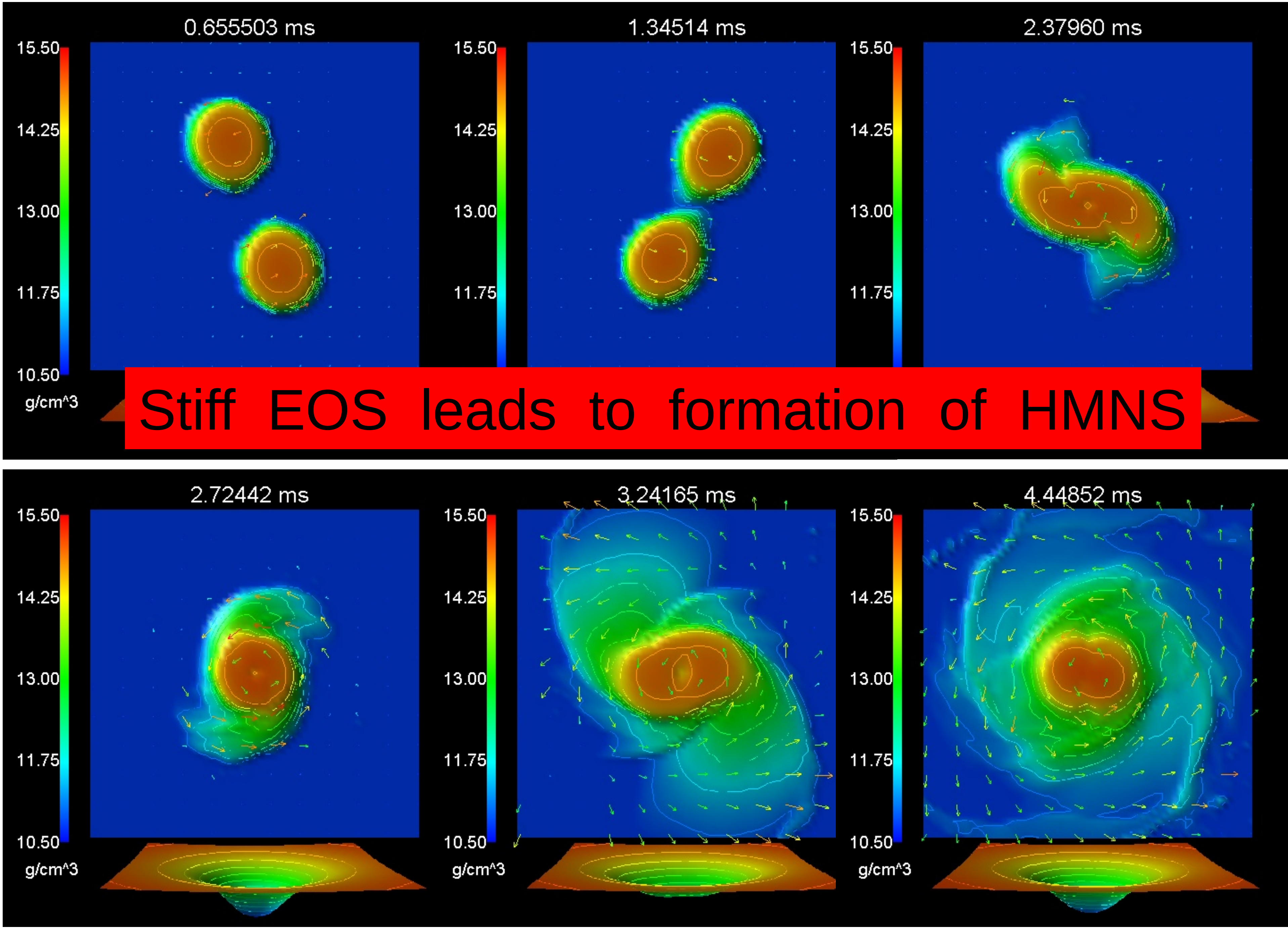
GRB duration distribution




Hyperaccreting Black Holes
NS - NS merger
0.01 $M_\odot$ torus
very, very fast jet
BH - NS merger
0.1 $M_\odot$ torus
?
BH - WD merger
1 $M_\odot$ torus
few $M_\odot$ torus
Clarifying sources is one of the Central issues in astrophysics
Long
collapsar = rotating, collapsing "failed" supernova
M. Ruffert, H.-Th. Janka, 1998

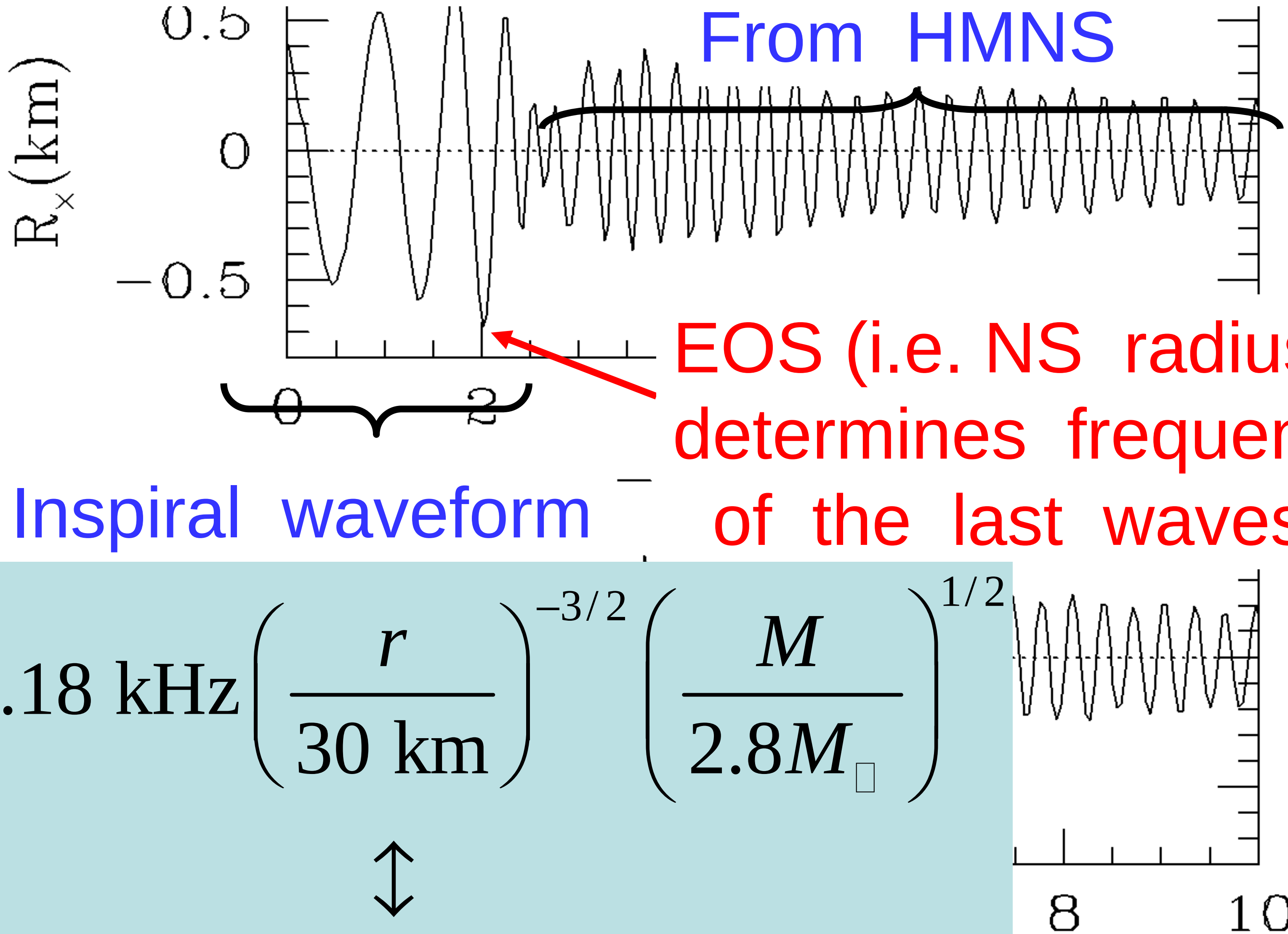
Next targets

- NS-NS merger waveform
  ⇒ Physics of high-density matter
- Supernova → Mechanism


0.655503 ms
1.34514 ms
2.37960 ms
2.72442 ms
3.24165 ms
4.44852 ms
15.50
14.25
13.00
11.75
10.50
g/cm^3
Stiff EOS leads to formation of HMNS

