Thermalization of axion dark matter

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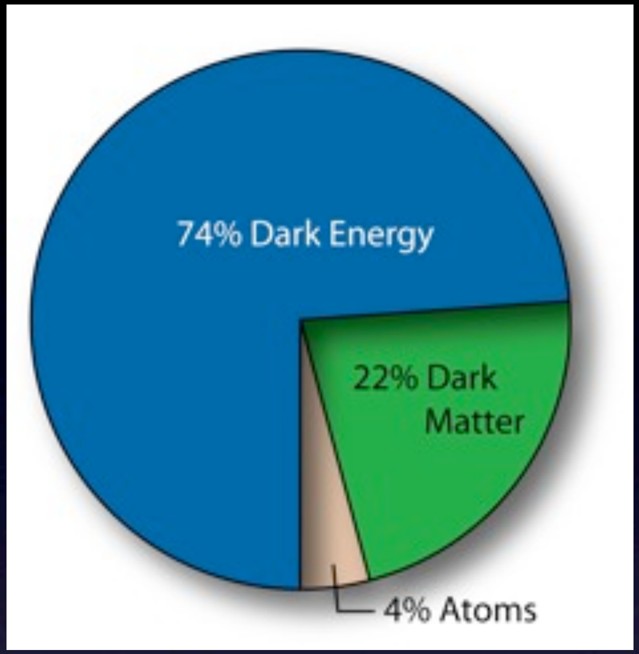
Reference: KS and M. Yamaguchi, arXiv:1210.7080 [hep-ph]

Abstract

- Discuss the possibility that QCD axions form a Bose-Einstein condensate (BEC)
- Calculate time evolution of occupation number of axions in the condensed regime
  - Derive a formula for thermalization rate
  - Revisit axion cosmology

Dark Matter

- Recent astrophysical observations
  - 22% of the total energy of universe is occupied by unknown matter
  - “invisible”
    (interaction with ordinary matters is weak)



http://map.gsfc.nasa.gov/

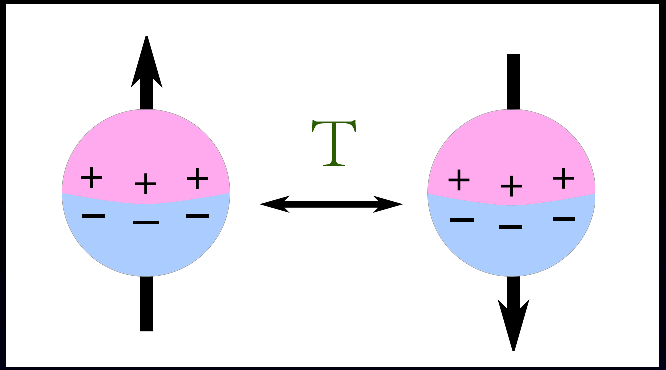
- Physics beyond the standard model
  - a well motivated candidate : axion

Strong CP problem

- How they are produced, and how they evolved ?
  → key to understand the nature of dark matter

Strong CP problem

- Neutron electric dipole moment d_n



$$H = -\mu_n \mathbf{B} \cdot \frac{\mathbf{S}}{S} - d_n \mathbf{E} \cdot \frac{\mathbf{S}}{S}$$

time-reversal

$$T(\mathbf{B} \cdot \mathbf{S}) = \mathbf{B} \cdot \mathbf{S}, \qquad T(\mathbf{E} \cdot \mathbf{S}) = -\mathbf{E} \cdot \mathbf{S}$$

  - non zero value of d_n → violation of T (violation of CP)

  Experiments: $d_n < 3 \times 10^{-26} e\mathrm{cm}$ Baker et. al. (2006)

- θ term in Quantum chromodynamics (QCD)
  - violates CP $\mathcal{L}_\theta = \frac{\theta}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu}$

Theoretical estimation: $d_n \simeq 10^{-16} \theta e\mathrm{cm}$

$\longrightarrow$ $\boxed{|\theta| < 10^{-10}}$

- Why θ is so small ?

Peccei-Quinn mechanism

Peccei, Quinn (1977)

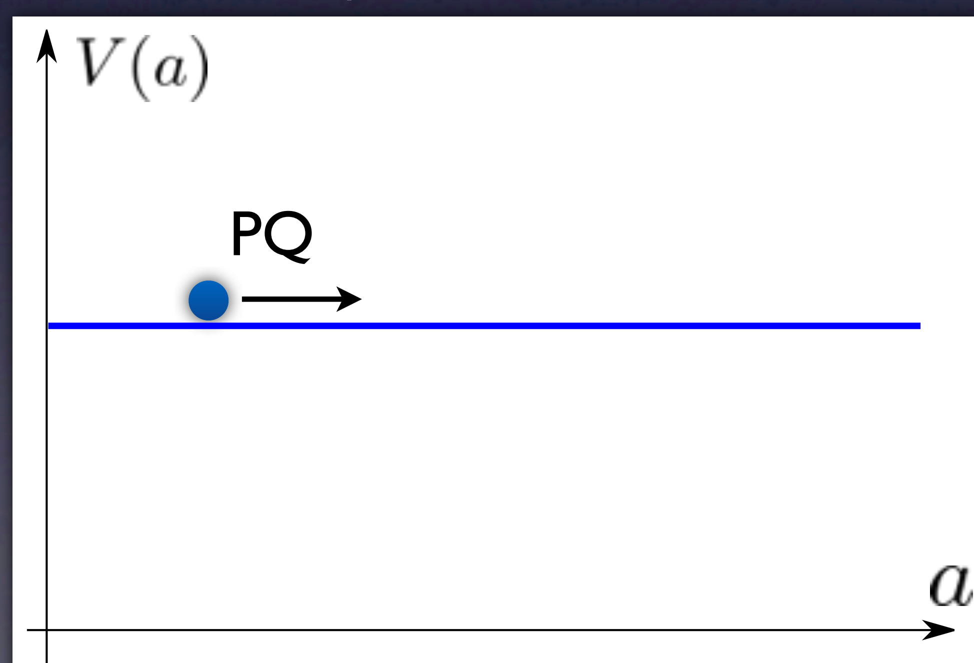
- Take θ as a dynamical variable (field)

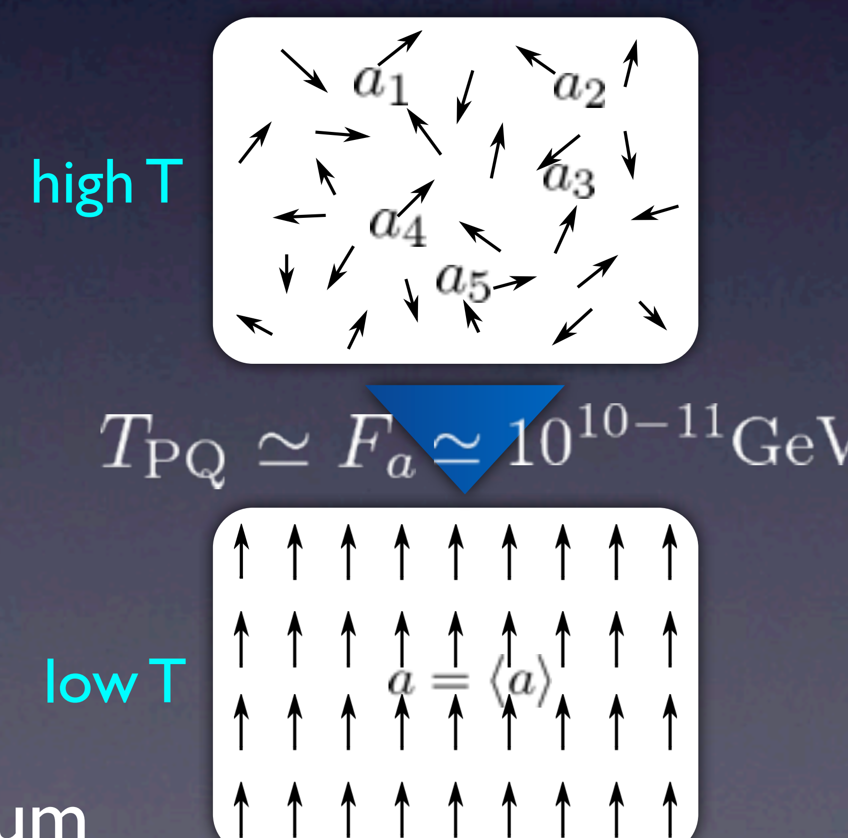
$$\theta \quad \to \quad \theta_{\text{eff}}(x) = \theta + a(x)/F_a$$

