Forecast constraints on cosmic string parameters from observations of gravitational waves and CMB

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1. Introduction

Cosmic strings • • • "Strings" which contain large energy and randomly stretch across the universe.

As a result of repeating collisions and reconnection, they form a "string network".

Allen+, PRL 64, 119('90)
1. introduction

◆ How are they generated?

1. spontaneous symmetry breaking (SSB) (Kibble, J. Phys. A9, 1387 ('76))
   → appearance of 1-dim. topological defects = strings
   
   prediction from Grand Unified Theory (GUT) etc

2. inflation based on superstring theory
   → “strings” with cosmological length
   (cosmic superstring)

◆ Why are they important?

   • They are related to
     - particle physics beyond Standard Model
     - inflation

   • observational signals

Cosmic strings are important observational probe for particle physics and cosmology
1. introduction

- signals from cosmic string

1. gravitational wave (GW)
   
   (Damour+ PRD 64, 064008 ('01) etc)

   main GW source: loops

   They emit "GW bursts"

   - instantaneous emission
   - beam-like

   detected as

   1. rare bursts: strong enough to be detected singly, but come to the earth infrequently.

   2. GW background (GWB): consists of overlapping weak bursts.

2. anisotropy of cosmic microwave background (CMB)
   
   generated by complicated motion of strings

   fluc. of temperature & polarization

   E-mode & B-mode

(Pogosian+, PRD 60, 083504 ('99) etc)
1. introduction

3 parameters which characterize cosmic strings

1. tension (mass per unit length) $G\mu$ ($G$: Newton constant)

\[ \mu \sim E_{SSB}^2 \quad (E_{SSB} : \text{energy scale of SSB}) \]

2. loop size $\alpha$

The size of a loop formed at time $t$ is $l = \alpha t$

$\alpha$ is not a parameter, but there is an uncertainty over many orders of magnitude (from $\alpha \sim (G\mu)^n$ to $\alpha \sim 0.1$).

3. reconnection probability $p$

$p = 1$ for strings from SSB

$p \ll 1$ for cosmic superstrings

As $p$ decreases, interval and curvature radius of strings decrease.
1. introduction

**3 parameters which characterize cosmic strings**

1. **tension** (mass per unit length) \( G \mu \) (\( G \): Newton constant)
   \[
   \mu \sim \frac{E_{SSB}^2}{G^2} \quad (E_{SSB} : \text{energy scale of SSB})
   \]

2. **loop size** \( \alpha \)
   
   The size of a loop formed at time \( t \) is \( l = \alpha t \)
   
   \( \alpha \) is not a parameter, but there is an uncertainty over many orders of magnitude (from \( \alpha \sim (G \mu)^n \) to \( \alpha \sim 0.1 \)).

3. **reconnection probability** \( p \)
   
   \( p = 1 \) for strings from SSB
   
   \( p \ll 1 \) for cosmic superstrings

   As \( p \) decreases, interval and curvature radius of strings decrease.

   deeply related to background physics
1. introduction

- experiments which can search cosmic strings

GW experiments

- direct detection by interferometers
  - eg) ground: LIGO
  - space: DECIGO

- burst detection
  - GWB measurement

- pulsar timing
  - eg) SKA

  - extract the signal of GWB from fluctuations of time of arrival of pulses from pulsars

- observation of CMB
  - eg) Planck, CMBPol

- How strongly will each experiment constrain $G\mu, \alpha, p$?
- Different experiments give us different information
  - How much will combining them tighten constraints?

investigate using Fisher analysis
2. signals from cosmic strings
2. signals from cosmic strings

- rare bursts

\[
\frac{dR}{dh}(f, h) : \text{detection rate of bursts with frequency } f \text{ and amplitude } h
\] (burst rate)

- parameter dependency

\[f = 220\text{Hz} \text{ (to which LIGO is sensitive)}\]

\[p \downarrow \rightarrow \text{burst rate} \uparrow\]
2. signals from cosmic strings
   - GW background

\[ \Omega_{GW}(f) \equiv \frac{1}{\rho_{cr}} \frac{d\rho_{GW}(f)}{d \ln f} \]