Gravitational waveforms

+ mode



From HMNS

EOS (i.e. NS radius determines frequency of the last waves

Inspiral waveform

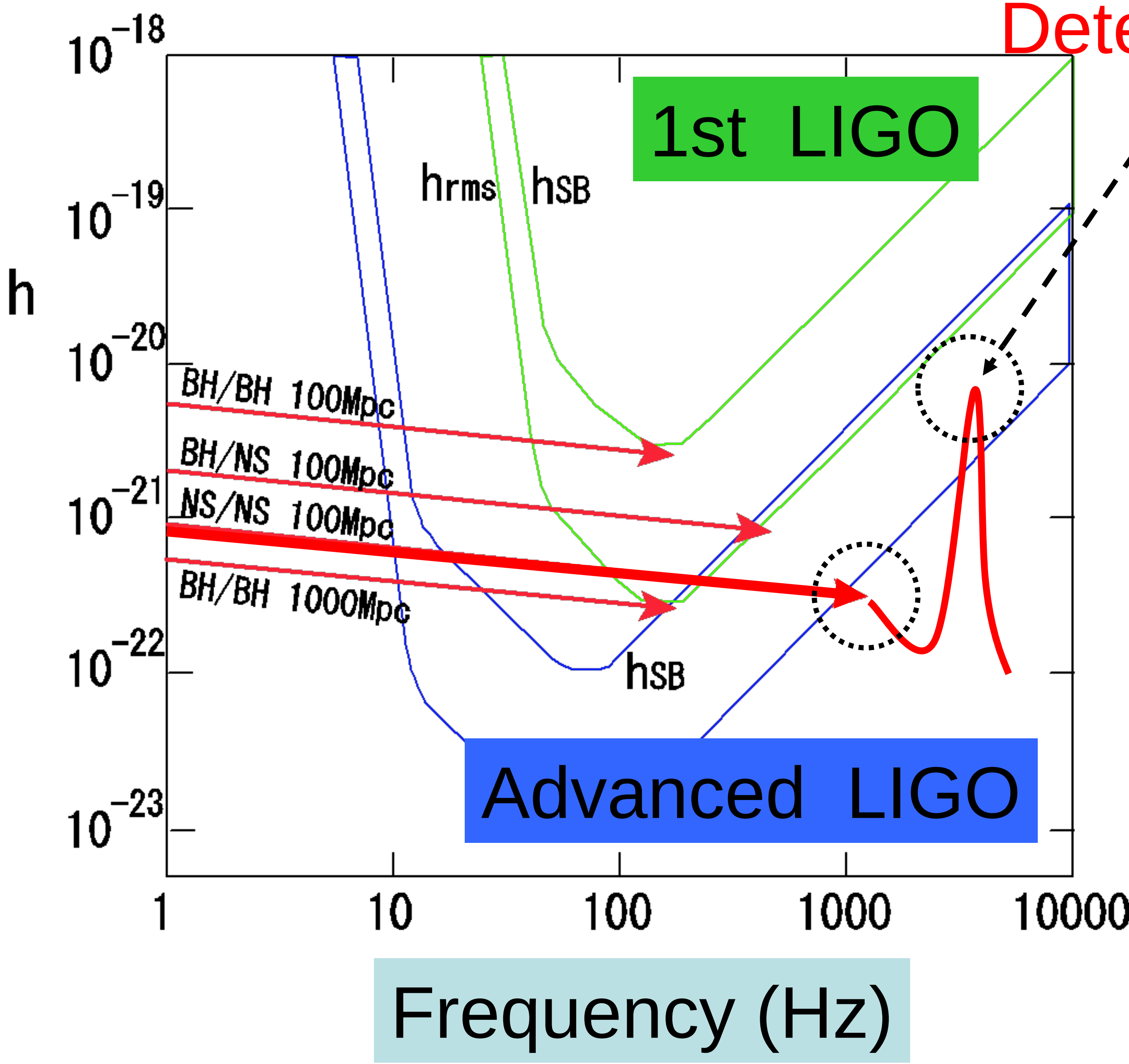
$$f = 1.18\ \text{kHz}\left(\frac{r}{30\ \text{km}}\right)^{-3/2}\left(\frac{M}{2.8M_{\odot}}\right)^{1/2}$$

$$\updownarrow$$

$$r \approx 2R$$

GW signal

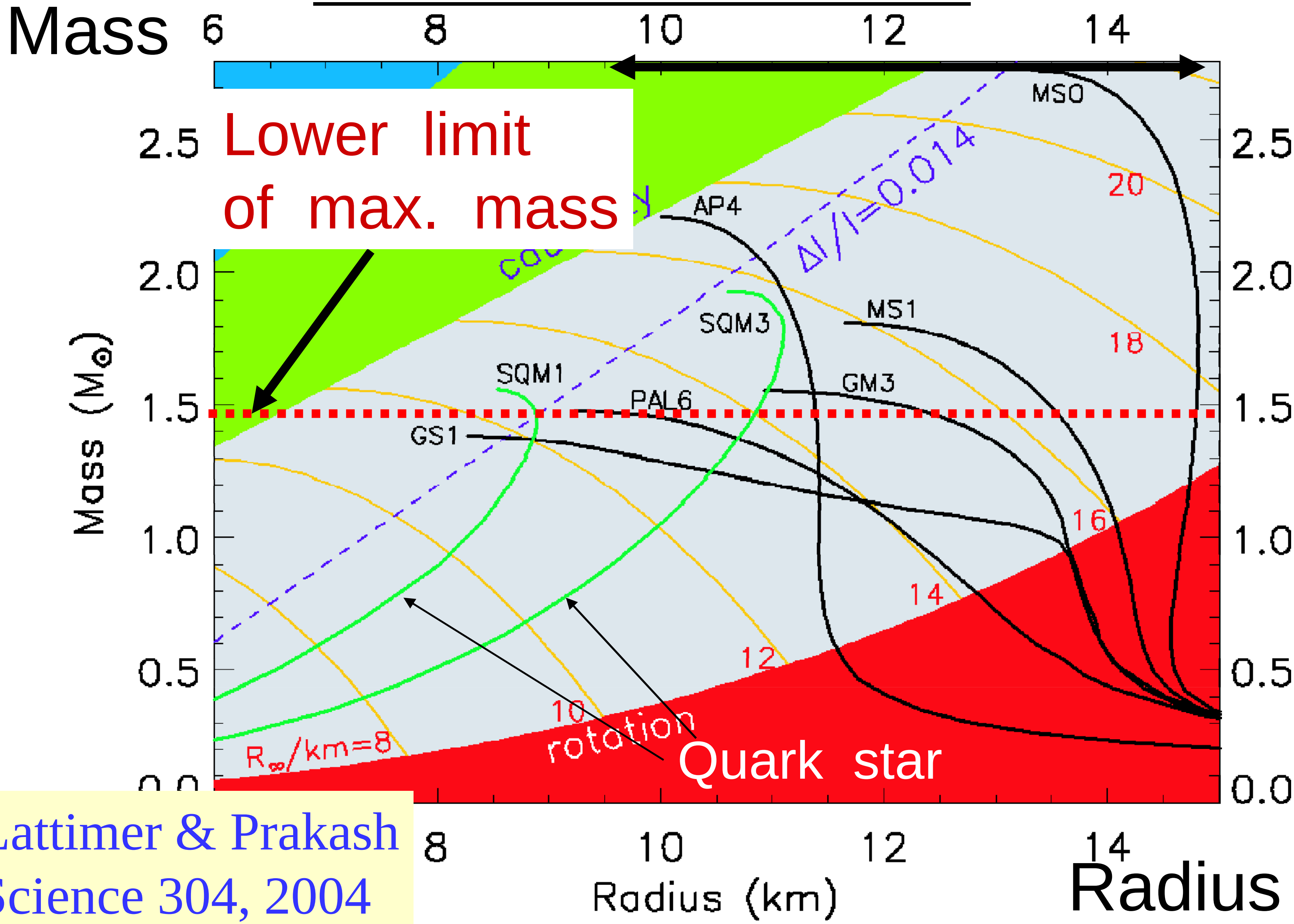
For $r < 50\text{Mpc}$
Detectable !



