Magnetic Fields of Primordial Origin

Ryo Namba

McGill University

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In collaboration with

T. Fujita, Y. Tada, N. Takeda,

H. Tashiro, M. Peloso, N. Barnaby

Outline

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1. Introduction
   - Observations of extragalactic magnetic fields
   - Difficulties in cosmological generation
   - One (possibly) successful scenario

2. Axion Inflation
   - Production
   - Post-inflationary evolution
   - Present magnetic field amplitude

3. Summary and outlook

Observed large-scale magnetic fields

Large-scale magnetic field observed

- **Galactic scale** $\sim$ kpc: $10^{-6} - 10^{-5}\,\mathrm{G}$
- **Extragalactic scales** $\sim$ **Mpc:** $B_{\mathrm{eff}}^{\mathrm{obs}} \gtrsim 10^{-17}\,\mathrm{G}$

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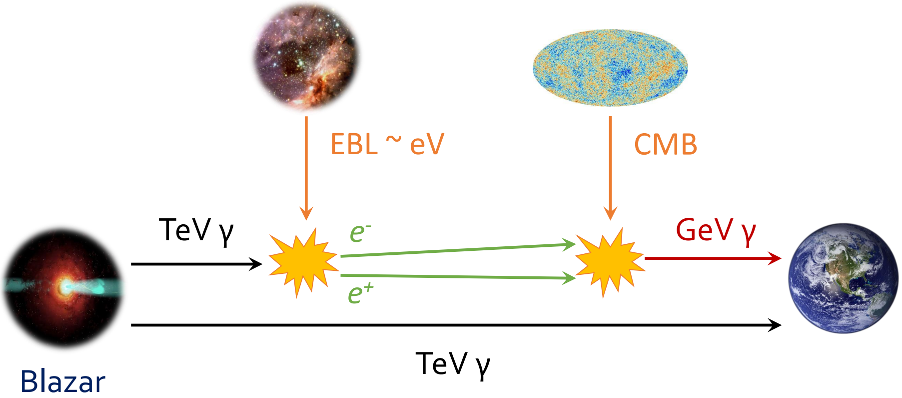
  - **Blazar TeV-GeV γ ray observation**

Neronov & Vovk '10, Essey et al. '11, Takahashi et al. '13

Observational method for extragalactic $\vec{B}$

No magnetic fields



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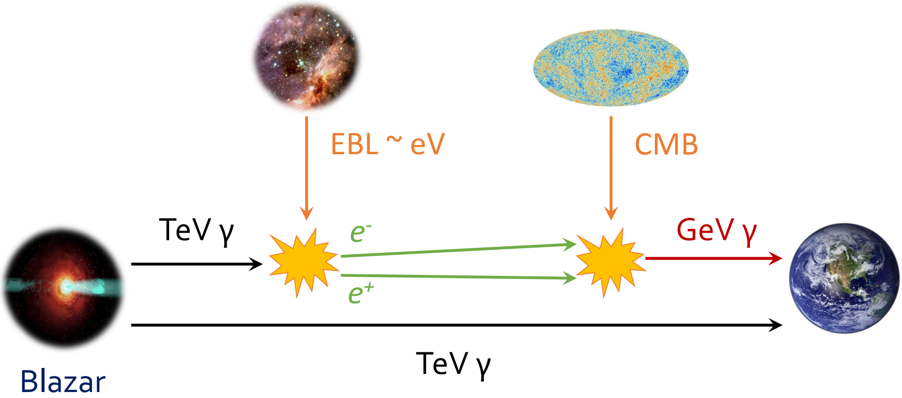
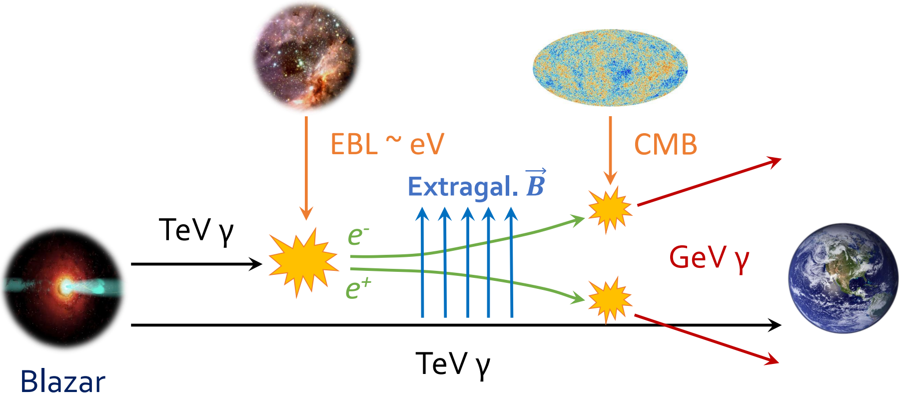


