Nonlinear Dynamics of Collective Neutrino Oscillation in Core-Collapse Supernovae

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ICRR seminar at 2022/07/21

Outlines

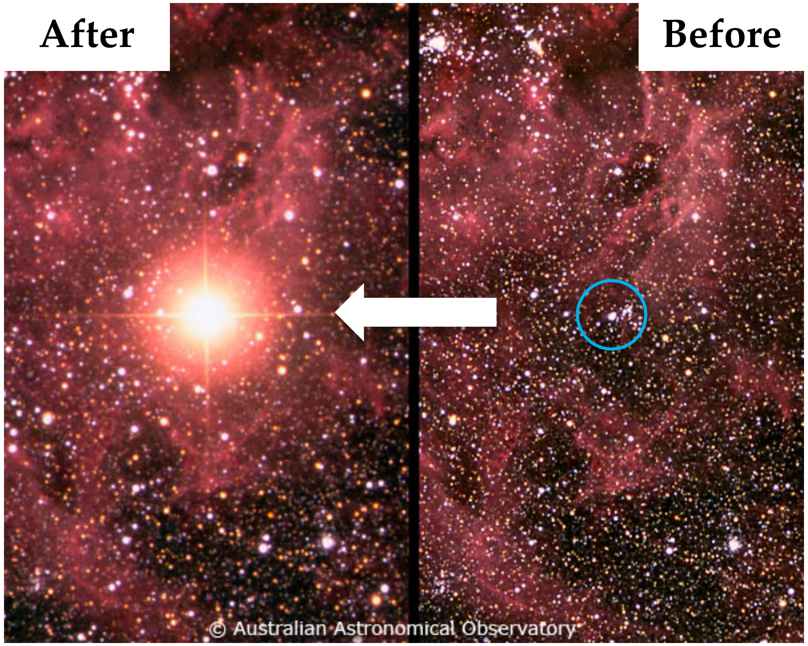
1. Introduction:
   - Supernova neutrinos & Flavor conversions
   - Collective neutrino oscillation

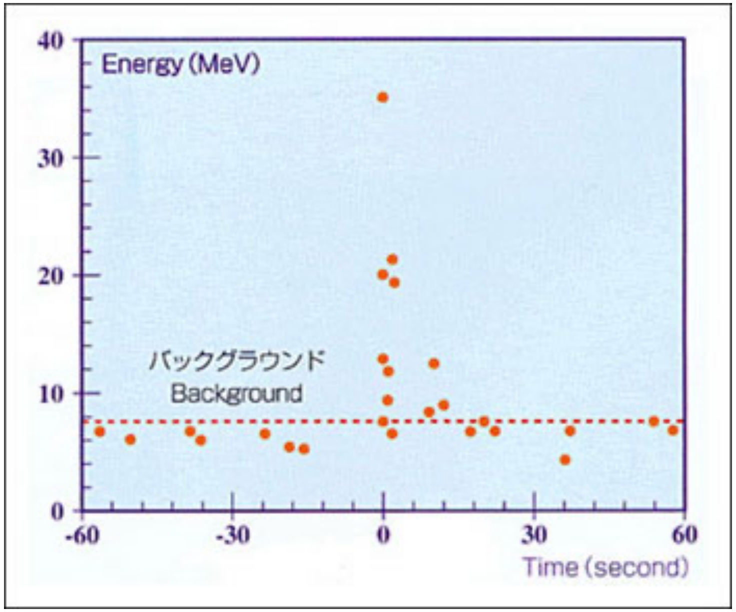
2. Slow Flavor Conversion
   - Suppression & Symmetry breaking

3. Fast Flavor Conversion
   - Dynamical evolution & Asymptotic behaviors

4. Summary

Historical Supernova Event





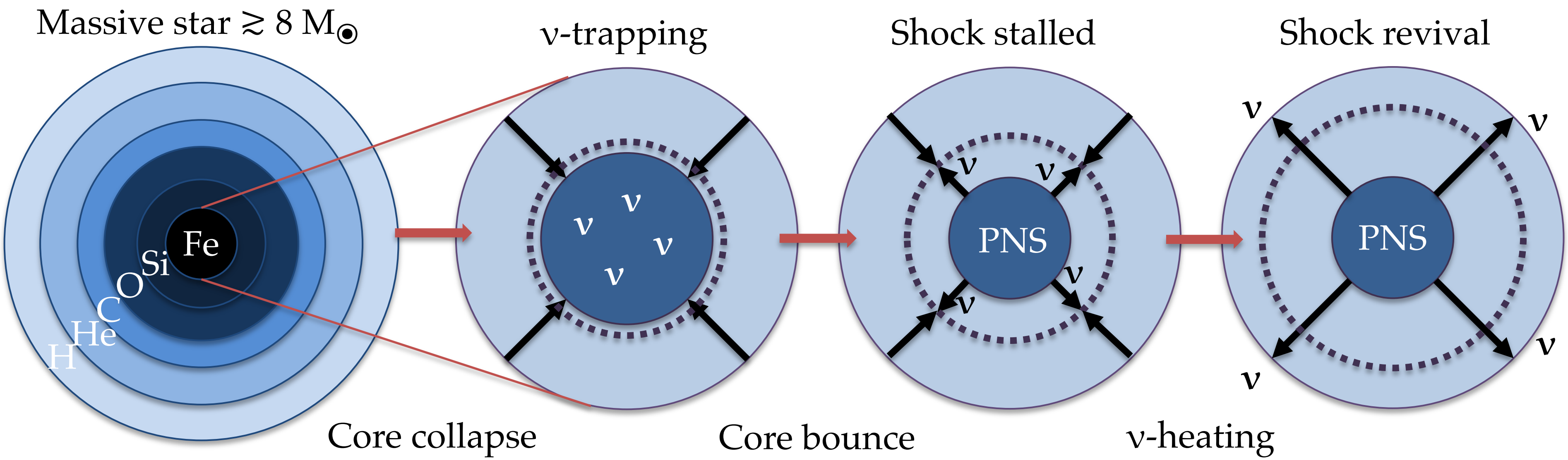
SN1987A in LMC

20 events @ Kamiokande-II and IMB detectors

- $L_\nu \sim 3 \times 10^{53}$ erg
- $E_\nu \sim 10$ MeV
- $t_\nu \sim 10$ sec

➢ Confirmed our understanding of CCSN physics.

Supernovae & Neutrinos



Neutrino-heating process:

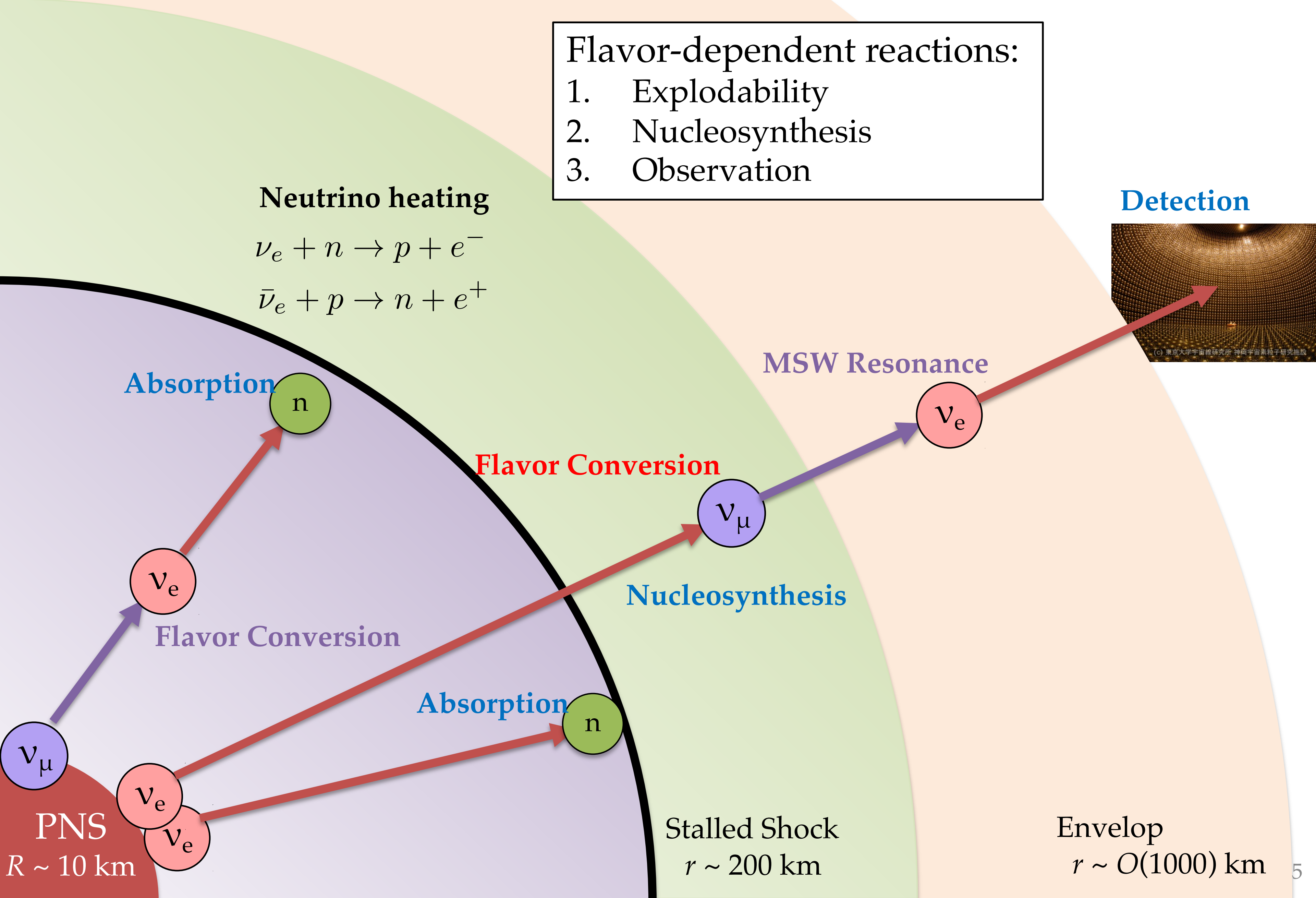
- Shock wave stalls due to the accreting matters and fails to explode.
- Neutrinos deposit the energy to the stalled shock and power the explosion.

SN Event rate ~ O(1/100) yr.

- **Theoretical studies/predictions are essential.**

Neutrino Oscillations vs. CCSNe

Flavor-dependent reactions:
1. Explodability
2. Nucleosynthesis
3. Observation

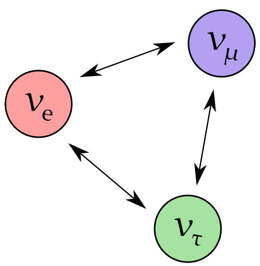


Flavor mixing in CCSNe

Three types of neutrino oscillations:

(Collision term)

$$(\partial_t + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}})\rho_\nu = -\mathrm{i}\,[H_{\mathrm{vac}} + H_{\mathrm{mat}} + H_{\nu\nu},\ \rho_\nu] + \mathcal{C}$$



Vacuum
(Neutrino mass)

Matter
(Background matter)

Collective
(ν-ν interactions)

Oscillation scale:

- Vacuum : $\omega \propto E_\nu^{-1} \sim O(1)$ km (for 10 MeV neutrinos)
- Matter : $\lambda \propto n_e \lesssim O(1)$ cm (in the decoupling region)
- **Collective : $\mu \propto n_\nu \lesssim O(1)$ cm (in the decoupling region)**

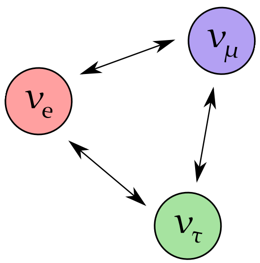
Much smaller (& faster) than stellar evolution.

