Gravitational lensing of gravitational waves

In collaboration with
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seminar@ICRR, 2012 June 26

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Research : Post-Newton → LIGO Data Analysis → Cosmology → KAGRA

Contents

-1. Announcements

- 1st KAGRA Data Analysis School

@ RESCEU, 9/3(Mon.)-5(Wed.), 2012

  - ✓ Introductory lectures by the Data analysis subgroup
  - ✓ In Japanese (Sorry!)
  - ✓ Travel support,
  - ✓ Data analysis demonstration,
  - ✓ Data analysis practice (octave?),
  - ✓ Social gathering (Most important!)

- 2nd KAGRA Data Analysis School

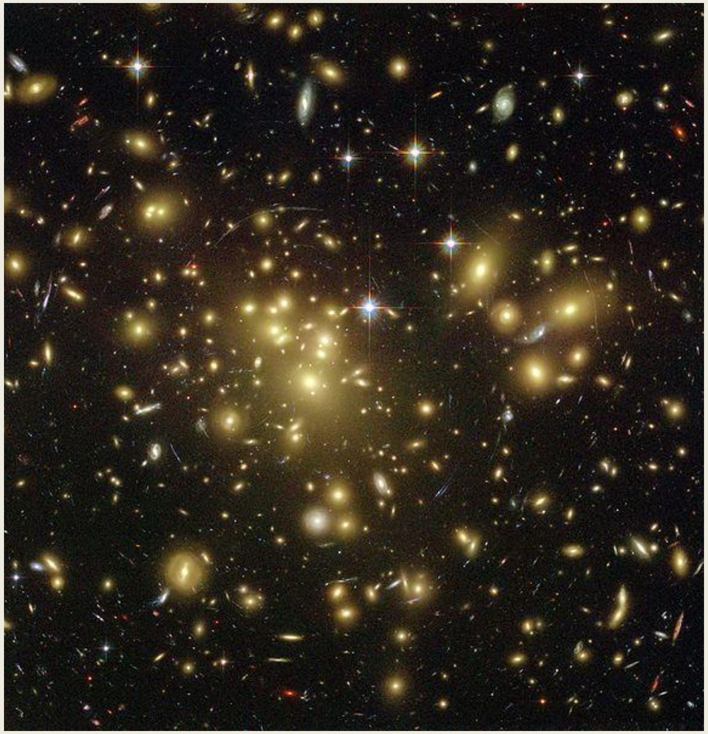
@ NAOJ (Hopefully …)

Part 0:
Short Introduction

Just in case …

- Gravitational lens?

http://spiff.rit.edu/classes/phys230/lectures/planets/Lens_Nav.swf



HST image of Abell 1689

Chandra (pinkish) & Weak lensing (bluish, pseudo-) images of the bullet cluster http://chandra.harvard.edu/photo/2006/1e0657/

Introduction

•GW suffers from GL effect by galaxies and clusters

•GWs are coherent in many cases and we may be able to see interference pattern.

•**Takahashi & Nakamura ApJ 595 1039 (2003)**

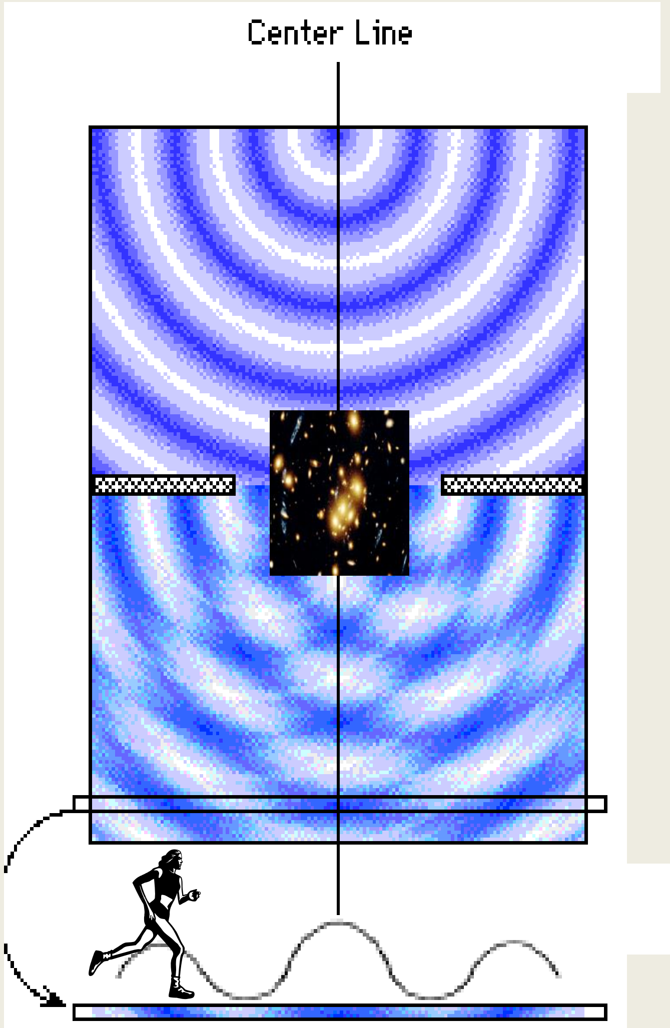
- GL of GWs
- 1e6 ~ 1e9 $M_\odot$ Lens (for the obsolete LISA detector)
- Diffraction effect
- Lens mass & source position relative to optical axis.

•**Any other astrophysical information from GL of GW phenomena?**

Part 1:

Young's (cosmological) double slit experiment

Motion in a spatial interference pattern



$$I(t,x) \;=\; \left|\frac{1}{r_1}e^{i\phi_1} + \frac{1}{r_2}e^{i\phi_2}\right|^2 \simeq 2\frac{1}{r^2}(1+\cos(\phi_1-\phi_2))$$

$$\begin{aligned}\phi_1(t,x) - \phi_2(t,x) &= 2\pi \int_{t_r}^{t_r+t_d(t,x)} f(t')dt'. \\ &= 2\pi f_c t_d(t,x)\end{aligned}$$

$$I(t,x) \sim \cos\left(2\pi\frac{f_c v\theta}{c}t + const.\right) + const.$$

$$\frac{\Delta x}{v} \;=\; 2.5\text{years}\left(\frac{370\text{km/s}}{v}\right)\left(\frac{10''}{\theta}\right)\left(\frac{0.1\text{Hz}}{f}\right)$$

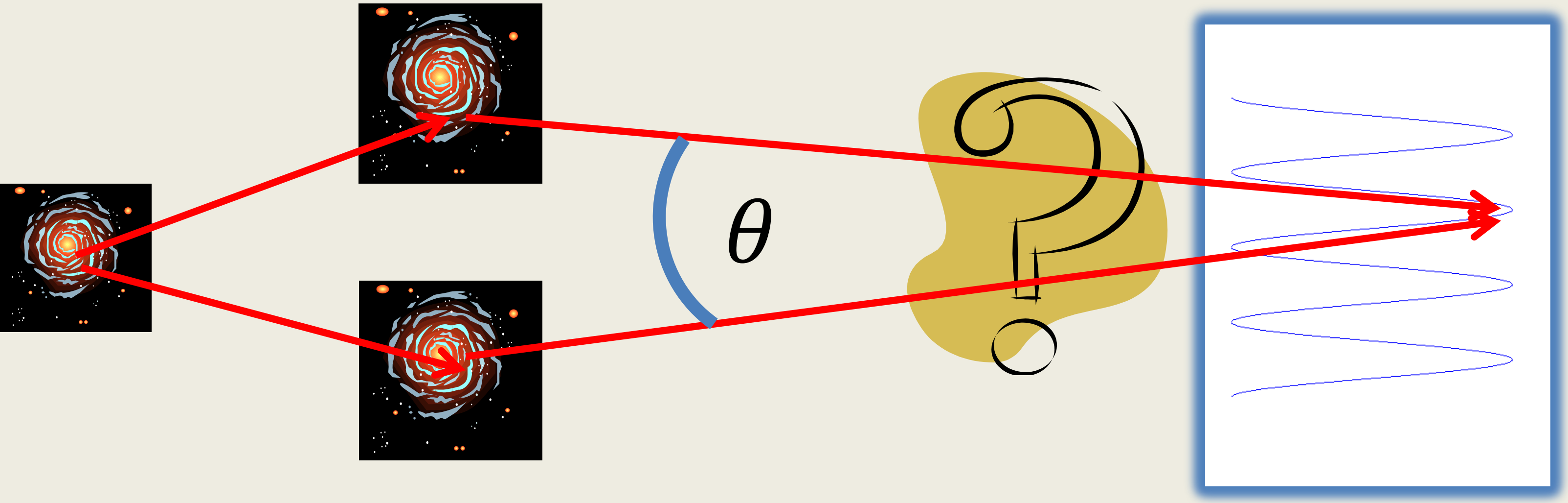
→ Transverse velocity

http://www.bottomlayer.com/bottom/reality/chap2.html

Part 2:

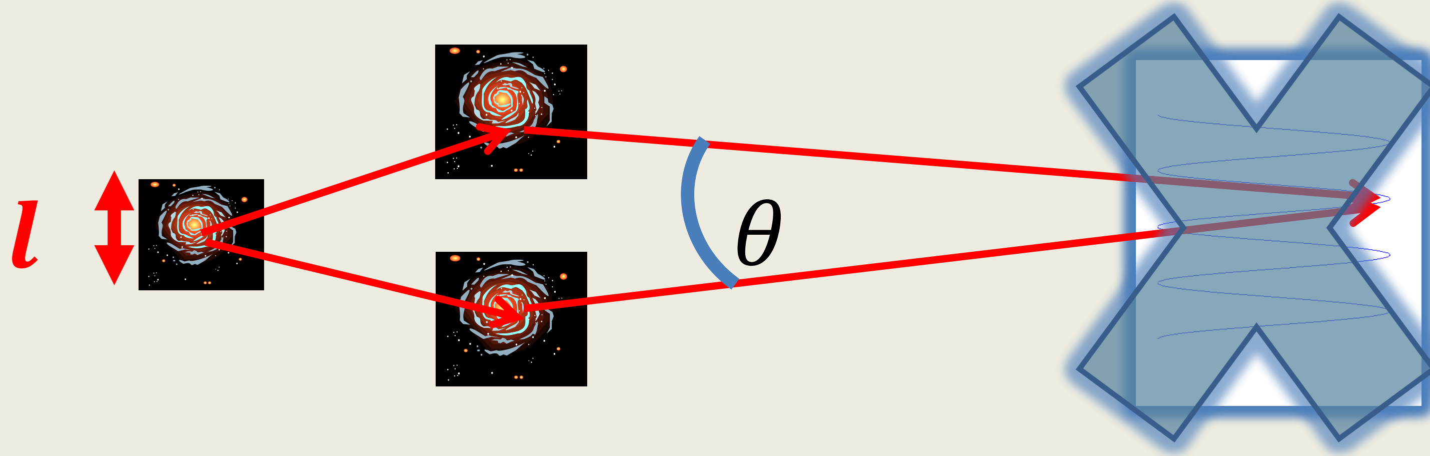
Can we see interference pattern?

