2D $\mathcal{N} = (2, 2)$ SYM on the lattice — a status report —

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- I. Kanamori and H.S., Nucl. Phys. B **811** (2009) 420 [arXiv:0809.2856 [hep-lat]].
- I. Kanamori and H.S., Phys. Lett. B **672** (2009) 307 [arXiv:0811.2851 [hep-lat]].
- I. Kanamori, H.S. and F. Sugino, Phys. Rev. D **77** (2008) 091502 [arXiv:0711.2099 [hep-lat]].
- I. Kanamori, F. Sugino and H.S., Prog. Theor. Phys. **119** (2008) 797 [arXiv:0711.2132 [hep-lat]].

INTRODUCTION

Nonperturbative formulation of SUSY theories?

- widely believed that SUperSYmmetry play an important role in particle physics beyond SM
  - hierarchy (naturalness) problem
  - consistency of string theory (gauge/gravity correspondence)
- nonperturbative phenomena?
  - color confinement, bound states, spontaneous chiral symmetry breaking, quantum tunneling, . . .
  - dynamical spontaneous SUSY breaking
- nonperturbative formulation? lattice?

SUSY on the lattice?

- manifest SUSY would be *impossible*, because

$$\left\{ Q_\alpha^A, (Q_\beta^B)^\dagger \right\} = 2\delta^{AB}\sigma^m_{\alpha\dot{\beta}}P_m$$

but *no* infinitesimal translations P_m defined for lattice fields

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- however, at least a linear combination Q of Q_α^A and $(Q_\beta^B)^\dagger$ such that

$$\{Q, Q\} = 2Q^2 = 0$$

  could be realized even on the lattice

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- however, at least a linear combination Q of Q_α^A and $(Q_\beta^B)^\dagger$ such that

$$\{Q,Q\}=2Q^2=0$$

  could be realized even on the lattice
- moreover, *if* the target continuum action S can be written as

$$S=QX$$

  Q-invariance of S could be promoted to a lattice symmetry!

SUSY on the lattice? (cont'd)

- (partial) list of SUSY gauge theories with $S = QX$
  - 4D $\mathcal{N} = 4$ SYM
  - 3D $\mathcal{N} = 8$ SYM
  - 3D $\mathcal{N} = 4$ SYM
  - 2D $\mathcal{N} = (8, 8)$ SYM
  - 2D $\mathcal{N} = (4, 4)$ SYM
  - 2D $\mathcal{N} = (2, 2)$ SYM (+ matter multiplet)
- lattice formulations with an exact fermionic symmetry Q
  - Cohen, Endres, Kaplan, Katz, Ünsal (2002–)
  - Sugino (2003–)
  - Catterall (2004–)
  - D'Adda, Kanamori, Kawamoto, Nagata (2005–)
  - Damgaard, Matsuura
  - Kikukawa, Sugino
  - Kadoh, Sugino, H.S.

2D $\mathcal{N} = (2, 2)$ SYM: CONTINUUM THEORY

2D $\mathcal{N} = (2,2)$ SYM

- action (dimensional reduction of 4D $\mathcal{N} = 1$ SYM to 2D)

$$S_{\text{2DSYM}} = \frac{1}{g^2} \int d^2x \, \text{tr} \left[\frac{1}{2} F_{MN} F_{MN} + \Psi^T C \Gamma_M D_M \Psi + \tilde{H}^2 \right]$$

- SUSY

$$\delta A_M = i\epsilon^T C \Gamma_M \Psi, \qquad \delta\Psi = \frac{i}{2} F_{MN} \Gamma_M \Gamma_N \epsilon + i\tilde{H} \Gamma_5 \epsilon$$

$$\delta\tilde{H} = -i\epsilon^T C \Gamma_5 \Gamma_M D_M \Psi$$

- we set $\Gamma_0 = \begin{pmatrix} -i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix}$, $\Gamma_1 = \begin{pmatrix} i\sigma_3 & 0 \\ 0 & -i\sigma_3 \end{pmatrix}$, $\Gamma_2 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$, $\Gamma_3 = C = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$$\Psi^T \equiv (\psi_0, \psi_1, \chi, \eta/2), \qquad \epsilon^T \equiv -\left(\varepsilon^{(0)}, \varepsilon^{(1)}, \tilde{\varepsilon}, \varepsilon \right)$$

and decompose

$$\delta \equiv \varepsilon^{(0)} Q^{(0)} + \varepsilon^{(1)} Q^{(1)} + \tilde{\varepsilon} \tilde{Q} + \varepsilon Q$$