Flavor mixing in CCSNe

Three types of neutrino oscillations:

(Collision term)

$$(\partial_t + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}})\rho_\nu = -\mathrm{i}\left[H_{\mathrm{vac}} + H_{\mathrm{mat}} + H_{\nu\nu},\ \rho_\nu\right] + \mathcal{C}$$



Vacuum
(Neutrino mass)

Matter
(Background matter)

Collective
(ν-ν interactions)

Neutrino density matrix:

$$\rho_\nu = \begin{pmatrix} \rho^{ee} & \rho^{e\mu} & \rho^{e\tau} \\ \rho^{\mu e} & \rho^{\mu\mu} & \rho^{\mu\tau} \\ \rho^{\tau e} & \rho^{\tau\mu} & \rho^{\tau\tau} \end{pmatrix}$$

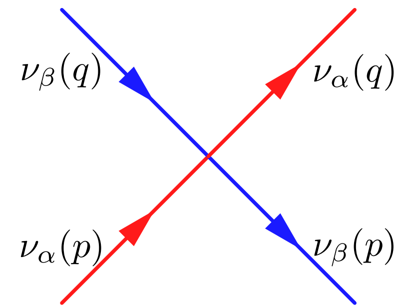
- $\rho^{\alpha\alpha}$ is a flavor content of α.
- $\rho^{\alpha\beta}$ is a flavor correlation between α and β.

The growth of $\rho^{\alpha\beta}$ indicates the occurrence of flavor conversion.

Collective Neutrino Oscillation

In the dense neutrino media $N\nu$~10^{58} (L ~ 10^{53}erg/s),
νν interactions can't be ignored.

Momentum exchange

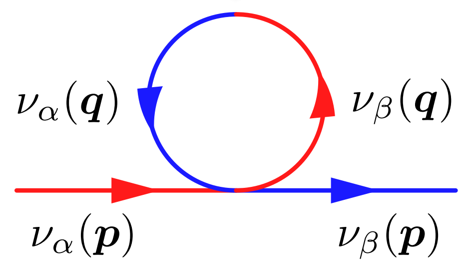


$$H \propto \sum a^\dagger a + \sum (1 - \cos\theta_{\rm int}) a^\dagger a^\dagger a a$$

Many-body problem.
(e.g., Balantekin+ '06)

Mean-field approx.

$$H = U \frac{M^2}{2E_\nu} U^\dagger + v^\mu \Lambda_\mu + \sqrt{2} G_{\rm F} \int {\rm d}\Gamma' v^\mu v'_\mu \rho'_\nu$$



Vacuum oscillation
(Neutrino mass)

Matter oscillation
(Electron density)

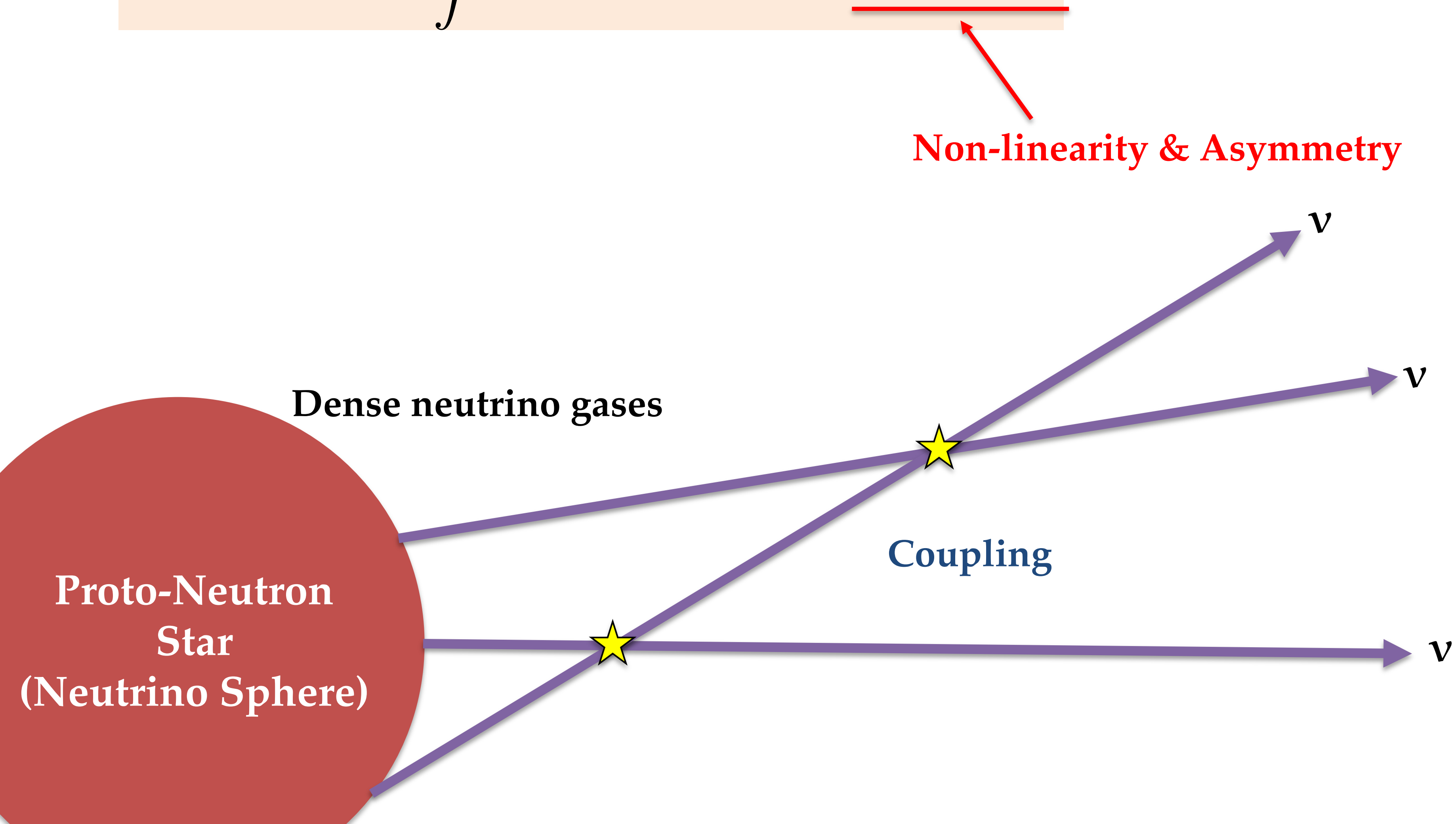
Collective neutrino oscillation
(ν-ν forward scattering)

Collective Neutrino Oscillation

In the dense neutrino media, the neutrino self-interaction has

$$H_{\nu\nu} = \sqrt{2}G_{\mathrm{F}} \int \mathrm{d}^3q(1 - \boldsymbol{v}_p \cdot \boldsymbol{v}_q)(\rho_\nu - \bar{\rho}_\nu)$$

c.f., Pantalone 1992
Duan+ '06

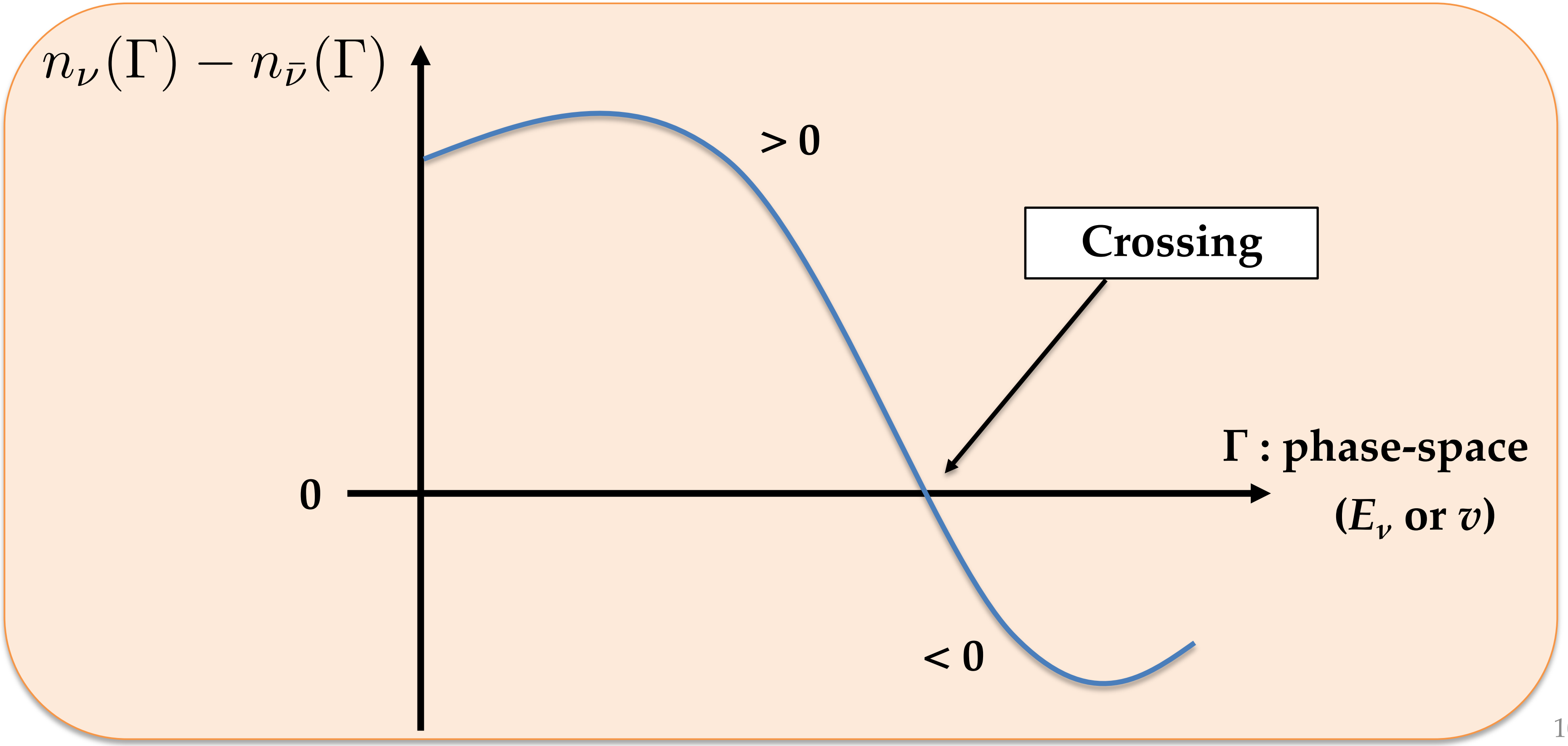


Spectral Crossings

Collective neutrino oscillation is triggered by the spectral difference $(\rho - \bar{\rho})$.

The existence of **``crossing"** in the difference between phase-space distributions of ν & $\bar{\nu}$.