Observe both TeV + GeV γ rays

Observational method for extragalactic $\vec{B}$

Magnetic fields are present



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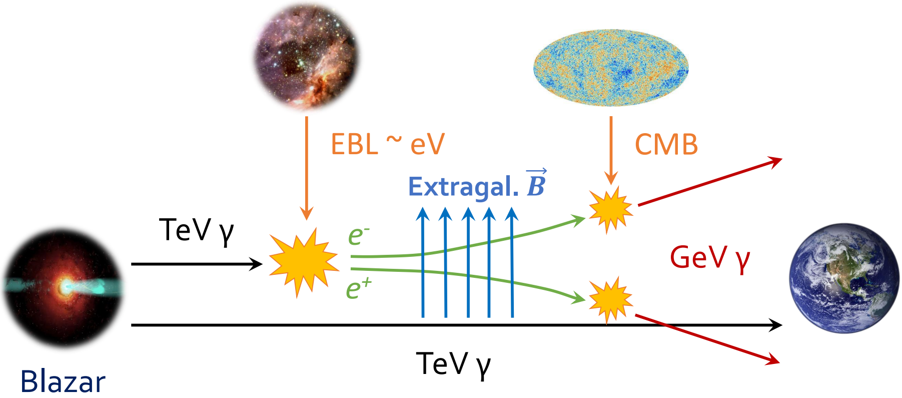
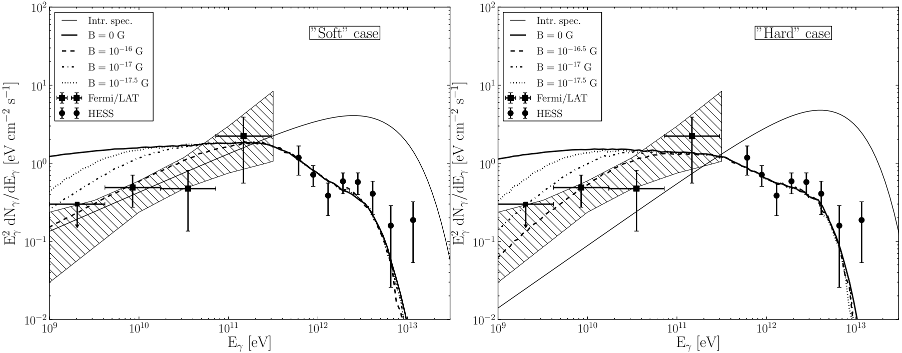


Observe TeV γ rays, lack of GeV γ rays

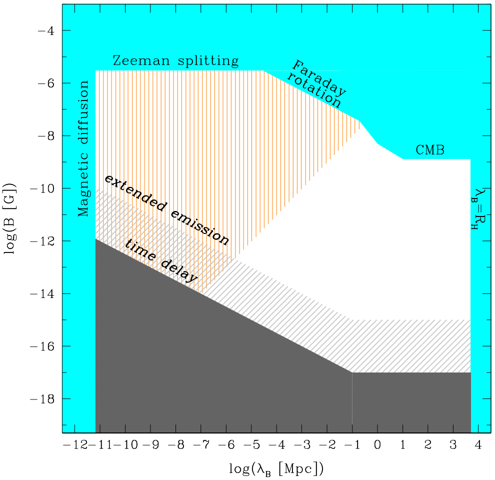
Spectrum of blazar photons



Vovk et al. '12

Observed extragalactic $\vec{B}$

Taylor, Vovk & Neronov '11



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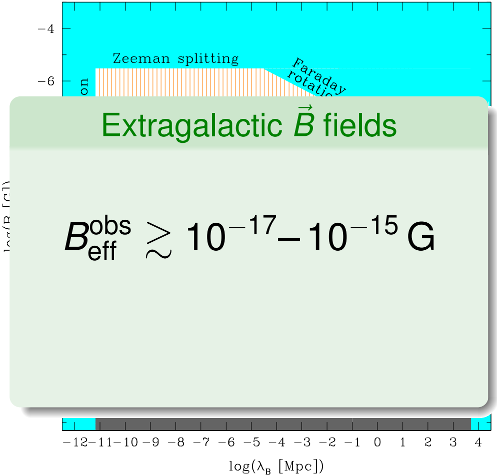