Detection
= HMNS
exists

⇒
Stiff EOS
→
Constrain
EOS

R-M relation of NSs

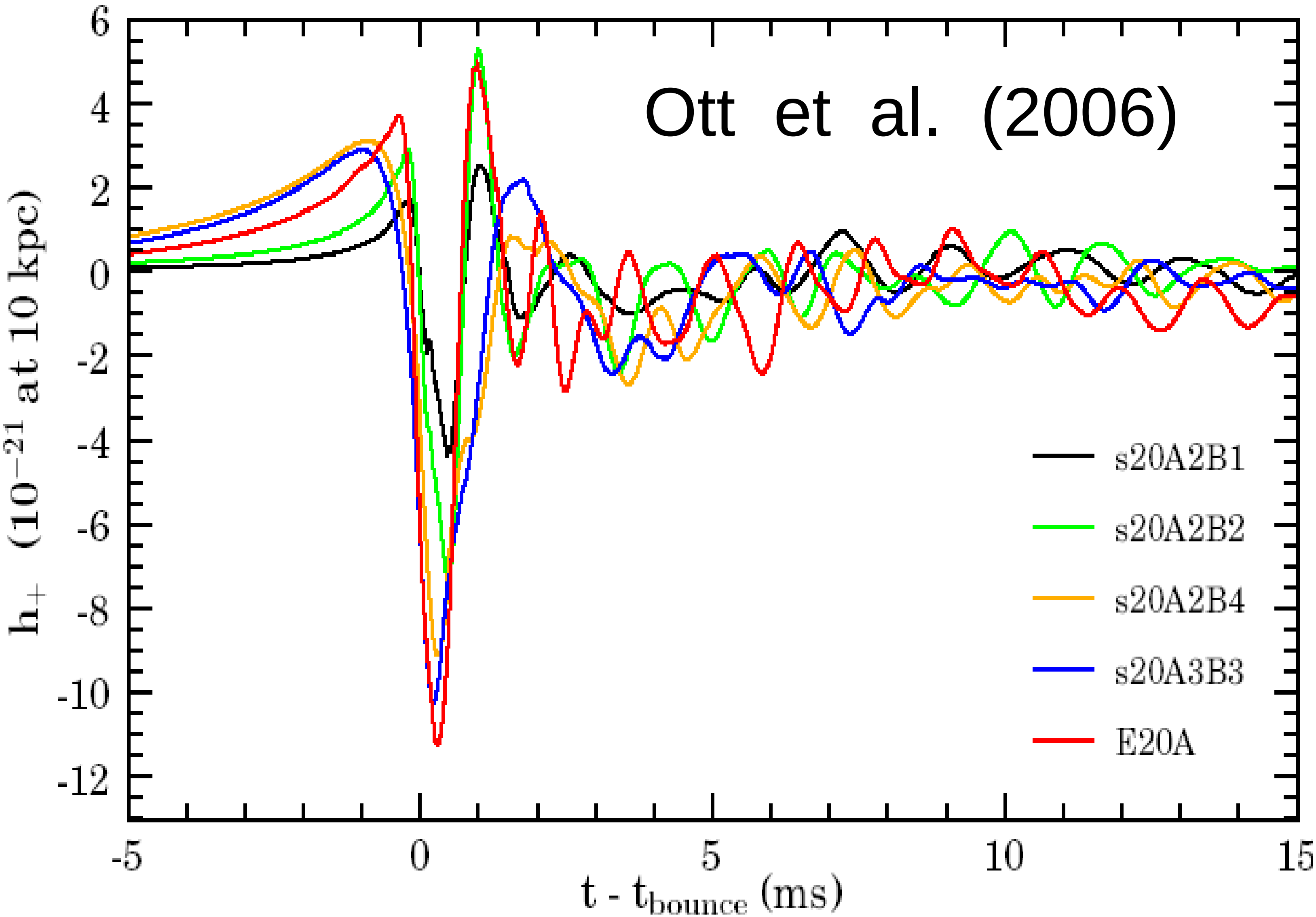


Lattimer & Prakash
Science 304, 2004

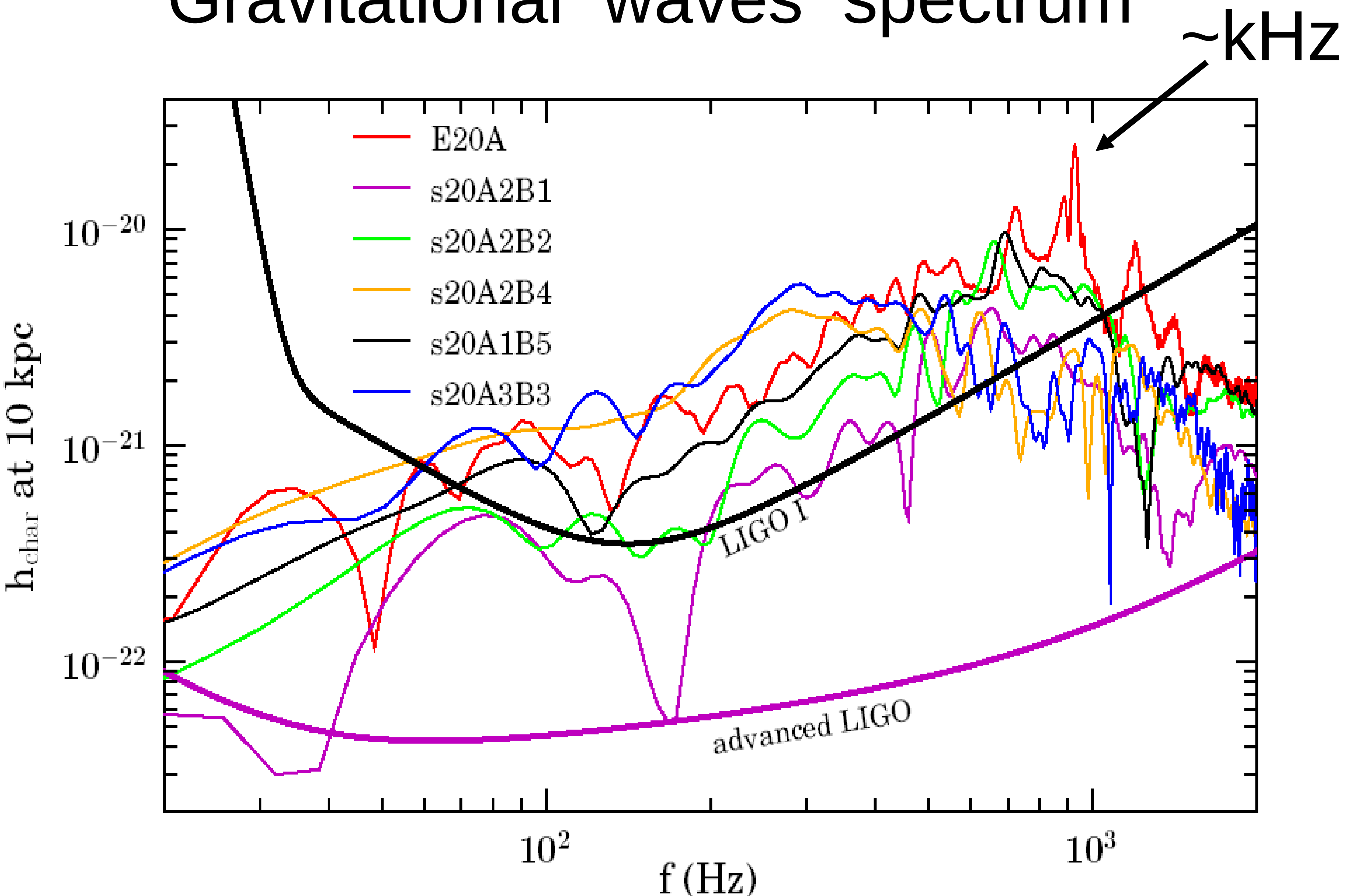
GW from supernovae

- No one knows mechanism for explosion
- GWs may carry information on the mechanism as well as on the EOS, rotation rate, …

Gravitational waves at bounce



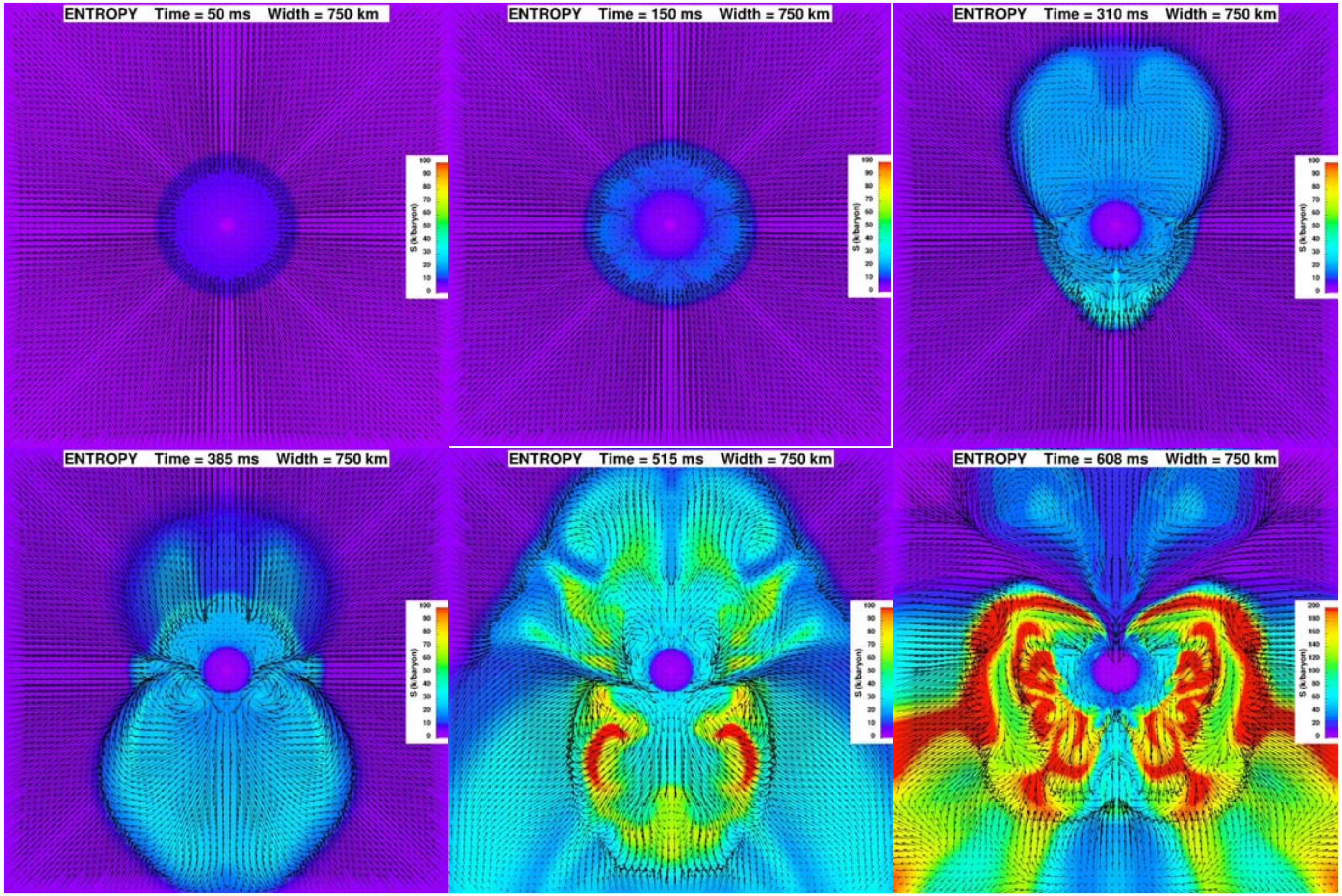
Gravitational waves spectrum



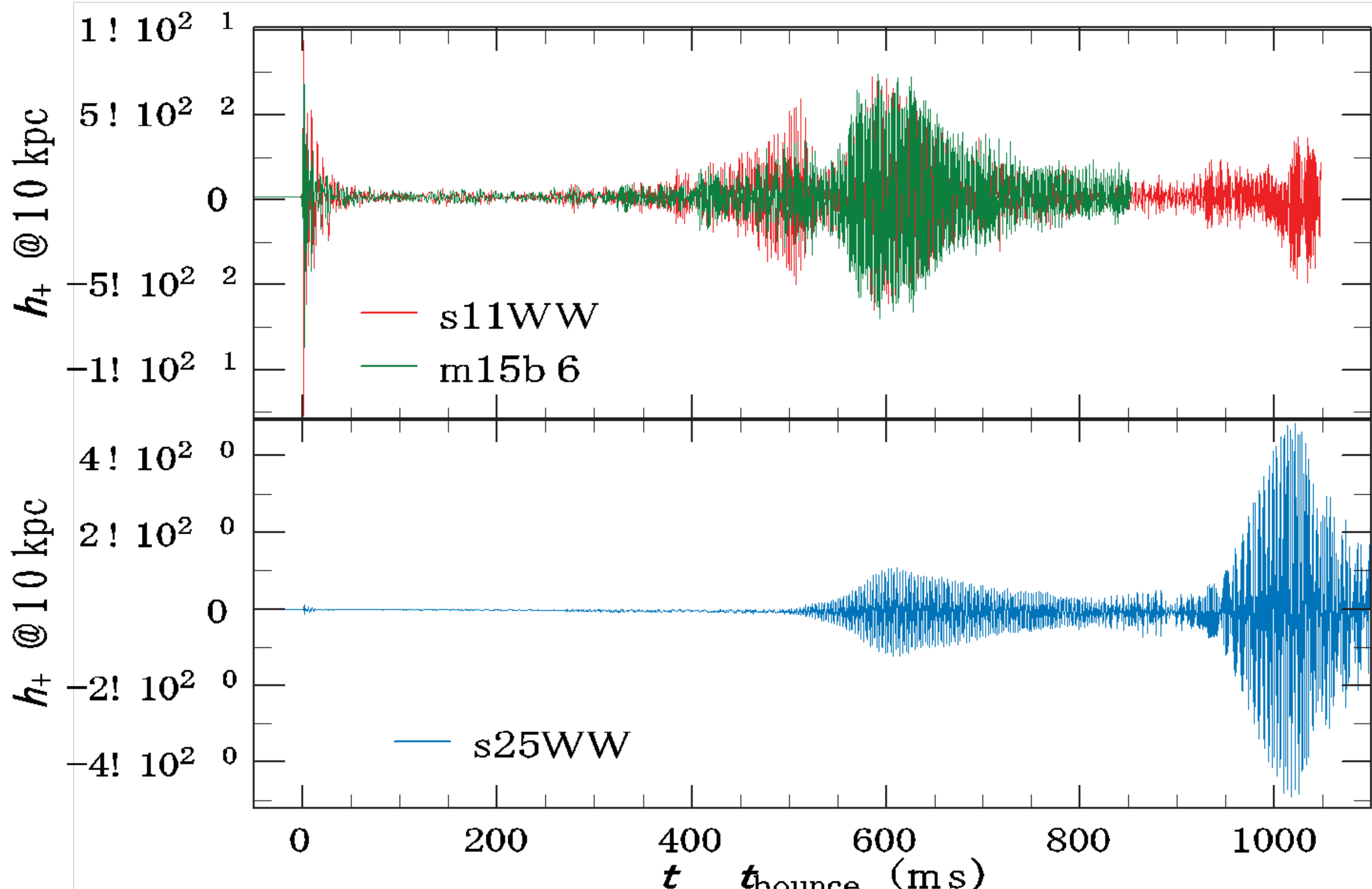
Ott et al. (2006)