(Morinaga '21 and Dasgupta '21)



Oscillation Modes

Slow mode
(Slow Flavor Conversion)

$$\mathcal{H} = H_{\mathrm{vac}} + H_{\nu\nu}$$

$$\mu_s = \sqrt{\frac{\Delta m^2}{2E_\nu}\sqrt{2}G_{\mathrm{F}}n_\nu} \sim \mathcal{O}(1)\ \mathrm{m}^{-1}$$

- Crossing in the energy dist.
- Driven by ν-ν & **vacuum** terms.
- Relatively long (slow) scale
  - Likely to be suppressed by the other physical scales.
    - ex.) matter oscillation

e.g.,
- Duan+ '06
- Chakraborty+ '16

Fast mode
(Fast Flavor Conversion, FFC)

$$\mathcal{H} = H_{\nu\nu}$$

$$\mu_f = \sqrt{2}G_{\mathrm{F}}n_\nu \sim \mathcal{O}(1)\ \mathrm{cm}^{-1}$$

- Crossing in the angular dist.
- Driven **only** by the self-interaction
  - independent of mass term
- Short (fast) scale.
  - Can evolve promptly.

e.g.,
- Sawyer '05 & '16
- Izaguirre+ '17

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Oscillation Modes

Slow mode (Slow Flavor Conversion)

$$\mathcal{H} = H_{\mathrm{vac}} + H_{\nu\nu}$$

$$\mu_s = \sqrt{\frac{\Delta m^2}{2E_\nu}\sqrt{2}G_{\mathrm{F}} n_\nu} \sim \mathcal{O}(1)\ \mathrm{m}^{-1}$$

- **Crossing in the energy dist.**
- Driven by ν-ν & **vacuum** terms.
- **Traditionally**, slow flavor conversion has been investigated.

e.g.,
- Duan+ '06
- Chakraborty+ '16

Fast mode (Fast Flavor Conversion, FFC)

$$\mathcal{H} = H_{\nu\nu}$$

$$\mu_f = \sqrt{2}G_{\mathrm{F}} n_\nu \sim \mathcal{O}(1)\ \mathrm{cm}^{-1}$$

- Crossing in the angular dist.
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- Short (fast) scale.
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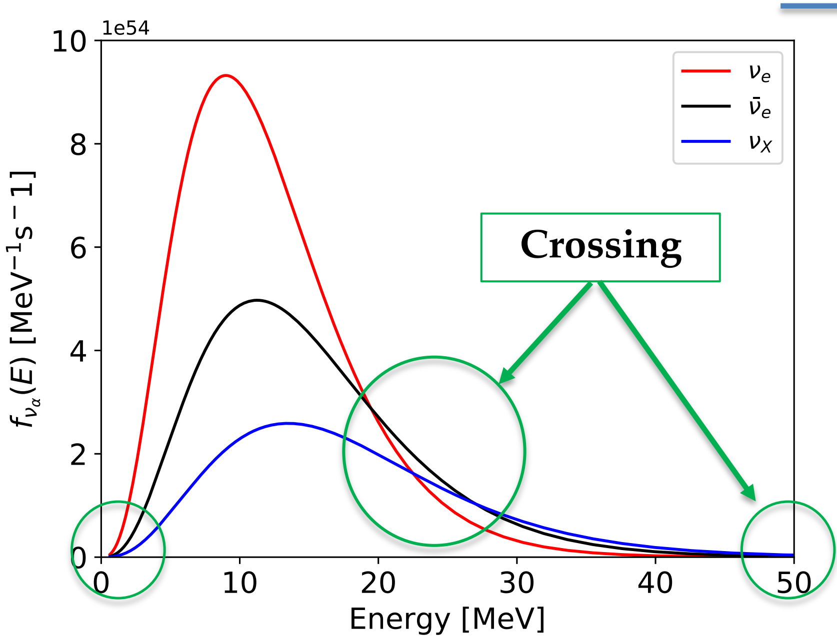
e.g.,
- Sawyer '05 & '16
- Izaguirre+ '17

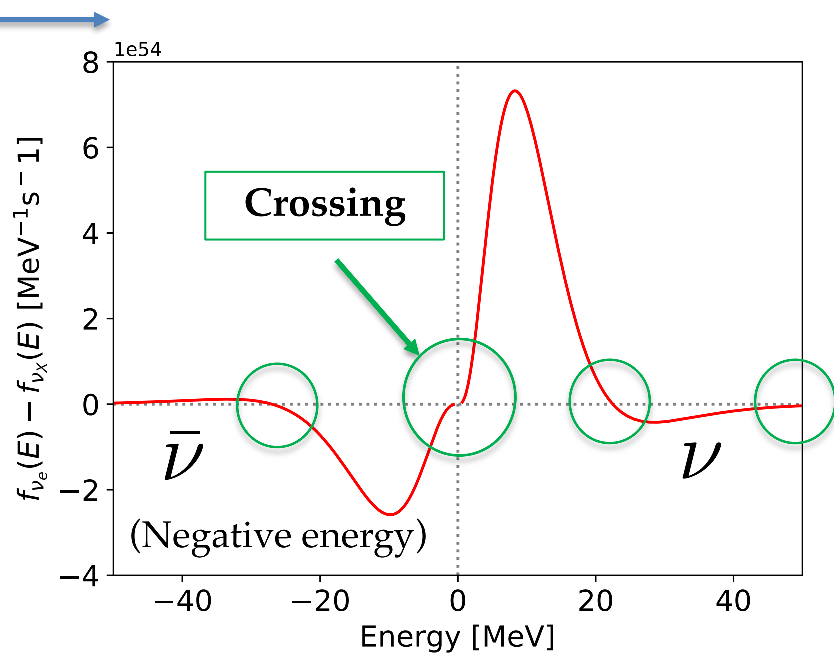
Crossings in Energy

- **Energy distribution.**

$$g(E) = \begin{cases} f_{\nu_e}(E) - f_{\nu_X}(E) & (\text{for } E > 0) \\ -f_{\bar{\nu}_e}(E) + f_{\bar{\nu}_X}(E) & (\text{for } E < 0) \end{cases} \qquad (\rho - \bar{\rho})$$

There always exists at least one crossing at E=0 or ∞.
= **Globally satisfied.**





Bulb Model

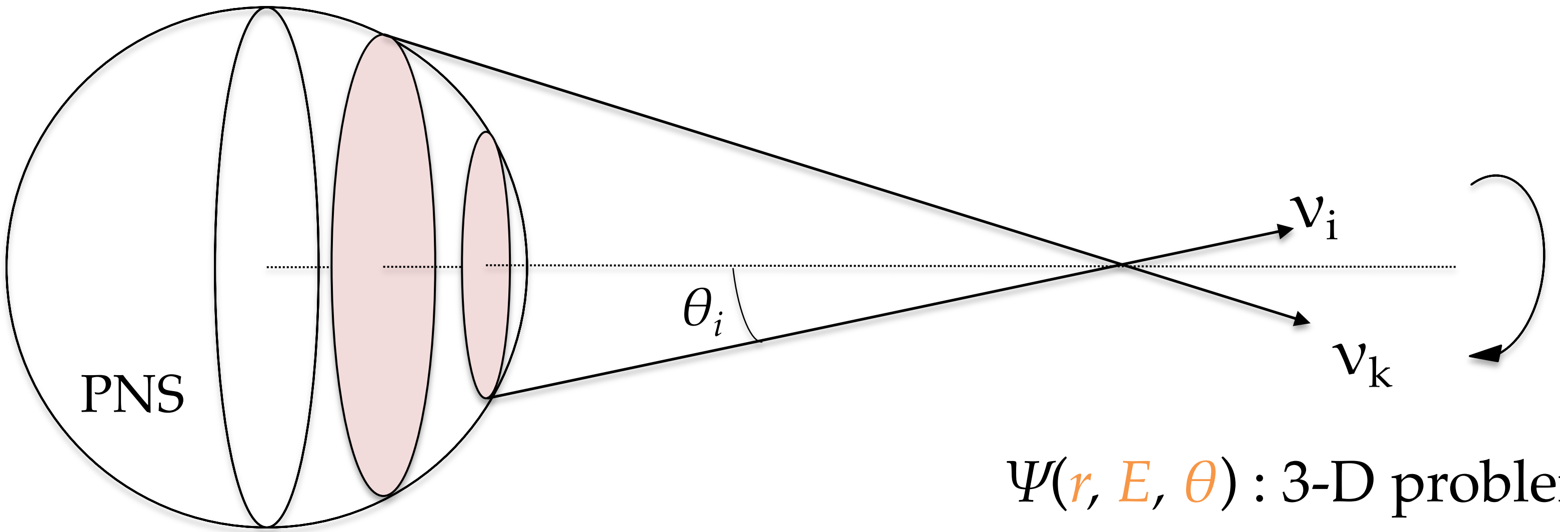
$\rho_\nu(\{t\}, \{r, \Theta, \Phi\}, \{E, \theta, \phi\})$: (1+3+3)-D, 7-dimensional systems

Stationary → $\rho_\nu(\{r, \Theta, \Phi\}, \{E, \theta, \phi\})$: (3+3)-D

Spherical symmetry → $\rho_\nu(\{r\}, \{E, \theta, \phi\})$: (1+3)-D

Axial symmetry → **$\rho_\nu(\{r\}, \{E, \theta\})$: (1+2)-D**

(Bulb model)

