Proton Decay and Related Topics

K.S. Babu
Oklahoma State University

ICRR/CRC Future Plan Symposium
University of Tokyo, Kashiwa
August 28-29, 2007

Baryon number violation in the Standard Model

- B is a global symmetry of SM classically
- Nonperturbative weak interaction effects lead to B violation

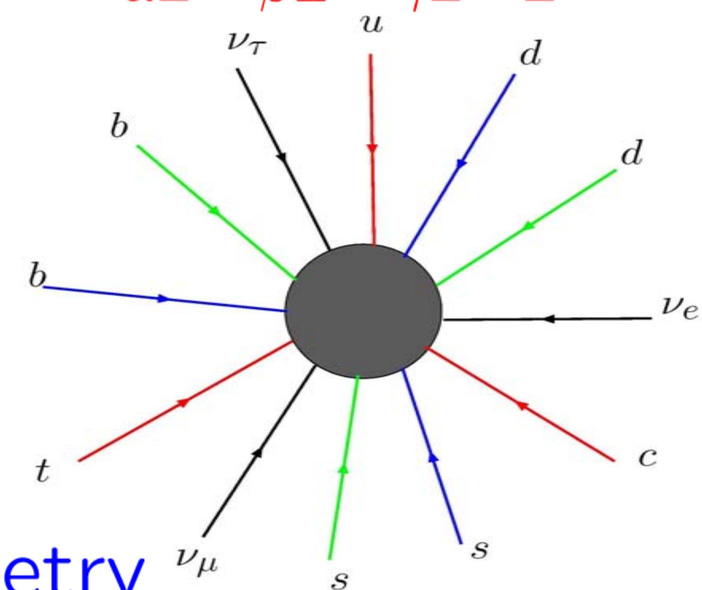
't Hooft (1976)

$$\mathcal{O}_{\text{eff}} = c\left(\frac{1}{m_W}\right)^{14} e^{\frac{-2\pi}{\alpha_2}} \prod_{i=1}^{3} (\epsilon^{\alpha\beta\gamma} Q^i_{\alpha L} Q^i_{\beta L} Q^i_{\gamma L} L^i_L)$$

- $\Delta B = \Delta L = 3$

Eg: $ppn \to e^+ e^+ \overline{\nu}$

$$\text{Rate} \quad \propto \quad \left| e^{-\frac{2\pi}{\alpha_2}} \right|^2 \sim 10^{-165}$$



- Relevant for baryon asymmetry

Kuzmin, Rubakov, Shaposhnikov (1985)

Gravity--induced Nucelon Decay

- Gravity expected to violate all global symmetries including B
- Leading B violating operators:

$$\mathcal{O}_{\Delta B \neq 0} = \frac{c}{M_{\rm Pl}^2} QQQL$$

- Proton lifetime:

$$\Gamma(p \to e^+ \pi^0) \sim c^2 \frac{m_p^5}{M_{\rm Pl}^4}$$

$$\Rightarrow$$

$$\tau(p \to e^+ \pi^0) \sim 10^{48} \text{ yr.}$$

Structure of Matter Multiplets in Standard Model

$$Q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim (3, 2, \frac{1}{6})$$

$$u^c = \left(u_1^c \quad u_2^c \quad u_3^c \right) \sim (\overline{3}, 1, \frac{-2}{3})$$

$$d^c = \left(d_1^c \quad d_2^c \quad d_3^c \right) \sim (\overline{3}, 1, \frac{1}{3})$$

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix} \sim (1, 2, \frac{-1}{2})$$