Extragalactic $\vec{B}$ fields

$$B_{\rm eff}^{\rm obs} \gtrsim 10^{-17} \text{–} 10^{-15}\,{\rm G}$$

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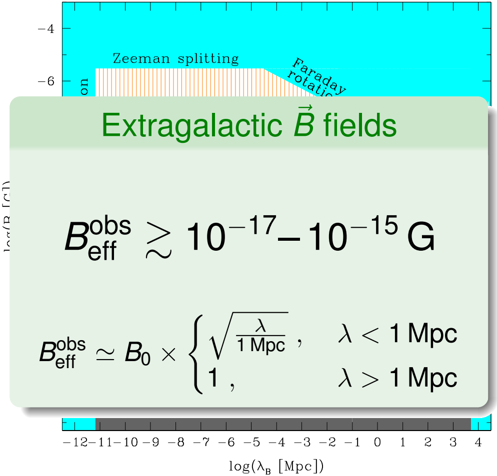

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$$B_{\rm eff}^{\rm obs} \simeq B_0 \times \begin{cases} \sqrt{\dfrac{\lambda}{1\,{\rm Mpc}}} \,, & \lambda < 1\,{\rm Mpc} \\ 1 \,, & \lambda > 1\,{\rm Mpc} \end{cases}$$

Observed extragalactic $\vec{B}$

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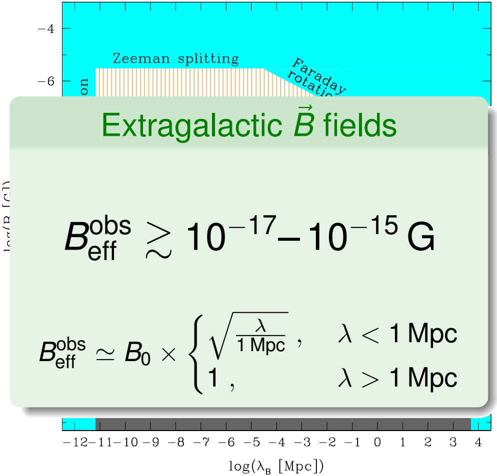

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- Indications of non-zero helicity of intergalactic $\vec{B}$ field

Cosmological origins ?

Free EM photon is *conformally* coupled to gravity in 4D

$$
\begin{aligned}
g_{\mu\nu} \quad &\rightarrow \quad \Omega^2 \, g_{\mu\nu} \; , \\
S_{\rm EM}^{\rm free} \propto \sqrt{-g}\, g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \quad &\rightarrow \quad \Omega^{4-2-2} \sqrt{-g}\, g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}
\end{aligned}
$$

Equation of motion in (flat) FLRW spacetime

$$
\left(\frac{\partial^2}{\partial \tau^2} + k^2 \right) \vec{A} = 0 \; , \qquad ds^2 = a^2 (-d\tau^2 + d\vec{x}^2)
$$

◇ **EM field is insensitive to the background expansion**

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◇ **EM field is insensitive to the background expansion**

No production of free EM by cosmic expansion

Several proposed mechanisms

▷ Cosmological phase transition — Vachaspati '91 , Enqvist & Olsen '93

▷ Second-order perturbation theory

Matarrese et al. '04, Ichiki et al. '07, Maeda et al. '09, Fenu et al. '11, Saga et al. '15

○ **Inflationary magnetic field production**

Turner & Widrow '88, Ratra '91, Bamba & Yokoyama '04, Martin & Yokoyama '08, Kunze '10, . . .

Most studied model

$$\mathcal{L} = -\frac{I^2(a)}{4} F_{\mu\nu} F^{\mu\nu}$$

Ratra '91

- $\partial_t I \neq 0$ breaks conformal invariance

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$\vec{B}$ field spectrum in this model

$$\frac{d\left\langle B^2 \right\rangle}{d \ln k} \sim H^4 \left(\frac{k}{aH} \right)^{5-2|n-\frac{1}{2}|} , \qquad I \propto a^{-n}$$

- Scale invariance is preferable for extragalactic $\vec{B}$

— Scale-invariance $\Longleftrightarrow$ $n = 2, -3$

Difficulties in large-scale magnetogenesis

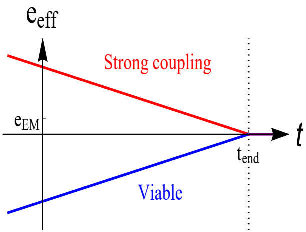
1. Strong coupling problem — Demozzi, Mukhanov & Rubinstein '09