- Peccei-Quinn (PQ) symmetry breaking

PQ: $\quad a \to a + \lambda, \qquad \lambda = \text{const.}$

  - potential for $a(x)$ has a minimum at $\langle a \rangle = -\theta F_a$ (QCD effect)





  - naturally predict $\theta_{\text{eff}} \simeq 0$ in our vacuum

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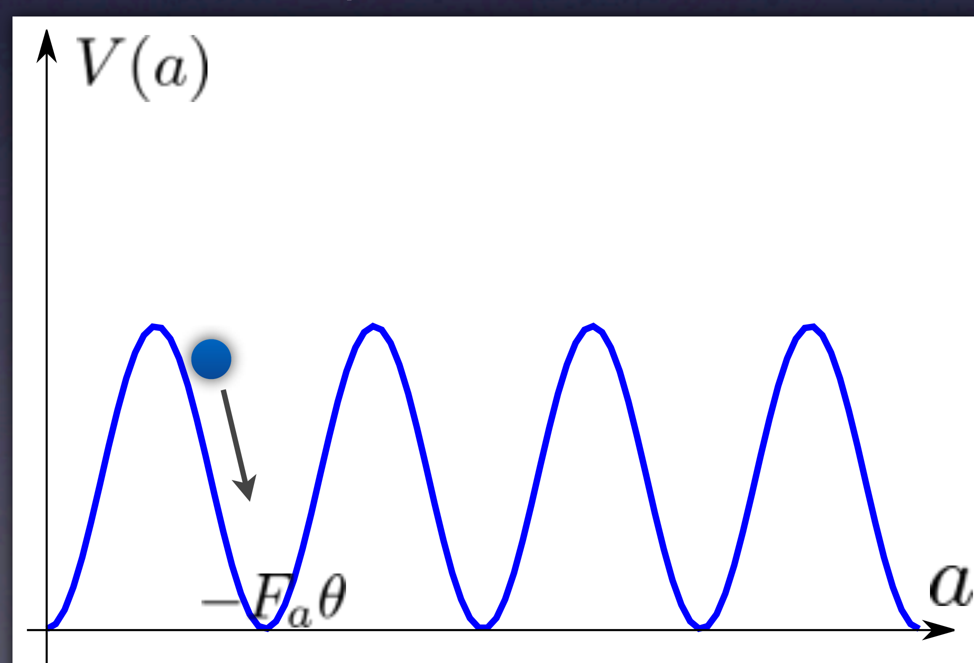
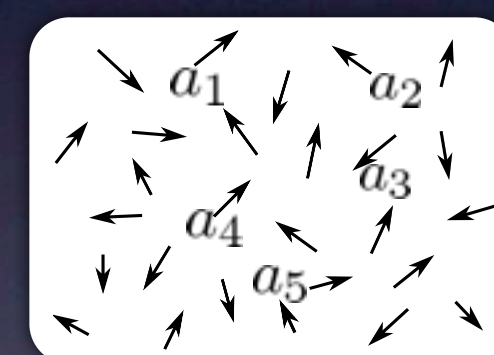
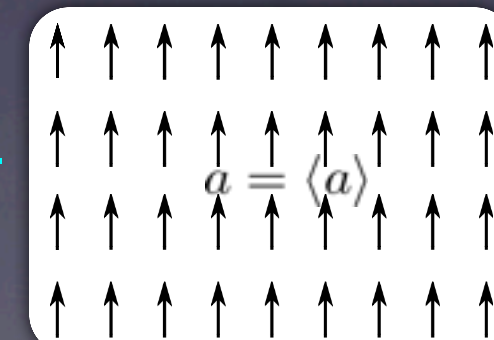


high T



$$T_{\text{PQ}} \simeq F_a \simeq 10^{10-11}\text{GeV}$$

low T



  - naturally predict $\theta_{\text{eff}} \simeq 0$ in our vacuum

Axion

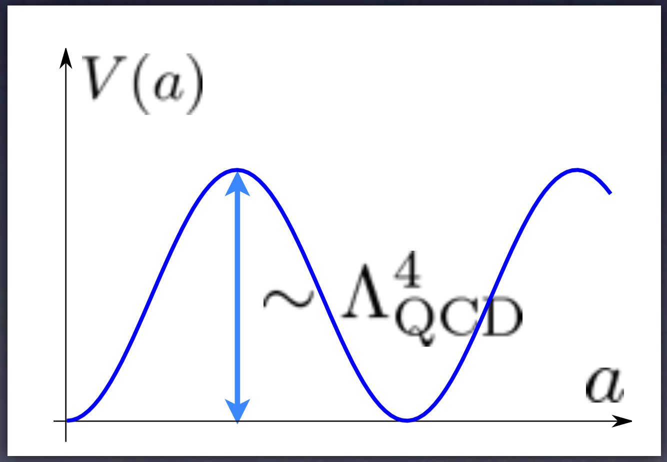
- Spontaneous breaking of continuous Peccei-Quinn symmetry at $T \simeq F_a \simeq 10^{9-12}\mathrm{GeV}$ "axion decay constant"
  - Nambu-Goldstone theorem
    → emergence of the (massless) particle ≡ axion

Weinberg(1978), Wilczek(1978)

- Axion has a small mass (QCD effect)
  → pseudo-Nambu-Golstone boson

$$m_a \sim \frac{\Lambda_{\mathrm{QCD}}^2}{F_a} \sim 6 \times 10^{-6}\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{F_a}\right)$$