$$e^c \sim (1, 1, +1)$$

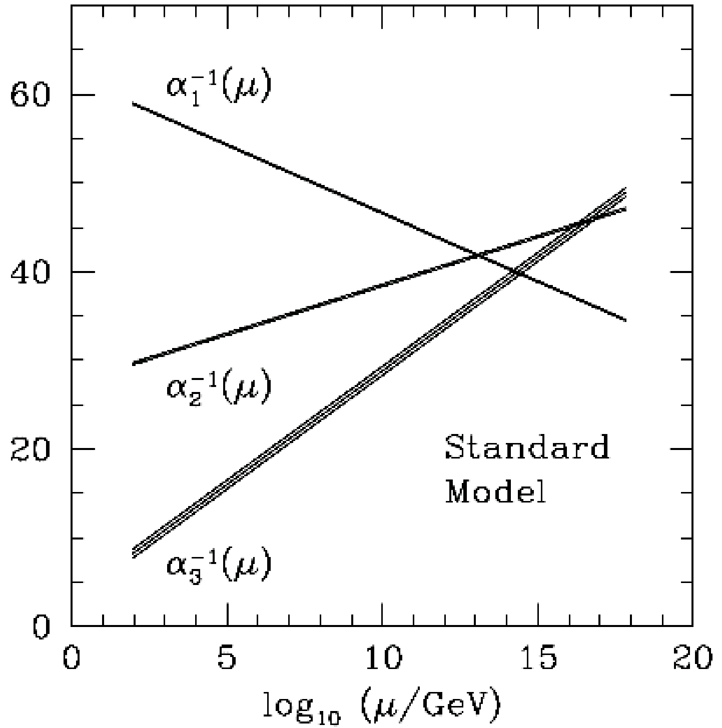
$$\nu^c \sim (1, 1, 0)$$

Matter Unification

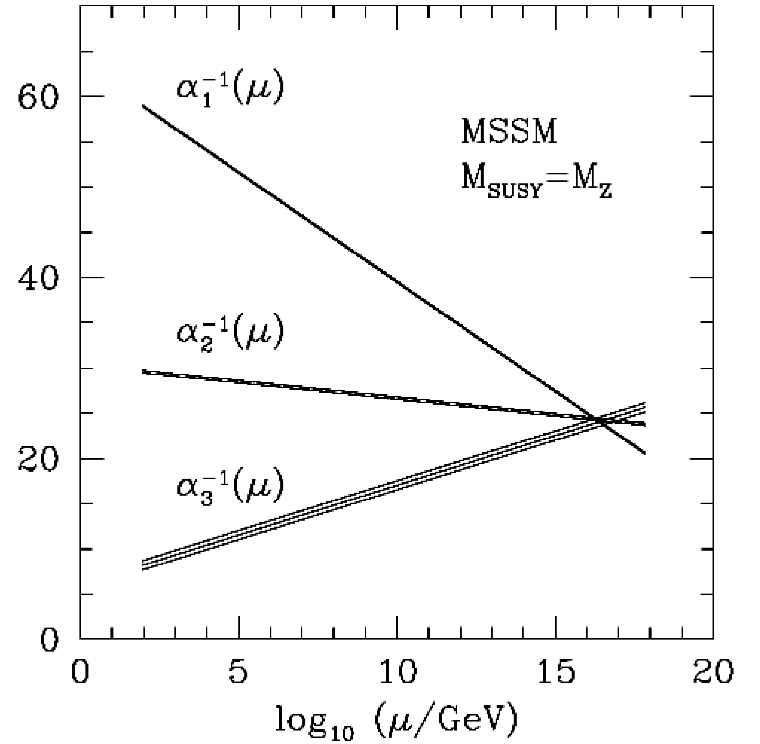
in 16 of SO(10)

u_1	:	$\|\uparrow\downarrow\uparrow\uparrow\downarrow>$
u_2	:	$\|\uparrow\downarrow\uparrow\downarrow\uparrow>$
u_3	:	$\|\uparrow\downarrow\downarrow\uparrow\uparrow>$
d_1	:	$\|\downarrow\uparrow\uparrow\uparrow\downarrow>$
d_2	:	$\|\downarrow\uparrow\uparrow\downarrow\uparrow>$
d_3	:	$\|\downarrow\uparrow\downarrow\uparrow\uparrow>$
u^c_1	:	$\|\downarrow\downarrow\uparrow\downarrow\downarrow>$
u^c_2	:	$\|\downarrow\downarrow\downarrow\uparrow\downarrow>$
u^c_3	:	$\|\downarrow\downarrow\downarrow\downarrow\uparrow>$
d^c_1	:	$\|\uparrow\uparrow\uparrow\downarrow\downarrow>$
d^c_2	:	$\|\uparrow\uparrow\downarrow\uparrow\downarrow>$
d^c_3	:	$\|\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν	:	$\|\uparrow\downarrow\downarrow\downarrow\downarrow>$
e	:	$\|\downarrow\uparrow\downarrow\downarrow\downarrow>$
e^c	:	$\|\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν^c	:	$\|\uparrow\uparrow\uparrow\uparrow\uparrow>$