   - $\vec{A} \to I^{-1} \vec{A}$

     $$\mathcal{L} = -\frac{I^2}{4} F^2 + e\, A_\mu J^\mu \to -\frac{1}{4} F^2 + \frac{e}{I} A_\mu J^\mu + \ldots$$

   - Effective coupling $e/I \lesssim 1$ is needed
   - Requires $I \gtrsim 1$ at all times

     $$\Longleftrightarrow \quad n > 0$$



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   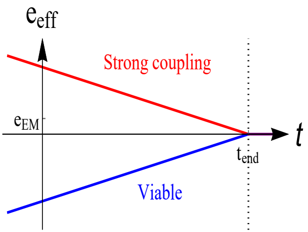

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   $$\iff \quad n > 0$$

2. Strong backreaction problem
   - Large-scale $\vec{B}$ $\Leftrightarrow$ magnetic spectral index $n_B \leq 0$
   - Weak coupling + red-tilted $\vec{B}$ $\Rightarrow$ $\rho_E \gg \rho_B$
   - ρ_E **back-reacts to inflationary dynamics and curvature perturbations !**

   Barnaby et al. '12, Fujita & Mukohyama '12, Fujita & Yokoyama '13, Ferreira et al. '14

Model independent limit

$$\rho_{\rm inf}^{1/4} < 300\,{\rm MeV} \left(\frac{1\,{\rm Mpc}}{L_B}\right)^{5/4} \left(\frac{10^{-15}\,{\rm G}}{B_{\rm obs}}\right) \,, \quad (L_B \leq 1\,{\rm Mpc})$$

Fujita & Yokoyama '14

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Fujita & Yokoyama '14

Premises made

- **Production only during inflation**
- Adiabatic evolution after inflation, $B_{\rm phy} \propto a^{-2}$
- $A_i \sim a^n$ at the last e-folding of inflation

For sufficient production

Must overcome the obstacles

- **Substantial dilution after inflation**
- Too large electromagnetic energy spoiling inflation
- Induced curvature perturbations

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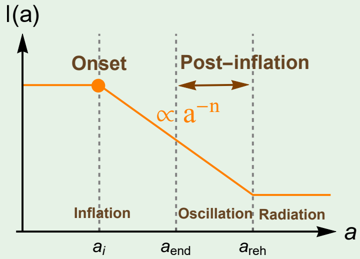
Post-inflationary evolution

One successful scenario

Fujita & RN '16

$$\mathcal{L}_{\rm int} = -\frac{I^2(a)}{4} F_{\mu\nu} F^{\mu\nu}$$

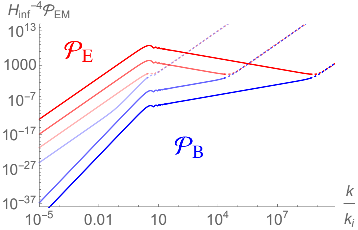


◇ **Sudden onset of $I(a)$ moving** $\Longrightarrow$ suppresses constraints from CMB

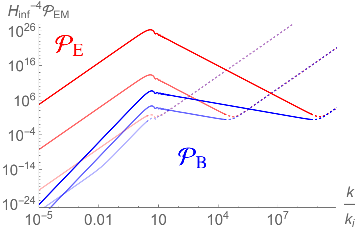
◇ **Post-inflationary coupling** $\Longrightarrow$ suppresses the dilution after inflation

EM power spectra

During inflation

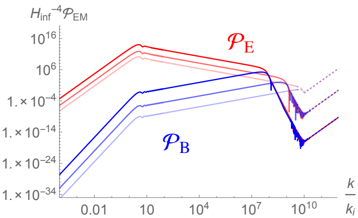


$I \propto a^{-2.5}$



$I \propto a^{-3.5}$

After inflation



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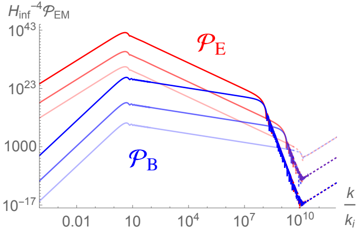