2D $\mathcal{N} = (2, 2)$ SUSY algebra

- SUSY algebra in this spinor basis,

$$Q^2 = \tilde{Q}^2 = \delta_\phi, \qquad (Q^{(0)})^2 = (Q^{(1)})^2 = -\delta_{\overline{\phi}}$$
$$\{Q, Q^{(\mu)}\} = -2i\partial_\mu + 2\delta_{A_\mu}, \quad \{\tilde{Q}, Q^{(\mu)}\} = -\epsilon_{\mu\nu}(-2i\partial_\nu + 2\delta_{A_\nu})$$
$$\{Q, \tilde{Q}\} = \{Q^{(0)}, Q^{(1)}\} = 0$$

  where

$$\phi \equiv A_2 + iA_3, \qquad \bar{\phi} = A_2 - iA_3, \qquad \epsilon_{01} \equiv 1$$

  and δ_φ denotes the infinitesimal gauge transformation with the parameter φ: $\delta_\varphi = [\varphi, \cdot]$ for matter fields and $\delta_\varphi A_\mu = iD_\mu \varphi$

- Q-transformation is nilpotent, on gauge invariant combinations:

$$Q^2 = \delta_\phi \simeq 0$$

Q-transformation

- Q-transformation ($H \equiv \tilde{H} + iF_{01}$)

$$
\begin{aligned}
QA_\mu &= \psi_\mu, & Q\psi_\mu &= iD_\mu\phi \\
Q\phi &= 0 & & \\
Q\bar{\phi} &= \eta, & Q\eta &= [\phi, \bar{\phi}] \\
Q\chi &= H, & QH &= [\phi, \chi]
\end{aligned}
$$

is nilpotent on gauge invariant combinations

$$
Q^2 = \delta_\phi \simeq 0
$$

- moreover, the continuum action is Q-exact

$$
S_{\text{2DSYM}} = Q\frac{1}{g^2}\int d^2x \,\text{tr}\left[-2i\chi F_{01} + \chi H + \frac{1}{4}\eta\,[\phi, \bar{\phi}] - i\psi_\mu D_\mu\bar{\phi}\right]
$$

R-symmetries

- $U(1)_A$ symmetry ($\longleftrightarrow$ 2-3 plane rotation in 4D)

$$\Psi \to \exp(\alpha\Gamma_2\Gamma_3)\,\Psi, \qquad \phi \to \exp(2i\alpha)\,\phi, \qquad \overline{\phi} \to \exp(-2i\alpha)\,\overline{\phi}$$

- $U(1)_V$ symmetry ($\longleftrightarrow$ $U(1)_R$ symmetry in 4D SYM)

$$\Psi \to \exp(i\alpha\Gamma_5)\,\Psi$$
$$S: \Psi \to i\Gamma_5\Psi, \qquad (\alpha = \pi/2)$$

- a $\mathbb{Z}_2$ symmetry ($\longleftrightarrow$ reflection in 2-direction in 4D)

$$R: \Psi \to i\Gamma_2\Psi, \qquad \phi \to -\overline{\phi}, \qquad \overline{\phi} \to -\phi, \qquad H \to -H + 2iF_{01}$$

- a useful relation

$$Q^{(0)} = RSQS^{-1}R^{-1}, \qquad Q^{(1)} = RQR^{-1}, \qquad \tilde{Q} = SQS^{-1}$$

Is this theory trivial? Not quite!

- This is a "toy" field theory, but no obvious low-energy description
- in 2D, no SSB of bosonic global symmetries (no chiral lagrangian)
- super-renormalizable, but perturbation theory in infinite volume suffers from severe IR divergence
- gauge coupling g simply provides a mass scale, like Λ_{QCD}
- no expansion parameter at low energies except possibly the number of colors N_c (and $1/N_c$ expansion is nontrivial)
- there exist flat directions $[\phi, \overline{\phi}] = 0$, but (probably) no vacuum modulus in 2D
- the Witten index is unknown (SSUSYB?, Hori-Tong)

2D $\mathcal{N} = (2, 2)$ SYM:
LATTICE FORMULATION

Sugino's lattice formulation

- 2D lattice (a: lattice spacing)

$$\Lambda = \left\{ x \in a\mathbb{Z}^2 \mid 0 \leq x_0 < \beta,\ 0 \leq x_1 < L \right\}$$

- lattice action ($U_\mu(x)$: link variables)

$$S_{\text{2DSYM}}^{\text{LAT}} = Q \frac{1}{a^2 g^2} \sum_{x \in \Lambda} \operatorname{tr} \Big[-i\chi(x)\hat{\Phi}(x) + \chi(x)H(x) + \frac{1}{4}\eta(x) \left[\phi(x), \bar{\phi}(x)\right] - i \sum_{\mu=0}^{1} \psi_\mu(x) \left(U_\mu(x)\bar{\phi}(x + a\hat{\mu})U_\mu(x)^{-1} - \bar{\phi}(x) \right) \Big]$$

where the lattice field strength $\hat{\Phi}(x)$ ($\simeq 2F_{01}$) is given basically by the plaquette

$$\hat{\Phi}(x) \simeq -iU_0(x)U_1(x + a\hat{0})U_0(x + a\hat{1})^{-1}U_1(x)^{-1} + \text{h.c.}$$