Evolution of Gauge Couplings

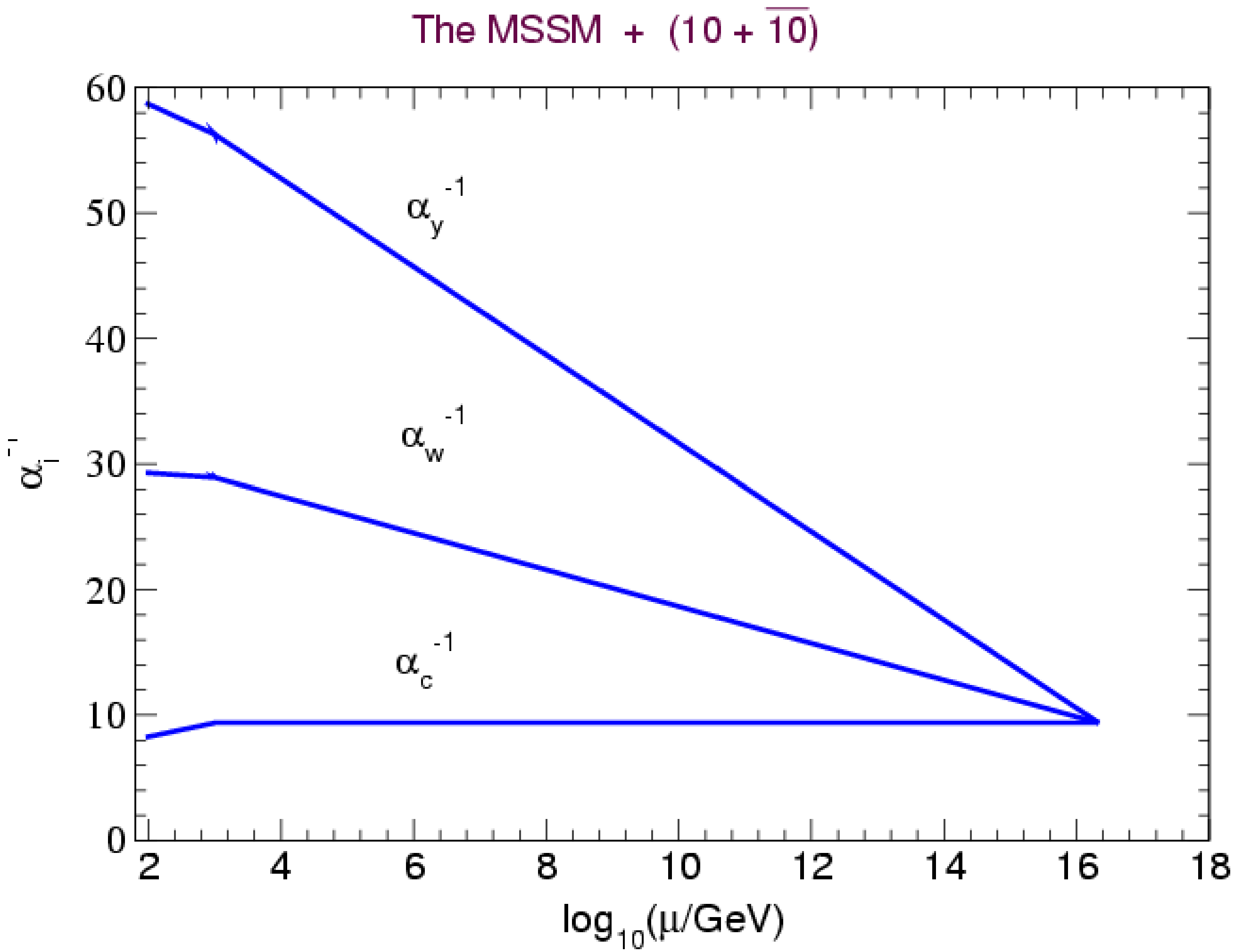


Standard Model



Minimal Supersymmetry

Unification with MSSM plus additional 10+10*

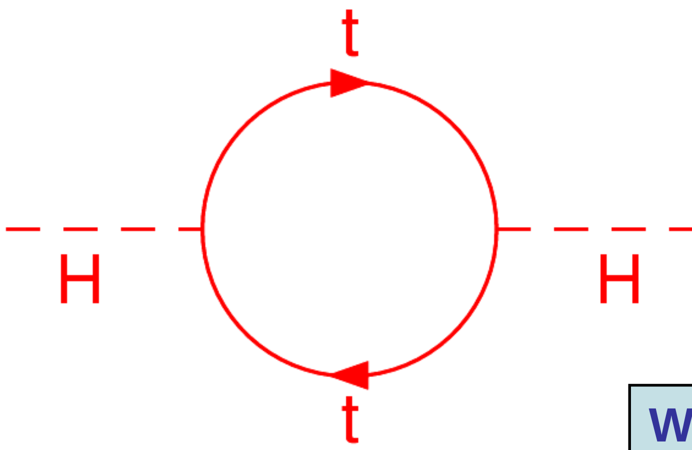


SUSY Spectrum

	SM Particles	SUSY Partners	
Spin = 1/2	Q	$\tilde{Q}$	Spin = 0
	u^c	$\tilde{u}^c$	
	d^c	$\tilde{d}^c$	
	L	$\tilde{L}$	
	e^c	$\tilde{e}^c$	
Spin = 0	H_u	$\tilde{H}_u$	Spin = 1/2
	H_d	$\tilde{H}_d$	
Spin = 1	g	$\tilde{g}$	Spin = 1/2
	W	$\tilde{W}$	
	B	$\tilde{B}$	

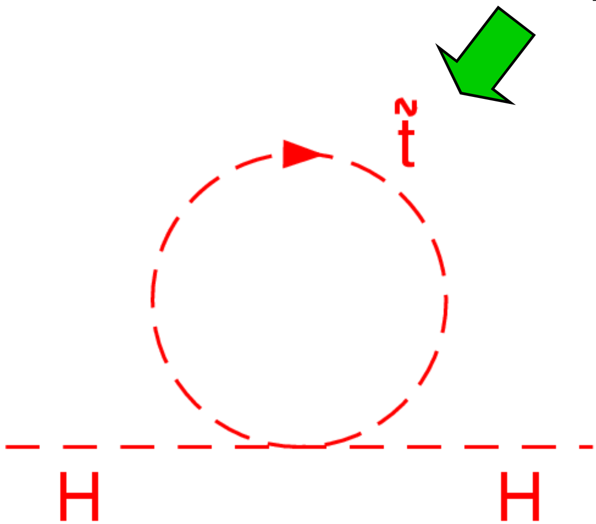
$$R = (-1)^{3B+L+2S}$$

Stability of Higgs mass



$$\Delta m_H^2 = -\frac{\lambda_t^2}{8\pi^2}\Lambda^2$$

With SUSY, Quadratic Divergence Cancels



$$\Delta m_H^2 = +\frac{\lambda_t^2}{8\pi^2}\Lambda^2$$

$$m_{\tilde{t}}^2 - m_t^2 \lesssim (\text{TeV})^2$$

Neutrino mass scale and unification

$$\mathcal{L}_{\Delta L=2} = \frac{LLHH}{M_R}$$

$$\langle H \rangle \sim 246 \text{ GeV and } m_{\nu_3} \sim 0.05 \text{ eV}$$

from atmospheric neutrino oscillation data

$$\Longrightarrow \quad m_R \sim 10^{14} - 10^{15}\,\text{GeV}$$

Very Close to the GUT scale.

Leptogenesis via ν_R decay explains cosmological baryon asymmetry

Fukugita, Yanagida (1986)

Other evidences for unification

- **Electric charge quantization:** $|Q_p| = |Q_e|$
- **Anomaly freedom automatic in many GUTs**
- **Nonzero neutrino masses required in most GUTs**
- **Baryon number violation natural in GUTs – needed for generating cosmological baryon asymmetry**
- **Relates masses of quarks, leptons and neutrinos**

  $M_d = M_\ell^T$ **works well for 3rd family** $(m_b = m_\tau)$

SU(5) GUT

Georgi, Glashow (1974)

Matter multiplets: $\{\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}\}$

$$\mathbf{10}: \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}$$

$$\bar{\mathbf{5}}: (d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

$$\mathbf{1}: \nu^c$$

Higgs: $\mathbf{24}_H, \quad \{\mathbf{5}_H, \quad \bar{\mathbf{5}}_H\}$ ⇨ Contain color triplets $\{H_C, \quad \bar{H}_C\}$

Yukawa Couplings $Y_u^{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{5}_H + Y_d^{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H$

$$M_\ell = M_d^T \Rightarrow m_b = m_\tau, m_s = m_\mu, m_d = m_e$$

MSSM Higgs doublets have color triplet partners in GUTs.

$$H(1,2,1/2) \oplus H_c(3,1,-1/3) = \mathbf{5} \text{ of } SU(5)$$

$$\bar{H}(1,2,-1/2) \oplus \bar{H}_c(\bar{3},1,1/3) = \bar{\mathbf{5}}$$

$H,\ \bar{H}$ **must remain light**

$H_c, \bar{H}_c$ **must have GUT scale mass to prevent rapid proton decay**

Doublet-triplet splitting

Even if color triplets have GUT scale mass, d=5 proton decay is problematic.

