Efficient methods to probe

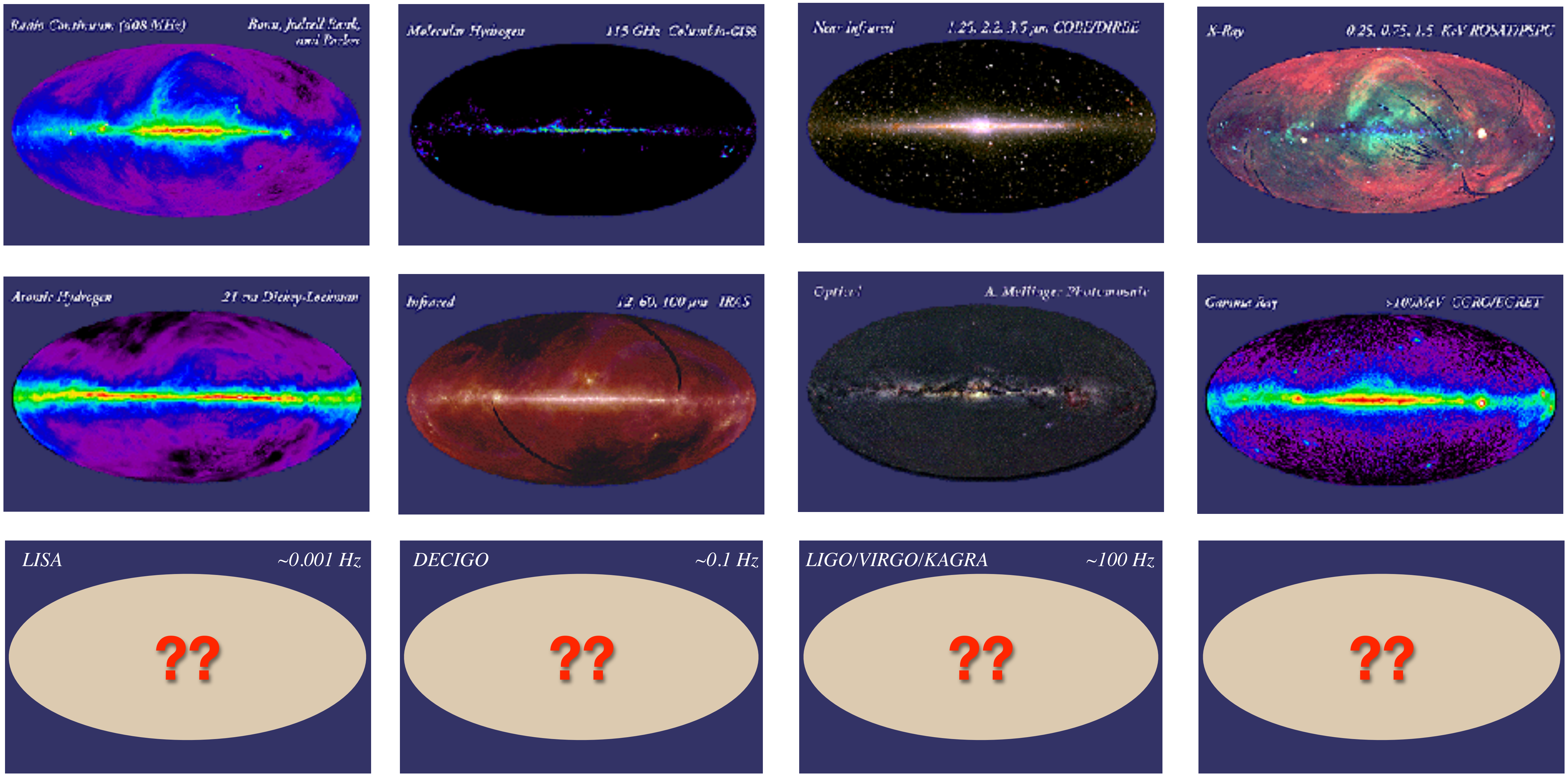
Stochastic Gravitational Wave Background Anisotropy

With Ground-based Detectors

ICRR seminar, March 11,2019
Jishnu Suresh

To Map the GW Sky

http://mwmw.gsfc.nasa.gov/mmw_allsky.html

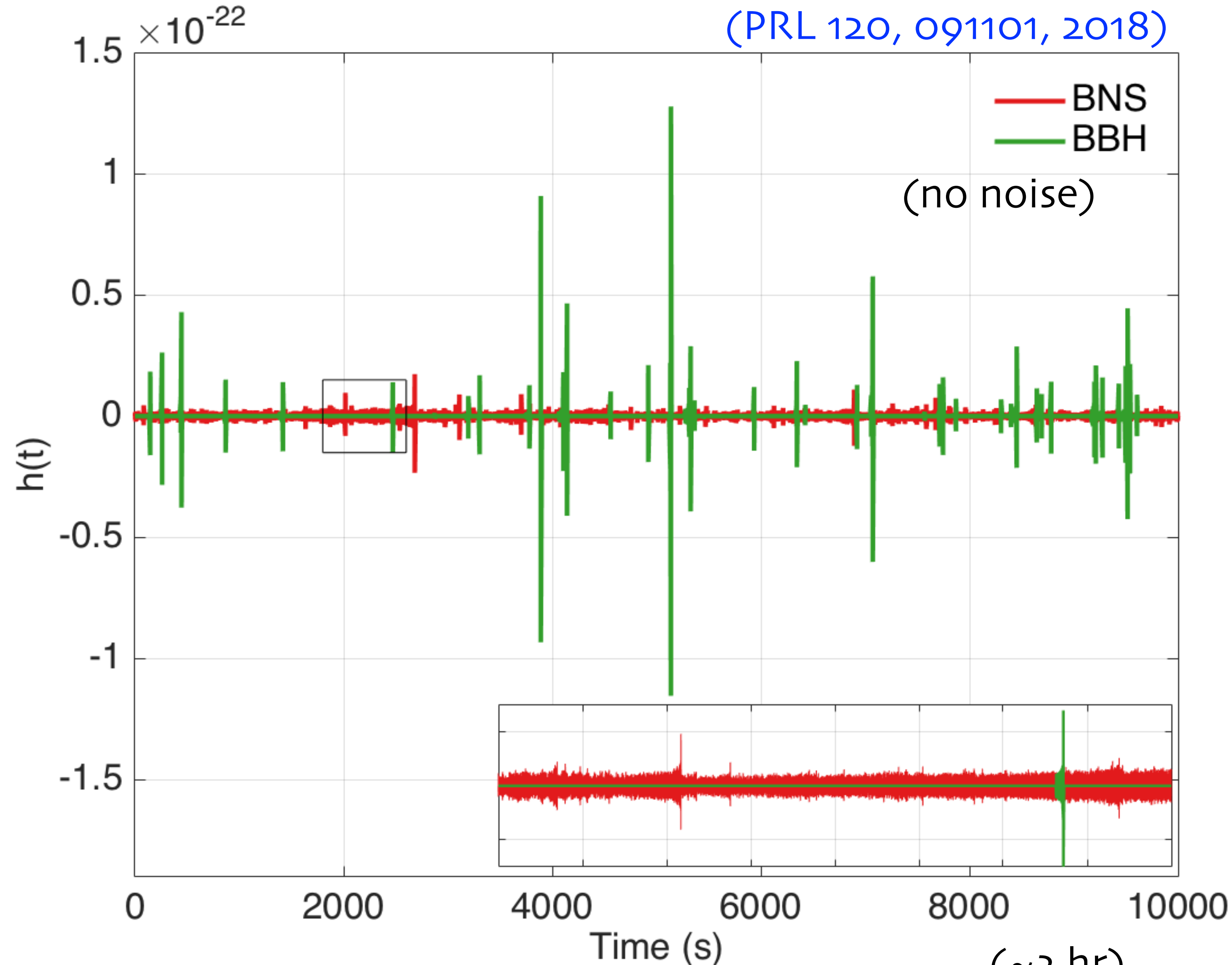


slide: Sanjit Mitra

Stochastic Gravitational Wave Background (SGWB)

- individually undetectable (subthreshold)
- but detectable as a collectivity via their common influence on multiple detectors
- combined signal described statistically—stochastic gravitational-wave background

(PRL 120, 091101, 2018)



(~3 hr)

Stochastic Gravitational Wave Background (SGWB)

- Unresolved astrophysical or cosmological sources
  - popcorn or continuous
- Carry information not accessible in electro-magnetic astronomy
  - astrophysical sources
    - information on the anisotropic local universe
  - primordial cosmological background (CGWB)
    - direct probe of inflation
- Persistent unknown isotropic or anisotropic sources

Characterizing the stochastic background

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_c}\frac{d\rho_{\mathrm{GW}}}{d\log f}$$

$$\rho_c = \frac{3c^2H_0^2}{8\pi G_N} = 7.8\times 10^{-9}\mathrm{erg/cm^3}$$

Critical energy density to close the universe

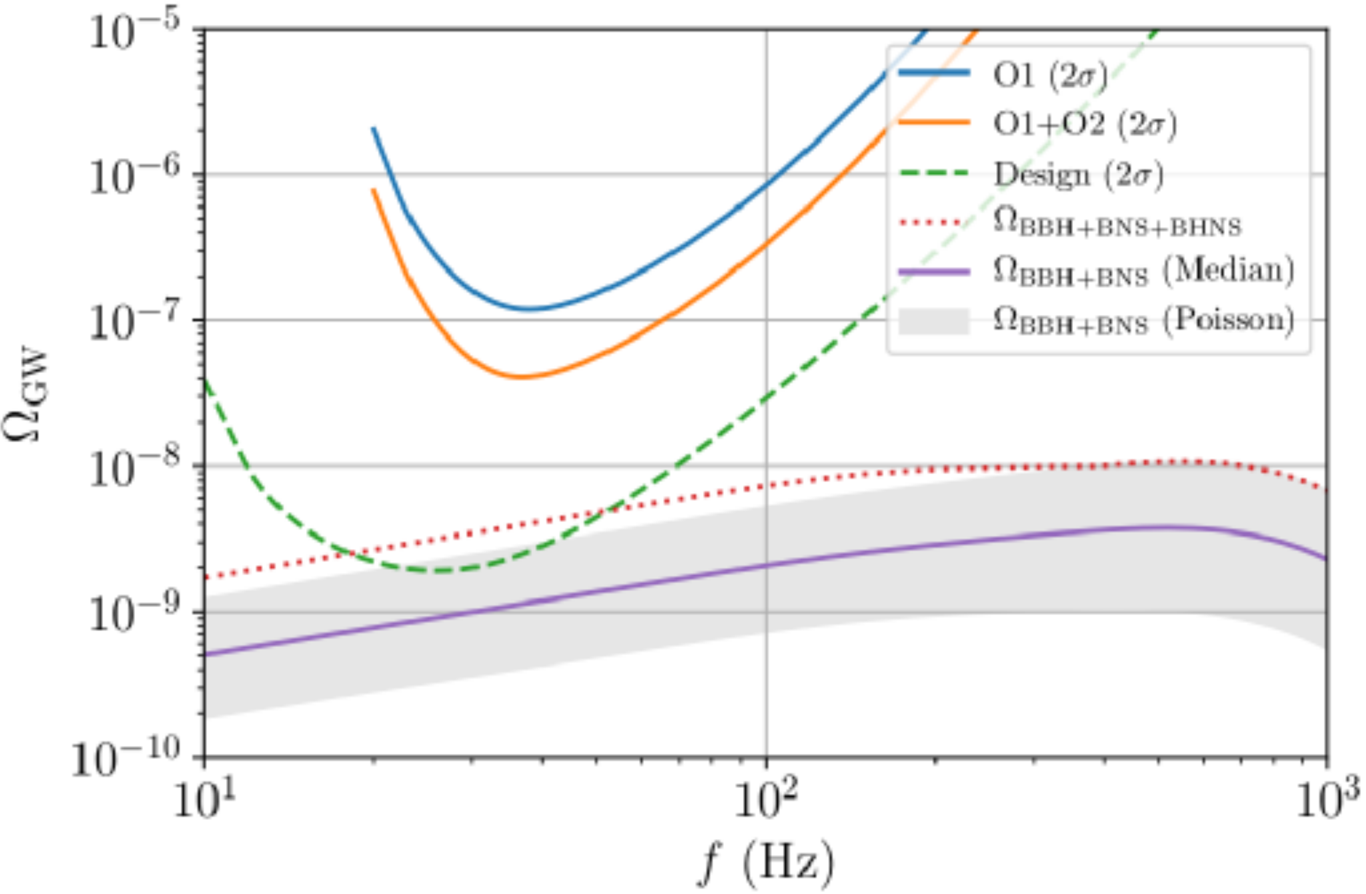
Many models give power law spectra in our band

$$\Omega_{\mathrm{GW}}(f) = \Omega_\alpha\left(\frac{f}{f_{\mathrm{ref}}}\right)^\alpha$$

$\alpha = 0$	Flat (inflation, cosmic strings in our band…)
$\alpha = 2/3$	Binary inspiral (BBH, BNS)

Potentially detectable with advanced LIGO/Virgo

(ArXiv:1903.02886)



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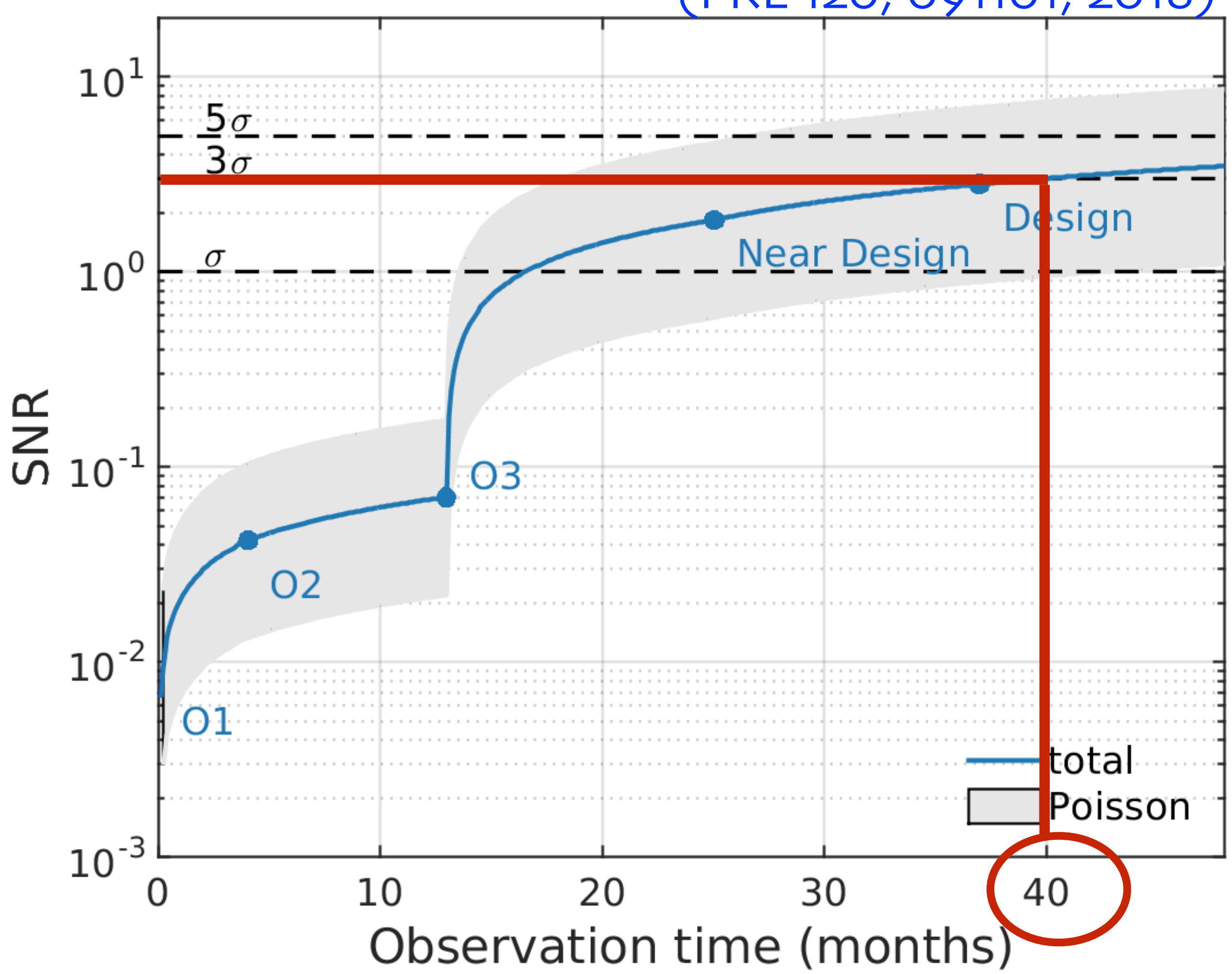
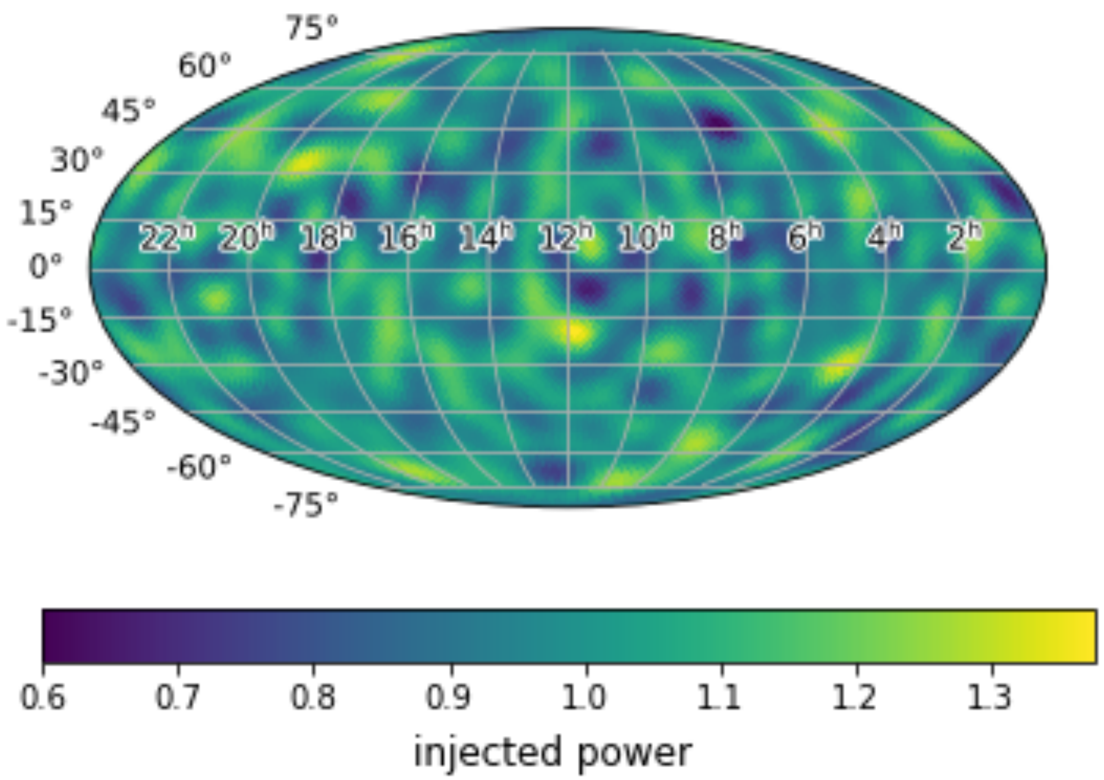


Types of Stochastic Gravitational Wave Background

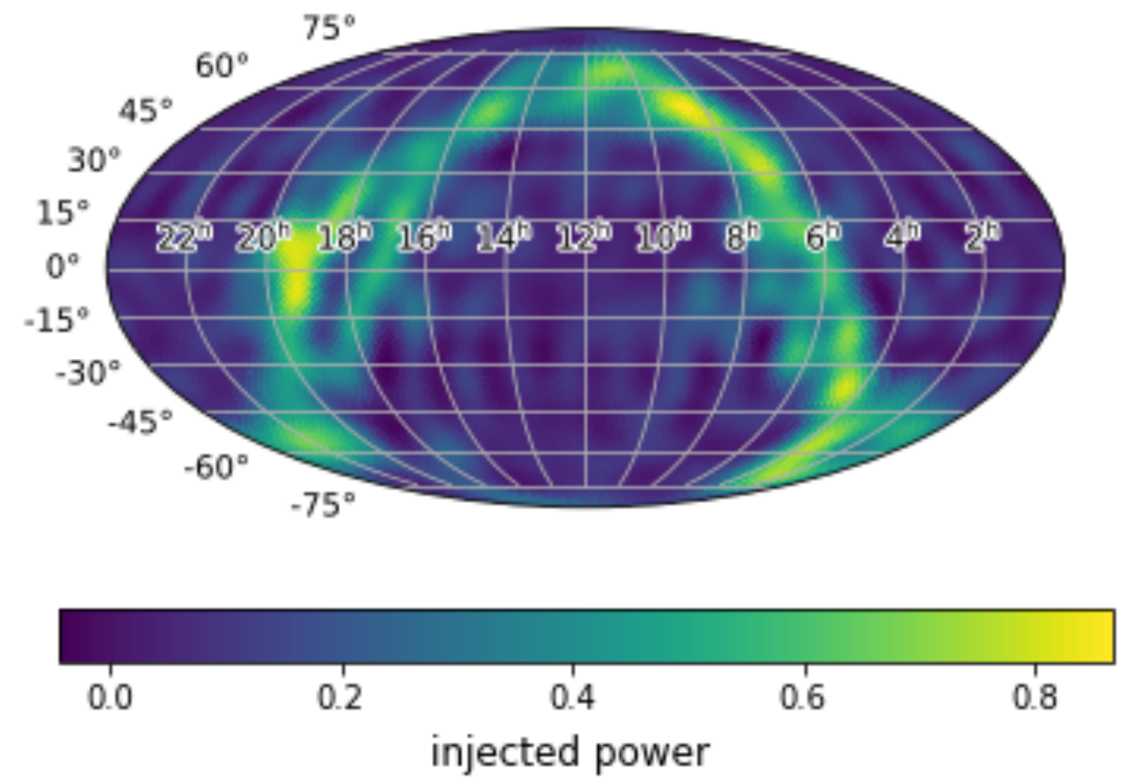
(i) Stochastic backgrounds can differ in spatial distribution

<u>(statistically) isotropic</u>



(like cosmic microwave background)