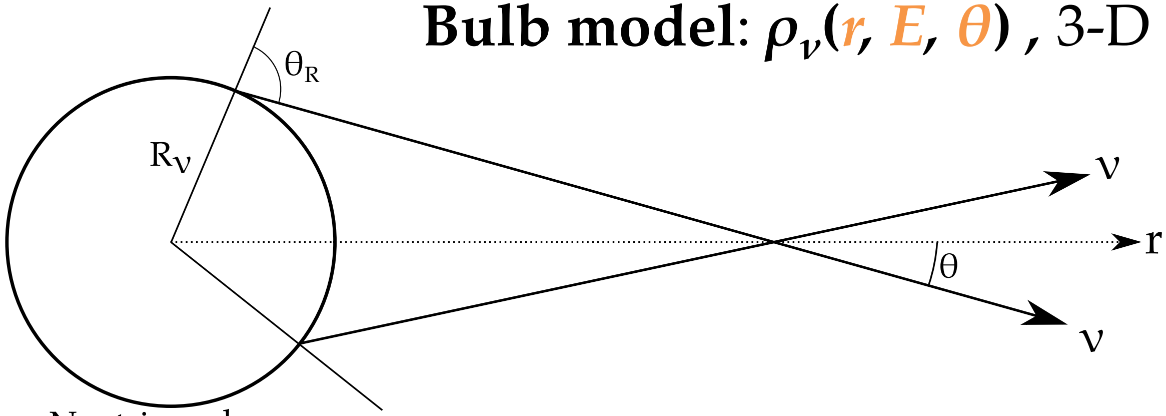


$\Psi(r, E, \theta)$: 3-D problem

Duan+ '06

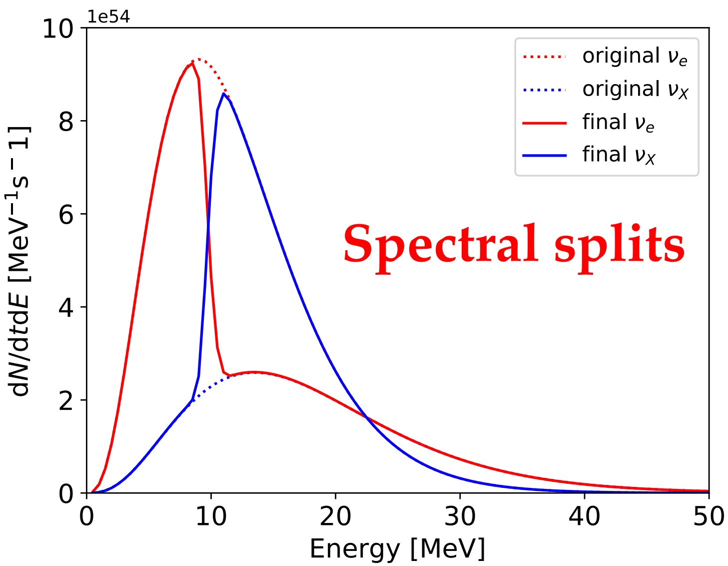
Bulb Model

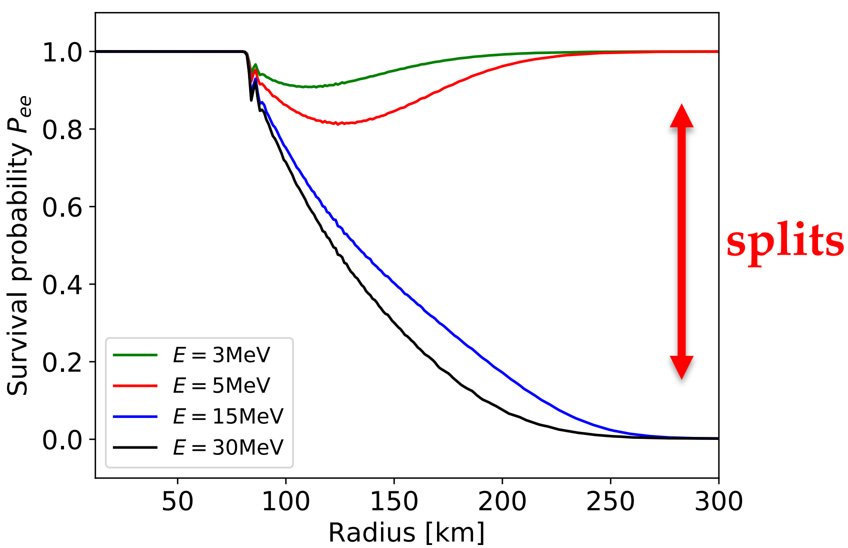
Bulb model: $\rho_\nu(r, E, \theta)$, 3-D problem



Duan+ '06, Fogli+ '07

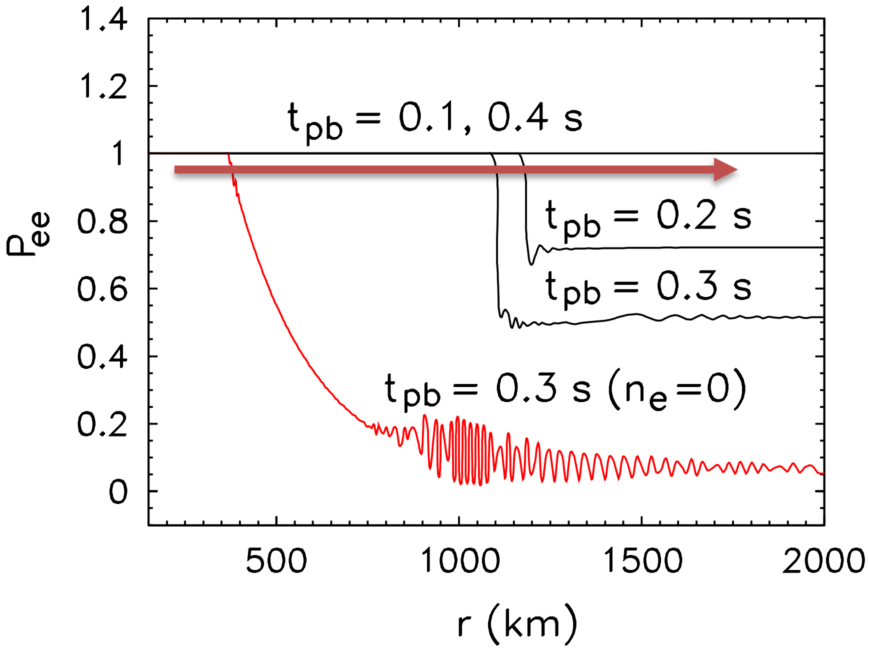
Nontrivial flavor evolution



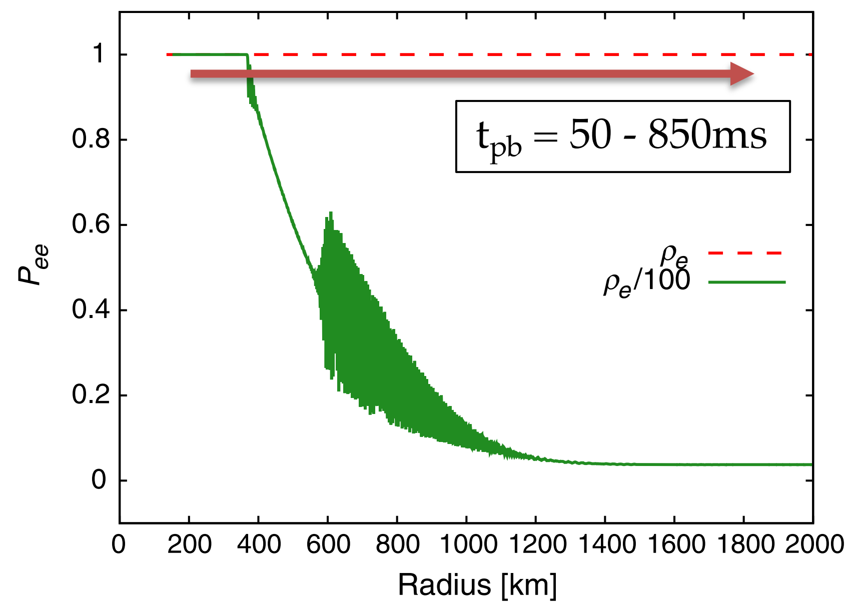


Matter Suppression

CCSNe with 10.8 $M_\odot$



Failed SNe with 40 $M_\odot$



(Chakraborty+ '11, Zaizen+ '18)

Dense background matter can suppress the collective effects.

(Esteban-Pretel+'08, Chakraborty+'11, Dasgupta+'12, Wu+'14, MZ+'18 & '20, Sasaki+'20)

$$\lambda \lesssim \sqrt{\omega\mu}$$

Collective slow flavor oscillation is likely to be suppressed in massive progenitors.

Symmetry Breaking

Spatio-Temporal instability

$\rho_\nu(t, \{r, \Theta\}, \{E_\nu, \theta_\nu, \phi_\nu\})$: (1+2+3)-D

Dasgupta+'15, Capozzi+'16, **Zaizen+ *in prep.***

Halo effects (Inclusion of scattering)

$\rho_\nu(\{r\}, \{E_\nu, \theta_\nu\}) + \mathcal{C}$: (1+2)-D

Cherry+'12 & '13, Cirigliano+'18, **Zaizen+'20**

Axial symmetry breaking

$\rho_\nu(\{r\}, \{E_\nu, \theta_\nu, \phi_\nu\})$: (1+3)-D

Raffelt+'13, Chakraborty+'14, **Zaizen+'21**

Slow mode

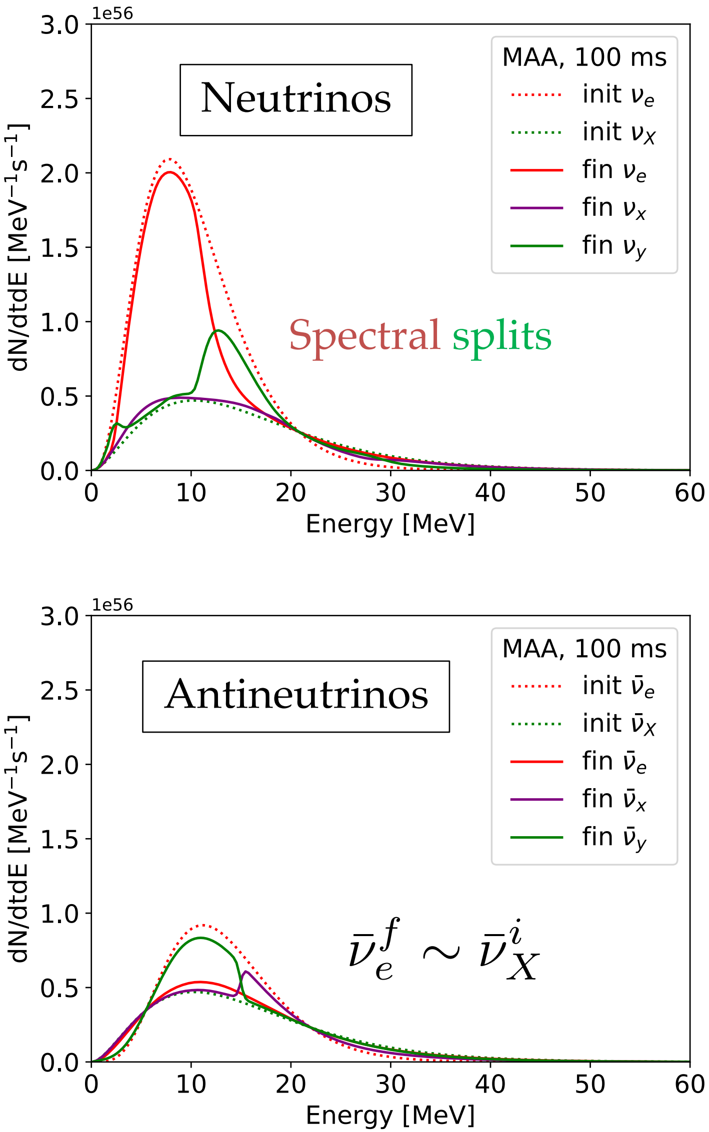
$\rho_\nu(\{r\}, \{E_\nu, \theta_\nu\})$: (1+2)-D

(Bulb model)

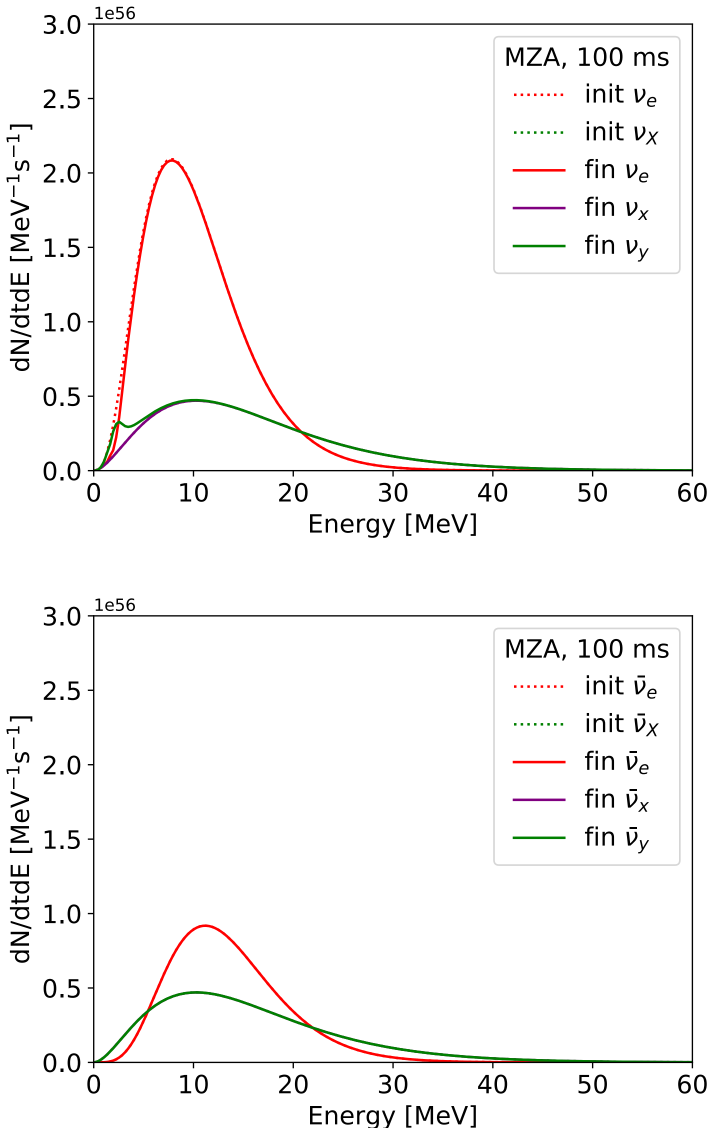
Symmetry breakings can enhance the neutrino self-interaction or weaken the matter suppression.