Symmetry breaking

Doublet-triplet splitting in SU(5)

$$W_{D-T} = \bar{\mathbf{5}}_H(\lambda \mathbf{24}_H + M)\mathbf{5}_H$$

$$< 24_H >= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix} V$$

FINE-TUNED TO *$O(M_W)$*

$$M_{H_c} = \lambda V + M \sim O(M_{GUT}) \quad M_H = -\frac{3}{2}\lambda V + M$$

The GOOD

(1) Predicts unification of couplings

(2) Uses economic Higgs sector

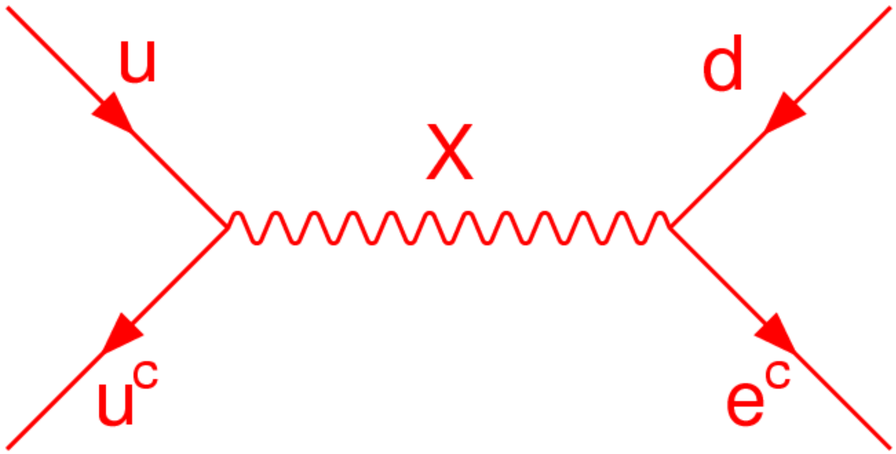
The BAD

(1) Unnatural fine tuning

(2) Large proton decay rate

Nucleon decay in SUSY GUTs

Gauge boson Exchange



$$p \to e^+\pi^0,\ \tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 \approx [2 \times 10^{35\pm1} yr]^{-1}$$

Sources of uncertainty:

Threshold effects, matrix element, $\alpha_s(m_Z)$

SUSY SU(5) Prediction

J. Hisano (2000)

$$\frac{1}{\Gamma(p \to e^+\pi^0)} = (2.0 \times 10^{35}\ \text{yr}) \times \left(\frac{\alpha_H}{0.01\ \text{GeV}^3}\right)^{-2} \left(\frac{\alpha_G}{1/25}\right)^{-2} \left(\frac{A_R}{2.5}\right)^{-2} \left(\frac{M_X}{10^{16}\ \text{GeV}}\right)^{4}$$

$$(-2\alpha_3^{-1} - 3\alpha_2^{-1} + 3\alpha_Y^{-1})(M_Z) = \frac{1}{2\pi}\left\{36\ \ln\left(\frac{M_X}{M_Z}\left(\frac{M_\Sigma}{M_X}\right)^{1/3}\right) + 8\ \ln\left(\frac{M_{\text{SUSY}}}{M_Z}\right)\right\}$$

- Color octect Higgs boson mass is near GUT scale, but it is not precisely known.

$$\frac{M_\Sigma}{M_X} \leq 1.8$$

(Perturbation theory)

Current SuperK limit:

$$\tau(p \to e^+\pi^0) > 8 \times 10^{33}\ \text{yr}$$

SUSY SO(10) Prediction

SO(10) has additional gauge bosons X', Y' that mediate proton decay

Spectrum of gauge bosons

$$\begin{aligned} M^2(V_{u^c}) &= 4g^2(c^2+a^2) \\ M^2(X',Y') &= g^2(4c^2+a^2) \\ M^2(W_R^\pm) &= 4g^2c^2 \\ M^2(X,Y) &= g^2a^2 \end{aligned}$$

$c = \langle 16_H \rangle$ breaks $SO(10)$ to $SU(5)$
$a = \langle 45_H \rangle$ breaks $SU(5)$

- $c < a$ prefered from α_s prediction

X', Y' degenerate with $X, Y \Rightarrow$
enhances proton decay rate by a factor of 2

SUSY SO(10) Prediction (cont)

Unification scale should be above the heaviest gauge boson mass

(X, Y) and (X', Y') are naturally lighter in $SO(10)$ by factor 2 compared to $SU(5)$

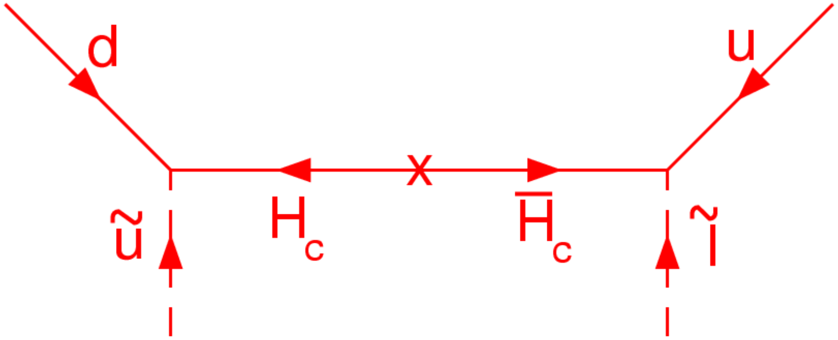
Expectation for SUSY $SO(10)$ for $p \to e^+\pi^0$:

$$\frac{1}{\Gamma(p \to e^+\pi^0)} = (10^{34} - 10^{35})\ \text{yr}$$

Higgsino Exchange

Sakai, Yanagida (1982)

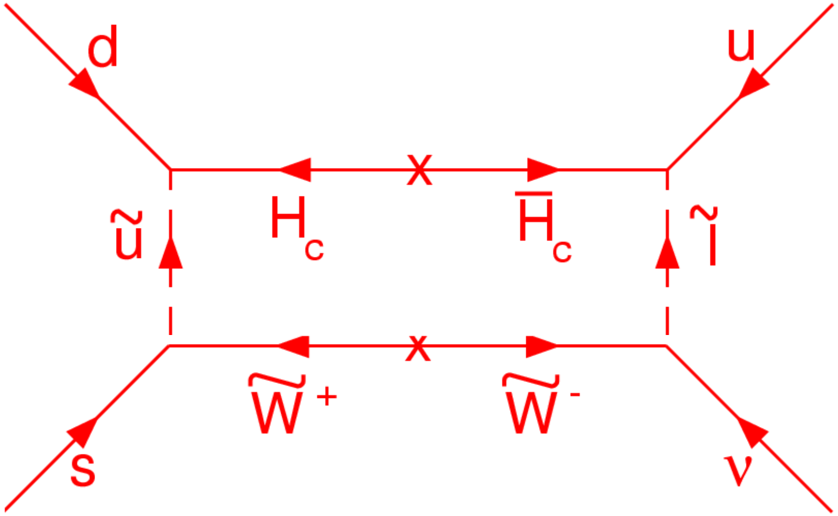
Weinberg (1982)