<u>anisotropic</u>

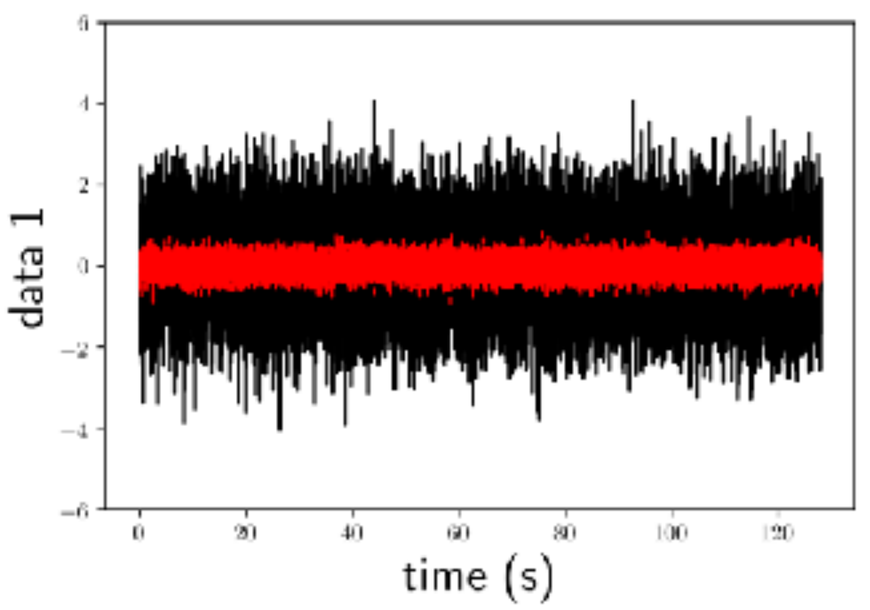


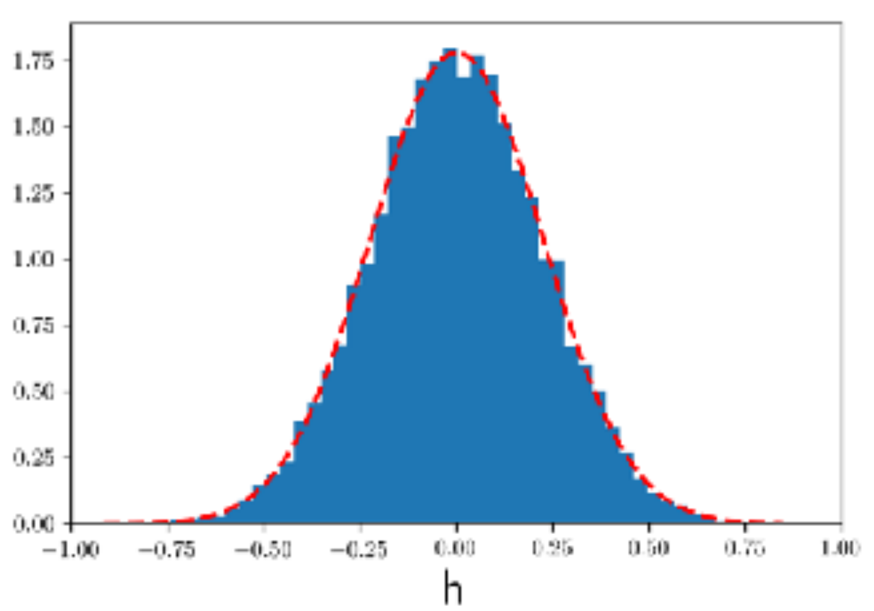
(galactic plane in equatorial coords)

Joe's lectures;Les Houches Summer School 2018

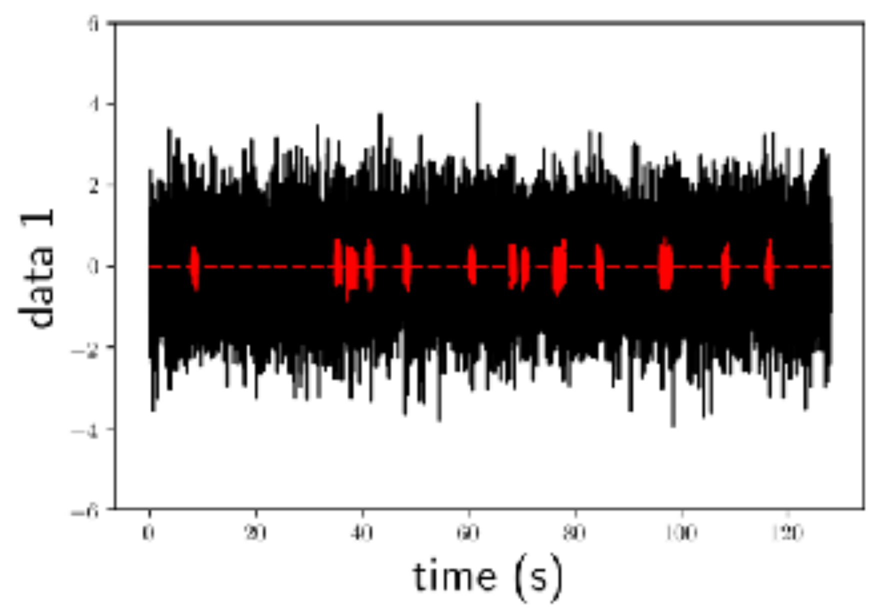
(ii) differ in temporal distribution and amplitude

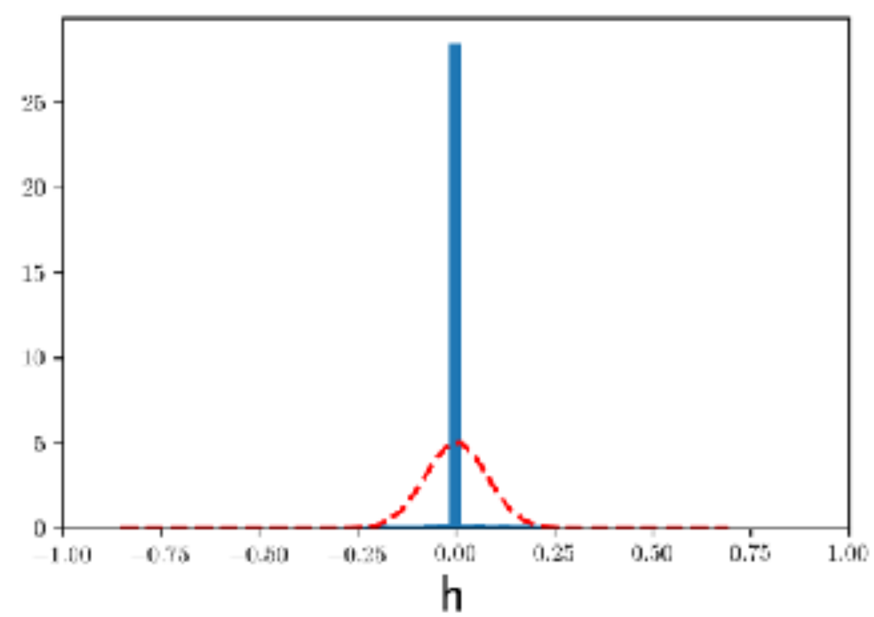
Stationary Gaussian



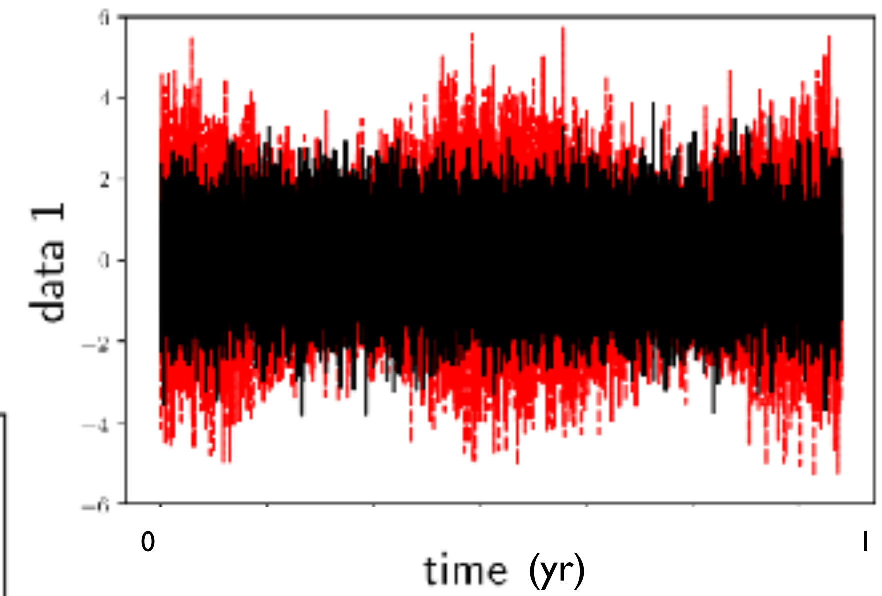


Non-stationary (non-gaussian)





Foreground



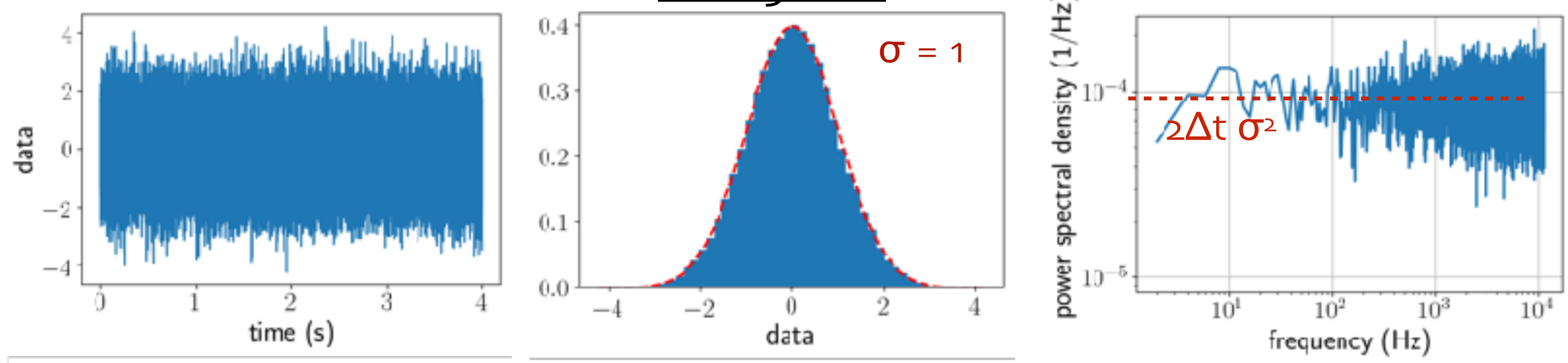
(e.g., from galactic white dwarf binaries; modulated by LISA's orbital motion)

Joe's lectures; Les Houches Summer School 2018

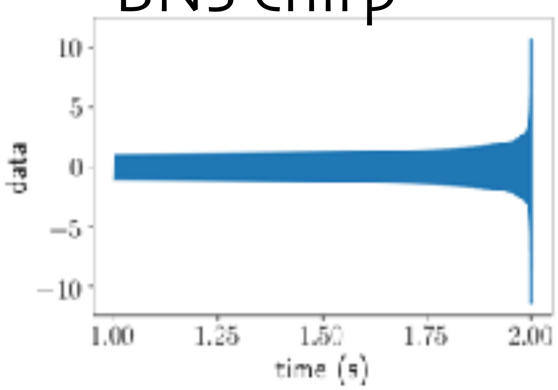
(iii) differ in power spectra depending on source

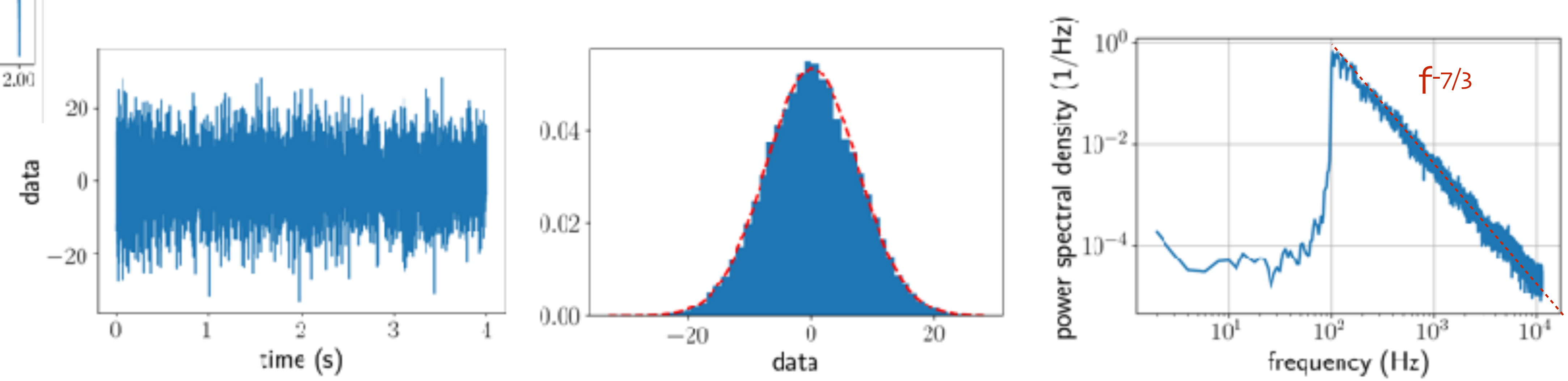
white noise

histograms



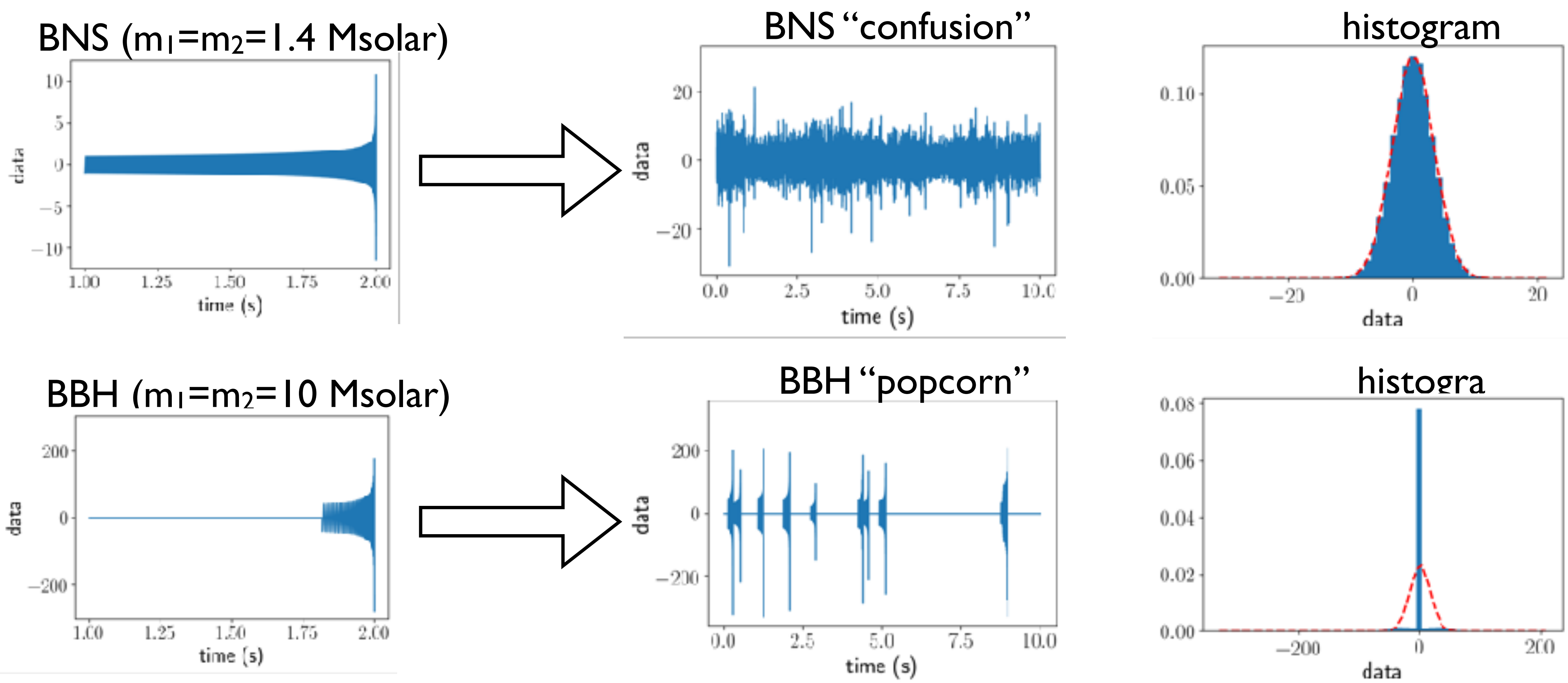
BNS chirp





Joe's lectures;Les Houches Summer School 2018

Example: Rate estimates and signal durations imply BNS "confusion" & BBH "popcorn" for LIGO / Virgo



Joe's lectures;Les Houches Summer School 2018

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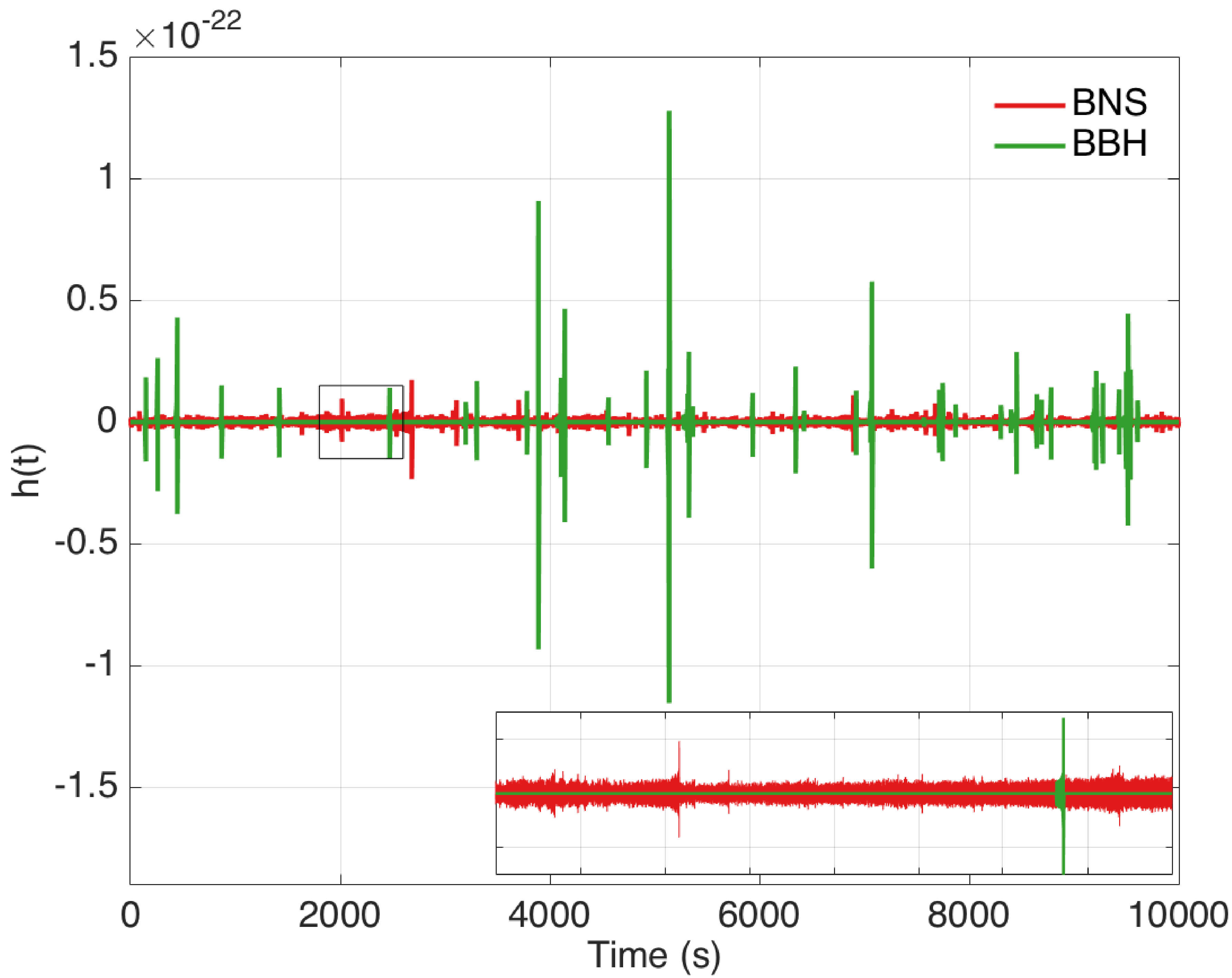


res;Les Houches Summer School 2018

Cross-Correlation Search

Allen & Romano (2001)

- Detector output = true signal + noise
- Normally detector noise are uncorrelated for far away detectors and times:
- Cross-correlation (CC) statistic is the best choice for unmodeled sources
  - one detector's signal is the filter for other detector's data