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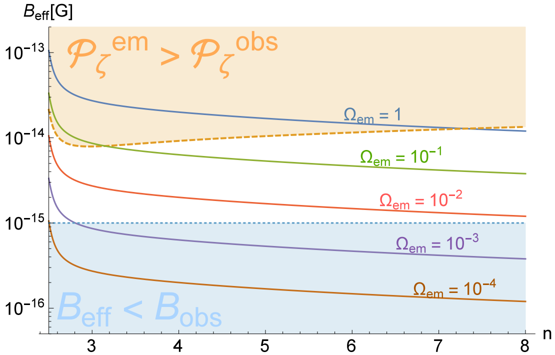
Allowed amplitudes of present $\vec{B}$



$$T_{\rm reh} = 1\,{\rm GeV},\ \ k_i = 1\,{\rm Mpc}^{-1},\ \ \Omega_{\rm em} \equiv \frac{\rho_E}{\rho_{\rm tot}}(t_{\rm reh})$$

Allowed amplitudes of present $\vec{B}$

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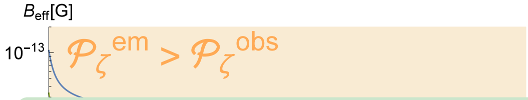

Bottom line

$$B_{\rm obs} \gtrsim 10^{-15}\,{\rm G}$$

can be realized **without**

- **Strong coupling problem**
- **Backreaction problem**
- **Curvature perturbation problem**

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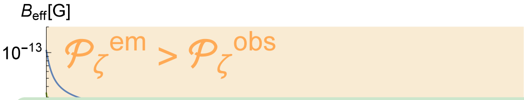

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- **Any natural model ?**

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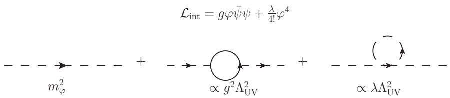
Axion inflation

◇ **Slow roll** of inflaton φ is necessary for inflation

◇ Slow roll is **UV sensitive**

▷ Radiative corrections



▷ η problem, e.g. in supergravity

$$|\eta| \ll 1 \text{ is needed but } V_{\rm SG} \sim V\frac{\varphi^2}{M_p^2} \text{ leads } \eta \sim \mathcal{O}(1)$$

◇ One solution – **shift symmetry**: invariance under $\varphi \to \varphi + c$

▷ Symmetry exact ⇔ completely flat potential $V(\varphi) = \text{const.}$

▷ Mild breaking ⇒ flat $V(\varphi)$ is technically natural

Helical $\vec{B}$ production by axion-gauge interaction

Coupling to EM fields — gauge invariance, shift symmetry, parity

Axion-gauge coupling

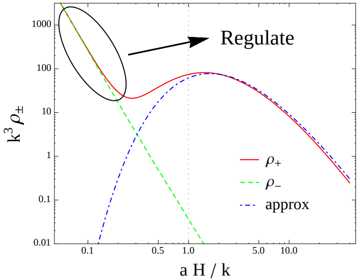
$$\mathcal{L}_{\rm int} = -\frac{\alpha}{4f}\,\varphi\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Modified dispersion of the EM field

$$\frac{\partial^2}{\partial\tau^2}A_\pm + \left(k^2 \mp a\frac{\alpha}{f}k\dot{\varphi}\right)A_\pm = 0$$

◇ **Only one helicity** grows exponentially

Anber & Sorbo '09



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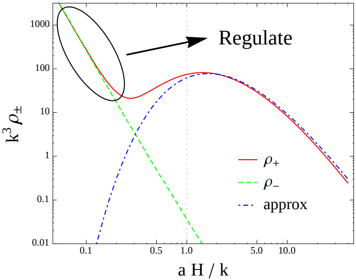
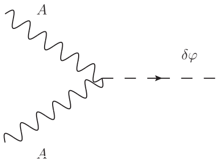


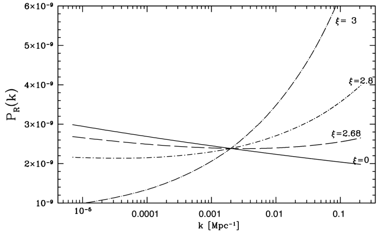
Large production of helical magnetic fields !