Higgsino Exchange

Sakai, Yanagida (1982)

Weinberg (1982)



$$p \to \bar{\nu} K^+$$

$$\tau_p^{-1} \approx [\frac{f^2}{M_{H_c} M_{SUSY}}]^2 (\frac{\alpha}{4\pi})^2 m_p^5 \approx [10^{28} - 10^{32} yr]^{-1}$$

Minimal SUSY SU(5) is highly disfavored

Hisano, Murayama, Yanagida (1993)

Murayama, Pierce (2001)
Bajc, Perez, Senjanovic (2002)

SO(10) GUT

☆ **Quarks and leptons ~ {*16*$_i$}**

☆ **Contains ν_R and Seesaw mechanism**

Model with Non-renormalizable Yukawa Couplings

Higgs: $\{45_H + 10_H + 16_H + \bar{16}_H\}$

$$\mathcal{L}_{\text{Yukawa}} = f_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H + h_{ij}16_i16_j\bar{16}_H\bar{16}_H/M_{Pl}$$

$$\Rightarrow \quad m^D_{\nu_\tau} \simeq m_t;\ m^M_{\nu_{\tau_R}} \simeq h_{33}\frac{M^2_{GUT}}{M_{Pl}}$$

$$m_{\nu_\tau} = \frac{m_t^2}{m_{\nu_{\tau_R}}} \simeq 0.05 \text{ eV},\ h_{33} \sim 1$$

Fits the atmospheric neutrino data well

- Small Higgs rep ⇒ small threshold corrections for gauge couplings
- R-parity not automatic (needs a Z_2 symmetry)

SUSY SO(10)

$$W_{D-T} = \lambda(\mathbf{\bar{10}}_H \mathbf{45}_H \mathbf{10}'_H) + \ldots$$

$$< 45_H >= \begin{pmatrix} a & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes i\tau_2 \quad \propto B - L$$

⇨ B-L VEV gives mass to triplets only (DIMOPOULOS-WILCZEK)

⇨ If 10_H only couples to fermions, no d=5 proton decay

⇨ Doublets from 10_H and $10'_H$ light

4 doublets, unification upset

⬇

Add mass term for *10'*$_H$

$$W_{D-T} = \lambda(\mathbf{\bar{10}}_H \mathbf{45}_H \mathbf{10}'_H) + M\mathbf{10}'_H \mathbf{10}'_H$$

Realistic SO(10) GUT

Babu, Pati, Wilczek, (1998)

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon+\sigma \\ 0 & -\epsilon+\sigma & 1 \end{pmatrix} m_U, \quad D = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & \epsilon+\eta \\ 0 & -\epsilon+\eta & 1 \end{pmatrix} m_D,$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon+\sigma \\ 0 & 3\epsilon+\sigma & 1 \end{pmatrix} m_U, \quad L = \begin{pmatrix} 0 & -3\epsilon'+\eta' & 0 \\ 3\epsilon'+\eta' & 0 & -3\epsilon+\eta \\ 0 & 3\epsilon+\eta & 1 \end{pmatrix} m_D$$

$$M_\nu^R = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R$$

$$\begin{aligned} ''1'' &: \ 16_3 16_3 10_H \\ ''\epsilon'' &: \ 16_2 16_3 (10_H \times 45_H)/M \\ ''\sigma'' &: \ 16_2 16_3 (10_H \times 1_H)/M \\ ''\eta'' &: \ 16_2 16_3 16_H 16_H/M \end{aligned}$$

$$\langle 45_H \rangle \propto (B-L)$$

Predictions

$$m_b^0 \approx m_\tau^0$$

$$m_s(1\ GeV) \approx 116\ MeV$$

$$V_{cb} \approx 0.043$$

$$\sin^2 2\theta_{\mu\tau} = (0.96,\ 0.91,\ 0.86,\ 0.83\ ,0.81)$$

$$\frac{m_{\nu_\mu}}{m_{\nu_\tau}} = (1/10,\ 1/15,\ 1/20,\ 1/25,\ 1/30)$$

$$m_d(1\ GeV) \approx 8\ MeV$$

$$\theta_c \approx |\sqrt{m_d/m_s} - e^{i\phi}\sqrt{m_u/m_c}|$$

$$\left|\frac{V_{us}}{V_{cs}}\right| \approx \sqrt{\frac{m_u}{m_c}} \approx 0.07$$

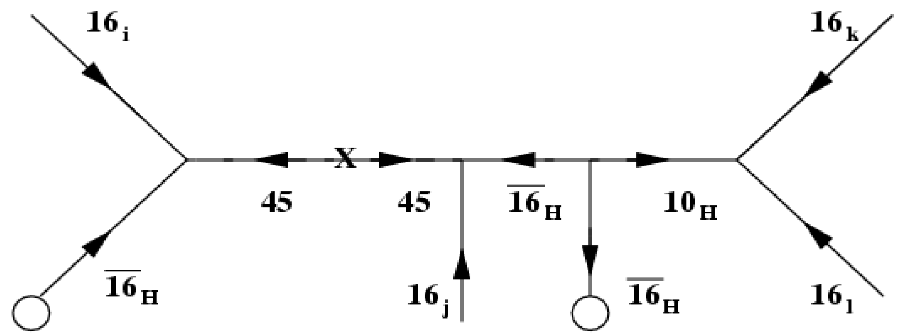
$$\tau(p \to \bar{\nu} K^+) \lesssim 10^{34}\ \text{yr}$$