Probably no
in Electro Magnetic Astronomy …



- Summation of incoherent emitters (bunch of electrons …)
- interfere distractively
- Can see interference only when …

Probably no
in Electro Magnetic Astronomy …

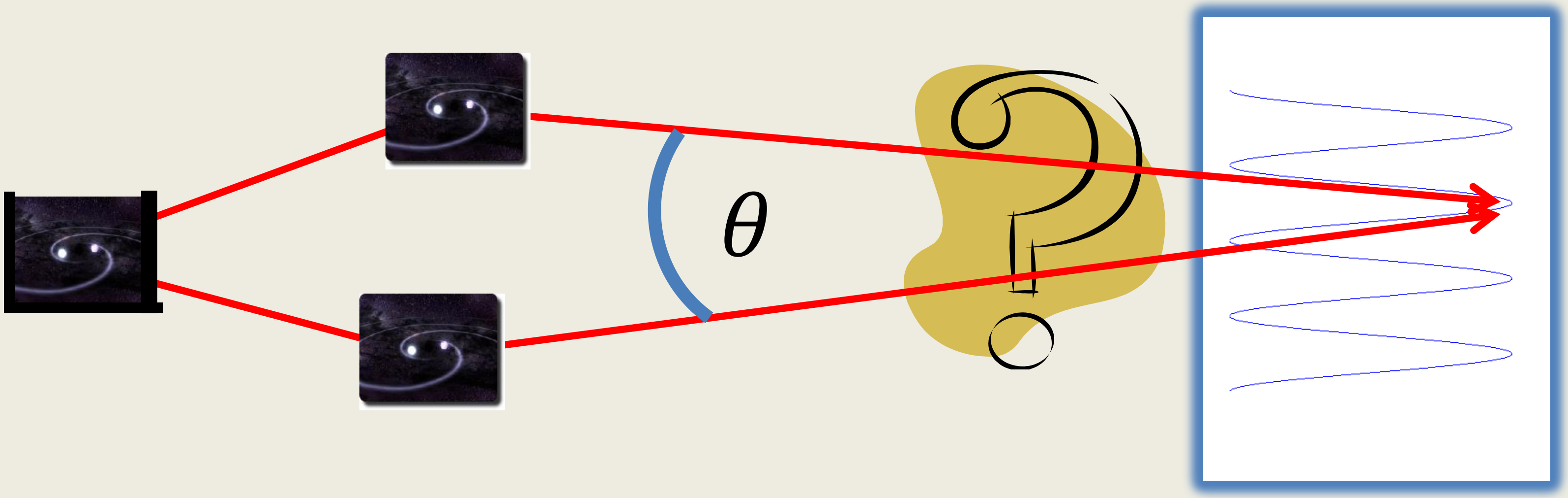


$$l \lesssim \ \kappa\lambda/\theta \equiv \kappa l_0 = 2\kappa \text{ cm } \left(\frac{10''}{\theta}\right)\left(\frac{\lambda}{1\mu m}\right)$$

We can see interference pattern only when the linear dimension of the source (galaxy) is less than ~ 2 cm.

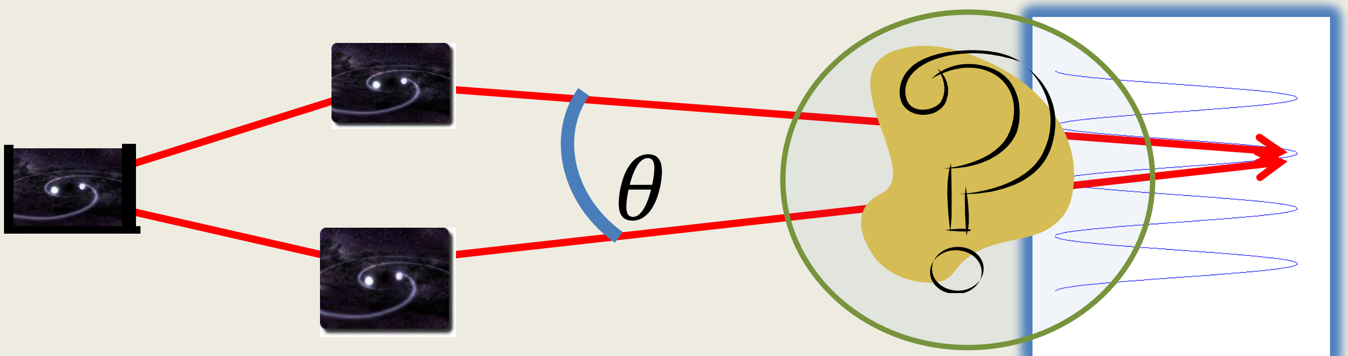
Impossible!!

Probably yes
in Gravitational Wave Astronomy …?



- Coherent emitter (single source if we are lucky (?) …)
- Interfere constructively.
- Can see interference if …

Probably yes
in Gravitational Wave Astronomy …?



$$l = 10^4 \mathrm{km} \left(\frac{2M_{NS}}{2.8M_{\odot}}\right)^{1/3} \left(\frac{0.1\mathrm{Hz}}{f}\right)^{2/3}$$

$$l_0 = 10^2 \kappa\ \mathrm{AU} \left(\frac{10''}{\theta}\right) \left(\frac{0.1\mathrm{Hz}}{f}\right) \gg l$$

Cosmological Young's experiment in GWA possible in reality?

- Pulsars in globular clusters

$$\Delta x \simeq 3 \times 10^{10}\mathrm{km}\left(\frac{1\mathrm{mas}}{\theta}\right)\left(\frac{1\mathrm{kHz}}{f_c}\right) \simeq 200\mathrm{AU}.$$

  - But diffraction obscures the interference pattern when $M_L \lesssim 10^2 M_\odot (f/1\mathrm{kHz})^{-1}$.

$$I(t,x) \sim \cos\left(2\pi\frac{fv\theta}{c}t + \mathrm{const.}\right)$$

- NS/NS binaries

$$\frac{df}{dt} = \frac{96\pi^{8/3}}{5}\left(\frac{GM_{\mathrm{chirp}}}{c^3}\right)^{5/3} f^{11/3} \quad \& \quad \int_0^{T_{\mathrm{obs}}} I(t', vt' + \mathrm{const.})dt'.$$

  - But freq. derivative & SNR complicates the issue.

No (ﾉω･､) in space, but …
Yes (♪ d(⌒o⌒)b ♪) in a computer

- Time delay
  - **Reach the observer separately in time**
  - **Extract the interference pattern in a computer**

- SNR
  - **Integrate to get SNR**

$$\begin{aligned}\int I(t,x)dt &\sim const. + \int \cos\left(2\pi \frac{f_c v\theta}{c} t.\right) dt \\ &\sim v\ independetnt.\end{aligned}$$

  - **Interference disappears**
  - **Filtered output**

$$\begin{aligned}\int I(t,x)\cos(2\pi f_c \Gamma^T t)dt &\sim \int \cos\left(2\pi \frac{f_c v\theta}{c} t.\right)\cos(2\pi f_c \Gamma^T t)dt \\ &\sim \mathrm{sinc}\left(\pi f_c \left(\frac{v\theta}{c} - \Gamma^T\right) T_{obs}\right)\end{aligned}$$

  - **Chirp**
    - **Intrinsic frequency (time-)derivative**

Part 3:

Gravitational lensing of gravitational waves

Lensed GW from inspiraling binary

No relative motion geometrical optics approximation (Takahashi & Nakamura 2003)

$$h_\alpha^L(f) = \frac{\sqrt{3}}{2}\sum_j |\mu_j|^{1/2}\Lambda_{\alpha,j}(t_j)e^{-i\pi n_j - i\Phi_{\alpha,j}(t_j)}Af^{-7/6}e^{i\Psi_j(f)},$$

Dopper shift in mass and time due to relative motion

$$M_{cz,j} \to \gamma_j M_{cz}, t_{c,jk} \to \gamma_k t_{c,j},$$

$$\gamma_j = \left(1 + \vec{N}_j \cdot \frac{\vec{v}}{c}\right)$$

Γ is dopper shift difference between the two images

$$\begin{aligned}\Gamma &= \frac{\gamma_1}{\gamma_2} = 1 + (\vec{N}_1 - \vec{N}_2)\cdot\vec{\beta} + O(\beta^2). \\ &\simeq 1 + 1.8\times 10^{-7}\left(\frac{\theta}{10''}\right)\left(\frac{v}{370\mathrm{km/s}}\right),\end{aligned}$$

If you like equations (copy from my paper).

Detector index

GL amplification

Half integer: saddle = ½ etc

$$h^L_\alpha(f) = \frac{\sqrt{3}}{2}\sum_j |\mu_j|^{1/2}\Lambda_{\alpha,j}(t_j)e^{-i\pi n_j - i\Phi_{\alpha,j}(t_j)}Af^{-7/6}e^{i\Psi_j(f)},$$

$$\boxed{\Lambda_{\alpha,j}(t) = \left((1+(\vec{L}\cdot\vec{N}_j)^2)^2F_\alpha^{+}(t) + 4(\vec{L}\cdot\vec{N}_j)^2F_\alpha^{\times}(t)\right)^{1/2}}$$

Amplitude of the detector beam pattern function

$$\boxed{\Phi_{\alpha,j}(t) = \phi_{D,j}(t) + \phi_{p,j,\alpha}(t)}$$

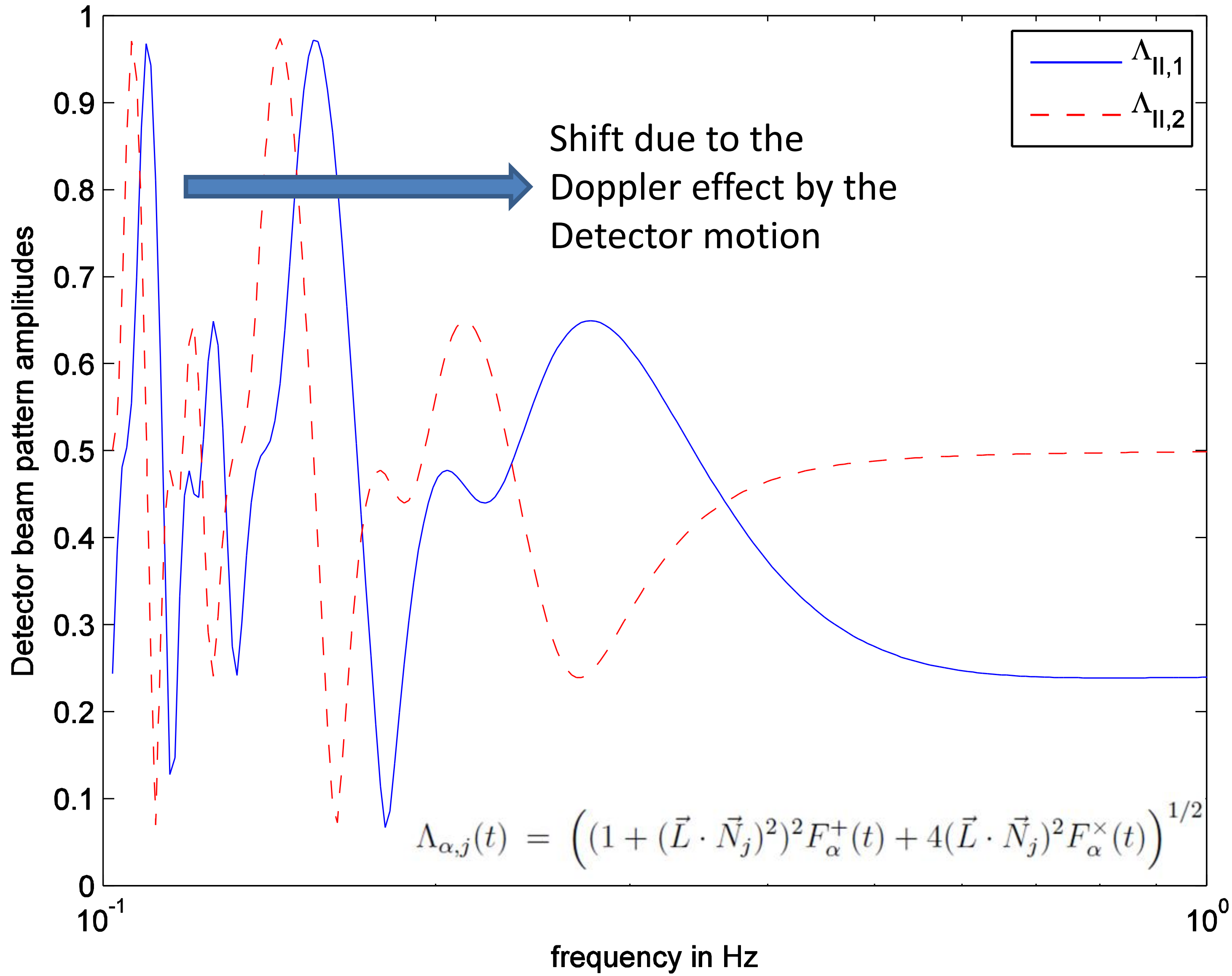
$$\phi_{p,j,\alpha}(t) = \tan^{-1}\left(\frac{2(\vec{L}\cdot\vec{N}_j)F_\alpha^{\times}(t)}{(1+(\vec{L}\cdot\vec{N}_j)^2)F_\alpha^{+}(t)}\right)$$

Phase of the detector beam pattern function.