Standing Accretion Shock Instability

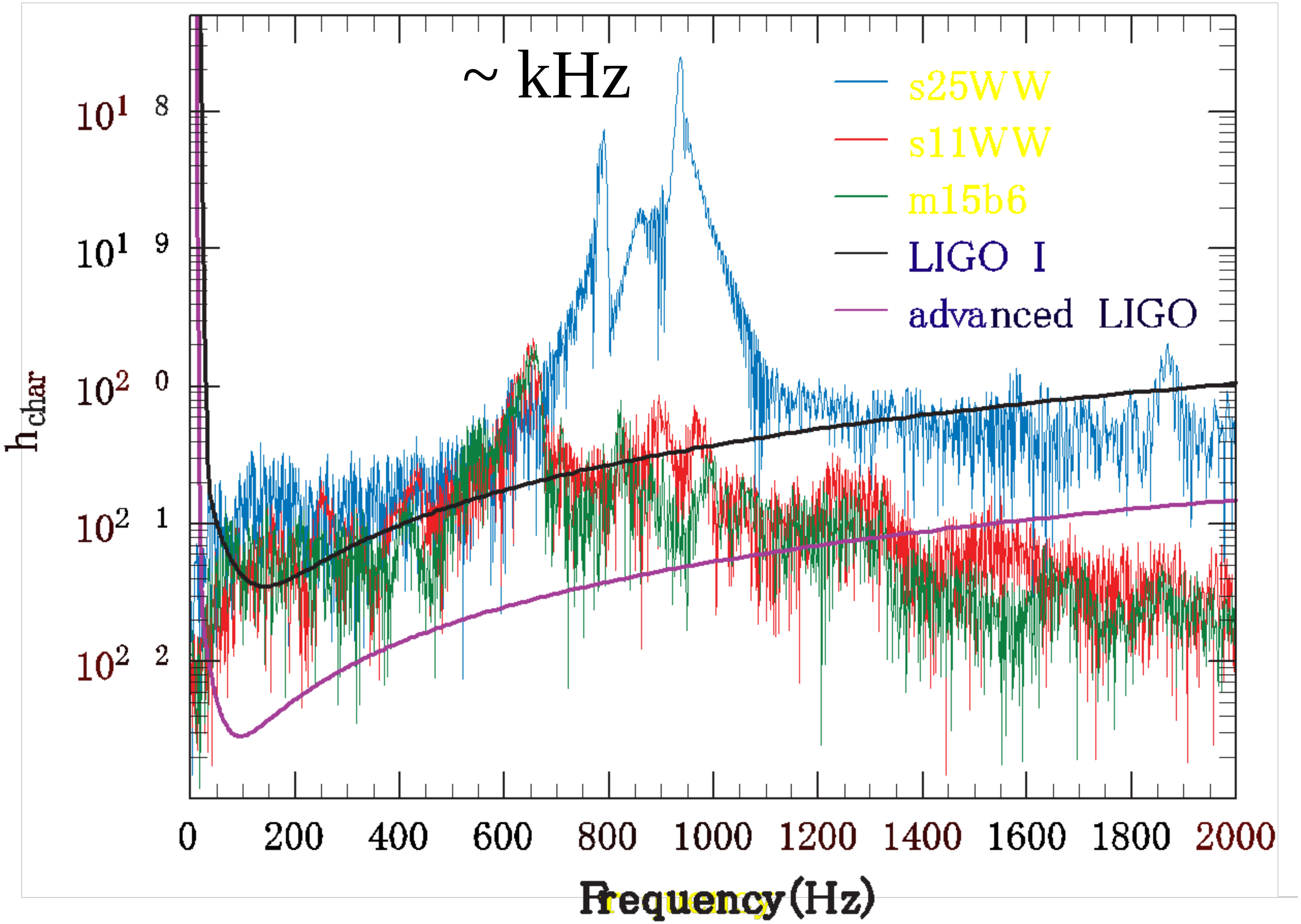
- Longterm simulation (Burrows et al. 2006)



GWs from SASI



Fourier spectrum @ 10kpc

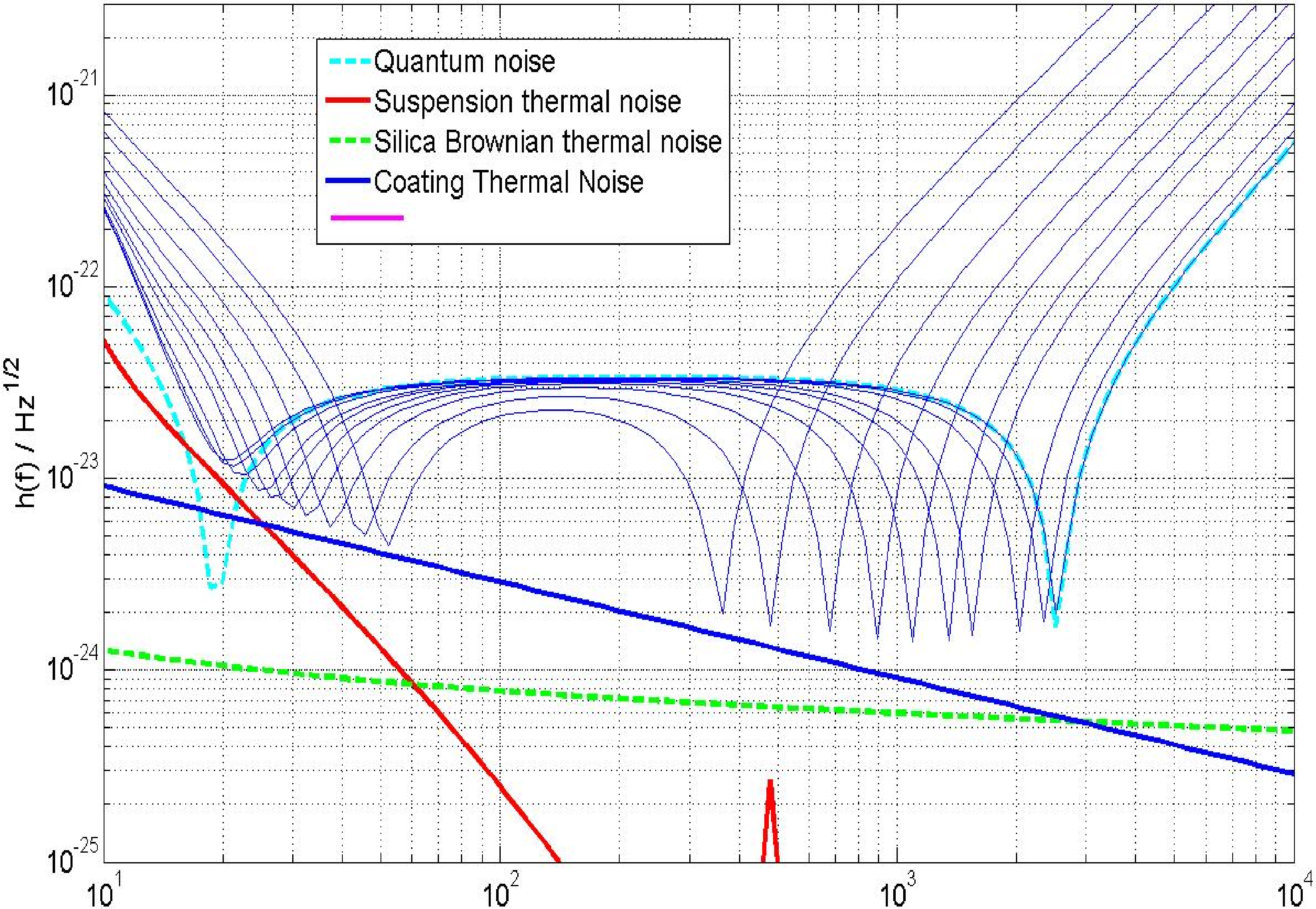


For this next step, …

- Interesting sources for ~1kHz – ~3kHz
- However, A-LIGO and LCGT are not high sensitive enough for detection

→ Improve the sensitivity in the high-frequency region using, e.g., resonant side-band extraction technique.

RSE



10 100 1000 Hz

B Space interferometer

- For detecting GWs of $f < 10\text{Hz}$, detectors in space are necessary

$$f \approx \frac{v^3}{\pi G M} \approx 4\ \text{mHz}\left(\frac{M}{10^6 M_\odot}\right)^{-1}\left(\frac{v}{0.4c}\right)^3$$

$\Rightarrow$ Massive object, like SMBH, can be detected.

Furthermore, amplitude is large

$$h_{eff} \approx 2\times 10^{-18}\left(\frac{10\ \text{Gpc}}{D}\right)\left(\frac{M}{3\times 10^6 M_\odot}\right)\left(\frac{v}{0.4c}\right)^{-1/2} \quad \text{for } T_{obs} > \tau$$

$$\approx 2\times 10^{-18}\left(\frac{10\ \text{Gpc}}{D}\right)\left(\frac{M}{3\times 10^6 M_\odot}\right)\left(\frac{v}{0.4c}\right)^{2} \varepsilon\sqrt{f T_{obs}} \quad \text{for } T_{obs} < \tau$$

Mildly relativistic objects are also sources

$$f \approx \frac{v^3}{\pi G M} \approx 1\ \mathrm{Hz} \left(\frac{M}{4 M_\odot} \right)^{-1} \left(\frac{v}{0.04c} \right)^3$$

And, if $T_{obs} > \tau$,

$$h_{eff} \approx 3 \times 10^{-21} \left(\frac{10\ \mathrm{Gpc}}{D} \right) \left(\frac{M}{4 M_\odot} \right) \left(\frac{v}{0.04c} \right)^{-1/2}$$

where $\tau \sim \left(\frac{0.4c}{v} \right)^5 \frac{1}{f} = 10^5\ \sec \left(\frac{0.04c}{v} \right)^5 \left(\frac{1\ \mathrm{Hz}}{f} \right)$

Why is the amplitude large ?