$$Br(p \to \mu^+ K^0) \sim 10\%$$

Sample amplitude calculation for p decay

$$
\begin{aligned}
\hat{A}[(ud)(s\nu_\tau)] \;\simeq\;& M_{\text{eff}}^{-1} h_{33}^2 [1.9\times10^{-5}\eta_{cd}\eta_{ts}\eta_{\epsilon'} - 9.0\times10^{-6}\eta_{cd}\eta_{cb}\eta_{\epsilon'} \\
-\;& 6.1\times10^{-6}\eta_{td}\eta_{ts}\eta_{cb}\eta_{\epsilon'} + 4.8\times10^{-6}\eta_{ts} - 2.5\times10^{-6}\eta_{cb} \\
+\;& 2.1\times10^{-6}\eta_{td}\eta_{\eta'} + 3.0\times10^{-6}\eta_{cd}\eta_{ts}\eta_{\eta'} + 2.2\times10^{-6}\eta_{cd}\eta_{cb}\eta_{\eta'}] \\
+\;& (M_{16}\tan\gamma)^{-1} h_{33}\hat{f}_{33}[3.1\times10^{-7}\eta_{cd}\eta_{ts}\eta_{\epsilon'} + 6.0\times10^{-7}\eta_{td}\eta_{ts}\eta_{cb}\eta_{\epsilon'} \\
+\;& 4.3\times10^{-7}\eta_{td}\eta_{ts}\eta_{cb}\eta_{\eta'} + 1.5\times10^{-7}\eta_{td}\eta_{\eta'} \\
+\;& 2.3\times10^{-7}\eta_{cd}\eta_{ts}\eta_{\eta'} - 0.0411\eta_{cb}x + 3.1\times10^{-5}\eta_{cd}\eta_{\epsilon'}y \\
+\;& 2.2\times10^{-5}\eta_{cd}\eta_{ts}\eta_{cb}\eta_{\epsilon'}y + 1.1\times10^{-5}\eta_{td}\eta_{cb}\eta_{\eta'}y \\
+\;& 1.6\times10^{-5}\eta_{cd}\eta_{ts}\eta_{cb}\eta_{\eta'}y + 1.1\times10^{-5}\eta_{cd}\eta_{\eta'}y \\
+\;& z\{-2.47\times10^{-4}\eta_{td}\eta_{cb} - 1.28\times10^{-4}\eta_{cd} \\
+\;& 9.94\times10^{-5}\eta_{ts}\eta_{cb}\eta_{\epsilon'} + 7.23\times10^{-5}\eta_{ts}\eta_{cb}\eta_{\eta'}\}]
\end{aligned}
$$

Neutrino mass related proton decay



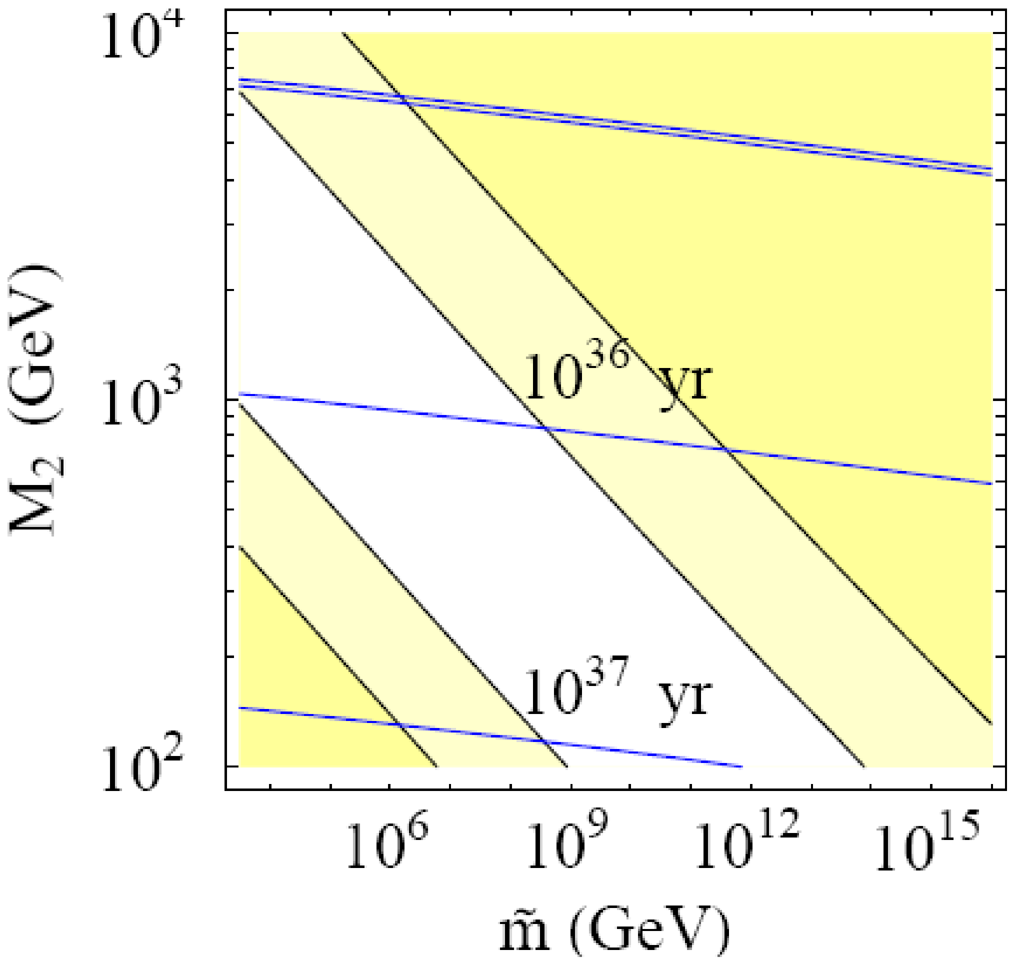
Split Supersymmetry

Arkani-Hamed, Dimopoulos, 2004

- Squarks and sleptons are all superheavy, while a single Higgs and the SUSY fermions have masses of order 10^2 GeV
- Unification works well, as in MSSM, and there is cold dark matter
- Higgsino mediated nucleon decay unobservable
- Gauge boson mediated proton decay predicts

$$\tau(p \to e^+\pi^0) \sim (10^{35} - 10^{37})\ \text{yr}$$

Proton lifetime in Split SUSY



Arkani-Hamed et al, 2004

$$\tau_p^{-1} \equiv \Gamma(p \to e^+\pi^0) = \frac{\pi\, m_p}{4 f_\pi^2}\alpha_N^2 (1 + D + F)^2 \left(\frac{\alpha_{\rm GUT}}{M_V^2}\right)\ \left[A_R^2 + A_L^2\left(1 + |V_{ud}|^2\right)^2\right]$$