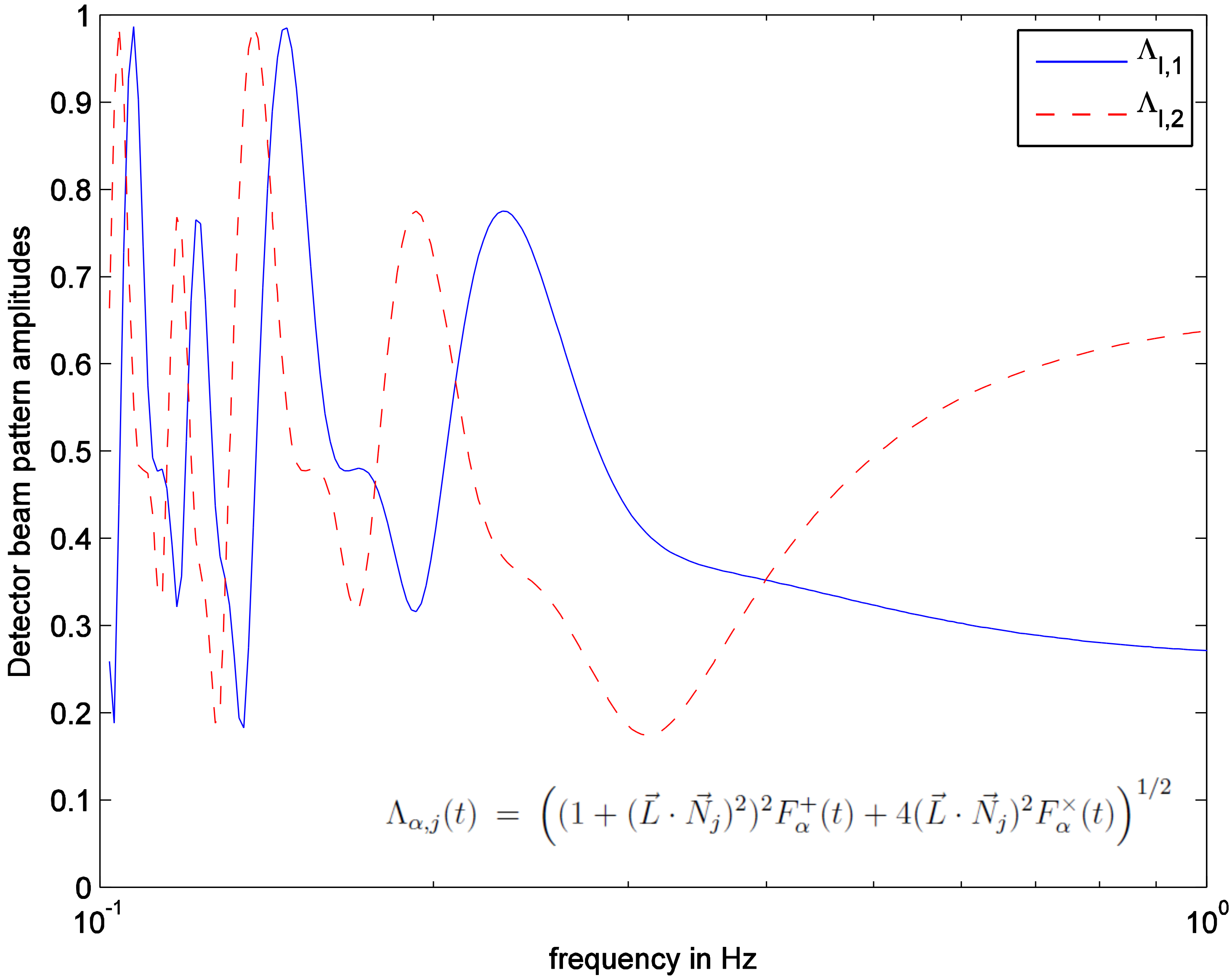
$$\phi_{D,j}(t) = 2\pi f(t)R\sin\bar{\theta}_{S,j}\cos\left(\bar{\phi}(t) - \bar{\phi}_{S,j}\right)$$

Doppler phase due to the DECIGO orbital motion.

Beam pattern function (II)



Beam pattern function (I)



Do you still like equations?

Image index

time variable

GL amplitude

$$h^L_\alpha(f) = \frac{\sqrt{3}}{2}\sum_j |\mu_j|^{1/2}\Lambda_{\alpha,j}(t_j)e^{-i\pi n_j - i\Phi_{\alpha,j}(t_j)}Af^{-7/6}e^{i\Psi_j(f)},$$

GL amplitude

$$\boxed{A = \frac{1}{D_S(1+z_S)^2}\left(\frac{5}{96}\right)^{1/2}\pi^{-2/3}M_{cz}^{5/6}}$$

$$\boxed{t_j(f) = t(f) + t_{d,j} = t_{c,j} - 5(8\pi f)^{-8/3}M_{cz}^{-5/3}}$$

$$t(f) = = t_c - 5(8\pi f)^{-8/3}M_{cz}^{-5/3}$$

$$\boxed{\Psi_j(f) = 2\pi f t_{d,j} + \Psi(f) = 2\pi f t_{c,j} - \phi_c - \frac{\pi}{4} + \frac{3}{4}(8\pi M_{cz} f)^{-5/3}}$$

We would like to determine for the following parameters (in case of no relative motion.)

$$\{M_{cz}, \phi_c, t_c, D_S, \bar{\theta}_{S,j}, \bar{\phi}_{S,j}, \vec{L}, n_1 - n_2, \Delta t_d = t_{d,1} - t_{d,2}, |\mu_1|/|\mu_2|\}$$

1. Chirp mass
2. Phase of coalescence
3. Time of coalescence
4. Luminosity distance to the source
5. Source direction
6. Binary orbital plane inclination
7. Lens image relative parity
8. Time delay
9. Relative magnification

Geometrical Optics Approx. (GOA) $|f\Delta t_d| >> 1$

$$h^L_\alpha(f) = \frac{\sqrt{3}}{2}\sum_j |\mu_j|^{1/2}\Lambda_{\alpha,j}(t_j)e^{2\pi i f t_{d,j} - i\pi n_j} e^{-i\Phi_{\alpha,j}(t_j)} A f^{-7/6} e^{i\Psi(f)}.$$

$$h^T_\alpha(f) = \frac{\sqrt{3}}{2}\Lambda^T_\alpha(t^T)e^{-i\Phi^T_\alpha(t^T)} A^T f^{-7/6} e^{i\Psi^T(f)}.$$

Lens

Template

$$\rho^2_\alpha = 4Re\int_{f_i}^{f_e} \frac{h^L_\alpha(f) h^{T*}_\alpha(f)}{S_h(f)} df$$

Geometrical Optics Approx. (GOA) $|f\Delta t_d| >> 1$

$$h_\alpha^L(f) = \frac{\sqrt{3}}{2}\sum_j |\mu_j|^{1/2}\Lambda_{\alpha,j}(t_j)e^{2\pi i f t_{d,j} - i\pi n_j}e^{-i\Phi_{\alpha,j}(t_j)}Af^{-7/6}e^{i\Psi(f)}.$$

$$h_\alpha^T(f) = \frac{\sqrt{3}}{2}\Lambda_\alpha^T(t^T)e^{-i\Phi_\alpha^T(t^T)}A^T f^{-7/6}e^{i\Psi^T(f)}.$$

Lens

Template

$|f\Delta t_d| >> 1$

$$\rho_\alpha^2 = \sum_j \rho_{\alpha,j}^2,$$

$$\rho_{\alpha,j}^2 = 3|\mu_j|A^2 Re\int_{f_i}^{f_e}\frac{f^{-7/3}}{S_h(f)}\Lambda_{\alpha,j}^2(t_j)df,$$

We can separately detect the two images.

To extract interference in GOA (no noise).

$$
\begin{aligned}
s_A(t) &= \sum_j h_j(t) \ \text{ for } \ t_{e,1} - \Delta t_d \le t \le t_{e,1}, \\
s_B(t) &= h_2(t) \ \text{ for } \ t_{e,1} \le t \le t_{e,2}.
\end{aligned}
$$

Even without detector noise

$$
4Re \int_{f_s}^{f_e} \frac{s_1(f) s_2^*(f)}{S_h(f)} df \ = \ 4Re \sum_\alpha \int_{f_s}^{f_e} \frac{h^L_{\alpha,1}(f) h^{L*}_{\beta,2}(f)}{S_h(f)} df = 0
$$