Sugino's lattice formulation (cont'd)

- lattice Q-transformation

$$QU_\mu(x) = i\psi_\mu(x)U_\mu(x)$$

$$Q\psi_\mu(x) = i\psi_\mu(x)\psi_\mu(x) - i\left(\phi(x) - U_\mu(x)\phi(x+a\hat{\mu})U_\mu(x)^{-1}\right)$$

$$Q\phi(x) = 0$$

$$Q\bar{\phi}(x) = \eta(x), \qquad Q\eta(x) = \left[\phi(x), \bar{\phi}(x)\right]$$

$$Q\chi(x) = H(x), \qquad QH(x) = [\phi(x), \chi(x)]$$

is nilpotent on gauge invariant combinations on the lattice

$$Q^2 = \delta_\phi \simeq 0$$

- Q is a manifest lattice symmetry, $QS_{\text{2DSYM}}^{\text{LAT}} = 0$
- $U(1)_A$ is another manifest symmetry

$\Psi(x)^T \equiv (\psi_0(x), \psi_1(x), \chi(x), \eta(x)/2)$

$$\Psi(x) \to \exp(\alpha\Gamma_2\Gamma_3)\,\Psi(x),$$

$$\phi(x) \to \exp(2i\alpha)\,\phi(x), \qquad \bar{\phi}(x) \to \exp(-2i\alpha)\,\bar{\phi}(x)$$

Restoration of full SUSY (and R symmetries)?

- this lattice formulation possesses manifest lattice symmetries Q and $U(1)_A$
- but how about other $Q^{(0)}$, $Q^{(1)}$, $\widetilde{Q}$? (and $U(1)_V$, $\mathbb{Z}_2$)?
- the best thing we can hope is that these are restored in the continuum limit $a \to 0$
- does this really realize? the most important issue to be settled before going into physics
- perturbative argument on the basis of the effective action (Sugino; cf. Kaplan et al.)

RESTORATION of SUSY?

What is the most useful characterization of SUSY restoration?

- scalar 2-point function? ($\Leftarrow$ not gauge invariant)
- degeneracy of mass spectra? ($\Leftarrow$ cannot be distinguished from the spontaneous SUSY breaking)
- (local) SUSY Ward-Takahashi (WT) identity would be best
- in the target continuum theory, we expect

$$\partial_\mu \langle s_\mu(x)\,\mathcal{O}(y_1,\ldots,y_n)\rangle \qquad s_\mu\text{: supercurrent}$$

$$= \frac{\mu^2}{g^2}\langle f(x)\,\mathcal{O}(y_1,\ldots,y_n)\rangle - i\frac{\delta}{\delta\epsilon(x)}\langle\mathcal{O}(y_1,\ldots,y_n)\rangle\,,$$

in the presence of a SUSY breaking scalar mass term

$$S_{\text{mass}} = \frac{\mu^2}{g^2}\int d^2x\,\operatorname{tr}\left[\bar\phi\phi\right]\,, \qquad f \equiv 2iC\left(\Gamma_\uparrow \operatorname{tr}\left[\phi\Psi\right] + \Gamma_\downarrow \operatorname{tr}\left[\bar\phi\Psi\right]\right)$$

and

$$\Gamma_{\uparrow,\downarrow} \equiv \frac{i}{2}\left(\Gamma_2 \mp i\Gamma_3\right)$$

SUSY WT identity in the continuum

- the (local) SUSY WT identity

$$\partial_\mu \langle s_\mu(x)\,\mathcal{O}(y_1,\ldots,y_n)\rangle = \frac{\mu^2}{g^2}\langle f(x)\,\mathcal{O}(y_1,\ldots,y_n)\rangle - i\frac{\delta}{\delta\epsilon(x)}\langle\mathcal{O}(y_1,\ldots,y_n)\rangle$$

  holds, irrespective of
  - boundary conditions ($\because$ used localized SUSY transformations)
  - spontaneous SUSY breaking ($\Rightarrow$ Nambu-Goldstone fermion)

  provided that the regularization respects SUSY
- is the WT identity reproduced in the continuum limit $a \to 0$?
- renormalization/mixing of composite operators?

SUSY WT identity on the lattice?