$$\Lambda_{\mathrm{QCD}} \simeq \mathcal{O}(100)\mathrm{MeV}$$

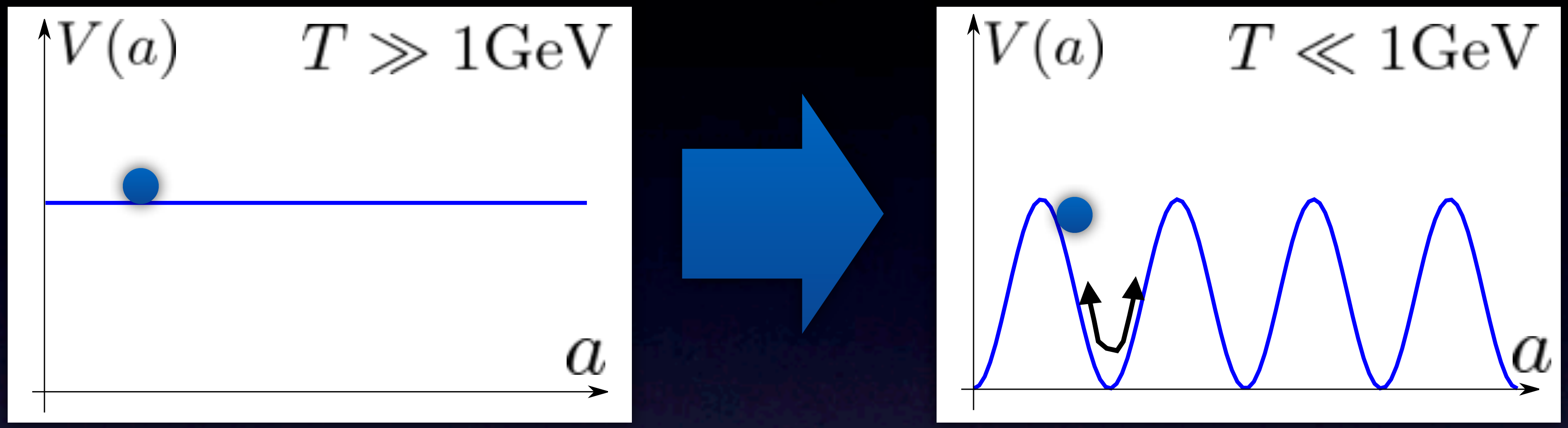


- Tiny coupling with matter + non-thermal production
  → good candidate of cold dark matter

Production mechanism

Preskill, Wise, Wilczek (1983); Abbott, Sikivie (1983); Dine, Fischler (1983)

- Misalignment mechanism



The axion mass "turns on" at $m_a(t_1) = H(t_1)$ ($T_1 \sim 1\text{GeV}$)

- EOM for homogeneous axion field

$$\left(\frac{d^2}{dt^2} + \frac{3}{2t}\frac{d}{dt} + m_a^2\right)\langle a \rangle = 0$$

$$\Rightarrow m_a A^2 \propto R^{-3}(t) \quad , \quad \langle a \rangle = A(t)\cos(m_a t)$$

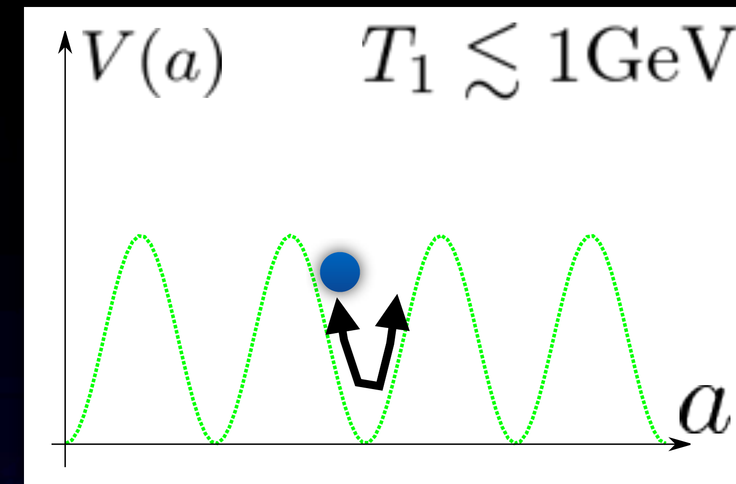
$R(t)$: scale factor of the universe

$$\Rightarrow \rho_a(t) = \frac{1}{2} m_a^2 \langle a \rangle^2 \propto R^{-3}(t)$$

behave like non-relativistic matter

Peculiarities of axion dark matter

- Non-thermal production



$$H \lesssim m_a \quad (t = t_1)$$

$$\Rightarrow \quad t_1 \sim 10^{-7}\mathrm{sec}$$

$$\delta v \sim \frac{\delta p}{m_a} \sim \frac{R(t_1)}{R(t_0)} \frac{1}{m_a t_1} \sim 3 \times 10^{-17} \left(\frac{F_a}{10^{12}\mathrm{GeV}} \right)^{0.81}$$

"cold" dark matter

- Large occupation number

$$\mathcal{N} \sim n_a \frac{(2\pi)^3}{\frac{4\pi}{3}(m_a \delta v)^3} \sim 10^{61} \left(\frac{F_a}{10^{12}\mathrm{GeV}} \right)^{2.75}$$

($n_a \sim m_a F_a^2 (R(t_1)/R(t_0))^3$: number density of axions)

c.f. $\mathcal{N} \sim 10^{-18} \left(\frac{100\mathrm{GeV}}{m_{\mathrm{wimp}}} \right)^4$ for WIMPs

Do axions form a BEC ?

- Bose-Einstein condensate
  - Large fraction of bosons are in the lowest-energy state
  - Critical temperature

$$T_c = \left(\frac{\pi^2 n_a}{\zeta(3)}\right)^{1/3} \simeq 2\times 10^2\mathrm{GeV}\left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{0.54}\left(\frac{R(t_1)}{R(t)}\right)$$

$$\overset{!}{\gg} \delta\omega \sim \frac{1}{2}m_a(\delta v)^2 \sim 4\times 10^{-13}\mathrm{eV}\left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{0.25}\left(\frac{R(t_1)}{R(t)}\right)^2$$

- Assumptions

1. Particles are bosons
2. Number is conserved
3. Large occupation number
4. In thermal equilibrium

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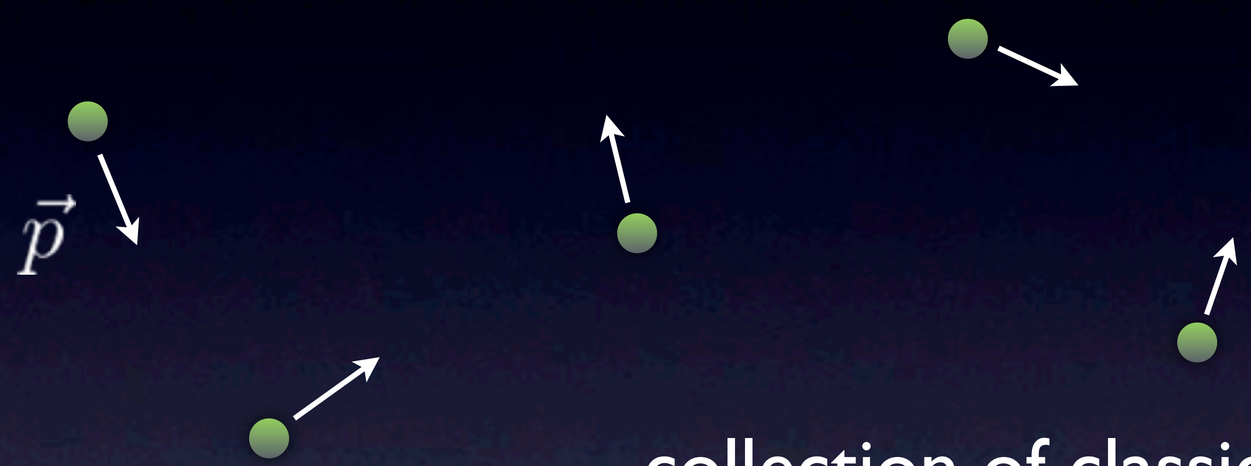
- Assumptions

	For axions
1. Particles are bosons	→ satisfied
2. Number is conserved	→ satisfied
3. Large occupation number	→ satisfied
4. In thermal equilibrium	→ ???