Axial-symmetry Breaking

Symmetry breaking (1+3D)

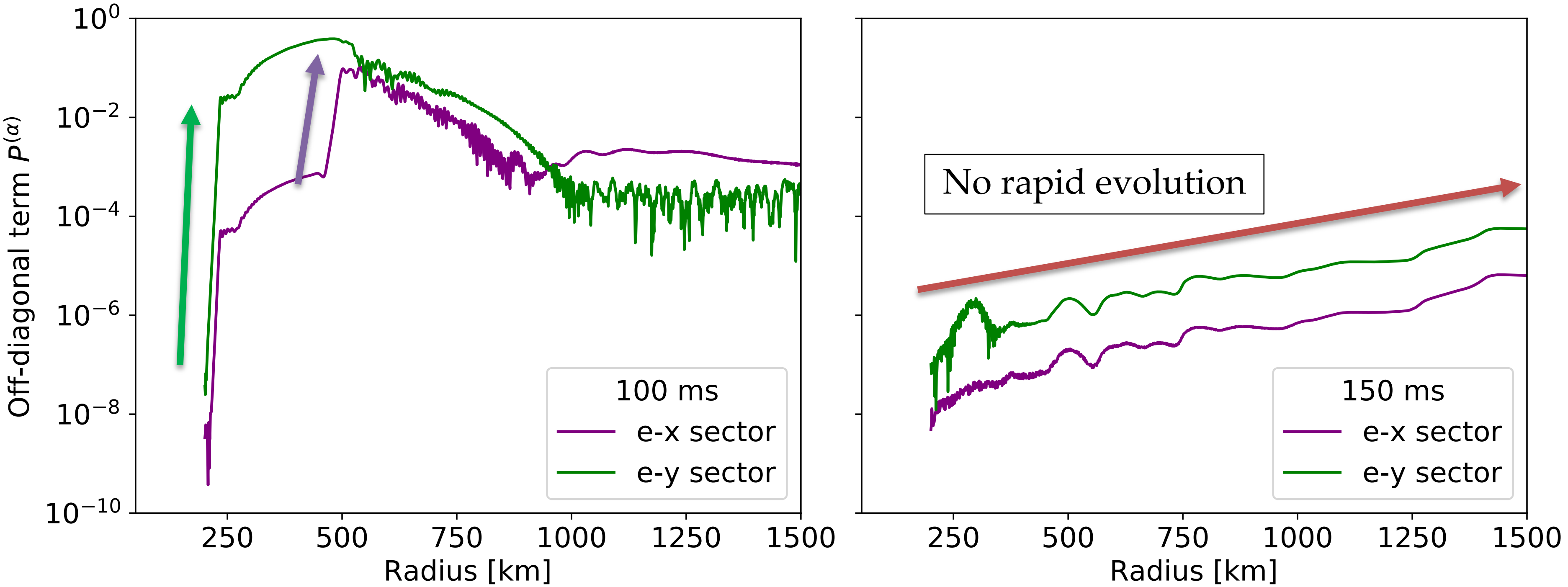


Axisymmetry (1+2D)



Zaizen+ '21

Symmetry Breaking vs. Suppression



$$P^{(\alpha)} = \int d\Gamma' |\rho_{e\alpha}| \;\; \cos\varphi'$$

Zaizen+ '21

Left panel is the same time snapshot as the previous slide.
Flavor instability exponentially evolves and break the matter suppression.

But, at the other snapshots, more dense matter suppresses the rapid evolution.

Symmetry Breaking

Spatio-Temporal instability

$\rho_\nu(t, \{r, \Theta\}, \{E_\nu, \theta_\nu, \phi_\nu\})$: (1+2+3)-D

Dasgupta+'15, Capozzi+'16, **Zaizen+ *in prep.***

Halo effects (Inclusion of scattering)

$\rho_\nu(\{r\}, \{E_\nu, \theta_\nu\}) + \mathcal{C}$: (1+2)-D

Cherry+'12 & '13, Cirigliano+'18, **Zaizen+'20**

Axial symmetry breaking

$\rho_\nu(\{r\}, \{E_\nu, \theta_\nu, \phi_\nu\})$: (1+3)-D

Raffelt+'13, Chakraborty+'14, **Zaizen+'21**

Slow mode

$\boldsymbol{\rho_\nu(\{r\}, \{E_\nu, \theta_\nu\})}$ **: (1+2)-D**

(Bulb model)

Symmetry breakings can enhance the neutrino self-interaction or weaken the matter suppression.

But,
Matter suppression is still dominant against slow flavor conversion.

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$$\mu_s = \sqrt{\frac{\Delta m^2}{2E_\nu}\sqrt{2}G_{\rm F}n_\nu} \sim \mathcal{O}(1)\ {\rm m}^{-1}$$

- Crossing in the energy dist.
- Driven by ν-ν & vacuum terms.
- Relatively long (slow) scale
  - Likely to be suppressed by the other physical scales.
    - ex.) matter oscillation

e.g.,
- Duan+ '06
- Chakraborty+ '16

Fast mode
(Fast Flavor Conversion, FFC)

$$\mathcal{H} = H_{\nu\nu}$$

$$\mu_f = \sqrt{2}G_{\rm F}n_\nu \sim \mathcal{O}(1)\ {\rm cm}^{-1}$$

- **Crossing in the angular dist.**
- Driven **only** by the self-interaction
  - independent of mass term.
- **Newly** found.

e.g.,
- Sawyer '05 & '16
- Izaguirre+ '17

Crossings in Angle

- **Neutrino-flavor lepton number (NFLN) angular distribution.**

$$G_{\boldsymbol{v}}^{ex} = \sqrt{2}G_{\mathrm{F}} \int \frac{E^2 \mathrm{d}E}{2\pi^2} \left[(f_{\nu_e} - f_{\bar{\nu}_e}) - (f_{\nu_x} - f_{\bar{\nu}_x}) \right]$$

$(\rho - \bar{\rho})$

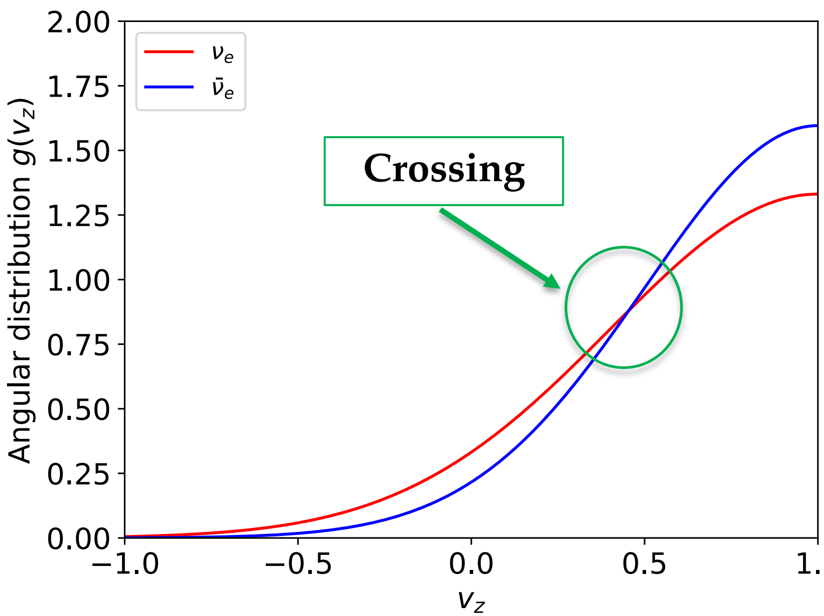
= ELN - XLN

Angular crossings are not always generated.
= **Locally satisfied.**

If $\nu_X = \bar{\nu}_X$, NFLN is reduced to **ELN**.

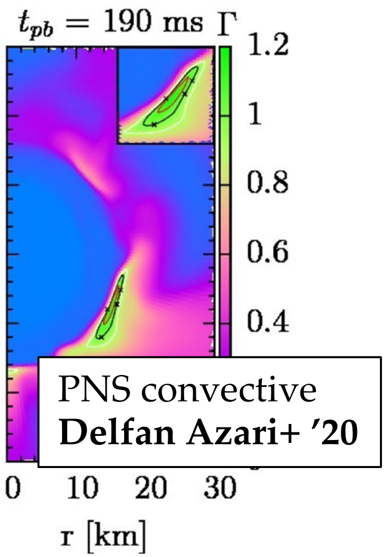
Electron lepton number (ELN)

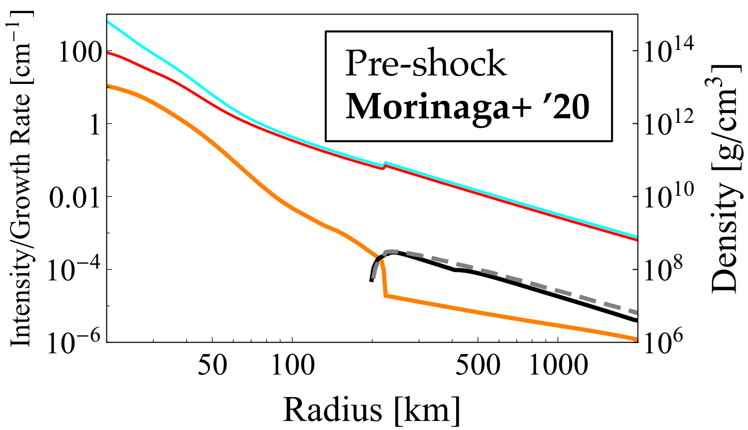
$$G_{\boldsymbol{v}} \propto g_{\nu_e}(\boldsymbol{v}) - g_{\bar{\nu}_e}(\boldsymbol{v})$$



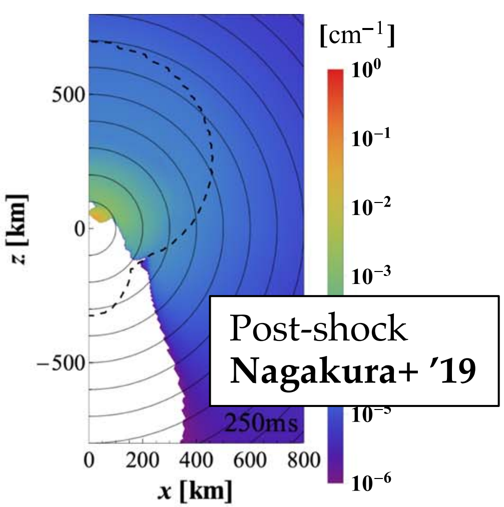
Crossing Search

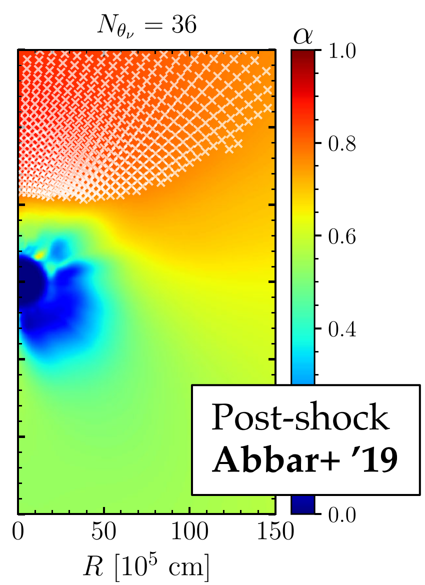
Search for ELN crossings in neutrino data of CCSN simulations.