(\(\rho_{cr}\): critical density)

- parameter dependency

\[ \Omega_{GW} \equiv \frac{d\rho_{GW}(f)}{d \ln f} \]

\[ \rho_{GW}(f) \equiv \frac{1}{\rho_{cr}} \frac{d\rho_{GW}(f)}{d \ln f} \]

\[ p \downarrow \rightarrow \Omega_{GW} \uparrow \]
2. signals from cosmic strings

- **CMB anisotropy**

  method to calculate angular power spectra of CMB anisotropies

  unconnected segment model

  infinite string network

  approximate

  ensemble of unconnected segment

  calculate $T_{\mu\nu}$

  $\rightarrow$ calculate CMB fluc.

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**Inflation ($r = 0.1$)**

- $G_\mu = 10^{-7}$
- $p = 1$

**String**

- $G_\mu = 10^{-7}$
- $p = 1$

---

**B-mode polarization**

- $G_\mu = 10^{-7}$
- $p = 1$
2. signals from cosmic strings

- parameter dependency of temperature power spectrum
  (not depend on $\alpha$)

\[ p \Downarrow \rightarrow \begin{cases} \text{string # density} \\ \text{fluc. on small scale} \end{cases} \]
3. future constraints on cosmic strings from gravitational wave direct detection experiments
3. future constraints on cosmic strings from GW direct detection

First, we consider constraints from future ground-based GW detectors

colored : excluded
above lines : future GW detectors (e.g. Adv. LIGO) can probe

fiducial parameter set for Fisher analysis
Fisher analysis: method to forecast the determination accuracy of parameters in future experiments

\[ P[p|D] : \text{probability that the parameter is } p \text{ when the data is } D \]

\[ P[D|p] : \text{probability that the data is } D \text{ when the parameter is } p \]  

\[ \mathcal{L}(p, D) \equiv P[D|p] : \text{likelihood} \]

If there are multiple parameters, the determination accuracy is given by the Fisher matrix

\[ F_{ij} = - \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial p_i \partial p_j} \right\rangle \]
3. future constraints on cosmic strings from GW direct detection

- **Fisher matrix for rare burst detection**

  expectation value of # of bursts with amplitude $h_i \sim h_i + dh_i$

  $$N_i = \Phi(h_i)dh_i, \Phi(h_i) = \frac{dR}{dh}(h_i)T \quad (T: \text{observation time})$$

  we observe $k_i$ bursts in i-th amplitude bin

  $$\mathcal{L} = \prod_i \frac{(N_i)^{k_i}e^{N_i}}{k_i!}$$

  (Poisson distribution)

  **Fisher matrix for GW background detection**

  $$\mathcal{L} = \prod_{I>J} \frac{1}{\sqrt{2\pi\sigma^2_{IJ}}} \exp \left[ \frac{(S_{IJ} - \langle S_{IJ} \rangle)^2}{2\sigma^2_{IJ}} \right]$$

  $S_{IJ}$ : cross correlation signal between detector I and J
  $\sigma_{IJ}$ : noise determined by the sensitivity curves of detector I and J

  $$F_{ij} = \left( \frac{3H_0^2}{10\pi^2} \right)^2 2T \sum_{I>J} \int_0^\infty df \frac{\gamma_{IJ}|^2\partial_{p_i}\Omega_{GW}(f)\partial_{p_j}\Omega_{GW}(f)}{f^6S_{n,I}(f)S_{n,J}(f)}$$

  (Miyamoto+, PRD 86, 023503 ('12))

  (Seto, PRD 73, 063001 ('06), Kudo+, PRD 73, 064006 ('06))
3. Future constraints on cosmic strings from GW direct detection

- Constraints from the GW detector network (LIGO, Virgo, KAGRA)
  - Fiducial parameter: $G \mu = 10^{-7}, \alpha = 10^{-16}, \rho = 1$

LIGO etc. detect $1.8 \times 10^5$ bursts and GWB with SNR=187

The burst rate and the GWB spectrum give us different information

Combining them leads to breaking of parameter degeneracy and better constraints
4. future constraints on cosmic strings from gravitational wave direct detection, pulsar timing and CMB experiments
4. constraints on cosmic strings from GW direct detection, pulsar timing and CMB