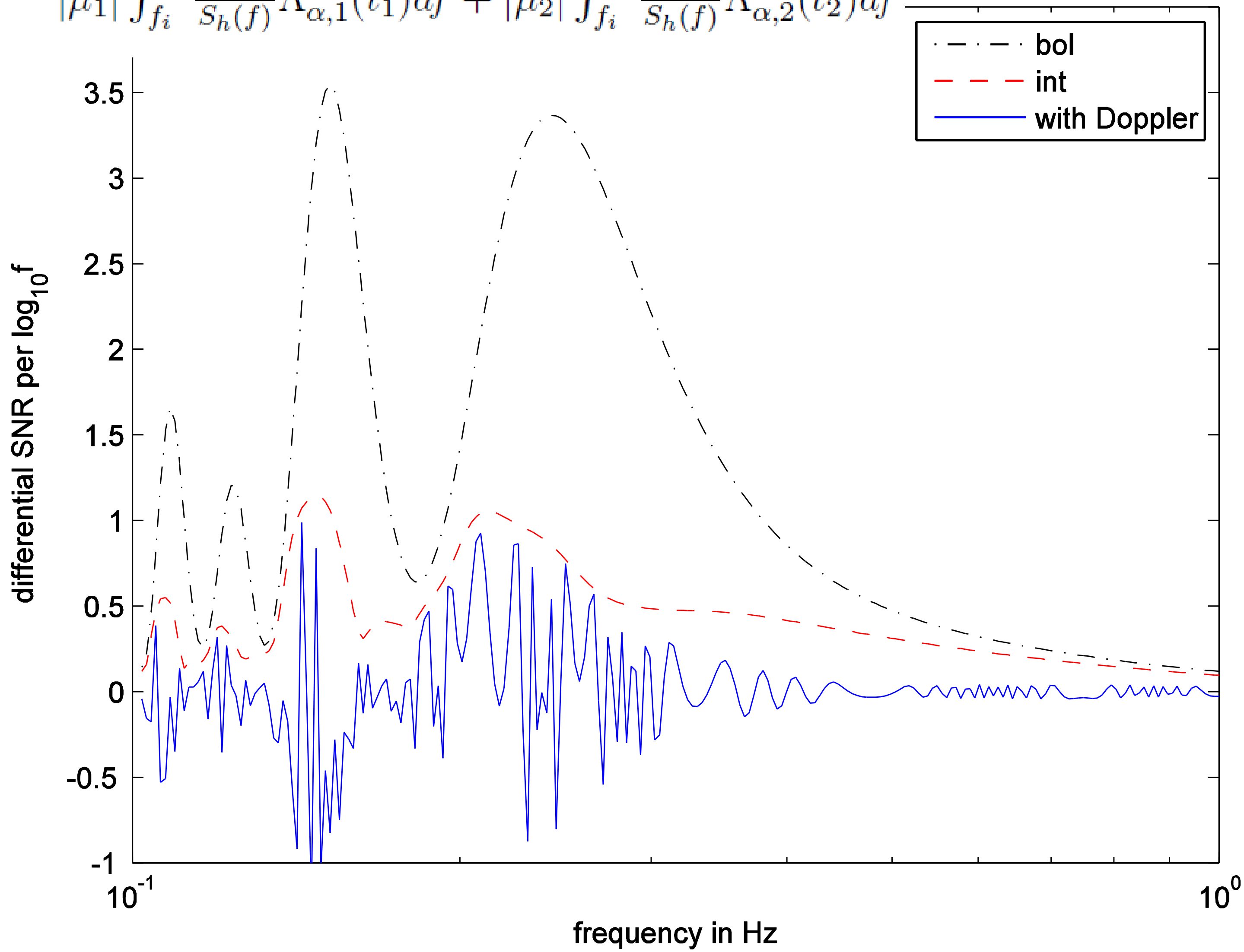
In stead,

$$
\Theta(\mathbf{p}) \equiv \Phi_{\alpha,j}(t_2) - \Phi_{\alpha,j}(t_1) + \pi(n_1 - n_2)
$$

$$
\begin{aligned}
\zeta_\alpha &= 4Re \int_{f_s}^{f_e} \frac{h^L_{\alpha,1}(f) h^{L*}_{\alpha,2}(f)}{S_h(f)} \cos(2\pi f \Delta t_d^T + \Theta(f, \mathbf{p}^T)) df \\
&\simeq 3|\mu_1|^{1/2} |\mu_2|^{1/2} A^2 \\
&\times \int_{f_s}^{f_e} \frac{f^{-7/3}}{S_h(f)} \Lambda_{\alpha,1}(t_1) \Lambda_{\alpha,2}(t_2) \cos(2\pi f (\Delta t_d - \Delta t_d^T) + \Theta(f, \mathbf{p}) - \Theta(f, \mathbf{p}^T)) df.
\end{aligned}
$$

Differential SNR

$$R^2 = \frac{\rho^2_{\alpha,int}}{\rho^2_{\alpha,bol}} = \frac{|\mu_1\mu_2|^{1/2}\int_{f_i}^{f_e}\frac{f^{-7/3}}{S_h(f)}\Lambda_{\alpha,1}(t_1)\Lambda_{\alpha,2}(t_2)df}{|\mu_1|\int_{f_i}^{f_e}\frac{f^{-7/3}}{S_h(f)}\Lambda^2_{\alpha,1}(t_1)df + |\mu_2|\int_{f_i}^{f_e}\frac{f^{-7/3}}{S_h(f)}\Lambda^2_{\alpha,2}(t_2)df}$$



Transverse velocity

$$\vec{v} = \vec{v}_o + \frac{1+z_L}{1+z_S}\frac{D_L}{D_{LS}}\vec{v}_s - \frac{D_{LS}+D_L}{D_{LS}}\vec{v}_L,$$

- When relative transverse velocity is not zero,
  - Doppler shift to frequency, unobservable *a priori.*
  - Doppler shift to the Masses and time variables

  Unobservable *a priori*.
  - Time variation in the source direction

  Unobservable for astronomical sources.
  - But relative Doppler factor is observable

$$\Gamma = \frac{\gamma_1}{\gamma_2} = 1 + (\vec{N}_1 - \vec{N}_2)\cdot\vec{\beta} + O(\beta^2) \simeq \theta\beta_\perp$$

For those who like equations (if any)…

Dopper factor

$$\gamma_j = (1 + \vec{N}_j \cdot \vec{\beta})$$

$$\tilde{M}_{cz,j} = \gamma_j M_{cz},$$

$$\tilde{t}_{c,jk} = \gamma_k t_{c,j},$$

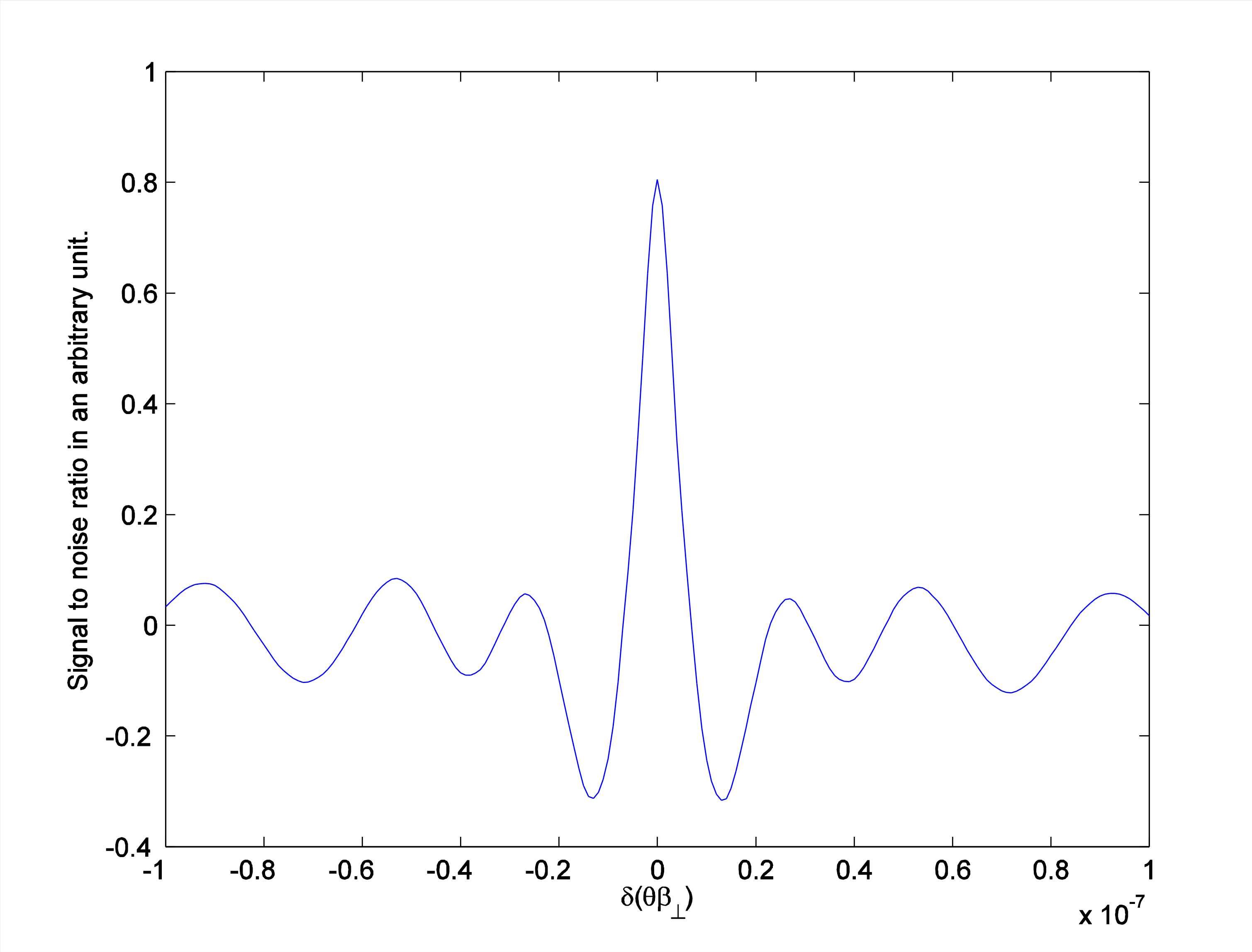
$$\tilde{A}_j = \gamma_j^{-7/6} A$$

$$\tilde{t}_{jk}(f) = \tilde{t}_{c,jk} - 5(8\pi f)^{-8/3} \tilde{M}_{cz,k}^{-5/3},$$

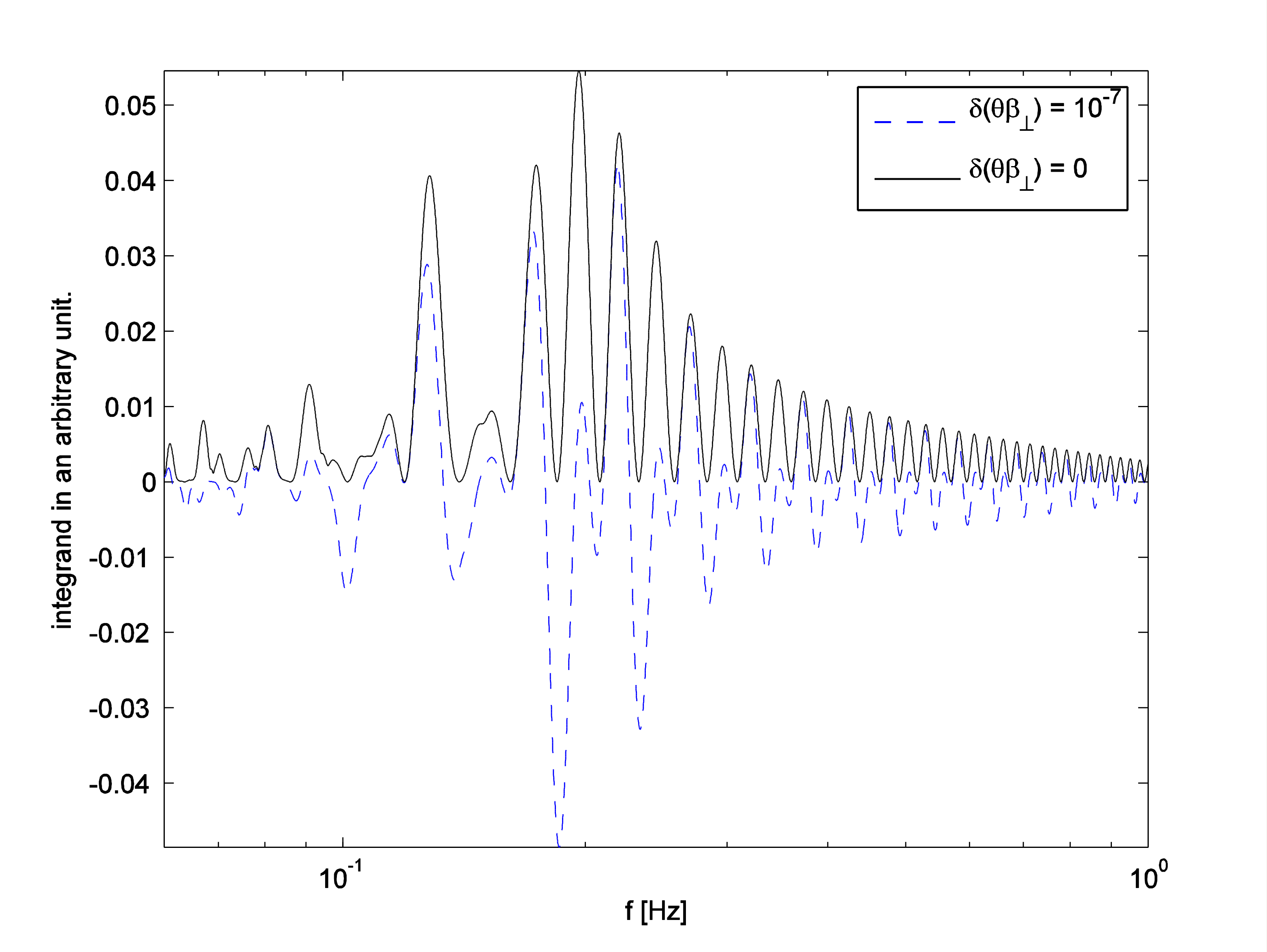
$$\tilde{h}^L_{\alpha,j}(f) = \frac{\sqrt{3}}{2} |\mu_j|^{1/2} \Lambda_{\alpha,j}(\tilde{t}_{jj}) e^{-i\pi n_j} e^{-i\Phi_{\alpha,j}(\tilde{t}_{jj})} \tilde{A}_j f^{-7/6} e^{i\tilde{\Psi}_{jj}(f)},$$

$$\boxed{\tilde{\Psi}_{jk}(f) = 2\pi f \tilde{t}_{c,jk} - \phi_c - \frac{\pi}{4} + \frac{3}{4}\left(8\pi \tilde{M}_{cz,k} f\right)^{-5/3}}$$