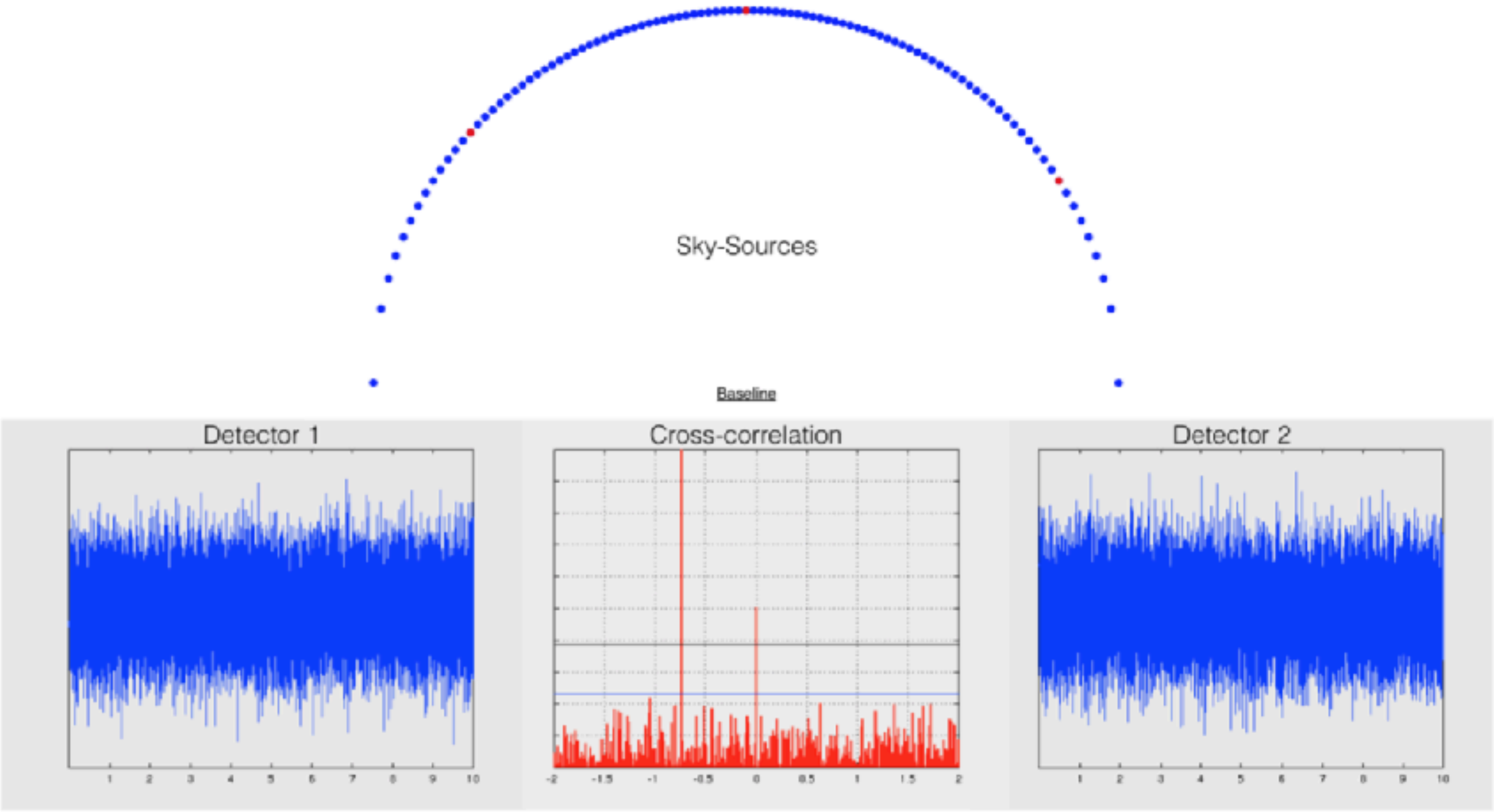
$$h_1 = n_1 + s$$
$$h_2 = n_2 + s$$

signal and noise are uncorrelated

$$\langle h_1 h_2 \rangle = \langle s^2 \rangle + \langle s n_1 \rangle + \langle s n_2 \rangle + \langle n_1 n_2 \rangle$$

signal

noise at H1 and L1 is uncorrelated



There are three sources of different strength (strong, medium and weak) marked in red.

cross-correlation is essentially a one dimensional map of the sky

Stochastic search

For a power law background…

$$\Omega_{\rm GW}(f) = \Omega_\alpha \left(\frac{f}{f_{\rm ref}}\right)^\alpha$$

We can write an optimal estimator for the energy density

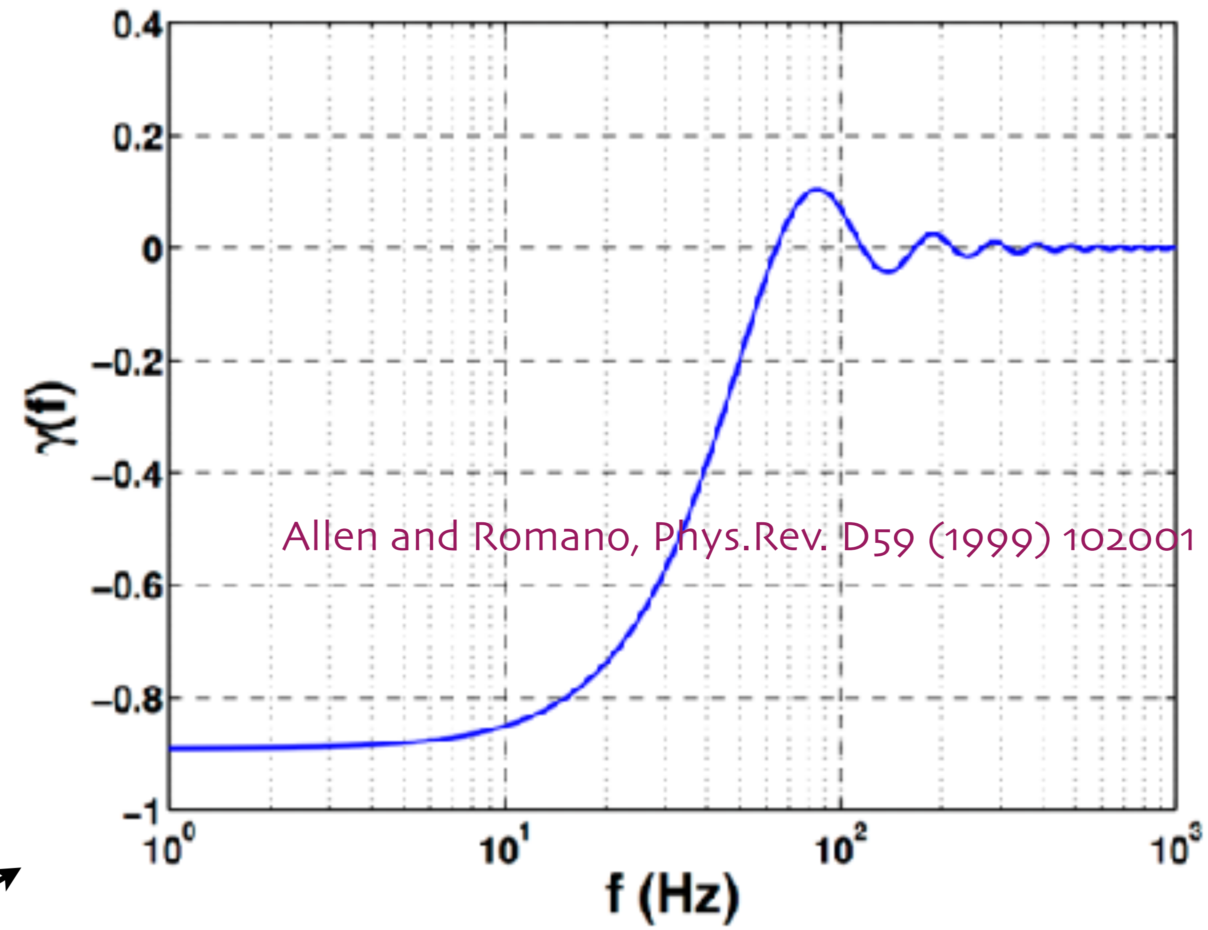
$$\hat{\Omega}_\alpha = \int_{-\infty}^{\infty} \tilde{h}_1^\star(f)\tilde{h}_2(f)\tilde{Q}(f)df$$

Choose Q to maximize SNR for fixed spectral shape:

Overlap reduction function for H1/L1

$$\langle \tilde{h}_1(f)\tilde{h}_2^*(f')\rangle = \frac{1}{2}\delta(f-f')\Gamma_{12}(f)S_h(f)$$

Accounts for separation and orientation of detectors
Note we are most sensitive below 100 Hz



$$\tilde{Q}(f) \propto \frac{\Gamma_{12}(f)H(f)}{P_1(f)P_2(f)}$$

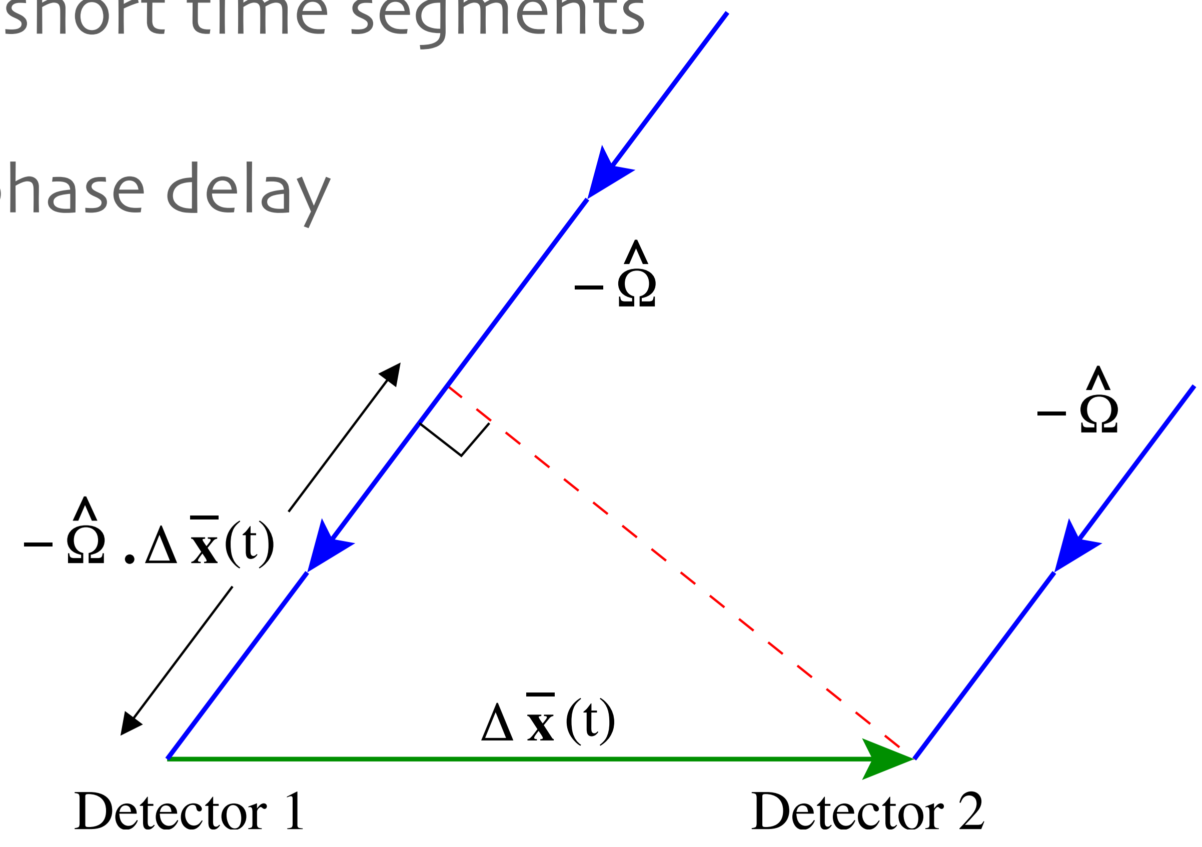
expected signal spectrum

de-weight correlation when noise is large or overlap is small

Radiometer Algorithm

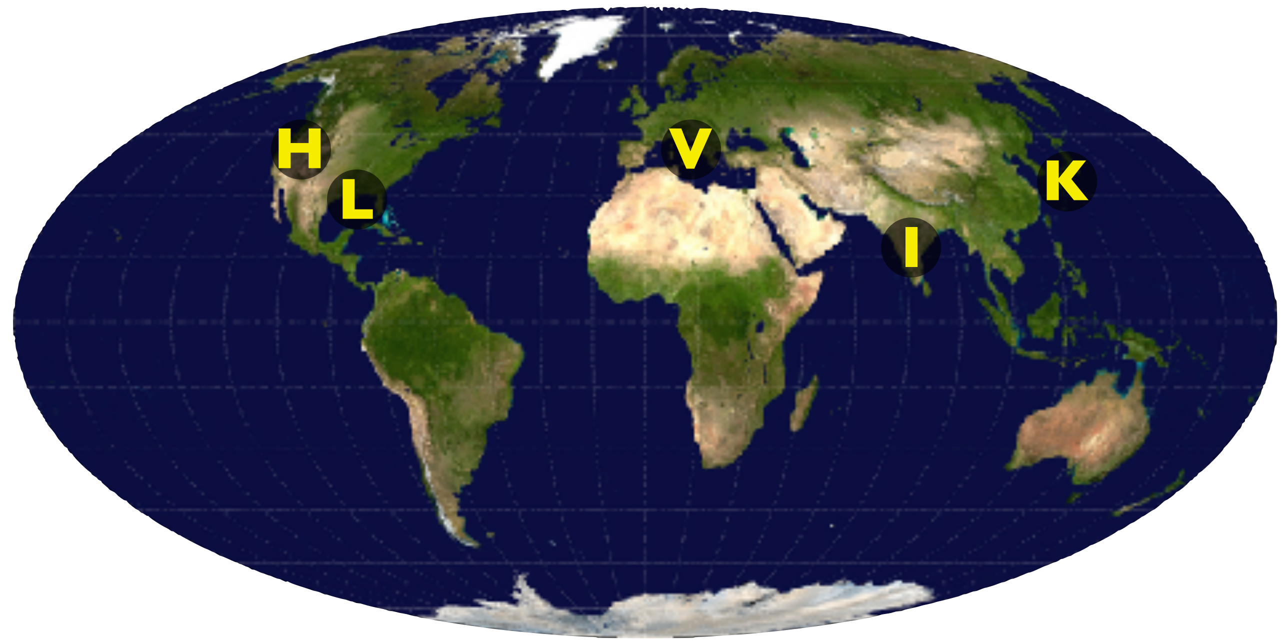
$$\int \mathrm{d}t \int \mathrm{d}t' \, s_1(t) \, s_2(t') \, q(t,t') \equiv \int \mathrm{d}f \, \tilde{s}_1^*(t,f) \, \tilde{s}_2(t,f) \, \tilde{q}(t,f)$$

- Essentially Earth Rotation Synthesis Imaging
  - Cross-correlate detector outputs in short time segments
  - map making: use time dependent phase delay
- Use spectral filters
  - to enhance signal power
  - to reduce noise power

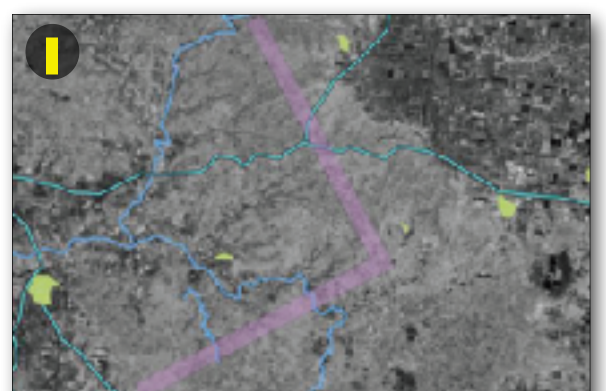


Lazzarini & Weiss (2004)
Ballmer (2006)
Mitra et al (2007)

Terrestrial Network

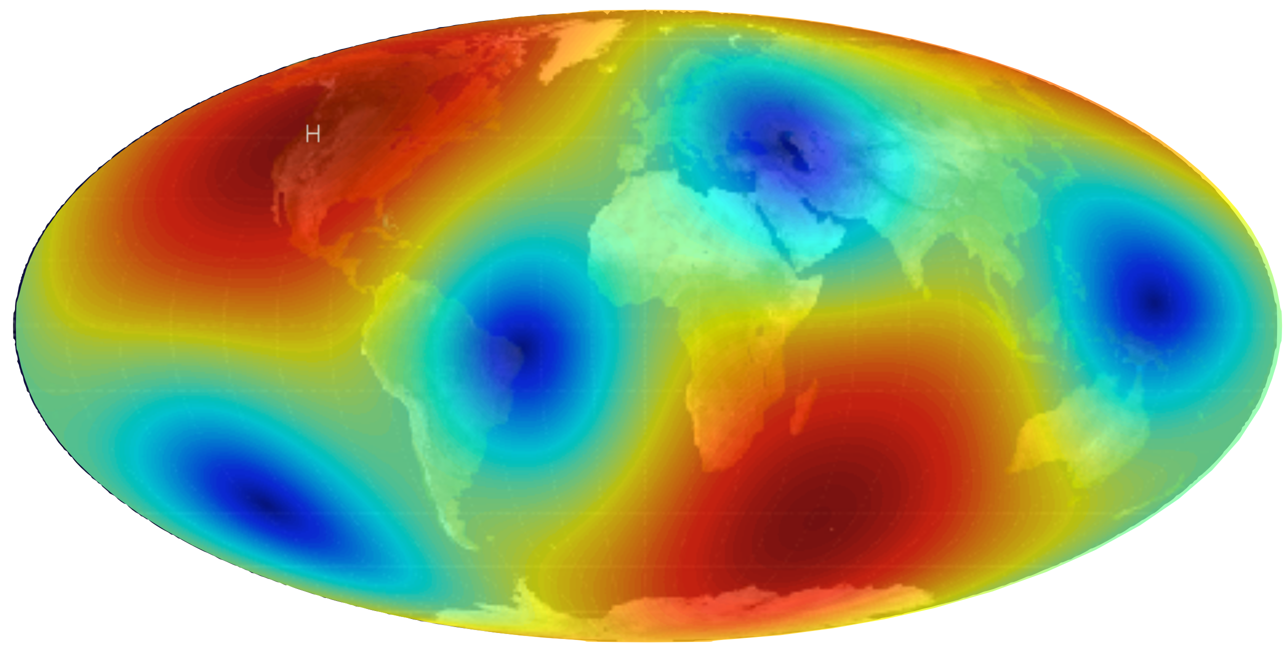


(credit: N. Cornish)



Sky coverage of individual detectors →

Terrestrial Network



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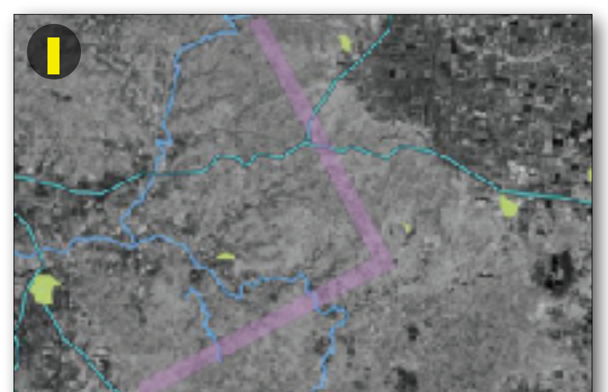
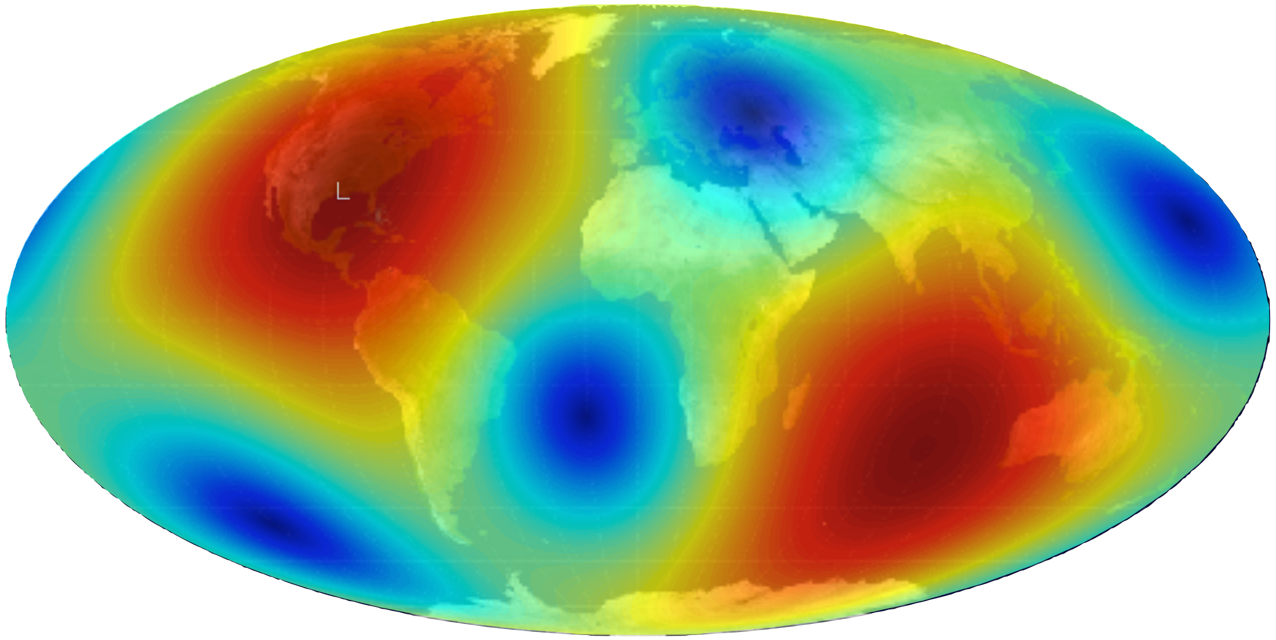


Terrestrial Network



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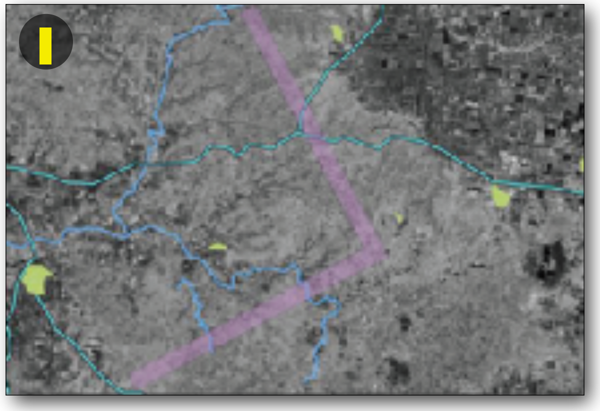

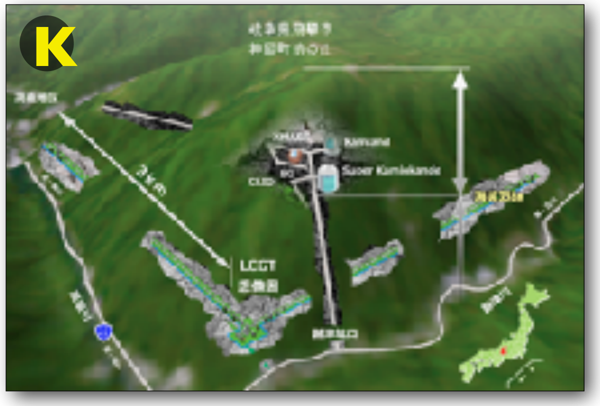

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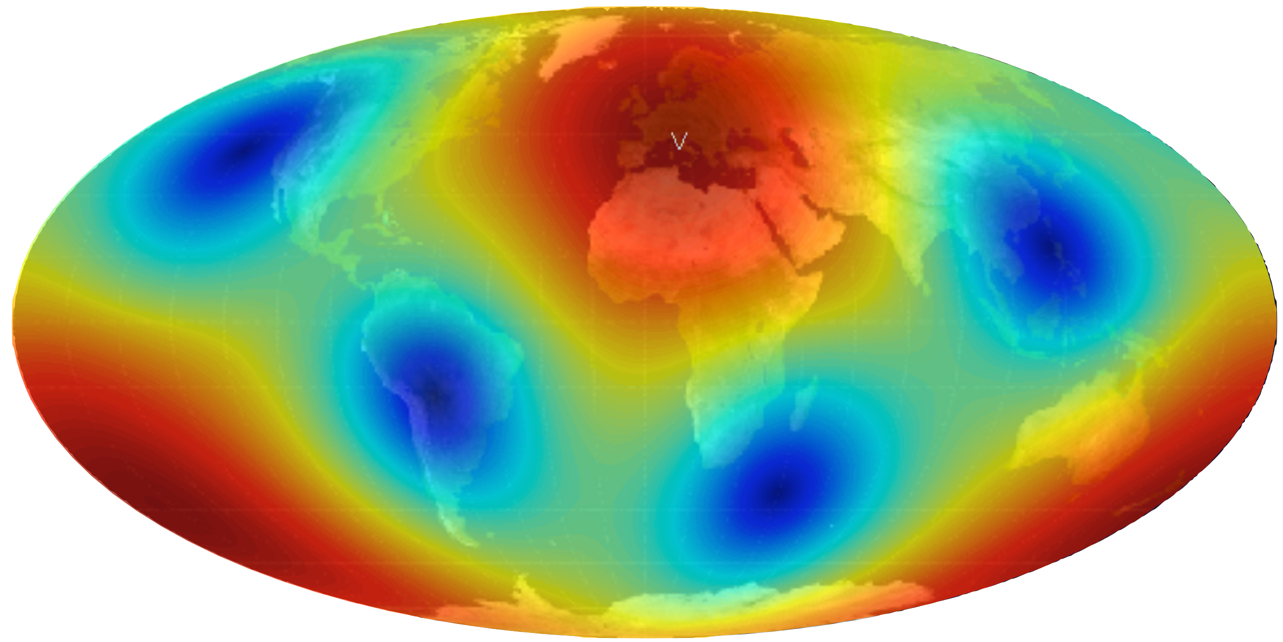