Extra dimensional unification

Hall, Nomura, 2001
Kawamura, 2000

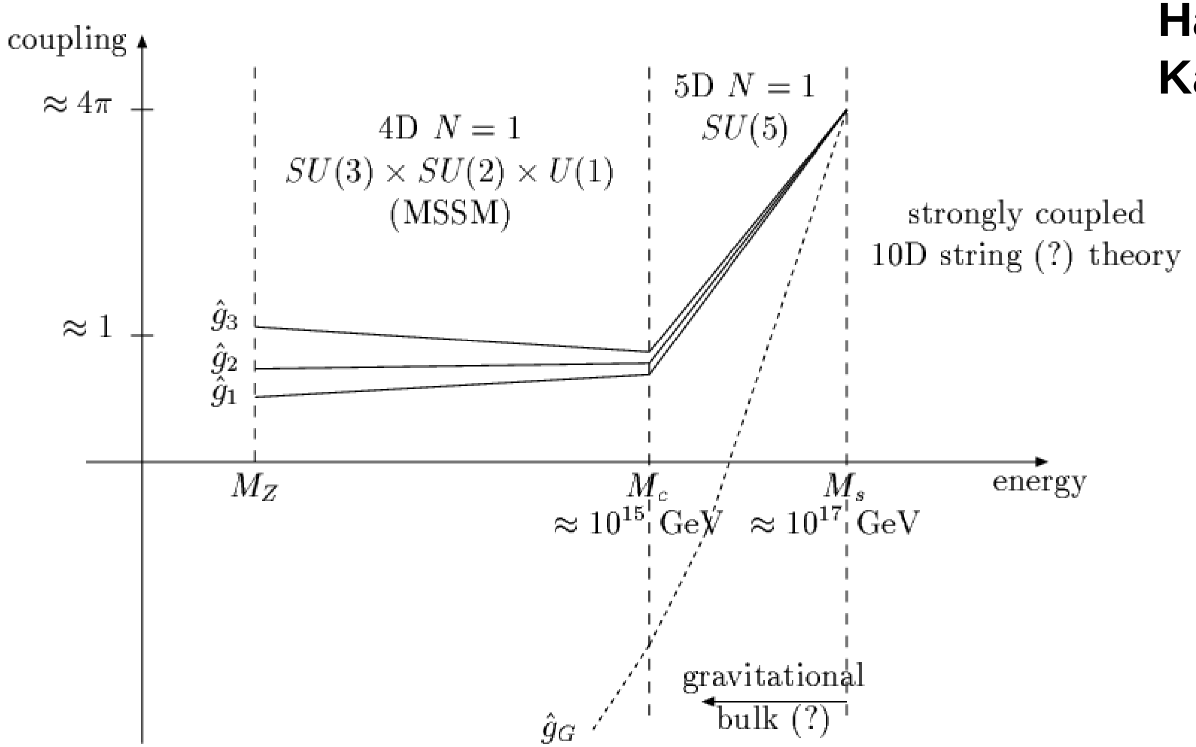


Figure 6: The energy dependence of the strengths of the gauge interactions.

$d = 5$ nucleon decay can be eliminated

$$\tau_p \sim 10^{34} \text{ yr}$$

u_1	:	$\vert\uparrow\downarrow\uparrow\uparrow\downarrow>$
u_2	:	$\vert\uparrow\downarrow\uparrow\downarrow\uparrow>$
u_3	:	$\vert\uparrow\downarrow\downarrow\uparrow\uparrow>$
d_1	:	$\vert\downarrow\uparrow\uparrow\uparrow\downarrow>$
d_2	:	$\vert\downarrow\uparrow\uparrow\downarrow\uparrow>$
d_3	:	$\vert\downarrow\uparrow\downarrow\uparrow\uparrow>$
u_1^c	:	$\vert\downarrow\downarrow\uparrow\downarrow\downarrow>$
u_2^c	:	$\vert\downarrow\downarrow\downarrow\uparrow\downarrow>$
u_3^c	:	$\vert\downarrow\downarrow\downarrow\downarrow\uparrow>$
d_1^c	:	$\vert\uparrow\uparrow\uparrow\downarrow\downarrow>$
d_2^c	:	$\vert\uparrow\uparrow\downarrow\uparrow\downarrow>$
d_3^c	:	$\vert\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν_e	:	$\vert\uparrow\downarrow\downarrow\downarrow\downarrow>$
e	:	$\vert\downarrow\uparrow\downarrow\downarrow\downarrow>$
e^c	:	$\vert\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν_e^c	:	$\vert\uparrow\uparrow\uparrow\uparrow\uparrow>$

$+$

c_1	:	$\vert\uparrow\downarrow\uparrow\uparrow\downarrow>$
c_2	:	$\vert\uparrow\downarrow\uparrow\downarrow\uparrow>$
c_3	:	$\vert\uparrow\downarrow\downarrow\uparrow\uparrow>$
s_1	:	$\vert\downarrow\uparrow\uparrow\uparrow\downarrow>$
s_2	:	$\vert\downarrow\uparrow\uparrow\downarrow\uparrow>$
s_3	:	$\vert\downarrow\uparrow\downarrow\uparrow\uparrow>$
c_1^c	:	$\vert\downarrow\downarrow\uparrow\downarrow\downarrow>$
c_2^c	:	$\vert\downarrow\downarrow\downarrow\uparrow\downarrow>$
c_3^c	:	$\vert\downarrow\downarrow\downarrow\downarrow\uparrow>$
s_1^c	:	$\vert\uparrow\uparrow\uparrow\downarrow\downarrow>$
s_2^c	:	$\vert\uparrow\uparrow\downarrow\uparrow\downarrow>$
s_3^c	:	$\vert\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν_μ	:	$\vert\uparrow\downarrow\downarrow\downarrow\downarrow>$
μ	:	$\vert\downarrow\uparrow\downarrow\downarrow\downarrow>$
μ^c	:	$\vert\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν_μ^c	:	$\vert\uparrow\uparrow\uparrow\uparrow\uparrow>$