SNR as a function of template mismatch in Γ



Differential SNR w or w/o mismatch in Γ



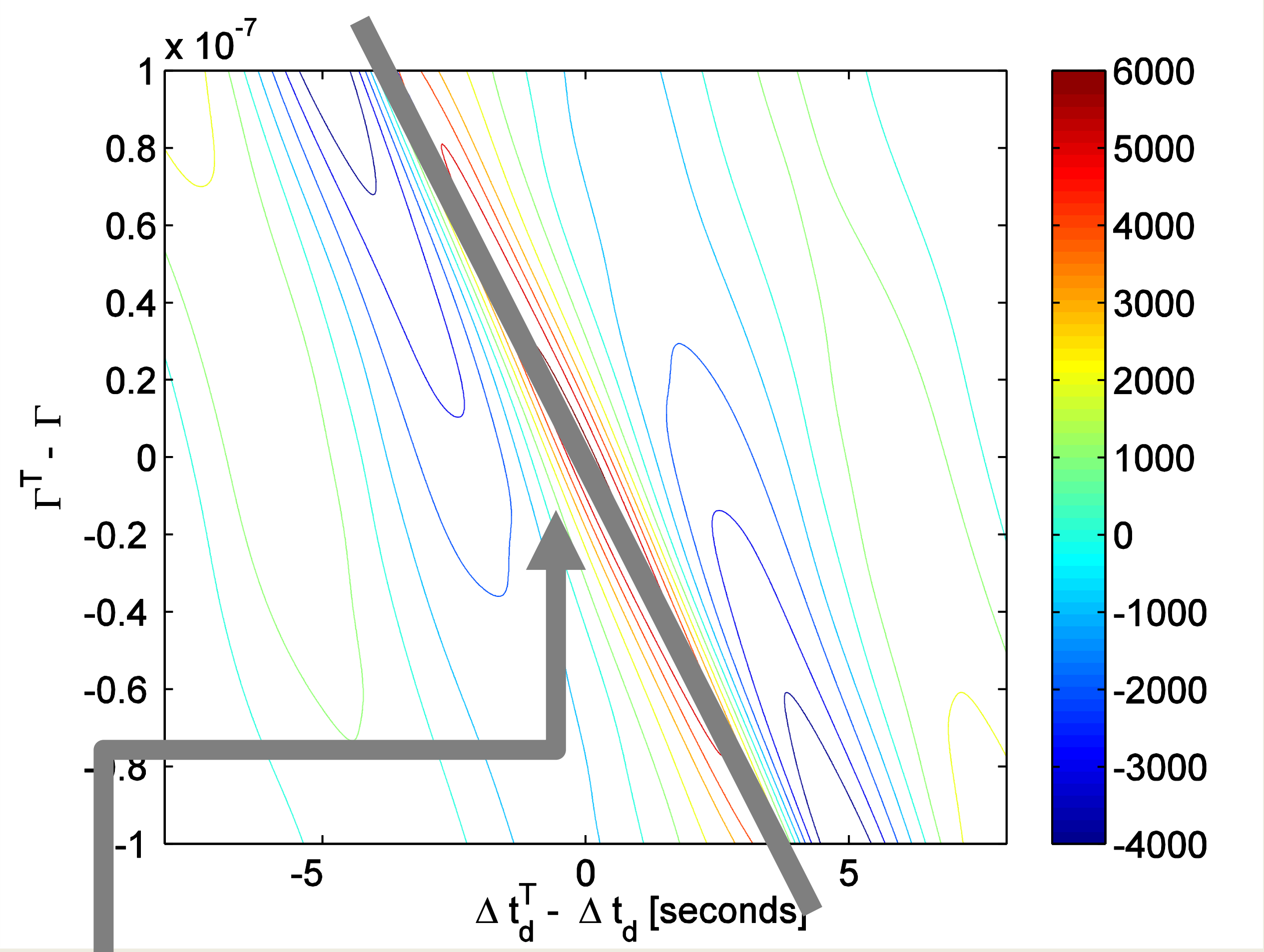
Example parameters (Q0957+561)

f_s [Hz]	f_e [Hz]	T_{obs} [yrs]	Δt_d [yrs]	v [km/s]	θ [$''$]	$\Gamma - 1$
0.113	1	1.137	1.14	480	6.26	4.9×10^{-8}

μ_1, μ_2	t_c [yrs]	m_1, m_2 [$M_\odot$]	θ_L [rad]	ϕ_L [rad]	θ_S [rad]	ϕ_S [rad]
1.5, 1	1.14	1.4, 1.4	1.09	2.90	2.62	0.99

ϕ_0 [rad]	ϕ_C [rad]	α_0 [rad]	z_S	z_L
2.64	2.55	2.06	1.41	0.36

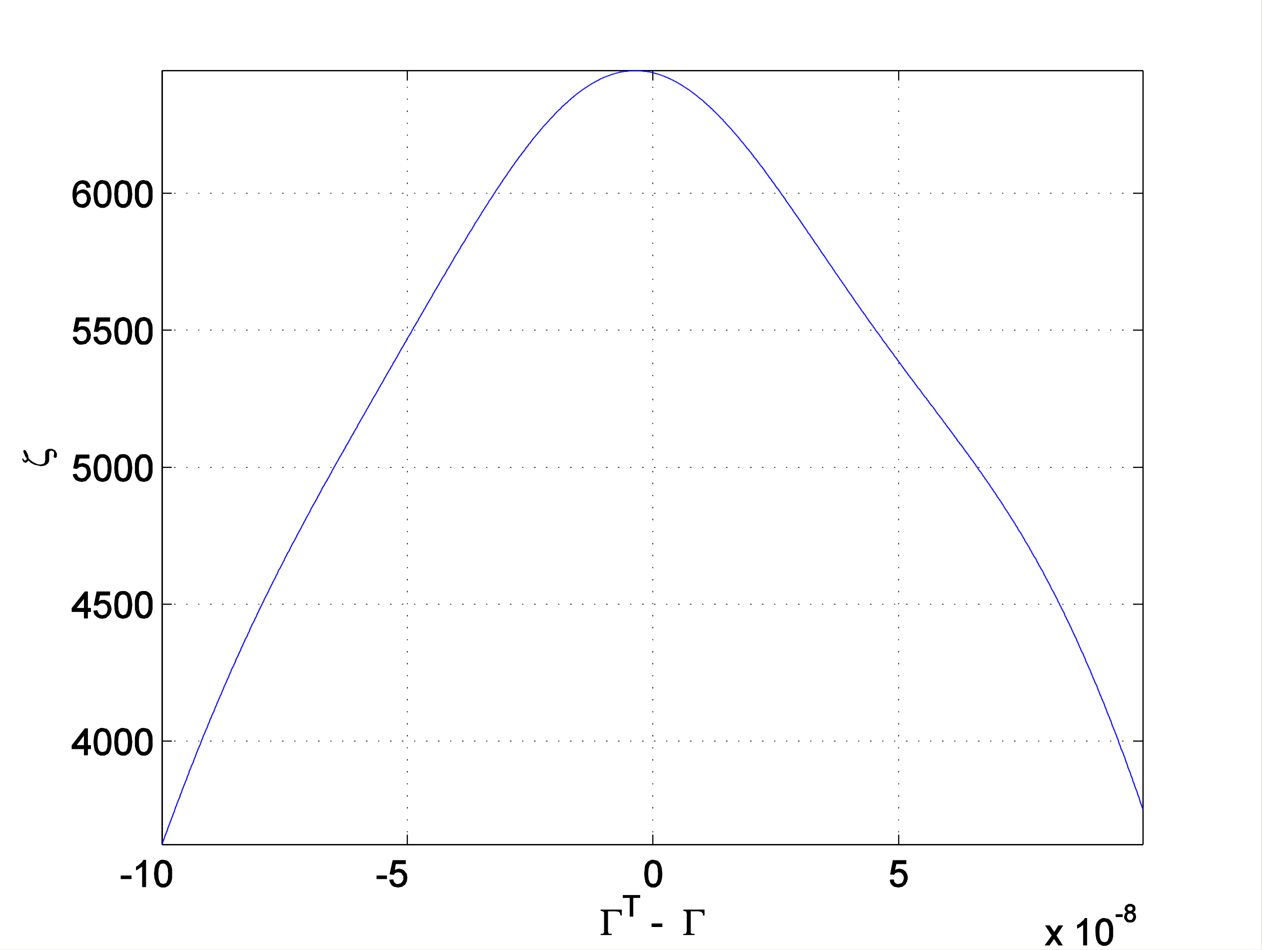
Correlation between $\theta\beta$ and time delay



$$\Delta t_d^T - \Delta t_d = -(\Gamma^T - \Gamma)T_{\rm obs}$$

Along the "ridge"