- Mass is large
- A large number of wave cycles can be accumulated ← near periodic (stationary) objects can be the sources

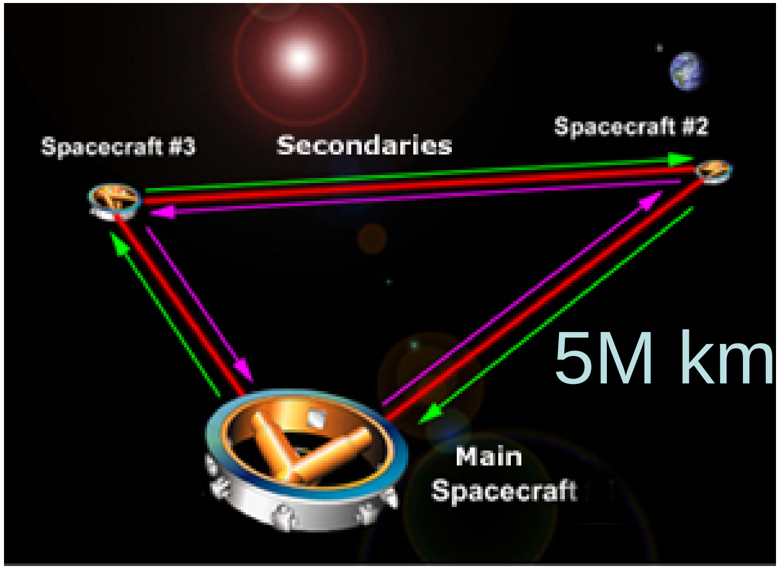
GW		corresp. EM
kHz →	Burst-like sources	γ-ray
		X-ray
<Hz →	Stationary & Burst	Optical
		IR

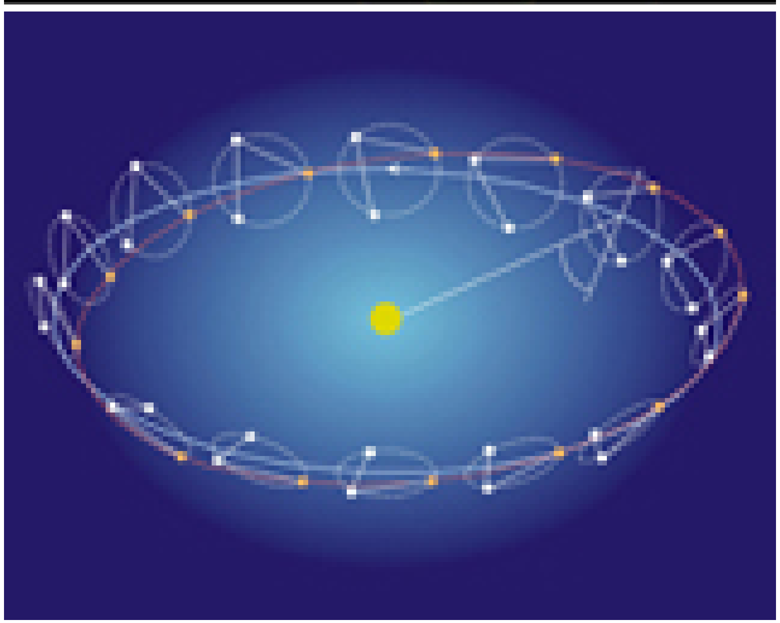
GW astronomy starts from ‘γ-ray’ band, but it is natural to develop ‘optical’ band.

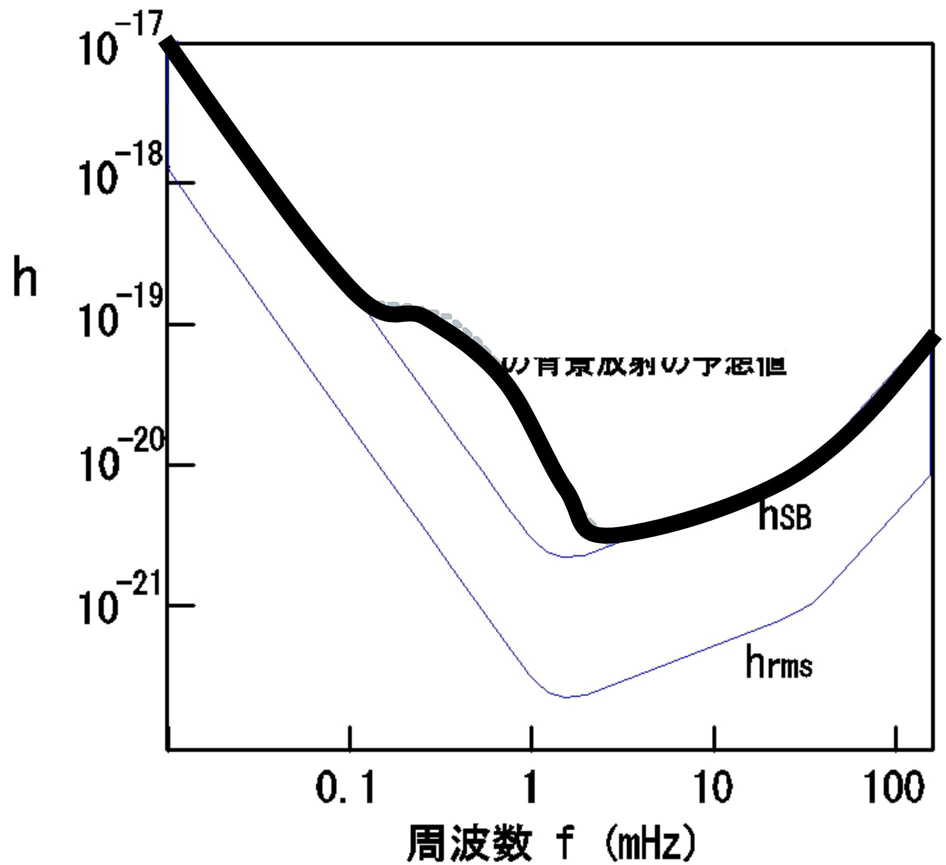
Sources for mHz – Hz band

- Coalescence of supermassive-BH binaries
- Coalescence of intermediate-mass BH binaries (if they really exist)
- Merger of SMBH and stellar mass BH/NS/white dwarf
- Stellar mass BH-BH, BH-NS, NS-NS are also potential sources for $f \sim 1$ Hz.