Axions vs WIMPs

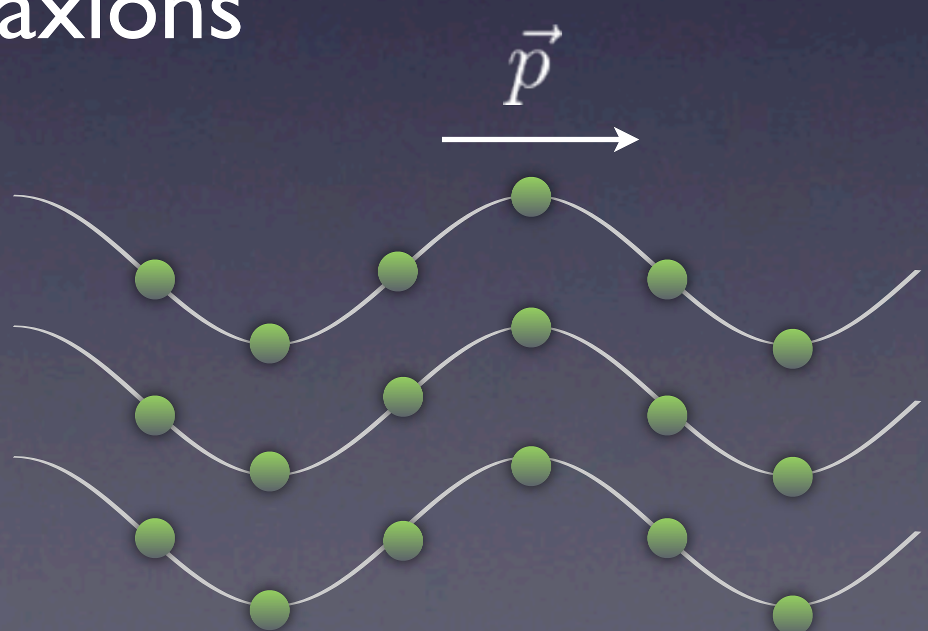
- Thermalize if $\Gamma \sim \dot{\mathcal{N}}(p)/\mathcal{N}(p) > H$
- WIMPs



collection of classical "point particles"

→ evolution : use Boltzmann eq.

- axions



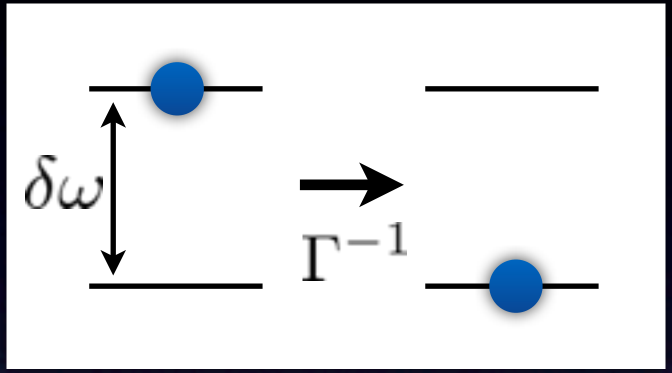
"wavy fields"
rather than point particles

→ cannot use Boltzmann eq.

Previous study (1)

Erken, Sikivie, Tam, Yang, PRD85, 063520 (2012)

- Consider transitions between different quantum states.



- Two different regimes
- WIMPs $\delta\omega \to \mathrm{large}$

$$\delta\omega \gg \Gamma$$

energy exchanged in the transitions — transition rate

"particle kinetic regime"

- axions $\delta\omega \to \mathrm{small}$

$$\delta\omega \ll \Gamma$$

"condensed regime"

A transition makes sense if $\mathcal{N}\delta\omega \gg \Gamma$

Previous study (2)

Erken, Sikivie, Tam, Yang, PRD85, 063520 (2012)

- Time evolution of quantum operators in the Heisenberg picture

l : label of the state (momentum)

$$H = \sum_i \omega_i a_i^\dagger a_i + \sum_{i,j,k,l} \frac{1}{4} \Lambda_{kl}^{ij} a_k^\dagger a_l^\dagger a_i a_j \qquad \mathcal{N}_l = a_l^\dagger a_l$$

$$\begin{aligned}
\dot{\mathcal{N}}_l &= i[H, \mathcal{N}_l] \\
&= i \sum_{i,j,k} \frac{1}{2} (\Lambda_{ij}^{kl} a_i^\dagger a_j^\dagger a_k a_l e^{-i\Omega_{ij}^{kl} t} - \mathrm{H.c.}) \\
&\quad + \sum_{k,i,j} \frac{1}{2} |\Lambda_{ij}^{kl}|^2 [\mathcal{N}_i \mathcal{N}_j (\mathcal{N}_l + 1)(\mathcal{N}_k + 1) \\
&\quad - \mathcal{N}_l \mathcal{N}_k (\mathcal{N}_i + 1)(\mathcal{N}_j + 1)] \frac{2}{\Omega_{ij}^{kl}} \sin(\Omega_{ij}^{kl} t) + \ldots
\end{aligned}$$

$$\Omega_{ij}^{kl} \equiv \omega_k + \omega_l - \omega_i - \omega_j$$

- What about the quantum-mechanical averages $\langle \dot{\mathcal{N}}_l(t) \rangle$?

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$$\dot{\mathcal{N}}_l = i[H, \mathcal{N}_l]$$

$$= \boxed{i \sum_{i,j,k} \frac{1}{2} (\Lambda_{ij}^{kl} a_i^\dagger a_j^\dagger a_k a_l e^{-i\Omega_{ij}^{kl} t} - \mathrm{H.c.})}$$

Leading contribution in the condensed regime

$$\Omega_{ij}^{kl} t \ll 1$$

$$\dot{\mathcal{N}}_l \sim \mathcal{O}(\Lambda_{ij}^{kl})$$

$$+ \sum_{k,i,j} \frac{1}{2} |\Lambda_{ij}^{kl}|^2 [\mathcal{N}_i \mathcal{N}_j (\mathcal{N}_l + 1)(\mathcal{N}_k + 1)$$

$$- \mathcal{N}_l \mathcal{N}_k (\mathcal{N}_i + 1)(\mathcal{N}_j + 1)] \frac{2}{\Omega_{ij}^{kl}} \sin(\Omega_{ij}^{kl} t) + \dots$$

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\end{aligned}$$

reduce to Boltzmann eq.
in the particle kinetic regime

$$\Omega^{kl}_{ij} t \gg 1$$

$$\dot{\mathcal{N}}_l \sim \mathcal{O}(|\Lambda^{kl}_{ij}|^2)$$

$$\Omega^{kl}_{ij} \equiv \omega_k + \omega_l - \omega_i - \omega_j$$

- What about the quantum-mechanical averages $\langle \dot{\mathcal{N}}_l(t) \rangle$?

Effects on cosmological parameters ?

- Thermalization rate is enhanced in the condensed regime → leads to axion BEC
- Thermalization rate with other species is also enhanced (?) Erken, Sikivie, Tam, Yang, PRD85, 063520 (2012); PRL108, 061304 (2012)
  - axions and photons have thermal contact

$$\rho_{\gamma i} = \frac{\pi^2}{15} T_{\gamma i}^4 = \rho_{\gamma f} + \rho_{a f} = \frac{\pi^2}{30} T_{\gamma f}^4 (2+1)$$

$$\Rightarrow \quad T_{\gamma f} = (2/3)^{1/4} T_{\gamma i}$$

baryon-to-photon ratio at BBN $\eta_{\rm BBN} = (2/3)^{3/4} \eta_{\rm std.}$

effective # of neutrino d.o.f. $N_{\rm eff} = 6.77$ (obs. $N_{\rm eff} \simeq 3-4$)

- Is it true ? Does axion BEC conflict with standard cosmology?