- first, we want to define lattice analogues of $Q^{(0)}$, $Q^{(1)}$ and $\tilde{Q}$, such that

$$(Q^{(0)})^2 = (Q^{(1)})^2 = \tilde{Q}^2 = 0$$

and, under the $U(1)_A$ transformation,

$$(Q^{(0)}, Q^{(1)}, \tilde{Q}, Q) \to (e^{-i\alpha} Q^{(0)}, e^{-i\alpha} Q^{(1)}, e^{i\alpha} \tilde{Q}, e^{i\alpha} Q)$$

- these can be accomplished, with the help of lattice analogues of S and R, as

$$Q^{(0)} \equiv RSQS^{-1}R^{-1}, \qquad Q^{(1)} \equiv RQR^{-1}, \qquad \tilde{Q} \equiv SQS^{-1}$$

- furthermore, using representations

$$\begin{aligned} S_{\text{2DSYM}}^{\text{LAT}} &= QX \\ &= Q^{(0)} RSX + (1 - RS)\, S_{\text{2DSYM}}^{\text{LAT}} \\ &= Q^{(1)} RX + (1 - R)\, S_{\text{2DSYM}}^{\text{LAT}} \\ &= \tilde{Q} SX + (1 - S)\, S_{\text{2DSYM}}^{\text{LAT}} \end{aligned}$$

SUSY WT identity on the lattice

- we have an identity on the lattice,
  $\partial^*_\mu g(x) \equiv (1/a)(g(x) - g(x - a\hat{\mu}))$

$$\partial^*_\mu \langle s_\mu(x)\mathcal{O}(y_1,\ldots,y_n)\rangle \qquad s_\mu(x)\text{: lattice supercurrent}$$
$$= \frac{\mu^2}{g^2}\langle f(x)\mathcal{O}(y_1,\ldots,y_n)\rangle - i\frac{\delta}{\delta\epsilon(x)}\langle \mathcal{O}(y_1,\ldots,y_n)\rangle$$
$$+ \langle B(x)\mathcal{O}(y_1,\ldots,y_n)\rangle, \qquad B(x) = O(a),$$

where $f(x) \equiv 2iC(\Gamma_\uparrow \operatorname{tr}[\phi(x)\Psi(x)] + \Gamma_\downarrow \operatorname{tr}[\bar{\phi}(x)\Psi(x)])/a^{5/2}$, s.t., under the $U(1)_A$ transformation and

$$s_\mu(x) \to \exp(-\alpha\Gamma_2\Gamma_3)\, s_\mu(x), \qquad B(x) \to \exp(-\alpha\Gamma_2\Gamma_3)\, B(x)$$

- then the crucial issue is

$$B(x)^T = (*, *, *, 0) \xrightarrow{a\to 0} 0?$$

Argument based on the formal perturbation theory

- $B(x)$ is $O(a)$, but could become $O(1)$ through radiative corrections
- we assume that $\mathcal{O}$s are gauge invariant operators
- we further assume that $x \neq y_i$ $(i = 1, \ldots, n)$
- $B(x)$ can then mix with gauge invariant, fermionic, mass dimension $\leq 5/2$, $U(1)_A$ covariant operators
- assuming that the gauge group is $G = SU(N_c)$ and $B(x)$ can be cancelled by local counterterms (i.e., SUSY has no intrinsic anomaly),

$$B(x) \xrightarrow{a\to 0} \text{const.} C \left(\Gamma_\uparrow \operatorname{tr}[\phi\Psi] + \Gamma_\downarrow \operatorname{tr}\left[\bar{\phi}\Psi\right] \right) = \text{const.} \begin{pmatrix} * \\ * \\ * \\ -\operatorname{tr}\{\phi\eta/2\} \end{pmatrix}$$

- but, because of lattice Q-symmetry

$$B(x)^T = (*, *, *, 0) \Rightarrow B(x) \xrightarrow{a\to 0} 0$$

Lattice SUSY WT identity in the continuum limit

- So, in the continuum limit, when $x \neq y_i$ $(i = 1, \ldots, n)$

$$\partial_\mu \langle s_\mu(x)\mathcal{O}(y_1, \ldots, y_n)\rangle = \frac{\mu^2}{g^2} \langle f(x)\mathcal{O}(y_1, \ldots, y_n)\rangle$$

  and SUSY is automatically restored!