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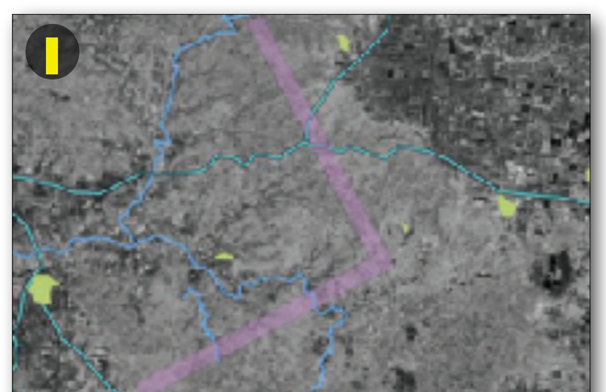

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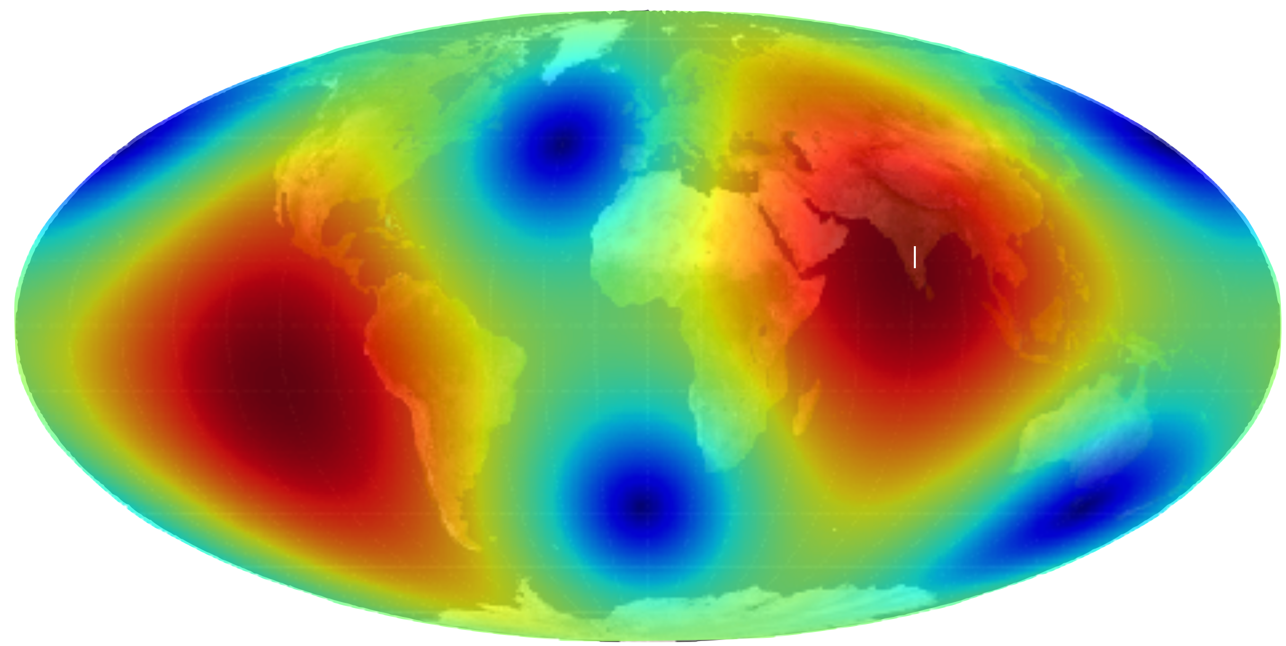

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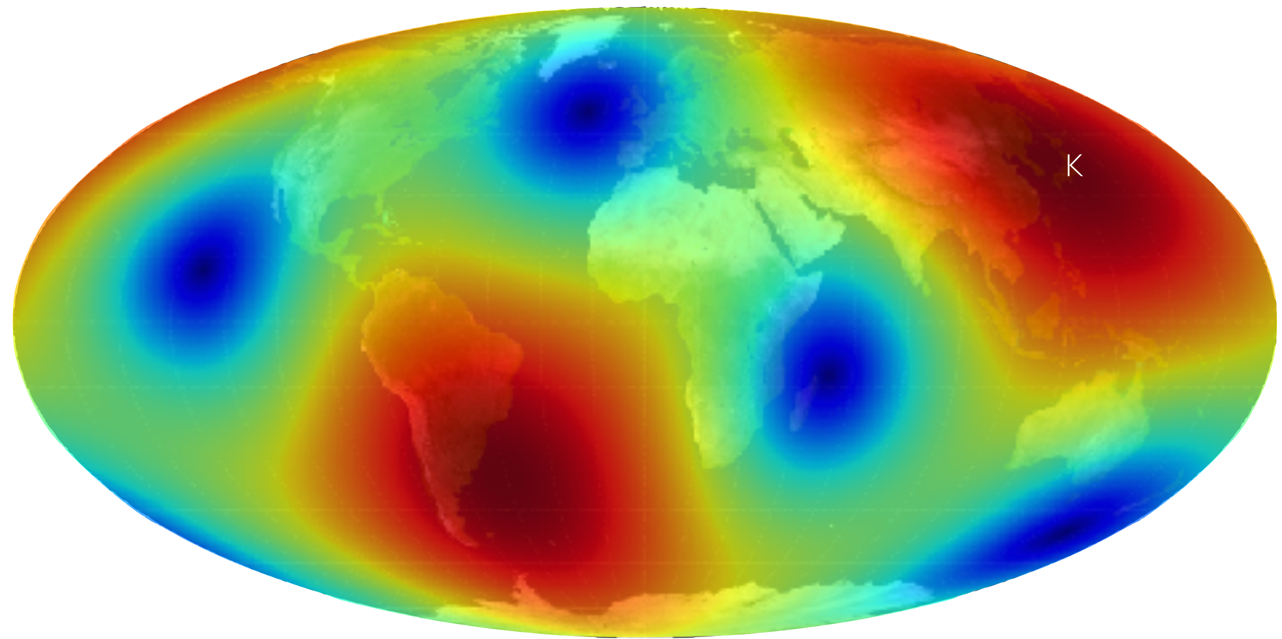

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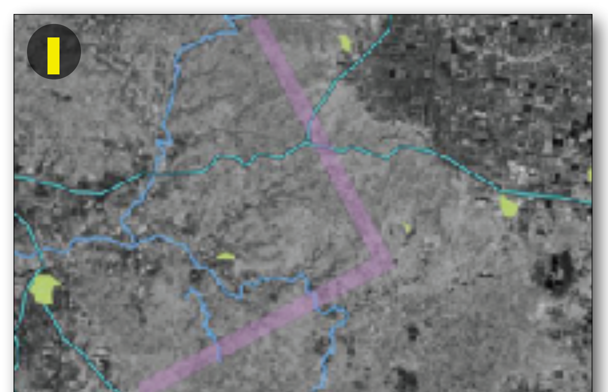


Radiometric Search for SGWB

- Observed data := Cross Spectral Density := product of SFT's

$$\mathbf{C}^I \equiv C^I_{ft} := \widetilde{s}^*_{I_1}(t;f)\,\widetilde{s}_{I_2}(t;f)$$

- Noise (in the small signal limit):

$$\mathbf{n}^I \equiv n^I_{ft} := \widetilde{n}^*_{I_1}(t;f)\,\widetilde{n}_{I_2}(t;f)$$

- Covariance matrix:

$$\mathbf{N} \equiv \mathrm{Cov}(C^I_{ft}, C^{I'}_{f't'}) \approx \frac{(\Delta T)^2}{4}\,\delta_{II'}\,\delta_{tt'}\,\delta_{ff'}\,P_{I_1}(t;f)\,P_{I_2}(t;f)$$

CSD Observed from an Anisotropic Background

- Anisotropic SGWB in some basis: $\mathcal{P}(\hat{\mathbf{\Omega}}) \;:=\; \sum_{\alpha} \mathcal{P}_\alpha \, e_\alpha(\hat{\mathbf{\Omega}}); \;\; \boldsymbol{\mathcal{P}} \;\equiv \mathcal{P}_\alpha$

- Observed CSD = convolution of anisotropic background with additive noise

$$C^I_{ft} \;:=\; \sum_\alpha K^I_{ft,\alpha} \mathcal{P}_\alpha \;+\; n^I_{ft}$$

Low signal limit

- the "kernel" or "beam":

$$\mathbf{K}^I \;\equiv\; K^I_{ft,\alpha} \;:=\; \Delta T \, H(f) \, \gamma^I_\alpha(f,t)$$

  * generalized overlap reduction function

$$\gamma^I_\alpha(f,t) \;:=\; \sum_{A=+,\times} \int_{S^2} \mathrm{d}\hat{\mathbf{\Omega}} \, F^A_{I_1}(\hat{\mathbf{\Omega}},t) \, F^A_{I_2}(\hat{\mathbf{\Omega}},t) \, e^{2\pi \mathrm{i} f \hat{\mathbf{\Omega}} \cdot \mathbf{\Delta x}(t)/c} \, e_\alpha(\hat{\mathbf{\Omega}})$$

ML Estimation of SGWB Anisotropy

- ML estimate of SGWB anisotropy in any basis with a network of detectors:

$$\mathcal{P}(\hat{\mathbf{\Omega}}) := \sum_\alpha \mathcal{P}_\alpha \, e_\alpha(\hat{\mathbf{\Omega}}); \quad \boldsymbol{\mathcal{P}} \equiv \mathcal{P}_\alpha$$

$$\hat{\mathcal{P}}_\alpha \equiv \hat{\boldsymbol{\mathcal{P}}} = \boldsymbol{\Sigma} \cdot \mathbf{X}$$

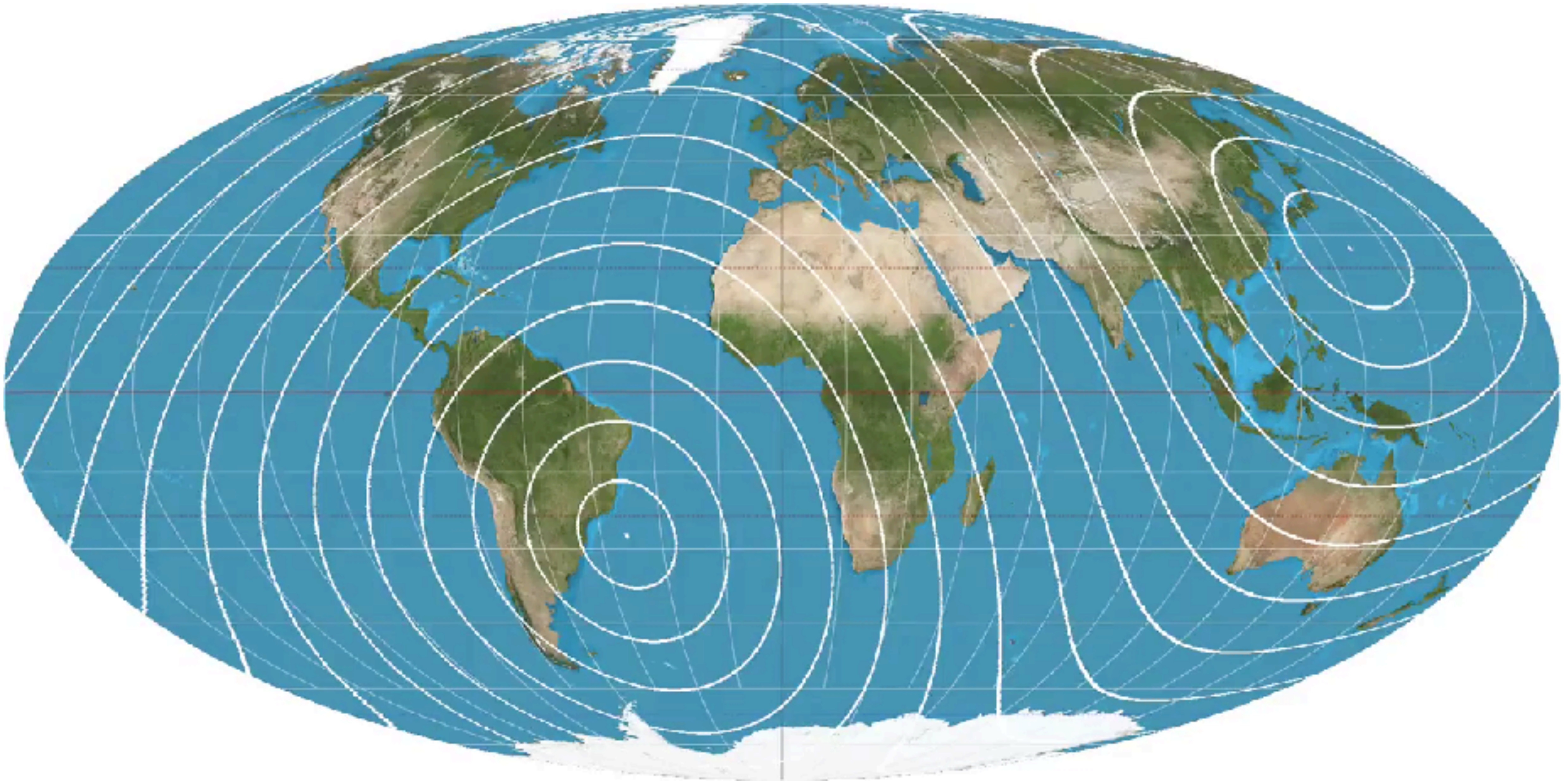
- "Dirty" map (essentially filtered output):

$$\mathbf{X} := \mathbf{K}^\dagger \cdot \mathbf{N}^{-1} \cdot \mathbf{C} \;\Rightarrow\; X_\alpha = \frac{4}{\Delta T} \sum_{I,ft} \frac{H(f)\, \gamma^{I*}_{ft,\alpha}}{P_{I_1}(t;f)\, P_{I_2}(t;f)} \, \widetilde{s}^*_{I_1}(t;f)\, \widetilde{s}_{I_2}(t;f)$$

- Fisher information matrix:

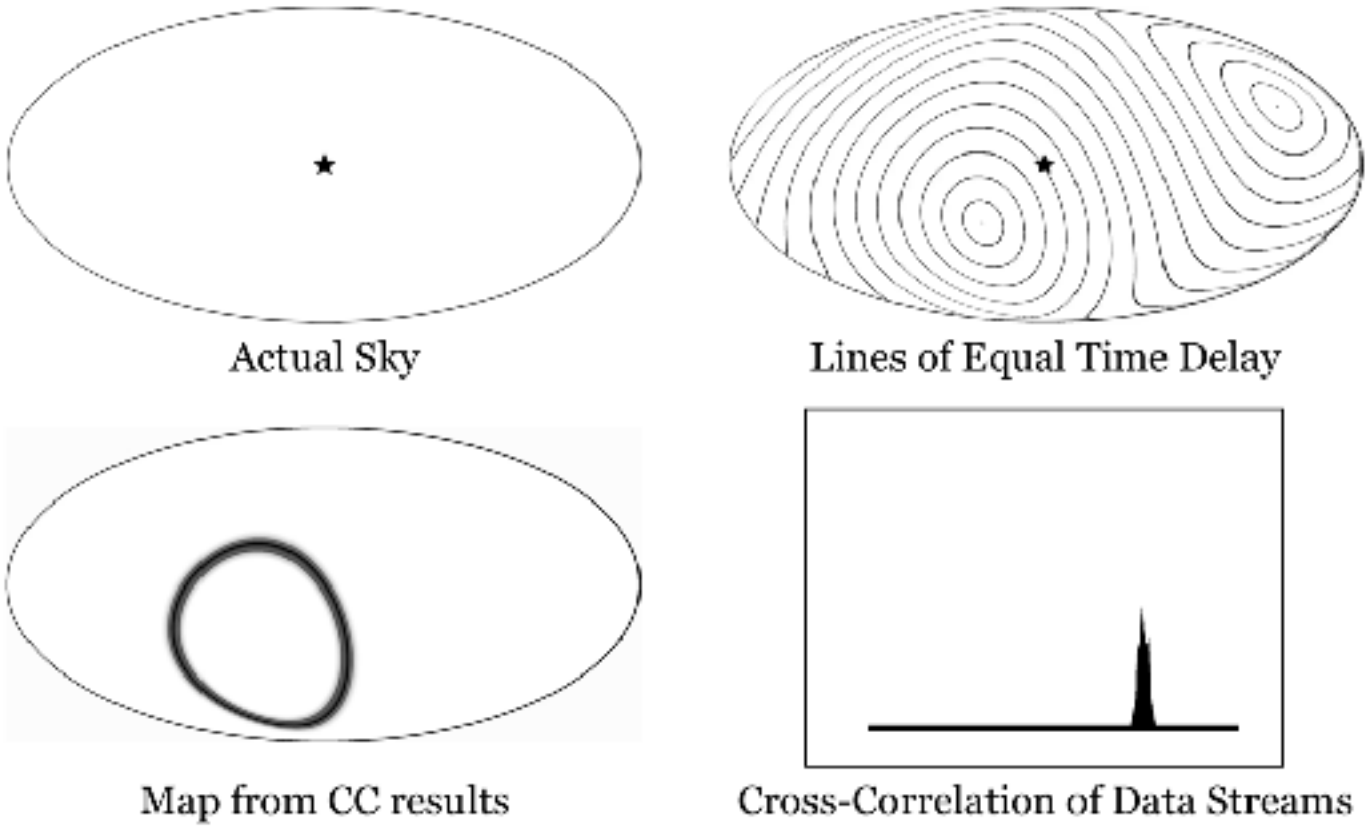
$$\boldsymbol{\Sigma}^{-1} := \mathbf{K}^\dagger \cdot \mathbf{N}^{-1} \cdot \mathbf{K} \;\Rightarrow\; [\Sigma^{-1}]_{\alpha\alpha'} = 4 \sum_{I,ft} \frac{H^2(f)}{P_{I_1}(t;f)\, P_{I_2}(t;f)} \, \gamma^{I*}_\alpha(f,t)\, \gamma^I_{\alpha'}(f,t)$$