Analysis method

In-in formalism

Weinberg, PRD72, 043514 (2005)

- Calculate expectation value of a quantum operator via perturbative expansion

$$\langle \mathcal{O}(t) \rangle = \langle \mathrm{in} | \mathcal{O}(t) | \mathrm{in} \rangle$$
$$= \langle \mathcal{O} \rangle + i \int_{t_0}^{t} dt_1 \langle [H_I(t_1), \mathcal{O}] \rangle$$
$$+ (i)^2 \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \langle [H_I(t_1), [H_I(t_2), \mathcal{O}]] \rangle + \ldots$$

hereafter,

ϕ : axion field

$$\mathcal{O} = \mathcal{N}_n \equiv \frac{a_n^\dagger a_n}{V}$$: number operator

$$\int \frac{d^3p}{(2\pi)^3} \to \frac{1}{V} \sum_{n_x, n_y, n_z}$$

(label becomes discrete)

$$H_I(t) = \frac{1}{V^4} \sum_{ijkl} \frac{1}{4} \Lambda_{kl}^{ij} e^{-i\Omega_{kl}^{ij} t} a_k^\dagger a_l^\dagger a_i a_j$$

(ignore axion # violating process)

(1) scalar self-coupling $\quad \Lambda_{kl}^{ij} = -\frac{\lambda}{4m_a^2} V \delta_{i+j,k+l} \quad \Leftarrow \quad H_I = -\int d^3x \frac{\lambda}{4!} \phi^4$

(2) gravity $\quad \Lambda_{kl}^{ij} = -4\pi G m_a^2 \left(\frac{1}{|\mathbf{p}_k - \mathbf{p}_i|^2} + \frac{1}{|\mathbf{p}_k - \mathbf{p}_j|^2} \right) V \delta_{i+j,k+l} \quad \Leftarrow \quad H_I = -\frac{G}{2} \int d^3x d^3x' \frac{\rho(\mathbf{x},t)\rho(\mathbf{x}',t)}{|\mathbf{x} - \mathbf{x}'|}$

In state

- $|\mathrm{in}\rangle$ = a state which represents the coherent oscillation of axions
- Use a coherent state

$$|\{\alpha\}\rangle = \prod_i e^{-\frac{1}{2}|\alpha_i|^2} \sum_{n=0}^{\infty} \frac{\alpha_i^n}{n!\sqrt{V^n}} (a_i^\dagger)^n |0\rangle$$

with $a_i|0\rangle = 0$

$$a_i|\alpha_i\rangle = V^{1/2}\alpha_i|\alpha_i\rangle$$

- Field amplitude

$$\phi = \frac{1}{V}\sum_n \frac{1}{\sqrt{2E_{p_n}}}(e^{ip_n\cdot x}a_n + e^{-ip_n\cdot x}a_n^\dagger)$$

$$\langle\{\alpha\}|\phi|\{\alpha\}\rangle = \sum_n \frac{1}{\sqrt{2m_aV}}(e^{-im_at+i\mathbf{p_n}\cdot\mathbf{x}}\alpha_n + e^{im_at-i\mathbf{p_n}\cdot\mathbf{x}}\alpha_n^*)$$

$$= \sum_n \sqrt{\frac{2}{m_aV}}|\alpha_n|\cos(m_at - \mathbf{p_n}\cdot\mathbf{x} - \beta)$$

classical field trajectory

- c.f. number state for other species (photons, baryons,...)

$$|\{\mathcal{N}\}\rangle = \prod_k \frac{1}{\sqrt{\mathcal{N}_k!V^{\mathcal{N}_k}}}(a_k^\dagger)^{\mathcal{N}_k}|0\rangle$$

"Zero modes"

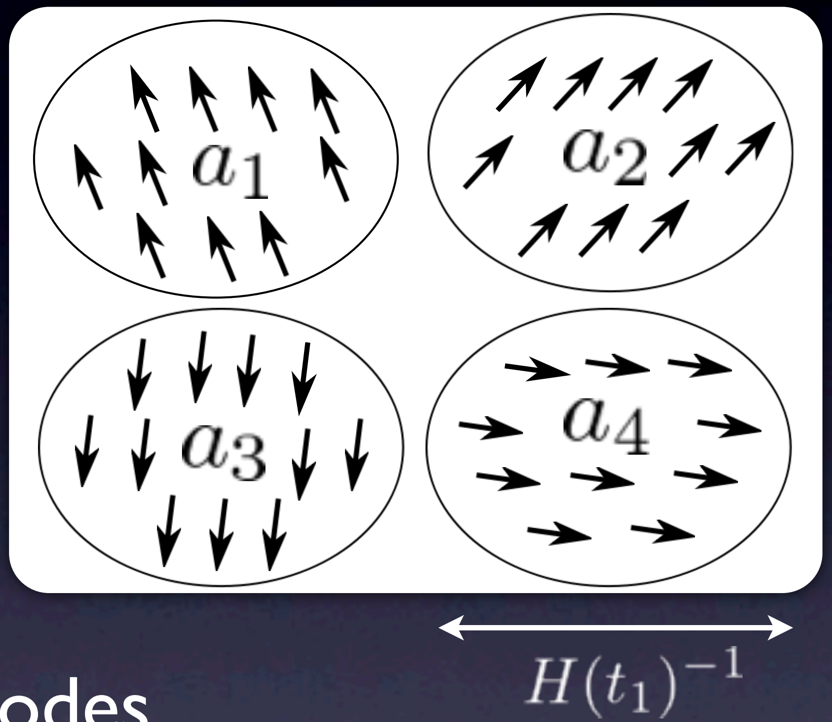
- Initial time t_1 (QCD phase transition) :