LISA

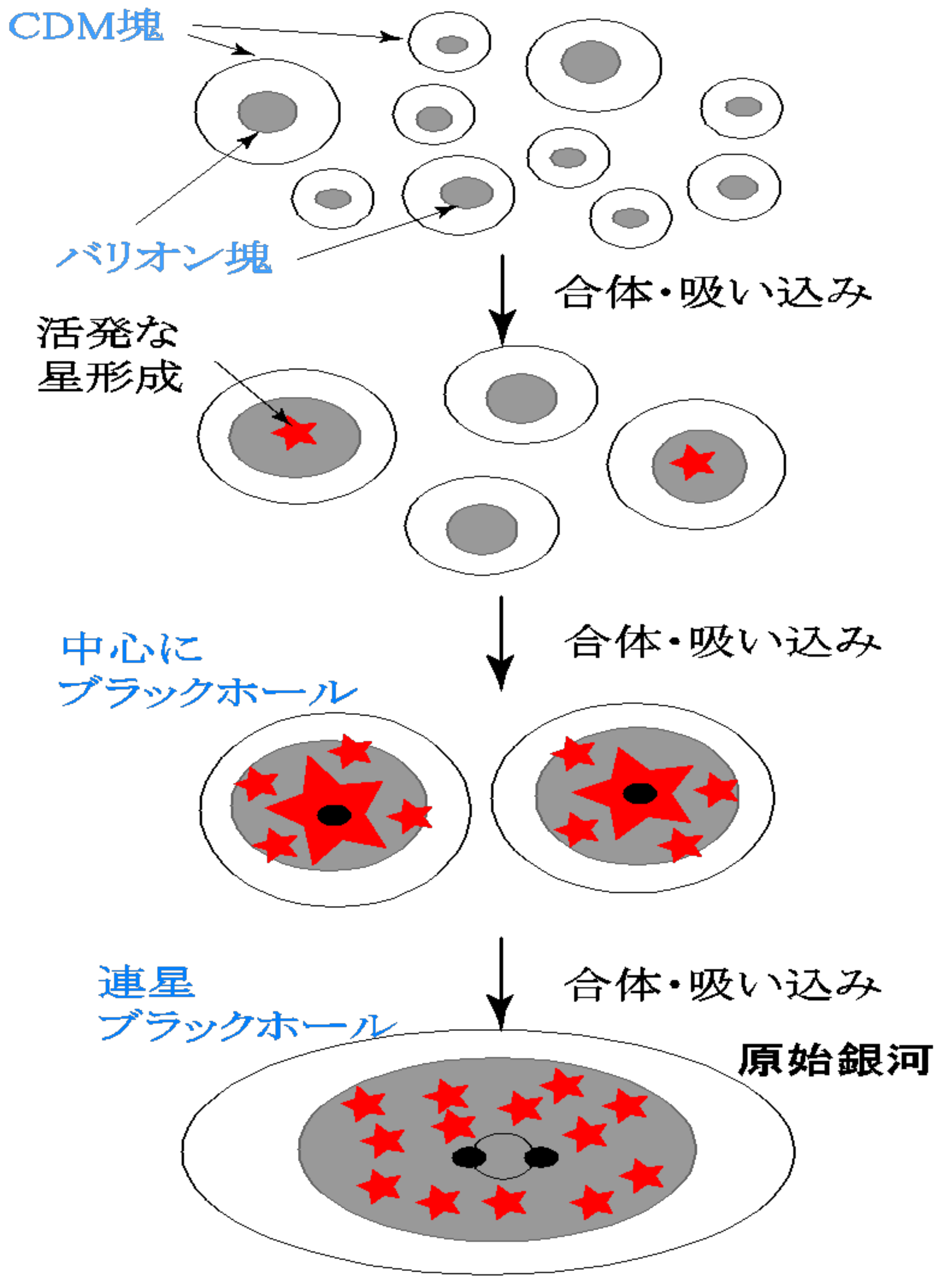




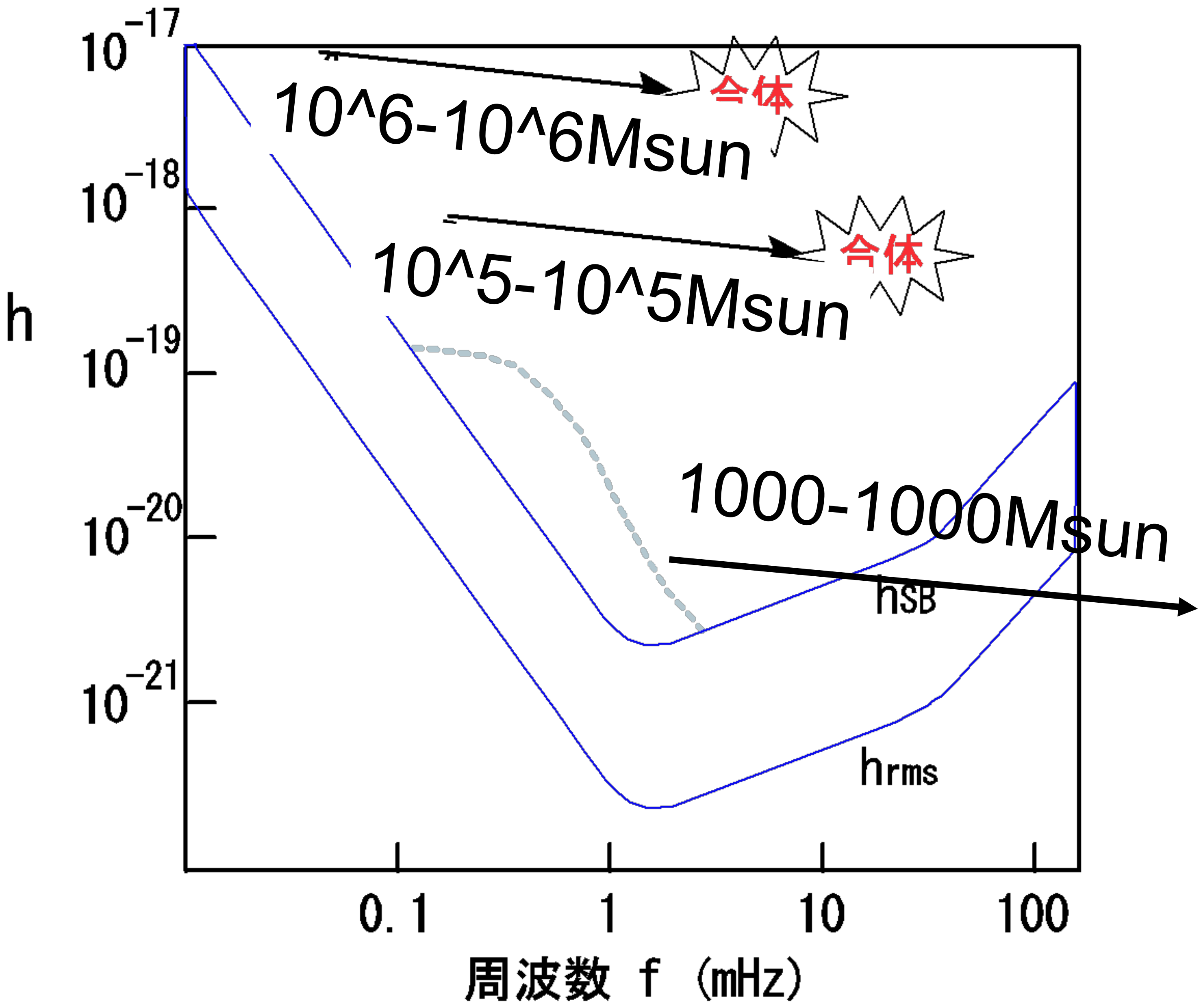


$f \sim 1$—100 mHz

Hierarchical clustering scenario

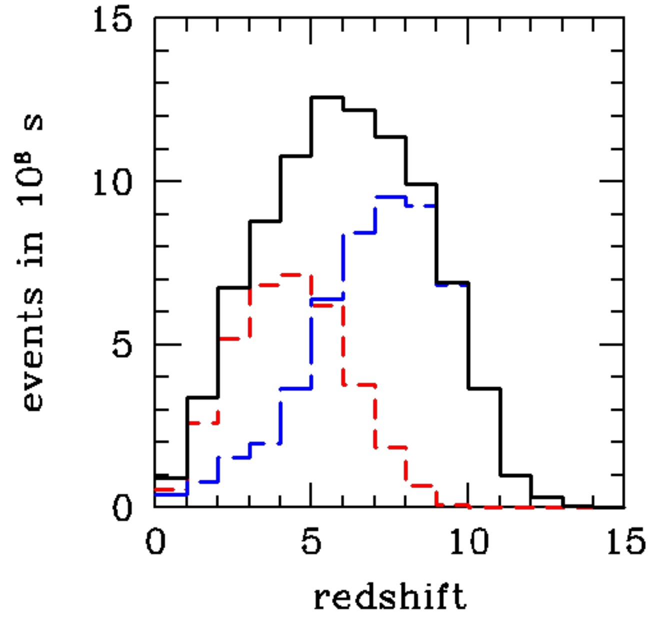


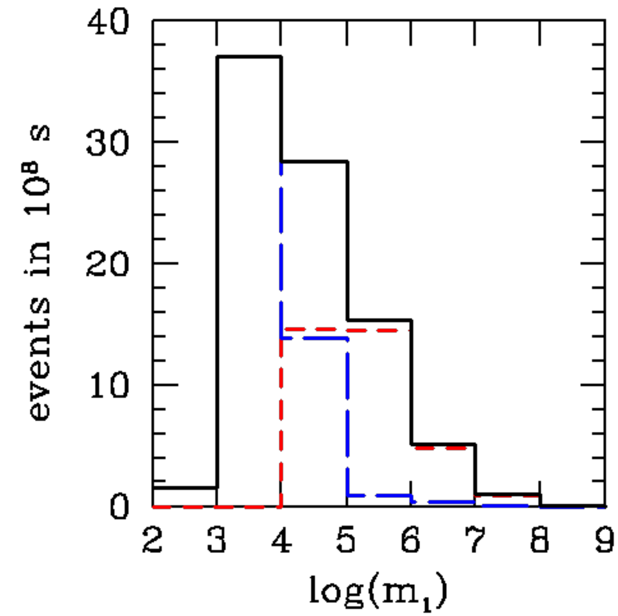
GW from SMBH binary

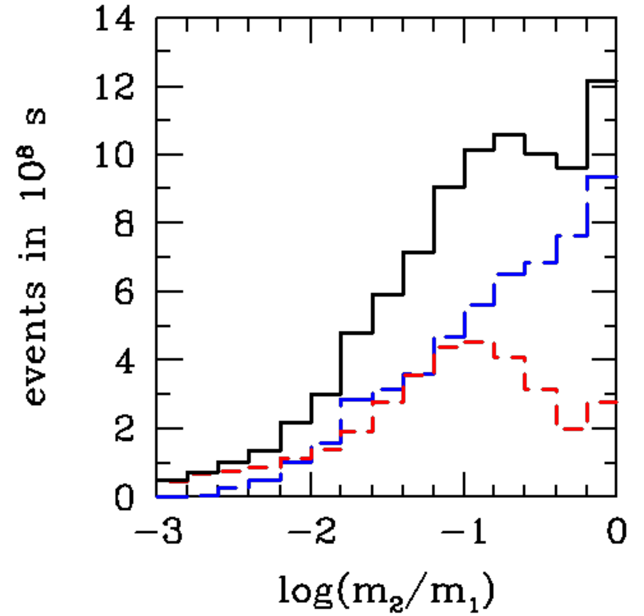


Detection rate of GWs from MBH binaries

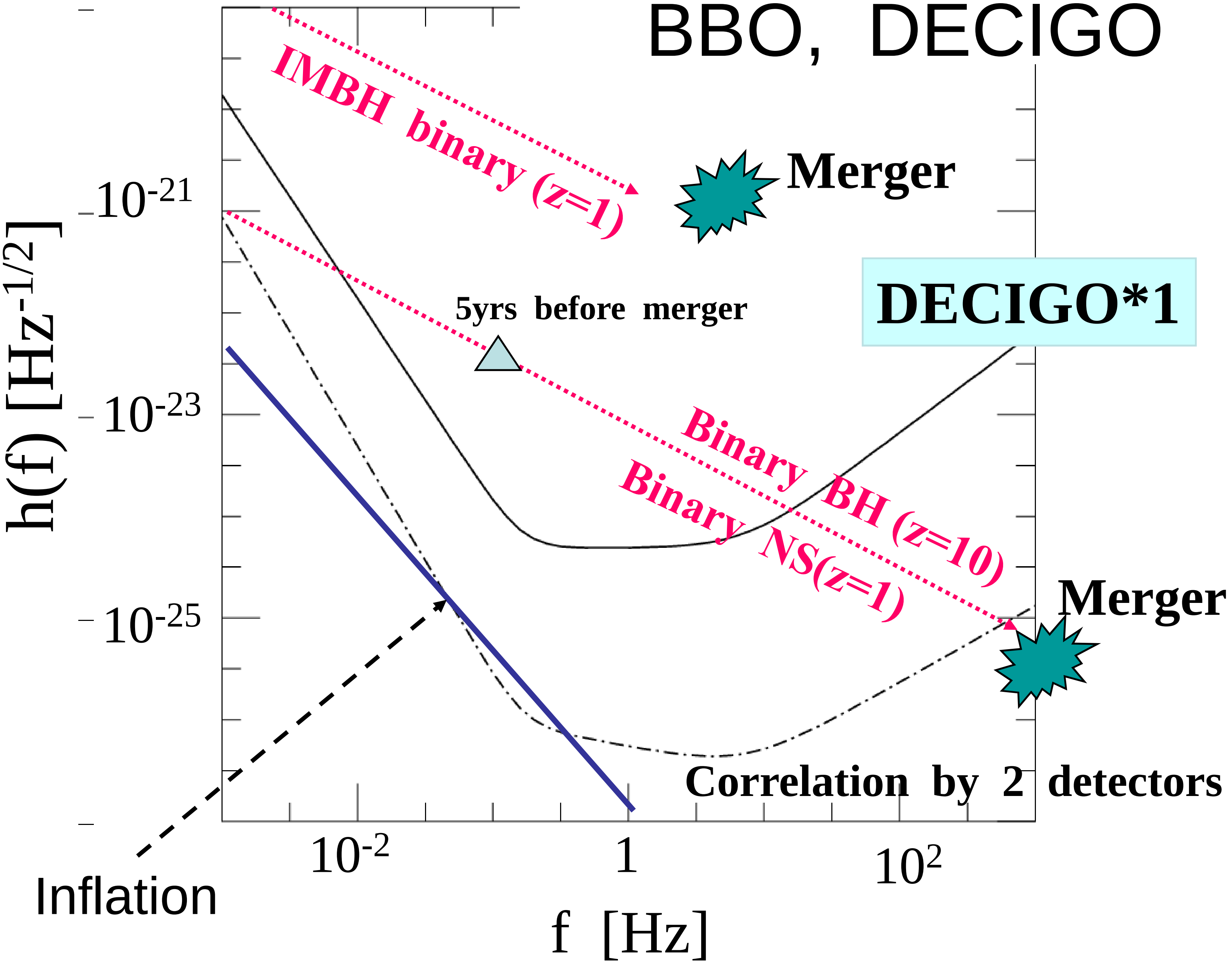
Sesena et al. (2005)