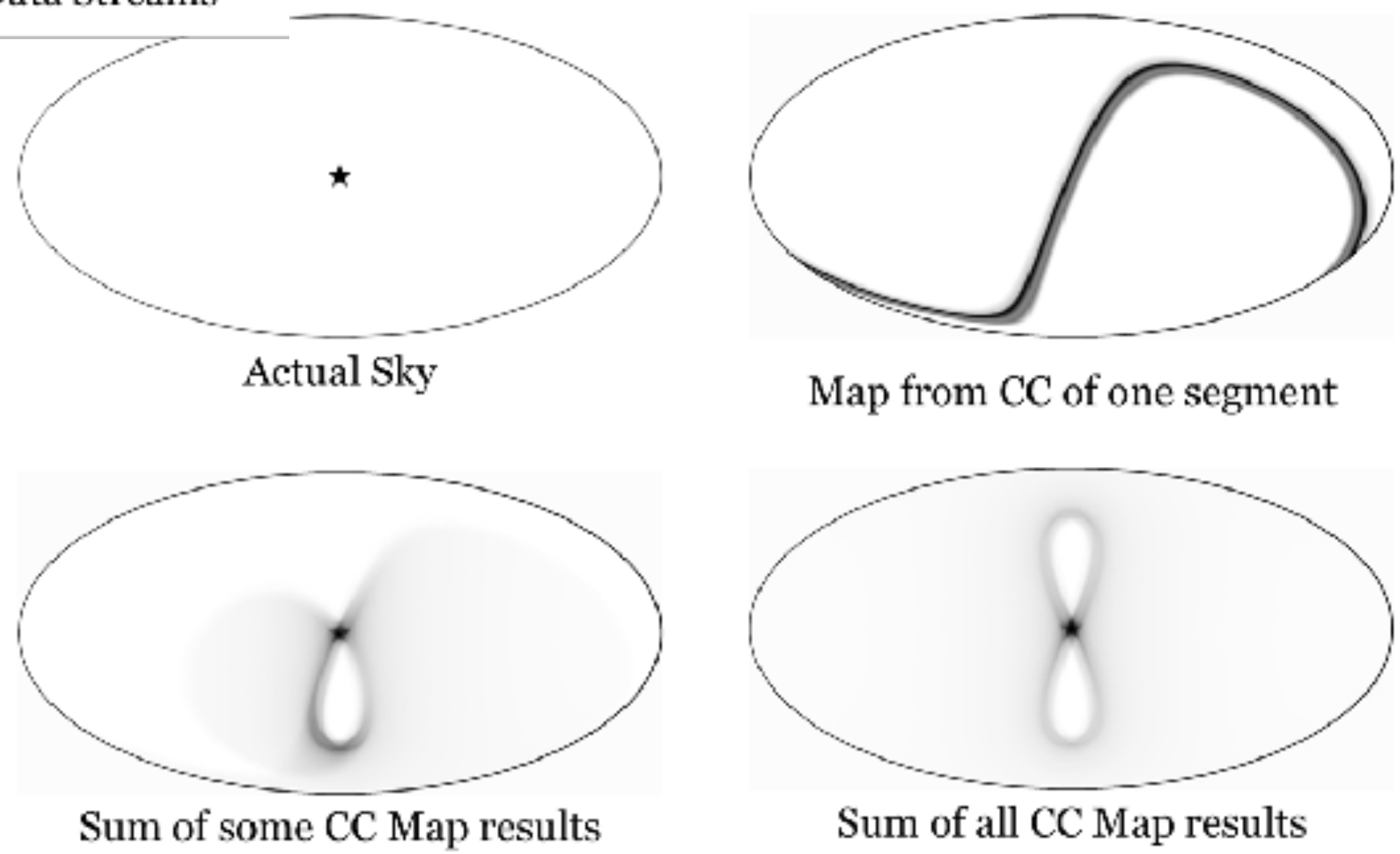
Base Line Sidereal Rotation



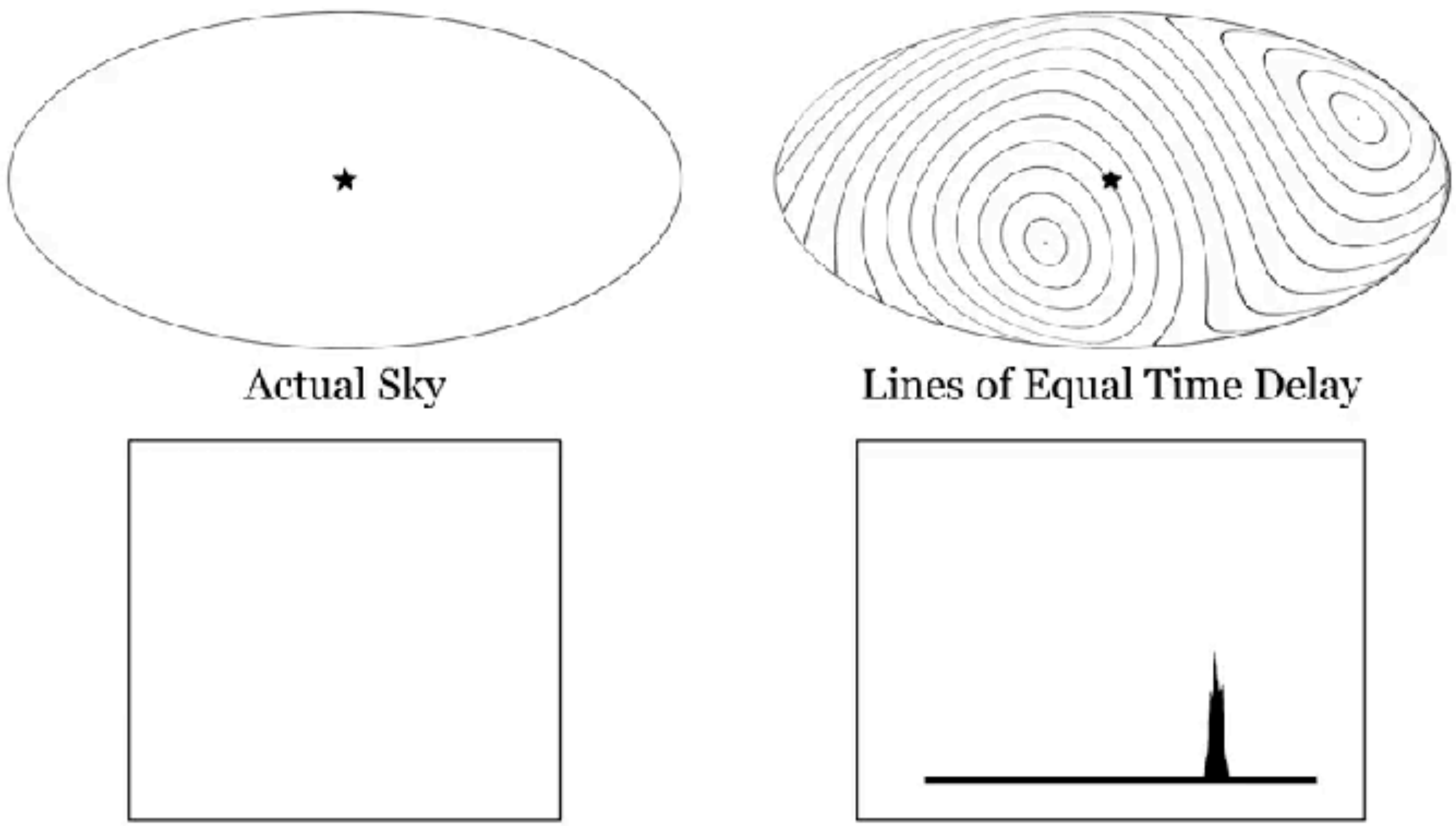
The white circles indicate positions in the sky map which will have equal time/phase delay when the signal from that part of the sky arrive in the LIGO Livingston and Hanford detectors

SGWB point source pre-processing





Pre-Processing



Actual Sky

Lines of Equal Time Delay

CC results of one Day

Cross-Correlation of Data Streams

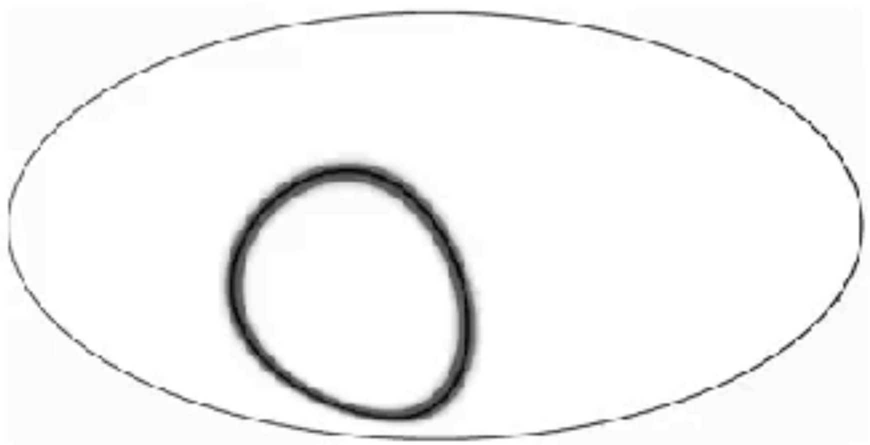
For a point source in sky (top-left) the cross-correlation results are calculated (bottom right) and stored (bottom left).

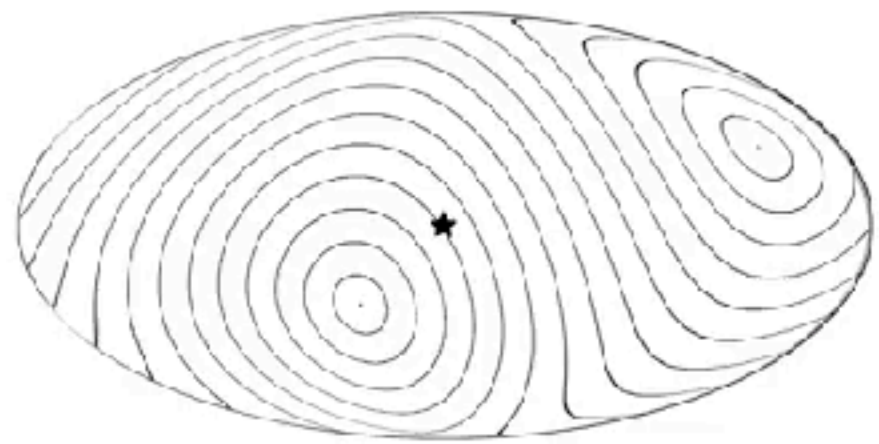
Visualising the Stochastic Pipeline - Anirban Ain, IUCAA

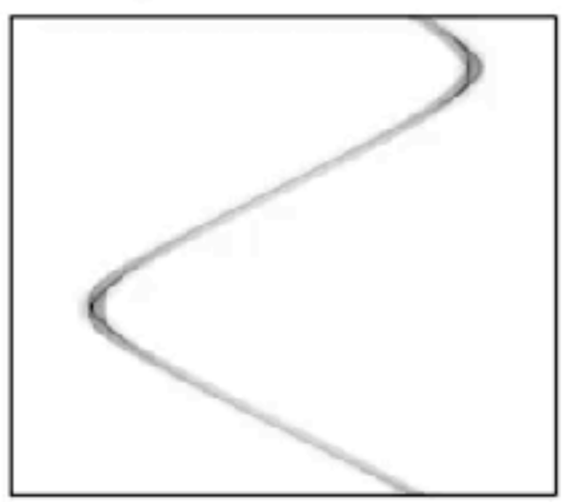
Visualising the Stochastic Pipeline - Anirban Ain, IUCAA

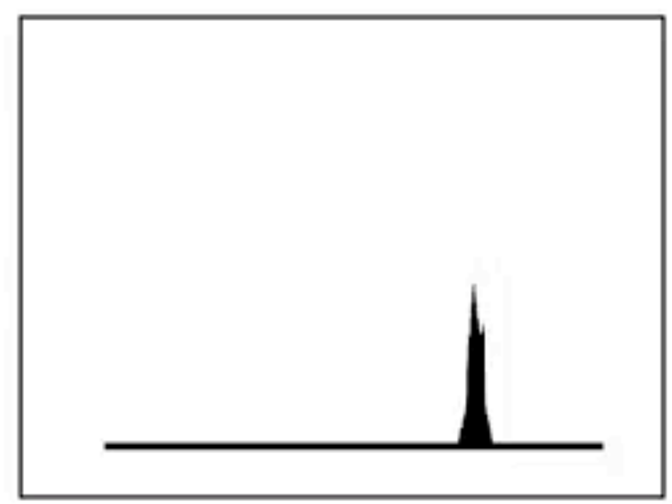
SGWB point source processing

Processing


Map from CC results


Lines of Equal Time Delay


CC results of one Day

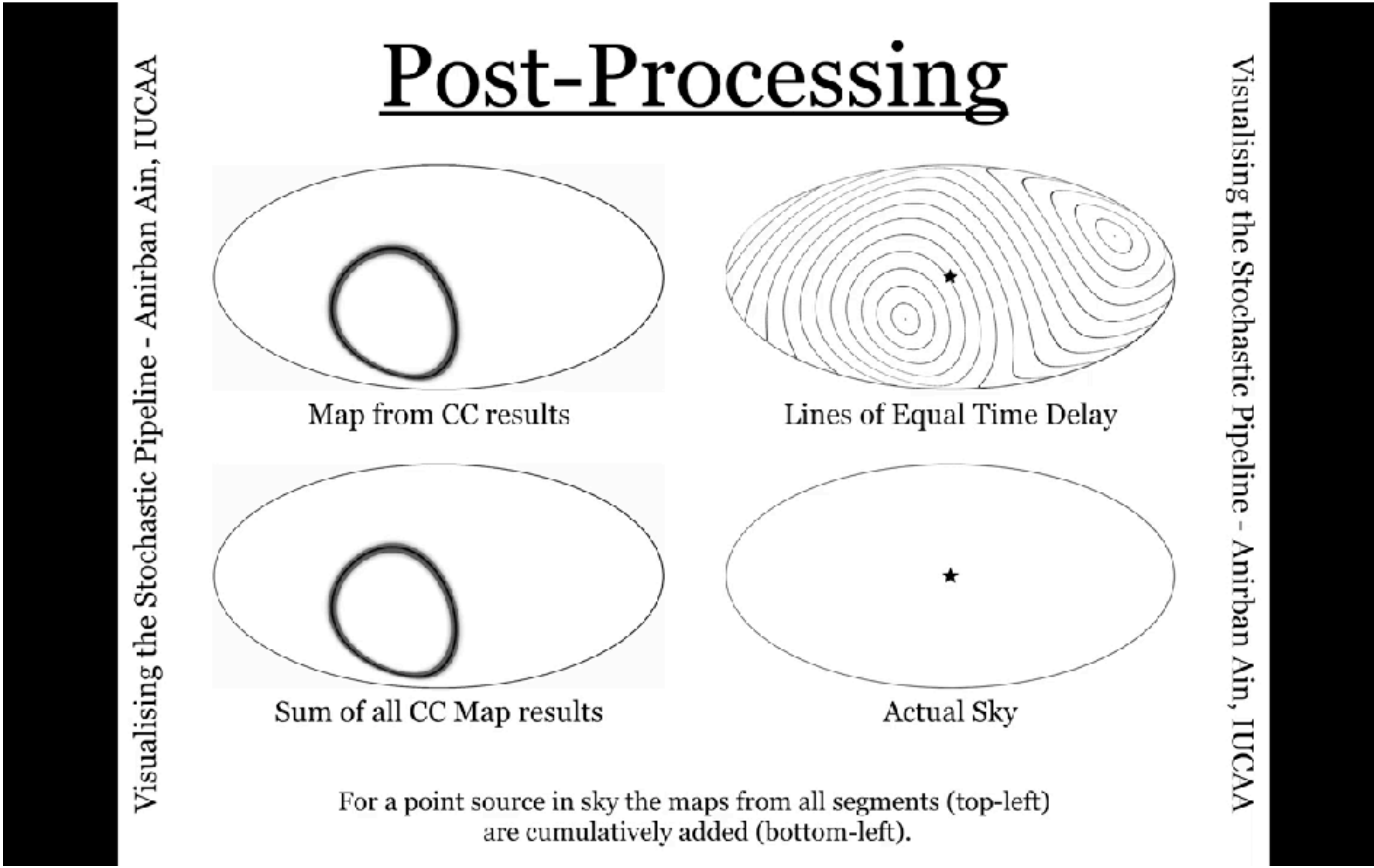

Cross-Correlation of Data Streams

For a point source in sky the cross-correlation results are turned into maps (top-left).

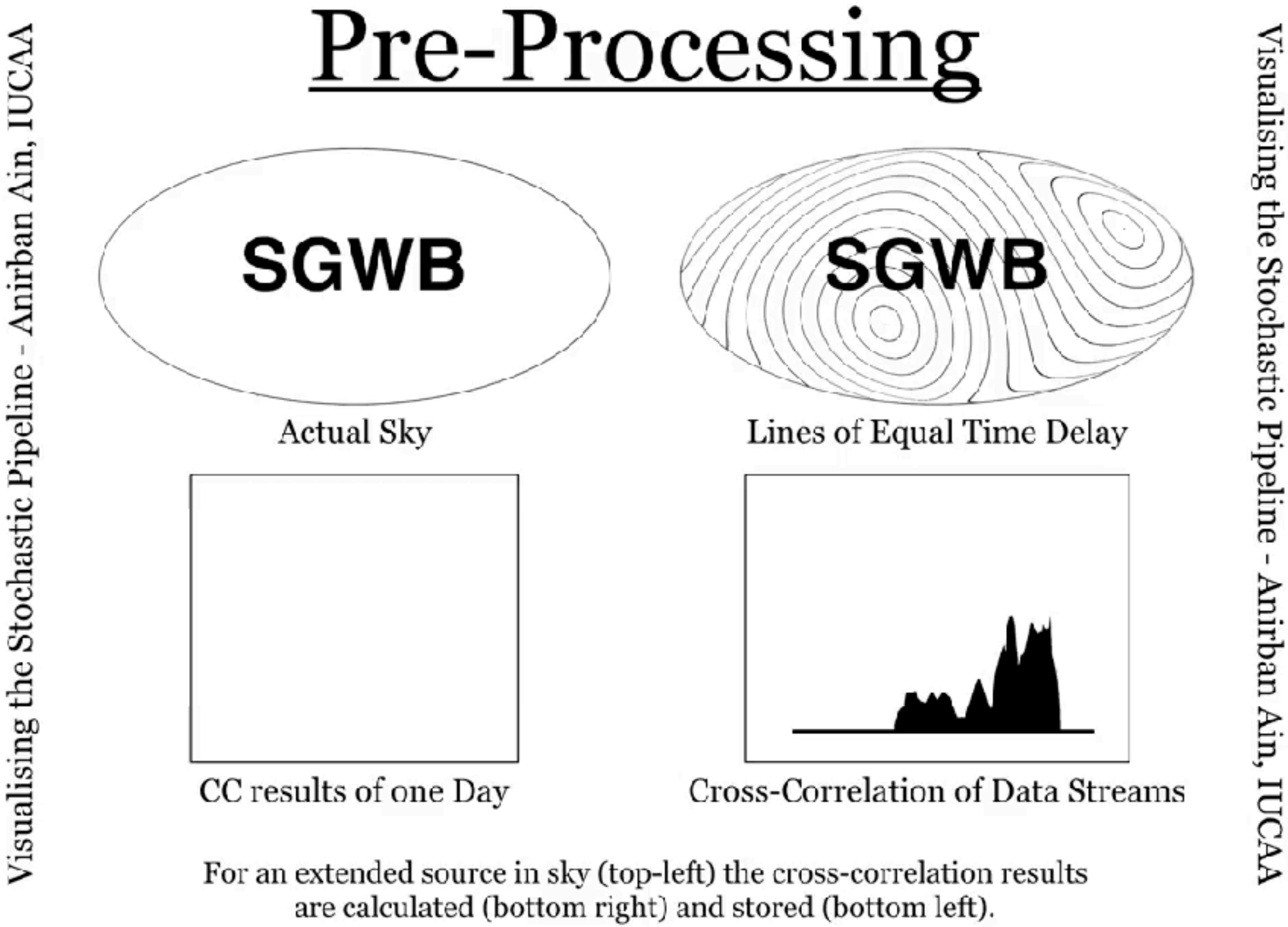
Visualising the Stochastic Pipeline - Anirban Ain, IUCAA