CMB bounds on the coupling

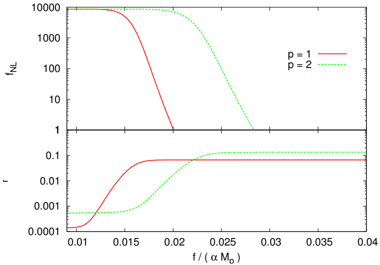
▷ Produced photons **inverse-decay** to inflaton quanta



$\Longrightarrow$ **curvature perturbations ζ**



Meerburg & Pajer '12



Barnaby, RN & Peloso '12

CMB bounds on the coupling

- **Bounds from CMB observations**

CMB bounds

$$\frac{\alpha}{f} \leq 35 - 48 M_p^{-1}$$

Planck collaboration '15

- **Potential signals at terrestrial GW interferometers**
  - ▷ No constraints from current (1^{st} generation) detectors
  - ▷ Future (2^{nd} & 3^{rd} gen.) have potential **to detect helical GWs**

Crowder et al. '12; c.f. Seto & Taruya '07

Evolution of magnetic fields

Gauge field e.o.m.: $\ddot{A}_\pm + H\dot{A}_\pm + \left(\frac{k^2}{a^2} \mp \frac{\alpha}{f}\frac{k}{a}\dot{\phi}_0\right) A_\pm = 0$

Background $\begin{cases} \ddot{\phi}_0 + 3H\dot{\phi}_0 + V_\phi(\phi_0) = \frac{\alpha}{f}\langle \vec{E}\cdot\vec{B}\rangle \\ 3M_p^2 H^2 = \frac{1}{2}\dot{\phi}_0^2 + V(\phi_0) + \frac{\langle \vec{E}^2 + \vec{B}^2\rangle}{2} \end{cases}$

Unique post-inflationary evolution

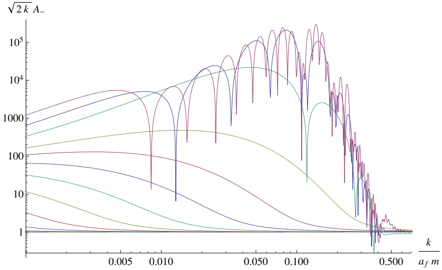
Coupling to inflaton φ until reheating $\quad \mathcal{L}_{\rm int} \sim \varphi F\tilde{F}$

- **Slow roll breaks down** $\quad \dot{\varphi} \nearrow$
- **Inflaton oscillation** after inflation $\quad \dot{\varphi} \sim \cos(m_\phi t)$
- **Helical** $\vec{B}$ fields $\quad B_+ \neq B_-$

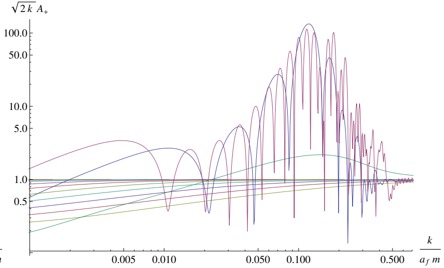
Growth near end of inflation

1. **Tachyonic growth**
   - towards the end of inflation
   - growth only in one helicity state
2. **Parametric resonance**
   - lasts a few e-folds after inflation
   - growth in both helicity states

Spectrum of growing $(-)$ state

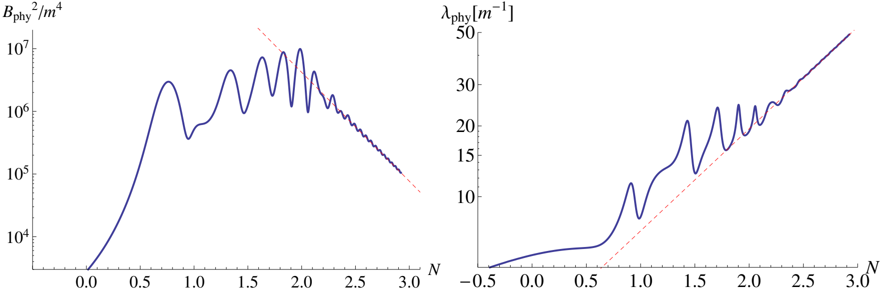


Spectrum of non-growing $(+)$ state



Evolution of amplitude and correlation length

- **Tachyonic growth** and **parametric resonance**
- Once parametric resonance ceases, $\vec{B}$ fields evolve adiabatically