$$\Delta t_d^T - \Delta t_d \;=\; -(\Gamma^T - \Gamma)T_{\rm obs}$$



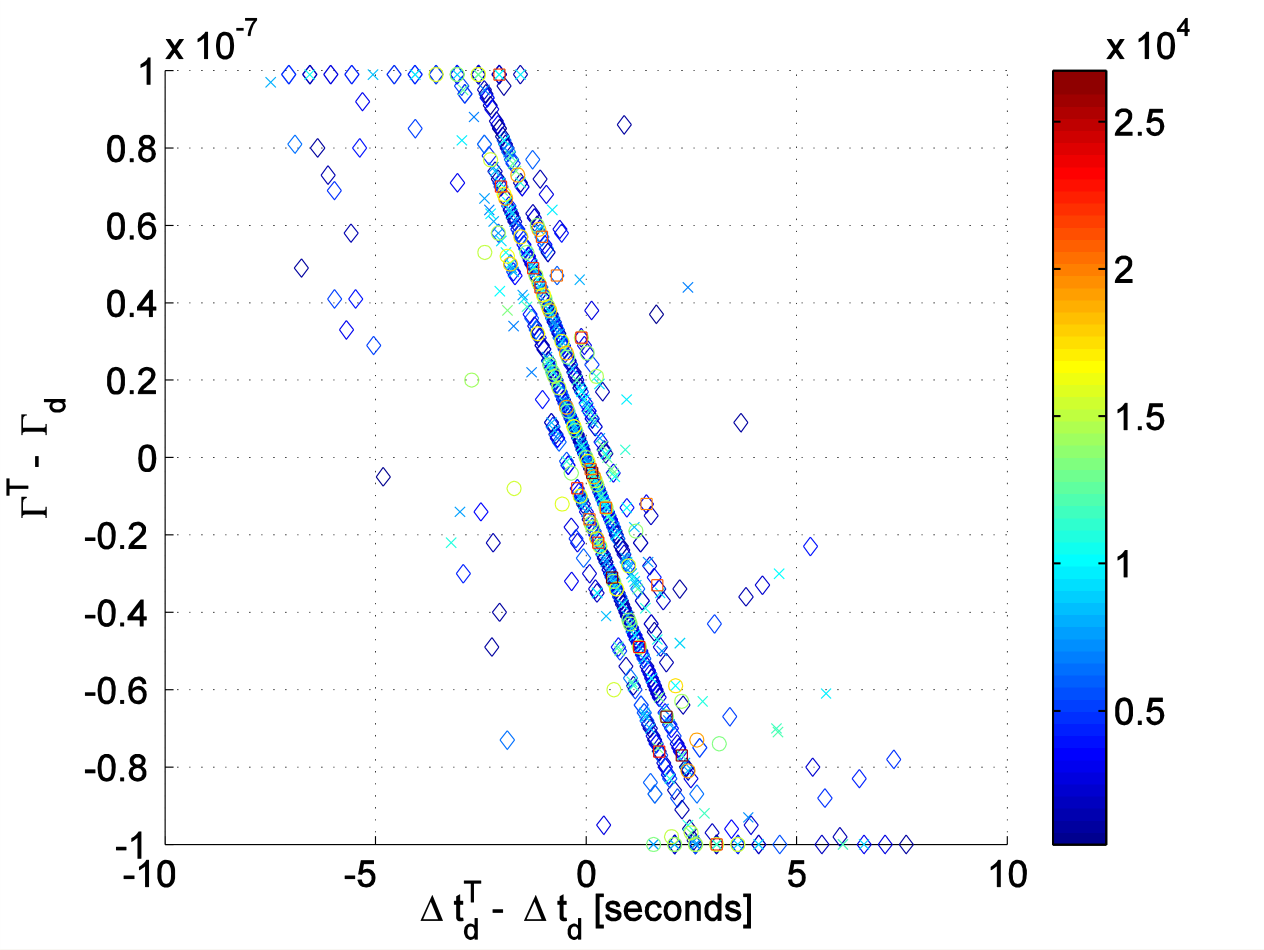
Part 4:

Real world

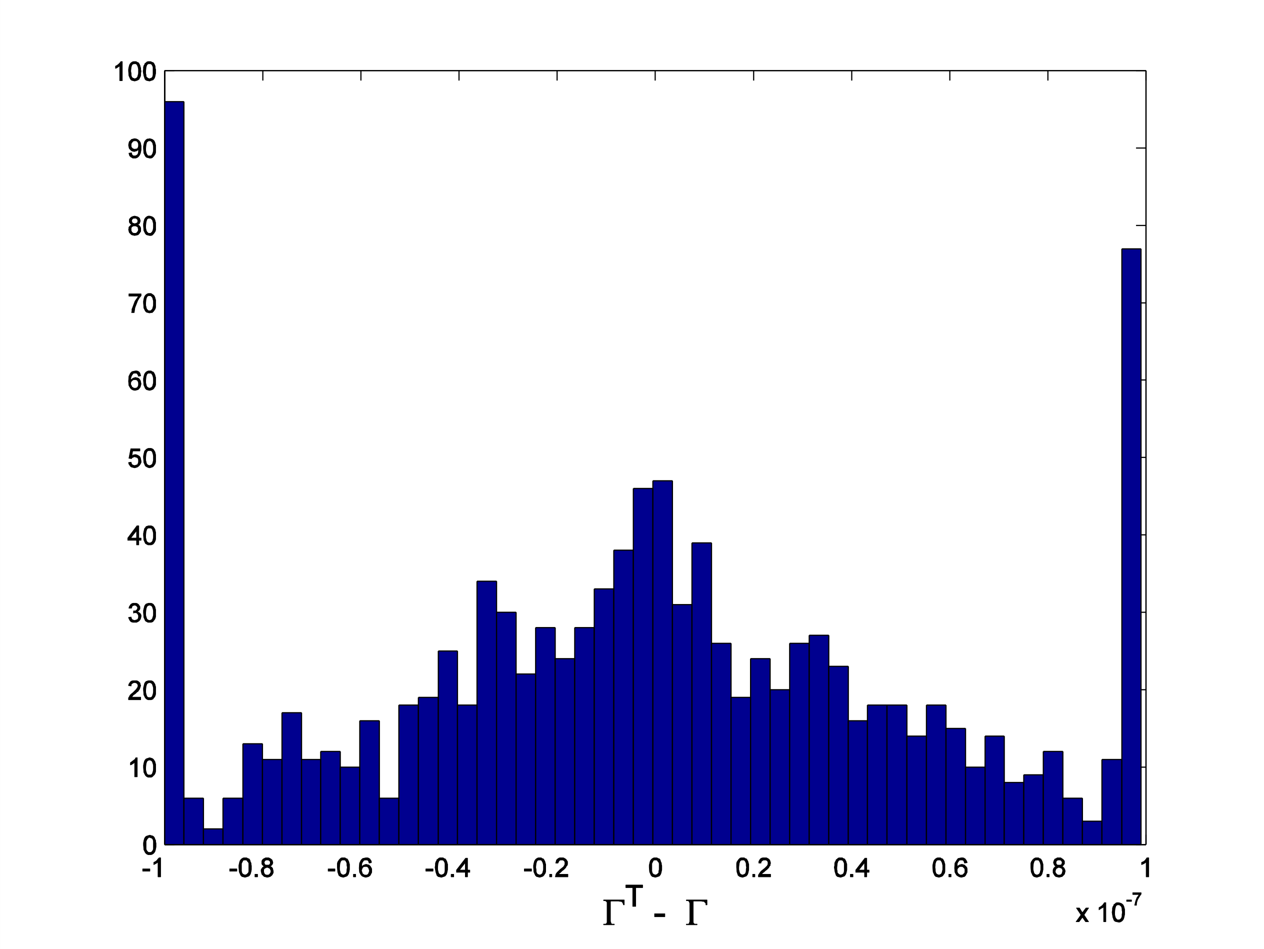
Noises

1. Parameters estimation noises
2. Detector noises

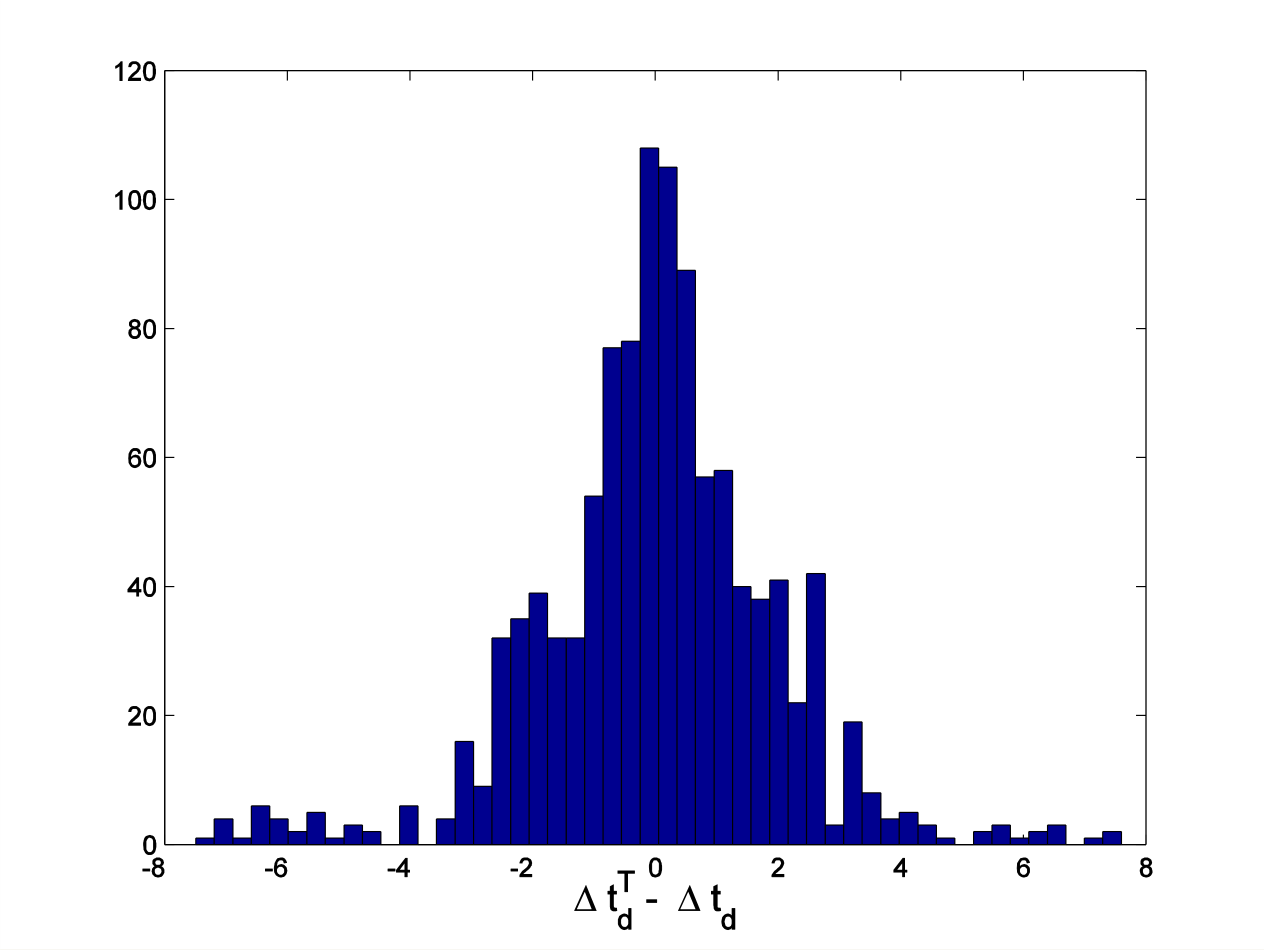
Effect of nuisance parameters estimation errors (td = 1.14 years)