Visualising the Stochastic Pipeline - Anirban Ain, IUCAA

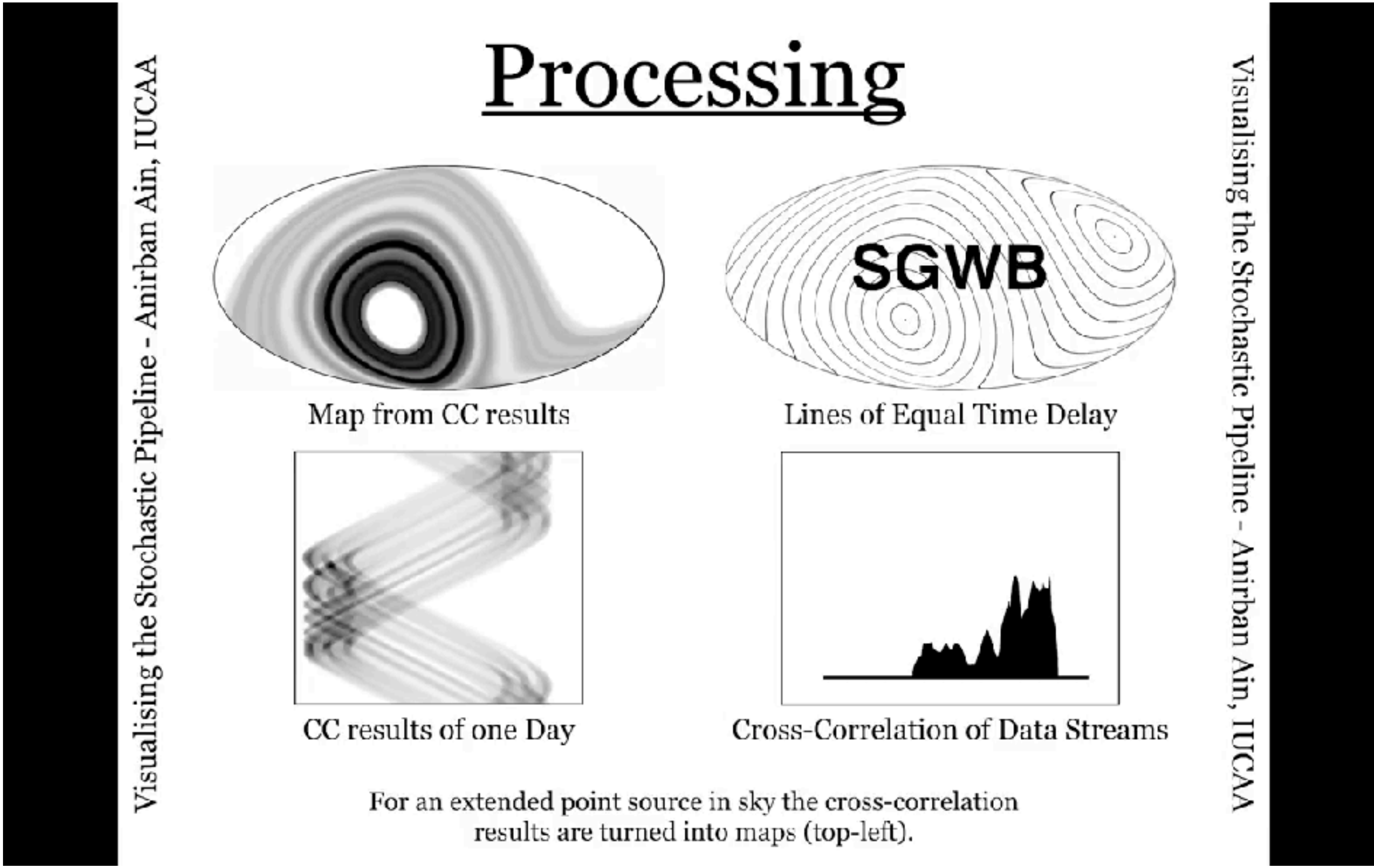
SGWB point source post-processing



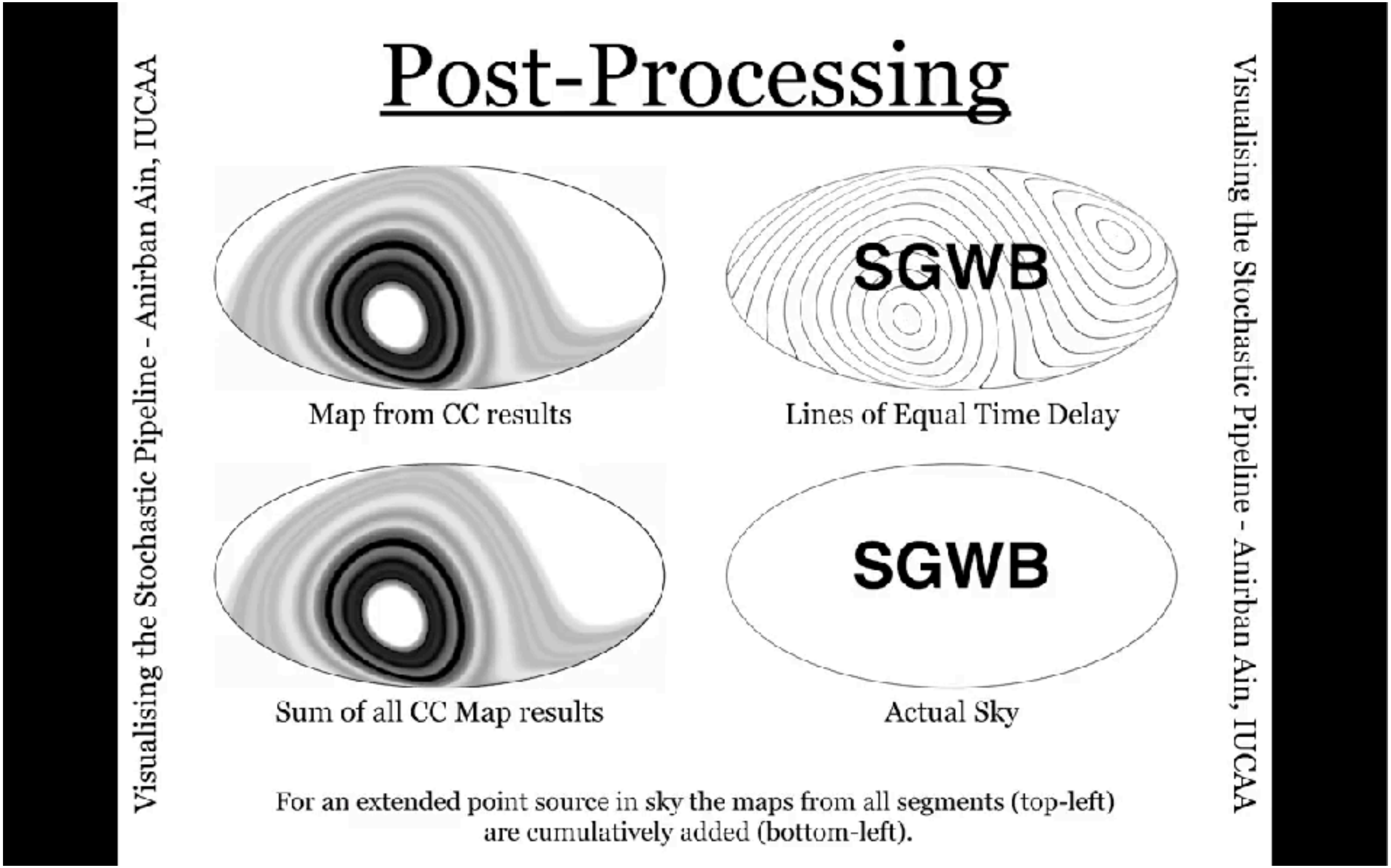
SGWB extended source pre-processing



SGWB extended source processing



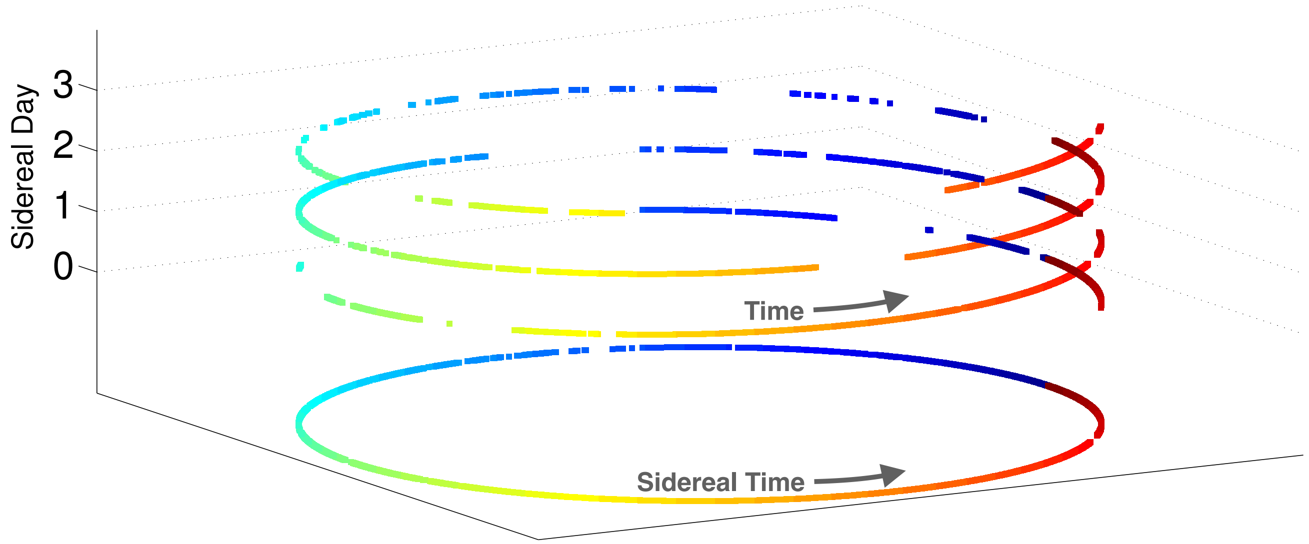
SGWB extended source post-processing



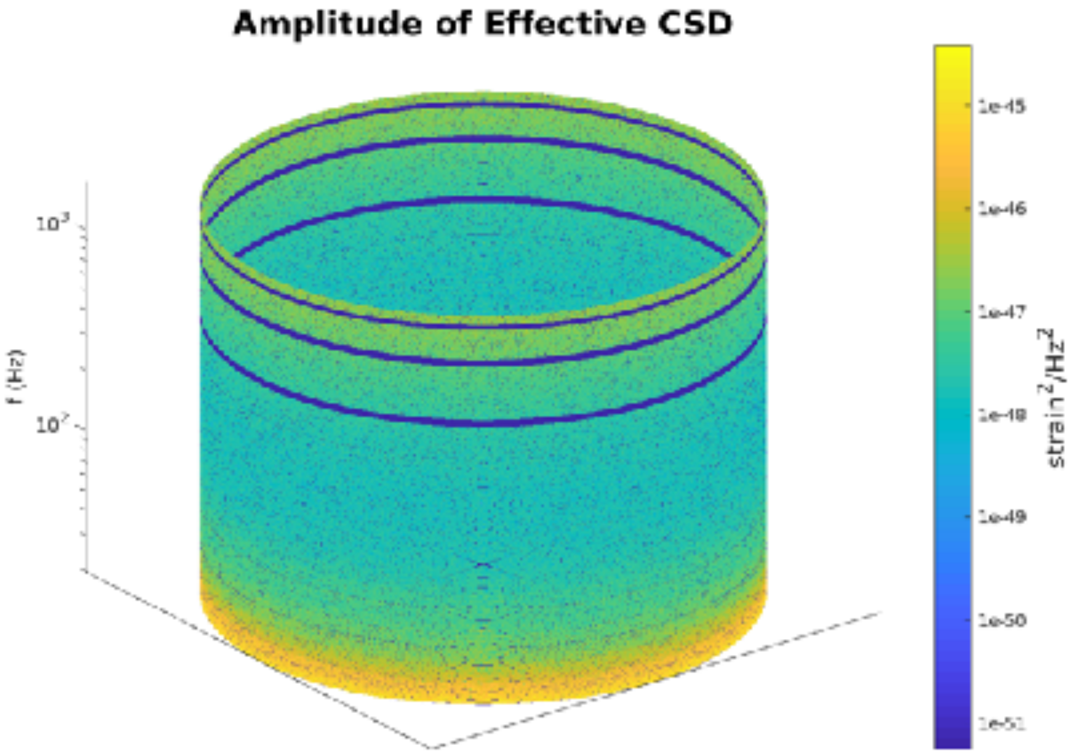
Folding stochastic data to one sidereal day

- No approximation, based on a mathematical symmetry

- Incorporates all the dirty stuff (quality cuts, overlapping window correction)



Ain, Dalvi & Mitra, PRD 92, 022003 (2015)



Folded full LIGO O2 CSD (abs)

Folding Data

- The estimator ("dirty map")

$$X_\alpha = \sum_{Ift} K^{I*}_{\alpha,ft} \left[\sigma^{-2}_{Ift} C^I_{ft} - \frac{1}{2}\varepsilon^I_{t-1} \left\{ \sigma^{-2}_{Ift} + \sigma^{-2}_{If(t-1)} \right\} C^I_{f(t-1)} - \frac{1}{2}\varepsilon^I_{t+1} \left\{ \sigma^{-2}_{Ift} + \sigma^{-2}_{If(t+1)} \right\} C^I_{f(t+1)} \right]$$

$$= \sum_{Ift_{sid}} K^{I*}_{\alpha,ft_{sid}} \sum_{i_{day}} \left[\sigma^{-2}_{If(i_{day}+t_{sid})} C^I_{f(i_{day}+t_{sid})} - \frac{1}{2}\varepsilon^I_{i_{day}+t_{sid}-1} \left\{ \sigma^{-2}_{If(i_{day}+t_{sid})} + \sigma^{-2}_{If(i_{day}+t_{sid}-1)} \right\} C^I_{f(i_{day}+t_{sid}-1)} - \frac{1}{2}\varepsilon^I_{i_{day}+t_{sid}+1} \left\{ \sigma^{-2}_{If(i_{day}+t_{sid})} + \sigma^{-2}_{If(i_{day}+t_{sid}+1)} \right\} C^I_{f(t_{sid}+t_{sid}+1)} \right]$$

- The Fisher information matrix (three data streams)

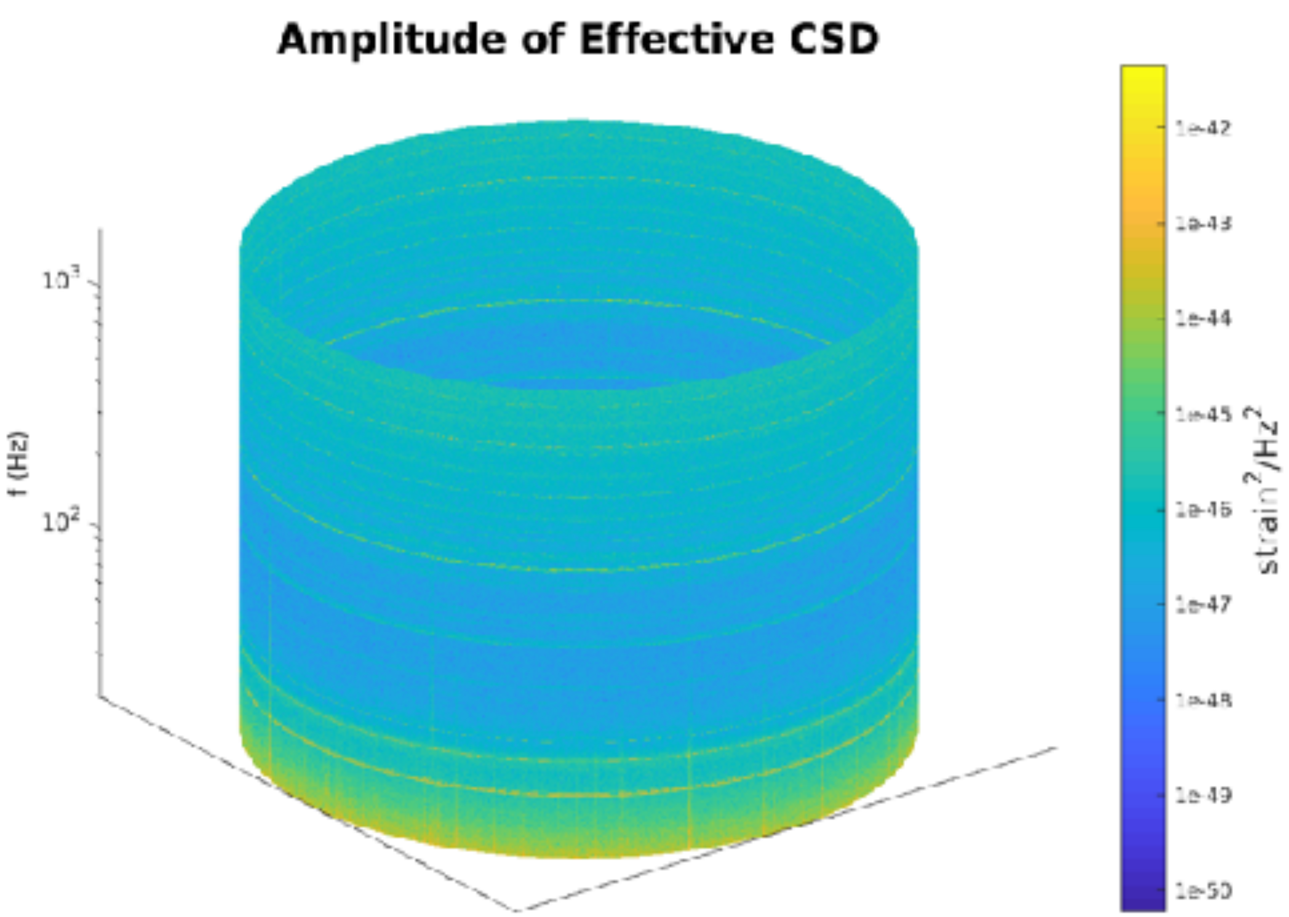
$$\Gamma_{\alpha\alpha'} = \sum_{Ift} K^{I*}_{\alpha,ft} \left[\sigma^{-2}_{Ift} K^{I}_{ft,\alpha'} - \frac{1}{2}\varepsilon^{I}_{t-1} \left\{ \sigma^{-2}_{Ift} + \sigma^{-2}_{If(t-1)} \right\} K^{I}_{f(t-1),\alpha'} - \frac{1}{2}\varepsilon^{I}_{t+1} \left\{ \sigma^{-2}_{Ift} + \sigma^{-2}_{If(t+1)} \right\} K^{I}_{f(t+1),\alpha'} \right]$$

$$= \sum_{Ift_{sid}} K^{I*}_{\alpha,ft_{sid}} K^{I}_{ft_{sid},\alpha'} \sum_{i_{day}} \sigma^{-2}_{If(i_{day}+t_{sid})}$$

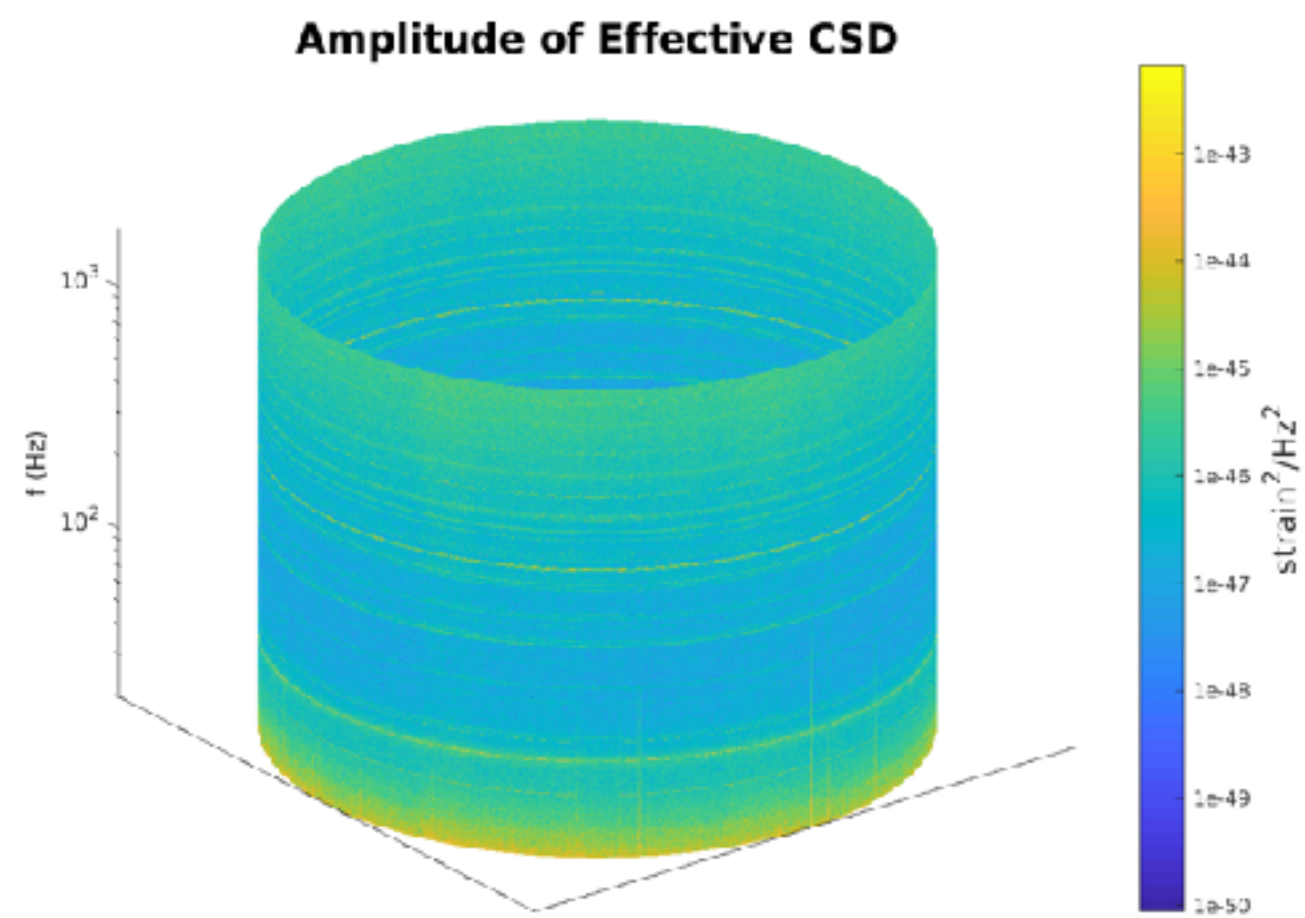
$$- \sum_{Ift_{sid}} K^{I*}_{\alpha,ft_{sid}} K^{I}_{f(t_{sid}-1),\alpha'} \sum_{i_{day}} \frac{1}{2}\varepsilon^{I}_{i_{day}+t_{sid}-1} \left\{ \sigma^{-2}_{If(i_{day}+t_{sid})} + \sigma^{-2}_{If(i_{day}+t_{sid}-1)} \right\}$$

$$- \sum_{Ift_{sid}} K^{I*}_{\alpha,ft_{sid}} K^{I}_{f(t_{sid}+1),\alpha'} \sum_{i_{day}} \frac{1}{2}\varepsilon^{I}_{i_{day}+t_{sid}+1} \left\{ \sigma^{-2}_{If(i_{day}+t_{sid})} + \sigma^{-2}_{If(i_{day}+t_{sid}+1)} \right\}$$

(HV baseline)

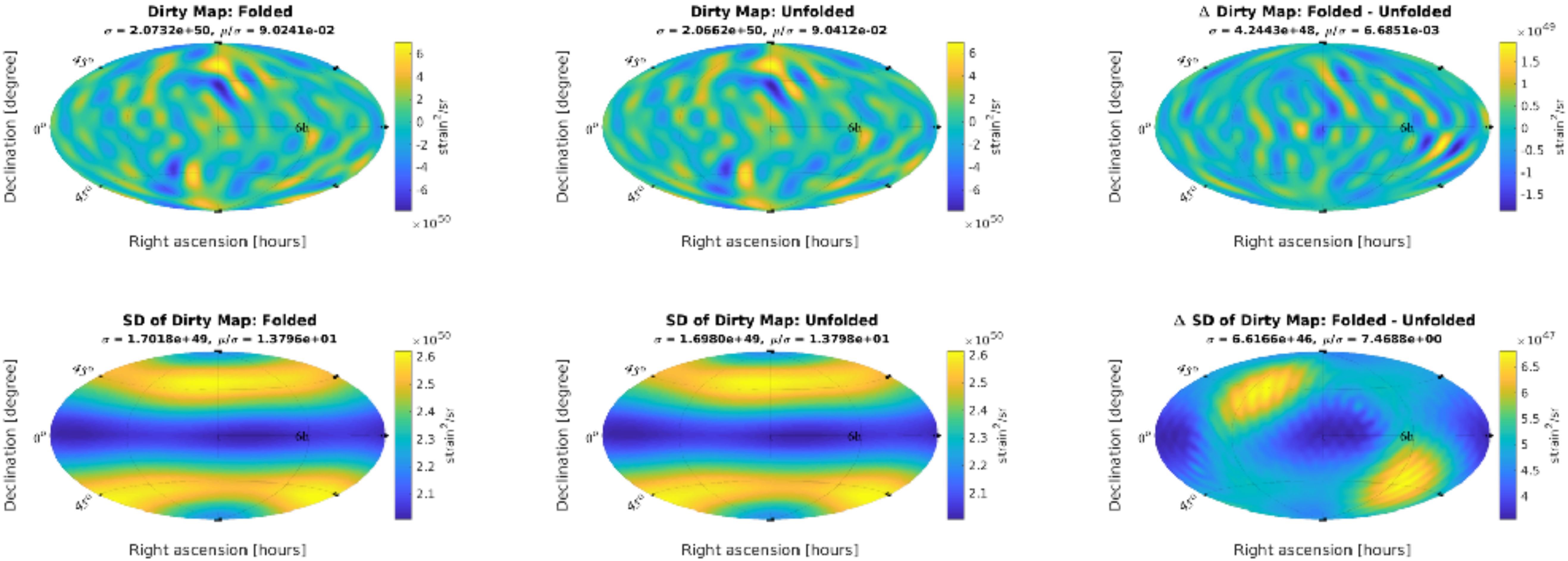


(LV baseline)



- No data quality cuts has been incorporated to these SIDs
- One can incorporate either in folding code or SID code

Validation of Folded Data



RMS Differences = RMS (folded - unfolded) / RMS (unfolded)

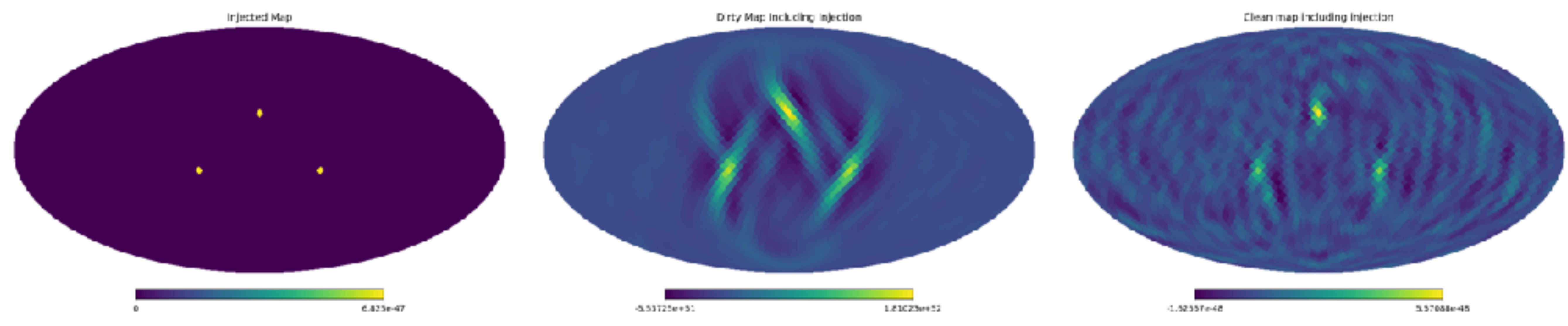
Real parts of FSID Fisher matrix	= 4.51e-03
Imaginary parts of FSID Fisher matrix	= 4.92e-03
Dirty SpH	= 2.38e-02
Clean SpH	= 2.44e-02
Dirty map	= 2.05e-02
Clean map	= 2.25e-02
Variance map	= 2.12e-03
SNR map	= 2.08e-02