- For other choices of supercurrent $s'_\mu(x)$ such that $\Delta s_\mu(x) \equiv s'_\mu(x) - s_\mu(x) = O(a)$ is gauge invariant,

$$\Delta s_\mu(x) \xrightarrow{a\to 0} \text{const.} M \operatorname{tr}[\Psi] \equiv 0, \qquad \text{for } G = SU(N_c)$$

  and the SUSY WT identity holds also for $s'_\mu(x)$:

$$\partial_\mu \left\langle s'_\mu(x)\mathcal{O}(y_1, \ldots, y_n)\right\rangle = \frac{\mu^2}{g^2} \langle f(x)\mathcal{O}(y_1, \ldots, y_n)\rangle$$

- by perturbative argument, one sees that $s_\mu(x)$ and $f(x)$ are finite (and thus correctly normalized) operators

Explicit confirmation of the SUSY WT identity

- the argument so far is rather formal, because perturbation theory (in massless theory) in infinite volume suffers from IR divergence
- we may avoid the IR divergence by putting the system into a finite box of size L
- perturbation theory then becomes an expansion w.r.t. Lg, i.e., we have a small volume expansion (IR divergence is reproduced as $L \to \infty$)

We have two choices:

- can use perturbative expansion for small volume $Lg \ll 1$
- for large physical volume $Lg \gtrsim 1$, perturbation theory is useless. use instead the Monte Carlo simulation

Semi-perturbative expansion

- however, since

$$S_{\text{2DSYM}}^{\text{LAT}} = \frac{N^2}{a^2 g^2} \operatorname{tr} \left[-\frac{1}{2} [\tilde{A}_\mu(0), \tilde{A}_\nu(0)]^2 + \tilde{\Psi}(0)^T C \Gamma_\mu i [\tilde{A}_\mu(0), \tilde{\Psi}(0)] + \cdots \right.$$

constant modes do not allow a perturbative expansion and a naive order counting is modified:

$$\square = \tilde{\Psi}(0) = O((ag)^{3/4}) \qquad \circ = \tilde{A}_\mu(0) \text{ or } \tilde{\phi}(0) = O((ag)^{1/2})$$

- semi-perturbative analysis of a scalar 2-point function in Kaplan's model (Onogi-Takimi, PRD 72 (2005))
  - perturbative integration over non-zero momentum modes
  - nonperturbative numerical integration over constant modes

Semi-perturbative expansion (cont'd)

- we take a dimension $1/2$ operator

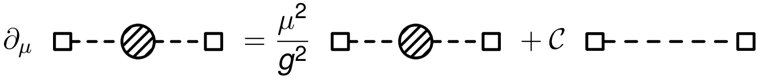
$$\mathcal{O}(y) = f_\nu(y) \equiv -\frac{1}{2g^2}\Gamma_\nu C^{-1} f(y)$$

and want to see whether the SUSY WT identity

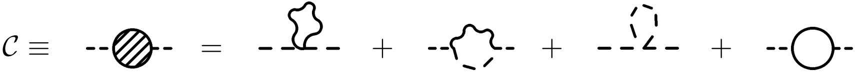
$$\partial_\mu \langle s_\mu(x) f_\nu(y)\rangle = \frac{\mu^2}{g^2}\langle f(x) f_\nu(y)\rangle\,, \qquad \text{for } x \neq y$$

holds or not

- the first nontrivial order turns to be $O((ag)^{3/2})$ and, schematically,



where $\mathcal{C}$ denotes the scalar one-loop self energy



Semi-perturbative expansion (cont'd)

- somewhat lengthy one-loop calculation yields ($N \equiv L/a$, λ is the gauge parameter)

$$\mathcal{C} = N_c \frac{2}{N^2} \sum_{(n_0,n_1)\neq(0,0)} \left[\frac{1}{2}\left(1+\frac{1}{\lambda}\right)\frac{1}{\hat{k}^2} + \frac{1}{2}\left(1-\frac{1}{\lambda}\right)\frac{1}{\hat{k}^2 + a^2\mu^2} - \frac{1}{\hat{k}^2} \right]$$

where

$$\hat{k}^2 \equiv \sum_{\mu=0}^{1} (\hat{k}_\mu)^2, \qquad \hat{k}_\mu \equiv 2\sin\frac{k_\mu}{2},$$

and $k_\mu \equiv \frac{2\pi n_\mu}{N}$, $n_\mu = 0, 1, 2, \dots, N-1$

- we may further neglect $a^2\mu^2 = (\mu^2/g^2)a^2g^2$ in the denominator and then

$$\mathcal{C} = 0$$

- the SUSY WT identity really holds in the first nontrivial order in the semi-perturbative expansion

Monte Carlo simulation (brief sketch)

- simulation with a dynamical Majorana spinor ($N_f = 1/2$)
- partition function

$$\mathcal{Z} = \mathcal{N} \int [d(\text{fields})]\ e^{-S} = \mathcal{N}' \int [d(\text{bosonic fields})]\ e^{-S_B}\, \text{Pf}\{D\}$$

- pseudo-fermion

$$\begin{aligned}\text{Pf}\{D\} &= e^{i\,\text{Arg}\,\text{Pf}\{D\}} (\det D^\dagger D)^{1/4} \\ &= e^{i\,\text{Arg}\,\text{Pf}\{D\}} \int [d\varphi]\,[d\overline{\varphi}]\ e^{-\overline{\varphi}(D^\dagger D)^{-1/4}\varphi}\end{aligned}$$

- rational approximation (RHMC)

$$x^{-1/4} \simeq \alpha_0 + \sum_{i=1}^{N} \frac{\alpha_i}{x + \beta_i}$$

Remez algorithm, multi-shift solver, ...