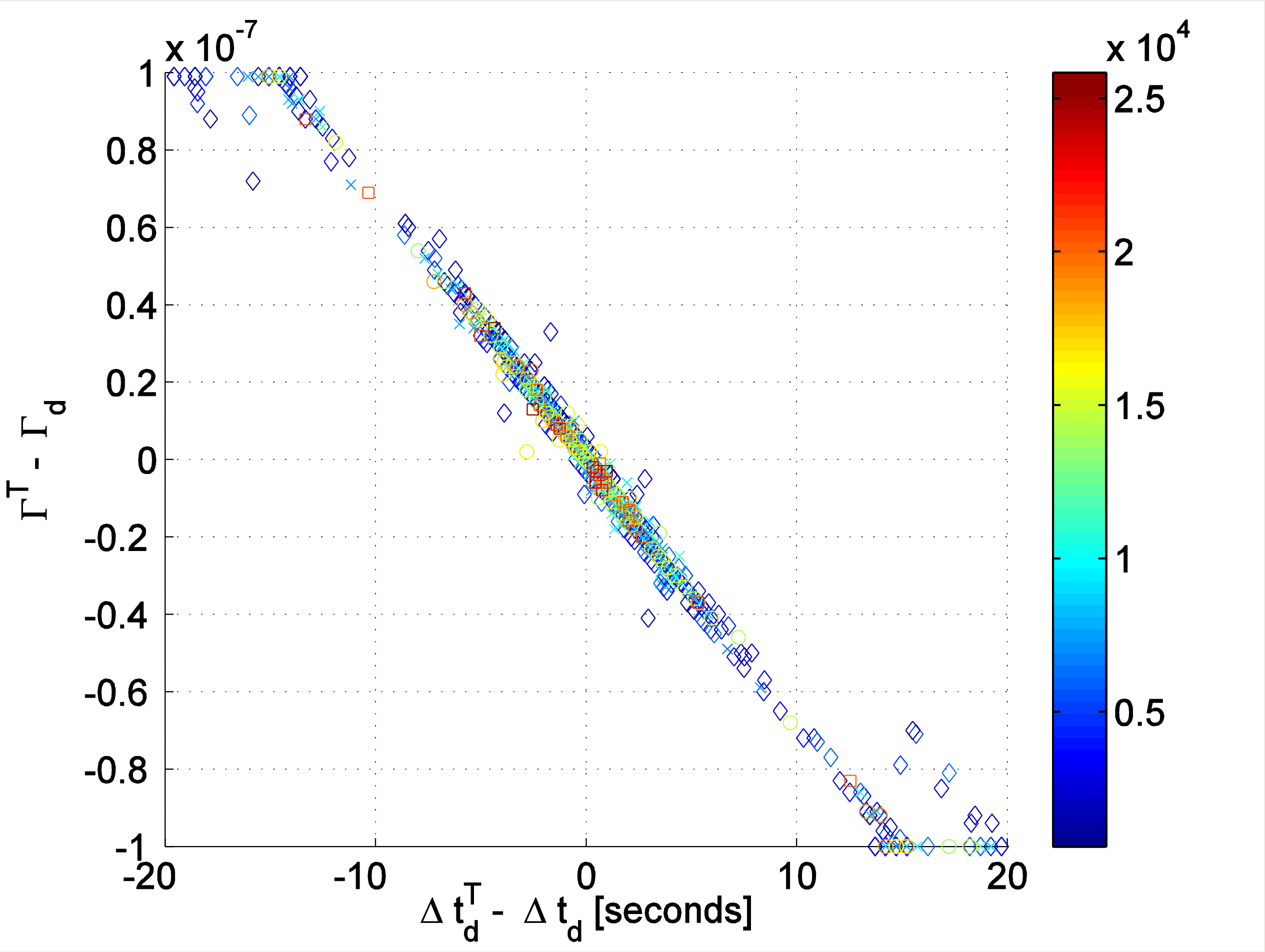
Effect of nuisance parameters estimation errors to Γ (td = 1.14 years)



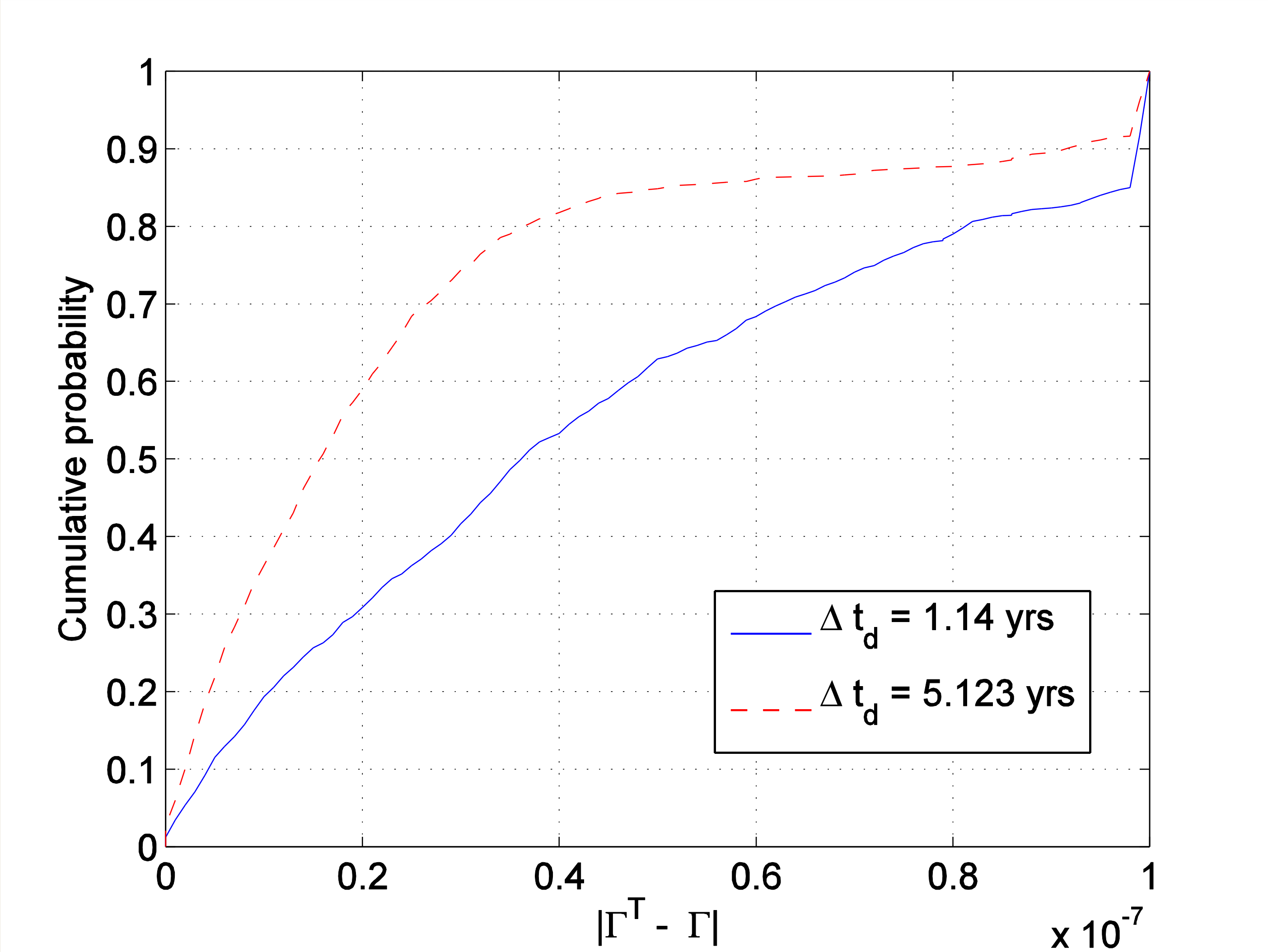
Effect of nuisance parameters estimation errors to td (td = 1.14 years)



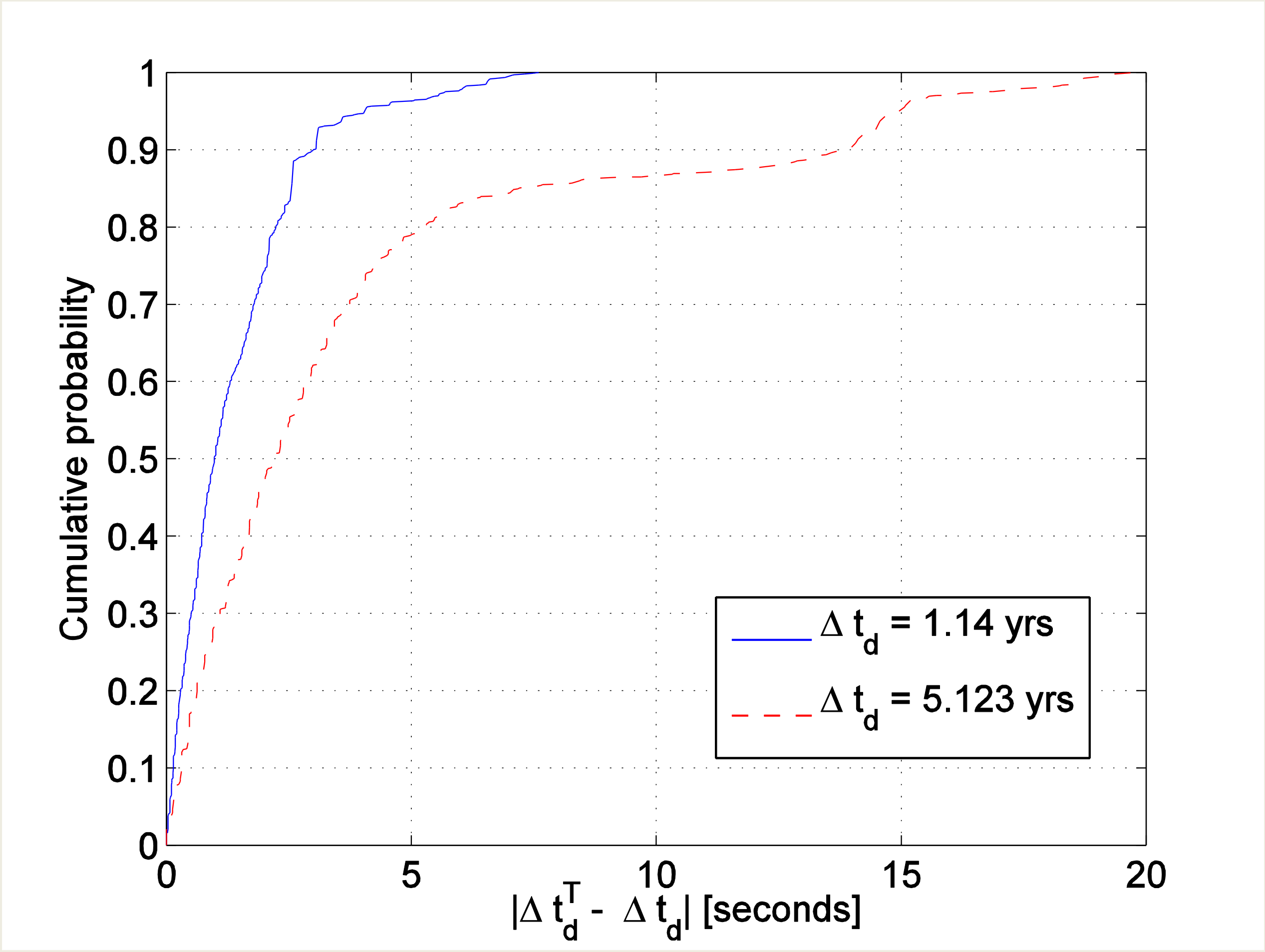
Effect of nuisance parameters estimation errors (td = 5.1 years)