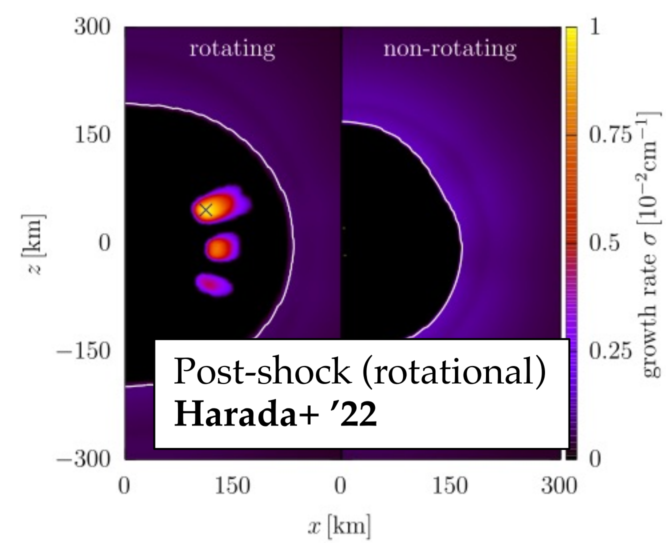




There exist angular crossings in any regions of CCSNe.



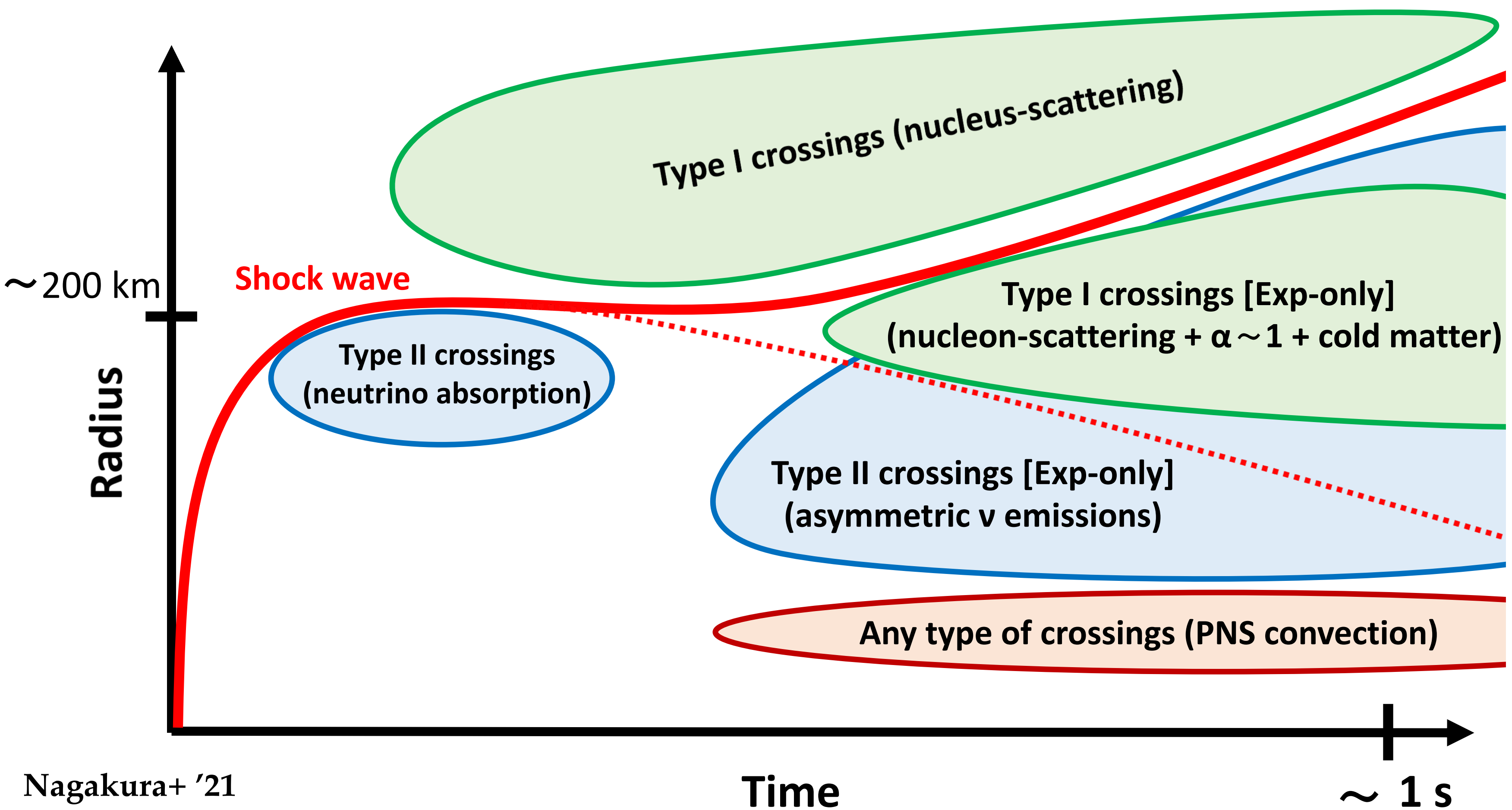




Possibility of ELN Crossings

Where / when / Why do ELN crossings appear in CCSNe?

Space-time diagram of ELN-angular crossings in CCSNe

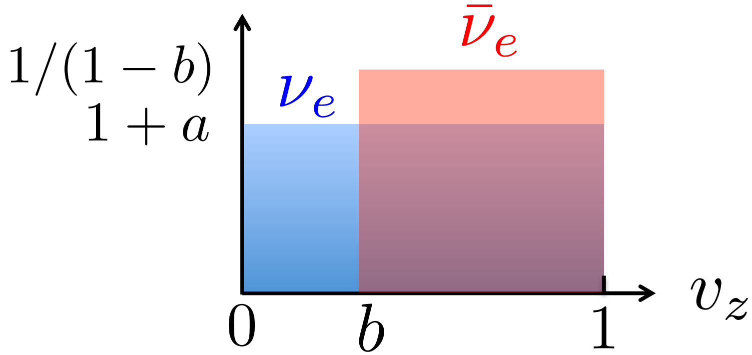


Nagakura+ '21

Nonlinear with homogeneity

First numerical simulation for FFC.

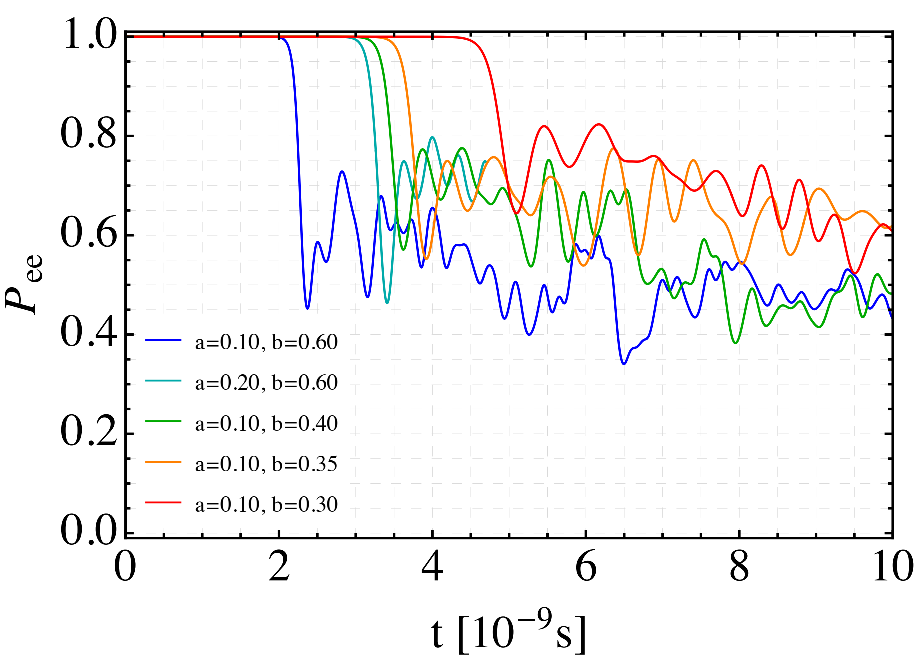
Dasgupta+ '17



Angular distribution (Toy model)

Single-energy distribution.

$$g_{\nu_e}(\omega, v_z) = \delta(\omega - \omega_0) B(v_z)$$



FFC is induced by **local** angular dist.

Neutrino dist.: $\rho(t, v_z)$

Homogeneous space

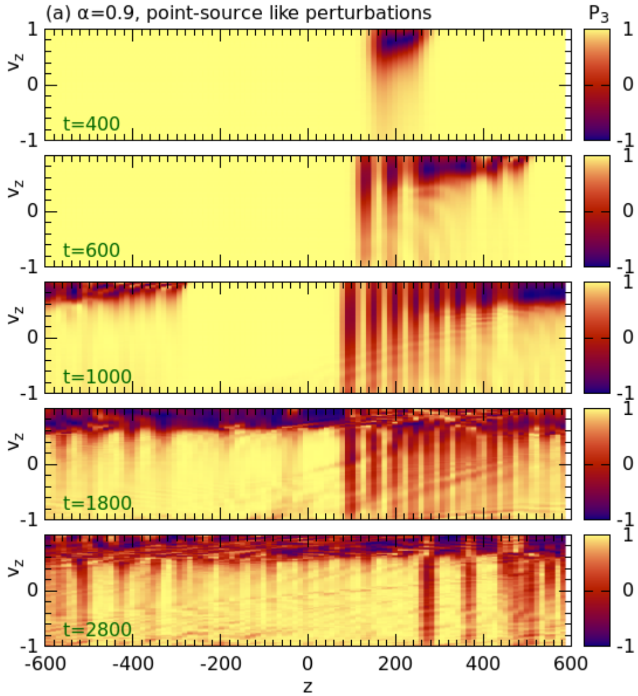
→ Only time evolution

Oscillation timescale = $\mathcal{O}(1)$ ns.