Simulation parameters

- $G = SU(2)$, antiperiodic BC, $Lg = 1.414$, $\beta = 2L$
- Lattice sizes

$$12 \times 6, \quad 16 \times 8, \quad 20 \times 10$$

- Lattice spacings

$$ag = 0.2357, \quad 0.1768, \quad 0.1414$$

- Scalar masses

$$\mu^2/g^2 = 0.04, \quad 0.25, \quad 0.49, \quad 1.0, \quad 1.69$$

- Number of uncorrelated configurations

800–1800

- $\sim 20,000\,\mathrm{CPU} \cdot \mathrm{hour}$

Monte Carlo confirmation! (Kanamori-H.S., NPB 811 (2009))

- Continuum limit of the ratio

$$\frac{\partial_\mu \left\langle (s'_\mu)_i(x)(f_0)_i(y)\right\rangle}{\langle (f)_i(x)(f_0)_i(y)\rangle} \xrightarrow{a\to 0} \frac{\mu^2}{g^2} \qquad \text{for } x \neq y?$$

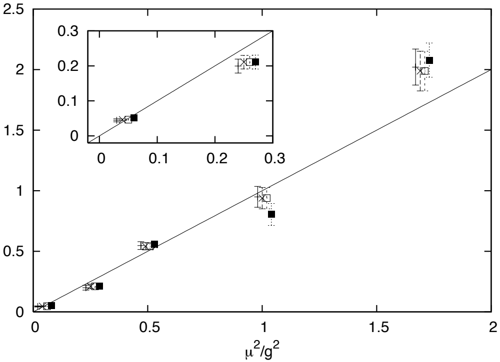


Figure: $i = 1$ $(+)$, $i = 2$ $(\times)$, $i = 3$ $(\square)$, $i = 4$ $(\blacksquare)$

Monte Carlo confirmation (cont'd)

- it appears that, at least for $\mu^2/g^2 > 0$, with antiperiodic BC, the SUSY WT identity holds in the continuum limit
- breaking of SUSY (and other symmetries) owing to lattice regularization disappears
- the target theory (2D $\mathcal{N} = (2,2)$ SYM with a SUSY breaking scalar mass) seems to be realized in the continuum limit
- this is the first (and so far unique) example in lattice gauge theory in which the restoration of SUSY was observed!

PHYSICS

Correlation functions with power-like behavior

- this system has no mass gap (Witten) ($\hookleftarrow$ 't Hooft anomaly matching condition)
- more definitely, on $\mathbb{R}^2$ (Fukaya-Kanamori-H.S.-Hayakawa-Takimi, PTP 116 (2007))

$$
\begin{aligned}
&-\frac{i}{2}\langle j_\mu(x)\epsilon_{\nu\rho}j_{5\rho}(0)\rangle \\
&=\frac{1}{4\pi}(N_c^2-1)\int\frac{d^2p}{(2\pi)^2}\,e^{ipx}\left\{-\frac{1}{p^2}(p_\mu p_\nu-\epsilon_{\mu\rho}\epsilon_{\nu\sigma}p_\rho p_\sigma)+\widetilde{c}\delta_{\mu\nu}\right\} \\
&=\frac{1}{4\pi}(N_c^2-1)\left\{\frac{1}{\pi}\frac{1}{(x^2)^2}(x_\mu x_\nu-\epsilon_{\mu\rho}\epsilon_{\nu\sigma}x_\rho x_\sigma)+\widetilde{c}\delta_{\mu\nu}\delta^2(x)\right\},
\end{aligned}
$$

where j_μ and $j_{5\rho}$ are $U(1)_V$ and $U(1)_A$ currents, respectively ($\widetilde{c}$ is ambiguity in operator definition)

Can we see this massless bosonic state?