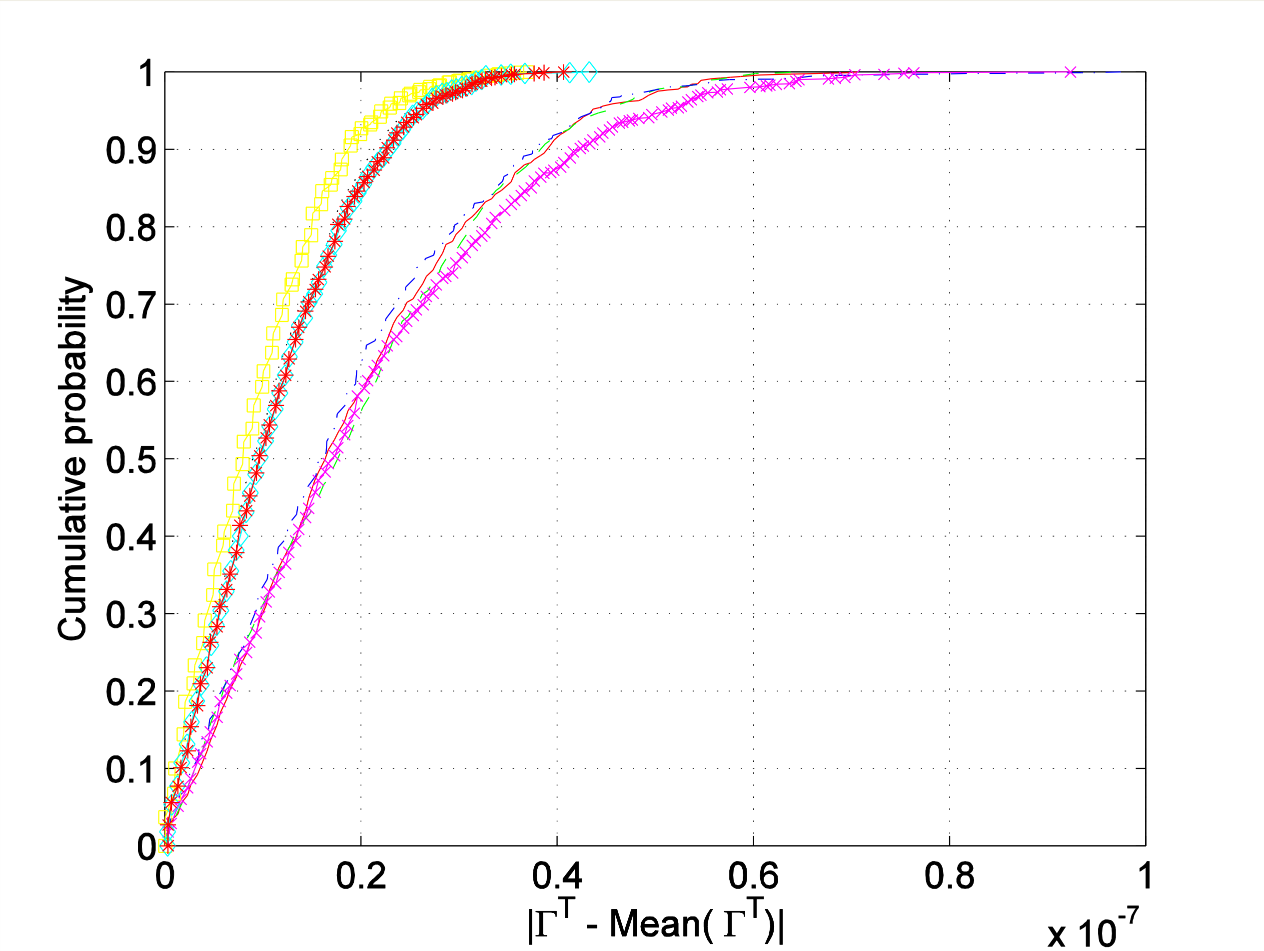
Effect of nuisance parameters estimation errors to Γ



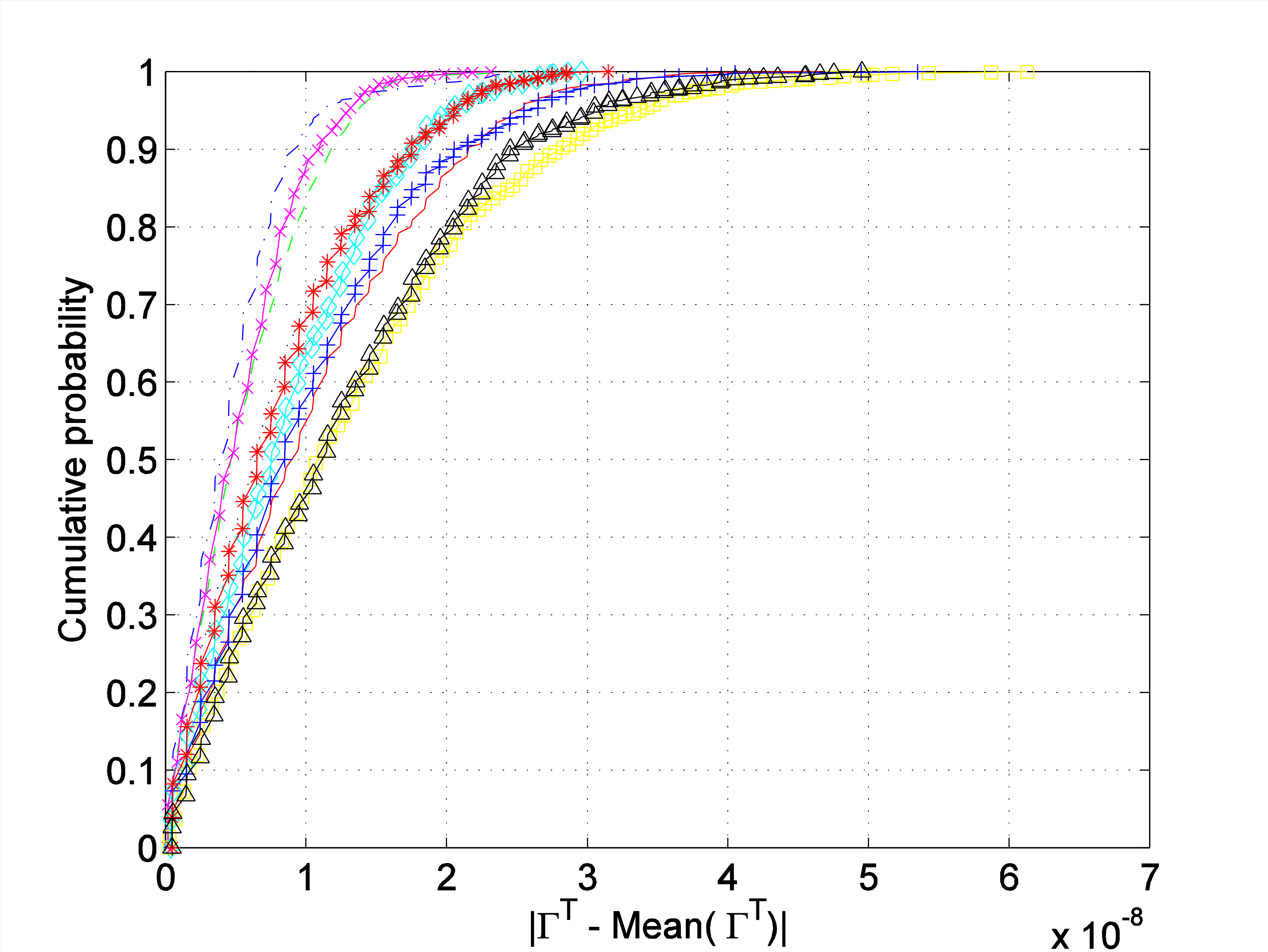
Effect of nuisance parameters estimation errors to td



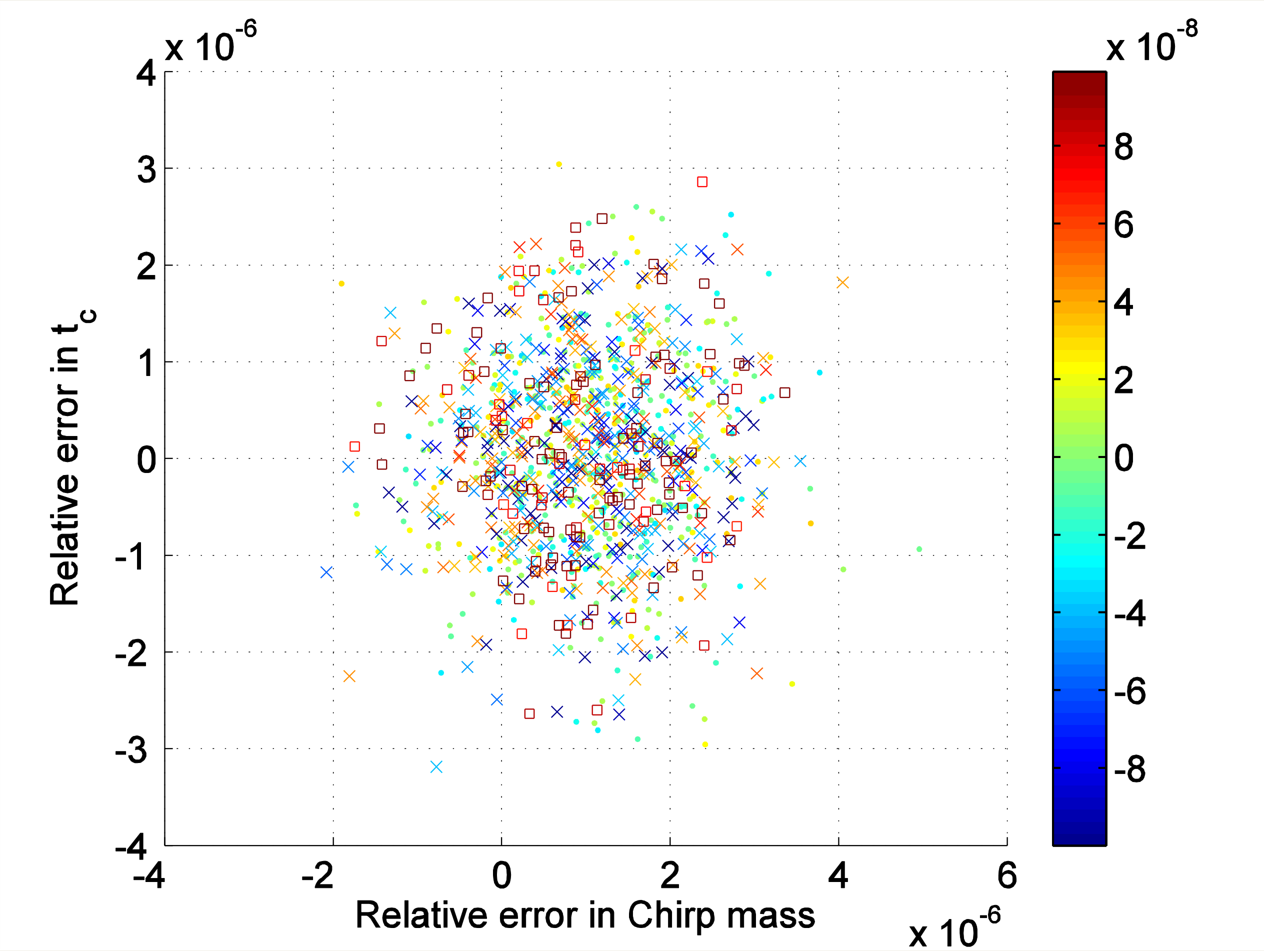
10 different nuisance parameters set, 1000 noise realizations each (td = 1.4 years)



10 different nuisance parameters, 1000 noise realizations each (td = 5.1 years)



If you think mass/tc should give Γ



It is mathematically true but at lesser accuracy.

Summary (as of 10:10, June 26, 2012)

- It is possible that gravitational wave can be gravitationally lensed in principle.
- I could not come up with any natural source that shows observable spatial interference pattern.
- On a computer, we can extract information contained in the interference term.
- For this purpose, I propose a new statistic ζ .
- With ζ , we can measure the transverse velocity of the sources which is hard to detect in a conventional EM GL in the DECIGO/BBO era.