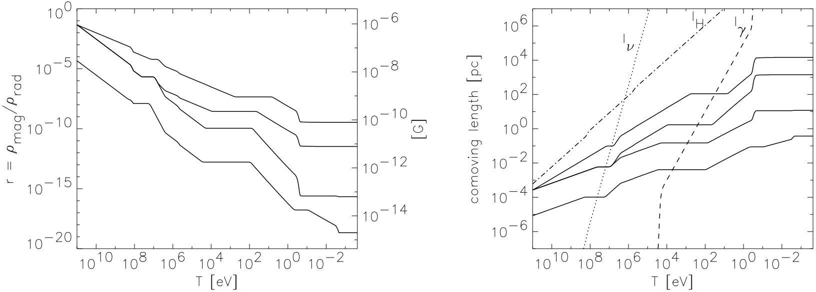
$$B_{\rm phys} \simeq \left(8 \cdot 10^{22}\, a^{-2}\right)\ {\rm G}\ , \quad \lambda_{\rm phys} \simeq \left(9 \cdot 10^{-52}\, a\right)\ {\rm Mpc}\ , \quad \left(\frac{\alpha}{f} = 8\, M_p^{-1}, N \gtrsim 2\right)$$

Evolution after reheating

Magneto-hydro dynamics (MHD)

◇ Magnetic fields in plasma

Banerjee & Jedamzik '04



Presence/absence of helicity $\Longrightarrow$ different evolution

Inverse cascade in turbulent plasma

Inverse cascade $=$ helicity conservation

- **Non-linear evolution** of MHD dynamics
- **Conserved Helicity** of magnetized fluid with high conductivity
- **Partial energy transfer** to larger scales

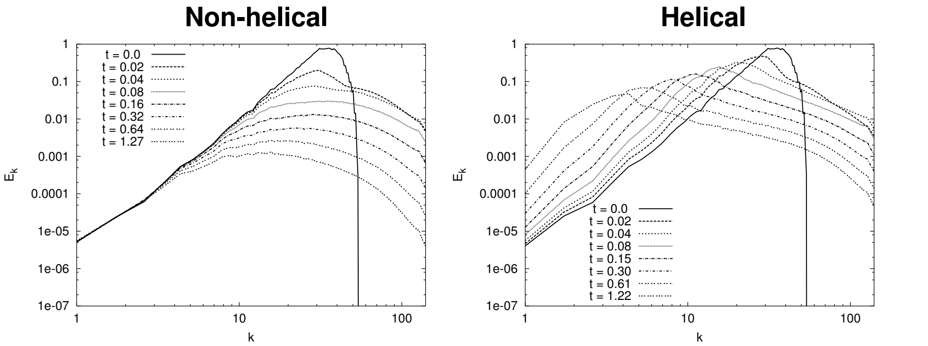
$$\dot{\vec{v}} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} - \left(\vec{v}_A\cdot\vec{\nabla}\right)\vec{v}_A \approx 0$$

$$\dot{\vec{v}}_A + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}_A - \left(\vec{v}_A\cdot\vec{\nabla}\right)\vec{v} \approx 0$$

Alfvén velocity: $\vec{v}_A \equiv \dfrac{\vec{B}}{\sqrt{\rho + P}}$

Helicity: $\mathfrak{h} \equiv \dfrac{1}{V}\displaystyle\int d^3x \left\langle \vec{A}\cdot\vec{B}\right\rangle$

Inverse cascade in turbulent plasma



Banerjee & Jedamzik '04

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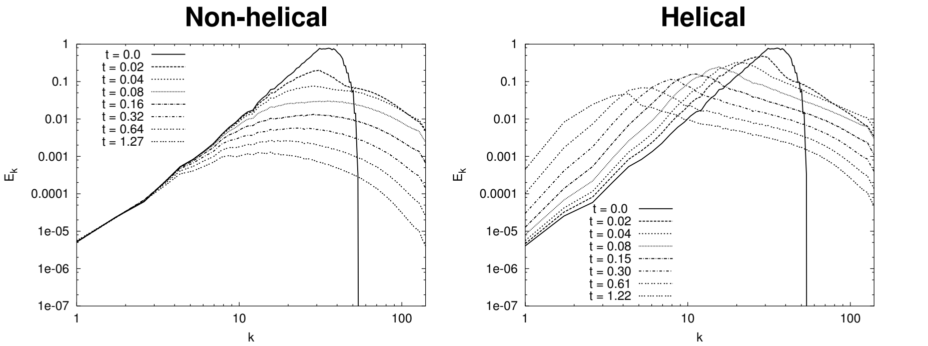