  amplitudes of oscillation might be uncorrelated

  beyond the horizon

  → axions have non-zero (but tiny) momenta

  $$p(t_1) \lesssim H(t_1) \sim m_a(t_1)$$



- Assume plural (say K) oscillating modes

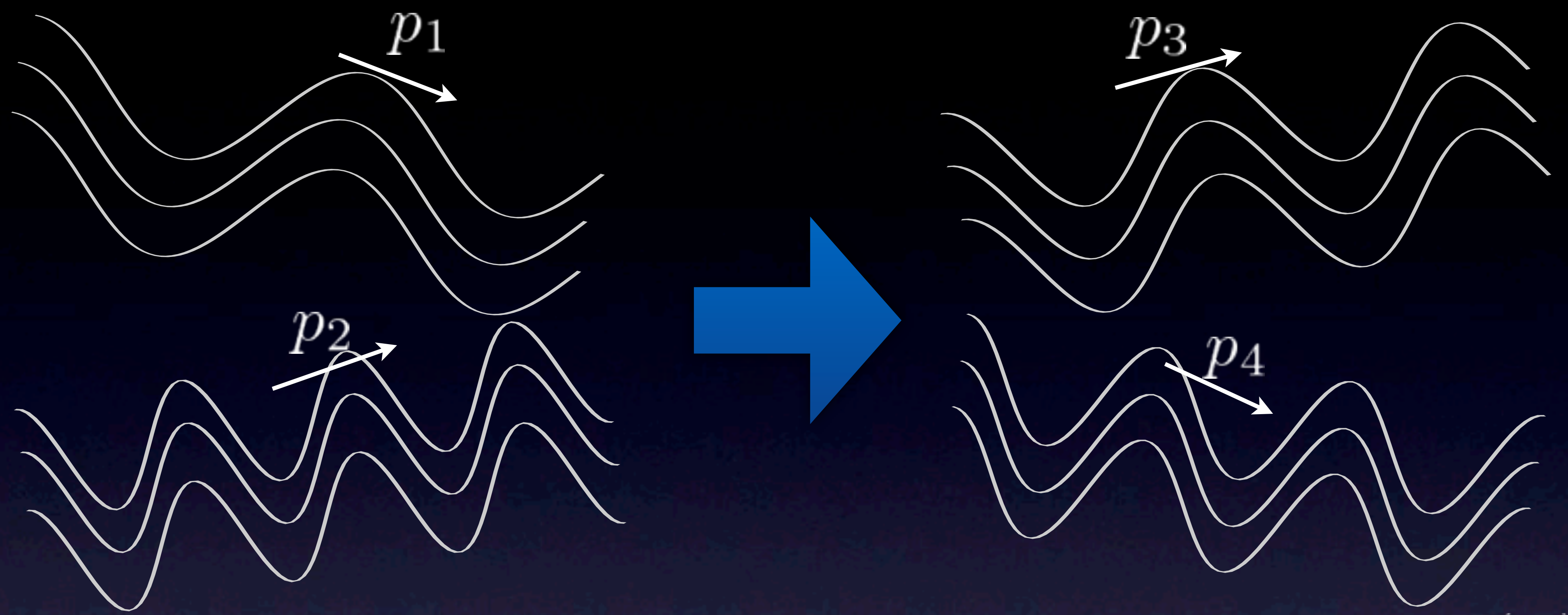
$$|\{\alpha\}\rangle = \prod_i^K e^{-\frac{1}{2}|\alpha_i|^2} \sum_{n=0}^{\infty} \frac{\alpha_i^n}{n!\sqrt{V^n}} (a_i^\dagger)^n |0\rangle$$

$|\mathbf{p}_i| \lesssim H(t_1) \sim m_a(t_1)$ for $i = 1, \ldots, K$

- $|\alpha_i|^2 \leftrightarrow$ momentum distribution

$$n_a = \frac{1}{V}\sum_n \langle\{\alpha\}|\mathcal{N}_n|\{\alpha\}\rangle = \frac{1}{V}\sum_i^K |\alpha_i|^2 \equiv \sum_i^K n_{c,i}$$

Cosmic axion thermalization



$p_1, p_2, \ldots, p_K < H(t_1)$

- Question : how these plural oscillating modes (“zero modes”) reach thermal equilibrium ?
  - decoupled axions
    = each of K modes oscillates independently
  - self-interacting axions
    = transition between plural modes becomes significant
    → change the distribution $|\alpha_i|$

Evolution of occupation number

$$\langle \mathrm{in}|\mathcal{N}_p(t)|\mathrm{in}\rangle = \langle \mathcal{N}_p\rangle + i\int_{t_0}^{t} \langle [H_I(t_1), \mathcal{N}_p]\rangle + \mathcal{O}(H_I^2) + \dots$$

$$i\int_{t_0}^{t} dt_1 \langle [H_I(t_1), \mathcal{N}_p]\rangle \xrightarrow{t-t_0\to\infty} -\frac{1}{2V^2}\sum_j^K \sum_k^K \sum_l^K \left[\Lambda_{kl}^{pj}\frac{e^{-i\Omega_{kl}^{pj}t}}{\Omega_{kl}^{pj}}\alpha_k^*\alpha_l^*\alpha_j\alpha_p + \mathrm{c.c.}\right]$$

for $$|\mathrm{in}\rangle = \prod_i^K e^{-\frac{1}{2}|\alpha_i|^2}\sum_{n=0}^{\infty}\frac{\alpha_i^n}{n!\sqrt{V^n}}(a_i^\dagger)^n|0\rangle$$

coherent state

$$i\int_{t_0}^{t} dt_1 \langle [H_I(t_1), \mathcal{N}_p]\rangle = 0$$ for $$|\mathrm{in}\rangle = \prod_k \frac{1}{\sqrt{\mathcal{N}_k! V^{\mathcal{N}_k}}}(a_k^\dagger)^{\mathcal{N}_k}|0\rangle$$

number state

- First order term is relevant if

  (1) condensed regime $\Omega_{kl}^{pj}t \ll 1$

  (c.f. $e^{-i\Omega_{kl}^{pj}t} \approx 0$ for particle kinetic regime $\Omega_{kl}^{pj}t \gg 1$)

  (2) coherent state representation $|\mathrm{in}\rangle = |\{\alpha\}\rangle$

Application to axion cosmology

Thermalization rate

- Thermalization rate of coherently oscillating components

$$\Gamma \equiv \frac{1}{\mathcal{N}_p(t)} \frac{d\mathcal{N}_p(t)}{dt} \simeq \Lambda n_a$$

n_a : number density of axions

$\Lambda_{pj}^{kl} = \Lambda V \delta_{k+l,p+j}$: coefficient in the interaction term

scalar phi^4 $\quad \Gamma_s \simeq \dfrac{\lambda n_a}{4m_a^2}$

gravity $\quad \Gamma_g \simeq \dfrac{4\pi G m_a^2 n_a}{(\delta p)^2}$

$$\delta p \sim m_a \delta v \propto 1/R(t)$$

- Exceed the expansion rate at

$$\Gamma_g \gtrsim H \Rightarrow \boxed{T_{\mathrm{BEC}} \simeq 2 \times 10^3 \mathrm{eV} \left(\frac{F_a}{10^{12}\mathrm{GeV}} \right)^{0.56}}$$

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gravity $\quad \boxed{\Gamma_g \simeq \dfrac{4\pi G m_a^2 n_a}{(\delta p)^2} \quad \propto 1/R(t)}$