- power-like behavior on $\mathbb{R}^2$

$$-\frac{i}{2}\left\langle j_0(x)\epsilon_{0\rho} j_{5\rho}(0)\right\rangle = \frac{3}{4\pi^2}\frac{1}{(x_0)^2}, \qquad \text{for } N_c = 2 \text{ along } x_1 = 0$$

- if so, the $U(1)_V$ symmetry is restored

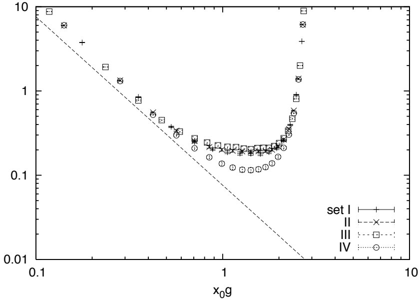


Figure: IV: antiperiodic BC, 20×16, $ag = 0.1414$, $\mu^2/g^2 = 0.25$

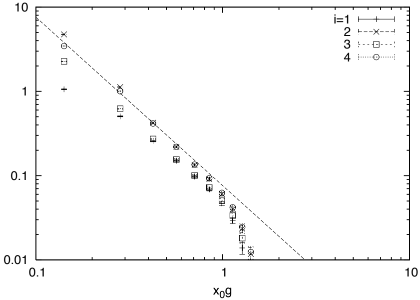
Almost degenerated fermionic state

- a (global) SUSY WT identity

$$\langle (s_0)_i(x)(f_0)_i(0)\rangle = -\frac{i}{2}\langle j_0(x)\epsilon_{0\rho}j_{5\rho}(0)\rangle$$

$$\overbrace{-\left\langle j_0(x)\epsilon_{0\rho}\frac{1}{g^2}\,\mathrm{tr}\left\{A_3(0)F_{\rho 2}(0)-A_2(0)F_{\rho 3}(0)\right\}\right\rangle}^{O(g^2);\ \text{no massless singularity}}$$

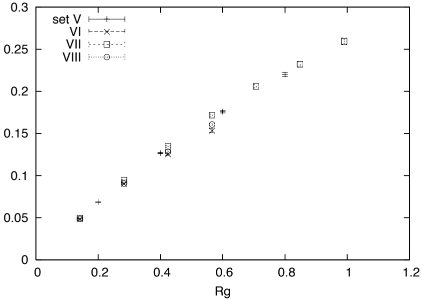
(this follows from $\delta\langle j_\mu(x)f_\nu^T(0)\rangle = 0$, neglecting μ^2 and aPBC)



Static potential between charges in fund. rep.

- static potential between charges in the fundamental representation $V(R)/g$

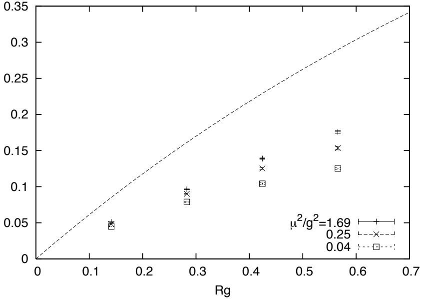
$$-\ln\{W(T,R)\} = V(R)T + c(R)$$



- this confining behavior appears distinct with a conjecture in (Armoni-Frishman-Sonnenschein, PLB 449 (1999))

Static potential (cont'd)

- static potential between charges in the fundamental representation $V(R)/g$ for various scalar masses



- the broken line: (Gross-Klebanov-Matytsin-Smilga, NPB 461 (1996)) for $\mu^2/g^2 \to \infty$

Hamiltonian density: order parameter of SSUSYB!

- in the lattice SUSY WT identity, set

$$\mathcal{O}(y) = \left(s_0'\right)_{i=1}(y),$$

where $i = 1$ spinor component corresponds to the $Q^{(0)}$-transformation

- then the lattice WT identity (for $\mu^2 \to 0$) provides SUSY current algebra among correctly normalized current operators (recall that $B_{i=4}(x) = 0$)

$$\partial_\mu^* \left\langle (s_\mu)_{i=4}(x) \left(s_0'\right)_{i=1}(y) \right\rangle = i\frac{1}{a^2}\delta_{x,y} \left\langle Q \left(s_0'\right)_{i=1}(x) \right\rangle$$

- the right-hand side can be regarded as the hamiltonian density

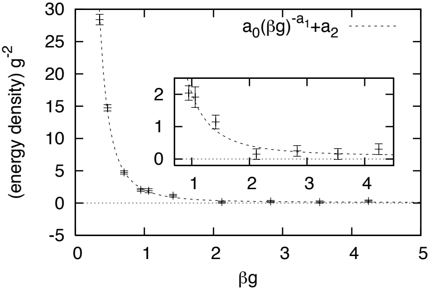
$$\left\langle Q \left(s_0'\right)_{i=1}(x) \right\rangle = 2 \left\langle \mathcal{H}(x) \right\rangle \quad \Leftrightarrow \quad \{Q, Q^{(0)}\} = -2i\partial_0 + 2\delta_{A_0}$$

- this is the prescription for the hamiltonian density, advocated in (Kanamori-Sugino-H.S., PRD 77 (2008))

Vacuum energy density $\mathcal{E}_0$ (Kanamori, PRD 79 (2009))

- can be obtained from the zero temperature limit $\beta \to \infty$ of $\langle \mathcal{H} \rangle$

$$\mathcal{E}_0/g^2 = 0.09 \pm 0.09(\text{sys})\,^{+0.10}_{-0.08}(\text{stat})$$



- it appears that the dynamical spontaneous SUSY breaking in this system (Hori-Tong, JHEP 0705 (2007)) is unlikely. . .