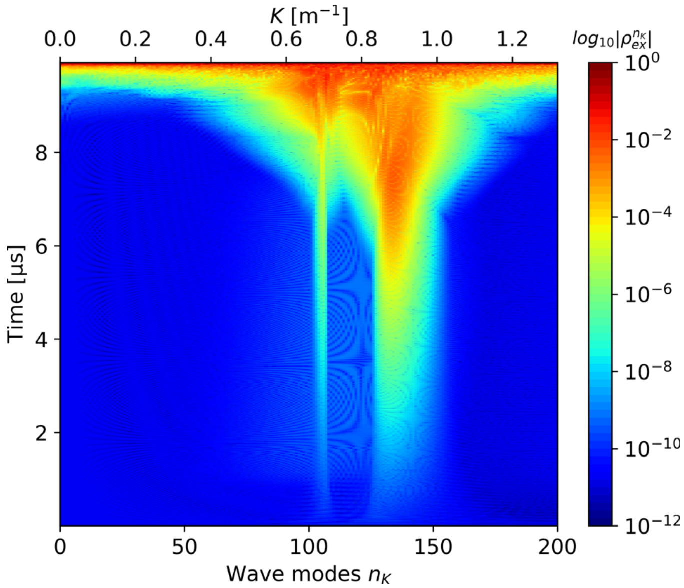
Much shorter!

Nonlinear with inhomogeneity

Wu+ '21



Zaizen+ '21



Consider spatial structure recently.

Neutrino dist.: $\rho(t, z, v_z)$

But it remains

- 1D-space
- Periodical box.

Flavor equilibrium. $P_{ee} = 0.5$
Interference with flavor waves.

Cascade in spatial modes (+inverse)

Quantum Kinetic Equation

QKE for flavor evolution :

$$\mathrm{i}(\partial_t + v_z\partial_z)\rho = [\mathcal{H}, \rho]$$

$$\rho = \frac{\mathrm{Tr}(\rho)}{2} + \frac{1}{2}\boldsymbol{\sigma}\cdot\boldsymbol{P}$$

$$(\partial_t + v_z\partial_z)\boldsymbol{P} = \boldsymbol{\mathcal{H}} \times \boldsymbol{P}$$

$$\boldsymbol{\mathcal{H}}(t,z,u) = \mu\int_{-1}^{+1} \mathrm{d}v'_z(1 - v_z v'_z)\left[g_{v'_z}\boldsymbol{P}(t,z,v'_z) - \bar{g}_{v'_z}\bar{\boldsymbol{P}}(t,z,v'_z)\right]$$

μ is a ubiquitous dimensional quantity.

$$\mu = \sqrt{2}G_\mathrm{F}n_\nu = 0.8\ \mathrm{cm}^{-1}\left(\frac{L_\nu}{10^{52}\ \mathrm{erg/s}}\right)\left(\frac{10\ \mathrm{MeV}}{\langle E_\nu\rangle}\right)\left(\frac{50\ \mathrm{km}}{R_\nu}\right)^2$$

(Example)

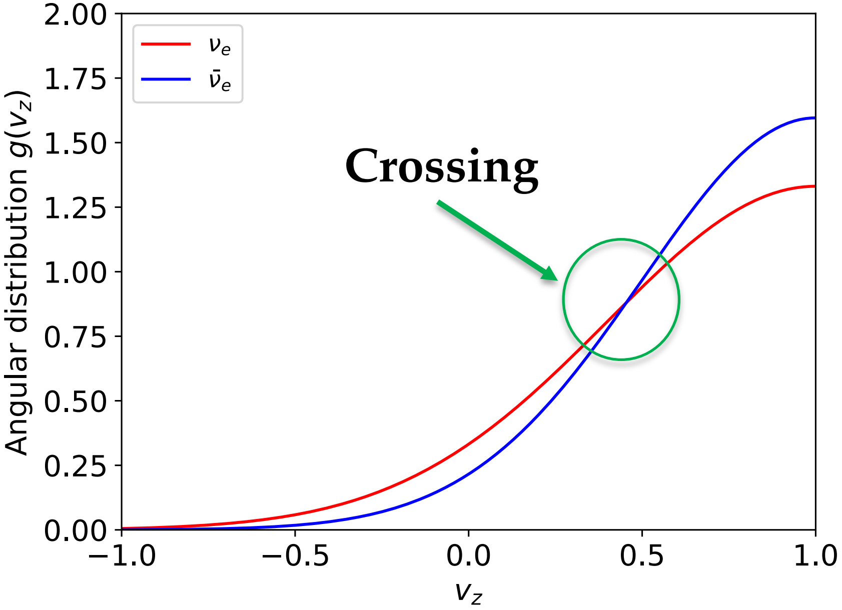
We can recast it dimensionless by setting μ=1.

Model

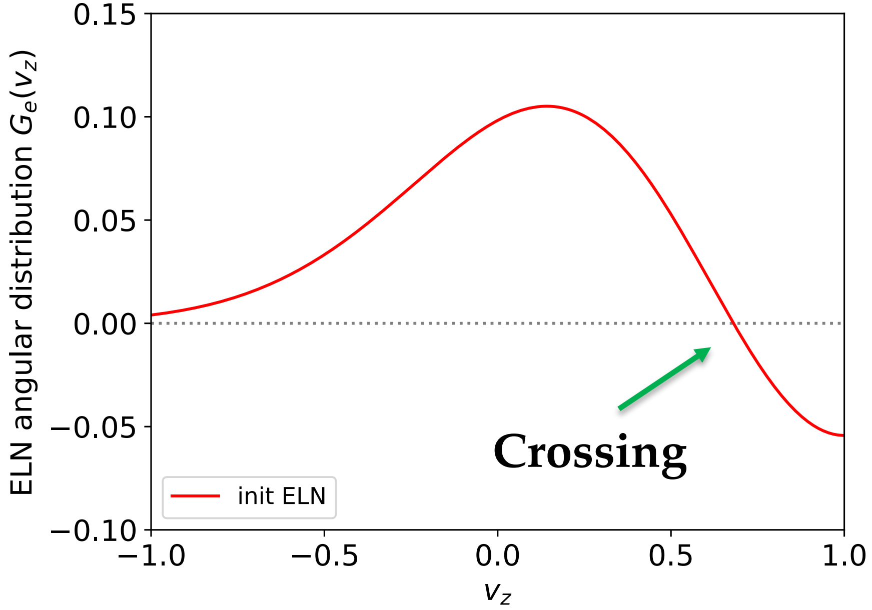
$$G^e_{\boldsymbol{v}} = \sqrt{2} G_{\mathrm{F}} \int \frac{E^2 \mathrm{d}E}{2\pi^2} \left[f_{\nu_e}(\boldsymbol{v}) - f_{\bar{\nu}_e}(\boldsymbol{v}) \right]$$

Pure electron state. XLN is zero.