$$\delta p \sim m_a \delta v \propto 1/R(t)$$

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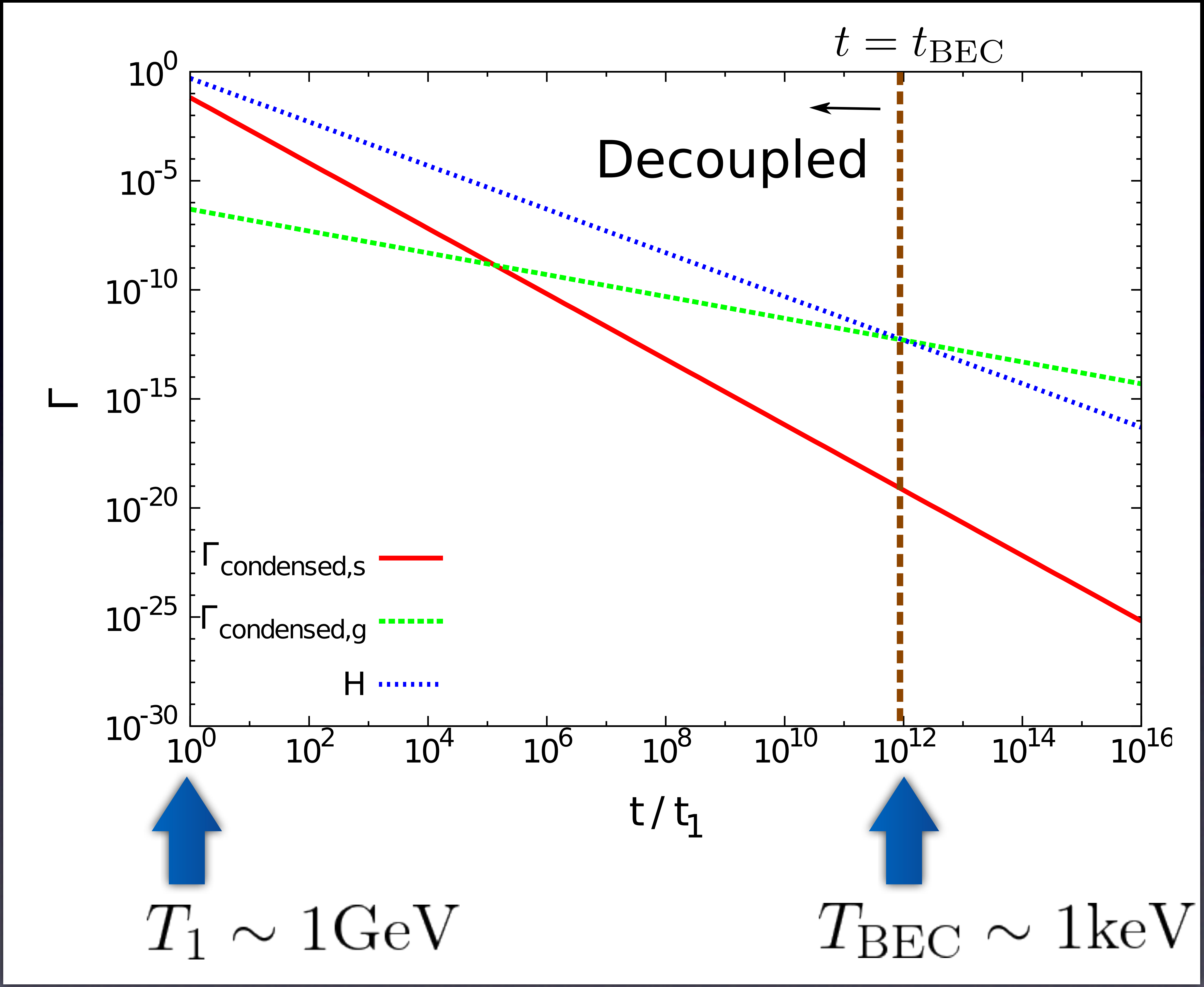


$t = t_{\rm BEC}$
Decoupled
Γ
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
$\Gamma_{\rm condensed,s}$
$\Gamma_{\rm condensed,g}$
H
10^{0} 10^{2} 10^{4} 10^{6} 10^{8} 10^{10} 10^{12} 10^{14} 10^{16}
t/t_1
$T_1 \sim 1{\rm GeV}$
$T_{\rm BEC} \sim 1{\rm keV}$

No photon cooling

- Interaction with other species b

$$H_{I,b}(t) = \frac{1}{V^4} \sum_{ijkl} \frac{1}{4} \Lambda_b{}^{ij}{}_{kl} e^{-i\Omega^{ij}_{kl} t} a^\dagger_k b^\dagger_l a_i b_j$$

- Assume b particles are represented as a number state

$$|\mathrm{in}\rangle = \prod_k \frac{1}{\sqrt{\mathcal{N}_k! V^{\mathcal{N}_k}}} (b^\dagger_k)^{\mathcal{N}_k} |\{\alpha\}\rangle$$

while $|\{\alpha\}\rangle = \prod_i^K e^{-\frac{1}{2}|\alpha_i|^2} \sum_{n=0}^{\infty} \frac{\alpha_i^n}{n!\sqrt{V^n}} (a^\dagger_i)^n |0\rangle$

- First order term exactly vanishes

$$\langle [H_{I,b}(t), \mathcal{N}_p] \rangle = 0$$

- Thermalization with other species is second order effect.
  - BEC axions do not have thermal contact with photons
    → does not affect cosmological parameters

Summary

- Estimate thermalization rate in the condensed regime

Initial states	rate
coherent ⇔ coherent	1st order in coupling Λ (enhanced)
coherent ⇔ number	2nd order in coupling Λ (negligible)
number ⇔ number	2nd order in coupling Λ (negligible)

with axions $\approx$ coherent state

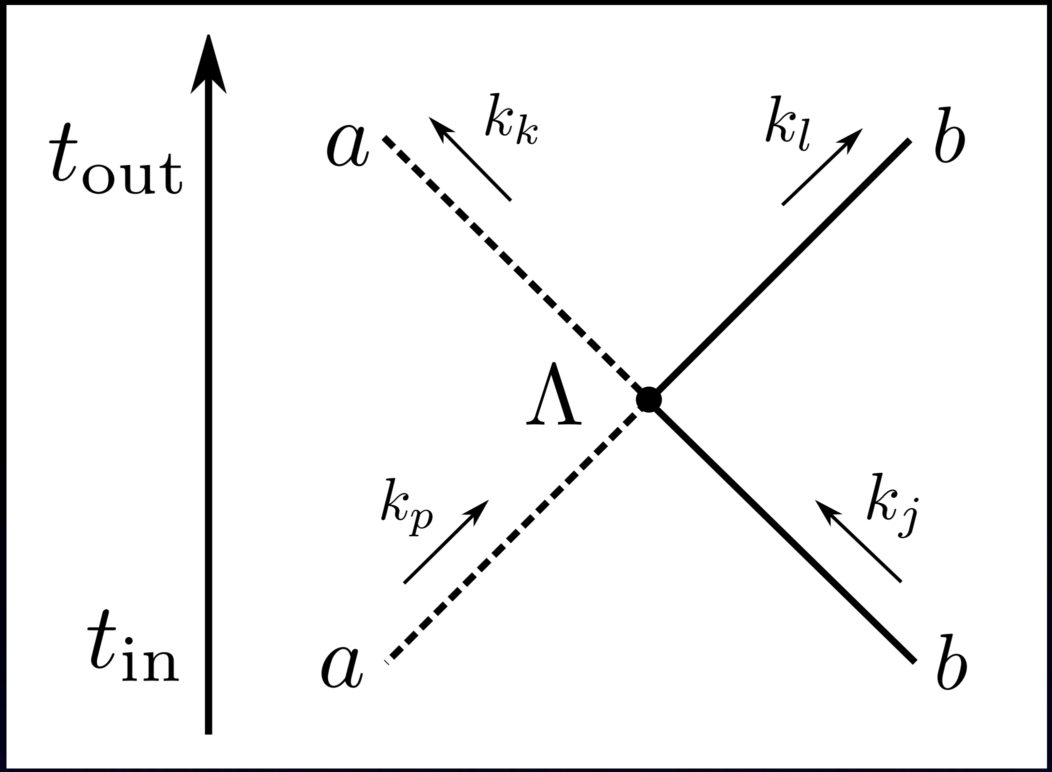
other particles (baryons, photons,...) $\approx$ number state

- Self-interaction of cold axions becomes relevant at $T \sim \mathcal{O}(1)\mathrm{keV}$