BBO, DECIGO



IMBH binary (z=1)
Merger
DECIGO*1
5yrs before merger
Binary BH (z=10)
Binary NS(z=1)
Merger
Correlation by 2 detectors
10-21
10-23
10-25
h(f) [Hz-1/2]
10-2
1
102
f [Hz]
Inflation

Study for theory of gravity

- Relativity: lowest order = quadrupole emission
- Scalar tensor theory: lowest order = dipole emission

$$\dot{P}_{\rm orb} = -\frac{192\pi}{5}\frac{\mu}{M}\left(\frac{GM\Omega}{c^3}\right)^{5/3} - 4\pi\frac{\mu}{M}\left(\frac{GM\Omega}{c^3}\right)\frac{\sigma^2}{\omega+2}$$

M : Total mass, μ: Reduced mass

Ω: Orbital angular velocity, $\left(\frac{GM\Omega}{c^3}\right)^{1/3} = \frac{v}{c}$

σ : Difference of scalar charge ~ 0.1 (BH-NS)

ω : Brans-Dicke parameter

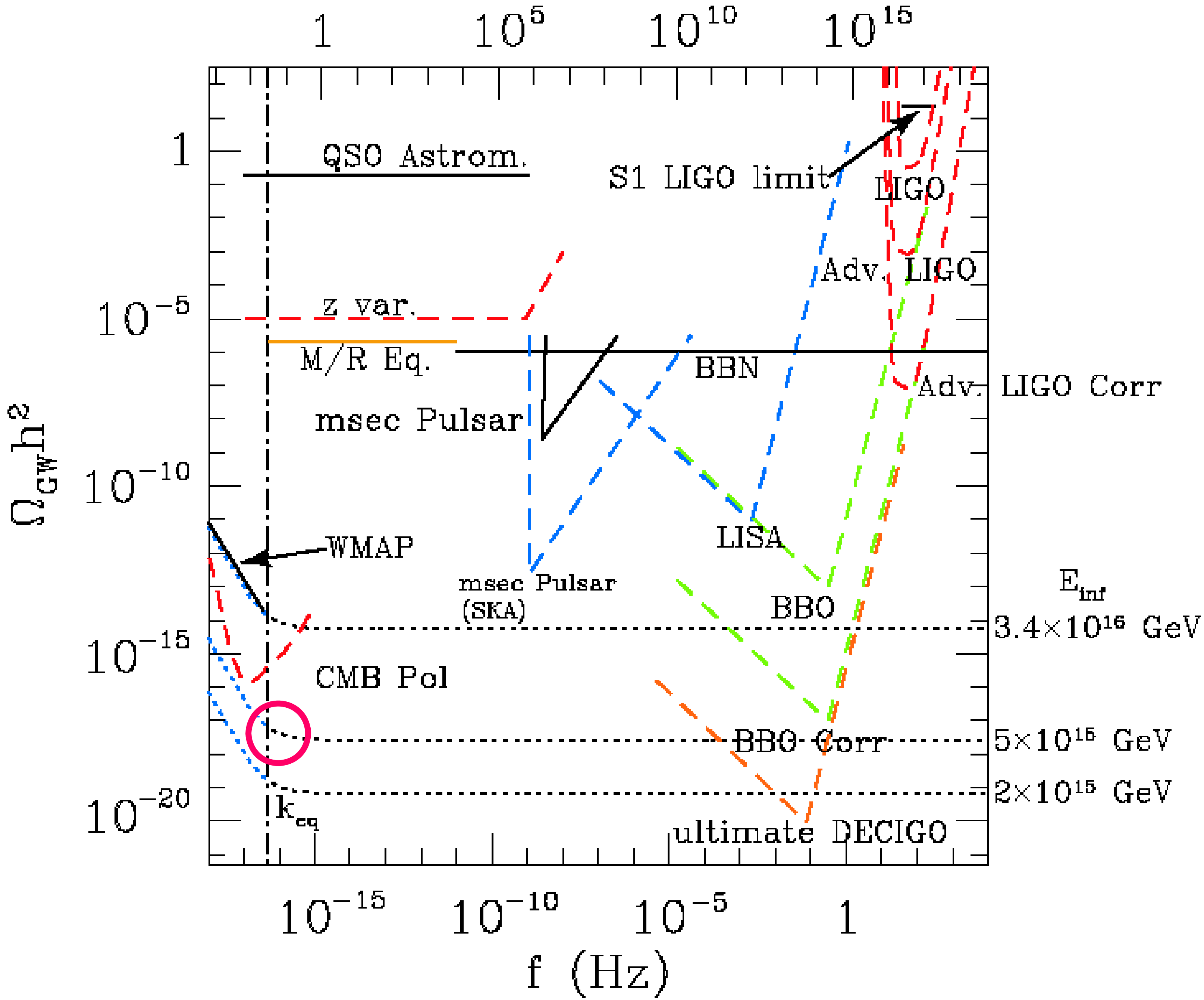
Ratio

$$\frac{\text{Scalar}}{\text{GW}} = \frac{5}{48}\left(\frac{GM\Omega}{c^3}\right)^{-2/3} \frac{\sigma^2}{\omega+2}$$

$$= 2.36\times10^{-7}\left(\frac{M}{3M_{\odot}}\right)^{-2/3}\left(\frac{f}{0.2\text{ Hz}}\right)^{-2/3}\left(\frac{\sigma}{0.1}\right)^{2}\left(\frac{\omega}{10^7}\right)^{-1}$$

1 yrs integration at $f = 0.2$ Hz $\rightarrow$ 6×10^6 cycles
$\Rightarrow$ 1 phase difference by scalar-waves emission

If 1 phase difference can be measured (S/N = 10),
ω is constrainted as $\omega \geq 10^7$.



1
10^5
10^10
10^15
QSO Astrom.
S1 LIGO limit
LIGO
Adv. LIGO
z var.
M/R Eq.
BBN
Adv. LIGO Corr
msec Pulsar
WMAP
LISA
msec Pulsar
(SKA)
E_inf
BBO
3.4×10^16 GeV
CMB Pol
BBO Corr
5×10^15 GeV
2×10^15 GeV
k_eq
ultimate DECIGO
Ω_GW h^2
1
10^−5
10^−10
10^−15
10^−20
10^−15
10^−10
10^−5
1
f (Hz)