$$g_{\nu_e} \propto \exp\left[-(v-1)^2 / 2\sigma_{\nu_e}^2 \right]$$

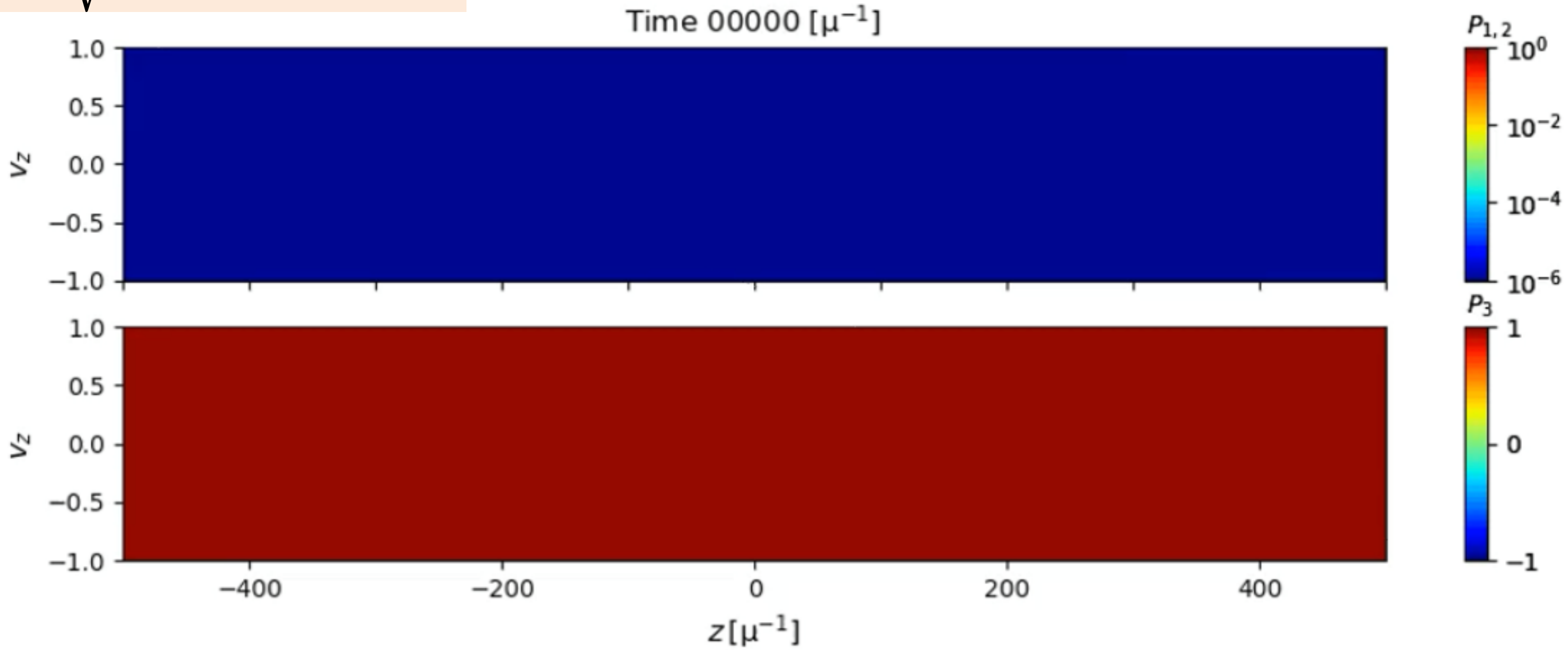


ELN angular distribution



Nonlinear simulation

$$P_\perp = \sqrt{P_1^2 + P_2^2} = \rho_\nu^{\alpha\beta}$$



$$P_{ee} = \frac{1}{2}(1 + P_3)$$

Simulation box

$$L_z[\mu^{-1}] = 1000\mu^{-1} = 1250 \text{ cm}$$

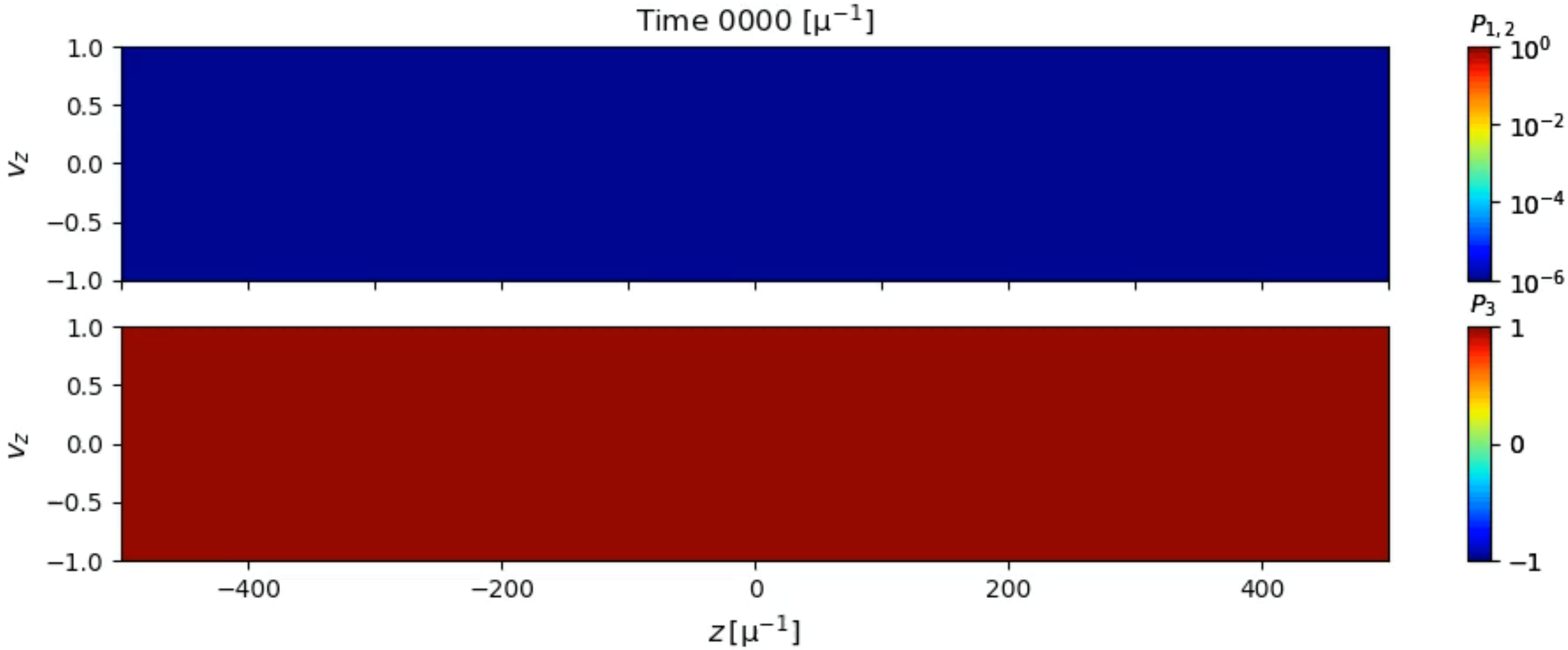
$$t_{\text{max}}[\mu^{-1}] = 5000\mu^{-1} \sim 200 \text{ ns}$$

$$\mu = \sqrt{2}G_{\text{F}}n_\nu = 0.8 \text{ cm}^{-1}$$

Randomly spatial perturbation

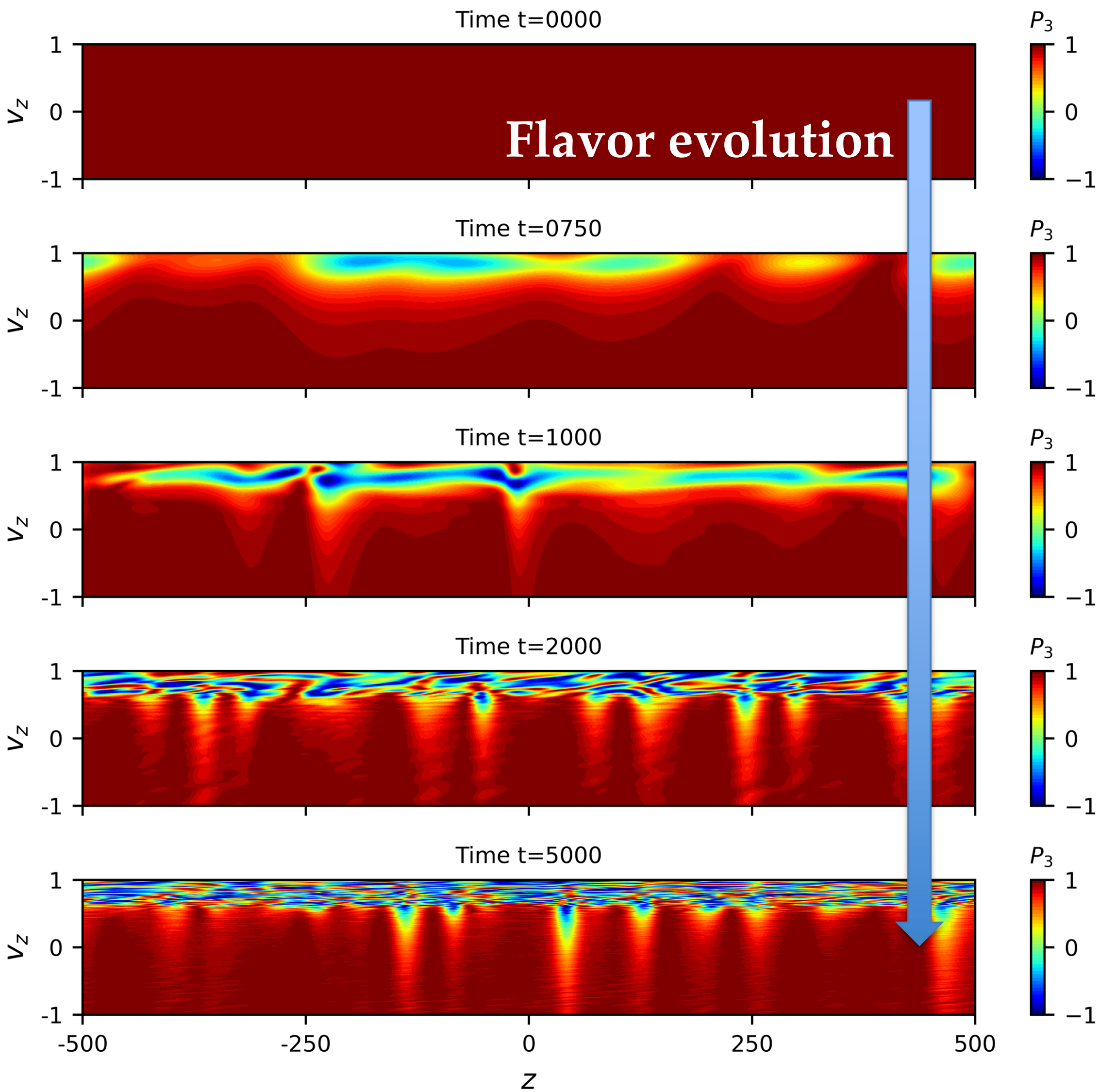
Nonlinear simulation

$$P_\perp = \sqrt{P_1^2 + P_2^2} = \rho_\nu^{\alpha\beta}$$

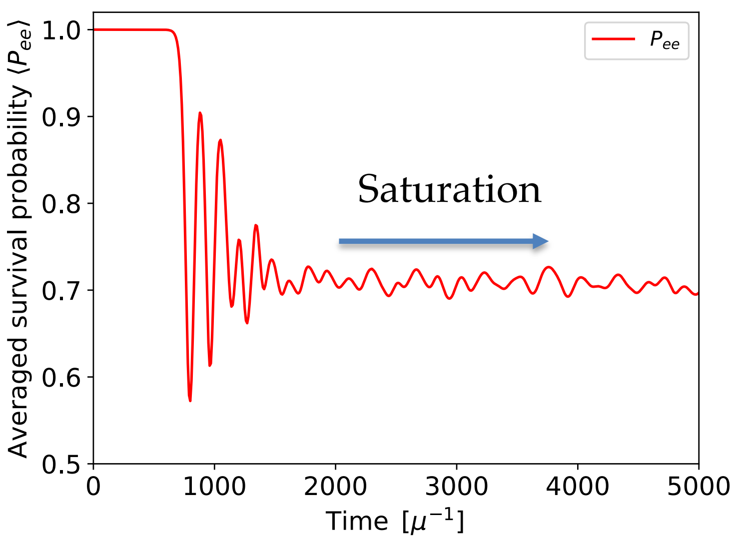


$$P_{ee} = \frac{1}{2}(1 + P_3)$$

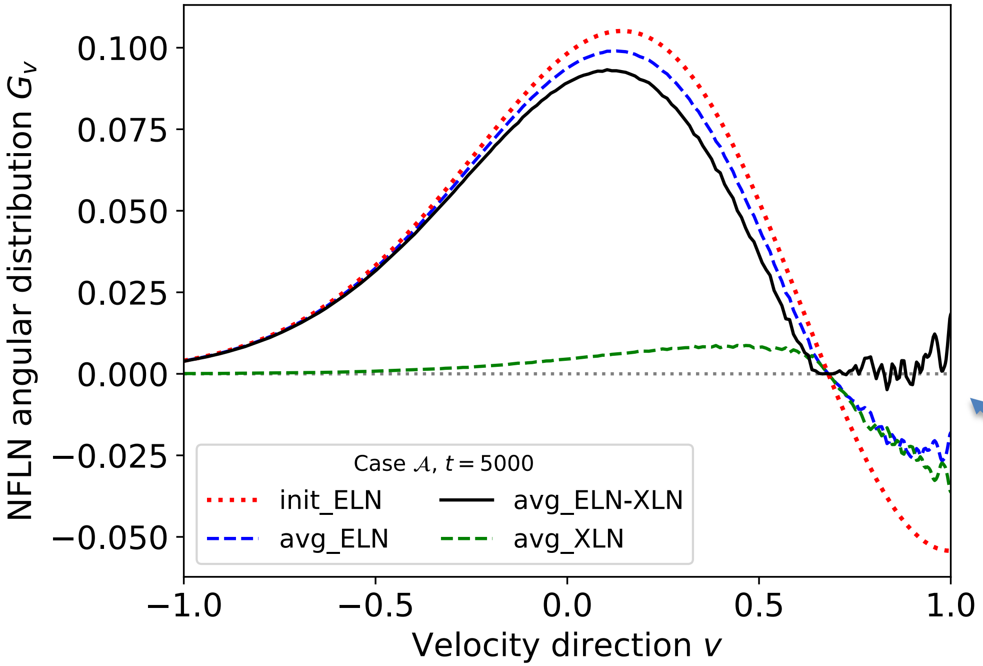
Spatial Structure



- Generate smaller-scale structure in time.
- FFC occurs only within crossing.
- FFC reaches nonlinear saturation at t ~ 2000.



Spatial Average



ELN still has a crossing.

No Crossing in **ELN-XLN** !!

FFC reaches nonlinear saturation due to **the disappearance of angular crossing**.
Further flavor conversion does not occur.

FFC works to eliminate the angular crossings.
And the saturation timescale is in O(ns).
We can employ FFC as sub-grid models in CCSN simulation!!

Summary & Future Prospect

- Flavor mixings in core-collapse supernovae
  - May affect the dynamics and the observables.
- Slow & Fast flavor conversions
  - Triggered by spectral crossings in phase-space distributions.
  - Slow: dramatic spectral splits, but likely to be suppressed.
  - Fast : locally but evolve promptly due to the faster timescale.
- Nonlinear saturation in fast flavor conversion
  - Eliminate the angular crossings.
- Sub-grid model of nonlinear flavor conversion in CCSN simulation?