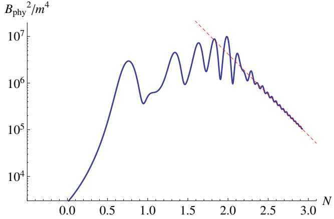
Banerjee & Jedamzik '04

Helicity conservation

$$\mathfrak{h} \propto a^3 \, \lambda_{\rm phy} \, B_{\rm phy}^2 = \text{const.} \qquad (\sigma_c \to \infty)$$

Present $\vec{B}$ amplitude from axion inflation

$$\mathcal{L}_{\rm int} = -\frac{\alpha}{4f}\,\varphi\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$



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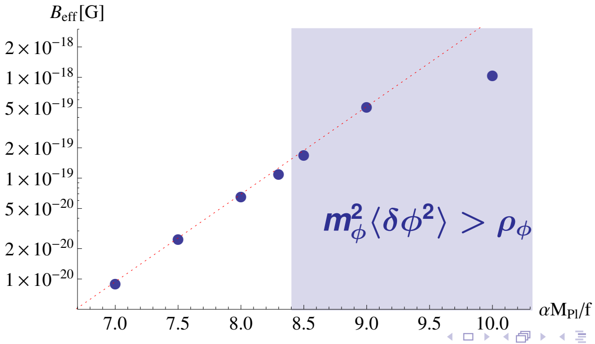

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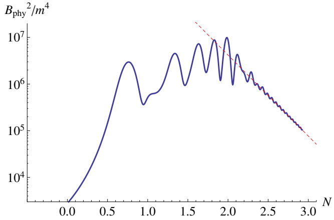

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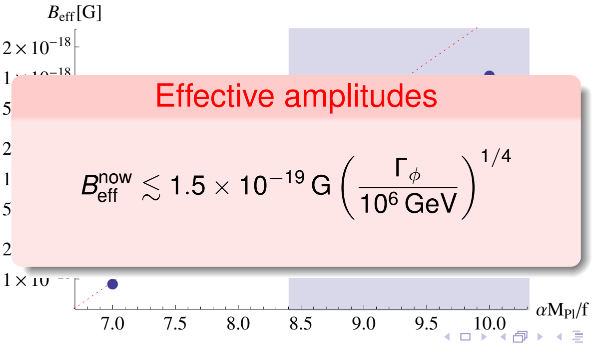

Effective amplitudes

$$B_{\rm eff}^{\rm now} \lesssim 1.5\times10^{-19}\,{\rm G}\left(\frac{\Gamma_\phi}{10^6\,{\rm GeV}}\right)^{1/4}$$

Recent work by Adshead et al. '16

- Lattice simulation in nonlinear regime $\delta\phi > \phi_0 \;\Leftrightarrow\; \alpha/f \in [7, 12]\, M_p^{-1}$

- Results: for $\alpha/f \gtrsim 10\, M_p^{-1}$,

$$B_{\rm eff}\,(t_{\rm now}) \approx 2.5 \cdot 10^{-14}\, {\rm e}^{-3N_{\rm reh}/2}\, {\rm G}$$

$$\textbf{IF}\;\; N_{\rm reh} \approx 1 \;\;\Longrightarrow\;\; B_{\rm peak}(t_{\rm now}) \approx 10^{-13}\, {\rm G}\,, \quad \lambda_{\rm phys} \approx 10\,{\rm pc}$$

- **Sufficient $\vec{B}$ amplitude for blazar observations ?** Reasonable $N_{\rm reh}$?

- Distinguishability: **Helicity of the $\vec{B}$ field ?** Tashiro et al. '13, '14; Chen et al. '14

Outline

Summary and outlook

- Blazars observations $\Rightarrow B_{\rm eff} \gtrsim 10^{-17}$–$10^{-15}$ G at ~ 1 Mpc !
- Challenging to find an **inflation-only** origin
- Post-inflationary evolution is necessary to save the situation
  - Enhancement/production after inflation
  - Magneto-hydro dynamical (MHD) evolution
- Theoretically motivated **axion inflation**
  - Generation mechanism of $\vec{B}$ naturally implemented
  - Rich physics
    - Tachyonic enhancement at the end of inflation
    - Parametric resonance
    - Parity violation $\Rightarrow$ helical $\vec{B} \Rightarrow$ Inverse cascade
  - Maybe sufficient production in nonlinear regime ?

Summary and outlook

- Other possibilities
  - ◇ Non-trivial coupling in matter sector — Tasinato '14
  - ◇ Breaking of gauge invariance — Domènech, Lin & Sasaki '15
  - ◇ Non-trivial coupling to gravity sector — Mukohyama '16

Prospects for future observational constraints