(previous) Stochastic Pipelines

	Conventional pipeline	Folding pipeline
Intermediate data	450 GB	1.5 GB
Processing time	10 CPU years	10 CPU days
Intermediate results	800 TB	2.5 TB
Final results	500 MB	500 MB

PyStoch - fast HEALPix based SGWB mapmaking

- Folding + PyStoch = Few thousand times speed up
    - **perform the whole analysis on a laptop in few minutes!**
- Produces the narrowband maps as intermediate result
    - so separate search for different frequency spectra becomes redundant

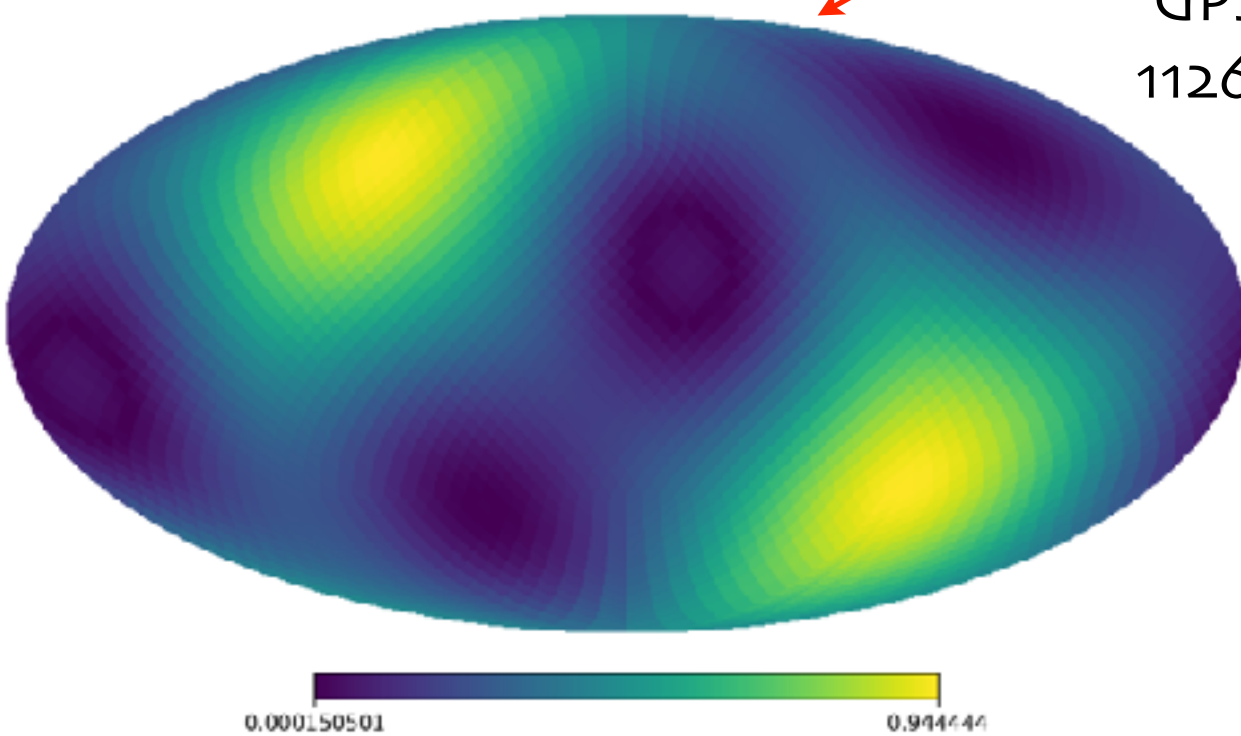


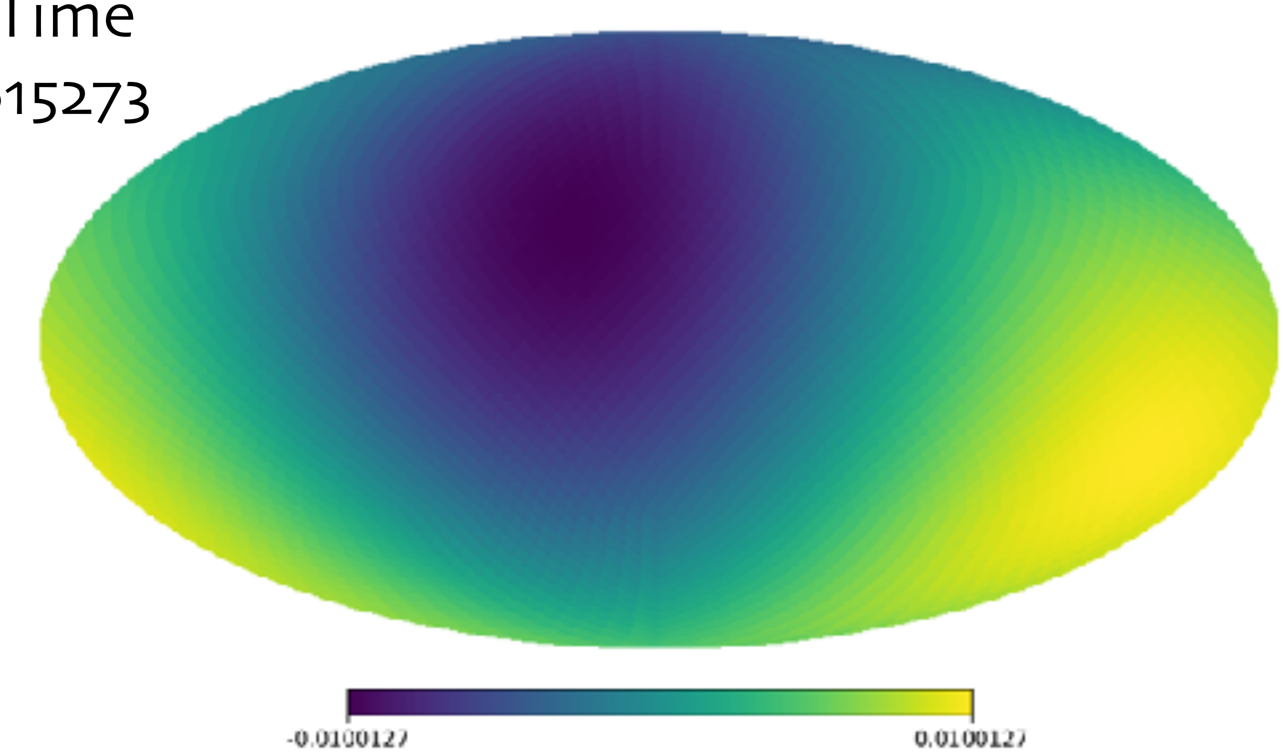
Efficient Overlap Reduction Function Computation

Breaking Down the ORF

$$\gamma^{I}_{ft,\alpha} := \sum_{A} \int_{S^2} d\hat{\mathbf{\Omega}} F^{A}_{\mathcal{I}_1}(\hat{\mathbf{\Omega}}, t) F^{A}_{\mathcal{I}_2}(\hat{\mathbf{\Omega}}, t) e^{2\pi i f \frac{\hat{\mathbf{\Omega}} \cdot \mathbf{\Delta x}_I(t)}{c}} e_{\alpha}(\hat{\mathbf{\Omega}})$$

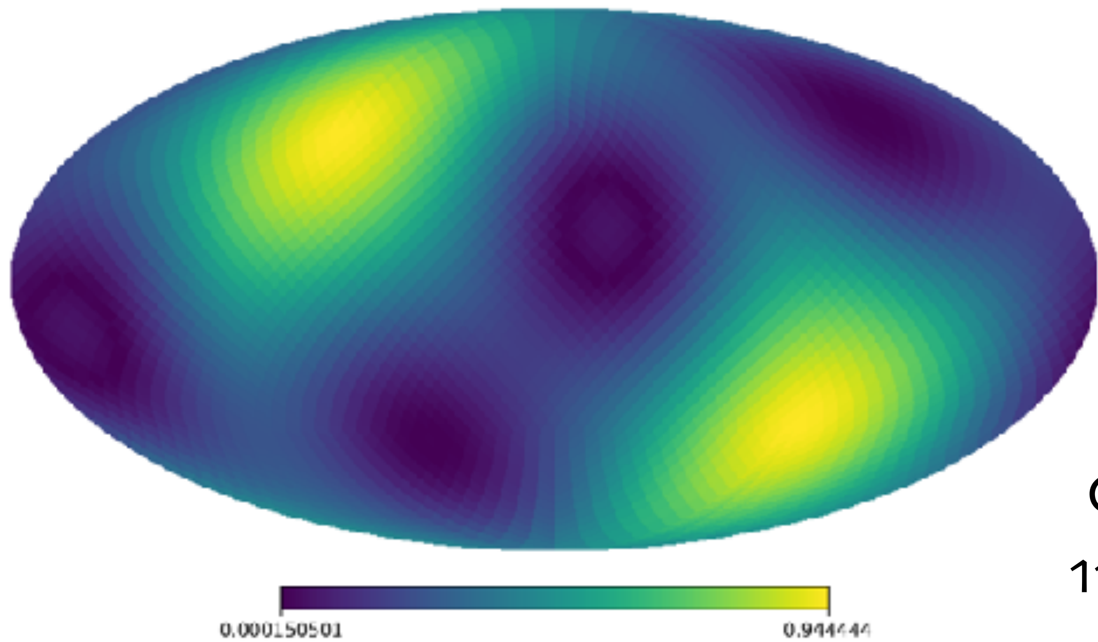
GPS Time
1126615273



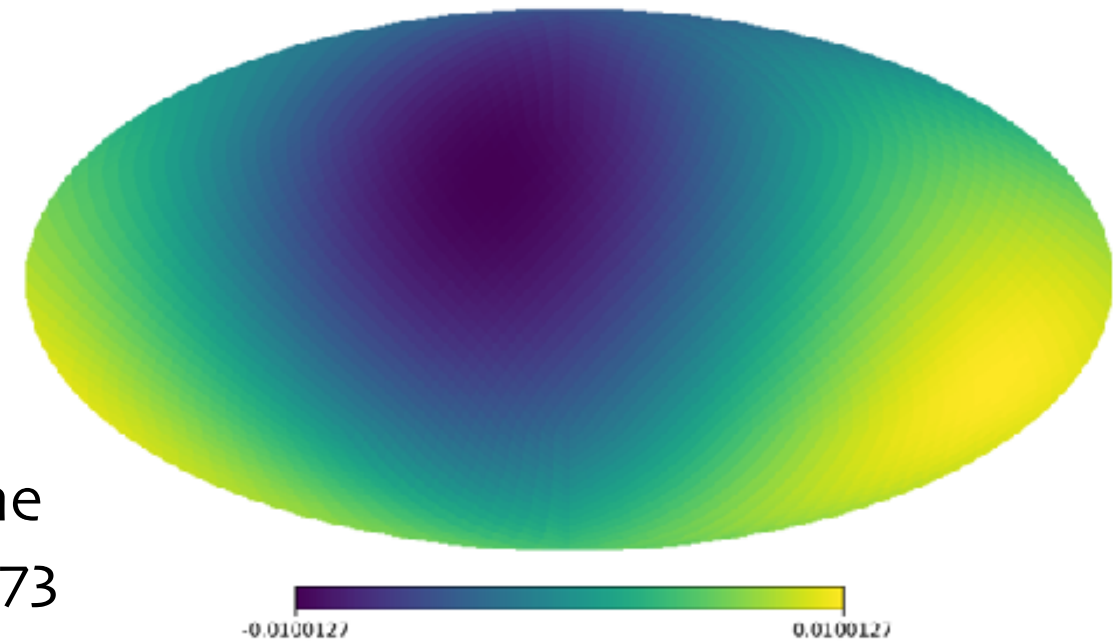


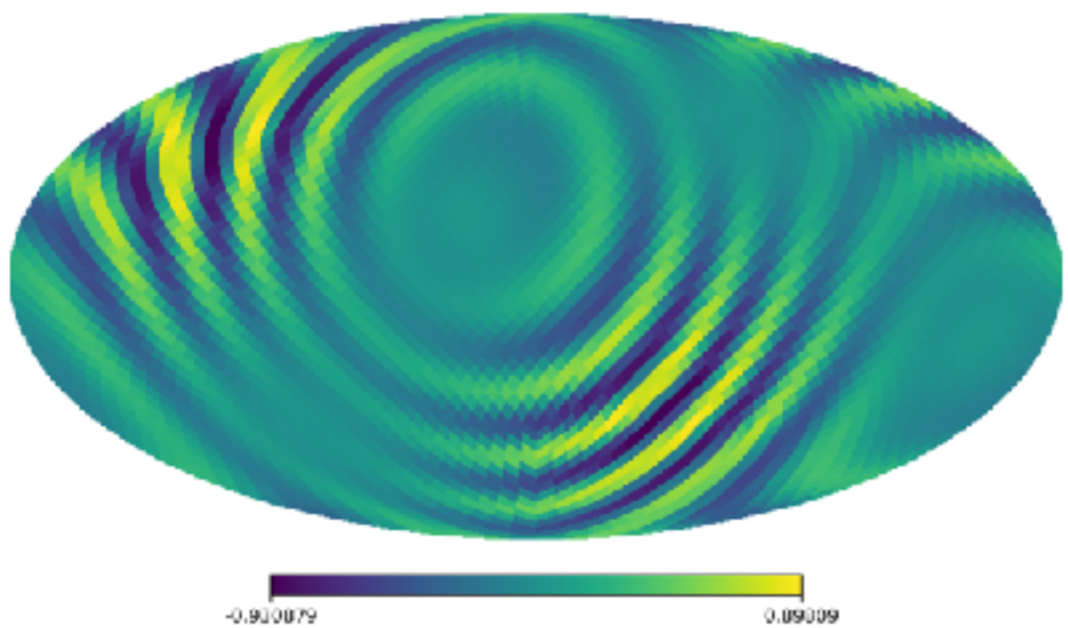
3314 pairs of seed maps (for one sidereal day) produced in a laptop in 20 seconds.

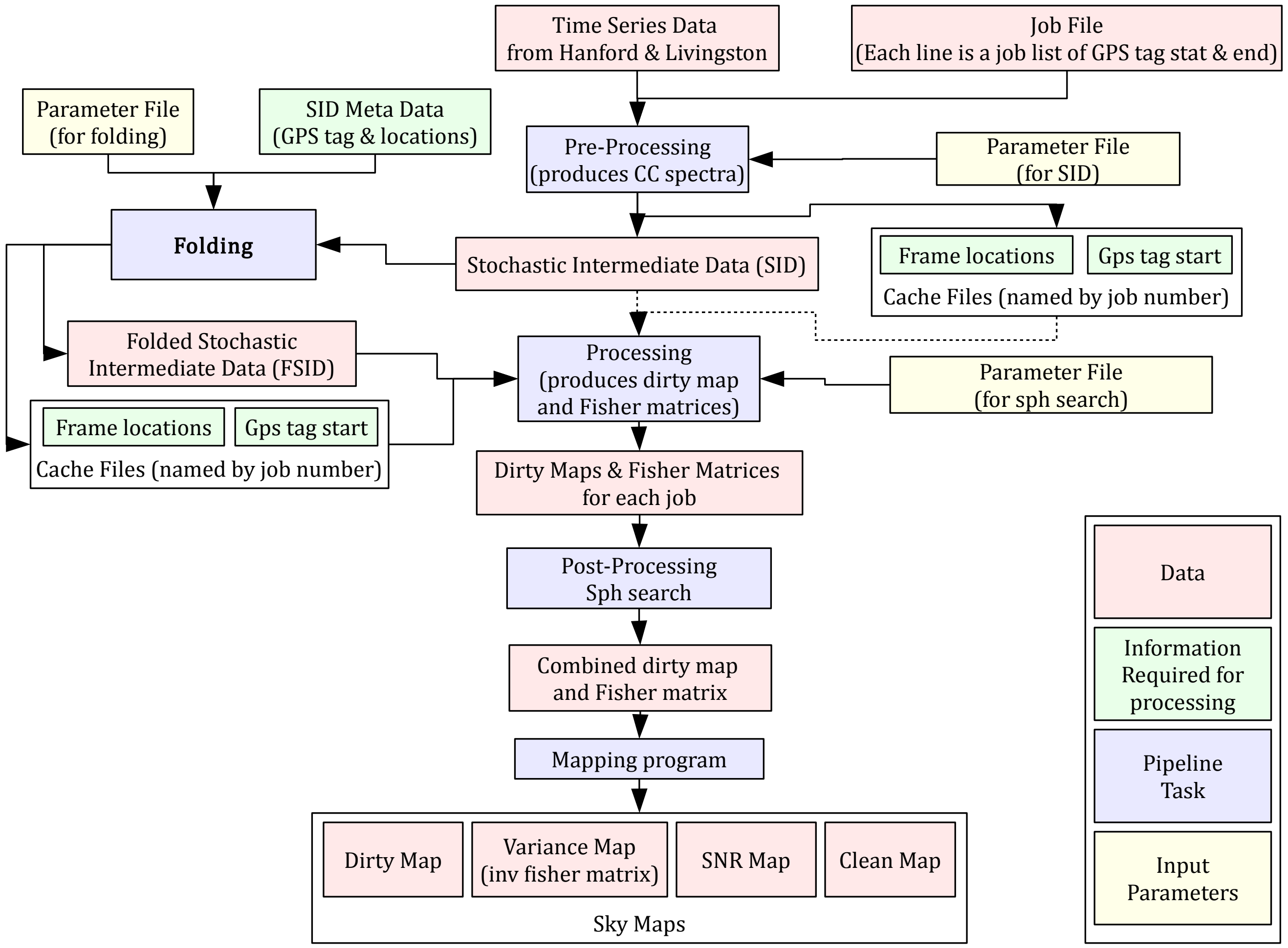
ORF from ORF seed maps



GPS Time
1126615273

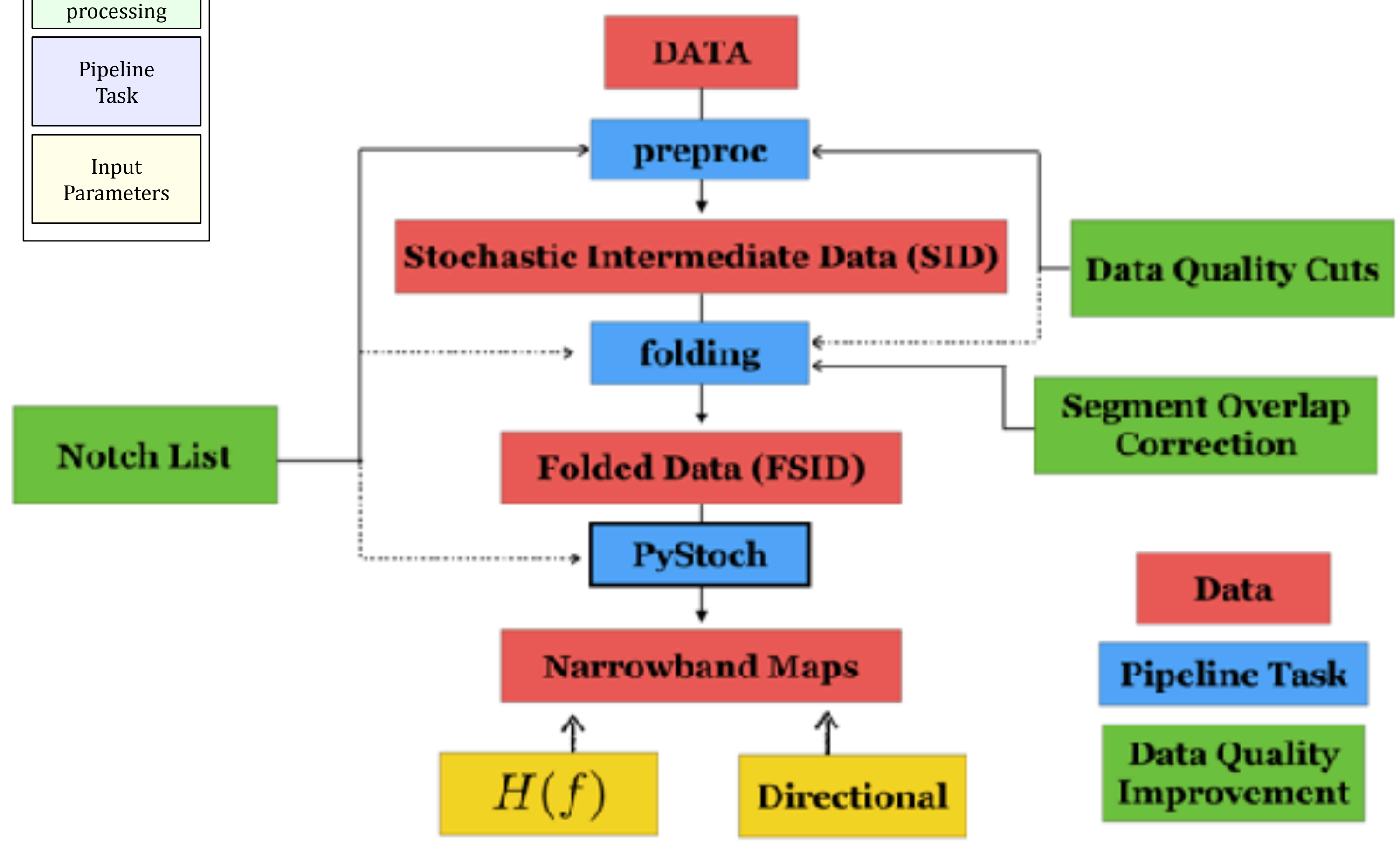






Stochastic pipeline

Stochastic pipeline - PyStoch



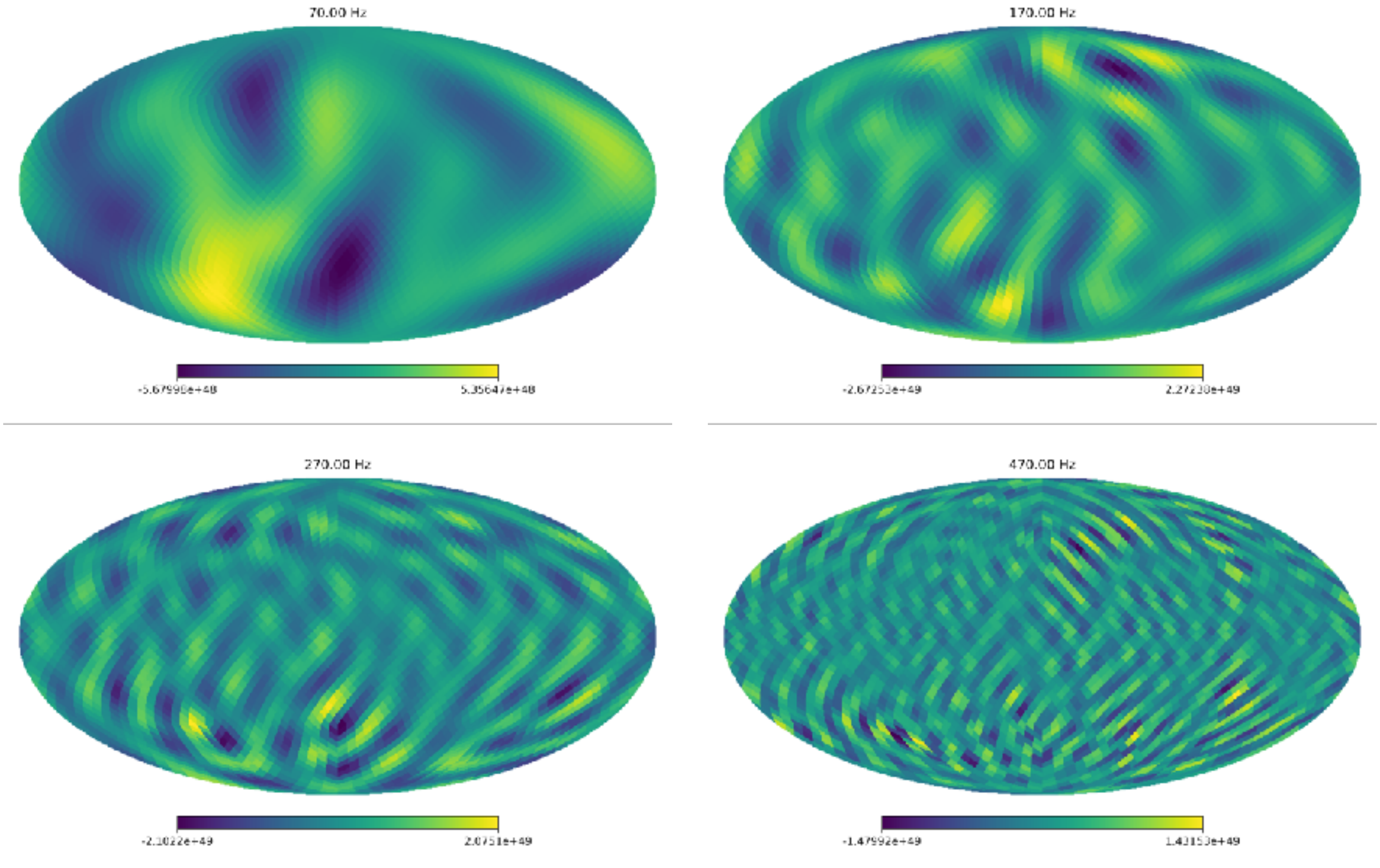
(New) Stochastic Pipeline - PyStoch

	Conventional pipeline	Folding pipeline	Folding and PyStoch
Intermediate data	450 GB	1.5 GB	1.5 GB
Processing time	10 CPU years	10 CPU days	40 CPU minutes
Intermediate results	800 TB	2.5 TB	Not required
Final results	500 MB	500 MB	500 MB