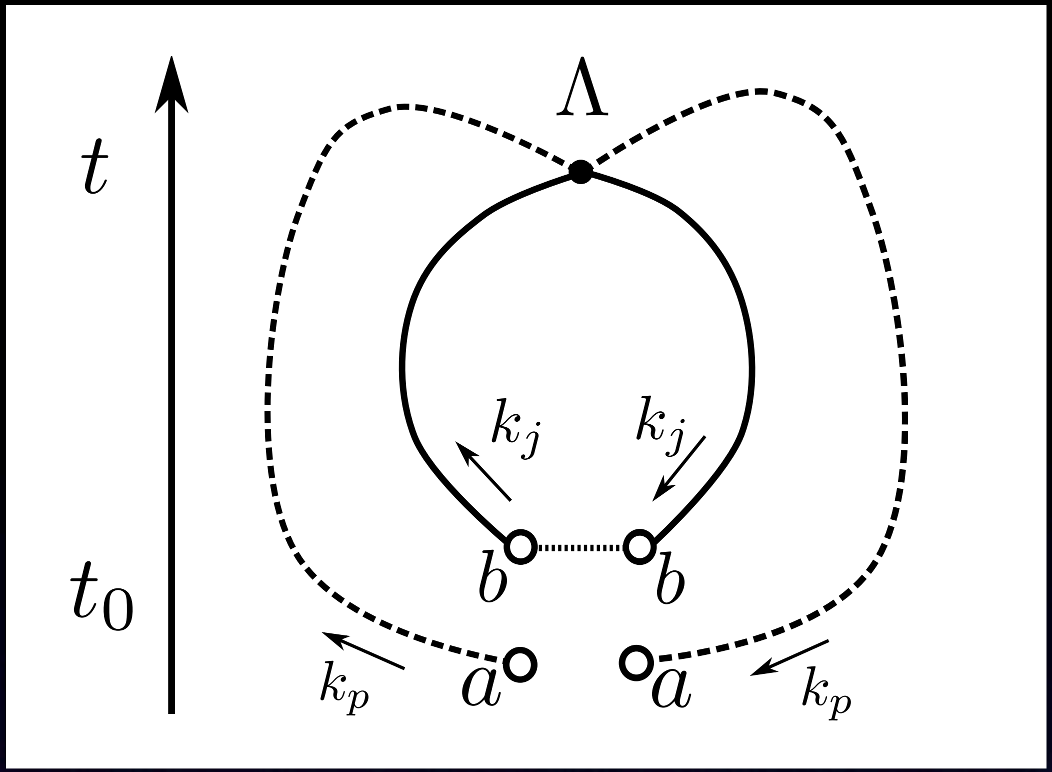
  → implies the formation of axion BEC (?)

- Thermal contact with photons is negligible

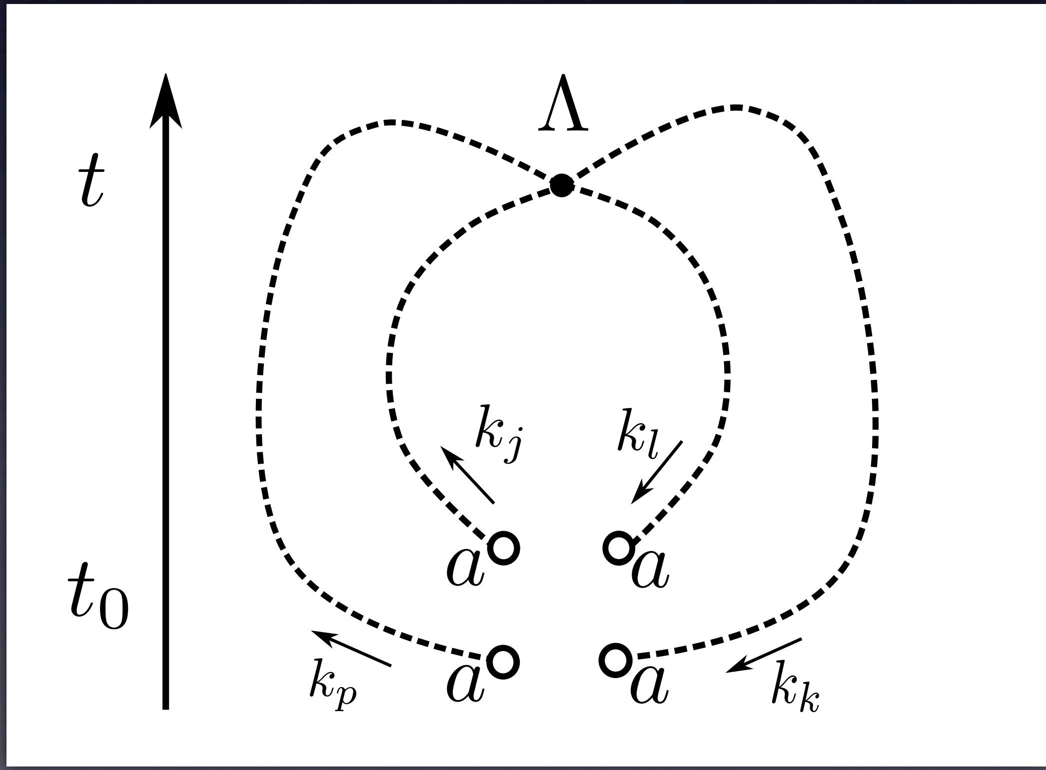
  → It does not modify standard cosmological parameters



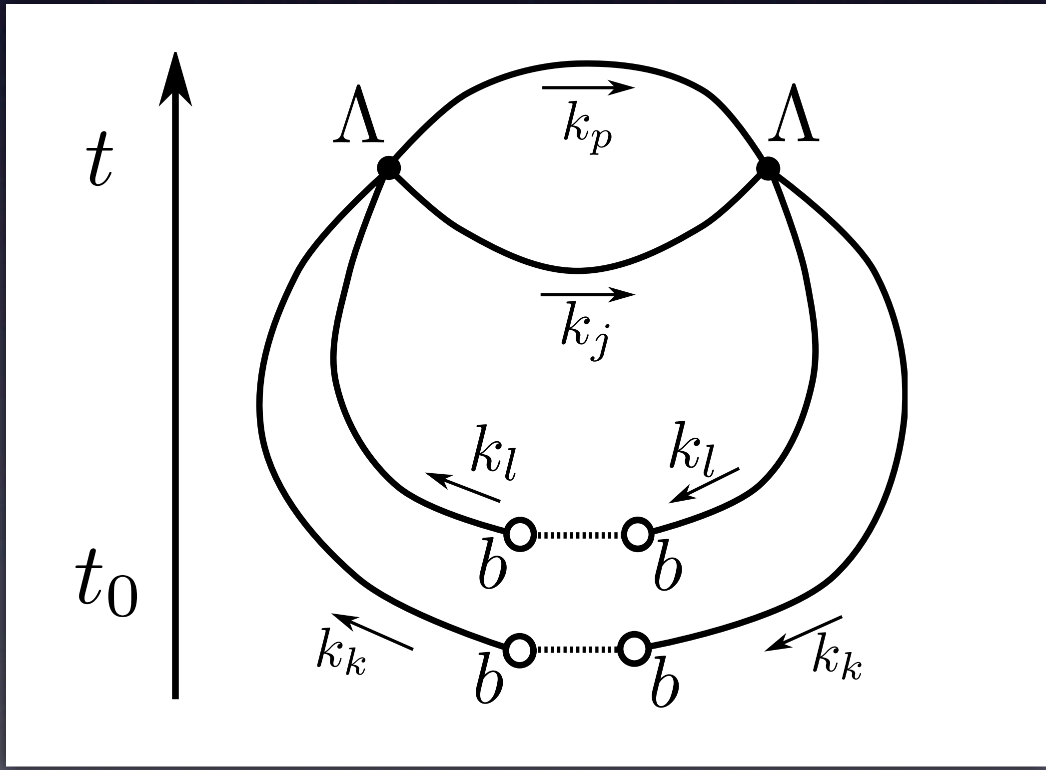
usual a+b→a+b process



1st order a+b→a+b (vanish)



1st order a+a→a+a (non-zero)



2nd order b+b→b+b (non-zero)