$+$

t_1	:	$\vert\uparrow\downarrow\uparrow\uparrow\downarrow>$
t_2	:	$\vert\uparrow\downarrow\uparrow\downarrow\uparrow>$
t_3	:	$\vert\uparrow\downarrow\downarrow\uparrow\uparrow>$
b_1	:	$\vert\downarrow\uparrow\uparrow\uparrow\downarrow>$
b_2	:	$\vert\downarrow\uparrow\uparrow\downarrow\uparrow>$
b_3	:	$\vert\downarrow\uparrow\downarrow\uparrow\uparrow>$
t_1^c	:	$\vert\downarrow\downarrow\uparrow\downarrow\downarrow>$
t_2^c	:	$\vert\downarrow\downarrow\downarrow\uparrow\downarrow>$
t_3^c	:	$\vert\downarrow\downarrow\downarrow\downarrow\uparrow>$
b_1^c	:	$\vert\uparrow\uparrow\uparrow\downarrow\downarrow>$
b_2^c	:	$\vert\uparrow\uparrow\downarrow\uparrow\downarrow>$
b_3^c	:	$\vert\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν_τ	:	$\vert\uparrow\downarrow\downarrow\downarrow\downarrow>$
τ	:	$\vert\downarrow\uparrow\downarrow\downarrow\downarrow>$
τ^c	:	$\vert\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν_τ^c	:	$\vert\uparrow\uparrow\uparrow\uparrow\uparrow>$

Family Unification

$$G = SO(10) \times SO(10) \times SO(10) \times \mathcal{F}$$

- $\mathcal{F}$: Family Parity
- G: Maximally symmetric unification group with 3 families – with no exotics
- For one family, $SO(10)$ is the maximally symmetric unification group

  Anomaly free
  Chiral

- Can realistic models with three family unification be constructed?

Babu, Barr, Gogoladze (2007)

$$G = SO(10) \times SO(10) \times SO(10) \times \mathcal{F}$$

Fermion content

$$\{(16,1,1) + (1,16,1) + (1,1,16)\}$$

With Family Parity $\mathcal{F}$ all 48 components of fermions are indistinguishible

- How does G break down to SM?
- Can realistic fermion masses and mixings arise?
- What predictions?

Symmetry Breaking

- Assume supersymmetry
- Majorana neutrino mass generation and R parity conservation motivates use of a bispinor Higgs:

$$\Delta : \{(16,16,1)+(1,16,16)+(16,1,16)\}$$

$$\overline{\Delta} : \{(\overline{16},\overline{16},1)+(1,\overline{16},\overline{16})+(\overline{16},1,\overline{16})\}$$

$\langle\Delta\rangle + \left\langle\overline{\Delta}\right\rangle$ break

$$\boxed{SO(10)^3 \times \mathcal{F} \rightarrow SU(5)^3 \times \mathcal{F}}$$

$$\boxed{SO(10)^3 \times \mathcal{F} \to SU(4)_c \times SU(2)_L \times SU(2)_R}$$

Bifundamental Higgs:

$$\Omega_i = \{(10,10,1) + (1,10,10) + (10,1,10)\}$$

Combined effect:

$$SO(10)^3 \times \mathcal{F} \to SU(3)_c \times SU(2) \times U(1)_Y$$

Two Ω fields needed for $\mathcal{F}$ breaking
and natural doublet-triplet splitting

Fundamental Higgs:

$$H = \{(10,1,1) + (1,10,1) +)1,1,10)\}$$

for electroweak symmetry breaking

Doublet-Triplet Splitting

$$W_{DT} = \alpha[H_1 H_2 \Omega_{12} + H_2 H_3 \Omega_{23} + H_3 H_1 \Omega_{31}]$$

Doublet Mass Matrix:

$$M_D \quad = \quad \begin{pmatrix} 0 & \alpha b & 0 \\ \alpha b & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

A pair of doublets from H_3 remain light

Other doublets are heavy

Triplet Mass Matrix:

$$M_T \quad = \quad \begin{pmatrix} 0 & \alpha a & \alpha a \\ \alpha a & 0 & \alpha a \\ \alpha a & \alpha a & 0 \end{pmatrix}$$

All triplets are heavy

Fermion Masses

$$\begin{aligned} W_Y &= Y[\psi_1\psi_1 H_1 + \psi_2\psi_2 H_2 + \psi_3\psi_3 H_3] \\ &+ F(\psi_1\psi_2\overline{\Delta}_{12} + \psi_2\psi_3\overline{\Delta}_{23} + \psi_3\psi_1\overline{\Delta}_{31}) \end{aligned}$$

H_3 has light doublets, $H_{1,2}$ do not

$\Rightarrow$ First two family masses $= 0$

Third family mass $\neq 0$

Right handed neutrino Majorana Mass Matrix:

$$M_R = M_0 \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Eigenvalues: $M_0 \times \{-1, -1, 2\}$

$\Rightarrow$ Resonant Leptogenesis!

Other Predictions

There is no SUSY flavor problem

All scalar fields have same mass-squared
Gaugino mass is universal

Proton lifetime

Threshold corections need to be computed

Best estimate for $d = 6$ proton lifetime:
$\tau_p \sim 10^{34}$ years

Neutrino oscillation data seems consistent

$\tan\beta$ is large ~ 40

Summary and Conclusions

There is strong circumstancial evidence for unification

Nulceon decay is the missing link

Search for supersymmetric modes for nucleon decay should continue vigorously

Experimental program should aim for the more model independent gauge boson mediated proton decay

Megaton Water Cerenkov detector is the best way to probe proton decay

It will be great to reach sensitivity of $10^{34} - 10^{36}$ yr