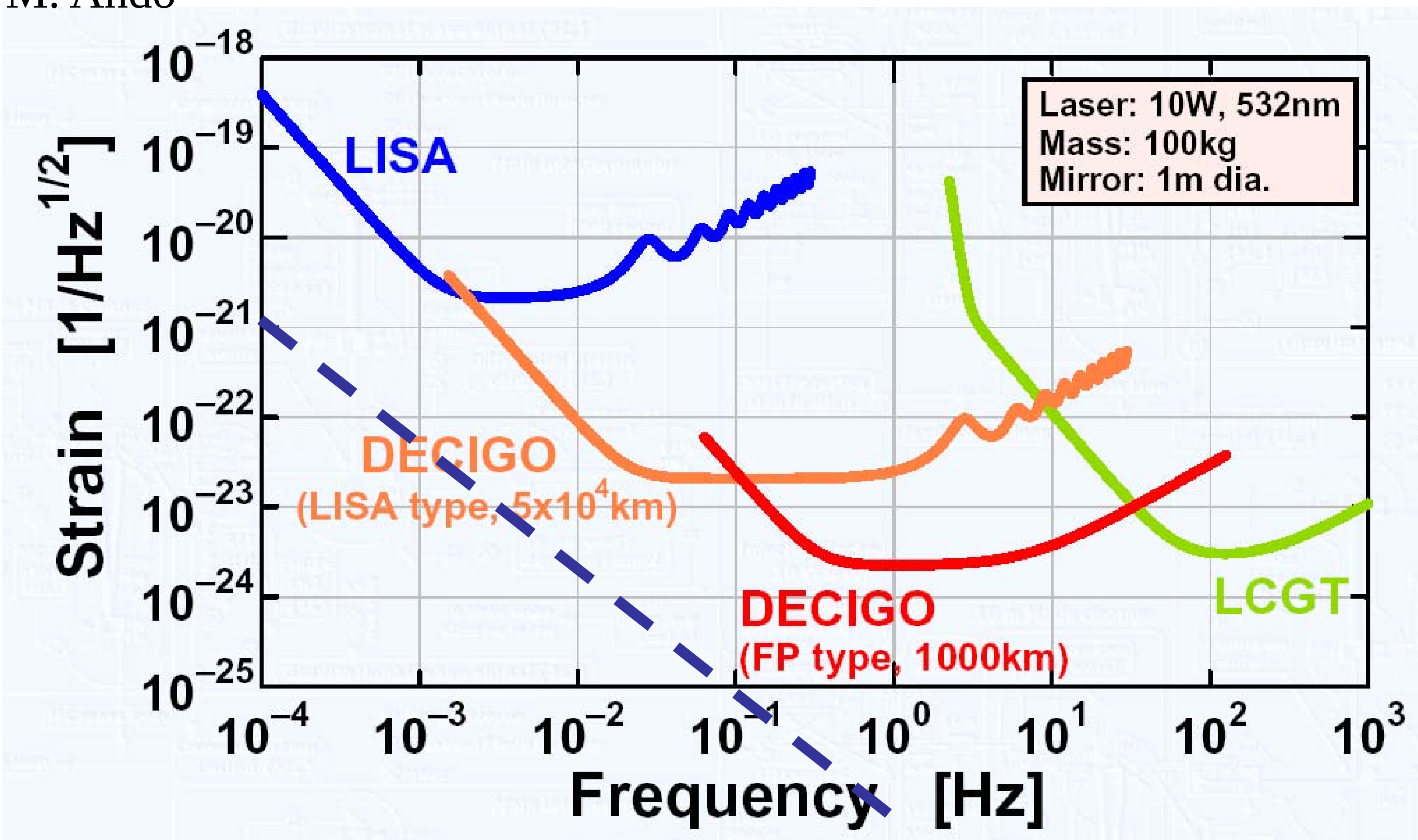
Summary

- Step 0: Detection (< 2015)
- Step 1: Study of stellar-mass BHs and NSs using grand-based detectors (~'15)
- Step 1.5: Study of details of NSs and nuclear matter using kHz band (~ '20)
- Step 2: Observation of low frequency GWs using space-detectors (2020 ~ ?): Sources = SMBHs, evolution of galaxies, theory of gravity,
- Step ??: Detect GW background (2048?)

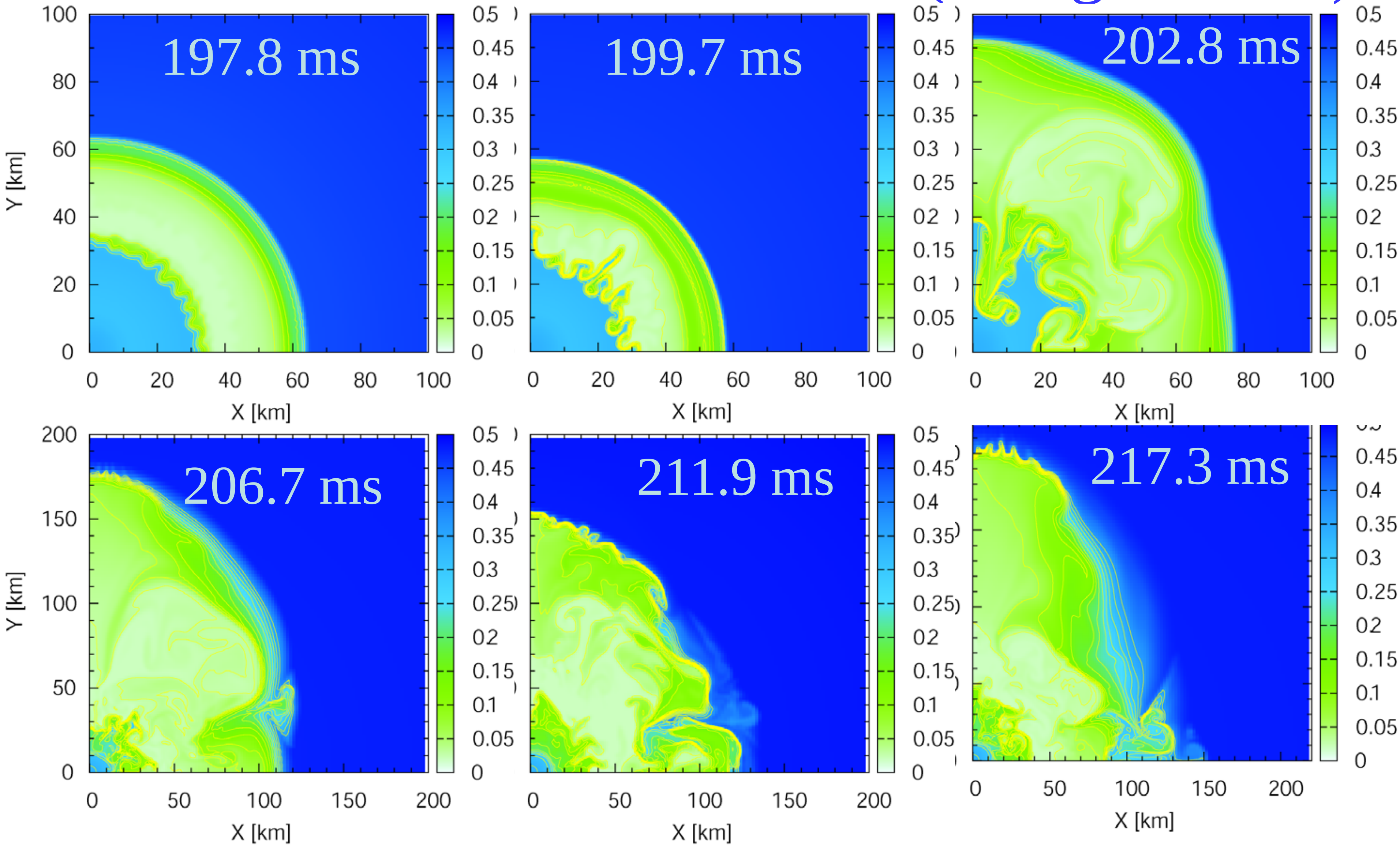
GWBs of inflationary origins

$(\Omega_{gw}=10^{-14}) \propto f^{-3/2}$

By M. Ando

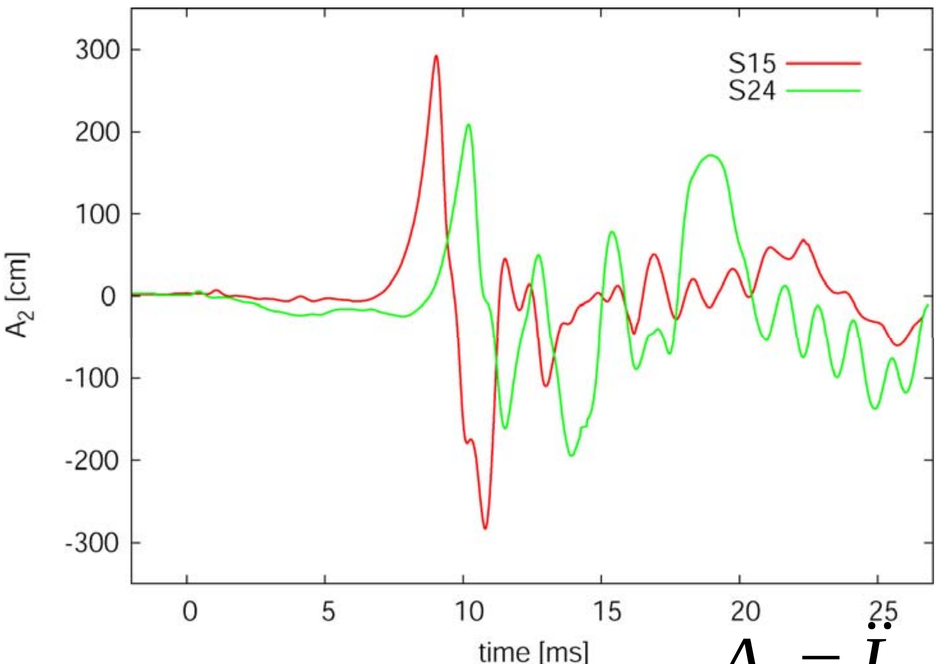


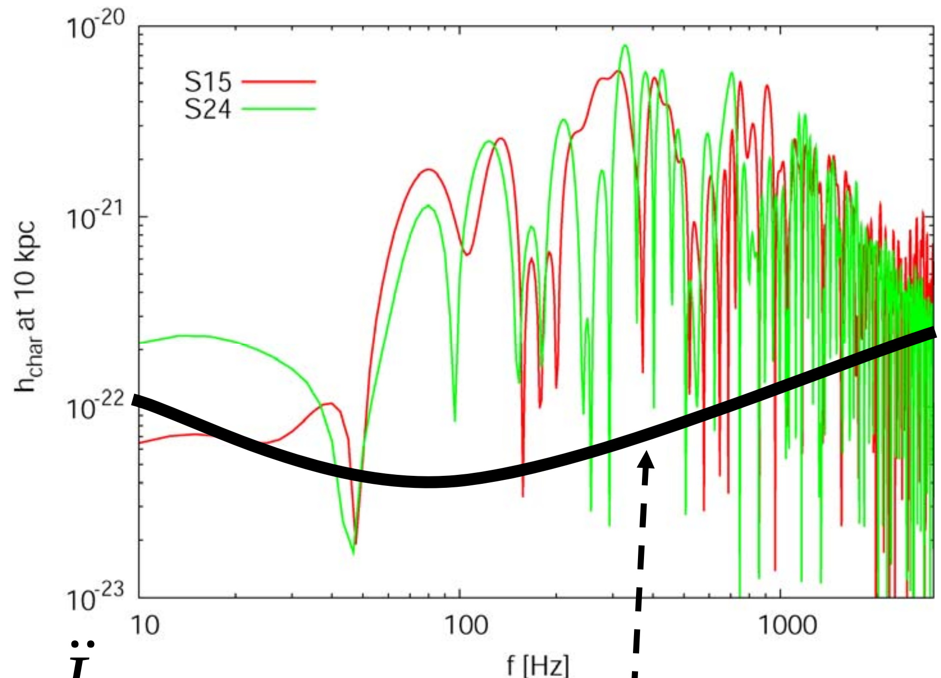
Contours of electron fraction (Sekiguchi 07)



Unstable to convection → energy transport

Gravitational waveforms (Sekiguchi 07)





Advance LIGO

$$A_2 \equiv \ddot{I}_{zz} - \ddot{I}_{xx}$$

$$h = \frac{A_2}{D}\sin^2\theta$$

$$\approx 10^{-20}\left(\frac{10\ \mathrm{kpc}}{D}\right)\left(\frac{A_2}{3\ \mathrm{m}}\right)\sin^2\theta$$