Summary

- although our research so far is on a 2D SUSY gauge theory, we have really realized the steps

nonperturbative formulation of SUSY gauge theory

$\Downarrow$

confirmation of SUSY restoration in the continuum limit

$\Downarrow$

study of nonperturbative phenomena from first principles

Summary

- further targets
  - 2D $\mathcal{N} = (2,2)$ SQCD
  - (2D $\mathcal{N} = (2,2)$ WZ model)
  - 2D $\mathcal{N} = (4,4)$ SYM
  - 4D $\mathcal{N} = 1$ SYM

Perturbative argument (Sugino; cf. Kaplan et al.)

- in the continuum limit, SUSY breaking owing to the lattice regularization should be able to be removed by *local* counterterms (i.e., absence of SUSY anomaly)
- possible local term in the effective action in the ℓ-loop

$$a^{p+2\ell-4}(g^2)^{\ell-1}\int d^2x\,\varphi^a\partial^b\psi^{2c},\quad p\equiv a+b+3c\geq 0$$

(up to some powers of $\ln a$)

- operators with $p+2\ell-4\leq 0$ survive in the continuum limit $a\to 0$. it is enough to consider $\ell=0$, 1, 2
- for $\ell=0$, the continuum limit coincides with the target theory

Perturbative argument (Sugino; cf. Kaplan et al.)

- for $\ell = 1$, only $p = 0, 1, 2$ could survive

$$
\begin{aligned}
p = 0 &\Rightarrow 1 \text{ (identity operator)} \leftarrow \text{no dynamical effect} \\
p = 1 &\Rightarrow \mathrm{tr}\{\phi\} = \mathrm{tr}\{\bar{\phi}\} = 0 \\
p = 2 &\Rightarrow \mathrm{tr}\{F_{01}\} = \mathrm{tr}\{D_\mu\phi\} = \mathrm{tr}\{D_\mu\bar{\phi}\} = \mathrm{tr}\{H\} = 0 \\
&\Rightarrow \mathrm{tr}\{\phi\phi\}, \mathrm{tr}\{\bar{\phi}\bar{\phi}\} \leftarrow \text{prohibited by } U(1)_A \\
&\Rightarrow \mathrm{tr}\{\bar{\phi}\phi\} \leftarrow \text{prohibited by the } Q \text{ symmetry}
\end{aligned}
$$

- for $\ell = 2$, only $p = 0$ is marginal (i.e., the identity 1)

Derivation of the lattice identity

- identity

$$\int [d(\text{fields})]\ \delta\left[e^{-S_{\text{2DSYM}}^{\text{LAT}}-S_{\text{mass}}^{\text{LAT}}}\mathcal{O}(y_1,\ldots,y_n)\right]=0$$

and thus

$$\left\langle\frac{\delta}{\delta\epsilon(x)}(S_{\text{2DSYM}}^{\text{LAT}}+S_{\text{mass}}^{\text{LAT}})\mathcal{O}(y_1,\ldots,y_n)\right\rangle=\left\langle\frac{\delta}{\delta\epsilon(x)}\mathcal{O}(y_1,\ldots,y_n)\right\rangle$$

- setting

$$\delta S_{\text{2DSYM}}^{\text{LAT}}\equiv -ia^2\sum_{x\in\Lambda}\epsilon(x)^T\left[-\partial_\mu^* s_\mu(x)+B(x)\right]$$

$$\delta S_{\text{mass}}^{\text{LAT}}\equiv -ia^2\sum_{x\in\lambda}\epsilon(x)^T\frac{\mu^2}{g^2}f(x)$$

we have the lattice SUSY WT identity

$$\partial_\mu^*\langle s_\mu(x)\mathcal{O}(y_1,\ldots,y_n)\rangle$$

$$=\frac{\mu^2}{g^2}\langle f(x)\mathcal{O}(y_1,\ldots,y_n)\rangle-i\frac{\delta}{\delta\epsilon(x)}\langle\mathcal{O}(y_1,\ldots,y_n)\rangle+\langle B(x)\mathcal{O}(y_1,\ldots,y_n)\rangle$$