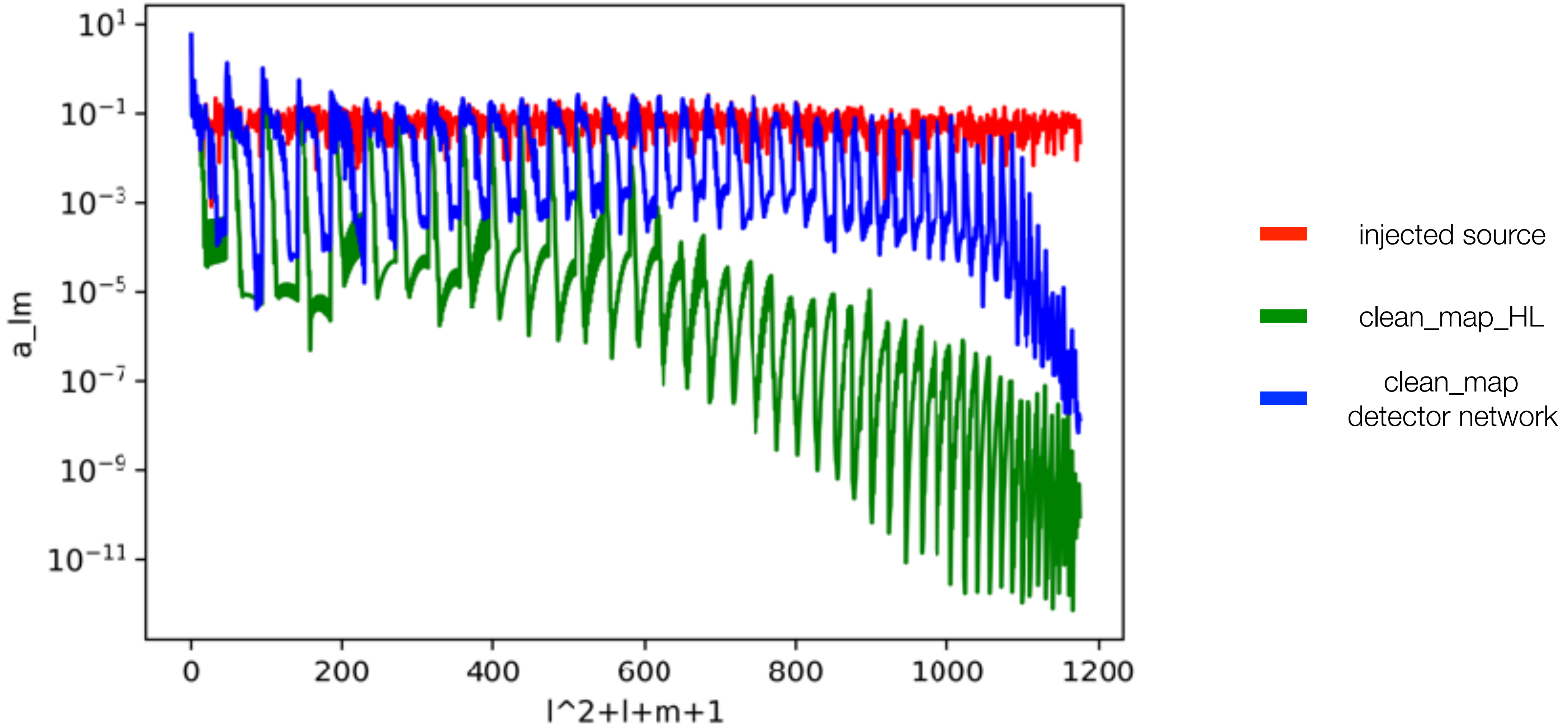
Ain, Suresh & Mitra, PRD 98 024001 (2018)

Narrowband Maps

1920 Narrowband maps (20 Hz to 500 Hz, 0.25 Hz bins) produced in a laptop in less than 10 minutes.



Multi-baseline studies



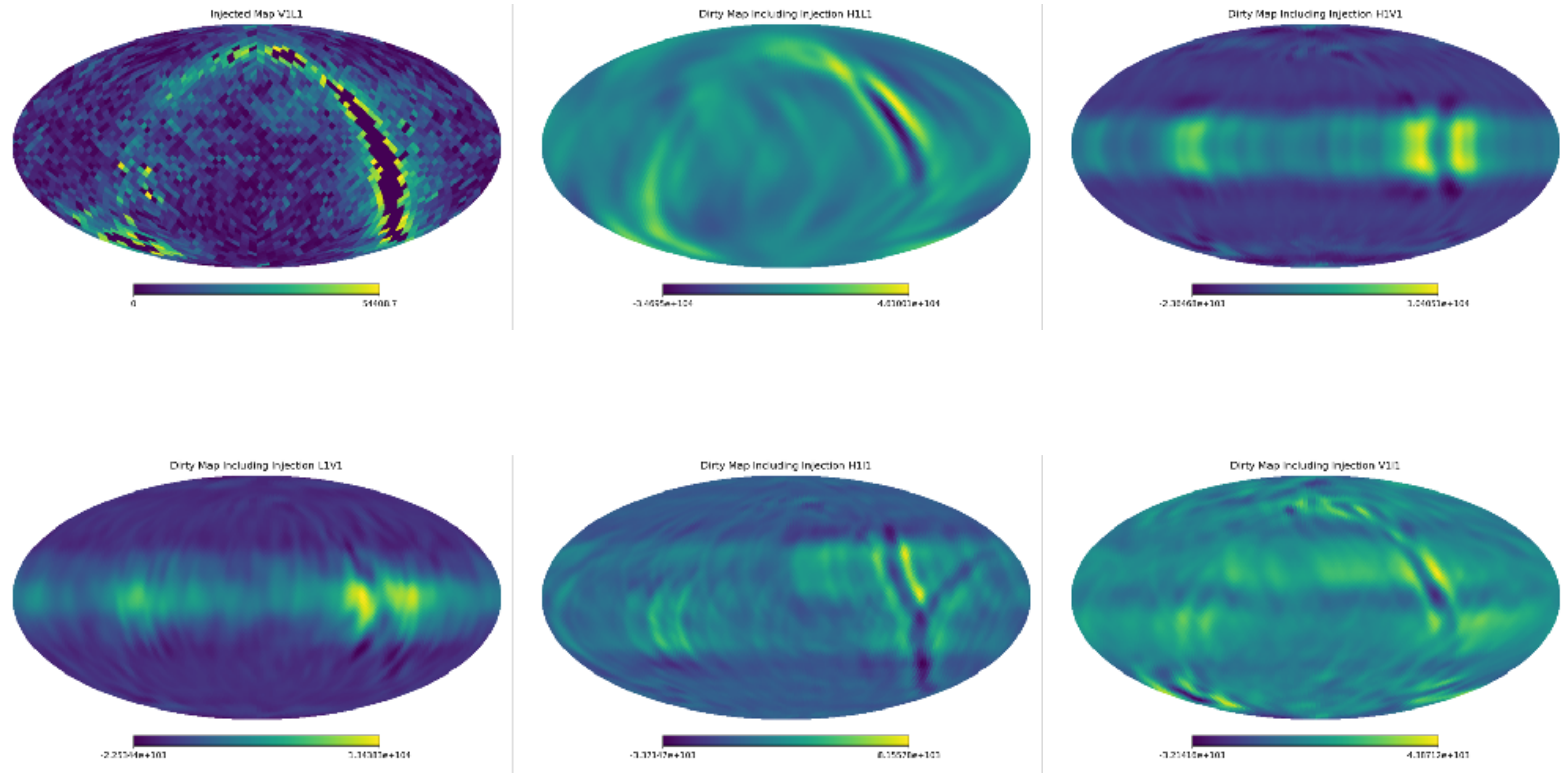
Multi baseline improvements :

Phys. Rev. D 83, 063002, 2011

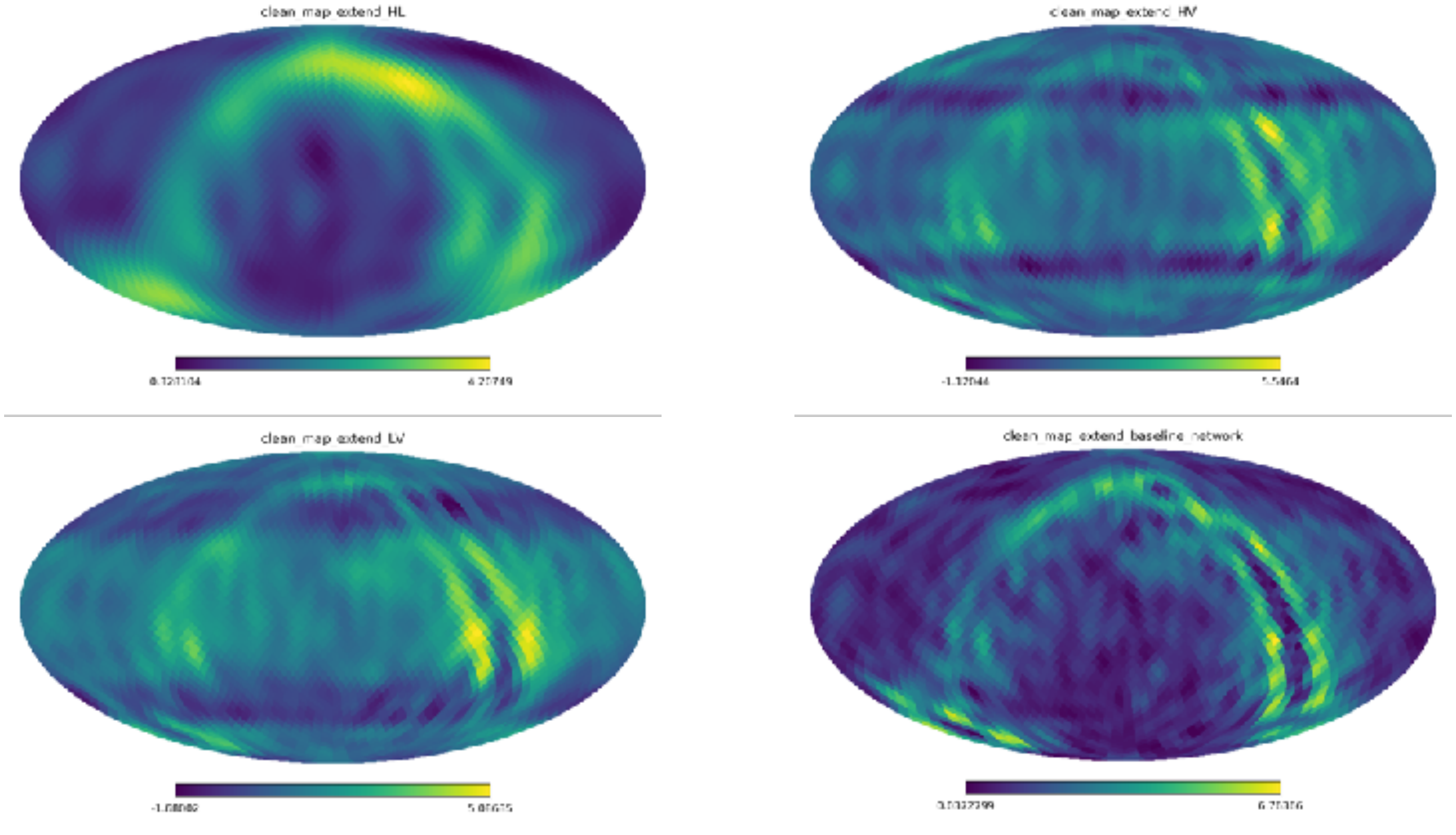
- Sensitivity improvement
- Sky coverage
- Parameter accuracy
- Quality of the sky maps

How the stochastic analysis is going to improve with KAGRA coming online ?

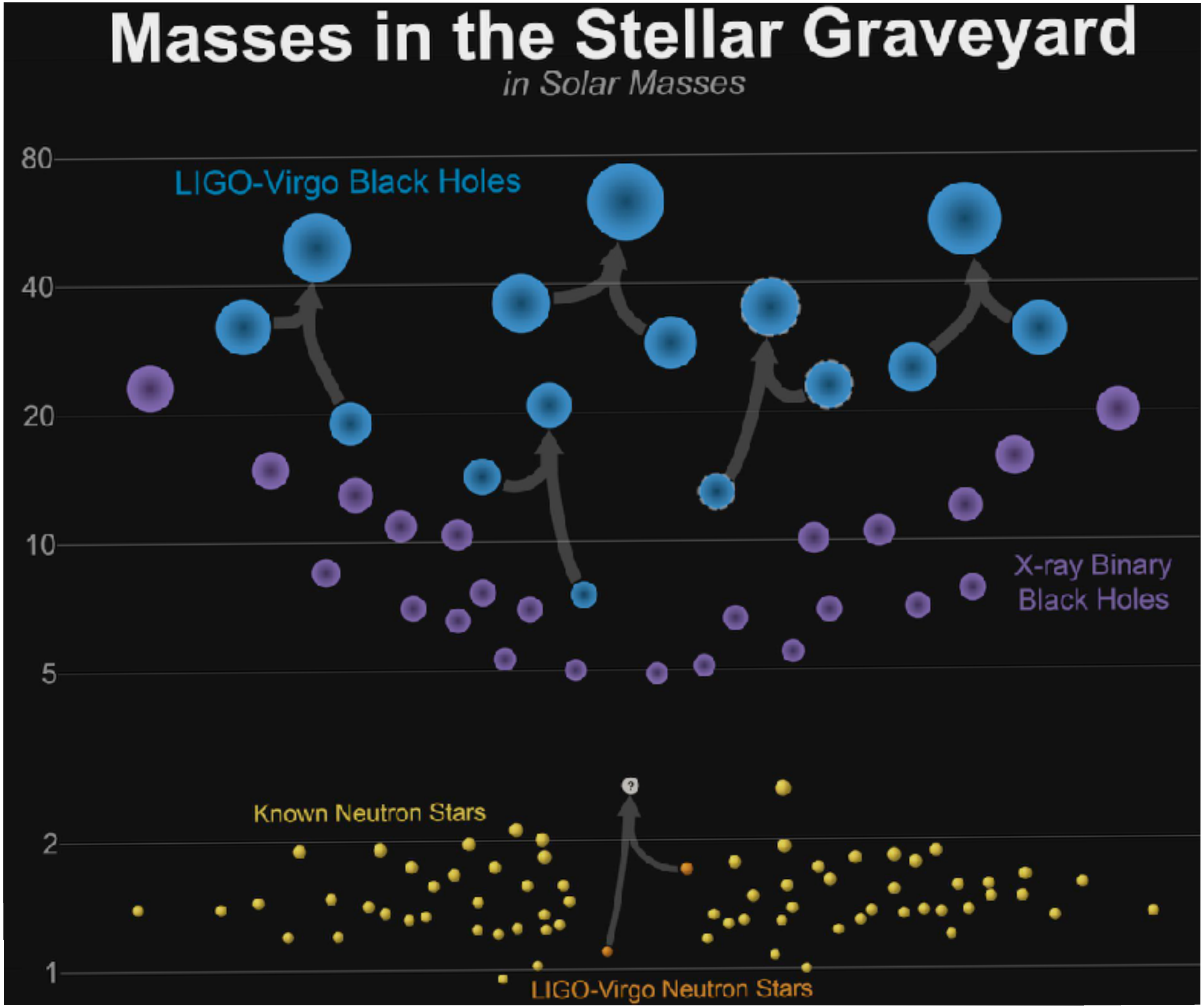
Multi-baseline Maps : Broad Source



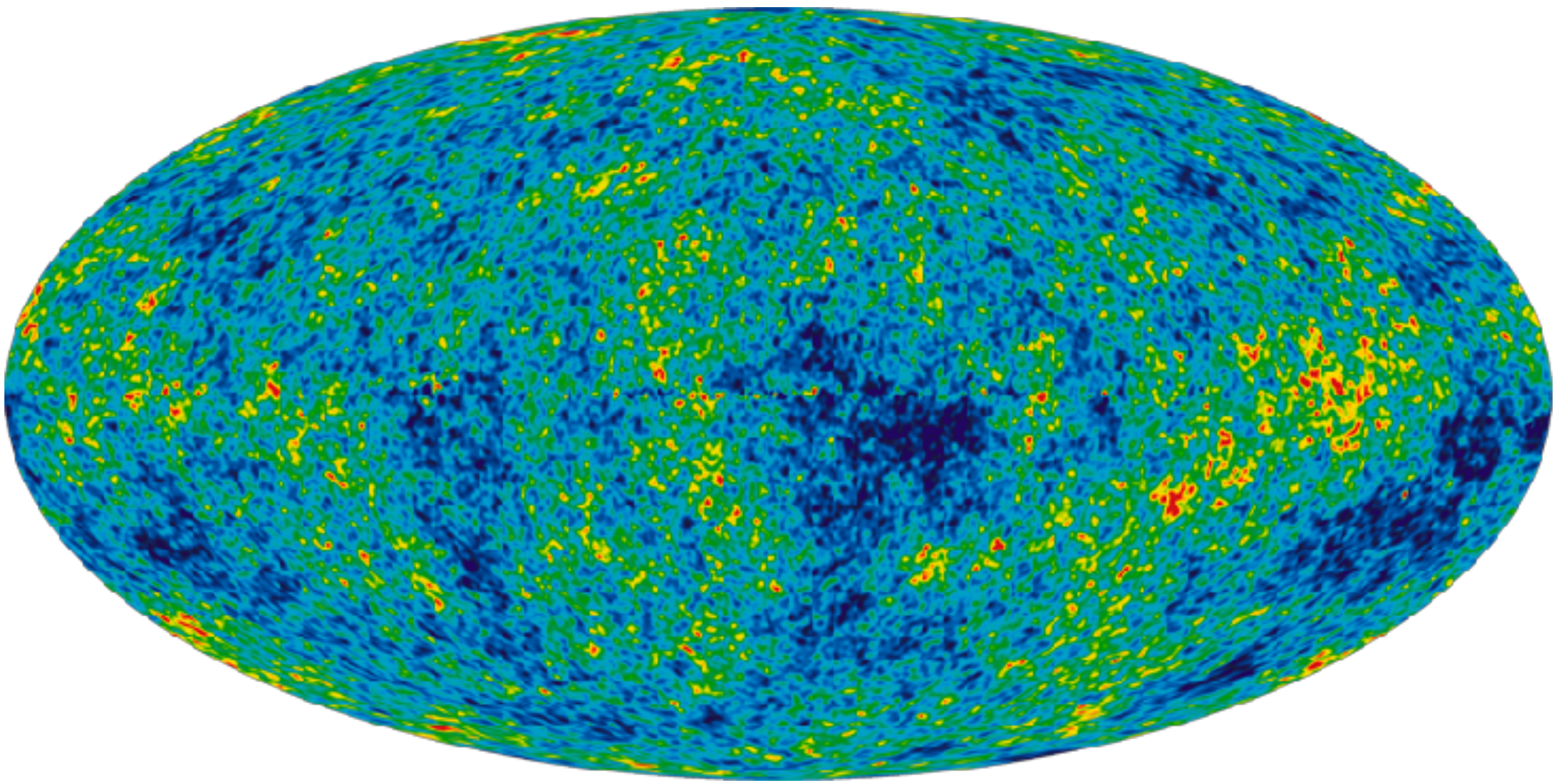
Multi-baseline Maps : Clean Maps



- The GW radiometer algorithm is optimal to search for backgrounds
  - **folding+PyStoch has made the algorithm few thousand times faster**
  - PyStoch is handy to study multi baseline efficiency for stochastic searches
  - can also search for persistent unknown sources
  - **can rapidly follow up the outliers**


Masses in the Stellar Graveyard
in Solar Masses
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
X-ray Binary
Black Holes
Known Neutron Stars
LIGO-Virgo Neutron Stars
?

…and we expect many more weaker signals…
Stay tuned for..



thank you