We consider pulsar timing exp. and CMB exp. in addition to GW direct detection exp.

colored : excluded
above lines : future exp. can probe

![Graph showing constraints on $G\mu$](image)

current CMB constraint: $G\mu < 1.4 \times 10^{-7}$

Planck B-mode measurement can probe $G\mu > 2.4 \times 10^{-8}$

We perform Fisher analysis choosing some fiducial parameter sets

(Miyamoto+, PRD 87, 023522 ('13))
Fisher matrix for pulsar timing experiment

fluctuation of arrival time of pulses from i-th pulsar: \( R_i(t) \)

cross correlation between 2 pulsars: \( r_i \equiv \frac{1}{N} \sum_{a=0}^{N-1} R_{i_1}(t_a) R_{i_2}(t_a) \)

extract contribution from GW

signal: \( S = \frac{1}{N_p} \sum_{i=0}^{N_p-1} \frac{(r_i - \bar{r})(\zeta(\theta_i) - \bar{\zeta})}{\sigma_r \sigma_\zeta} \)

\[
F_{ij} = \frac{1}{N^2} \frac{\partial \langle S \rangle}{\partial p_i} \frac{\partial \langle S \rangle}{\partial p_j}, \quad N^2 = \langle S^2 \rangle - \langle S \rangle^2
\]

Fisher matrix for CMB experiment (Zaldarriaga+, ApJ 448, 1 ('97))

\[
F_{ij} = \sum_l \sum_{X,X'} \frac{\partial C^X_l}{\partial p_i} \text{Cov}^{-1}(C^X_l, C^X'_l) \frac{\partial C^{X'}_l}{\partial p_j}
\]

(X = TT, EE, BB, TE)

\[
\text{Cov}(C^X_l, C^{X'}_l) = \langle (C^X_l - \langle C^X_l \rangle) (C^{X'}_l - \langle C^{X'}_l \rangle) \rangle
\]

determined by noise power spectra
4. constraints on cosmic strings from GW direct detection, pulsar timing and CMB

**case 1:** \( G\mu = 10^{-9}, \alpha = 10^{-9}, p = 1 \)

😊 CMB experiments cannot detect strings even in the future
😊 Pulsar timing exp. and ground-based interferometers (LIGO, KAGRA, Virgo) can detect GWs from strings

LIGO etc. detect 168 bursts and SKA detects GWB with SNR=33

🌟 constraints from 3yr run of ground-based interferometers and 10yr run of SKA
4. constraints on cosmic strings from GW direct detection, pulsar timing and CMB

*Case 2:* \( G\mu = 10^{-7}, \alpha = 10^{-16}, p = 1 \)

- Pulsar timing exp. cannot detect GWs from string \((f\text{ is too high})\)
- Ground-based GW detectors (LIGO, KAGRA, Virgo) and CMB exp. can detect string signals.

LIGO etc. detect \(1.8 \times 10^5\) bursts and GWB with SNR=187
CMB satellites detect string signals (small scale \(T\) fluc. and B-mode)

- constraints from ground-based GW detectors (3yr run) and CMB satellites
4. constraints on cosmic strings from GW direct detection, pulsar timing and CMB

\[
\text{case 3: } G_\mu = 10^{-14}, \alpha = 10^{-13}, p = 1
\]

\(G_\mu\) is so small that only space-borne interferometers, such as BBO and DECIGO, can detect string signatures.

BBO/DECIGO etc. detect 35 bursts and GWB with SNR=510

◆ constraints from 3yr run of BBO/DECIGO
5. summary
Cosmic strings are important probes of cosmology and particle physics (if they exist).

We investigated how strongly future GW direct detection experiments, pulsar timing experiments and CMB experiments will constrain the cosmic string parameters, tension $G_\mu$, loop size $\alpha$ and reconnection probability $p$.

We found that different types of experiments complement each other for constraining string parameters. We saw that GW direct detection experiments are powerful, because they intrinsically have two ways of observation, burst detection and background measurement. (Especially, space-borne interferometers are extremely powerful.)